LQG Control via Wireless Sensor Networks with Minimal Transmission Power

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Abstract: This study concerns control of dynamical systems via wireless sensor nodes with minimal transmission power. In particular, we jointly design a controller and a transmission power control mechanism that adjusts the transmit power of a wireless sensor node. We develop a framework for achieving the minimum expected transmit power required for a specific level of the LQG control performance. We model the communication channel between the wireless sensor node and the controller by an AWGN channel compatible with the IEEE 802.15.4 standard, which was formed for the specification of low-data-rate and low-power wireless communication. We use dynamic programming to characterize the optimal policies, and show that there exists a separation between the designs of the optimal estimator, optimal controller, and optimal transmission power control mechanism.

Keywords: Transmission Power Control, Wireless Sensor Networks, AWGN communication channel, LQG Control, Optimal Policies, Separation Principle.

1. INTRODUCTION

This study concerns control of dynamical systems via wireless sensor nodes with minimal transmission power. Wireless sensor nodes are of significant interest because of their promising and innovative applications in the context of Internet of Things (IoT). In general, a wireless sensor node is very small and requires very low power, but it is often equipped with a limited power source that is not replaceable due to application-related constraints. The fact is that the power of a wireless sensor node is mainly consumed for communication. Hence, energy-efficient wireless communication for control purposes is vital. One effective technique to conserve energy in a wireless sensor node is transmission power control. In this study, we would like to design a mechanism for minimizing the transmit power of a wireless sensor node subject to a constraint on the quality of control. In fact, the transmit power influences signal-to-noise ratio and subsequently the packet dropout rate. Packet dropouts, as shown by Sinopoli et al. (2004) and Schenato et al. (2007), can cause instability in the estimation and the control problems. Therefore, we need to jointly design a controller and a transmission power control mechanism. Intuitively speaking, a transmission power control mechanism should adjust the transmit power such that a high level of transmit power is used only for transmission of measurements that contain important information for the controller.

In this paper, we develop a framework for achieving the minimum expected transmit power required for a specific level of the LQG control performance. We model the communication channel between the wireless sensor node and the controller by an AWGN channel compatible with the IEEE 802.15.4 standard, which was formed for the specification of low-data-rate and low-power wireless communication. We use dynamic programming to characterize the optimal policies, and show that there exists a separation between the designs of the optimal estimator, optimal controller, and optimal transmission power control mechanism. Transmission power control for estimation problems over a fading channel have been investigated by Leong et al. (2011), Wu et al. (2015), and Ren et al. (2018). Closely related to our study, the joint design of a state-feedback controller and a transmission power control mechanism for fading channels is addressed by Gatsis et al. (2014). On the contrary, here we jointly design an output-feedback controller and a transmission power control mechanism.

The outline of this paper is as follows. We formulate the problem in Section 2. In Section 3, we obtain the optimal policies and show the separation principle. We illustrate numerical and simulation results in Section 4. Finally, we make concluding remarks in Section 5.

1.1 Notations

In this paper, we represent an n dimensional vector with $x = [x_1, \dots, x_n]^T$ where x_i is its ith component. We write x^T to denote the transpose of the vector x. The normal distribution with mean μ and covariance Σ is denoted by $N(\mu, \Sigma)$. The expected value and the covariance of the random variable x are denoted by $\mathsf{E}[x]$ and $\mathsf{cov}[x]$, respectively. For matrices A and B, we write $A \succ 0$ and $B \succeq 0$ to mean that A and B are positive definite and positive semi-definite respectively.

2. PROBLEM FORMULATION

2.1 Dynamical System and Quantization

Consider a discrete-time dynamical system generated by the following linear state equation:

$$x_{k+1} = Fx_k + Bu_k + w_k, \tag{1}$$

for time $k \in \mathbb{N}_+$ and with initial condition x_0 where $x_k \in \mathbb{R}^n$ is the state of the system, F is the state matrix, B is the input matrix, $u_k \in \mathbb{R}^m$ is the control input, $w_k \in \mathbb{R}^n$ is a Gaussian white noise with zero mean and covariance R_1 where $R_1 \succ 0$. At each time step, the output of the system is measured by a wireless sensor node. The measurement of the sensor is given by

$$y_k = Hx_k + v_k, \tag{2}$$

where $y_k \in \mathbb{R}^p$ is the output of the system, H is the output matrix, and $v_k \in \mathbb{R}^p$ is a Gaussian white noise with zero mean and covariance R_2 where $R_2 \succ 0$. It is assumed that the initial state x_0 is a Gaussian vector with mean m and covariance R_0, and that x_0, w_k, and v_k are mutually independent. In addition, it is assumed that (F, B) is controllable and (F, H) is observable.

