Classification of human-robot team interaction paradigms ⋆

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Abstract: Human-robot team interaction is challenging in terms of system complexity and control synthesis. Classifying different interaction paradigms between a human and a robot team eases the formal analysis. The challenge is to classify the paradigms appropriately, w.r.t. the setting and the task to be performed. In this paper three interaction paradigms are formally defined and analyzed using controllability. It is shown that a straightforward classification of interaction paradigms, based on the mapping properties of the input space to the tangent of the state space is possible. Specific examples in the human-robot team interaction for cooperative manipulation tasks validate the proposed classification methods.

1. INTRODUCTION

Human-robot interaction is a wide area of research that benefits from a number of scientific fields: psychology, control theory, haptics, etc.. As humans are able to perform tasks which require cognitive capabilities such as planning and adapting to uncertainties, robots are able to conduct tasks which require high precision. Therefore, it is reasonable to exploit their complementary abilities in a way that human operator(s) conduct high-level sub-tasks and robot(s) conduct low-level sub-tasks, working together in this way to achieve a final goal.

In order to perform multiple sub-tasks (constituting a complex task) simultaneously, interaction between multiple humans and/or multiple robots is necessary. As a result the interactive system is (highly) redundant. The specific set of sub-tasks can be *dynamically* assigned either to human(s), robot(s) or both. Depending on the distribution of sub-tasks among human(s) and robot(s) in specific stages of a general task, it is reasonable to define a number of interaction paradigms. In order to make a distinction between interaction paradigms, the suitable tools for classification are the system properties such as controllability and observability.

Different types of human-robot interaction are summerized in Yanco and Drury (2004). However, the literature on human-robot team interaction mainly analyzes physical human-robot interaction scenarios (Lawitzky et al. (2010)). Non-physical interaction, or more specifically, teleoperation of robot teams is considered in a classic setting of coupling the human to the master robot (Lee and Spong, 2005), (Lin et al., 2015). Different modes of interaction together with the concept of adaptable semi-autonomy are introduced in Laschi et al. (2001). Laschi et al. claim that the involvement of human(s) in decision process and autonomous behavior of robots for repetitive tasks is a desirable combination. The levels of autonomy ranging from teleoperation to full autonomy are proposed in Baker and Yanco (2004). However, the formal analysis of the levels of autonomy is not provided. Novel forms of interaction between humans and robots are possible thanks to the availability of wearable haptic devices (e.g. Chinello et al. (2015)). For example, it is possible to establish the direct teleoperation without using the master robot. The human moves freely, which allows for a transition from uncoupled (teleoperative) to coupled (physical) human-robot interaction. An example of a teleoperation experimental scenario with the human, equipped with wearable thimble devices (Chinello et al., 2015) is depicted in fig. 1. Additionally, brain-computer interface (BCI) also enables definition of novel ways in which humans and robots interact (Tonin et al., 2010).

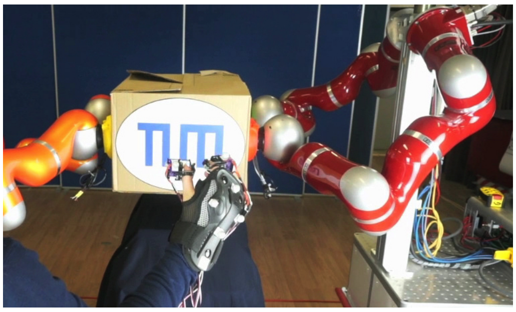

Fig. 1. Bilateral teleoperation with wearable haptic devices

In this paper we define and formally classify interaction paradigms between a single human and multiple robots that form a team and use the controllability property to distinguish between the paradigms. The defined interaction paradigms differ depending on the distribution of sub-tasks and on the level of autonomy. Therefore, the provided analysis enables the theoretical consideration of extreme human-robot interaction modes; teleoperation and physical interaction. The classification of interaction paradigms is motivated by the cooperative manipulation task in which a team of multiple robots manipulates a single object over extended workspace (Erhart and Hirche, 2016). The contribution of this paper is the proposition and the analysis of three interaction paradigms: *direct, complementary and overlapping.* In Section 2 the prob-

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lem is formulated and necessary theoretical concepts are introduced. In Section 3 three interaction paradigms are defined and their properties are derived. The verification of the properties is conducted on the analytical examples of cooperative manipulation systems.

2. PROBLEM FORMULATION

Let us consider a *multi-input multi-output (MIMO)* non-linear, affine control system of the form:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + G(\boldsymbol{x})\boldsymbol{u} \tag{1}$$

where $\boldsymbol{x} \in \mathcal{M}$ is the state vector defined on an n-dimensional, smooth manifold $\mathcal{M}$, $\boldsymbol{u} \in \mathcal{K}$ is the input vector defined on an m-dimensional, smooth manifold $\mathcal{K}$ and $G = [\boldsymbol{g}_1, ..., \boldsymbol{g}_m]$. The real-valued mappings $\boldsymbol{f}, \boldsymbol{g}_1, ..., \boldsymbol{g}_m$ are smooth vector fields defined on the manifold $\mathcal{M}$.
Let us define the input vector $\boldsymbol{u}$ as a stacked vector of the input commands provided by *two control inputs*; the human and the autonomous controller:

$$\boldsymbol{u} = [\underbrace{u_1^h, ..., u_k^h}_{\boldsymbol{u}^h}, \underbrace{u_{k+1}^a, ..., u_m^a}_{\boldsymbol{u}^a}]^T \tag{2}$$

where superscripts h and a indicate the human and the autonomous control inputs, respectively. The input manifold $\mathcal{K}$ can be divided into two submanifolds: $\mathcal{K} = \mathcal{K}^h \cup \mathcal{K}^a$. Let us write G as: $G = [G^h \ G^a]$ where $G^h = [\boldsymbol{g}_1, ..., \boldsymbol{g}_k]$ and $G^a = [\boldsymbol{g}_{k+1}, ..., \boldsymbol{g}_m]$. The general feedback control input is assumed to be state dependent and is represented as:

