TECHNISCHE UNIVERSITÄT MÜNCHEN

Institut für Verkehrswesen,
Lehrstuhl für Verkehrstechnik

Modelling and location optimization of dynamic wireless charging infrastructure for electric vehicles

Raffaela Riemann, M.Sc.

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1. Prof. Dr.-Ing. Fritz Busch
2. Associate Prof. David Wang Zhiwei, Ph.D.

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Abstract

This research focuses on model formulation and optimization of facility locations on road networks, specifically for wireless charging infrastructure for electric vehicles. It encompasses a thorough account of the theoretical framework and creates the fundamentals to analyze and to determine the optimal location of charging infrastructure by taking different assumptions, boundaries and constraints into account.

What is of particular interest is to analyze and to determine where to locate charging infrastructure and simultaneously to ensure that the relevant boundary conditions and constraints are integrated. This is achieved by developing two different mathematical model formulations. The first model deals with the optimal location of charging infrastructure considering traffic flow, travel times and a predetermined number of possible charging locations (flow-capturing location model with stochastic user equilibrium. The second model (set-covering location model with charging system design) incorporates a full coverage and more technical specifications into the design of the wireless charging system while assuming a pre-determined traffic pattern.

For both, the flow-capturing location model with stochastic user equilibrium and the set-covering location model with charging system design, the modelling frameworks, as well as the underlying assumptions and considerations are presented. Subsequently, the model formulations are depicted, followed by the illustration of the proposed solution method. Subsequently, the respective input parameters and analysis of the models' results are presented.

Contents

1 Introduction

The focus of this research is the modelling and optimization of facility locations on road networks, particularly for dynamic wireless charging infrastructure for electric vehicles. The motivation underlying this research is presented in section 1.1. Section 1.2 describes the problem statement and the objectives while section 1.3 entails the outline of this work, including an overview of the contents of the remaining sections.

1.1 Motivation

Given the restrictions on CO_2 emissions, mobility is in a transition phase and electric vehicles offer the potential to reduce emissions as well as dependency on petroleum. Furthermore, in "the age of access" [RIFKIN, 2001], where facilities and services are more likely to be accessed by the users whenever a demand arises instead of everyone owning everything needed on an individual basis, the importance of the optimal location of facilities and services strongly increases. An example of an increasingly popular access service in the mobility sector is the publicly available charging station for electric vehicles. As the "access model" increases in economic importance, it is desirable that the given facilities or services are conveniently accessible to all parties that would like to make use of them.

Five factors that influence the comprehensive implementation and usage of electric vehicles can be summarized as (1) the political framework, (2) the human attitude and expectation towards electric vehicles, (3) the technical vehicle specifications, (4) the costs and (5) the characteristics of the charging infrastructure. The first factor comprises the political will and resulting incentives towards a broader usage of electric vehicles. These incentives can include reduction of taxes and parking fees for electric vehicles and priority usage of the transportation network. Human attitude and expectation towards electric vehicles mainly deals with safety concerns and range anxiety that is defined as the fear of not being able to complete trips with an electric vehicle due to its insufficient range [EGBUE AND LONG, 2012]. The third category, technical vehicle specifications, entails factors such as battery lifetime and range of the electric vehicles whereas the forth category encompasses boundaries which are related to cost of buying and maintaining an electric vehicle as well as the costs of deploying, operating and maintaining charging infrastructure. The fifth category, charging infrastructure, deals with the technical specification, location and availability of charging infrastructure for electric vehicles. While all concerns and boundaries are important, this research focuses mainly on the fifth category of concerns: charging infrastructure. The lack of charging infrastructure is considered one of the main barriers hindering a broader implementation of electric vehicles.

The deployment of electromobility is supported by political decision makers through policies and incentives targeted at electric vehicles, making the total cost of ownership of electric vehicles comparable or in the future even lower than that of a vehicle with a conventional internal combustion engine motor [WU ET AL., 2015]. However, without sufficient provision of charging infrastructure, the broad deployment of electric vehicles will remain challenging due to the state-of-the-art technical vehicle specifications, especially the limited range, and the resulting human attitude towards electric vehicles, manifesting itself in range anxiety [EGBUE AND LONG, 2012].

1.2 Problem statement and objectives

The provision of a comprehensive, convenient and cost-effective charging infrastructure is still under development. Hence, an analysis of technical and infrastructural constraints is crucial in order to determine the optimal locations of charging infrastructure for electric vehicles. Charging infrastructure for electric vehicles can be classified into several categories, including stationary plug-in charging (conductive), wireless charging (inductive) and battery swapping stations. Dynamic wireless charging infrastructure, a technical solution of charging infrastructure where vehicles charge while driving on the roadway, is not yet widely implemented and the existing systems operate mainly on a test and demonstration level. Hence, it is worthwhile to develop an optimization framework for determining optimal locations of dynamic wireless charging infrastructure beforehand in order to analyze and estimate the potential impact on the transportation system and the required number of charging facilities before the implementation. The first objective of this research is to set up a framework for modelling the problem of charging station placement for electric vehicles. Second, a focus is set on the determination of optimal locations of charging facilities for dynamic wireless charging and finally to identify the key factors for determining such optimal locations. Moreover, two approaches shall be followed: The first approach investigates how many electric vehicles can be covered when a predetermined number of facilities is assumed. The second approach examines how much charging infrastructure is required, given the condition that all demand must be covered. While the emphasis is firmly on wireless charging, stationary charging is included to illustrate specific points of the research. Taking these considerations into account the following research question is derived:

How can the location of dynamic wireless charging infrastructure be optimized on a discrete road network?

1. What percentage of charging demand of electric vehicles can be covered with a predetermined number of facilities?
2. What is the minimum number of facilities and their locations that results in a full coverage of charging demand from electric vehicles?

The objective function and the constraints vary depending on the location problem to be solved. Therefore, two different approaches with specific objectives and constraints for the optimal location of wireless charging infrastructure for electric vehicles are included. Two mathematical models derived from classical location science are proposed to determine the optimal location of wireless charging facilities for electric vehicles (EVs). The first model deals with the optimal location of charging infrastructure by taking into account traffic flow, travel times and a predetermined number of possible charging locations. The second model includes pre-assigned traffic while taking into consideration full coverage and additional technical specifications for the design of the wireless charging system. The models are tested with varying input parameters to give insights on the required infrastructure deployment based on different scenarios.

To summarize, two mathematical models are proposed to determine the optimal location of wireless charging facilities for EVs. The reasons to follow this approach are twofold: first, to fit the specific requirements of incorporating optimal location and equilibrium constraints and second, to model the technical and range requirements in detail.

1.3 Outline

The remainder of this work is organized as follows: In section 2, the literature relevant for location modelling of charging infrastructure of electric vehicles is collated and classified. First, methods for determining optimal locations and their applicability to charging infrastructure, second, different types of charging infrastructure and third, models for the placement of charging infrastructure are investigated.

Based on the conclusions drawn from the literature review and the state of the art analysis and taking the research questions into account, two location models for the placement of wireless charging infrastructure for electric vehicles including their mathematical formulation and the applied solution methods are presented in section 3.

The application of the proposed models is explored in section 4. Sets of numerical experiments are conducted taking a baseline network into consideration in order to demonstrate the model validity and solution quality. Furthermore, the computational modelling and optimization framework is described followed by the presentation of the model-specific applications and results.

Section 5 provides a summary and conclusions, including the key findings, contributions and limitations. Moreover, an outline of future research is presented. An overview on the research framework and structure of this research work is depicted in Fig. 1.1.

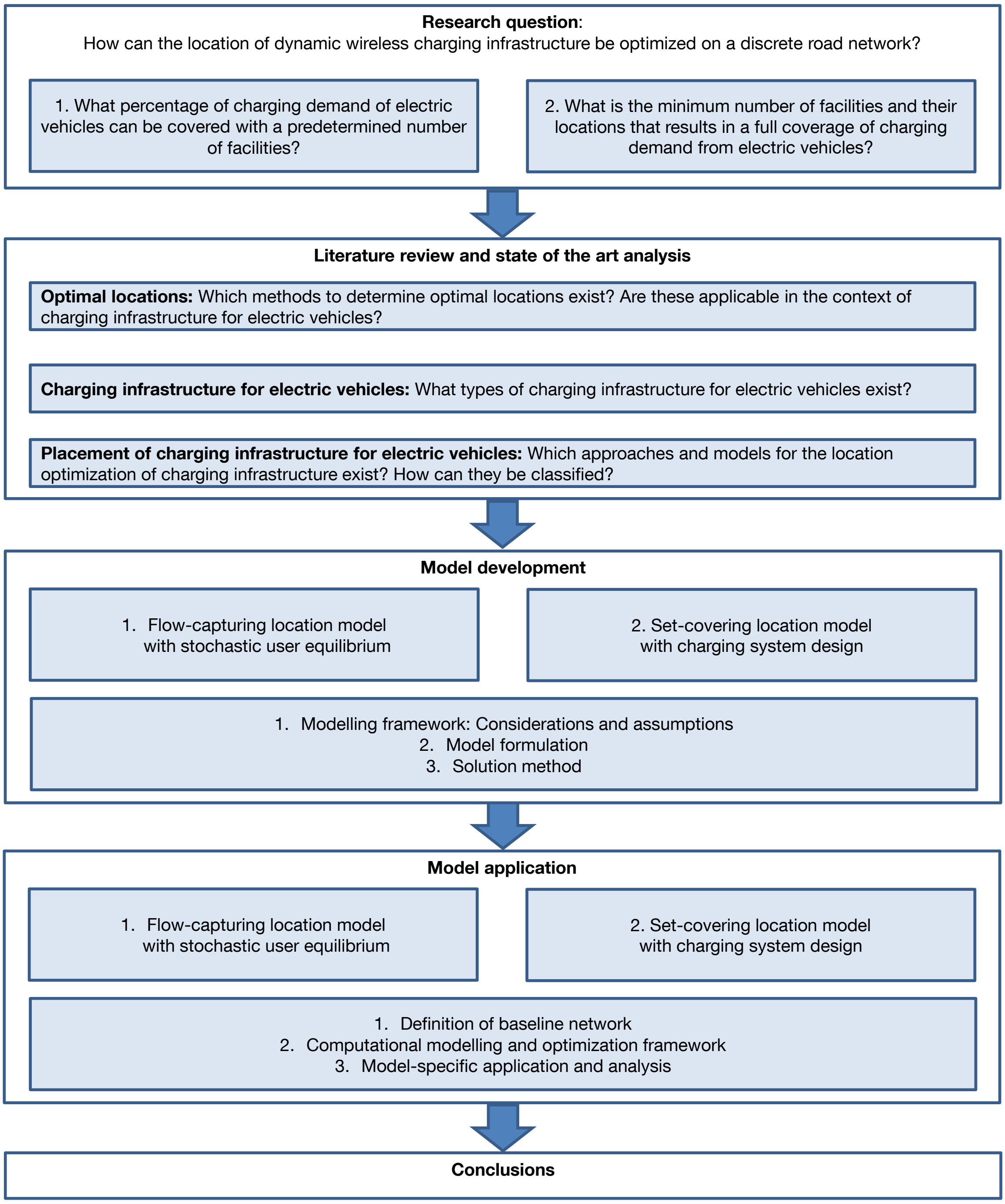

Fig. 1.1: Overview on research framework and structure

2 Literature review and state of the art analysis

In this section, the literature relevant for modelling and optimizing facility locations in discrete networks in the context of charging infrastructure for electric vehicles is collated and classified. The purpose of this literature review is threefold: (1) to set out the mathematical fundaments of optimization and location science (section 2.1), (2) to present an overview of basic information and current knowledge on electric vehicles and their charging infrastructure (section 2.2) and (3) to give an adequate account of the existing approaches and models for determining optimal locations of charging infrastructure for electric vehicles (section 2.3). For each of the three aforementioned domains, the relevant literature is discussed and analyzed and finally conclusions are drawn in section 2.4.

2.1 Optimal locations

The following two subsections contain the general formulation and brief description of mathematical optimization problems (section 2.1.1) and the application of such optimization problems in location science (section 2.1.2).

2.1.1 Mathematical optimization

"Optimization is everywhere" [YANG AND KOZIEL 2011, p. 1] and it is used throughout a wide range of applications in engineering and science as well as in industrial applications. Due to the fact that resources, time and budget are mostly limited, the "optimal" use of these resources is crucial, especially in practice [YANG AND KOZIEL, 2011]. Depending on the respective application, the parameters that are to be considered within the optimization model and whether costs are to be minimized or savings to be maximized, the mathematical formulation of such an optimization model and the appropriate solution method vary. Yet, in any effort of mathematical optimization fit, the problem is first formulated and a method for solving the problem is chosen subsequently.

2.1.1.1 Formulating optimization problems

The basic formulation of a nonlinear optimization problem can be described as follows [YANG AND KOZIEL, 2011]:

$$minimize\ f_i(x), (i = 1,2, \dots, M)$$

Subject to:

$$h_j(x), (j = 1,2, \dots, J)$$

$$g_k(x) \leq 0, (k = 1,2, \ldots, K)$$

In this notation f_i, h_j and g_k are general nonlinear functions, where the functions f_i depict the objective or the cost functions, h_j the equality constraints and g_k the inequality constraints. When f_i, h_j and g_k are linear functions the overall problem is defined as being a linear problem. Furthermore, optimization problems can be classified into convex and non-convex problems, where convex optimization is considered a generalization of linear programming [BOYD AND VANDENBERGHE, 2004].

If the specific vector x^* has the smallest objective value compared to all vectors which satisfy the constraints, it is called optimal or a solution of the problem [BOYD AND VANDENBERGHE, 2004]. The vector $x = (x_1, x_2, \ldots, x_n)$ can be continuous, discrete or mixed in n-dimensional space [YANG AND KOZIEL, 2011]. If some values of the vector x are integer values (for instance binary values) the overall optimization problem is considered to be a mixed integer problem [YANG AND KOZIEL, 2011]. Mixed integer problems can be classified further and are referred to as mixed integer linear problem (MILP) and mixed integer nonlinear problem (MINLP).