The measurements are quantized by a high-resolution quantizer into codewords of fixed length ℓ. We model the quantizer output by

$$z_k = y_k + n_k, \tag{3}$$

where n_k is assumed to be a Gaussian white noise with zero mean and covariance Λ where $\Lambda \succ 0$. We assume that each measurement codeword is carried by a single network packet.

2.2 Wireless Communication Channel

The wireless sensor node is connected to a controller via a wireless communication channel. The codeword corresponding to each measurement is modulated, transmitted as an analog signal in the channel, and then demodulated. We assume that the channel is an additive white Gaussian noise (AWGN) channel. Compatible with the IEEE 802.15.4 standard, we employ the offset quadrature phase-shift keying (O-QPSK) modulation with coherent detection and cyclic redundancy check (CRC).

The power of the transmitted signal is attenuated as the signal propagates through the channel. The power of the received signal is obtained by

$$p_k^{RX} = p_k^{TX} p_k^{D^{-1}}, \tag{4}$$

where p_k^{RX} is the received power, p_k^{TX} is the transmit power, and p_k^D is the power decay due to attenuation. We model attenuation in the channel with a path loss model (Goldsmith (2005)), i.e.,

$$p_k^D = \left(\frac{4\pi f d_0}{c}\right)^2 \left(\frac{d}{d_0}\right)^\eta, \tag{5}$$

where d_0 is the reference distance, d is the distance between the wireless sensor node and the controller, f is the carrier frequency, c is the speed of light, and η is the path loss exponent.

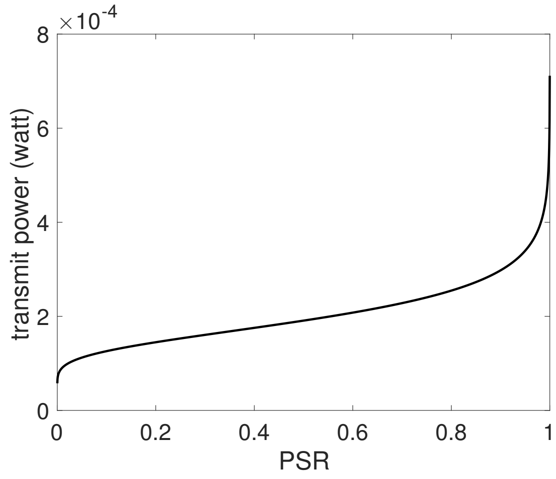

Fig. 1. The required transmit power versus packet success rate for the specific wireless channel described in Section 4.

In an AWGN channel, the received signal-to-noise ratio SNR_k of the channel is defined as the ratio of the received signal power to the noise power within the bandwidth of the transmitted signal (Goldsmith (2005)), i.e.,

$$\mathsf{SNR}_k = \frac{p_k^{RX}}{N_0 B_n} = \frac{E_k R_c}{N_0 B_n}, \tag{6}$$

where E_k is the received signal energy per bit, R_c is the communication rate, N_0 is the noise power spectral density, and B_n is the noise bandwidth. Using equations (4), (5), and (6), we obtain

$$\frac{E_k}{N_0} = \left(\frac{4\pi f d_0}{c}\right)^{-2} \left(\frac{d}{d_0}\right)^{-\eta} (N_0 R_c)^{-1} p_k^{TX}, \tag{7}$$

In addition, for O-QPSK modulation with coherent detection (Proakis (1995)) the performance of the communication channel is specified by

$$\mathsf{BER}_k = Q\left(\sqrt{\frac{2E_k}{N_0}}\right), \tag{8}$$

where BER_k is the bit error rate and $Q(.)$ denotes the Q-function. Following CRC error detection, all single bit errors in a packet can be detected. Hence, the packet success rate PSR_k for transmission of packets with length of ℓ bits is given by

$$\mathsf{PSR}_k = (1 - \mathsf{BER}_k)^\ell. \tag{9}$$

Inserting (8) in (9), we obtain

$$\mathsf{PSR}_k = \left(1 - Q\left(\sqrt{\frac{2E_k}{N_0}}\right)\right)^\ell. \tag{10}$$

Inserting (7) in (10), we can obtain the required transmit power at time k as a function of the packet success rate as

$$p_k^{TX} = \begin{cases} c_0 \left(Q^{-1}\left(1 - \mathsf{PSR}_k^{\frac{1}{\ell}}\right)\right)^2, & \mathsf{PSR}_k \in [a, b], \\ 0, & \mathsf{PSR}_k = 0, \end{cases} \tag{11}$$

where $[a, b]$ represents the operational range of PSR_k with specific a and b, and

$$c_0 = \frac{1}{2} N_0 R_c \left(\frac{4\pi f d_0}{c}\right)^2 \left(\frac{d}{d_0}\right)^\eta . \quad (12)$$