$$\boldsymbol{u}^{h,a} = \boldsymbol{\alpha}^{h,a}(\boldsymbol{x}) + \boldsymbol{\beta}^{h,a}(\boldsymbol{x})\boldsymbol{v}^{h,a} \tag{3}$$

where $\boldsymbol{\alpha}^{h,a}(\boldsymbol{x})$ and $\boldsymbol{\beta}^{h,a}(\boldsymbol{x})$ are defined on U_0 around point $\boldsymbol{x}$ and $\boldsymbol{\beta}^{h,a}(\boldsymbol{x})$ is nonsingular for all $\boldsymbol{x}$ while $\boldsymbol{v}^{h,a}$ is the new reference input. The feedback control law (3) modifies the system dynamics given by (1) into the form:

$$\dot{\boldsymbol{x}} = \tilde{\boldsymbol{f}}(\boldsymbol{x}) + \tilde{G}(\boldsymbol{x})\boldsymbol{v}_i \tag{4}$$

where: $\tilde{\boldsymbol{f}}(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) + G(\boldsymbol{x})\boldsymbol{\alpha}(\boldsymbol{x})$ and $\tilde{G}(\boldsymbol{x}) = \boldsymbol{G}(\boldsymbol{x})\boldsymbol{\beta}(\boldsymbol{x})$.
We propose three interaction paradigms between a human operator and a robot team: direct, complementary and overlapping (further categorized into cooperative and competitive). An overview of the interaction paradigms is given in fig. 2. A general architecture of the analyzed system is depicted in fig. 3. The *selection mechanism* block is in charge of selecting the appropriate interaction paradigm. The system exemplarily analyzed in the paper is a cooperative manipulation system, with the dynamics of a single manipulator in the task space:

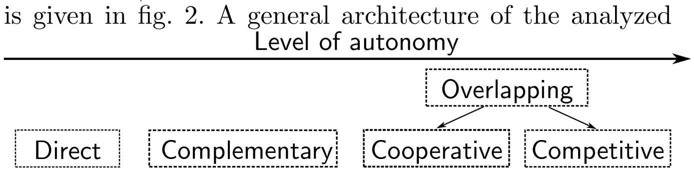

Fig. 2. Overview of the interaction paradigms.

$$M_i(\boldsymbol{p}_i)\ddot{\boldsymbol{p}}_i + \boldsymbol{c}_i(\boldsymbol{p}_i,\ \dot{\boldsymbol{p}}_i) + \boldsymbol{h}_i^g(\boldsymbol{p}_i) = \boldsymbol{h}_i \quad i = 1, 2 \tag{5}$$

with $\boldsymbol{p}_i \in SE(3)$ being the pose of the i-th end-effector in the task space, $M_i(\boldsymbol{p}_i) \in R^{6\times 6}$ its inertial matrix, $\boldsymbol{c}_i(\boldsymbol{p}_i,\ \dot{\boldsymbol{p}}_i) \in R^6$ its Coriolis terms, $\boldsymbol{h}_i^g(\boldsymbol{p}_i) \in R^6$ its gravitational forces and $\boldsymbol{h}_i \in R^6$ its wrench input. Transformation of (5) into the form (1) for two manipualtors, gives:

$$\underbrace{\begin{bmatrix} \dot{\boldsymbol{p}} \\ \ddot{\boldsymbol{p}} \end{bmatrix}}_{\dot{\boldsymbol{x}}} = \underbrace{\begin{bmatrix} \dot{\boldsymbol{p}} \\ M(\boldsymbol{p})^{-1}(-\boldsymbol{c}(\boldsymbol{p}, \dot{\boldsymbol{p}}) - \boldsymbol{h}^g(\boldsymbol{p})) \end{bmatrix}}_{\boldsymbol{f}(\boldsymbol{x})} + \underbrace{\begin{bmatrix} \boldsymbol{0} \\ M(\boldsymbol{p})^{-1} \end{bmatrix}}_{\boldsymbol{g}(\boldsymbol{x})} \boldsymbol{u} \tag{6}$$

where the states are $\boldsymbol{x} = [\boldsymbol{p}_1^T \ \boldsymbol{p}_2^T \ \dot{\boldsymbol{p}}_1^T \ \dot{\boldsymbol{p}}_2^T]^T$, $M(\boldsymbol{x}) = \text{bldiag}(M_1(\boldsymbol{p}_1)\ M_2(\boldsymbol{p}_2))$, $\boldsymbol{c}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = [\boldsymbol{c}_1^T(\boldsymbol{p}_1, \dot{\boldsymbol{p}}_1)\ \boldsymbol{c}_2^T(\boldsymbol{p}_2, \dot{\boldsymbol{p}}_2)]^T$, $\boldsymbol{h}^g(\boldsymbol{x}) = [\boldsymbol{h}_1^{gT}(\boldsymbol{p}_1)\ \boldsymbol{h}_2^{gT}(\boldsymbol{p}_2)]^T$ and $\boldsymbol{u} = [\boldsymbol{h}_1^T\ \boldsymbol{h}_2^T]^T$.

3. CLASSIFICATION OF INTERACTION PARADIGMS

3.1 Direct interaction paradigm

Definition 1. The interaction paradigm is called direct if the complete vector space of the system (1), $T\mathcal{M}$ is accessible by the control inputs $\boldsymbol{u}^h$.

