Development of a probabilistic model for the prediction of building damage due to tunneling induced settlements

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ABSTRACT: Tunnel construction can cause deformations of the surrounding ground, which endanger buildings and other structures located in the vicinity of the tunnel. The prediction of these deformations and the damages to the buildings is difficult due to limited knowledge of the geotechnical conditions and uncertainty in the response of the structures to the settlements. This paper presents a probabilistic model for prediction of the damage to buildings due to tunneling, which combines the volume loss method with the equivalent beam model. It furthermore proposes a probability-based method for determination of the limiting value of settlement that is used for control purposes during the tunnel construction. Updating of the limiting settlement with measurements gathered during the construction is described. The proposed methodology is applied to a masonry building affected by the construction of the L9 metro line tunnel in Barcelona.

1 INTRODUCTION

The prediction of damages to buildings caused by underground constructions such as tunnels entails uncertainty due to our limited knowledge of the geotechnical conditions and the response of the structures subjected to differential settlements. Prediction of damages is important as a basis for the design, the selection of the construction technology and for setting allowable limits on settlements. These limiting values of settlement are then used in the construction phase for control purposes: if the measured settlement exceeds the limiting values, the construction is stopped or additional safety measures must be taken. At present, settlement profiles and resulting damages in buildings are commonly modeled deterministically. Settlement profiles are typically predicted by means of 2D Finite Element (FE) models combining the soil, the tunnel and the foundations of a given building. Alternatively, the volume loss method (Peck, 1969; Attewell et al., 1986) can be used for approximation of the subsidence trough. The volume loss method is an empirical approach to determining the settlement profile; this empirical approach is computationally more efficient than 2D FE simulation and it is fully sufficient for many engineering applications. Once the settlement profile is calculated, it is possible to predict damages in buildings by means of empirical methods such as the equivalent beam method (Burland & Wroth, 1974; Boscardin &

Cording, 1989), which is widely used in tunnel engineering. This method determines the maximum tensile strain in the building by modeling it as a linear elastic beam subjected to a given deflection ratio. This strain value is then compared with limiting strain values, which define different categories of damage according to the severity of affection. An iteration process is performed in order to assess the limiting value of settlement that leads to damages below an acceptable level.

This paper proposes a computationally efficient probabilistic model for the estimation of building damage due to tunneling, combining the volume loss method for approximation of the subsidence trough and the equivalent beam method for modeling the response of the building (Sec. 2). The parameters of the volume loss method are usually selected based on expert judgment. The uncertainty connected to the choice of these parameters is typically high. The proposed methodology allows taking into account these uncertainties as well as the uncertainty in the building response.

The paper further proposes a novel methodology for the determination of the limiting settlement value on a probabilistic basis (Sec. 3). The limiting settlement is here defined as a settlement, for which the probability of damage to the building is acceptably low. Two approaches for setting this limiting value are proposed: (1) a simple approximate approach using a plot of the results of the probabilistic analysis, (2) an advanced approach based on reliability updating (Straub, 2011). Additionally, a procedure for updating the limiting values with observations gathered during the tunnel construction is described. The proposed methodology is applied to a case study

of masonry buildings affected by the construction of the L9 metro line in Barcelona.

2 PROBABILISTIC MODEL OF BUILDING DAMAGE DUE TO TUNNELING

The shape of the settlement profile in a plane, which is close to perpendicular to the tunnel axis, can be modeled by means of a Gaussian curve (Peck, 1969). The settlement at the distance y from the tunnel axis then equals:

$$s(y) = S_{max} \exp\left(-\frac{y^2}{2i^2 \cos\theta}\right) \tag{1}$$

where *i* is the location of the inflection point (horizontal distance from tunnel axis), θ is the angle between the modeled plane and the perpendicular plane and S_{max} is the maximum settlement in the center of the Gaussian curve, i.e. above the tunnel axis. S_{max} can be calculated as:

$$S_{max} = f_1(V_L, K) = \frac{V_L \cdot d^2}{3.192 \cdot K \cdot z_0}$$
(2)

where *d* and z_0 are the diameter and depth of the tunnel, respectively, V_L is the expected volume ground loss (i.e. the ratio between the area of the settlement trough and the cross-section area of the tunnel), which is dependent on the tunneling technology, and *K* is a shape parameter of the curve which depends on the type of soil. The product $K \cdot z_0$ determines the location of the inflection point *i* of the Gaussian curve with respect to tunnel centerline. V_L and *K* are modeled as random variables (RVs). The model error is considered as described later in Eq. (12).

Knowledge of the shape of the settlement trough allows determining the deflection ratios Δ/L that are affecting the building, where *L* is the distance between two reference points and Δ is the relative deflection between these two points.

The response of the building is modeled using the equivalent beam method, which represents the building by means of a weightless linear elastic rectangular beam. The aim is to calculate the tensile strains in the beam for a given deflected shape. The distribution of strains in the beam depends on the mode of deformation. Therefore, extreme modes of bending and shear are analyzed separately. The extreme fiber strains in bending and shear are given by the following equations:

$$\varepsilon_{br} = f_2 \left(V_L, K, \frac{E}{G} \right) \cdot E_{\varepsilon_{br}} = (\varepsilon_{bmax} + \varepsilon_h) \cdot E_{\varepsilon_{br}}$$
(3)

$$\varepsilon_{dr} = f_3 \left(V_L, K, \frac{E}{G} \right) \cdot E_{\varepsilon_{dr}} =$$

$$= \left[\varepsilon_h \left(1 - \frac{E}{4G} \right) + \sqrt{\frac{\varepsilon_h^2}{16} \left(\frac{E}{G} \right)^2 + \varepsilon_{dmax}^2} \right] \cdot E_{\varepsilon_{dr}}$$
(4)

where $\frac{E}{G}$ is the ratio between the Young and shear moduli of the building material, which is modeled as a RV, $E_{\varepsilon_{br}}$ and $E_{\varepsilon_{dr}}$ represent the model errors and ε_h is the horizontal strain at the base of the beam, which is obtained as the derivative of the horizontal displacements *u*:

$$u(y) = \frac{s(y) \cdot y}{z_0} \tag{5}$$

$$\varepsilon_h(y) = \frac{du(y)}{dy} \tag{6}$$

The model errors $E_{\varepsilon_{br}}$ and $E_{\varepsilon_{dr}}$ are considered as multiplicative RVs with mean value equal to 1. They result from the assumption of linear elasticity, the position of the neutral axis and the omission of the presence of openings.

Maximum bending (ε_{bmax}) and shear (ε_{dmax}) strains in the equivalent beam are calculated as:

$$\varepsilon_{bmax} = \frac{\frac{\Delta}{L}}{\left(\frac{L}{12t} + \frac{3I}{2aLH}\frac{E}{G}\right)}$$
(7)

$$\varepsilon_{dmax} = \frac{\frac{\Delta}{L}}{\left(1 + \frac{HL^2 G}{18I E}\right)} \tag{8}$$

where H is the beam height, I is the inertia per unit length, t is the assumed position of the neutral axis and a is the location of the fiber where strains are calculated.

The calculation of Eqs. (3)-(8) is performed separately for the zone of the building undergoing sagging deflection (upwards concavity) and for the zone undergoing hogging deflection (downwards concavity). The errors of the equivalent beam model in sagging, $E_{\varepsilon br}^{sag}$, $E_{\varepsilon dr}^{sag}$, and hogging, $E_{\varepsilon br}^{hog}$, $E_{\varepsilon dr}^{hog}$, are assumed to be independent. In case of sagging deflection, the neutral axis is assumed to be at middle height (t = H/2). In case of hogging deflection, the neutral axis is assumed to be at the top fiber (t = H). Strains are calculated in the most critical fiber from the position of the neutral axis, so that a = t in both cases. The damage on the buildings is determined depending on the maximum strain ε_{max} :

