A Regression-based Control Approach for Limited Slip Differential

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Abstract—This work represents an innovative approach for the control of a Limited Slip Differential (LSD). The limited slip differential transmits the power of the motor to the ground allowing the wheels to spin at different speed. Its task is dividing the transmitted torque between the driven wheels in different driving situations and scenarios. We start with covering the current trends and an introduction to the functionality and impact of a limited slip differential in driving dynamics. Since the current control system for such a differential is very complex and has no ability to adapt itself over time to the changes, this work proposes a new control approach, based on machine learning techniques. Due to the features of the data sets, gathered from real driving situations and are used for the training of the model, the supervised regression-based machine learning methods are selected for evaluation. To be able to choose the right regression method, the data for training the model is closely analyzed and an appropriate model that has the ability of improving the accuracy of a limited slip differential control while ensuring a safe, pleasant and high performance drive is chosen.

I. INTRODUCTION

A. Driving with a Limited Slip Differential (LSD)

An open differential enables the wheels to spin at different speed but transmits the same torque amount to both driven wheels. This implies a major drawback in case of driving on a ground where the wheels have different friction coefficients. In similar situations like the one at fig. 1, one of the wheels is being driven on a dry ground while the other one is on a different surface like ice, transmitting the same amount of torque to both wheels will cause the wheel on a low friction surface to slip and lower the speed and performance of the other wheel too and with it of the entire vehicle.

To overcome this problem limited slip differentials are introduced. A limited slip differential is a type of differential that grants the relative rotation of the output shafts while providing an asymmetrical distribution of the torque. Such limited slip differentials are used instead of standard ones, if a higher vehicle dynamic standards need to be achieved. The functionality of a clutch, based on limited slip differential can be defined in two situations:

- **Open clutch**: the wheels are not limited, they spin at different speeds and the same amount of torque is transmitted through both wheels to the ground.

- **Closed clutch**: the wheels are limited, they spin at the same speed but more torque is transmitted to the ground through the wheel with more grip.

The performance of a vehicle is defined by its ability to be driven at high speeds under safe conditions. The ideal torque transmitted to the ground is the maximal torque, that is not causing the wheels to slip. As soon as the wheels are slipping the driver has less control over the vehicle. To avoid a complete control loss over the vehicle the wheels are not allowed to slip, they have to always be able to transmit a minimal amount of torque to the ground. When driving a vehicle without a limited speed differential, in case of one wheel slipping, the stability control lets the brake system [6] intervene, thus causing a loss in speed. In the same situation a vehicle with a limited slip differential and an accurate control system, will be at their optimum and the cornering will happen at the maximal possible speed, without giving the driver the impression of losing control over the vehicle. As a result the driver will feel safe and in control but also will be able to enjoy a dynamic sporty drive. Limited slip differential plays a significant role in improving performance, driving pleasure and driving safety. In the situations like the one described at 1, using an open differential, the wheel on ice will not be able to transmit any torque to the ground and will continue slipping. By limiting the independence of the wheels and allowing different amounts of torque, the other wheel will transmit a higher amount of torque to the ground enabling the vehicle to still move at higher speed and provide a better performance. Current high
performance cars adopt electronically controlled limited slip differentials. A typical limited slip differential is depicted at figure 2. The first part presents the motor controlling the clutch mechanical functionality. The main objective of the control approach proposed in this work is to define an angular position for this motor. The second part of the figure depicts the shafts that by rotating press together the lamellas (shown in part three of figure 2). To control the slip speed in such a differential, real time data from wheel speed sensors, drivers torque request and the actual friction conditions of a lamella clutch (fig. 2) must be acquired beforehand. Orchestration of these factors which shape the structure of a limited slip differential brings a high mechanical and control complexity besides the high cost of the production materials and software components. To be able to provide an accurate control of such differential, a complex physical definition of the functionality has to be defined.

Furthermore, non-measurable real time data, like lamella temperature \( T_{clutch} \) or transmitted torque \( M_{Lock} \) in the clutch, are estimated by software models. Thus, simple controller algorithms, based on semi-physical models can control the clutch slip speed. Semi-physical models are a combination between a knowledge-based model and a black box model. The accuracy of these algorithms is strongly dependent on the calibration data defining the controller coefficients. A controller coefficient is a factor indicating the change ratio of a controlled variable [1]. We believe that modern control strategies like machine learning could provide a more qualitative control of such a limited slip differential. Therefore, we need to consider the benefits that a machine learning-based approach could bring to control accuracy and improve the vehicle performance.

\[ \Delta POS \]

**Figure 2:** A Typical Limited Slip Differential

**Figure 3:** Control workflow of the angular position

B. Control of a Limited Slip Differential

One possible way of controlling the workflow of such a limited slip differential is by using a motor, as an actuator. A motor is defined by any class of rotatory electrical machines that converts direct current electrical energy into mechanical energy.