According to the definition 1, the input vector (2) has the following structure: $\boldsymbol{u} = [u_1^h, ..., u_k^h, \boldsymbol{0}_{m-k}]^T$. Due to the structure of the input vector $\boldsymbol{u}$ it is sufficient to focus only on the properties of the vector fields denoted as G^h. The input vector $\boldsymbol{u}^h$ is mapped by the mapping matrix $G^h(\boldsymbol{x})$ onto the tangent space $T_{\boldsymbol{x}}\mathcal{M}$.The distribution spanned by the vector fields G^h is:

$$\Delta^h = \text{span}\{\boldsymbol{g}_1^h, ..., \boldsymbol{g}_k^h\}, \quad \forall \boldsymbol{x} \in \mathcal{M} \tag{7}$$

The distribution (7) is non-singular, i.e. the following equality is fulfilled:

$$\dim\{\Delta^h\} = \text{const.} = m \tag{8}$$

As a consequence of non-singularity, complete input vector $\boldsymbol{u}^h$ is mapped onto the tangent space $T\mathcal{M}$ for all $\boldsymbol{x}$.
The distribution (7) is *involutive*. This means that the Lie bracket of any $\boldsymbol{g}_i^h$ and $\boldsymbol{g}_j^h$ belongs to the distribution Δ^h:

$$\boldsymbol{g}_i^h \in \Delta^h,\ \boldsymbol{g}_j^h \in \Delta^h \ \Rightarrow [\boldsymbol{g}_i^h,\ \boldsymbol{g}_j^h] \in \Delta^h \tag{9}$$

where: $[\boldsymbol{g}_i^h,\ \boldsymbol{g}_j^h] = \frac{\partial \boldsymbol{g}_j^h}{\partial \boldsymbol{x}}\boldsymbol{g}_i^h - \frac{\partial \boldsymbol{g}_i^h}{\partial \boldsymbol{x}}\boldsymbol{g}_j^h$ is the Lie bracket. Controllability of non-linear systems can be analyzed locally using the *controllability distribution*.

$$\mathcal{R}(\boldsymbol{x}) = [\boldsymbol{G}^h(\boldsymbol{x}),\ ad_{\boldsymbol{f}}\boldsymbol{G}^h(\boldsymbol{x}), ..., ad_{\boldsymbol{f}}^{n-1}\boldsymbol{G}^h(\boldsymbol{x})] \tag{10}$$

where $ad_{\boldsymbol{f}}^0\boldsymbol{G}^h = \boldsymbol{G}^h$ and $ad_{\boldsymbol{f}}^k\boldsymbol{G}^h = [\boldsymbol{f},\ ad_{\boldsymbol{f}}^{k-1}\boldsymbol{G}^h]$.
Proposition 1. If the distribution Δ^h is non-singular and involutive and if:

$$\dim\{\mathcal{R}\} = n \tag{11}$$

where n is the number of the states, then the system 1 is controllable from the human input. For the proof of the proposition (1) we refer to the (Isidori (1995)).
Example 1. A classical example of the direct interaction paradigm in robotics is bilateral teleoperation. Let us consider a robotic system of 2 manipulators, given by (6). Furthermore, let us consider this robotic team is teleoperated by the human operator. The motion of the two human fingers is the desired motion of the manipulators in the task space, as depicted in the fig. 4. Let us consider the translational motions. The control inputs are position and translational velocity of the human fingers, $\boldsymbol{p}_1^h\ \boldsymbol{p}_2^h \in R^3$ and $\dot{\boldsymbol{p}}_1^h,\ \dot{\boldsymbol{p}}_2^h \in R^3$, respectively. Jointly, the inputs are represented as a stacked vector:

$$\boldsymbol{v}^h = [\underbrace{\boldsymbol{p}_1^h,\ \boldsymbol{p}_2^h}_{\boldsymbol{p}^h},\ \underbrace{\dot{\boldsymbol{p}}_1^h,\ \dot{\boldsymbol{p}}_2^h}_{\dot{\boldsymbol{p}}^h}]^T \ \in \ R^{12}$$

Using the impedance feedback control strategies for each subsystem, the human and autonomous input commands are mapped to the input wrenches of the system (6):

$$\boldsymbol{u}^h = \underbrace{[K\ \ D]\,\boldsymbol{x}}_{\boldsymbol{\alpha}(\boldsymbol{x})} + \underbrace{[-K\ \ -D]}_{\boldsymbol{\beta}}\,\boldsymbol{v}^h \tag{12}$$

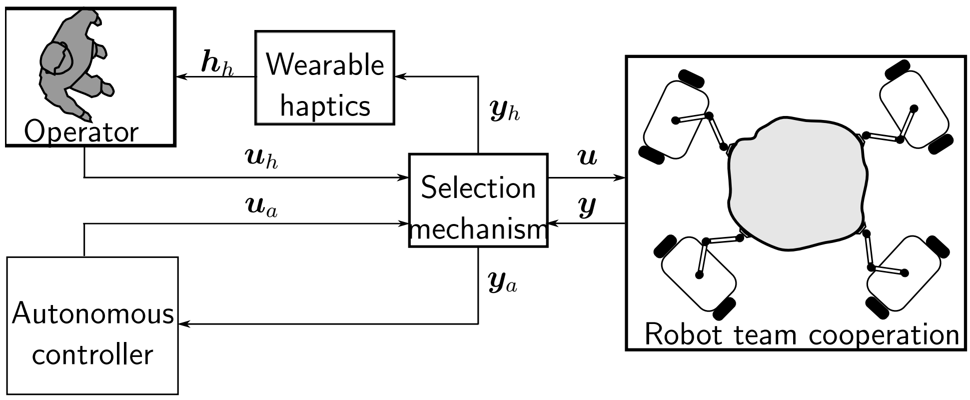

Fig. 3. General architecture for the human-robot team interaction in a cooperative manipulation task.

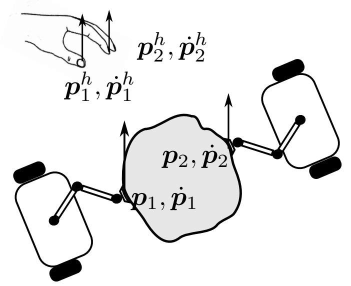

Fig. 4. Cooperative manipulation example of the direct interaction paradigm

where $D = \text{bldiag}(D_1 \;\; D_2)$ is a block diagonal damping matrix with $D_1,\ D_2 \in R^{3\times3}$ being the damping matrices for each manipulator and $K = \text{bldiag}(K_1 \;\; K_2)$ is a block diagonal stiffness matrix with $K_1,\ K_2 \in R^{3\times3}$ being the stiffness matrices for each manipulator. It is possible to show that the dimension of the controllability distribution (10) is $\dim\{\mathcal{R}\} = \dim\{\boldsymbol{x}\} = 12$ if the matrix $M(\boldsymbol{x})$ is positive-definite and for any choice of positive-definite damping and stiffness matrices.