$$\varepsilon_{max} = \max\left[\mathcal{E}_{\varepsilon_{br}}^{sag}, \mathcal{E}_{\varepsilon_{dr}}^{sag}, \mathcal{E}_{\varepsilon_{br}}^{hog}, \mathcal{E}_{\varepsilon_{dr}}^{hog}\right]$$
(9)

where $\varepsilon_{\varepsilon_{br}}^{sag}$ and $\varepsilon_{\varepsilon_{br}}^{hog}$ are the maximum bending strains in sagging and hogging respectively, both obtained using Eq. (3), and $\varepsilon_{\varepsilon_{dr}}^{sag}$ and $\varepsilon_{\varepsilon_{dr}}^{hog}$ are the maximum shear strains in sagging and hogging respectively, both obtained using Eq. (4).

Based on ε_{max} , one can estimate the size of the cracks in the building. The approach of Burland et al. (1977) is used in this paper for classification of the damage magnitudes as shown in Table 1.

Table 1. Classification of damage (Burland et al., 1977)

Category of damage	Normal degree of severity	Typical damage	Limiting tensile strain (\mathcal{E}_{lim})(%)
0	Negligible	Hairline cracks less than 0.1mm	0-0.050
1	Very slight	Fine cracks up to 1mm	0.050 - 0.075
2	Slight	Cracks easily filled up to 5mm	0.075 - 0.150
3	Moderate	Cracks from 5 to 15mm.	0.150 - 0.300
4	Severe	Extensive repair work. Cracks from 15 to 25mm.	> 0.300
5	Very severe	Partial or complete rebuilding. Cracks > 25mm.	

The different damage categories can be used for the definition of system failure $F_{\varepsilon_{lim}}$. Failure occurs if the maximum strain (ε_{max}) obtained from Eq. (9) exceeds a given limiting tensile strain value ε_{lim} for a target category of damage according to Table 1. For example, if cracks with a width larger than 0.1mm are considered inacceptable, the limiting strain defining the failure is $\varepsilon_{lim} = 0.05\%$. The limit state function (LSE) is then defined as

The limit state function (LSF) is then defined as

$$g(\mathbf{X}) = \varepsilon_{lim} - \varepsilon_{max} \tag{10}$$

where **X** is the vector of variables that are considered to be random. The LSF determines the failure domain $\Omega_F = \{g(\mathbf{x}) \le 0\}$. The probability of failure then equals the probability of **X** taking a value within the failure domain:

$$\Pr(F_{\varepsilon_{\lim}}) = \Pr\left(\mathbf{X} \in \Omega_F\right) \tag{11}$$

Note that this definition of LSF is suitable when applying sampling methods for the computation of probabilities. If methods such as First-Order Reliability Method (FORM) were used, separate LSFs for $\varepsilon_{\varepsilon_{br}}^{sag}$, $\varepsilon_{\varepsilon_{dr}}^{sag}$, $\varepsilon_{\varepsilon_{dr}}^{hog}$, should be defined and the failure event should be described as a series system.

3 DETERMINATION OF LIMITING SETTLEMENT

The measured maximal settlement above the crown of the tunnel S_m equals:

$$S_m = S_{max} + E_f + E_m = S_{max} + E_E \tag{12}$$

where S_{max} is the maximal settlement calculated using Eq. (2), E_f is the model error representing the deviation of the real settlement from the idealized Gaussian shape described by Eqs. (1) and (2), E_m is the error of measurement on site, which reflects imprecision of the instruments, human errors, effect of temperature changes, etc., and $E_E = E_f + E_m$.

The goal is to find the limiting value of settlement s_{lim} from the following condition:

$$\Pr\left(F_{\varepsilon_{lim}}|S_m = s_{lim}\right) = p_T \tag{13}$$

 p_T is the required (target) safety level. A measured settlement $S_m > s_{lim}$ thus implies an unacceptably high probability of failure $F_{\varepsilon lim}$ and would trigger further actions.