With abuse of notation we consider $\mathsf{PSR}_k = 0$ for the case in which the transmitter is in sleep mode. Therefore, if $\mathsf{PSR}_k = 0$ then nothing is transmitted at time k. In the sequel, we denote the function in (11) by $p_k^{TX} = \psi(\mathsf{PSR}_k)$ defined in the domain $\Gamma = 0 \cup [a, b]$. The required transmit power as a function of the packet success rate is depicted in Figure 1 for the specific wireless channel described in Section 4.

The packet loss in the channel is modeled by an i.i.d. arrival process γ_k with probability distribution $\mathsf{P}(\gamma_k = 1) = \mathsf{PSR}_k$ such that

$$\gamma_k = \begin{cases} 1, & \text{if } z_k \text{ is received successfully,} \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

Hence, $\mathsf{E}[\gamma_k] = \mathsf{PSR}_k$. We assume that packets are received with one-step delay, and packets that are not received successfully are not retransmitted.

2.3 Transmission Power Optimization for LQG Control

Let π and μ be a packet success rate policy and a control policy respectively. At time k, we assume that the information available at the controller is specified by:

$$\mathcal{I}_k = \{\gamma_{0:k-1}, u_{0:k-1}, z_{0:k-1}\}. \quad (14)$$

We assume that the controller decides also about the transmit powers, and notifies the wireless sensor node of transmit powers through an ideal communication channel.

We measure the control performance penalizing the state deviation and control effort over the time horizon N by the quadratic function:

$$\Phi = \mathsf{E}\Big[\|x_{N+1}\|^2_{Q_0} + \sum_{k=0}^{N} \|x_k\|^2_{Q_1} + \|u_k\|^2_{Q_2}\Big], \quad (15)$$

where $Q_0, Q_1 \succeq 0$ and $Q_2 \succ 0$ are weighting matrices.

We would like to find the optimal π and μ that minimize the required transmit power for a guaranteed level of control performance β, i.e.,

$$\Psi = \inf_{\pi,\mu:\ \Phi \le \beta} \mathsf{E}\Big[\sum_{k=0}^{N} \psi(\mathsf{PSR}_k)\Big], \quad (16)$$

which is equivalent to

$$\Psi = \inf_{\pi,\mu} \mathsf{E}\Big[\lambda \|x_{N+1}\|^2_{Q_0} + \sum_{k=0}^{N} \psi(\mathsf{PSR}_k) + \lambda \|x_k\|^2_{Q_1} + \lambda \|u_k\|^2_{Q_2}\Big], \quad (17)$$

where $\lambda > 0$ is a Lagrange multiplier.

3. OPTIMAL POLICIES

We first need to obtain the optimal state estimate at the controller given the information set $\mathcal{I}_k$. The following lemma gives the optimal state estimate.

Lemma 1. The conditional expected value of the state with the following dynamics is the minimizer of the mean square error (MSE) for the system defined by (1), (2), and (3) with the one-step-delay channel with arrival process specified by (13):

$$\hat{x}_{k+1} = F\hat{x}_k + Bu_k + \gamma_k K_k (z_k - H\hat{x}_k), \quad (18a)$$

$$P_{k+1} = FP_kF^T + R_1 - \gamma_k K_k H P_k F^T, \quad (18b)$$

where $\hat{x}_k = \mathsf{E}[x_k|\mathcal{I}_k]$, $P_k = \mathsf{cov}[x_k|\mathcal{I}_k]$, and

$$K_k = FP_kH^T(HP_kH^T + \bar{R}_2)^{-1}, \quad (18c)$$

with initial conditions $\hat{x}_0 = m_0$, $P_0 = R_0$, and $\bar{R}_2 = R_2 + \Lambda$.

Proof. The state estimate and error covariance are propagated as

$$\hat{x}_{k+1} = F\hat{x}_{k^+} + Bu_k, \quad (19)$$

$$P_{k+1} = FP_{k^+}F^T + R_1, \quad (20)$$

where $\hat{x}_k$ and P_k are the a priori state estimate and error covariance, and $\hat{x}_{k^+}$ and P_{k^+} are the a posteriori state estimate and error covariance. Moreover, the state estimate and error covariance are updated as

$$\hat{x}_{k^+} = \hat{x}_k + \gamma_k P_k H^T (HP_kH^T + \bar{R}_2)^{-1}(z_k - H\hat{x}_k), \quad (21)$$

$$P_{k^+} = (I_n - \gamma_k P_k H^T (HP_kH^T + \bar{R}_2)^{-1} H) P_k. \quad (22)$$

We obtain the result by substituting (21) and (22) in (19) and (20) respectively. ■

The following facts are used in the derivation of the optimal control policy.