3.2 Complementary interaction paradigm

Definition 2. The interaction paradigm is called complementary if it is possible to define a d-dimensional distribution, $T\mathcal{M}^h \subset T\mathcal{M}$ and an $(n-d)$-dimensional distribution $T\mathcal{M}^a \subset T\mathcal{M}$ such that: $T\mathcal{M}^h \cap T\mathcal{M}^a = \emptyset$ and $T\mathcal{M}^h \cup T\mathcal{M}^a = T\mathcal{M}$ and if the distribution $T\mathcal{M}^h$ is accessible to the control inputs $\boldsymbol{u}^h$ while the distribution $T\mathcal{M}^a$ is accessible to the control inputs $\boldsymbol{u}^a$.

According to the definition (2), a subspace reachable to the human control input is unreachable to the autonomous control input and vice versa. This means the influences from the human control input and the autonomous control input are mutually *complementary.* The mappings of the complementary interaction paradigm are depicted in the fig. 5. Let us write the mapping matrix G as:

$$G = \begin{bmatrix} G^h & G^a \end{bmatrix} = \begin{bmatrix} G^{1h} & G^{1a} \\ G^{2h} & G^{2a} \end{bmatrix} \tag{13}$$

where $G^{1h} = [\boldsymbol{g}_1^1, ..., \boldsymbol{g}_k^1]$ and $G^{1a} = [\boldsymbol{g}_{k+1}^1, ..., \boldsymbol{g}_m^1]$ are sets of d-dimensional mappings and $G^{2h} = [\boldsymbol{g}_1^2, ..., \boldsymbol{g}_k^2]$ and $G^{2a} = [\boldsymbol{g}_{k+1}^2, ..., \boldsymbol{g}_m^2]$ are sets of $(n-d)$-dimensional mappings. Let us assume, without loss of generality, the states are ordered so that the first d states are controllable by the human and the remaining $(n-d)$ states are controllable by the autonomous controller. In order to ensure the control inputs, $\boldsymbol{u}^h$ and $\boldsymbol{u}^a$ are appropriately mapped onto the distributions $T\mathcal{M}^h$ and $T\mathcal{M}^a$, respectively, it is necessary to ensure the off-diagonal terms of (13), G_i^{1a} and G_i^{2h}, vanish. The distribution spanned by G^h is given by (18) and the distribution spanned by G^a is:

$$\Delta^a = \text{span}\{\boldsymbol{g}_{k+1}^a, ..., \boldsymbol{g}_m^a\}, \quad \forall \boldsymbol{x} \in \mathcal{M} \tag{14}$$

Proposition 2. Let $T\mathcal{M}^h$ and $T\mathcal{M}^a$ be nonsingular, involutive distributions of dimensions d and $n-d$, respectively. Furthermore, assume $T\mathcal{M}^h$ is invariant under the vector fields $\boldsymbol{f}, \boldsymbol{g}_1, ..., \boldsymbol{g}_k$ and $T\mathcal{M}^a$ is invariant under the vector fields $\boldsymbol{f}, \boldsymbol{g}_{k+1}, ..., \boldsymbol{g}_m$. Moreover, suppose the distribution $\text{span}\{\boldsymbol{g}_1, ..., \boldsymbol{g}_k\} \subset T\mathcal{M}^h$ and the distributions $\text{span}\{\boldsymbol{g}_{k+1}, ..., \boldsymbol{g}_m\} \subset T\mathcal{M}^a$. Than, for each $\boldsymbol{x}_0$ it is possible to find a neighborhood U_0 of $\boldsymbol{x}_0$ and transformations $\boldsymbol{z}_1 = \boldsymbol{\phi}_1(\boldsymbol{x})$ and $\boldsymbol{z}_2 = \boldsymbol{\phi}_2(\boldsymbol{x})$ defined on U_0 such that in the new coordinates, the system is:

$$\begin{aligned} \dot{\boldsymbol{\xi}}^h &= \boldsymbol{f}^h(\boldsymbol{\xi}^h, \boldsymbol{\xi}^a) + G^h(\boldsymbol{\xi}^h, \boldsymbol{\xi}^a)\boldsymbol{u}_i^h \\ \dot{\boldsymbol{\xi}}^a &= \boldsymbol{f}^a(\boldsymbol{\xi}^h, \boldsymbol{\xi}^a) + G^a(\boldsymbol{\xi}^h, \boldsymbol{\xi}^a)\boldsymbol{u}_i^a \end{aligned} \tag{15}$$

where $\boldsymbol{\xi}^h = \{\boldsymbol{z}_1, ..., \boldsymbol{z}_d\}$ and $\boldsymbol{\xi}^a = \{\boldsymbol{z}_{d+1}, ..., \boldsymbol{z}_n\}$.

Proof. It is possible to construct a candidate transformation $\boldsymbol{z}_1 = \boldsymbol{\phi}_1(\boldsymbol{x})$ around $\boldsymbol{x}_0$ such that the last $n-d$ elements of its Jacobian span a distribution $(T\mathcal{M}^h)^T$ (Isidori, 1995). Since vector fields $\boldsymbol{f}, \boldsymbol{g}_1, ..., \boldsymbol{g}_k$ are in $T\mathcal{M}^h$ by assumption, by transforming them to the new coordinates:

$$\bar{\boldsymbol{f}}(\boldsymbol{z}) = [\frac{\partial \boldsymbol{\phi}_1}{\partial \boldsymbol{x}} \boldsymbol{f}(\boldsymbol{x})]_{\boldsymbol{x}=\boldsymbol{\phi}^{-1}(\boldsymbol{z})},$$

$$\bar{G}^h(\boldsymbol{z}) = [\frac{\partial \boldsymbol{\phi}_1}{\partial \boldsymbol{x}} G^h(\boldsymbol{x})]_{\boldsymbol{x}=\boldsymbol{\phi}^{-1}(\boldsymbol{z})}$$

the last $n-d$ elements of $\boldsymbol{f}$ and $\boldsymbol{g}^h$ vanish, yielding:

$$\bar{\boldsymbol{f}}(\boldsymbol{z}) = \text{col}(\bar{\boldsymbol{f}}_1(\boldsymbol{z}), ..., \bar{\boldsymbol{f}}_d(\boldsymbol{z}), \boldsymbol{0}_{n-d})$$