In the following, the value of s_{lim} will be determined using two different approaches. In Sec. 3.1, an approximate approach based on engineering judgment is utilized. In Sec. 3.2, the exact value of s_{lim} will be determined using a reliability-based approach. Finally, Sec. 3.3 describes the updating of the limiting settlement based on observations gathered during the tunnel construction.

3.1 Approximate approach

An approximate estimate of the limiting settlement s_{lim} can be determined based on evaluation of the probabilistic model described in Sec. 2 using Monte Carlo (MC) simulation. For each sample of the input variables **X**, the settlement trough is evaluated according to Eqs. (1) and (2). Based on the estimated settlement, the maximal tensile strain in the building is calculated using Eqs. (3) to (8).

For simplification, the error terms are disregarded and only V_L , K and E/G are considered as RVs. Eq. (12) then reduces to $S_m = S_{max}$. The limiting settlement is approximately determined from a scatter plot of the maximum tensile stains ε_{max} against the maximal settlement S_{max} as is shown later in Figure 4. The value is determined visually from the plot using engineering judgment.

3.2 Reliability-based approach

The conditional probability of Eq. (13) can be determined by means of Bayesian updating techniques with equality type information as proposed in Straub (2011) and applied to geotechnical safety in Papaioannou and Straub (2012). With this approach, all the model and measurement errors are included.

First, the likelihood of V_L and K for given measured settlement S_m is calculated:

$$L(v_L, k) \propto \Pr(S_m = s_m | V_L = v_L, K = k) =$$
 (14)

$$= f_E \big(s_m - f_1(v_L, k) \big)$$

where f_E is the probability density function (PDF) of the error E_E in Eq. (12). Following Straub (2011), this likelihood function can be expressed by a LSF:

$$h(v_L, k, u) = u - \Phi^{-1}[cL(v_L, k)] \le 0$$
(15)

where *u* is the realization of a standard Normal RV, Φ^{-1} is the inverse standard normal CDF and $c = \sigma_{E_E} \cdot \sqrt{2\pi}$ is a scaling constant chosen to ensure that $cL(v_L, k) \leq 1$ for all v_L, k . This LSF defines the observation domain $\Omega_0 = \{h(\mathbf{x}, u) \leq 0\}$ in a space that contains the original RVs $\mathbf{X} = (V_L, K, \frac{E}{C})$ and the standard Normal variable *U*. With this approach, the conditional probability of failure $F_{\varepsilon lim}$ for a given observed settlement s_m is computed as:

$$\Pr(F_{\varepsilon_{lim}}|S_m = s_m) = \frac{\Pr(F_{\varepsilon_{lim}} \cap S_m = s_m)}{\Pr(S_m)}$$
$$= \frac{\Pr([\mathbf{x}, u] \in \Omega_0 \cap \Omega_F)}{\Pr([\mathbf{x}, u] \in \Omega_0)}$$
(16)

This probability can be evaluated using a MC simulation for different values of s_m . The limiting settlement value s_{lim} ensuring Eq. (13) is then found iteratively.

3.3 Updating with the measurements gathered during construction

After the construction starts, N measurements of the settlements $\mathbf{s} = (s_1, s_2, \dots s_N)$ are obtained along the tunnel. These measurements can be used for updating the probabilistic model and the value of the limiting settlement. The measurements are carried out in the same quasi homogeneous geotechnical section of the tunnel, where also the analyzed building is located.