2.1.1.2 Solving optimization problems

To compute the solution of an instance of the optimization problem case-specific solution methods must be used [BOYD AND VANDENBERGHE, 2004]. For instance, if the functions f_i, h_j and g_k are all linear, the overall optimization problem becomes linear as well and efficient solution methods can be applied to solve the optimization problem. Generally, for convex optimization problems, one can use efficient methods such as interior point methods for finding the optimum. For linear problems Dantzig's simplex method can be used to solve the problem accordingly. Furthermore, for mixed integer linear problems, existing solution methods such as the branch and bound algorithms can be used for determining the solution of the optimization problem. For linear problems and mixed integer linear problems commercially available solvers such as the CPLEX optimization software package which includes IBM ILOG CPLEX Optimizer can be used [ILOG CPLEX OPTIMIZATION STUDIO, 2014]. Alternative solvers that can be used for mixed integer linear problems include Gurobi [GUROBI OPTIMIZATION, 2016] or FICO Xpress [FICO XPRESS OPTIMIZATION SUITE, 2014].

Despite implementing the optimization problem with commercial optimization software and making use of a state of the art hardware, solving difficult problems may involve long computation time [KLOTZ ET AL., 2013a]. A comprehensive overview with suggestions on how to efficiently use optimizers to solve difficult linear and mixed integer linear programs is given by KLOTZ ET AL. [2013a] and KLOTZ ET AL. [2013b].

Heuristics such as evolutionary algorithms can be applied to all types of problems, however, the algorithms must be designed to the optimization problem. Hence, it is generally not possible to utilize readily available software packages.

To conclude, depending on the nature of the optimization problem, case-specific solutions methods must be applied. The extensive discussion of solution algorithms for optimization problems is out of scope of this research work and the interested reader can refer to BOYD AND VANDENBERGHE [2004] for a comprehensive insight on convex optimization and to YANG AND KOZIEL [2011] for an overview on computational optimization, methods and algorithms.

2.1.2 Location science

This section gives an overview of location science concepts and models for facility location problems. The theoretical framework for location science was established in the 17^{th} century with a seemingly simple geometric problem, often referred to as Fermat's problem [LAPORTE ET AL., 2015]. It encompasses finding the point at which the sum of distances between three given points in the Euclidian plane is minimized [LAPORTE ET AL., 2015]. However, it is not part of the concern of this research work to dwell on the historical development of location science. The interested reader may consult REVELLE AND EISELT [2005]; EISELT AND MARIANOV [2011] and LAPORTE ET AL. [2015] for detailed reviews.

Typically, facility locations problems consist of "determining the "best" location for one or several facilities or equipments [sic] in order to serve a set of demand points" [LAPORTE ET AL., 2015, p. 1]. It should be noted that the denotation of "best" is subject to the model constraints and the objective considered [LAPORTE ET AL., 2015].

This literature review focuses on discrete network location problems in the public sector as these are the basis for the models developed within this research. Discrete problems are defined as problems in which facilities must be placed at the nodes of a network, while in continuous problems facilities can be placed anywhere on the plane [SNYDER, 2011]. More specifically, the review concentrates on the set-covering location problem, the maximal covering location problem and the flow-covering location problem, which constitutes a further development of the classic maximal covering model by taking network flows instead of nodes into account.

The basic components of a location problem are threefold: (1) a space with a defined distance measure, (2) a set of given points (customers or demands) and (3) candidate locations for the placement of new points (facilities) [EISELT AND MARIANOV, 2011]. Fig. 2.1 illustrates these basic components.

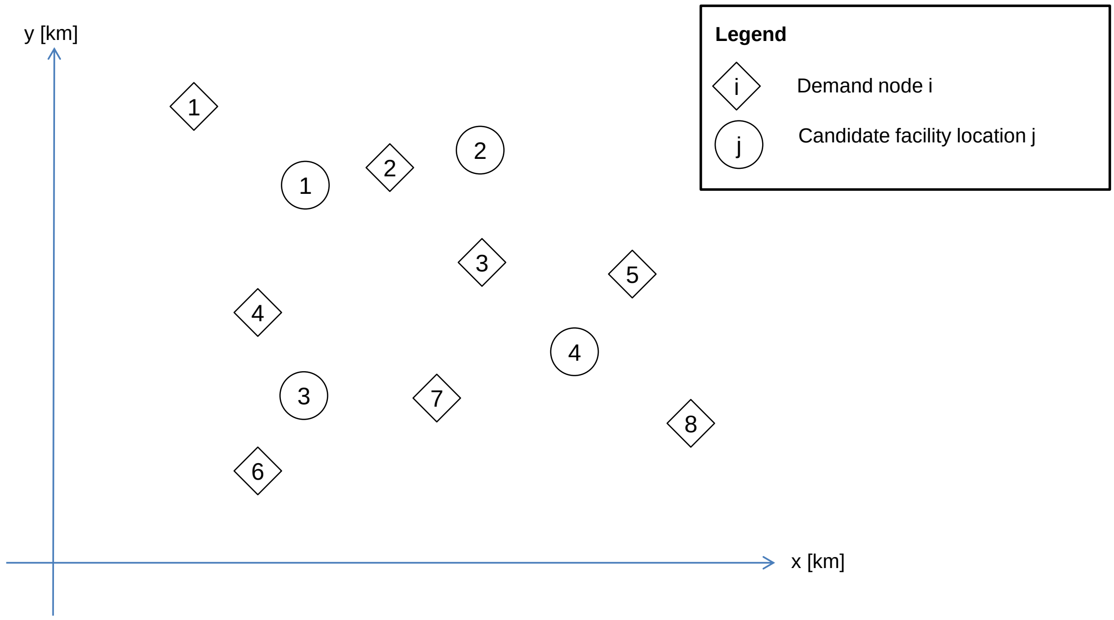

Fig. 2.1 Schematic view of basic location problem components

Due to the aforementioned differences in model constraints and objectives, which can be considered when determining the "best" location, location problems can be classified further. One major classification that can be employed is the application of these problems in different sectors. Classically, one would differentiate between location models applied in the private sector or in the public sector.

Typically, private location problems seek to maximize profit or to capture larger market shares from a competitor [MARIANOV ET AL., 2002]. Public location problems rather reflect social optima and seek to minimize social cost, guarantee universality of service, efficiency and equity [MARIANOV ET AL., 2002]. Measuring these objectives proves to be rather difficult, hence supporting frameworks for formulation of the public location problems are often considered to find optima for public location problems. These supporting frameworks for the formulation of public location models could for instance seek to optimize costs of location and operation to obtain full coverage or to maximize coverage with a predefined number of facilities or budget in order to implicitly capture social optima [MARIANOV ET AL., 2002].

A second common classification of locations problems is the space in which they are modelled [REVELLE AND EISELT, 2005]. REVELLE AND EISELT [2005] differentiate between location problems, which are modelled in "a subset of d-dimensional real space and networks" [REVELLE AND EISELT, 2005]. Both of these categories can be further classified into continuous or discrete location problems. In continuous location problems, facilities are to be placed anywhere in the space or the network, subsequently the mathematical formulations of those problems are likely to be nonlinear. On the other hand, discrete location problems usually incorporate a preselected set of candidate facility locations at which the facilities may be located. This requires preprocessing of the space or network and thereby the creation of the set mentioned previously. Consequently, binary values are employed to model the choice between these discrete facility locations mathematically. This results in location problems that fall under the category of mixed integer problems. An overview of the most common discrete network location models, their objectives and the according publication is presented in Tab. 2.1. In all these models, the underlying networks as well as the locations of the demands and the facilities are given as input parameters [CURRENT ET AL., 2001].

Distance or measures “functionally related to distance (e.g. travel time or cost, demand satisfaction)” [CURRENT ET AL., 2001, p. 2] are the basis of facility location problems. Subsequently such models can be differentiated into covering or maximum distance models and total or average distance models. Maximum distance models consider covering distances and determine demand satisfaction by ensuring that the nearest facility is within the covering distance. In cases where the closest facility is not within the covering distance, the demand is not satisfied [CURRENT ET AL., 2001]; while total or average distance models take the overall distance that must be traversed to reach the closest facility into account.

Considering the objectives and the research questions defined in section 1.2, the set-covering location problem that seeks to minimize the number of facilities to cover all demand and the maximal covering location problem which seeks to maximize covered demand with a predetermined number of facilities to be located are considered as adequate basic model formulations. Subsequently, a detailed overview of the model formulations including the inputs and sets, the decision variables, the objective function and the constraints are given for these two models. Furthermore, an overview of the flow-capturing location model that constitutes a special form of the maximal covering location model is given. The flow-covering location model includes origin-destination flows instead of static demand points.

Covering or Maximum Distance Models		
Model	Objective	Publication
Set-covering location problem	Minimize number of facilities required to cover all demand nodes	TOREGAS ET AL. [1971]
Maximal covering location problem	Maximize covered demand with predetermined number of facilities, p	CHURCH AND REVELLE [1974]
p-center problem	Minimize the maximum distance that demand is from its closest facility with a predetermined number of facilities, p	HAKIMI [1964], HAKIMI [1965]
Total or Average Distance Models		
Model	Objective	Publication
p-median problem	Minimize demand-weighted total distance between demand nodes and assigned facilities with a predetermined number of facilities, p	HAKIMI [1964], HAKIMI [1965]
Fixed charge location problem	Minimize total facility and transportation costs	(form of p-median)
Hub location problems	Minimize total cost (often as a function of distance) with a predetermined number of facilities, p	Numerous models (e.g. O'KELLY [1986a], O'KELLY [1986b], CAMPBELL [1994])
Maxisum location problem	Maximize total demand-weighted distance between demand nodes and assigned facilities with predetermined number of facilities, p	CURRENT ET AL. [2001]

Tab. 2.1 Discrete network location models (adapted from CURRENT ET AL. [2001] and DASKIN [2008])

2.1.2.1 Set-covering location problems

The objective of the set-covering location problems (SCLP) is to locate a minimum number of facilities to "cover" all of the demand nodes [CURRENT ET AL., 2001] under the constraint that each demand node is covered by at least one facility. The distances between the demand nodes and the candidate facility locations as well as the distance coverage of the candidate facility locations is taken into consideration when creating the set of candidate locations that can cover the respective demand point. To ensure that each demand node is covered by a facility, a binary constraint, which is set to 1 if a facility is located at one of the candidate facility locations and 0 otherwise, is included within the model formulation. The mathematical formulation of the SCLP is depicted in Tab. 2.2.

Sets and Parameters	
I	set of demand nodes indexed by i
J	set of candidate facility locations, indexed by j
d_{ij}	distance between demand node i and candidate site j
D_c	distance coverage
N_i	N_i={j |$d_{ij} \leq D_C$}, set of all candidate locations that can cover demand point i
Decision variables	
x_j	Binary variable, 1 if there is a facility at location j, 0 otherwise
Objective function	
	Minimize $\sum_{j \in J} x_j$
Constraints	
Subject to:	
	$\sum_{j \in Ni} x_j \geq 1 \; \forall \; i \in I$
	$x_j \in \{0, 1\} \forall \; j \in J$

Tab. 2.2 Set-covering location problem (adapted from CURRENT ET AL. [2001] and TOREGAS ET AL. [1971])

A schematic view of a set-covering location problem and its solution is depicted in Fig. 2.2. The set of demand nodes $I = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and the set of candidate facility locations

$J = \{1, 2, 3, 4\}$ are given as well as the distances between the demand nodes and the candidate sites (see Fig. 2.2) and the distance coverage $D_c = 1\,km$. When considering the set-covering location problem depicted in Fig. 2.2, the set of all candidate locations that can cover the demand points N_i can be derived by visual analysis. For each demand node it must be analysed whether it is located within the distance coverage radius of the candidate facility locations. Consequently, the analysis yields the following specific sets $N_1 = \{1\}$, $N_2 = \{1, 2\}$, $N_3 = \{1, 2, 4\}$, $N_4 = \{1, 3\}$, $N_5 = \{4\}$, $N_6 = \{3\}$, $N_7 = \{3, 4\}$ and $N_8 = \{4\}$.

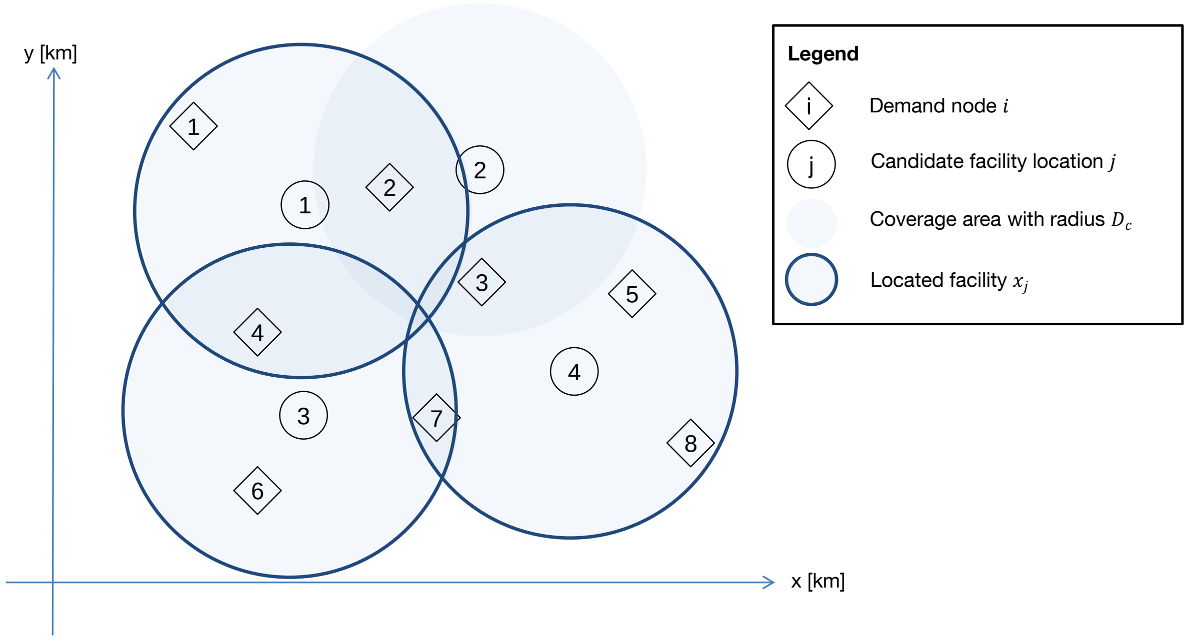

Fig. 2.2 Schematic view of a set-covering location problem

Using visual analysis and taking the aforementioned constraints into consideration, it is apparent that a minimum of three candidate facility locations placed at locations 1, 3 and 4 must be equipped with a facility to cover all demand nodes 1 to 8. The fourth facility at location 2 is not necessary.