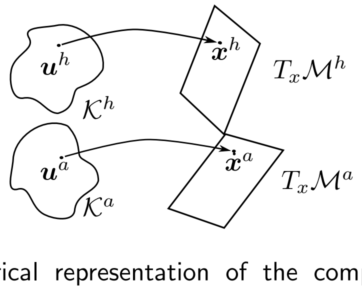

Fig. 5. Geometrical representation of the complementary interaction paradigm

$$\bar{G}^h(\boldsymbol{z}) = \mathrm{col}(\bar{\boldsymbol{g}}_1^1(\boldsymbol{z}), ..., \bar{\boldsymbol{g}}_d^1(\boldsymbol{z}), \boldsymbol{0}_{n-d})$$

Analogously, it is possible to construct a candidate transformation $\boldsymbol{z}_2 = \boldsymbol{\phi}_2(\boldsymbol{x})$ around $\boldsymbol{x}_0$ such that the first d elements of its Jacobian span a distribution $(T\mathcal{M}^a)^T$. Since vector fields $\boldsymbol{f}, \boldsymbol{g}_{k+1}, ..., \boldsymbol{g}_m$ are in $T\mathcal{M}^a$ by assumption, than by transformation to the new coordinates the first d elements of $\boldsymbol{f}$ and G^a vanish, yielding:

$$\bar{\boldsymbol{f}}(\boldsymbol{z}) = \mathrm{col}(\boldsymbol{0}_d, \bar{\boldsymbol{f}}_{d+1}(\boldsymbol{z}), ..., \bar{\boldsymbol{f}}_n(\boldsymbol{z}))$$

$$\bar{G}^a(\boldsymbol{z}) = \mathrm{col}(\boldsymbol{0}_d, \bar{\boldsymbol{g}}_{d+1}^2(\boldsymbol{z}), ..., \bar{\boldsymbol{g}}_n^2(\boldsymbol{z}))$$

This proves the proposition 2.

Let us define two controllability distributions:

$$\mathcal{R}^h(\boldsymbol{\xi}^h) = [\bar{G}^h(\boldsymbol{\xi}^h),\ ad_{\bar{\boldsymbol{f}}^h}\bar{G}^h(\boldsymbol{\xi}^h), ..., ad_{\bar{\boldsymbol{f}}^h}^{n-1}\bar{G}^h(\boldsymbol{\xi}^h)], \quad (16)$$

$$\mathcal{R}^a(\boldsymbol{\xi}^a) = [\bar{G}^a(\boldsymbol{\xi}^a),\ ad_{\bar{\boldsymbol{f}}^a}\bar{G}^a(\boldsymbol{\xi}^a), ..., ad_{\bar{\boldsymbol{f}}^a}^{n-1}\bar{G}^a(\boldsymbol{\xi}^a)] \quad (17)$$

Proposition 3. If the following holds:

$$\dim\{\mathcal{R}^h\} = d \quad (18)$$

where d is the number of the states $\boldsymbol{\xi}^h$ of the system 1, the states $\boldsymbol{\xi}^h$ are controllable w.r.t. the human input and the corresponding subsystem is controllable.

Proposition 4. If the following holds:

$$\dim\{\mathcal{R}^a\} = n - d \quad (19)$$

where $n-d$ is the number of the states $\boldsymbol{\xi}^a$ of the system 1, the states $\boldsymbol{\xi}^a$ are controllable w.r.t. the autonomous input and the corresponding subsystem is controllable.

If the control inputs, $\boldsymbol{u}^h$ and $\boldsymbol{u}^a$, are given as feedback control inputs of the form (3), the property of *controlled invariance* needs to be imposed.

Proposition 5. A distribution $T\mathcal{M}^h$ is controlled invariant on U if there exists a feedback pair $(\boldsymbol{\alpha}^h, \boldsymbol{\beta}^h)$ defined on U such that $T\mathcal{M}^h$ is invariant under the new vector fields (modified by a control law) $\tilde{\boldsymbol{f}}^h, \tilde{\boldsymbol{g}}_1^1, ..., \tilde{\boldsymbol{g}}_k^1$:

$$[\tilde{\boldsymbol{f}}^h, T\mathcal{M}^h] \subset T\mathcal{M}^h, \quad [\tilde{\boldsymbol{g}}_i^h, T\mathcal{M}^h] \subset T\mathcal{M}^h \quad i = 1, ...k$$

Local controlled invariance of the $T\mathcal{M}^h$ is guaranteed iff:

$$[\bar{\boldsymbol{f}}^h, T\mathcal{M}^h] \subset T\mathcal{M}^h + G^h$$

$$[\bar{\boldsymbol{g}}_i^h, T\mathcal{M}^h] \subset T\mathcal{M}^h + G^h \quad i = 1, ...k$$

Since by assumption $G^h \subset T\mathcal{M}^h$ (analogously $G^a \subset T\mathcal{M}^a$), the controlled invariance for the complementary interaction paradigm is always guaranteed.