Lemma 2. The following equalities hold:

$$(a)\ \mathsf{E}[\gamma_k^2|\mathcal{I}_k] = \mathsf{E}[\gamma_k|\mathcal{I}_k] = \mathsf{PSR}_k, \quad (23)$$

$$(b)\ \mathsf{E}[\gamma_k K_k (z_k - Hx_k)|\mathcal{I}_k] = 0, \quad (24)$$

$$(c)\ \mathsf{E}[x_k^T S x_k|\mathcal{I}_k] = \hat{x}_k^T S \hat{x}_k + \mathrm{tr}(SP_k), \quad \forall S. \quad (25)$$

Proof. (a) From the definition, the arrival process γ_k and its square γ_k^2 have the same mean. Moreover, since γ_k is independent of the information set $\mathcal{I}_k$, we have

$$\mathsf{E}[\gamma_k|\mathcal{I}_k] = \mathsf{E}[\gamma_k] = \mathsf{PSR}_k . \quad (26)$$

Therefore, $\mathsf{E}[\gamma_k^2|\mathcal{I}_k] = \mathsf{PSR}_k$.

(b) From the definition, the arrival process γ_k and the innovation $z_k - Hx_k$ are independent. Therefore,

$$\begin{aligned} &\mathsf{E}[\gamma_k K_k (z_k - Hx_k)|\mathcal{I}_k] \\ &= \mathsf{E}[\gamma_k]\, \mathsf{E}[K_k(z_k - Hx_k)|\mathcal{I}_k] \\ &= \mathsf{E}[\gamma_k] K_k\, \mathsf{E}[(z_k - Hx_k)|\mathcal{I}_k] = 0, \end{aligned}$$

where we used the facts that K_k is $\mathcal{I}_k$-measurable and the innovation has zero mean.

(c) Applying algebraic operations, we obtain

$$\begin{aligned} &\mathsf{E}[x_k^T S x_k|\mathcal{I}_k] \\ &= \mathrm{tr}\left(S\, \mathsf{E}[x_k x_k^T|\mathcal{I}_k]\right) \\ &= \mathrm{tr}\left(S\, \mathsf{E}[x_k|\mathcal{I}_k]\, \mathsf{E}[x_k|\mathcal{I}_k]^T + S\, \mathsf{cov}[x_k|\mathcal{I}_k]\right) \\ &= \hat{x}_k^T S \hat{x}_k + \mathrm{tr}(SP_k). \end{aligned}$$

■

We define the cost-to-go function V_k as

$$V_k = \min_{\pi,\mu} \mathsf{E}\Big[\|x_{N+1}\|^2_{Q_0} + \sum_{t=k}^{N} \frac{1}{\lambda}\psi(\mathsf{PSR}_t) + \|x_t\|^2_{Q_1} + \|u_t\|^2_{Q_2}\Big|\mathcal{I}_k\Big]. \qquad (27)$$

Then, $\Psi(\pi^*, \mu^*) = \lambda\,\mathsf{E}[V_0]$. The next lemma provides the optimal control policy.

Lemma 3. The optimal control policy is a certainty-equivalence policy, i.e., $\mu^*_k = -L_k\hat{x}_k$ where

$$L_k = (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} F, \qquad (28)$$

and $S \succeq 0$ is the solution of the following Riccati equation

$$S_k = F^T S_{k+1} F + Q_1 - L_k^T(B^T S_{k+1} B + Q_2)L_k, \qquad (29)$$

with initial condition $S_{N+1} = Q_0$.