2.1.2.2 Maximal covering location problems

The objective of maximal covering location problems (MCLP) is to locate a given number of facilities p while simultaneously ensuring that the demand covered by the facilities is maximized. This approach differs from the set-covering location problem in that the number of facilities is given, introducing a budget constraint to the model. If not all demand can be covered by the new facilities, the facilities that cover the most demand will be selected by the model [CURRENT ET AL., 2001]. Constraining factors include that only covered demand is taken into account and the number of facilities to be located is predetermined. Furthermore, the decision to position a facility at a given location and demand coverage are both expressed using binary constraints as presented in Tab. 2.3.

Sets and Parameters	
I	set of demand nodes indexed by i
J	set of candidate facility locations, indexed by j
d_{ij}	distance between demand node i and candidate site j
D_c	distance coverage
N_i	N_i=$\{j \mid d_{ij} \leq D_C\}$, set of all candidate locations that can cover demand point i
h_i	demand at node i
p	number of facilities to locate
Decision variables	
x_j	Binary variable, 1 if there is a facility at location j, 0 otherwise
z_i	Binary variable, 1 if demand node i is covered, 0 otherwise
Objective function	
	Maximize $\sum_{i \in I} h_i \, z_i$
Constraints	
Subject to:	
	$\sum_{j \in Ni} x_j - z_i \geq 0 \; \forall \; i \in I$
	$\sum_{j \in J} x_j = p$
	$x_j \in \{0, 1\} \; \forall \; j \in J$
	$z_i \in \{0, 1\} \; \forall \; i \in I$

Tab. 2.3 Maximal covering location problem (adapted from CURRENT ET AL. [2001] and CHURCH AND REVELLE [1974])

Accordingly, a schematic view of a set-covering location problem and its solution is depicted in Fig. 2.3. The set of demand nodes $I = \{1,2,3,4,5,6,7,8\}$ and the set of candidate facility locations $J = \{1,2,3,4\}$ are given as well as the distances (see Fig. 2.3) between the demand nodes and the candidate sites and the distance coverage $D_c = 1\ km$. For simplification it is assumed that all demand points are characterized by equal amounts of demand ($h_i = 1$). Furthermore, the number of possible facilities to locate is restricted to $p = 2$.

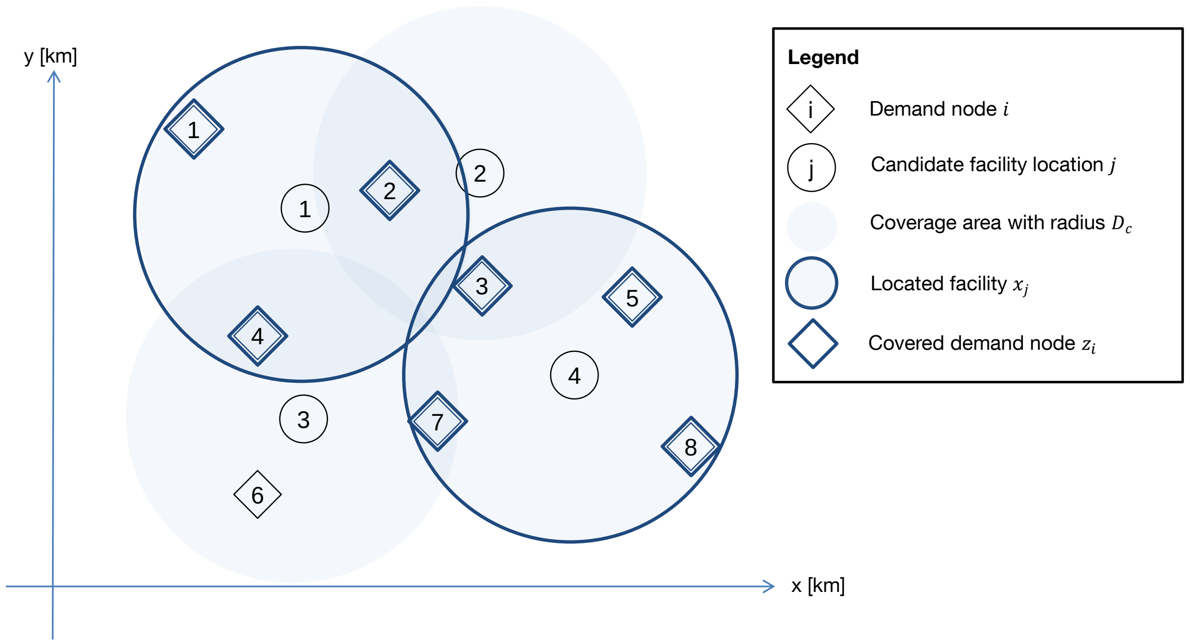

Fig. 2.3 Schematic view of a maximal covering location problem

Hence, the two candidate facility locations 1 and 4 are the optimal solution as those cover the maximum cumulated demand. To round off the picture and to give another example, in case the number of possible facilities to locate is increased to $p = 3$, locations 1, 3 and 4 would be the optimal solution and consequently, all given demand points would be covered. The resulting location pattern is exactly the same as the one for the set-covering location problem presented in Fig. 2.2 due to the circumstance that with an increased number of possible facilities to locate all the given demand points would be covered as well.

2.1.2.3 Flow-capturing location problems

The flow-capturing location problem (FCLP) depicts a further development of the maximal covering location problem. It includes origin-destination (OD) flows instead of static demand points in the model formulation and its objective is to maximize the captured flow, which is the flow that intersects one of the facilities located [HODGSON, 1990]. As in the maximal covering location problem, the facility location decisions and the flow covered are both expressed using binary constraints. The mathematical formulation of the FCLP is shown in Tab. 2.4.

Sets and Parameters	
Q	set of all OD pairs, indexed by q
K	set of all candidate facility locations, indexed by k
f_q	flow between OD pair q
N_q	set of nodes capable of capturing f_q between O_i and D_j
p	number of facilities to locate
Decision variables	
x_k	Binary variable, 1 if there is a facility at location k, 0 otherwise
y_q	Binary variable, 1 if demand node f_q is covered, 0 otherwise
Objective function	
	Maximize $\sum_{q \in Q} f_q \, y_q$
Constraints	

Subject to:

$$\sum_{k \in Nq} x_k \geq y_q \quad \forall \, q \in Q$$

$$\sum_{k \in K} x_k = p$$

$$x_k \in \{0, 1\} \, \forall \, k \in K$$

$$y_q \in \{0, 1\} \, \forall \, q \in Q$$

Tab. 2.4 Flow-capturing location problem [HODGSON, 1990]

For illustration, a schematic view of a flow-location problem and its solution is depicted in Fig. 2.4. The following sets and parameters are given: The flow fq moves between the OD pair using the given network, where the link width indicates the amount of flow on the link, the set of candidate facility locations $K = \{1, 2, 3, 4\}$, the number of facilities to locate $p = 1$ as well as the set of nodes capable of capturing the demand.

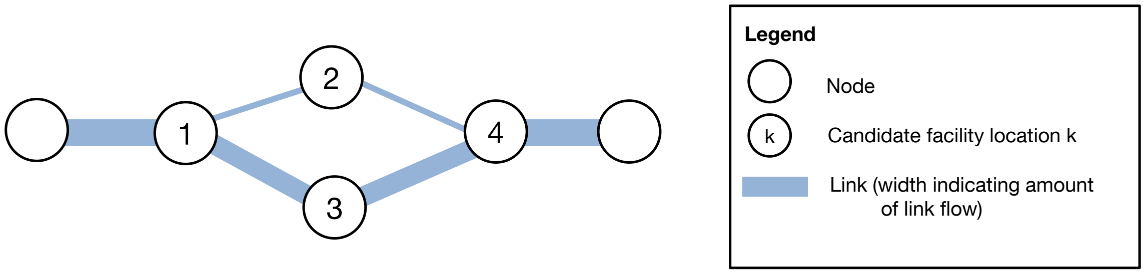

Fig. 2.4 Schematic view of a flow-covering problem

The solution of this schematic example yields more than one optimal solution. Candidate facility locations 1 or 4 both satisfy the constraints equally well as locating a facility at both location could ensure capturing all demand and hence capture a maximum of the flow in the network.

2.2 Charging infrastructure for electric vehicles

Charging infrastructure for electric vehicles can be classified into different types such as stationary plug-in charging (conductive), wireless charging (inductive) and battery swapping stations. The following review focuses solely on plug-in conductive charging and wireless inductive charging, hence in the next sections the system approaches for plug-in conductive charging and wireless inductive charging are specified in detail in sections 2.2.1 and 2.2.2, respectively. More specifically, the system architecture, implementation examples and typical locations for plug-in and wireless charging infrastructure are presented.

2.2.1 Plug-in charging infrastructure

An overview of the different system approaches for plug-in charging infrastructure and their classification according to different standardization bodies is given in Fig. 2.5. Generally, approaches for plug-in conductive charging systems can be distinguished between alternating current (AC) and direct current (DC) charging and their according plugs and power levels. Plug types 1, 2 and 3 are specified for AC charging with power levels varying between 3.7 kW and 43 kW. The three most common plug types for DC charging currently are CHAdeMO (50 kW), Type 2 (up to 35 kW) and Combo Plug (200 kW).

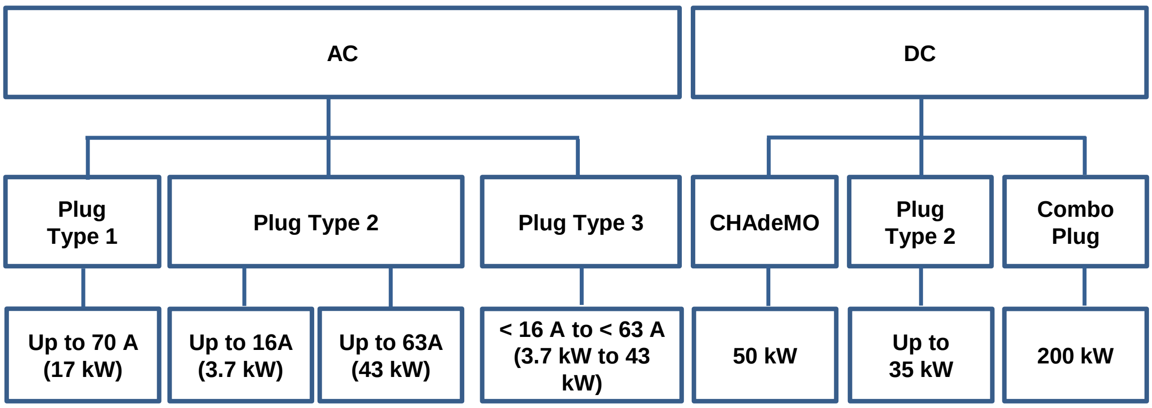

Fig. 2.5 System approaches for plug-in charging infrastructure for electric vehicles (adapted from German National Platform for Electric Mobility (NPE) [2014])

This type of charging infrastructure is designed to transfer the energy while the vehicle is parked as a cable must be plugged in to transfer energy and supply the battery within the vehicle.

2.2.1.1 System architecture

Plug-in conductive charging infrastructure receives commercial energy from the grid and subsequently the energy transfer is handled through a cable and direct contact to the vehicle as depicted in Fig. 2.6. The energy is then converted within the vehicle and provided to the vehicle battery that powers the electric motor.

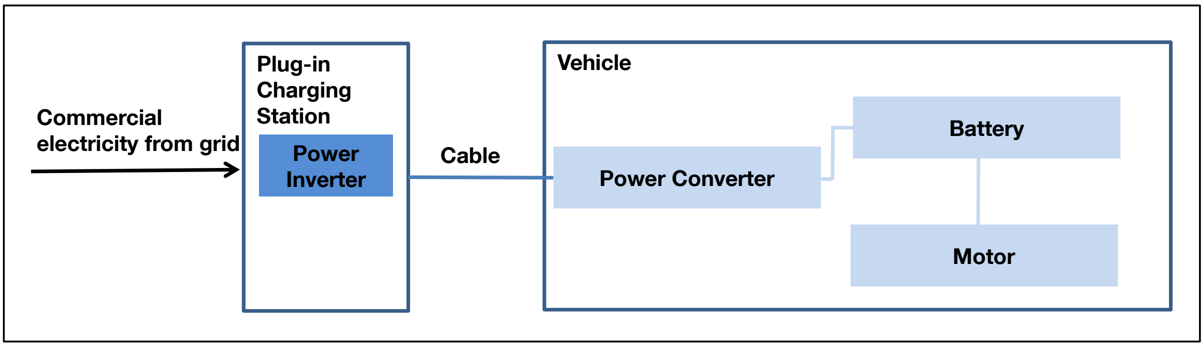

Fig. 2.6 Plug-in charging system architecture (adapted from German National Platform for Electric Mobility (NPE) [2014])

2.2.1.2 Implementation and typical locations

Charging infrastructure is typically located at either private or publicly accessible locations [German National Platform for Electric Mobility (NPE), 2015]. Typical locations for plug-in charging at private locations include garages or parking places at home, residential sites, apartment buildings or company car parks. Publicly accessible plug-in charging infrastructure is typically located at motorway service stations, shopping centers, car parks and curbside public parking spaces [German National Platform for Electric Mobility (NPE), 2015].

2.2.2 Dynamic wireless charging infrastructure

Dynamic wireless charging infrastructure employs an electromagnetic field to transmit energy to a vehicle system that is equipped with a pick-up coil capable of collecting the energy and charging on-board batteries [Ko, 2012]. This type of infrastructure is designed to transfer the energy while the vehicle is in motion; hence the vehicles are not required to stop during charging [Highways England, 2015]. Reported power levels transferred to the pick-up coil vary between 17 kW (Online Electric Vehicle (OLEV)) and 200 kW depending on the implementation [Highways England, 2015].

2.2.2.1 System architecture

The system architecture for dynamic wireless charging infrastructure based on the Online Electric Vehicle (OLEV) system is depicted in Fig. 2.7. The OLEV system entails a roadside system, the primary energy transmitter, which is embedded in the road, and the vehicle system. The primary energy transmitter is powered by commercial electricity from the grid and the electricity is picked up by the vehicle system through a pickup coil (power pick-up module) which provides the energy to the vehicle battery. The primary energy transmitters are embedded in the road in segments, which vary in length from 1 m to 1 km [BRECHER ET AL., 2014].