The ECU (Electronic Control Unit) controlling the limited slip differential receives a set of signals:

- Required torque
- Oil temperature
- The speed of the wheels
- Clutch characteristics (ex.: age, lamella count)

With the help of these signals, the responsible ECU computes a required position to control the motor (fig. 3). The required position is sent through an PI-control (fig. 3, block 2), that transforms the received data into a Pulse Wide Modulation (PWM) Signal and transmits it to the motor. The motor reaches the defined position and sends the actual position measured by the position sensors (fig. 3, block 3). The actual position is transformed by an encoder (fig. 3, block 4) and subtracted from the required position (fig. 3, block 1) resulting in a position difference \( \Delta POS \) defining the control loop.

Through this rotation of the motor, a force will be transmitted to the lamella clutch and forcing the lamellas to be pressed together or to be opened. The motor reaches a defined position generating a friction between pairs of traction surfaces and generating an axial force. The angular position of the motor is physically correlated to the force measured in a ball ramp system (fig. 4). A ball ramp system is composed of a frictional surfaces set, which are rolling and forcing the elements of the system in a frictional engagement [4].

The result of this action is a locking torque between the wheels. The actual value of the torque is computed from the angular position of the motor and compared to the torque request. As soon as the lamellas of the clutch are pressed together and spin at different speed, they generate high energy input due to the high friction rising the temperature in the clutch. The existing thermic flow is equivalent to the friction power. A part of the heat will be absorbed and transferred to the oil in the clutch. For a safe operation of the clutch the temperature must stay under a threshold of 250°C.
C. Standard Automatic Control

The current controlling algorithm is defined as follows:

1. **DSC Torque:**
   - Oil Temp
   - Compute Drag Torque

2. **Temperature Model:**
   - Compute Position
   - Look-up Table
   - Friction coeff.

3. **Position/PWM:**
   - Actual Torque
   - Friction torque computation
   - DC Motor

4. **Actual Position:**
   - Actual Torque
   - Actual Position

**Figure 5:** Control System Workflow.

In the current controlling algorithms, the DSC requires a torque, and from this torque the drag torque influenced by the oil temperature is subtracted. The drag torque defines the remaining torque from the environment unrelated to the clutch states, even if the clutch is open there still exists a small amount of drag torque (part 1). Because of the absence of a temperature sensor in the clutch, the temperature will be estimated by a thermal model. All the Look-up tables and factors are computed offline, as part of the calibration (part 2). The final position is defining the rotation of the motor, which will close or open the clutch (part 3). This position is measured by the hall sensors [3] in the motor and reversed computed to an actual torque (part 4).

This system is currently tested in vehicles and test bench. Because of the offline computation of the calibration and absence of adaptive behavior of the system, the outputs provided by this control approach are not fulfilling the expectations of the test driving experts.

II. Problem Statement

To provide high performance, driving dynamics, safety and driving pleasure, an adaptive and accurate controlling of a limited slip differential is required. According to the current state of the development, the limited slip differential is controlled by a semi-physical model, which is highly dependent on the calibration data defining the controller coefficients. As mentioned before the controller coefficients are scalars for observed values in the clutch. To be able to define this coefficients a mapping process has to be executed for each clutch. Also during the lifetime of a clutch the surfaces of the mechanical components are affected by friction resulting in a deviation from the initial controller coefficients. The process of generating the maps of coefficients is complex, time consuming and expensive. This high dependence on the controller coefficients results in a control of the differential that is not adaptive which implies a low performance of the vehicle. Such a complex semi-physical model implies complex implementations with thousand lines of code, a high computation power and memory space for the calibration parameters of the control system. Moreover, all these factors lead to a possible increase in erroneous torque transmission.

As it can be observed in the figure 6 the torque transmitted by the current system varies from the torque request, conveys a not perfect accuracy of the control system. In the figure it can be observed how the transmitted torque (orange) deviates from the torque request (blue) mostly at low values and has an unstable behavior, this declares a major problem of the system since average drivers are usually driving in this value ranges. Figure 6 compares the torque request to the transmitted torque of all the measurements used for this research. Each signal from the measurements was filtered and the deviation, influenced by noise and mechanics characteristics of the system are not considered in this visualization. The approximately 70% of erroneous torque deviation in the current system is shown at figure 6.