Proof. We can write:

$$V_k = \min_{\mathsf{PSR}_k, u_k} \mathsf{E}\Big[\frac{1}{\lambda}\psi(\mathsf{PSR}_k) + x_k^T Q_1 x_k + u_k^T Q_2 u_k + V_{k+1}\Big|\mathcal{I}_k\Big].$$

We shall prove that the cost-to-go is of the form $V_k = \hat{x}_k^T S_k \hat{x}_k + s_k$ where $S_k \succeq 0$ and s_k does not depend on x_k and $\hat{x}_k$. For time $N+1$, we see

$$V_{N+1} = \mathsf{E}[x_{N+1}^T Q_0 x_{N+1}|\mathcal{I}_{N+1}] = \hat{x}_{N+1}^T Q_0 \hat{x}_{N+1} + \mathrm{tr}(Q_0 P_{N+1}).$$

We assume that the hypothesis holds for time $k+1$, and show that it also holds for time k. From the hypothesis, we have

$$V_k = \min_{\mathsf{PSR}_k, u_k} \Big\{\frac{1}{\lambda}\psi(\mathsf{PSR}_k) + \hat{x}_k^T Q_1 \hat{x}_k + \mathrm{tr}(Q_1 P_k) + u_k^T Q_2 u_k + (F\hat{x}_k + Bu_k)^T S_{k+1}(F\hat{x}_k + Bu_k) + \mathsf{PSR}_k\,\mathrm{tr}\big(S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T\big) + s_{k+1}\Big\}. \qquad (30)$$

In the above derivation, we used

$$\mathsf{E}[x_k^T Q_1 x_k|\mathcal{I}_k] = \hat{x}_k^T Q_1 \hat{x}_k + \mathrm{tr}(Q_1 P_k),$$

and

$$\begin{aligned}
&\mathsf{E}[\hat{x}_{k+1}^T S_{k+1}\hat{x}_{k+1}|\mathcal{I}_k] \\
&= (F\hat{x}_k + Bu_k)^T S_{k+1}(F\hat{x}_k + Bu_k) + \mathsf{E}[\gamma_k^2|\mathcal{I}_k]\,\mathrm{tr}\big(S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T\big) \\
&= (F\hat{x}_k + Bu_k)^T S_{k+1}(F\hat{x}_k + Bu_k) + \mathsf{PSR}_k\,\mathrm{tr}\big(S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T\big),
\end{aligned}$$

where in the first and the second equalities we used (18a) and Lemma 2. We observe that u_k does not depend on PSR_k. Hence, taking the derivative of the expression in the minimum in (30) with respect to u_k and setting it equal to zero, we obtain $u^*_k = -L_k\hat{x}_k$ where

$$L_k = (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} F,$$

and

$$S_k = F^T S_{k+1} F + Q_1 - L_k^T(B^T S_{k+1} B + Q_2)L_k,$$

$$s_k = \frac{1}{\lambda}\psi(\mathsf{PSR}^*_k) + \mathrm{tr}(Q_1 P_k) + \mathsf{PSR}^*_k\,\mathrm{tr}(S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T) + \mathsf{E}[s_{k+1}|\mathcal{I}_k].$$

where PSR^* does not depend on x_k and $\hat{x}_k$. The minimum with respect to u_k exists for all k, because the relevant functions are quadratic and $Q_2 \succ 0$ and $S_{k+1} \succeq 0$. Hence, $V_k = \hat{x}_k^T S_k \hat{x}_k + s_k$. This completes the proof. ■

Lemma 4. The following equality holds:

$$\begin{aligned}
&\mathsf{E}[\mathrm{tr}(P_{N+1}Q_0) - \mathrm{tr}(R_0 S_0)] = \\
&+\mathsf{E}\Big[\sum_{k=0}^{N} \mathrm{tr}(S_{k+1}R_1) + \mathrm{tr}(P_k L_k^T(B^T S_{k+1} B + Q_2)L_k)\Big] \\
&-\mathsf{E}\Big[\sum_{k=0}^{N} \mathrm{tr}(P_k Q_1) + \mathsf{PSR}_k\,\mathrm{tr}(S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T)\Big].
\end{aligned} \qquad (31)$$

Proof. We can rewrite the random Riccati equation in (18b) as

$$P_{k+1} = FP_kF^T + R_1 - \gamma_k K_k(HP_kH^T + \bar{R}_2)K_k^T. \qquad (32)$$

Let us multiply (32) by S_{k+1} and (29) by P_k, and take the trace of difference. We obtain

$$\begin{aligned}
&\mathrm{tr}(P_{k+1}S_{k+1}) - \mathrm{tr}(P_k S_k) \\
&= \mathrm{tr}(S_{k+1}R_1) - \mathrm{tr}(\gamma_k S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T) \\
&\quad - \mathrm{tr}(P_k Q_1) + \mathrm{tr}(P_k L_k^T(B^T S_{k+1} B + Q_2)L_k).
\end{aligned}$$