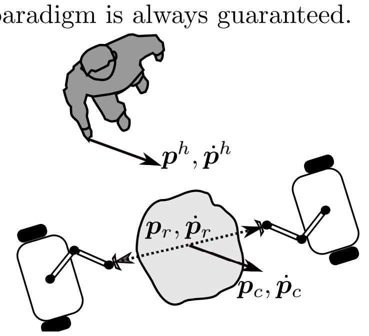

Fig. 6. Cooperative manipulation example of the complementary interaction paradigm

Example 2. Let us consider a cooperative manipulation system (6) in a setting depicted in fig. 6. Let us assume the two manipulators perform *cooperative* and *relative* behaviors. The cooperative behavior is a team behavior of the manipulators and the relative behavior is a relative motion between the manipulators. With the cooperative behavior of the manipulators it is possible to achieve object manipulation and with the relative behavior it is possible to approach to the object and maintain the grasp of the object. Let us assume the human commands the cooperative motion, while the autonomous controller commands the relative motion. The human control inputs are the position and the translational velocity of the human hand: $\boldsymbol{v}^h = [\boldsymbol{p}^h, \dot{\boldsymbol{p}}^h]^T$ and the autonomous control inputs are the desired relative position and the relative translational velocity between the manipulators: $\boldsymbol{v}^a = [\boldsymbol{p}^a, \dot{\boldsymbol{p}}^a]^T$. Let us introduce coordinate transformations $\boldsymbol{z}_1 = \boldsymbol{\phi}_1(\boldsymbol{x})$ and $\boldsymbol{z}_2 = \boldsymbol{\phi}_2(\boldsymbol{x})$:

$$\boldsymbol{\xi}^h = \begin{bmatrix} \boldsymbol{p}_c \\ \dot{\boldsymbol{p}}_c \end{bmatrix} = \underbrace{\frac{1}{2}\begin{bmatrix} \boldsymbol{p}_1 + \boldsymbol{p}_2 \\ \dot{\boldsymbol{p}}_1 + \dot{\boldsymbol{p}}_2 \end{bmatrix}}_{\boldsymbol{\phi}_1(\boldsymbol{x})},\ \boldsymbol{\xi}^a = \begin{bmatrix} \boldsymbol{p}_r \\ \dot{\boldsymbol{p}}_r \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{p}_1 - \boldsymbol{p}_2 \\ \dot{\boldsymbol{p}}_1 - \dot{\boldsymbol{p}}_2 \end{bmatrix}}_{\boldsymbol{\phi}_2(\boldsymbol{x})} \quad (20)$$

where $\boldsymbol{p}_c \in R^3$ and $\dot{\boldsymbol{p}}_c \in R^3$ are a mean of positions of the end-effectors (position of the mid-point between the manipulators) and the mean velocity of the team, respectively. Relative position and relative velocity of the robots are $\boldsymbol{p}_r \in R^3$ and $\dot{\boldsymbol{p}}_r \in R^3$, respectively. Time derivative of (20) gives:

$$\begin{bmatrix} \dot{\boldsymbol{\xi}}^h \\ \dot{\boldsymbol{\xi}}^a \end{bmatrix} = \underbrace{\begin{bmatrix} \dfrac{\partial \boldsymbol{\phi}_1}{\partial \boldsymbol{x}} \\ \dfrac{\partial \boldsymbol{\phi}_2}{\partial \boldsymbol{x}} \end{bmatrix}}_{T(\boldsymbol{x})} \dot{\boldsymbol{x}} = \begin{bmatrix} \frac{1}{2}I_3 & \frac{1}{2}I_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & \frac{1}{2}I_3 & \frac{1}{2}I_3 \\ I_3 & -I_3 & 0_3 & 0_3 \\ 0_3 & 0_3 & I_3 & -I_3 \end{bmatrix} \dot{\boldsymbol{x}} \quad (21)$$

The transformation of the system, $T(\boldsymbol{x})$, complies with the proposition (2) and ensures decoupling of the system (6). Furthermore, the following relations hold:

$$T^{-T}(\boldsymbol{p})M(\boldsymbol{p})T^{-1}(\boldsymbol{p}) = \begin{bmatrix} M_c(\boldsymbol{p}) & \boldsymbol{0} \\ \boldsymbol{0} & M_r(\boldsymbol{p}) \end{bmatrix} \quad (22)$$

where $M_c(\boldsymbol{p}) \in R^{d \times d}$ is the inertial matrix of the cooperative subsystem and $M_r(\boldsymbol{p}) \in R^{(n-d)\times(n-d)}$ is the inertial matrix of the subsystem for the relative behavior. Furthermore:

$$T^{-T}M\frac{d}{dt}T^{-1} + T^{-T}\boldsymbol{c}T^{-1}(\boldsymbol{p}) = \begin{bmatrix} \boldsymbol{c}_c(\boldsymbol{p}, \dot{\boldsymbol{p}}) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{c}_r(\boldsymbol{p}, \dot{\boldsymbol{p}}) \end{bmatrix} \quad (23)$$

where $\boldsymbol{c}_c \in R^d$ is a vector of Coriolis terms for the cooperative subsystem. Analogously, $\boldsymbol{c}_r \in R^{(n-d)}$ is a vector of Coriolis terms for the subsystem for the relative behavior. The decoupled dynamics given by (6) is represented by the subsystem for cooperative and relative behavior:

$$\underbrace{\begin{bmatrix} \dot{\boldsymbol{p}}_c \\ \ddot{\boldsymbol{p}}_c \end{bmatrix}}_{\dot{\boldsymbol{\xi}}^h} = \underbrace{\begin{bmatrix} \dot{\boldsymbol{p}}_c \\ M_c(\boldsymbol{p})^{-1}(-\boldsymbol{c}_c(\boldsymbol{p}, \dot{\boldsymbol{p}}) - \boldsymbol{h}_c^g(\boldsymbol{p})) \end{bmatrix}}_{\boldsymbol{f}^h(\boldsymbol{\xi}^h, \boldsymbol{\xi}^a)} + \underbrace{\begin{bmatrix} \boldsymbol{0} \\ M_c(\boldsymbol{p})^{-1} \end{bmatrix}}_{\boldsymbol{g}(\boldsymbol{\xi}^h, \boldsymbol{\xi}^a)} \boldsymbol{u}^h \quad (24)$$

$$\underbrace{\begin{bmatrix} \dot{\boldsymbol{p}}_r \\ \ddot{\boldsymbol{p}}_r \end{bmatrix}}_{\dot{\boldsymbol{\xi}}^a} = \underbrace{\begin{bmatrix} \dot{\boldsymbol{p}}_{rel} \\ M_r(\boldsymbol{p})^{-1}(-\boldsymbol{c}_r(\boldsymbol{p}, \dot{\boldsymbol{p}}) - \boldsymbol{h}_r^g(\boldsymbol{p})) \end{bmatrix}}_{\boldsymbol{f}_2(\boldsymbol{\xi}^h, \boldsymbol{\xi}^a)} + \underbrace{\begin{bmatrix} \boldsymbol{0} \\ M_r(\boldsymbol{p})^{-1} \end{bmatrix}}_{\boldsymbol{g}(\boldsymbol{\xi}^h, \boldsymbol{\xi}^a)} \boldsymbol{u}^a \quad (25)$$