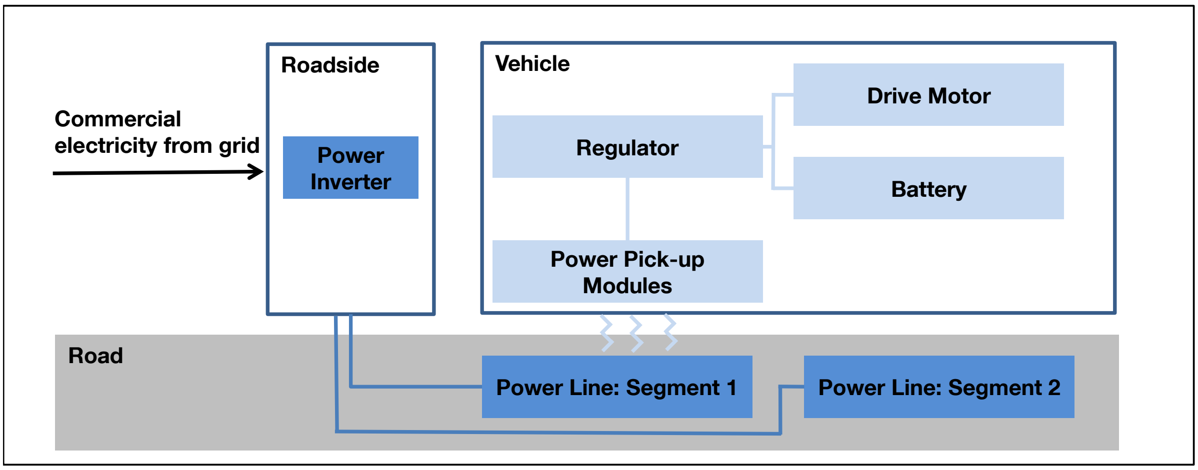

Fig. 2.7 OLEV dynamic wireless charging system architecture (adapted from BRECHER ET AL. [2014])

2.2.2.2 Implementation and typical locations

Dynamic wireless charging infrastructure is in the stage of research and early development, hence only a few services are implemented and operational. The measurement in Technology Readiness Levels (TRL) is often used to describe different stages of technology from basic principles to actual system and BRECHER ET AL. [2014] give an overview of wireless charging pilots and a classification of their TRLs.

TRLs are based on the following scale [BRECHER ET AL., 2014]:

- TRL 1: Basic principles observed and reported
- TRL 2: Technology concept and/or application formulated
- TRL 3: Analytical and experimental critical function and/or proof of concept
- TRL 4: Component and system validation in laboratory environment
- TRL 5: Laboratory scale, similar system validation in relevant environment
- TRL 6: Engineering/pilot-scale validation in relevant environment
- TRL 7: Full-scale validation in relevant environment
- TRL 8: Actual system completed and qualified through test and demonstration
- TRL 9: Actual system operated over full range of expectations

Tab. 2.5 presents an overview of wireless charging pilots, their application and TRL levels. In general the wireless charging pilots are implemented to show the capability of wireless charging for high capacity public transportation such as electric buses and high capacity light rail. Reported TRL levels vary between the engineering/pilot scale validation and the actual completion of the system including tests and demonstration and most of the wireless charging pilots depicted in Tab. 2.5 are categorized as TRL 8. However, a full range operational system is yet to be implemented.

Supplier	Application	TRL
IPT Technology (former Conductix-Wampfler)	Electric Buses	TRL 8
OLEV SMFIR	Electric bus and high capacity light rail vehicle	TRL 8 (Bus) TRL 6-7 (Rail)
WAVE	Electric buses	TRL 7-8
Bombardier PRIMOVE	Electric buses and light rail vehicles	TRL 8
EATON HyperCharger	Electric buses	TRL 7

Tab. 2.5 Wireless charging pilots (adapted from BRECHER ET AL. [2014])

Due to the insufficient technological maturity of dynamic wireless charging, it is not yet widely distributed and is only implemented currently in private locations and roads such as small scale testing areas and amusement parks [KO ET AL., 2012]. However, potential use cases in the long term could include publicly accessible roads such as a motorway or urban street network.

2.3 Placement of charging infrastructure for electric vehicles

The following section presents an account of different modelling approaches for locations of charging infrastructure for electric vehicles. First, modelling approaches for the optimal locations of plug-in charging infrastructure are reviewed in section 2.3.1, followed by modelling approaches for wireless charging infrastructure in section 2.3.2.

2.3.1 Optimal locations for plug-in charging infrastructure

Problems related to the optimal location of stationary charging facilities have received much attention from researchers. In this section, various modelling approaches for the placement of charging infrastructure for stationary charging that are relevant for this research work are presented. They are categorized based on their objective and the main inputs and outputs are summarized. Here, the models' objectives are divided into four categories: (1) set-covering approach (2) maximal covering approach, (3) flow covering approach and (4) equilibrium-based approach.

2.3.1.1 Set-covering approach

WANG [2007] formulated a model as set-covering location problem that focuses on the placement of charging infrastructure for electric scooters in a recreational context. The integer problem minimizes the cost of placing charging stations under the constraints that the time for recharging must be greater than the minimum recharge time for completing the trip, that the battery is fully charged at the beginning of a trip, that the number of scooters to be recharged is not greater than the number of recharge stations at a certain location and that the number of recharging stations at each location is not greater than their capacities. However, no routing costs are assumed as it is argued that the charging stations are located at trip destinations.

Furthermore, WANG AND LIN [2009] present a set-covering model that incorporates a vehicle refueling logic into the model formulation. WANG AND WANG [2010] use the refueling logic constraints developed by WANG AND LIN [2009] and develop a mixed integer problem with a set-covering approach and the objective of minimizing the total cost of installing charging stations while maximizing the coverage. The inputs of the model are potential station locations, set of all arcs, set of all vehicles traveling along paths, costs of locating a station at a node, refueling capacity and the distance between nodes.

2.3.1.2 Maximal covering approach

FRADE ET AL. [2011] formulated a model with the objective to maximize the demand covered within a given distance or service level, by locating a fixed number of charging stations. Furthermore, a differentiation between daytime and nighttime demand is incorporated. The function includes a penalty to prevent unnecessary charging point locations. The inputs for this model are (1) the number of statistical divisions, (2) the number of the possible locations, (3) the proportion of users assigned to a charging station, (4) the coverage distance, (5) whether there is a charging station or not, (6) the number of refueling events during the day and night, (7) the maximum number of stations to be installed, (8) the capacity of the charging station, and (9) the number of charging points for the certain charging station. As model constraints, FRADE ET AL. [2011] consider the maximum number of charging stations to be installed, a maximum demand covered of 100 percent, a defined coverage distance for which the demand is not covered if the charging station is farther away than this distance, the capacity and the according number of charging points to be installed. The output of the model is the number of charging stations to be installed as well as the number of charging points to be installed at each possible location. Additionally, the level of coverage of the sections is given.

2.3.1.3 Flow-covering approach

Several modelling approaches extend the flow-capturing location problem for the application of optimizing charging infrastructure for electric vehicles. KUBY AND LIM [2005] introduce the flow-refueling locations model (FRLM) by incorporating a flow refueling logic to the classical flow-capturing location problem. The FRLM is continuously extended for example by KUBY AND LIM [2007] who present a method to determine possible candidate locations for the FRLM on links or by LIM AND KUBY [2010] who propose to solve the FRLM more efficiently by making use of heuristic algorithms. UPCHURCH ET AL. [2009] develop a capacitated flow refueling location model that ensures that no facility refuels more than its capacity and limits the number of vehicles refueled at each station. As an input, the model requires precise knowledge about where the flows need to be refueled. The model objective is to maximize the total flows refueled under consideration of the combination of facilities. A capacity constraint ensures that it is impossible for a single combination to refuel all flows of a particular OD pair and the fraction of the flow refueled is indicated. Furthermore, the capacity constraint is introduced. It is ensured that the total use of the facilities by all flows and all combinations of facilities for each flow does not exceed the capacity of the facility.

Additionally, KIM AND KUBY [2012] introduce a deviation-flow model capable of taking deviations from the shortest path into account. Both KIM AND KUBY [2013] and HUANG ET AL. [2015] publish further extensions of the FRLM including deviation paths. The arc-cover path-cover FRLM (AC-PC FRLM), which constitutes a more efficient formulation of the FRLM is introduced by CAPAR ET AL. [2013]. Moreover, WANG AND LIN [2013] introduce the formulation of a capacitated multiple-recharging-station-location model including a vehicle-refueling logic which takes the maximum number of types of charging stations that can be located at a specific node and the length of stay at nodes into account.

Another modelling approach that takes the flow-covering approach into consideration is suggested by SHUKLA ET AL. [2011]. The objective function of their binary integer linear problem maximizes the "intercepted" traffic flow. The constraints taken into account are a budget constraint which includes an investment budget to cost of charging station ratio that is not to be exceeded and a constraint which determines the intercepted traffic flow. A path is seen as intercepted if one or more facilities are located on the path.

2.3.1.4 Equilibrium-based approach

HE ET AL. [2013a] develop a model that incorporates facility locations for plug-in charging facilities and a coupled transportation and power network to determine the equilibrium state. The objective of the model is to "maximize social welfare of the coupled networks for an average hour" by taking the total expected utility of electric vehicle drivers, the charging expenses, the total generation cost of electricity and the total construction cost of charging stations into consideration. The model determines the optimal number of charging stations for dedicated metropolitan areas from a macroscopic perspective. It optimizes the placement of charging stations by considering the total number of charging stations to be located among a set of possible locations under the constraints of an equilibrium state of the traffic and power flows distributions and the price of electricity.

Furthermore, HE ET AL. [2013b] use an equilibrium-based approach to formulate a model that incorporates electricity prices and road pricing as mechanisms to control and influence the management of transportation and power networks. Additionally, HE ET AL. [2014] develop three different network equilibrium models to integrate different flow dependencies and energy consumptions of electric vehicles and their recharging time under the assumption that drivers seek to minimize either trip time or cost while completing the trip without running out of energy.

2.3.2 Optimal locations for dynamic wireless charging infrastructure

KO ET AL. [2012] develop an optimization model for the placement of wireless charging technology. They assume that several identical vehicles are traveling on a circular fixed route, which is split into different segments depending on the properties of the road. The battery is charged when the vehicles pass over a road segment equipped with a power transmitter. Furthermore, they assume that the velocity profiles of the individual vehicles are known in advance. The formulated mixed integer problem aims to minimize the sum of the battery costs and the costs of the power transmitters (including inverter and inductive cable). The outputs of the model are the segments which are to be equipped with wireless charging technology and the capacity of the battery. The constraints take the energy level of the battery, which should be fully charged when starting and ending the trip and should not fall below a certain threshold during the trip, and the energy consumption during the trip into consideration.

An equilibrium-based approach to determine the maximum social welfare for different scenarios of full or partial electrification of roads is developed by HE ET AL. [2013c]. The optimal prices of electricity and wireless charging roads are determined by coupling the transportation and the power network.

FULLER [2016] uses a flow-based set-covering approach based on the seminal research work of WANG AND LIN [2009] and TOREGAS ET AL. [1971] to analyze the potential for wireless dynamic charging taking range and recharge issues into account. The objective of his model is to minimize the capital cost of the dynamic wireless charging facilities under the constraints of the design of the dynamic wireless charging system, the battery capacity, the routes of the electric vehicles and the according range requirements.

2.4 Summary and conclusions

The previous sections present an account of the literature and state of the art technologies relevant for determining how to optimize locations of dynamic wireless charging infrastructure on a discrete road network. The concept of mathematical optimization is presented with a subsequent detailed overview of the location optimization models used commonly in the context of the optimization of charging infrastructure: the set-covering location problem, the maximal covering location problem and the flow-covering location problem. When using mathematical optimization, it is desirable to formulate optimization problems as linear programs as these type of formulations allow for the use of existing solution algorithms and the use of readily available optimization software packages.

Moreover, an account of the basics of the charging infrastructure for electric vehicles for both plug-in and dynamic wireless charging infrastructure is given. Dynamic wireless charging infrastructure is not yet widely implemented and the existing systems mainly operate on a test and demonstration level. However, it is worthwhile to develop an optimization framework that determines optimal locations for this type of charging infrastructure to analyze the potential impact and the required extent of such systems before they are implemented.

The final part of the literature review provides an overview of previous research that deals with the placement of charging infrastructure for electric vehicles for plug-in and dynamic wireless charging infrastructure. The literature reviewed shows that throughout different research works the placement of charging infrastructure for electric vehicles is modelled with various different inputs and assumptions. Generally, there are a wide variety of model objectives ranging from economic, travel demand, distance, energy and traffic flow approaches. However, only the main objectives are used to classify the models and even though the objective of the model is for instance minimizing the cost, other factors such as travel demand or energy related objectives can be considered in the constraints of the model as well. Most of the reviewed model approaches take an economic viewpoint and seek to optimize the placement of charging infrastructure based on the cost. The type of costs taken into consideration differs between the models. Furthermore, compared to location optimization models which consider plug-in charging, models for dynamic wireless charging are not yet widely researched.

To conclude, the essence of the literature review leads to three conclusions concerning the potential for research that is pursued to answer the formulated research questions. First, the optimal locations of dynamic wireless charging infrastructure can be investigated on a discrete road network by employing mathematical optimization, more specifically facility location models can incorporate the specifics of traffic flow and charging infrastructure. Various fundamental models for determining optimal facility locations are available and the application of these well-established models allows for the exemplary investigation of different scenarios. This is specifically advantageous for exploring the effects of locating facilities under different objectives and constraints. Considering the basic components of a location problem (a space with a defined distance measure, a set of given demand and candidate locations), it can be concluded that they are applicable to the context of optimal locations for charging infrastructure for electric vehicles. More specifically, the space with a defined distance measure is depicted by the transportation network, the set of given demand is depicted by the charging demands of electric vehicles and the candidate locations are depicted by the possible locations of charging infrastructure. Furthermore, the fundamental concepts of set-covering and maximal covering location problems are well suited for answering the formulated research questions. Set-covering location models seek to minimize the number of facilities required to cover all demand nodes. Hence, they can be applied directly to the context of the optimization of charging infrastructure deployment when the objective is to seek the minimum number of facilities required to cover all charging demand. However, in cases when handed a budget and hence potentially only a limited number of locations is available and with which charging demand is to be met, the maximal-covering location model, which seeks to maximize the covered demand with a predetermined number of facilities, is the optimization model of choice. Both set-covering and maximal-covering models include integer decision variables in their basic model formulation. Furthermore, depending on the nature of the optimization problem, case-specific solutions methods must be applied. In the context of mixed integer linear optimization problems methods for solving these models already exist. Hence, the approach of formulating the optimization problem as a mixed integer linear problem is pursued further in this research work.