Summing from 0 to N, and then taking expectation we obtain

$$\begin{aligned}
&\mathsf{E}[\mathrm{tr}(P_{N+1}S_{N+1})] - \mathsf{E}[\mathrm{tr}(P_0 S_0)] = \\
&+ \mathsf{E}\Big[\sum_{k=0}^{N} \mathrm{tr}(S_{k+1}R_1) + \mathrm{tr}(P_k L_k^T(B^T S_{k+1} B + Q_2)L_k)\Big] \\
&- \mathsf{E}\Big[\sum_{k=0}^{N}(\mathrm{tr}(P_k Q_1) + \mathrm{tr}(\gamma_k S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T)\Big].
\end{aligned}$$

However, from independency of γ_k we have

$$\begin{aligned}
&\mathsf{E}[\mathrm{tr}(\gamma_k S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T)] \\
&= \mathsf{E}[\gamma_k]\,\mathsf{E}[\mathrm{tr}(S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T)] \\
&= \mathsf{PSR}_k\,\mathsf{E}[\mathrm{tr}(S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T)] \\
&= \mathsf{E}[\mathsf{PSR}_k\,\mathrm{tr}(S_{k+1}K_k(HP_kH^T + \bar{R}_2)K_k^T)].
\end{aligned}$$

Moreover, $S_{N+1} = Q_0$ and $P_0 = R_0$. This completes the proof. ■

Proposition 1. The cost function $\Psi(\pi, \mu^*)$ associated with the optimal control policy μ^* and a packet success rate policy π that is independent of the state and state estimate is equal to

$$\Psi(\pi, \mu^*) = \lambda m^T S_0 m + \lambda\,\mathrm{tr}(R_0 S_0) + \mathsf{E}\Big[\sum_{k=0}^{N} \psi(\mathsf{PSR}_k) + \lambda\,\mathrm{tr}(S_{k+1}R_1) + \lambda\,\mathrm{tr}(\Theta_k P_k)\Big], \qquad (33)$$

where $\Theta_k = F^T S_{k+1} B L_k$.

Proof. From the proof of Lemma 3 and by considering V_k^π as the cost-to-go V_k in (27) for a fixed π, we have

$$\Psi(\pi,\mu^*) = \lambda\,\mathsf{E}[V_0^\pi] = \lambda\,\mathsf{E}[\hat{x}_0 S_0 \hat{x}_0 + s_0]$$
$$= \lambda m^T S_0 m + \mathsf{E}\Big[\lambda\,\mathrm{tr}(Q_0 P_{N+1}) + \sum_{k=0}^{N}\psi(\mathsf{PSR}_k)$$
$$+ \lambda\,\mathrm{tr}(Q_1 P_k) + \lambda\,\mathsf{PSR}_k\,\mathrm{tr}(S_{k+1}K_k(HP_kH^T + R_2)K_k^T)\Big]$$
$$= \lambda m^T S_0 m + \mathsf{E}\Big[\lambda\,\mathrm{tr}(R_0 S_0) + \sum_{k=0}^{N}\psi(\mathsf{PSR}_k)$$
$$+ \lambda\,\mathrm{tr}(S_{k+1}R_1) + \lambda\,\mathrm{tr}(P_k L_k^T(B^T S_{k+1} B + Q_2)L_k)\Big],$$

where in the last equality we used Lemma 4. We can now obtain the result by using the definition of L_k. ∎

Let us define the value function W_k as

$$W_k(P_k) = \min_{\pi} \mathsf{E}\Big[\sum_{t=k}^{N}\psi(\mathsf{PSR}_t) + \lambda\,\mathrm{tr}(\Theta_t P_t)\Big|\mathcal{I}_k\Big], \quad (34)$$

with $W_{N+1} = 0$. Next, we characterize the optimal packet success rate policy of the optimization problem in (17).

Lemma 5. The optimal packet success rate policy is provided by

$$\pi_k^* = \underset{\mathsf{PSR}_k\in\Gamma}{\mathrm{argmin}}\Big\{\psi(\mathsf{PSR}_k) + (1-\mathsf{PSR}_k)W_{k+1}(FP_kF^T + R_1)$$
$$+ \mathsf{PSR}_k\,W_{k+1}(FP_kF^T + R_1 - K_kHP_kF^T)\Big\}. \quad (35)$$