The uncertain geotechnical conditions in this quasihomogeneous section, characterized by volume loss V_L and shape parameter K, are now described as stationary spatial stochastic processes with constant auto correlation functions $R_K(l) = \rho_K$ and $R_{VL}(l) =$ ρ_{V_l} , where l is the distance between two locations within the section. In other words, K has the same marginal distribution at any location within the section and the values of K at any two locations are correlated with correlation coefficient ρ_K , independent of the distance between them. The same holds for V_L . This simple correlation model was selected based on a preliminary analysis of data from a constructed tunnel; its validity should be tested in the future based on a more detailed analysis. The new measurements at locations 1, ..., N can be expressed by separate likelihood functions $L_1, ..., L_N$, following Eq. (14). For each likelihood function L_i , one can find the corresponding observation domain Ω_i defined by means of a LSF $h_i(v_{L,i}, k_i, u_i)$ as described in

Eq. (15). Here, $v_{L,i}$ and k_i are the realizations of the random processes *K* and V_L at the location of measurement *i*.

To update the limiting value of settlement for S_m conditional on the existing measurements **s**, the failure probability conditional on S_m and on **s** is computed (compare with Eq. (16)):

$$\Pr\left(F_{\varepsilon_{\lim}}|S_m = s_m, \mathbf{S} = \mathbf{s}\right) = \frac{\Pr\left(F_{\varepsilon_{\lim}} \cap S_m = s_m \cap \mathbf{S} = \mathbf{s}\right)}{\Pr(S_m = s_m \cap \mathbf{S} = \mathbf{s})}$$

$$= \frac{\Pr\left([\mathbf{x}, u, u_1, \dots u_N] \in \Omega_F \cap \Omega_0 \cap \Omega_1 \cap \dots \cap \Omega_N\right)}{\Pr\left([\mathbf{x}, u, u_1, \dots u_N] \in \Omega_0 \cap \Omega_1 \cap \dots \cap \Omega_N\right)}$$
(17)

Analogous to the procedure in Sec. 3.2, this conditional probability is evaluated for different values of s_m . The updated limiting settlement value s_{lim}^* ensuring Eq. (13) is found iteratively.

4 CASE STUDY

The proposed method is applied to a case study of the L9 metro line construction in Barcelona. The damage produced by the tunnel construction to a complex of masonry buildings from the late 1920's located in the *Bon Pastor* area is studied. An equivalent beam analysis of the buildings was already performed in Camós et al. (2012), showing the validity of this model. The location of the building and the tunnel is shown in Figure 1.



Figure 1. Location of buildings and tunnel track

4.1 Model parameters

The tunnel diameter d in the studied section is 12m, the depth of the tunnel is $z_0 = 23m$. The length of the building complex is L = 46m, the angle between the building wall and the plane perpendicular to the tunnel axis is $\theta = 26^\circ$, the building height is H = 3mand thus, the inertia per unit length of the crosssection of the building is equal to $I = 2.25m^4/m$. The parameter t equals 1.5m in the sagging zone and 3m in the hogging zone and a = t for both zones.

The probabilistic model is summarized in Table 2. The shape parameter of the settlement profile K usually varies from 0.2 to 0.3 for granular soils to 0.4 to 0.5 for stiff clays to values as high as 0.7 for soft silty clays (Burland, 2008). The ground in the ana-

lyzed tunnel section is formed by typical alluvial soil with coarse sand, limes and a small quantity of gravel. K is likely to be in the interval from 0.2 to 0.4, which is thus assumed to be a 90% confidence interval. The mean is assumed to be 0.3 and coefficient of variation (c.o.v.) is assumed equal to 0.2. K is non-negative and the lognormal distribution is thus an appropriate model for this RV.