III. Proposed Approach

After carefully analyzing the data, existing features and the environment where the training of the model will take place, a machine learning-based algorithm must be selected which fits properly into the functionality of
a LSD. A set of important notes, listed in following, will help in narrowing down the selection process:

- What is the quality, size and nature of features of the data?
- How much computational power and time is available?
- What is the final use of the model and how high should its accuracy level be?

A brief explanation of three main domains of machine learning is given in the following:

Supervised Learning - based algorithms make predictions based on labeled data. This algorithm uses well described inputs scenarios consisting of training data and labeled output variables. The goal is to find a mathematical model that approximates the input scenarios accurately and is able, under new inputs, to provide a desired prediction of the output. Supervised learning problems are divided into two groups:

- **Classification** which comes into use when the data used for the training of the model is discrete and divided into categories. For example a typical classification would be assigning colors to images, one imagine is red the other one is green and,
- **Regression** which is used when the data for the training contains continuous values. For example predicting house prices considering their properties: space, location, construction year.

Unsupervised Learning - based algorithms are only provided with input data and do not have any correct output values. These algorithms try to autonomously recognize structures in the data and predict outputs. One well-known unsupervised learning problem is clustering. Clustering algorithms try to divide data in different groups with similar characteristics. Each group contains in the end similar data defining a pattern. A clustering example is grouping customers by their purchasing behavior.

Reinforcement Learning - based algorithms define an agent who must decide by itself what is the next best action based on its current state. The main goal of reinforcement algorithms is to find automatically the ideal set of actions describing the behavior of an agent and to maximize its performance. During the training process there are no defined correct outcomes. The action policy is described by learning steps, which will be updated according to the result of a performed action. Taking into consideration the problem addressed by in this work and the characteristics of the data defining the training of the machine learning model a supervised learning algorithm is required. The continues nature of the data values also suggests the regression-based algorithms as a suitable candidate. The physical model described in the previous section, defining the current control of such a differential, will be partly replaced by a regression-based component, computing the angular position of a motor. As in the current control system, the computation of the clutch temperature will remain unchanged. Furthermore, the temperature in the clutch will be used as an input for the desired algorithm.

A regression-based approach is trying to find the best mathematical model approximating best the desired outcome. Algorithms of such approaches map a set of independent inputs to a dependent output.

The independent features defining the computation of the angular position \((y)\) for the motor are listed at table I.

|\(x_1\) | Requested torque |
|\(x_2\) | Speed difference in the clutch (also known as slip speed) |
|\(x_3\) | Clutch temperature |
|\(x_4\) | Oil temperature |
|\(y\)  | Predicted position for the motor |

Table I: Required features for the computation of angular position

The desired algorithm must generate a reasonable prediction of the angular position \((y)\) for the motor controlling the clutch. Executing this action will build the desired locking torque between the wheels. The transmitted torque to the wheels must not vary more than \(\pm 10\% \pm 15Nm\) from the requested torque.

IV. Evaluation

The transmitted torque defined by the angular position, predicted by the proposed approach, should not vary more than \(\pm 10\% \pm 15Nm\) from the torque request. For the evaluation of each regression method, root mean squared error (RMSE) is considered as one of the factors. As high as the RMSE is as inaccurate is the prediction, implying a high deviation from the observed samples.

The angular position predicted by the implemented algorithms is sent to the motor which has its own control unit. This control unit sets the position that will then be measured and reversed computed to an actual torque. As described in the previous chapters the angular position is dependent on the torque request, clutch temperature,
A. Polynomial Regression with Gradient Descent Evaluation

This approach generates a third degree polynomial and fits its coefficients by applying the Gradient Descent method. The features of the polynomial will describe the input variables torque request, oil temperature, clutch temperature, slip speed and will predict an angular position for the motor controlling the clutch. The polynomial defined by the proposed model manages to approximate the ideal behavior by an \( \text{RMSE} \) of 29.68 on the training data and 36.0433 on the test data. The time needed for this approach to train the model is 49.169 s and for predicting a new angular position only \( 2 \times 10^{-5} \) ms are needed. In the next two figures the deviation of the predicted angular position (red) from the observed angular position (blue) is visualized. Although this method shows substantial improvement of 60% compared to the previous version of Polynomial Regression with GD it still delivers a worst approximation of the ideal behavior than K-Nearest Neighbors Regression by 77%. Compared to the K-Nearest Neighbors Regression this method provides an improvement in the training and prediction time, but since this are not crucial factors in the problem proposed by this work the best choice remains K-Nearest Neighbors Regression.
C. Lasso Regression Evaluation