Proof. Notice that only the third and last terms of $\Psi(\pi,\mu^*)$ in (33) depend on PSR_k and P_k. Moreover, the dynamics of P_k is governed by the random Riccati equation in (18b). Hence, we can obtain the optimal packet success rate policy by solving the following optimization problem:

$$\min.\ \mathsf{E}\Big[\sum_{k=0}^{N}\psi(\mathsf{PSR}_k) + \lambda\,\mathrm{tr}(\Theta_k P_k)\Big],$$
$$\text{s.t. } P_{k+1} = FP_kF^T + R_1 - \gamma_k K_k H P_k F^T, \quad (36)$$

with initial condition $P_0 = R_0$. Let us define

$$\rho(P_k,\mathsf{PSR}_k) = \psi(\mathsf{PSR}_k) + \lambda\,\mathrm{tr}(\Theta_k P_k),$$

and

$$\phi(P_k,\mathsf{PSR}_k) = FP_kF^T + R_1 - \gamma_k K_k H P_k F^T.$$

Since the stage cost $\rho(P_k,\mathsf{PSR}_k)$ is $\mathcal{I}_k$-measurable for any PSR_k, we can write

$$W_k(P_k) = \min_{\mathsf{PSR}_k\in\Gamma}\Big\{\rho(P_k,\mathsf{PSR}_k) + \mathsf{E}[W_{k+1}(\phi(P_k,\mathsf{PSR}_k))|\mathcal{I}_k]\Big\}.$$

Moreover, the transition function $\phi(P_k,\mathsf{PSR}_k)$ can take only two different values:

$$P_{k+1}^0 = FP_kF^T + R_1, \quad (37)$$
$$P_{k+1}^1 = FP_kF^T + R_1 - K_kHP_kF^T. \quad (38)$$

The probability of $P_{k+1} = P_{k+1}^1$ is PSR_k and the probability of $P_{k+1} = P_{k+1}^0$ is $1-\mathsf{PSR}_k$. Therefore, we can calculate the expected value of the cost-to-go $W_{k+1}\big(\phi(P_k,\mathsf{PSR}_k)\big)$ as

$$\mathsf{E}[W_{k+1}(\phi(P_k,\mathsf{PSR}_k))|\mathcal{I}_k]$$
$$= (1-\mathsf{PSR}_k)W_{k+1}(P_{k+1}^0) + \mathsf{PSR}_k\,W_{k+1}(P_{k+1}^1).$$

Hence, the optimal PSR_k is obtained by

$$\mathsf{PSR}_k^* = \underset{\mathsf{PSR}_k\in\Gamma}{\mathrm{argmin}}\Big\{g(P_k,\mathsf{PSR}_k) + (1-\mathsf{PSR}_k)W_{k+1}(P_{k+1}^0)$$
$$+ \mathsf{PSR}_k\,W_{k+1}(P_{k+1}^1)\Big\}.$$

Notice that the term $\mathrm{tr}(\Theta_k P_k)$ in $\rho(P_k,\mathsf{PSR}_k)$ does not depend on PSR_k. This completes the proof. ∎

Following Lemma 5, the optimal transmit power at time k can be obtained by

$$p_k^{*TX} = \psi(\mathsf{PSR}_k^*). \quad (39)$$

The next theorem shows that a separation exists in the design of the overall system.

Theorem 1. There is a separation between the designs of the optimal estimator, optimal controller, and optimal transmission power control mechanism.

Proof. The optimal state estimate, optimal control policy, and optimal packet success rate policy are separately designed by Lemma 1, Lemma 3, and Lemma 5. ∎

4. ILLUSTRATIVE EXAMPLE

In this section, we provide a simple example to illustrate the method we developed in this study. Consider the following unstable system observed by a wireless sensor node:

$$x_{k+1} = 1.01x_k + u_k + w_k,$$
$$y_k = 0.30x_k + v_k,$$

with the initial conditions $m = 0$ and $R_0 = 2$, and with the covariances $R_1 = 0.1$ and $\bar{R}_2 = 0.3$. The weighting matrices are $Q_0 = 1$, $Q_1 = 1$, and $Q_2 = 0.5$, and the weighting coefficient is $\lambda = 0.0004$. The communication channel is characterized by the following parameters: the data rate of the communication $R_c = 250$ Kbps, the carrier frequency $f = 2.45$ GHz, the receiver distance $d = 10.00$ m, the path loss exponent $\eta = 3.6$, the noise power spectral density $N_0 = 6\times10^{-18}$ watt/Hz, and the payload length $\ell = 100$ bits. This yields $c_0 = 3.149\times10^{-5}$.

We used dynamic programming to calculate the cost-to-go function $W(P)$. The spaces of PSR and P are discretized by grids with cardinality $M = 2500$ over the intervals $[0.0001, 0.9999]$ and $[0, 10]$ respectively. Figure 2 illustrates the optimal transmit power policy as a function of the covariance at time $k = 1$. As it is seen, the transmitter is in sleep mode for small values of the covariance. This result can justify the usage of a simple threshold event-triggering mechanism for transmission power control (see e.g., Soleymani et al. (2016)). We simulated the system over the horizon $N = 200$. Figure 3 and Figure 4 depict the trajectories of the estimation error and error covariance, and Figure 5 shows the trajectory of the transmit power. It can be observed that the transmit power is adaptive and at many time steps the transmitter is in sleep mode, which leads to an overall improvement in the energy conservation of the wireless sensor node while satisfying the required control quality.