Using the impedance feedback control strategies for each subsystem, the human and autonomous input commands are properly mapped to the input wrenches:

$$\boldsymbol{u}^{h,a} = \underbrace{[K_{c,r}\ D_{c,r}]\,\boldsymbol{\xi}^{h,a}}_{\boldsymbol{\alpha}_{c,r}(\boldsymbol{\xi}^{h,a})} + \underbrace{[-K_{c,r}\ -D_{c,r}]}_{\boldsymbol{\beta}_{c,r}} \boldsymbol{v}^{h,a} \quad (26)$$

It is possible to show that the dimension of the controllability distribution (16) is $\dim\{\mathcal{R}^h\} = \dim\{\boldsymbol{\xi}^h\} = d$ if the matrix $M_c(\boldsymbol{x})$ is positive-definite and for any choice of positive-definite matrices, D_c and K_c. It is also possible to show that the dimension of the controllability distribution (17) is $\dim\{\mathcal{R}^a\} = \dim\{\boldsymbol{\xi}^a\} = n - d$ if the matrix $M_r(\boldsymbol{x})$ is positive-definite and for any choice of positive-definite D_r and K_r.

3.3 Overlapping interaction paradigm

Definition 3. The interaction paradigm is called overlapping if there exists an intersection of distributions $T\mathcal{M}^h$ and $T\mathcal{M}^a$: $\Delta^{ha} = T\mathcal{M}^h \cap T\mathcal{M}^a \neq \emptyset$ and if the distribution Δ^{ha} is accessible to the control inputs $\boldsymbol{u}^h$ and $\boldsymbol{u}^a$.

The overlapping interaction paradigm considers a coupled system in which the human input commands and the autonomous input commands jointly steer states of the system, as depicted in fig. (7). Controllability of the system is unaffected if additional inputs are added. The interconnections between subsystems are expressed by the non-zero off-diagonal blocks of the $\boldsymbol{g}$ Jacobian.
It is important to distinguish two overlapping cases: *cooperative* and *competitive*. The cooperative systems are characterized by the overlapping of submanifolds onto which human and autonomous inputs act in the same direction. For a competitive system the influence from the human and the autonomous controller is opposite.

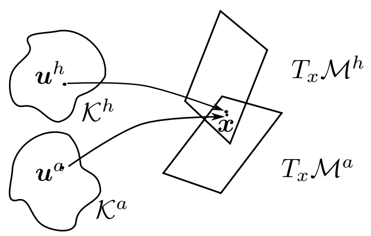

Fig. 7. Geometrical representation of the overlapping interaction paradigm

Definition 4. The vector field $\boldsymbol{g}$ is cooperative on a manifold $\mathcal{M}$ if the Jacobian matrix $\dfrac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}(\boldsymbol{x})$ has all non-negative off-diagonal elements for all $\boldsymbol{x} \in \mathcal{M}$. The system 1 is cooperative, if $\boldsymbol{g}$ is cooperative.

Definition 5. The vector field $\boldsymbol{g}$ is competitive on a manifold $\mathcal{M}$ if the Jacobian matrix $-\dfrac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}(\boldsymbol{x})$ has all non-negative off-diagonal elements for all $\boldsymbol{x} \in \mathcal{M}$. The system 1 is competitive, if $\boldsymbol{g}$ is competitive.

Example 3. A classical example of the cooperative overlapping interaction paradigm is load sharing in physical human-robot team interaction, depicted in the fig. 8. The human operator and the robot team cooperatively grasp the object. Let us consider only the translational motion. Furthermore, let us assume the object dynamics is known and is given with the following equation:

$$M_o(\boldsymbol{p}_o)\ddot{\boldsymbol{p}}_o + \boldsymbol{c}_o(\boldsymbol{p}_o,\ \dot{\boldsymbol{p}}_o) + \boldsymbol{h}^g(\boldsymbol{p}_o) = G\boldsymbol{u} \tag{27}$$

where $\boldsymbol{p}_o, \dot{\boldsymbol{p}}_o, \ddot{\boldsymbol{p}}_o \in R^3$ are position, translational velocity and translational acceleration of the common object.

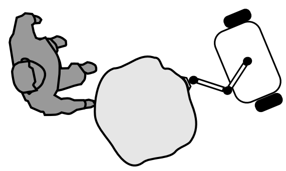

Fig. 8. Cooperative manipulation example of the overlapping interaction paradigm

Object inertial matrix, centrifugal and Coriolis forces and gravitational forces are given by $M_o(\boldsymbol{p}_o), \boldsymbol{c}(\boldsymbol{p}_o,\ \dot{\boldsymbol{p}}_o), \boldsymbol{g}(\boldsymbol{p}_o)$, respectively. Grasp matrix is $G = [I_3\ I_3]$ For more information on the physical human-robot interaction in a cooperative manipulation task see e.g. (Lawitzky et al., 2010). It is possible to define control inputs as linear homotopy between the forces applied on the object by the human, $\boldsymbol{f}^h$, and the robot, $\boldsymbol{f}^a$:

$$\boldsymbol{u} = \begin{bmatrix} \boldsymbol{u}^h \\ \boldsymbol{u}^a \end{bmatrix} = \begin{bmatrix} \boldsymbol{\alpha}_1 & 1-\boldsymbol{\alpha}_1 \\ 1-\boldsymbol{\alpha}_2 & \boldsymbol{\alpha}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{f}^h \\ \boldsymbol{f}^a \end{bmatrix} \tag{28}$$