Second, flow-capturing and flow-refueling location problems discussed in the literature review classify link traffic flow on the network as an exogenously given input parameter. Usually, the assignment of the origin-destination (OD) demand to the shortest paths is implemented through preprocessing using common shortest path algorithms.

Consequently, only the resistance weights (such as trip distance) assumed beforehand governs the route choice behavior of the motorists. The availability of charging stations and flow-dependent travel times cannot be depicted with these standard models. However, for both FCLM and FRLM, traffic flow on the network is included as a weight in the objective function and subsequently determines a crucial model parameter. Subsequently, the availability of charging facilities and the resulting effect on route choice of electric vehicle drivers are to be taken into consideration for the model formulation developed in this research work. This can be achieved by pursuing a modelling approach that simultaneously considers the optimal facility locations and the equilibrium traffic flow pattern within the transportation network.

Third, besides the fact that dynamic wireless charging infrastructure is not yet implemented on a large scale, limited research efforts concerned with modelling and optimization of this type of infrastructure have been undertaken. The flow-based set-covering model, which takes refueling of electric vehicles and dynamic wireless charging infrastructure into consideration can be extended by investigating the length of the segments of dynamic charging infrastructure and the energy to be transmitted as additional decision variables besides the facility location.

3 Model development

This section presents an insight into the modelling approaches and methodology applied within this research (section 3.1). The two models that are developed are explained in section 3.2 and section 3.3. These models can be applied to optimally locate charging facilities with wireless power transfer capabilities. The first is an extension of the flow-covering model including simultaneous traffic assignment with stochastic user equilibrium (SUE) and the second is an extension of the set-covering model with charging system design. For each model, this research work presents the modelling framework and the underlying assumptions and considerations. Subsequently, the model formulation and the applied solution method are depicted.

3.1 Modelling approaches

The main question to be answered is how the location of dynamic wireless charging infrastructure can be optimized on a discrete road network. To answer this question, two mathematical models to determine the optimal location of dynamic wireless charging infrastructure for electric vehicles are developed and applied. In line with the basic components of a facility location problem, for both models the following three building blocks are considered, first the transportation network, second the charging infrastructure and third the electric vehicles.

Existing models from literature serve as the basis and are refined and extended in order to formulate mathematical optimization models which capture the specifics of dynamic wireless charging infrastructure. The two different model formulations are derived based on the research questions presented in section 1.2.

Due to these model extensions, however, initially both optimization models are formulated as nonlinear models. Given the rationale presented in the literature review, it is desirable to create a modelling framework that allows for the application of existent solution methods already available in software packages. In summary, the further modelling approach after identifying the potential for extending existing location models can be described as follows:

1. Definition of the modelling framework
2. Formulation of the model based on the developed modelling framework
3. Application of a solution method that transforms the model into a mixed integer linear formulation of the optimization model

This approach is deployed to both models which were developed and it is reflected accordingly in the division of the subsections.

First, a newly developed mathematical model for determining the location of wireless charging infrastructure using a flow-covering approach that simultaneously assigns traffic flow and selects locations for charging infrastructure (referred to as the "flow-capturing location model with stochastic user equilibrium") is presented in section 3.2. This model can be deployed to give insights on the percentage of charging demand that can be covered when a predetermined number of facilities for dynamic wireless charging is available.

Second, a flow-based set-covering approach, the "set-covering location model with charging system design" which seeks to minimize the number and locations of facilities while ensuring full demand coverage and taking specifics of the charging system design into consideration, is described in section 3.3.

3.2 Flow-capturing location model with stochastic user equilibrium

Contents of this section were previously published in *Riemann, R., Wang, D. Z. W. and Busch, F., 2015. Optimal location of wireless charging facilities for electric vehicles: Flow-capturing location model with stochastic user equilibrium. Transportation Research Part C: Emerging Technologies, 58, pp.1–12.*

The flow-capturing location model with stochastic user equilibrium approaches the problem of optimally locating wireless charging facilities in discrete road networks. The model takes a predetermined number of wireless charging facilities to be located and electric vehicles' traffic flows into account. The modelling framework and the underlying assumptions and considerations are described in section 3.2.1.

Furthermore, it seeks to maximize the captured traffic flow, which implies that a maximum number of electric vehicles may use these links and access the wireless charging facilities. The AC-PC FRLM introduced by CAPAR ET AL. [2013] constitutes the basis of the model formulation that is illustrated and detailed in 3.2.2. Because of the interaction between the location of charging facilities on the network and the traffic flow assignment, the AC-PC FRLM is extended by equilibrium constraints, which capture the route choice behavior depending on preferences.

The proposed mathematical model is formulated as a mixed integer nonlinear problem. A global optimization method is applied in section 3.2.3 to the formulated model by transforming the nonlinear objective function into equivalent linear equations and by approximating the multinomial logit-based (MNL) model and the nonlinear travel time function with piecewise linearization techniques.

To summarize, the objective is to capture maximum traffic flow on the network under the constraints of a given number of wireless charging facilities which can be selected from a predetermined set of candidate facility locations and equilibrium traffic flow patterns. The traffic flow pattern follows the stochastic user equilibrium and incorporates drivers' routing choice behavior in the model. Both the wireless charging facility location and the traffic flow pattern are decision variables of the model that are attained endogenously from the model solution. The resulting model formulation is a mixed integer nonlinear problem (MINLP); therefore, a global optimization solution method is developed by applying linearization techniques to obtain the solution of the problem.

3.2.1 Modelling framework: Considerations and assumptions

The objective of the proposed model is to maximize the captured flow of electric vehicles, which is equivalent to that of the classical flow-capturing models. However, to capture and investigate changes in route choice and travel times due to the availability of charging infrastructure on the network, equilibrium constraints are incorporated to the model formulation.

In contrast to previous FCLM and FRLM, in the proposed model, both traffic flow and the availability of wireless charging facilities are considered simultaneously when locating the charging facilities in the network. It is assumed that charging facility availability might affect the route choice of electric vehicle drivers in such a way that they are more likely to choose routes with charging facilities. That is particularly true when dynamic wireless charging infrastructure is deployed as the electric vehicles will recharge their batteries while driving on the electrified roadway. This is a crucial factor for complementing the trips, as the range of the vehicles is limited. In addition, it may affect the attitude and expectations towards electric vehicles as it could mitigate range anxiety. Therefore, the interactions of facility location and traffic assignment are captured in the flow-capturing model with equilibrium constraints. The arc cover-path cover flow-refueling location model, which is a computationally more efficient formulation of the FRLM, is introduced by CAPAR ET AL. [2013] and constitutes the basis of this model formulation. However, the model is extended by taking equilibrium constraints into account.

Travel times are computed endogenously trough the Bureau of Public Roads (BPR) function [COMSIS CORP., 1983] which takes link free flow travel times, link capacities and the assigned continuous traffic flow on the links into account when computing flow dependent link travel times. Consequently, potential bottlenecks that may occur due to the availability of charging facilities and the resulting changes in route choice behavior are identified in advance as travel times are modelled as flow dependent variables.

Changes in route choice due to travel times and the availability of charging infrastructure in the network are modelled by using the concept of stochastic user equilibrium (SUE). This implies that the perceived travel times are modelled endogenously as flow and infrastructure dependent variables by applying a multinomial logit-based loading model. In the state of stochastic user equilibrium, no one can improve the utility of their route by "unilaterally changing routes" [SHEFFI, 1985]. Additionally, this implies that the utility on all routes will not be equal [SHEFFI, 1985]. In the proposed model formulation, the utility of choosing a route depends on the travel time and the availability of charging stations. In the MNL, the travel time is subject to a negative scaling parameter as a higher travel time implies a lower utility for the route. In contrast, the availability of a charging facility is subject to a positive scaling parameter as the possibility of charging the electric vehicles and therefore completing the routes (if the range is not sufficient to complete the trip) implies a higher utility.

Consequently, by utilizing this approach, the interaction between the location of charging facilities, route choice and therefore traffic flow on the links is incorporated explicitly in the proposed model formulation. To summarize, the optimal location of wireless charging facilities is influenced by the traffic flow, which in turn affects routing choice behavior, and thus the equilibrium traffic flow. Travel time and the availability of charging infrastructure are considered as the two factors influencing utility.

However, the utilization of a nonlinear travel time function and a MNL model indicates that the resulting model must be classified as mixed integer non-linear model (MINLP). The approach to formulate the flow-capturing location model with stochastic user equilibrium serves two purposes: First, to locate wireless charging facilities on the network and second, to incorporate both travel time and the availability of charging facilities endogenously when assigning traffic in the network.

The following assumptions have to be made to enable the modelling effort. Due to the incorporation of non-linear constraints the model formulation is quite complex. For this reason, all vehicles in the network are assumed to be electric vehicles. Furthermore, all of the vehicles start their trips with a fully charged battery, implying that at each node of trip origin access to a stationary charging facility exists. The possible candidate locations for the wireless charging facilities are all assumed to be located in the center of the links, at the centroid node of each link in the network. The charging process is conducted dynamically while the vehicles are driving over the links equipped with a wireless charging facility. Furthermore, as this model focuses on the macroscopic formulation of the location problem, instead of modelling the detailed technical properties of the wireless charging system, a full recharge is assumed when an electric vehicle uses a link hosting a wireless charging facility. Consequently, when the electric vehicles traverse a link equipped with a wireless charging facility, there is no time loss for recharging. If electric vehicles can complete a particular route without running out of energy, this particular route flow is regarded as "captured". However, a vehicle will run out of energy if not all the centroid-link segments can be refueled. These EV flows cannot complete their trips and are regarded as "not captured". This definition is according to the logic of the existent flow-capturing models. The parameters necessary for the calibration of the stochastic user equilibrium are assumed to be given beforehand as the focus of this research is on model formulation. Taking these considerations and assumptions into account, the notation presented in Tab. 3.1 is employed for the mathematical formulation of the sets and parameters of the FCLM with equilibrium constraints. Furthermore, the sets and parameters are classified according to the building blocks (transportation network, charging infrastructure and electric vehicles) taken into consideration for the model formulation.

Sets and Parameters	
Building block: transportation network	
W	Set of all OD pairs w
N	Set of all nodes n in network
A	Set of all links a in network
G	Network $G = (N, A)$ consisting of sets of nodes N and links A
O	Set of all origin nodes o , where $O \subseteq N$
D	Set of all destination nodes d, where $D \subseteq N$
Z	Set of all centroid nodes z of links a, $a \in A$
M	Set of all origin nodes, centroid nodes and destination nodes m, where $M = O \cup Z \cup D$, with $i, j \in M$
$a_{i,j}$	Segment starting from node i and ending at node j of neighboring links, $i, j \in M$
$A^{w}_{i,j,r}$	Ordered set of segments $a_{i,j}$ on route r of OD pair $w \in W$
C_a	Set of all link capacities c_a, $a \in A$
D_a	Set of all length of link distances d_a
T^0_a	Set of all link free flow travel times t^0_a, $a \in A$
R^w	Set of all feasible routes r of OD pair $w \in W$
$\delta^w_{a,r}$	Link-path incidence, $\delta^w_{a,r} = 1$ if link $a \in A$ belongs to route r between OD pair w, 0 otherwise
q^w	OD travel demand
α	Negative scaling parameter for travel time

Building block: charging infrastructure	
β	Positive scaling parameter for the availability of charging facilities
K	Set of all candidate facility locations k
$\Psi^{w}_{i,j,r}$	Set of candidate facility locations capable of recharging EV flows so that the EV can traverse the segment $a_{i,j}$ in $A^{w}_{i,j,r}$(EV range and link distances are taken into consideration when this set is generated)
p	Number of wireless charging facilities to be located
Building block: electric vehicles	
EVR	Range of EVs

Tab. 3.1 Sets and parameters FCLM with stochastic user equilibrium

The notation of the decision variables of the FCLM with equilibrium constraints is depicted below in Tab. 3.2.

Decision variables	
x_k	Binary variable, 1 if there is a wireless charging facility at location k, 0 otherwise
y^{w}_{r}	Binary variable, 1 if flow on route r is captured, 0 otherwise
f^{w}_{r}	Continuous flow variable on route r between OD pair $w \in W$
h_a	Continuous flow variable on link a
P^{w}_{r}	Probability of flow choosing route r between OD pair $w \in W$
t_a	Travel time when using link $a \in A$
t^{w}_{r}	Travel time when using route r between OD pair $w \in W$

Tab. 3.2 Decision variables FCLM with stochastic user equilibrium

3.2.2 Model formulation

In this section, the objective function and the constraints of the flow-capturing model with equilibrium constraints for the determination of wireless charging facilities in a network are presented. The illustrated problem formulation constitutes a mixed integer non-linear problem. Initially, the complete mathematical formulation is shown, followed by a detailed description of the model.

Maximize:

$$\sum_{w \in W} \sum_{r \in R^w} f_r^w y_r^w \tag{1}$$

Subject to:

$$\sum_{k \in \Psi_{i,j,r}^w} x_k \geq y_r^w, \qquad \forall\, a_{i,j} \in A_{i,j,r}^w, r \in R^w, \quad w \in W \tag{2}$$

$$\sum_{k \in K} x_k = p \tag{3}$$

$$h_a = \sum_{w \in W} \sum_{r \in R^w} f_r^w \delta_{a,r}^w, \qquad \forall a \in A \tag{4}$$

$$P_r^w = \frac{\exp(\alpha t_r^w + \beta y_r^w)}{\sum_{s \in R^w} \exp(\alpha t_s^w + \beta y_s^w)}, \qquad \forall\, r \in R^w, \quad w \in W \tag{5}$$

$$t_r^w = \sum_{a \in A} \delta_{a,r}^w t_a, \qquad \forall\, r \in R^w, \quad w \in W \tag{6}$$

$$t_a = t_a^0 \left[1 + 0.15 \left(\frac{h_a}{c_a}\right)^4\right], \qquad \forall a \in A \tag{7}$$

$$f_r^w = q^w P_r^w, \qquad \forall\, r \in R^w, \quad w \in W \tag{8}$$

$$x_a \in \{0|1\}, \qquad \forall a \in A \tag{9}$$

$$y_{a,r}^w \in \{0|1\}, \qquad \forall\, r \in R^w,\ w \in W,\ a \in A \tag{10}$$

The objective stated in (1) is to maximize the total captured traffic flow. The objective function is the first nonlinear term within this model formulation as both the flow-capturing variable y_r^w and the flow variable f_r^w are defined as decision variables that are solved endogenously. In the conventional AC-PC FRLM, the flow f_r^w is given as exogenous input, which is computed by solving the shortest path problem.

