Problem

\[
\begin{align*}
PDE: & \quad \nabla \cdot \sigma + b - p \ddot{u} = 0 \quad + \quad \text{BCs + ICs} \\
\text{FE:} & \quad M(p)\ddot{u}(t) + f(p, u(t)) = B(p)F(p, t)
\end{align*}
\]

Parameterization:

\[
\text{Parameter-Set: } p = \{p_1, p_2, \ldots, p_p\} \in P
\]

Model Reduction

Problem: Parameter studies and optimization of large models are very time consuming.

Idea: Reduce computational effort for solving equations of motion by applying model reduction.

Galerkin Projection

\[
\begin{align*}
& \quad u = Vq + e \approx Vq \\
\implies & \quad V^TMVq + V^Tf(Vq) = V^TB
\end{align*}
\]

Methods:

<table>
<thead>
<tr>
<th>Linear part $V_{na}$</th>
<th>Nonlinear part $V_{nl}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model truncation</td>
<td>Vibration modes</td>
</tr>
<tr>
<td>Static derivatives</td>
<td>Dimensional structures</td>
</tr>
<tr>
<td>Moment matching</td>
<td>Krylov directions</td>
</tr>
<tr>
<td>Static derivatives</td>
<td>Krylov derivatives</td>
</tr>
<tr>
<td>Other linear methods</td>
<td>Linear basis vectors</td>
</tr>
<tr>
<td>Exact derivatives</td>
<td>Static derivatives</td>
</tr>
</tbody>
</table>

Evaluation & optimization via system norm

Hyperreduction

\[
\begin{align*}
& \quad V^Tf(Vq) = \sum_{e \in E} VTL_eB_e^Tf_e(B_eVq) \\
\approx & \quad \sum_{e \in E} VTL_eB_e^Tf_e(B_eVq)
\end{align*}
\]

Methods:

- DEIM
- EGSW
- Polynomial Expansion

Parametric Reduction

Same methods at each parameter sampling point $p_i \in P$ ($i = 1, 2, \ldots, N$)

\[
\implies V(p_i)
\]

Research FE Code for Nonlinear Model Reduction

Module Structure:

<table>
<thead>
<tr>
<th>Solver</th>
<th>Mechanical System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Assembler (Data)</td>
</tr>
<tr>
<td></td>
<td>(Boundary)</td>
</tr>
<tr>
<td></td>
<td>Element</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Module Order Reduction module</th>
</tr>
</thead>
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<tr>
<td>amfe_stiffness</td>
</tr>
<tr>
<td>amfe_masses</td>
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<tr>
<td>amfe_nonlinear</td>
</tr>
<tr>
<td>amfe_linear</td>
</tr>
<tr>
<td>amfe_optim</td>
</tr>
<tr>
<td>amfe_oscil</td>
</tr>
</tbody>
</table>

Main features:

- Solve nonlinear structural dynamics problems
- Modular Structure
- Interpretative (no input files)
- Easy access to internal computations
- Rapid Prototyping of new model reduction methods

Model Reduction Features:

- Calculation of reduced bases
- Hyperreduction Techniques