where the $\boldsymbol{\alpha}_{1,2} \in [\mathbf{0}, \mathbf{1}]$ represent constant or time dependent mappings. The controller thus defined is termed as *homotopy-based* (Evrard and Kheddar, 2009). In this way a cooperative model of the system is obtained. Its transformation into the state space gives:

$$\begin{bmatrix} \dot{\boldsymbol{p}}_o \\ \ddot{\boldsymbol{p}}_o \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{p}}_o \\ -M_o^{-1}(\boldsymbol{c}_o + \boldsymbol{h}_o^g) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ M_o^{-1} \end{bmatrix} G\boldsymbol{u} \tag{29}$$

where the dependencies on the object position and velocities are omitted for brevity. The inertial matrix is positive-definite for robotic systems. The cooperative behavior is described by the control law with all non-negative terms (28) that are not dependent on the system states. Therefore, the system is cooperative.

3.4 Control design guidelines

If for a specific model the conditions are not satisfied they can, nevertheless, be met by introducing an appropriate feedback control strategy. For example, the controllability condition imposed by the proposition (1) can be met by *state-space exact linearization* which gives linearized and controllable system if the outputs of the system are chosen appropriately and the assumption of non-singularity is met. It is possible that the system (1) cannot be fully decoupled into subsystems (15). In this case the decoupling of the linearized system is achievable by designing the *noninteracting control*. Furthermore, it is necessary to ensure the reference inputs provided by the human, $\boldsymbol{v}^h$, and the autonomous controller, $\boldsymbol{v}^a$, *stabilize* the system (6). It is important to consider common cases in which the dimensionality of the input signal from the human, $\boldsymbol{u}^h$, is less than the number of states that need to be controlled. This can be due to the kinematic constraints of the human motion or because of the available undersensing measurement devices. In order to achieve the controllability of the system or a subsystem, the appropriate forward mapping of the human input commands is required, e.g. in the form of hand pose reconstruction, synergy based approach, etc.

3.5 Simulation results

Simulation results, depicted in fig. (9) and fig. (10), show the behavior of the cooperative system. Cooperative behavior is achieved in z direction, and the relative behavior

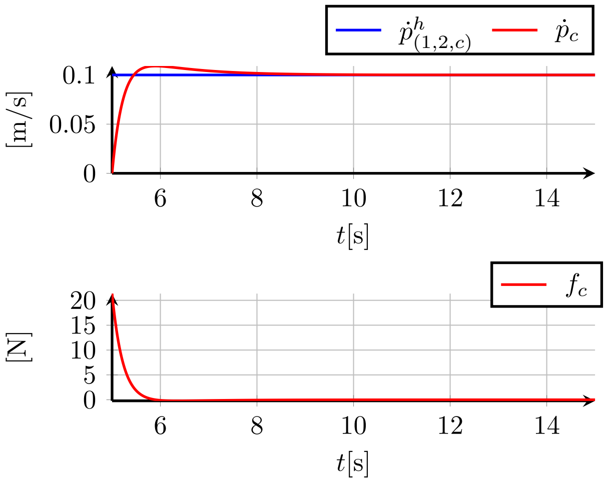

Fig. 9. Top: velocity of the object. Bottom: wrench on the object.

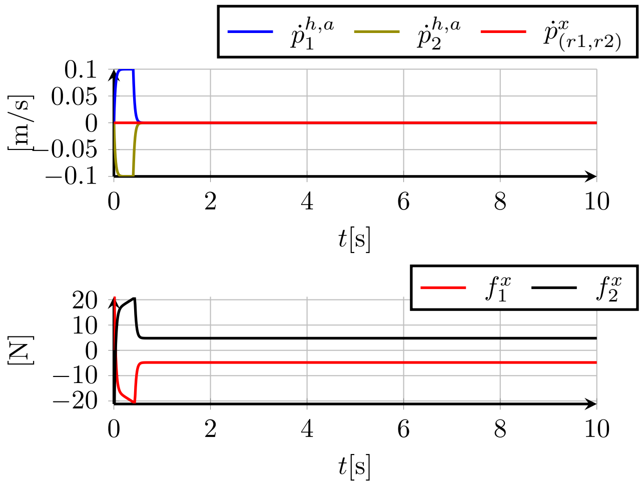

Fig. 10. Top: relative motion is blocked by the object. Bottom: internal forces applied by the manipulators.

in x direction. The results can represent all three interaction paradigms. We assume Coriolis, centrifugal and gravity terms to be compensated.

In the case of direct interaction paradigm, a desired object manipulation is achieved if the motion of the human fingers is in coordination. However, relative motion between the fingers can occur due to inherent uncertainty of the human behavior. This can cause an internal loading on the object and, hence, its undesirable motion. Therefore, the direct interaction paradigm is suitable when the robots are not in contact with the object (e.g. the grasping stage). When the robots are in contact it is reasonable to to achieve a precise, desired, relative behavior with the autonomous controller. Hence, the complementary interaction paradigm is suitable. It can be observed that the robots share equal load of the object as the forces applied in x direction are acting opposite and of the same intensity. Relative forces do not affect the cooperative sub-task of the system. When the agents are heterogeneous (e.g. humans and robots), the load sharing depends on the capabilities of the agents. For example, one may wish to minimize the effort exerted by the human and assign the effort completely to the robot. In this case the overlapping interaction paradigm is suitable.

4. CONCLUSION

In this paper three interaction paradigms between a single human and multiple robot systems are proposed: direct, complementary and overlapping interaction paradigms.It is shown that it is possible to distinguish the interaction paradigms using the controllability property and the structure of the Jacobian matrix of the mapping $\boldsymbol{g}$. The identified properties ease the selection of the controller objectives and strategies.In future work the required properties of the observability will be considered. Additionally, the identified properties will be used to perform synthesis of controllers.

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