Constraint (2) is based on the flow-cover constraint proposed by CAPAR ET AL. [2013] for the formulation of the AC-PC FRLM. It depicts the flow-cover component of the model for each specific route r. If an EV can utilize all the links on its route without running out of energy, this route is considered to be covered, thus $y_r^w = 1$. In contrast to CAPAR [2013] in the proposed model formulation, the set candidate facility locations capable of recharging EV flows so that the EV can traverse the segment $a_{i,j}$ in $A_{i,j,r}^w$ ($\Psi_{i,j,r}^w$) is generated by taking the distances between the centroid nodes of neighboring link as the segment that has to be traversed without running out of energy. This assumption is made as dynamic wireless charging facilities are to be installed on the links of a network; hence, the candidate facility locations are a set of the centroid-points of the links. Moreover, in contrast to CAPER ET AL. [2013], round-trips are not considered.

The predetermined EV range and the lengths of the links are taken as input parameters for generating the set $\Psi^{w}_{i,j,r}$, which is utilized by constraint (2). A schematic view of the set generation logic for the set of candidate facility locations capable of recharging EV flows so that the EV can traverse the segments, $\Psi^{w}_{i,j,r}$, is illustrated in Fig. 3.1. The available range of the electric vehicles is taken into account when generating the set.

The four diamond shapes in Fig. 3.1, which are located at the centroid point of each link, depict the set of candidate locations for charging facilities. With the assumption that a vehicle has a range of 60 km and that the battery is fully charged at the beginning of the trip, the route depicted would be considered traversable if a charging facility would to be located at the candidate facility location between nodes 3 and 4. If no facility is located at this candidate location, the route would not be traversable, hence it would not be considered as being a “captured” route.

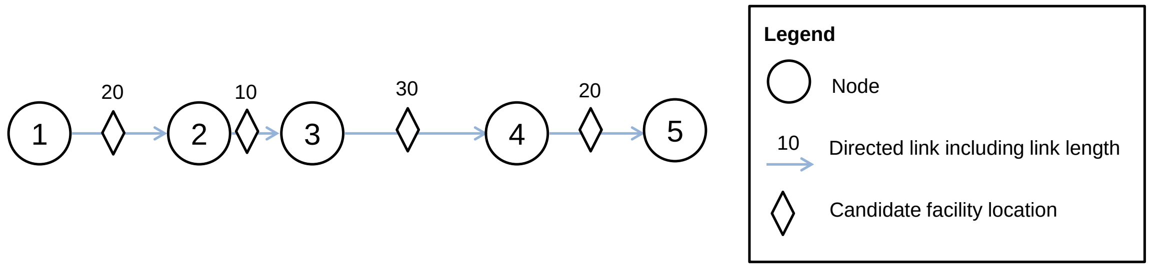

Fig. 3.1 Schematic view of the set generation logic for set $\Psi^{w}_{i,j,r}$

Constraint (3) determines the number of total wireless charging facilities p to be located. This is in line with the conventional FCLMs and FRLMs, for which the total number of facilities to be located is predetermined as well.

The incorporation of constraints (4), (5) and (8), which describe the stochastic user equilibrium [SHEFFI, 1985], allows for the determination of the traffic flow pattern and for the investigation of changes in route choice due to travel times and the availability of charging infrastructure in the network. The flow on each link in the network is computed as the sum over the products of the route flow, f^{w}_{r}, and the link-path incidence, $\delta^{w}_{a,r}$ for each link in constraint (4).

Constraint (5) entails the MNL model which determines the electric vehicles drivers’ routing choice behaviour. The MNL model depicts the interaction between equilibrium traffic flow assignment and location of charging facilities as it includes both decision variables into the utility function. The behaviour is assumed to be influenced by two factors: The flow-dependent route travel time t_r^w and the availability of charging facilities as incorporated by the variable y_r^w. Both of these are weighted with scaling parameters α and β, respectively, to incorporate routing decision sensitivity in the utility function for travel time and availability of charging facilities.

In practice, the values of parameters α and β must be calibrated and validated (for instance by utilizing survey data). However, as this research focuses on model formulation, α and β are assumed to be given as input parameters. The effects of the parameters on the overall model solution are analyzed in section 4.2.

Travel times are determined by constraints (6) and (7). The total travel time when using route r is determined by constraint (6) and constraint (7) allows for the computation of the link travel time, which is based on the Bureau of Public Roads' (BPR) travel time function [COMSIS CORP. 1983]. The BPR travel time function entails the interrelationship of link free flow travel time, t_a^0, link capacity c_a and flow on each link h_a.

The flow on each route is computed by using constraint (8), which takes the OD travel demand and the probability of the flow choosing a route between an OD pair into account. Finally, constraints (9) and (10) are added to express the binary properties of the decision variables x_k and y_r^w.

Taken together, the objective function and the model constraints enable the investigation of the interaction between the facility locations and the traffic flow in the network. However, the proposed model is a mixed integer nonlinear problem. The solution method and the resulting reformulated model are presented in subsection 3.2.3.

3.2.3 Solution method

In this section, the solution method to solve the model formulation is presented. It is intended to employ a global optimization method. Hence, the original nonlinear model formulation is transformed into a linear model to which common solution algorithms available through commercially available software can be applied. Nonlinearity is introduced into the model in three ways: First, the objective function entails the term $f_r^w y_r^w$, second, constraint (5) includes the nonlinear MNL model and third, constraint (7) entails the nonlinear BPR travel time function.

The objective function is linearized by applying the reformulation-linearization technique (RLT). Constraints (5) and (7) are linearized by transformation into logarithmic terms, followed by piecewise linear approximation.

3.2.3.1 Linearization of objective function

The first nonlinearity in the proposed model stems from the terms in the objective function, more specifically the variable f_r^w, which determines the continuous flow on route r between OD pair $w \in W$, and the binary variable y_r^w, which controls the flow that is considered to be captured, or otherwise. By employing the reformulation-linearization technique, originally proposed by SHERALI AND ADAMS [1994] the objective function can be substituted by a set of linear constraints. Another application of this technique in the context of transportation networks can be found for example in WANG ET AL. [2015].

The new set of linear constraints and the equivalence of this set to the original objective function are demonstrated in detail below. First, a new auxiliary variable φ_r^w as depicted in equation (11) is introduced to substitute the term in the objective function.

$$\varphi_r^w = f_r^w y_r^w, \qquad \forall\, r \in R^w, \quad w \in W \qquad (11)$$

Second, a parameter $\underline{f_r^w}$ to set the lower bound and a parameter $\overline{f_r^w}$ to set the upper bound of the route flow variable are introduced. These parameters must be set sufficiently small ($\underline{f_r^w}$) and sufficiently large ($\overline{f_r^w}$), respectively, to ensure that the feasible domain of f_r^w is not reduced.

Third, the four additional linear constraints shown in equations (12) - (15) incorporate the newly defined parameters $\underline{f_r^w}$ and $\overline{f_r^w}$ and must be employed to transform the objective function into equivalent linear constraints.

$$\varphi_r^w - \underline{f_r^w} y_r^w \geq 0, \qquad \forall\, r \in R^w, \quad w \in W \qquad (12)$$

$$\varphi_r^w - \overline{f_r^w} y_r^w \leq 0, \qquad \forall\, r \in R^w, \quad w \in W \qquad (13)$$

$$\varphi_r^w - f_r^w + \underline{f_r^w}(1 - y_r^w) \leq 0, \qquad \forall\, r \in R^w, \quad w \in W \qquad (14)$$

$$\varphi_r^w - f_r^w + \overline{f_r^w}(1 - y_r^w) \geq 0, \qquad \forall\, r \in R^w, \quad w \in W \qquad (15)$$

Variable y_r^w can only obtain the values zero or one due to its binary nature. Hence, in order to prove the equivalence of equation (11) and the linear constraints in equations (12) – (15), it must be proven for both cases of $y_r^w = 0$ and $y_r^w = 1$.

For the first case, if $y_r^w = 0$, the product $\varphi_r^w = f_r^w y_r^w = 0$. Accordingly, the linear constraints in equations (12) – (15) form equations (16) and (17) below. Both inequalities are only satisfied if $\varphi_r^w = 0$.

$$0 \leq \varphi_r^w \leq 0, \qquad \forall\, r \in R^w, \quad w \in W \qquad (16)$$

$$f_r^w - \overline{f_r^w} \leq \varphi_r^w \leq f_r^w - \underline{f_r^w}, \qquad \forall\, r \in R^w, \quad w \in W \tag{17}$$

For the second case, if $y_r^w = 1$, the product $\varphi_r^w = f_r^w\, y_r^w = f_r^w$. Accordingly, the linear constraints in equations (12) – (15) form equations (18) and (19). Both inequalities are only satisfied if $\varphi_r^w = f_r^w$.

$$\underline{f_r^w} \leq \varphi_r^w \leq \overline{f_r^w}, \qquad \forall\, r \in R^w, \quad w \in W \tag{18}$$

$$f_r^w \leq \varphi_r^w \leq f_r^w, \qquad \forall\, r \in R^w, \quad w \in W \tag{19}$$

To conclude, for both cases, $y_r^w = 0$ and $y_r^w = 1$, the equivalence is proven. Hence, the nonlinear objective function in equation (1) can be substituted by equation (11) as the new objective function and by equations (12) – (15) as additional linear model constraints.

3.2.3.2 Linearization of the route choice probability constraint

The MNL model in constraint (5) introduces the second nonlinearity. It is incorporated to govern the traffic flow choosing its route r between OD pair w. Constraint (5) can be converted into the following two equations (20) and (21) due to the property of Independence from Irrelevant Alternatives (IIA) which is inherent to the MNL model. This property entails that "*the ratio of probabilities of any two alternatives is independent of the choice set*" [BEN-AKIVA AND BIERLAIRE, 1999]. Equation (20) expresses the ratio of the probabilities of the flow choosing route r between OD pair w and equation (21) ensures that the sum of all probabilities for the relevant routes equals one which is according to any probability mass function.

$$\frac{P_r^w}{P_s^w} = \frac{\exp(\alpha t_r^w + \beta y_r^w)}{\exp(\alpha t_s^w + \beta y_s^w)}, \qquad \forall\, r,s \in R^w, r \neq s, \quad w \in W \tag{20}$$

$$\sum_{r \in R^w} P_r^w = 1, \qquad \forall\, r \in R^w, \quad w \in W \tag{21}$$

The linearization of the route choice probability constraint is performed according to the method presented by LIU AND WANG [2015]. According to their method, taking the logarithm of both sides of equation (20) results in equation (22).

$$ln\left(\frac{P_r^w}{P_s^w}\right) = ln\left(\frac{\exp(\alpha t_r^w + \beta y_r^w)}{\exp(\alpha t_s^w + \beta y_s^w)}\right), \qquad \forall\, r,s \in R^w, r \neq s, \quad w \in W \tag{22}$$

Then, employing standard logarithmic identities yields equation (23).

$$ln(P_r^w) - ln(P_s^w) = \alpha(t_r^w - t_s^w) + \beta(y_r^w - y_s^w), \qquad \forall\, r,s \in R^w, r \neq s, \quad w \in W \tag{23}$$

Consequently, equations (21) and (23) are used to express the nonlinear route choice probability constraint (5). The remaining nonlinearity lies in the logarithmic function, which is to be linearized using piecewise approximation and which is presented in the remainder of this section.

3.2.3.3 Linearization of link travel time

The following linearization of the link tavel time is performed according to the method presented by LIU AND WANG [2015]. It should be noted that the BPR function in equation (7), that is included for the computation of the link travel time, contains another nonlinear term. The continuous flow variable h_a which is a decision variable in the model, is exponential by a factor of 4 ($(h_a)^4$) in the BPR function. Subsequently, the auxiliary variable k_{f_a} is included to express this power function (see equation (24)).

$$k_{f_a} = (h_a)^4, \qquad \forall a \in A \tag{24}$$

First, the logarithm on both sides of equation (24) is taken, which yields equation (25). Similiar to the method of linearizing the route choice probability constraint, the nonlinearity of the power function is thus expressed using a logarithmic function.

$$ln\,(k_{f_a}) = 4\,(ln\, h_a), \qquad \forall a \in A \tag{25}$$

Next, the nonlinear term in the BPR function in equation (7) is replaced by the auxiliary variable k_{f_a}, which results in a linearization of the BPR link travel time function as shown in equation (26).

$$t_a = t_a^0 \left[1 + \frac{0.15}{(c_a)^4} k_{f_a}\right], \qquad \forall a \in A \tag{26}$$

Subsequently, equations (25) and (26) are included as supplementary constraints to the model.

3.2.3.4 Piecewise linearization of logarithmic terms

To conclude, the logarithmic functions constitute the remaining nonlinearity in the model. Logarithmic functions are globally concave, hence the linearization can be implemented straightforwardly using piecewise linear approximation. Several linearization methods are published to deal with this problem, for example the mixed integer linearization method of WANG AND LO [2010] and LIU AND WANG [2015].

Instead, here, the piecewise linearization is implemented using Special Ordered Sets Type 2. BEALE AND TOMLIN [1970] created the theoretical framework for Special Ordered Sets in order to facilitate the process of finding global optimum solutions to problems featuring piecewise linear approximations of nonlinear functions. They describe two types of

Special Ordered Sets (SOS): SOS Type 1 and SOS Type 2. SOS Type 1 are defined as "sets of variables of which not more than one member may be nonzero to the final solution" [BEALE AND FORREST, 1976]. Accordingly, in SOS Type 2 "not more than two members may be nonzero in the final solution" [BEALE AND FORREST, 1976] and furthermore, if two members of the set are nonzero they must be adjacent [BEALE AND FORREST, 1976]. With this definiton at hand, the formulation of the piecewise linearization of the logarithmic function is illustrated.