5. CONCLUSION

We studied the problem of minimizing the expected transmit power of a wireless sensor node required for a specific

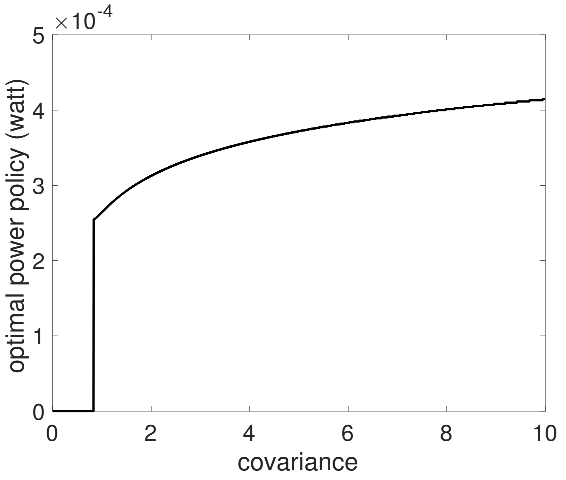

Fig. 2. Optimal transmit power policy at time $k = 1$.

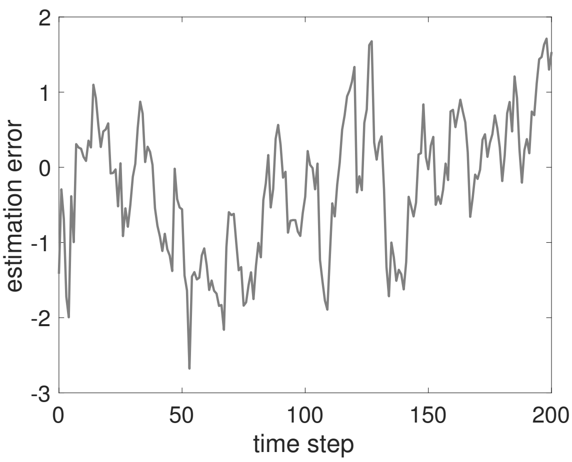

Fig. 3. Trajectory of the estimation error.

level of the LQG control performance over a finite horizon. We employed dynamic programming, obtained the optimal policies, and showed that there exists a separation between the designs of the optimal estimator, optimal controller, and optimal transmission power control mechanism.

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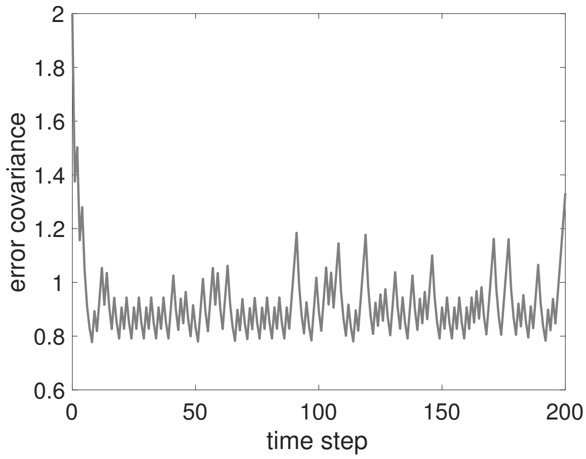

Fig. 4. Trajectory of the error covariance.

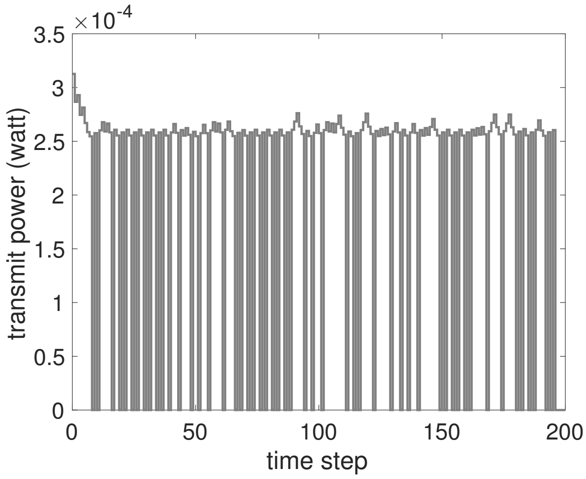

Fig. 5. Trajectory of the transmit power.

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