Experience from tunneling constructions in similar conditions (TYPSA, 2003) shows that the expected interval of volume loss V_L is in the range 0.1% to 0.6%. Nevertheless, the uncertainty on these values is high due to many unpredictable factors that influence ground losses (unexpected geological units, technical problems of the TBM, human errors, etc.). The interval of 0.1- 0.6% is thus assumed to be a 90% confidence interval and the c.o.v. is supposed to be 0.4. V_L is modeled by a lognormal distribution. A value equal to 2.5 is typically assumed for the ratio E/G of masonry buildings. Uncertainty is also present in this parameter due to the variety of orthotropic materials composing a building, yet this uncertainty is relatively small. Therefore, it is here modeled by a Beta distribution defined on the interval 2.4 to 2.6. The measurement error E_m and the model error E_f are represented with normal distribution with zero mean and standard deviations 0.5mm. The multiplicative model errors of the equivalent beam model $E_{\varepsilon_{br}}^{sag}$, $E_{\varepsilon_{dr}}^{sag}$, $E_{\varepsilon_{br}}^{hog}$, $E_{\varepsilon_{dr}}^{hog}$, are described by lognormal distributions with mean equal to 1.

Table 2. Random parameters of the model.

Parameter [units]	Distribution	Mean	St.dev.
K [-]	Lognormal (-1.22, 0.20)	0.3	0.06
V _L [%]	Lognormal (-0.99,0.39)	0.4	0.16
$\frac{E}{G}$ [-]	Beta (2,2,[2.4,2.6])	2.5	0.045
E_m, E_f [mm]	Normal (0.0,0.5)	0.0	0.50
$E_{\varepsilon_{br}}^{sag}, E_{\varepsilon_{dr}}^{sag}, E_{\varepsilon_{br}}^{hog}, E_{\varepsilon_{dr}}^{hog}$ [-]	Lognormal (0.0,0.05)	1.0	0.05

4.2 Results of the probabilistic analysis

The results of the MC simulation of the model described in Sec. 2 are presented here. They show the influence of the different uncertain parameters on the assessment of maximum strain ε_{max} calculated following Eq. (9) and on the associated damage category as defined in Table 1.

Figure 2 shows the influence of the volume loss V_L on the maximum strain ε_{max} . A positive linear correlation is observed; higher values of volume loss are likely to lead to more severe damages on the building. Figure 3 displays the influence of the shape parameter K on ε_{max} . Higher values of K produce flatter settlement troughs, which cause smaller tensile strains in the building and thus lead to milder damages. The relationship is clearly nonlinear.

The a-priori probability of the building damage being in category 0, which corresponds to negligible damages, is 0.6. The probability of only aesthetical damages, corresponding to categories 0 - 2, is 0.95.



Figure 2. Scatter plot of volume loss V_L and max. strain ε_{max} .



Figure 3. Scatter plot of parameter K and max. strain ε_{max} .

4.3 *Approximate determination of the limiting settlement value*

The limiting settlement s_{lim} is determined using the approximative approach described in Sec. 3.1. Only a negligible damage (category 0) is acceptable, as is usual in tunneling construction. More severe damages to buildings are considered as a failure, therefore the limiting tensile strain is set to $\varepsilon_{lim} = 0.05\%$. Figure 4 shows the scatter plot of settlement S_{max} and maximum strain ε_{max} obtained from the MC simulation. An approximate value of the limiting settlement is determined $s_{lim} = 22$ mm.



Figure 4. Scatter plot of settlement S_{max} and max. strain ε_{max} .

4.4 *Exact reliability-based determination of the limit value of settlement*

The reliability based approach shown in Sec. 3.2 is used to find the limiting settlement s_{lim} that satisfies Eq. (13) for p = 0.05 and $\varepsilon_{lim} = 0.05\%$ (the failure event is defined in accordance with the previous Sec. 4.3.). Figure 5 displays the conditional probability of failure for different values of measured settlement s_m from 20 to 30mm (denoted as prior estimate). The limiting settlement is determined as $s_{lim} = 23$ mm.



Figure 5. Conditional probability of failure for different values of measured settlement, Pr ($F_{\varepsilon_{lim}=0.050\%}|s_m$).