First, $l_{h_a} = ln\, h_a$ is introduced as an auxiliary variable. Then, the feasible domain of the logarithmic function is subdivided into intervals delimited by a set of n breakpoints BP $(h_a^n | ln\, h_a^n)$. Furthermore, the SOS Type 2 E_a^n with n members ε_a^n of non-negative convex combination weights associated with each breakpoint is introduced. By definition, not more than two members of the set E_a^n are strictly positive and if so, these two members must be adjacent. This is to ensure that the piecewise approximation of the feasible domain of the logarithmic function is triggered only for one specific interval.

Following the notation of SOS Type 2 [BEALE AND FORREST, 1976] the logarithmic function can be expressed by the following linear constraints (27) - (30):

$$h_a = \sum_{n \in N} h_a^n \, \varepsilon_a^n \, , \tag{27}$$

$$l_{h_a} = \sum_{n \in N} \ln(h_a^n) \, \varepsilon_a^n \, , \tag{28}$$

$$\sum_{n \in N} \varepsilon_a^n = 1 \, , \tag{29}$$

$$\text{SOS Type 2: } \varepsilon_a^1, \, \varepsilon_a^2, \ldots \varepsilon_a^n, \qquad \forall a \in A \tag{30}$$

To further illustrate the principle of piecewise linearization using SOS Type 2, a logarithmic function including a schematic view of the piecewise linear approximations of the respective logarithmic function including breakpoints is depicted in Fig. 3.2.

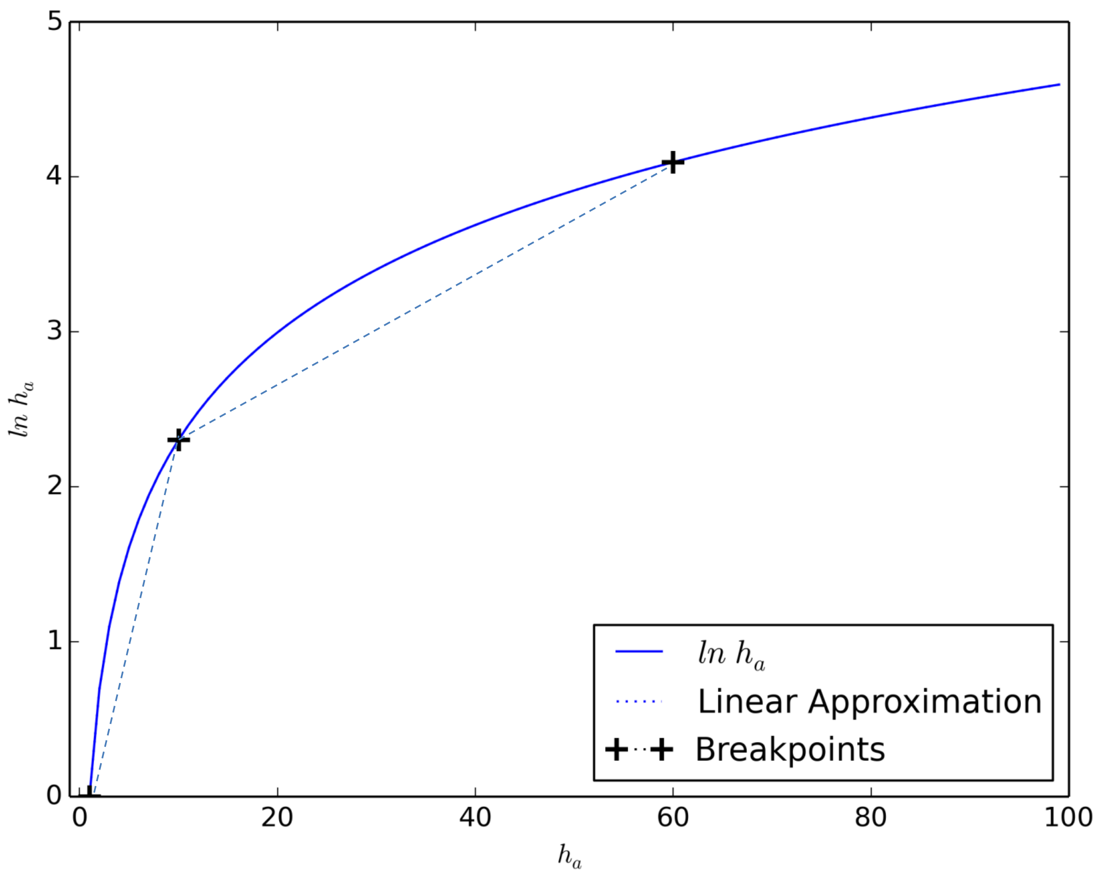

Fig. 3.2 Schematic view of piecewise linearization with SOS Type 2

Last, the logarithmic functions in equations (23) and (25) are expressed by the following auxiliary variables: $l_{P_r^w} = ln\,(P_r^w)$, $l_{P_s^w} = ln(\,P_s^w)$ and $l_{k_{fa}} = ln\,k_{fa}$. This leads in turn to the set of equations (31) and (32):

$$l_{P_r^w} - l_{P_s^w} = \alpha(t_r^w - t_s^w) + \beta(y_r^w - y_s^w) \tag{31}$$

$$l_{k_{fa}} = 4\,(l_{h_a}) \tag{32}$$

These equations in combination with the equations resulting from the piecewise approximation using SOS Type 2 (equations (27) – (30)) result in the linearization of the logarithmic terms.

3.2.3.5 Reformulated model

The model can be formulated as a mixed integer linear program by replacing the previously described non-linear objective function and the nonlinear constraints with the linearized versions of these equations and employed them as constraints. Objective function (33) and constraints (2) – (4), (6), (8) – (10), (12) – (15), (21), and (26) – (32) depict the framework for the linearized version of the flow-capturing model with equilibrium constraints.

The reformulated model is illustrated below.

Maximize:

$$\sum_{w \in W} \sum_{r \in R^w} \varphi_r^w \tag{33}$$

Subject to:

$$\sum_{k \in \Psi_{i,j,r}^w} x_k \geq y_r^w, \qquad \forall\, a_{i,j} \in A_{i,j,r}^w, r \in R^w, \quad w \in W \tag{2}$$

$$\sum_{k \in K} x_k = p \tag{3}$$

$$h_a = \sum_{w \in W} \sum_{r \in R^w} f_r^w \delta_{a,r}^w , \qquad \forall a \in A \tag{4}$$

$$t_r^w = \sum_{a \in A} \delta_{a,r}^w t_a , \qquad \forall\, r \in R^w, \quad w \in W \tag{6}$$

$$f_r^w = q^w P_r^w , \qquad \forall\, r \in R^w, \quad w \in W \tag{8}$$

$$\varphi_r^w - \underline{f_r^w} y_r^w \geq 0, \qquad \forall\, r \in R^w, \quad w \in W \tag{12}$$

$$\varphi_r^w - \overline{f_r^w} y_r^w \leq 0, \qquad \forall\, r \in R^w, \quad w \in W \tag{13}$$

$$\varphi_r^w - f_r^w + \underline{f_r^w}(1 - y_r^w) \leq 0, \qquad \forall\, r \in R^w, \quad w \in W \tag{14}$$

$$\varphi_r^w - f_r^w + \overline{f_r^w}(1 - y_r^w) \geq 0, \qquad \forall\, r \in R^w, \quad w \in W \tag{15}$$

$$\sum_{r \in R^w} P_r^w = 1 , \qquad \forall\, r \in R^w, \quad w \in W \tag{21}$$

$$t_a = t_a^0 \left[1 + \frac{0.15}{(c_a)^4} k_{f_a}\right] , \qquad \forall a \in A \tag{26}$$

$$h_a = \sum_{n \in N} h_a^n \, \varepsilon_a^n , \qquad \forall a \in A \tag{27}$$

$$l_{h_a} = \sum_{n \in N} \ln(h_a^n) \, \varepsilon_a^n , \qquad \forall a \in A \tag{28}$$

$$\sum_{n \in N} \varepsilon_a^n = 1 \,, \qquad \forall a \in A \tag{29}$$

$$l_{P_r^w} - l_{P_s^w} = \alpha(t_r^w - t_s^w) + \beta(y_r^w - y_s^w), \qquad \forall\, r, s \in R^w, r \neq s, \quad w \in W \tag{31}$$

$$l_{k_{f_a}} = 4\,(l_{h_a}), \qquad \forall a \in A \tag{32}$$

$$x_a \in \{0|1\}, \qquad \forall a \in A \tag{9}$$

$$y_{a,r}^w \in \{0|1\}, \qquad \forall\, r \in R^w, \quad w \in W, \quad a \in A \tag{10}$$

$$\text{SOS Type 2: } \varepsilon_a^1, \varepsilon_a^2, \dots \varepsilon_a^n, \qquad \forall a \in A \tag{30}$$

To conclude, the proposed flow-capturing location model with equilibrium constraints is reformulated into a mixed integer linear program. First, the objective function is linearized and expressed by equation (33) and (12) – (15). Equations (21) and (31) constitute the linearization of the route choice probability constraint and equations (26) and (32) the linearization of link travel time. Finally, the resulting logarithmic terms are linearized in a piecewise manner using Special Ordered Sets Type 2 by equations (27) – (30). By developing a fully linearized model, existing solution algorithms (e.g. Simplex, Branch-and-Bound) and commercially available software packages employing these algorithms for solving linear programs can be deployed.

3.2.3.6 Simplification of travel time function

In order to reduce complexity of the model, the BPR travel time function in constraint (7) can be replaced by a linear travel time function. This linear travel time function can be formulated by introducing an additional set S_a that includes capacity related scaling factors $S_a, a \in A$ which are specified for each link. These scaling factors are multiplied with the specific flow on each link (h_a) and added to the link free flow travel time (t_a^0). Consequently, constraint (7) is replaced by the linear travel time function expressed by equation (34):

$$t_a = S_a h_a + t_a^0 \qquad \forall a \in A \tag{34}$$

The advantages of assuming that the link travel time can be calculated by applying a linear travel time function can be summarized by the following aspects: First, no additional linearization of the BPR function is necessary. Second, the number of Special Ordered Sets in the optimization problem is reduced, which in return decreases the computation time required for solving the optimization problem.

3.3 Set-covering location model with charging system design

The set-covering location model with charging system design approaches the problem of optimally locating wireless charging facilities in discrete road networks from a different perspective than the flow-capturing model with equilibrium constraints. It tackles the limitation of the flow-covering model with equilibrium constraints, namely the assumption that vehicles can fully recharge when traversing a wireless charging station. Hence, a flow-based set-covering location model approach that incorporates the infrastructural constraints and the design of the wireless charging system is formulated.

This approach seeks to minimize the total length of the wireless charging system that is to be installed while covering all electric vehicles' charging demand and taking technical specifications of the dynamic wireless charging infrastructure and transportation infrastructure into account. The complete modelling framework and the underlying assumptions and considerations are described in section 3.3.1.

The interactions between facility locations, electric vehicles' charging demand, specifications of the charging system and costs are implicitly captured by integrating the length of wireless charging segments into the model formulation as decision variables and hence, providing them as endogenously computed variables of the model. The proposed model formulation extends the flow-based set-covering approach developed by FULLER [2016] and is described in section 3.3.2.

Again, the proposed mathematical model results in a formulation that is a mixed integer nonlinear problem. The global optimization method presented in section 3.2.3 is applied to the nonlinear terms of the set-covering location model with charging system design by transforming the nonlinear objective function into equivalent linear equations. The description of the solution method to linearize the set-covering model with charging system design is illustrated in section 3.3.3.

3.3.1 Modelling framework: Considerations and assumptions

The objective of the model is to minimize the total length of the wireless charging system that must be installed to cover all charging demand in the network. The modelled system simultaneously considers the following three building blocks: wireless charging infrastructure, electric vehicles and the transportation network. The decision variables of the model include the charging facilities' location plan, as well as the length of the wireless charging segments to be installed; both of which are obtained endogenously from the model solution.

The model seeks to cover all charging demand of electric vehicles in the network; hence no changes in route choice due to the availability of charging infrastructure are expected. Therefore, the assignment of the origin-destination (OD) demand to the shortest paths is implemented through preprocessing using common shortest path algorithms. The design of the charging system is modelled on a rather detailed level as power consumption, power level and vehicle speed are taken into consideration. The amount of energy that can be received by the electric vehicle while using a link with wireless charging infrastructure depends on the time the vehicle spends on this link. Hence, vehicle speed is a crucial factor for determining the design of the charging infrastructure.

The flow-based set-covering approaches developed by WANG AND LIN [2009] and by FULLER [2016] serves as basis for the model formulation. As an extension to the model by FULLER [2016] the length of the segments of dynamic charging infrastructure and the energy which has to be transmitted are incorporated into the model as decision variables in addition to the facility location. Furthermore, the proposed model entails a different approach for determining if the charging demand during a trip is considered to be satisfied or otherwise. For each sequential sub-route an additional constraint is added. These constraints determine if the charging demand is satisfied. Subsequently, as both the objective function and the constraint determining the added energy entail nonlinear terms, the model is classified as mixed integer nonlinear model (MINLP). However, by utilizing this approach, the facility location plan, the charging demand and the charging system design is determined simultaneously.

Again, assumptions are made before formulating the mathematical model. All vehicles on the network are assumed to be electric vehicles. The origin and destination relations are considered as input parameters and no dynamic traffic assignment to the network is assumed. These assumptions are justifiable because first, the model locates as many charging facilities as necessary to cover all the charging demand in the network and second, due to the full coverage there are no expected changes in route choice behavior as travelers can keep their planned routes without running out of energy in any case. Additionally, it is assumed that all electric vehicles have access to a charging facility at each node of trip origin and consequently, start their trips with a fully charged battery. All links in the network are considered as candidate facility location where a dynamic wireless charging infrastructure could potentially be located. Hence, for this model formulation, no additional set for the candidate facility locations is introduced. The required lengths of the charging segments are computed endogenously by the model taking vehicles speed, power consumption and power level received by the electric vehicle from dynamic charging segment on each individual link into consideration. The required length of charging facilities strongly influences the total cost of the infrastructure to be installed. Subsequently, the interaction of charging demand, the total length of installed dynamic wireless charging infrastructure and driving behavior is captured by the model formulation. The charging process is implemented as dynamic process and vehicles can charge without time loss while driving. Furthermore, it is assumed that the installed infrastructure ensures that the range of the vehicles never falls below 20% of the total range. This assumption is included in order to account for range anxiety by ensuring that a minimum range is always available. However, the amount of energy charged depends on the speed, the power level of the segment and the length of the dynamic wireless charging infrastructures' segment. The charging demand of the vehicles is satisfied if they can complete their routes without running out of energy. The range and the lengths of the links are taken into consideration when determining if they can complete their routes and consequently, added as model constraints.