This method is choosing parameters by shrinking their coefficients until reaching zero. The parameters which are not important for the prediction will be removed by optimizing their coefficients until reaching zero. Despite a fast training of 0.0291s and prediction of approximate 0.0ms this method has even worse results than Polynomial Regression when predicting an angular position for the motor. The root mean squared error is 35% higher on the train data and 25% on test data than the RMSE achieved by predicting the angular position with Polynomial Regression with GD. As depicted in the following figures (11 and 12) the method delivers a prediction with a high deviation over the entire range of values. Because of the reasons described above Lasso Regression is not the most suitable approach.

D. Ridge Regression Evaluation

Ridge Regression finds a model that predicts the desired outcome by using the ordinary least squared error and a regularization to avoid over-fitting the model. This approach has very similar outputs to the Lasso Regression. Compared to the Stochastic Gradient Descent Regression it shows an improvement in the deviation from the observed values of the angular position but still has a far more worse outcome than the K-Nearest Neighbors Regression. As visualized in the figures below (13 and 14) the prediction deviates from the observed values in both low and high torque range of the measurements. Therefore this is either not the most appropriate approach for the control of a limited slip differential.

E. Stochastic Gradient Descent Regression Evaluation

The Stochstic Gradient Descent Regression is a very similar approach to the Gradient Descent method used to compute the coefficients of the polynomial regression, the major difference is that the coefficients here are not computed by summation but only dependent on the actual train sample. This results in a less time consuming training, achieving a performance improvement of 95%. Despite this major improvement, the prediction accuracy is even lower than the previous ones. Comparing this method to the Ridge Regression and Lasso Regression, which have the lowest prediction accuracy, the present method has a 3.4% higher RMSE. Furthermore in the figures 15 and 16 a even higher deviation than in the previous predictions can be noticed. In conclusion the Stochastic Gradient Descent Regression is the least appropriate approach.
F. Decision Trees Regression Evaluation

Decision Trees Regression divides the training data set recursively into smaller subsets until reaching a leaf defining a continuous desired output. The major disadvantage is the complex structure of the tree. The complexity of this method implies a high computational power that would not be available in each running environment. Also, the complexity of the algorithm makes the implementation less understandable and traceable. Not taking into account the complexity, this method delivers a prediction with a RMSE of 4.653 on train data and 12.946 on the test data, placing the present method as second best approach after K-Nearest Neighbors Regression. Furthermore, the figures 17 and 18 show a good coverage of the observed samples.

G. K-Nearest Neighbors Regression Evaluation

K-Nearest Neighbors Regression predicts the desired output by dividing the training data in subsets of values with similar characteristics. When a new output has to be predicted the inputs are mapped to a set with specific characteristics by computing the distance to the nearest subset. Each subset has then a representative output. This method has the most accurate prediction with a RMSE of 2.733 on the train data and 11.783 on the test data. The training time and prediction time are longer compared to Decision Trees Regression, but comply the time requirements. As the following two figures (19 and 20) show this method approximates very well the desired angular position, it has a good coverage of the high and low values and no significant deviation can be observed. According to the results observed, K-Nearest Neighbors Regression is the most appropriate method for the control of a limited slip differential and the best approach to address the problem.

H. Torque Estimation

For evaluating the results, the reverse computation from the angular position to a torque will be needed. This reverse computation can be also estimated by a regression approach. The same approach as described before for the prediction of the angular position is used to estimate the actual torque transmitted to the wheels, since the same physical influences have to be modeled. The same features as before are used, additionally to this features, the actual measured torque at the wheels defines the observed output 21.

For the reverse computation of the transmitted torque the independent input variables are: Torque Request, Clutch Temperature, Oil Temperature, Slip Speed and the previous predicted Position. The current predicted output
is the measured torque at the wheels. This approach was chosen because both modules, position prediction and torque estimation, have to remain separate to comply with the simplicity requirements of the MISTRA test [5]. The position is needed for the control of the clutch, while an estimation of the transmitted torque is needed to enable the estimation of the temperature in the clutch. Furthermore, this estimated torque can be used to evaluate the position prediction, since the real transmitted torque can be only measured in a vehicle under special configuration and hardware. The simulation of the algorithm prediction will be defined as follows:

![Figure 17: Decision Tree Regression: prediction on training data](image1)

![Figure 18: Decision Tree Regression: prediction on test data](image2)

In order to evaluate the angular position prediction, the actual torque and the required torque are compared. The result of the comparison assess the torque accuracy and dynamics. Torque accuracy is provided when the actual torque is not varying more than $\pm 10\% \pm 15\text{Nm}$ from the requested torque. Furthermore, the torque dynamics describes the reaction time of the clutch meaning: how long does it takes for the actual torque to reach at least 90% of the value corresponding to the requested torque. As it can be observed in the figure below (fig. 23) for the torque accuracy, the actual torque (green) and the requested torque (blue) are compared to the allowed maximal variation values (red) fitting the requirements. Figure 24 illustrates the torque dynamics. The blue

![Figure 21: Actual transmitted torque estimation](image3)
Figure 23: Evaluation of the torque accuracy

Figure 24: Evaluation of the torque dynamic

line defines the torque request and the yellow one the measured torque. Between the two cursors (turquoise and yellow), the time needed for the achievement of 90\% of the torque request, is measured. The torque dynamics is influenced by the mechanics of the differential. This influences can only be seen in real vehicle measurements or test bench simulations where the real hardware is available. For the evaluation of this work only a computer simulation has been done, which will not take into consideration the time loss caused by the mechanics of the differential. Therefore the measurements used for the evaluation are filtered and the values influenced by the mechanics response time are removed.

Comparing the transmitted torque computed by the actual system to the achieved prediction of the torque the following results can be observed:

- While predicting the angular position an deviation from the actual angular position of 78.8276 [RMSE] is achieved
- Using this predicted angular position results in an estimated transmitted torque varying from the ideal behavior by a RMSE of 44.76
- Comparing the torque request to the actual torque transmitted by the current system results in a RMSE of 282.5354. This demonstrates that the trained model brings an improvement of approximately 80\%
- On average the prediction of the transmitted torque lies outside the requirement interval by 20.0778 Nm while the transmitted torque by the current system lies in average 165.0629 Nm outside the requirement interval. Taking into consideration this erroneous samples an improvement of 49.5\% can be observed. The average of the samples passing the accuracy requirements interval margins is computed as follows:

\[
\text{avgOutsideInterval} = \frac{\sum (\text{Torque}_{\text{transmitted}} - \text{Torque}_{\text{upperMargin}}) + \sum (\text{Torque}_{\text{lowerMargin}} - \text{Torque}_{\text{transmitted}})}{\text{Count}} \quad (1)
\]

The following figures visualize the results of the prediction on a random measurement for a more detailed analysis. As it can be observed in the figure 25 the prediction of the angular position has a high accuracy. Small deviations from the observed values can be observed generating a RMSE of 4.47.

To be able to evaluate the torque corresponding to this position the reverse computation from position to torque is performed. The estimated torque based on the newly predicted position approximates the torque request with an higher accuracy than the current control system estimation. As visualized in figure 26 can be observed how the approximation does not passes the requirements interval margins defined by the green lines. The torque transmitted by the current system only manages to keep inside the correct interval while transmitting high values (fig. 27). In average the transmitted torque by the current system passes the interval margins by 106.8805 Nm gener-
In this work, we have defined a new innovative control approach for a limited slip differential. This differential controls the locking torque between the wheels of a vehicle. It thus ensures that an optimum torque is transferred to the ground, without causing the wheels to slip. An accurate differential control implies the driver is able to corner at high speed without the impression of losing control over the vehicle. Starting with an inaccurate, non-adaptable control system, the need for a better control algorithm was proved. The actual system is defined by a complex semi-physical model with a high dependence on control coefficients. This control coefficients are generated by a difficult, time consuming and expansive process. To overcome the problems of the current system this research proposed the replacement of the semi-physical model by a machine learning-based control approach. For the purpose of this work we did use real vehicle measurements, representing the behavior of a limited slip differential in all possible driving scenarios. The data defining the behavior of a clutch was carefully analyzed and filtered. Following that,
six different regression methods were implemented and the results were compared according to their accuracy.

Afterwards, K-Nearest Neighbor Regression has shown better results and therefore is selected as the promising candidate among the other methods. With the angular position predicted by the regression model, the actual locking torque has been estimated. Furthermore the new locking torque has been compared to the torque computed by the current system showing an accuracy improvement of more than 70%. This achievement defines a more performant vehicle, where the driver is always in full control of the car and can enjoy a sporty and also a comfortable drive.

**References**