4.5 *Results of updating with observations from monitoring instruments*

The prior estimate of the limiting settlement described is now updated with the measurements gathered during the construction process, following the procedure described in Sec. 3.3. Two measurement of the settlement in the same quasi-homogeneous section are utilized: $s_1 = 14$ mm, $s_2 = 19$ mm. Correlation coefficients of the underlying normal distributions of shape parameter and volume loss are estimated by expert judgment: The shape of the settlement trough (described by parameter K) is dependent on the geotechnical conditions. A high correlation is therefore assumed within a geologically homogeneous section and $\rho_K = 0.7$. On the contrary, the volume loss V_L is strongly influenced by the construction process and it is typically highly variable within one homogeneous section. It is therefore assumed to be uncorrelated and $\rho_{V_I} = 0$. The updated conditional probabilities of failure for different values of the settlement measured at the vicinity of the building, s_m , are depicted in Figure 5. The updated value of limiting settlement is $s_{lim}^* = 27$ mm.

5 CONCLUDING REMARKS

The paper presented a computationally efficient model for probabilistic prediction of building damage due to tunnelling that is applicable in engineering practice (Sec. 2). Further, a novel method for determining the limiting settlement was presented (Sec. 3), which is a more systematic and traceable reliability-based approach with an explicit than the deterministic methodology rationale typically used in practice. Additionally, the reliability-based approach allows to incorporate measurements made during the construction. The

proposed procedure was demonstrated on a case study of a tunnel construction in Barcelona (Sec. 4). First, the influence of the uncertainty in the model parameters (volume loss, shape parameter of the settlement through, Young and shear moduli of the building material) on the estimated damage was presented. Second, the value of the limiting settlement was determined with the approximate approach as 22mm. Third, the limiting settlement was determined more precisely using an advance approach realiability-based as 23mm. Both approaches provide similar values and the simpler method appears to be satisfactory for practical applications. Both of these values are more strict than the value that was used in the real case, where a settlement of up to 24mm was considered to be safe. The reason for this difference is the fact that in the real case, the uncertainties in the ground parameters and building parameters were not considered and some unfavorable values of these parameters were thus not taken into account. Finally, the value of the limiting settlement was updated with observations gathered during the construction. The updated limiting settlement is 27mm and is thus higher that the prior value determined during the design phase. The increase of the the limit is possible thanks to the reduction of uncertainty after including the additional measurements.

REFERENCES

- Attewell, P.B., Yeates, J. and Selby, A.R. (1986) Soil movements induced by tunneling and their effects on pipelines and structures. Blackie Academic & Professional
- Boscardin, M.D. and Cording, E.J. (1989) Building response to excavation-induced settlement. *Jour of Geo. Eng.* 115(1-21)
- Burland. J.B. (2008) The assessment of the risk of damage to buildings due to tunnelling and excavations. J.T. Payma Cotas: *Mov. de edificios inducidos por excavaciones*. pp. 3.
- Burland, J.B., Broms, B. and De Mello, V.F.B. (1977) Behaviour of foundations and structures. *Proc. 9th Int. Conf. on Soil Mech. and Found. Eng.*, 2, 495-546.
- Burland, J.B. and Wroth, C.P., (1974). *Settlement of buildings and associated damage*. London: Pentech Press.
- Camós, C., Molins, C. and Arnau, O. (2012). A case study of damage on masonry buildings produced by tunneling induced settlements. *Int. Jour. of Architectural Heritage*
- Peck, R.B. (1969) *Deep excavations and tunneling in soft ground*. SOA Report, 7th Int. Conf. SM&FE.
- Papaioannou, I. and Straub. D (2012). Reliability updating in geotechnical engineering including spatial variability of soil. *Computers and Geotechnics* 42: 44–51
- Straub, D. (2011) Reliability updating with equality information. *Probabilistic Engineering Mech.* 26: 254-258
- TYPSA. (2003) Análisis de los movimientos del terreno producidos por la excavación mecánica del túnel de la L9 en la zona de Santa Coloma de Gramanet. Projecte de construcció de la Línia 9 de metro de Barcelona. Tram 4t Bon Pastor – Can Zam. Infraestructura i Estacions. Annex 8.