The notation presented in Tab. 3.3 is used for the mathematical formulation of the sets and parameters of the set-covering model with charging system design. Similarly, the building blocks (transportation network, charging infrastructure and electric vehicles) taken into account for the model formulation are taken as basis to classify the sets and parameters.

Sets and Parameters		
Building block: transportation network		
W		Set of all OD pairs w
N		Set of all nodes n in network, $i, j \in N$
A		Set of all links a in network
G		Network $G = (N, A)$ consisting of sets of nodes N and links A
R^w		Set of all feasible routes r of OD pair $w \in W$
M		Cardinality of set R^w
S_r^w		Set of all links on sub-routes s of all feasible routes r of OD pair $w \in W$, starting with first link of routes r
$\delta_{a,r}^w$		Link-path incidence, $\delta_{a,r}^w = 1$ if link $a \in A$ belongs to route r between OD pair w, 0 otherwise
D_a	$[km]$	Set of all length of link distances d_a
Building block: charging infrastructure		
PL_a	$[kW]$	Power level received by vehicle from dynamic charging segment on link a
Building block: electric vehicles		
R_{start}	$[km]$	Electric range of EVs at trip start
R_{max}	$[km]$	Maximum electric range of EVs
PC	$\left[\frac{kWh}{100km}\right]$	Average power consumption of EVs
V_a	$\left[\frac{km}{h}\right]$	Average vehicle speed on link a

Tab. 3.3 Sets and parameters set-covering location model with charging system design

The notation of the decision variables of the set-covering model with charging system design is presented below in Tab. 3.4.

Decision variables		
x_a		Binary variable, 1 if there is a wireless charging facility on link a, 0 otherwise
$y_{a,r}^{w}$		Binary variable, 1 if EVs are recharged on link a, on route r of OD pair $w \in W$, 0 otherwise
l_a	$[km]$	Length of wireless charging segment on link $a < d_a$
$paer_a$	$[km]$	Potentially added electric range from charging on link a
$aer_{a,r}^{w}$	$[km]$	Added electric range on link a, on route r of OD pair $w \in W$

Tab. 3.4 Decision variables set-covering location model with charging system design

3.3.2 Model formulation

The model formulation of the set-covering model with charging system design is demonstrated below taking the considerations and the assumptions for the modelling framework into consideration. The presented model is based on previous research of WANG AND LIN [2009] and FULLER [2016]. However, the objective function and specifics of the constraints were modified and extended to improve accounting for specifics of the installation of a dynamic wireless charging system. Specifically, the potentially variable length of dynamic wireless charging segments is incorporated to capture the charging demand more efficiently. In the section below, a detailed account of the models' objective and constraint is given.

Minimize:

$$\sum_{a \in A} l_a \, x_a \tag{35}$$

Subject to:

$$\sum_{a \in S_r^w} aer_{a,r}^{w} \geq 0.2 \ R_{max} - R_{start} + \sum_{a \in S_r^w} d_a \,, \qquad \forall \, a \in S_r^w \tag{36}$$

$$\sum_{a \in S_r^w} aer_{a,r}^{w} \leq R_{max} - R_{start} + \sum_{a \in S_r^w} d_a \,, \qquad \forall \ a \in S_r^w \tag{37}$$

$$aer_{a,r}^{w} = y_{a,r}^{w} \, paer_a, \qquad \forall a \ \in A \tag{38}$$

$$paer_a = \frac{PL_a}{v_a PC}\, l_a\,, \qquad \forall a \in A \tag{39}$$

$$\sum_{r \in R^w} y_{a,r}^w \leq M\, x_a, \qquad \forall a \in A \tag{40}$$

$$x_a \in \{0|1\}, \qquad \forall a \in A \tag{41}$$

$$y_{a,r}^w \in \{0|1\}, \qquad \forall\, r \in R^w, \quad w \in W, \quad a \in A \tag{42}$$

$$l_a \leq d_a, \qquad \forall a \in A \tag{43}$$

$$0 \leq aer_{a,r}^w \leq R_{max}\,, \qquad \forall\, r \in R^w, \quad w \in W, \quad a \in A \tag{44}$$

$$0 \leq paer_a \leq R_{max}\,, \qquad \forall\, r \in R^w, \quad w \in W, \quad a \in A \tag{45}$$

The objective function of the proposed model as presented in equation (35) seeks to minimize the length of the dynamic wireless charging infrastructure that is to be installed to cover all charging demand in the network. It presents the first nonlinear term of the model as it involves the product of the two decision variables l_a which determines the length the wireless charging segments to be installed, and x_a which determines whether a wireless charging segment is installed or not.

Constraints (36) and (37) incorporate the recharging logic of the electric vehicles by taking the maximum EV range (R_{max}), the range with which the EVs start their trip (R_{start}), the link distances (d_a) and the added electric range ($aer_{a,r}^w$) into consideration. Firstly, it is assumed that the remaining range after traversing any link on the network and the potentially received additional range from a wireless charging segment on that specific link must never fall below 20% of the maximum EV range. Second, this term must never exceed the maximum EV range:

$$0.2\ \mathrm{R}_{\max} \leq R_{a,r}^w - d_a + aer_{a,r}^w \leq \mathrm{R}_{\max}, \qquad \forall\, r \in R^w,\ w \in W,\ a \in A \tag{46}$$

The fundamental consideration presented in equation (46) is to be satisfied on each link by taking the length of previous links and previously added energy into account. Consequently, constraint (36) ensures that the actual range never falls below 20% of the maximal range when the vehicle is fully charged so that all vehicles can complete their trips. This is achieved by taking the distance traveled on each sub-route of all routes and the energy that is added on each sub-route by the wireless charging segment into consideration. To specify, each link sub-route is considered separately, allowing for the incorporation of route-specific path history as the constraint takes the link distances and the added energy on the wireless charging segments which were passed through previously into account. This allows for a truly flow-based depiction of the recharging.

In addition, constraint (37) restricts the sum of all added energy on each sub-route to always be less or equal as the sum of the distances traveled and the difference of the maximum range and the range with which the vehicles start their trips. Furthermore, constraint (38) is included to determine if an EV is recharged on a link and how much electric range is potentially added from charging on the specific link.

The amount of energy that can be supplied to the electric vehicles is calculated by constraint (39) which takes the power level received by the vehicle from the dynamic charging segment on the roadway, the average vehicle speed and the average vehicle power consumption as constant parameters into account. Furthermore, the length of the wireless charging segment to be installed on a link is incorporated as a decision variable, which allows the model to determine the optimal length endogenously. Constraint (40) determines which links are equipped with a wireless charging facility and furthermore ensures that the maximal possible sum of EV charging processes $y_{a,r}^{w}$ on a link is not exceeded. Constraints (41) and (42) ensure that the binary properties of the decision variables x_a and $y_{a,r}^{w}$ are implemented. Constraint (43) limits the maximum length of wireless charging segments to the according link length. Furthermore, constraints (44) and (45) define the boundaries of the potentially added energy and the added energy on the links between zero and the maximum range of the electric vehicles.

Subsequently, the objective function and the model constraints form the set-covering location model with charging system design, which allows for the analysis of the minimum length of wireless charging infrastructure under the assumption of full coverage, is defined. As well as the flow-capturing model with equilibrium constraints, the set-covering location model with charging system design is formulated as a mixed integer nonlinear problem. In the next section 3.3.3 the solution method for the set-covering location model is derived. Moreover, the reformulated model is presented.

3.3.3 Solution method

The solution method to solve the model formulation is described in this section. The proposed model is to be classified as a mixed integer nonlinear program (MINLP). The global optimization solution method developed in section 3.2.3 to linearize the model is applied to the set-covering model with charging system design as well.

The nonlinearity in the set-covering model with charging system design is twofold: First, the objective function includes the product of two decision variables: the length of the wireless charging segment on link a (l_a) and the binary variables x_a that determines whether a facility is to be located. Next, constraint (38) which determines the added electric range on link a on route r of OD pair w, $aerc_{a,r}^{w}$, contains a nonlinear term. It is expressed as the product of the two decision variables, first the variable to determine the potentially added electric range from charging on link a, $paerc_a$, and second, the the binary variable $y_{a,r}^{w}$ that determines if the electric vehicles are considered to be recharged on link a, on route r of OD pair w.

The objective function and constraint (38) are both linearized by applying the reformulation-linearization technique (RLT). The result of this linearization process is the transformation of the original mixed integer nonlinear model into a mixed integer linear problem which can be solved by making use of the appropriate solution algorithm such as branch and bound.

3.3.3.1 Linearization of objective function

The first nonlinearity of the set-covering model with charging system design stems from the objective function. Consequently, by employing the reformulation-linearization technique, originally proposed by SHERALI AND ADAMS [1994], the nonlinear term can be substituted by an equivalent set of linear constraints. The linearization approach is correspondent to the approach presented in section 3.2.3 for the linearization of the objective function of the flow-covering model with equilibrium constraints. Consequently, a condensed version of the linearization process is presented below.

The new auxiliary variable ϕ_a as depicted in equation (47) is added as the new objective function to express and substitute the original term in the objective function.

$$\phi_a = l_a x_a, \qquad \forall\, a \in A \tag{47}$$

The representation of the lower and upper bounds, respectively, of the variable l_a is conducted by introducing parameters $\underline{l_a}$ and $\overline{l_a}$. In order to ensure the retention of the feasible range of the decision variable l_a the selected parameters must be chosen sufficiently small ($\underline{l_a}$) and sufficiently large ($\overline{l_a}$).

Subsequently, the linearization of the objective function is expressed by equation (47) and the four additional linear equations shown in equations (48) - (51). These additional constraints are incorporated into the model formulation as well.

$$\phi_a - \underline{l_a} x_a \geq 0, \qquad \forall\, a \in A \tag{48}$$

$$\phi_a - \overline{l_a} x_a \leq 0, \qquad \forall\, a \in A \tag{49}$$

$$\phi_a - l_a + \underline{l_a}(1 - x_a) \leq 0, \qquad \forall\, a \in A \tag{50}$$

$$\phi_a - l_a + \overline{l_a}(1 - x_a) \geq 0, \qquad \forall\, a \in A \tag{51}$$

The proof of equivalence of these four additional constraints to the proposed objective function corresponds to the proof of equivalence provided in section 3.2.3 for the linearization of the objective function of the flow-covering model with equilibrium constraints.

3.3.3.2 Linearization of added energy constraint

The second nonlinearity of the set-covering model with charging system design is in equation (38) which determines the added energy constraint. However, as shown in the section above this nonlinear term can be converted into an equivalent set of linear constraints by adopting the reformulation-linearization technique. The term is expressed by the variable $aer_{a,r}^{w}$ as depicted in constraint (38):

$$aer_{a,r}^{w} = y_{a,r}^{w}\, paer_a, \qquad \forall\, r \in R^{w}, \quad w \in W \tag{38}$$

Next, parameters $\underline{paer_a}$ and $\overline{paer_a}$ are added as auxiliary variables to represent the lower and upper bounds, respectively, of the variable $paer_a$. Again, the variables must be sufficiently small ($\underline{paer_a}$) and sufficiently large ($\overline{paer_a}$) to avoid a reduction of the feasible range of the decision variable $paer_a$.

Lastly, the four additional linear constraints shown in equations (52) – (55) must be added to the model to transform constraint (38) into a set of equivalent linear constraints.

$$aer_{a,r}^{w} - \underline{paer_a}\, y_{a,r}^{w} \geq 0, \qquad \forall\, r \in R^{w}, \quad w \in W \tag{52}$$

$$aer_{a,r}^{w} - \overline{paer_a}\, y_{a,r}^{w} \leq 0, \qquad \forall\, r \in R^{w}, \quad w \in W \tag{53}$$

$$aer_{a,r}^{w} - paer_a + \underline{paer_a}(1 - y_{a,r}^{w}) \leq 0, \qquad \forall\, r \in R^{w}, \quad w \in W \tag{54}$$

$$aer_{a,r}^{w} - paer_a + \overline{paer_a}(1 - y_{a,r}^{w}) \geq 0, \qquad \forall\, r \in R^{w}, \quad w \in W \tag{55}$$

Again, the proof of equivalence of these four additional constraints to the original constraint (38) corresponds to the proof of equivalence provided in section 3.2.3 for the linearization of the objective function of the flow-covering model with equilibrium constraints.

3.3.3.3 Reformulated model

The model can be formulated as a mixed integer linear program when the nonlinear objective function and the nonlinear added energy constraint (38) are substituted by the equations derived from the linearization process. Objective function (56) and constraints (36), (37), (39) – (45), and (48) - (55) describe the linearized version of the set-covering model with charging system design. The reformulated model is depicted below.

Minimize:

$$\sum_{a \in A} \phi_a \tag{56}$$

Subject to:

$$\sum_{a \in S_r^w} aer_{a,r}^w \geq 0.2\ R_{max} - \mathrm{R_{start}} + \sum_{a \in S_r^w} d_a, \qquad \forall\, a \in S_r^w \tag{36}$$

$$\sum_{a \in S_r^w} aer_{a,r}^w \leq R_{max} - \mathrm{R_{start}} + \sum_{a \in S_r^w} d_a, \qquad \forall\ a \in S_r^w \tag{37}$$

$$paer_a = \frac{PL_a}{v_a PC}\ l_a, \qquad \forall a\ \in A \tag{39}$$

$$\sum_{r \in R^w} y_{a,r}^w \leq\ M\, x_a, \qquad \forall a\ \in A \tag{40}$$

$$\phi_a -\ \underline{l_a} x_a \geq 0, \qquad \forall\, a\ \in A \tag{48}$$

$$\phi_a -\ \overline{l_a} x_a \leq 0, \qquad \forall\, a\ \in A \tag{49}$$

$$\phi_a -\ l_a + \underline{l_a}(1 - x_a) \leq 0, \qquad \forall\, a\ \in A \tag{50}$$

$$\phi_a -\ l_a + \overline{l_a}(1 - x_a) \geq 0, \qquad \forall\, a\ \in A \tag{51}$$

$$aer_{a,r}^w -\ \underline{paer_a}\, y_{a,r}^w \geq 0, \qquad \forall\, r\ \in R^w, \quad w\ \in W \tag{52}$$

$$aer_{a,r}^w -\ \overline{paer_a}\, y_{a,r}^w \leq 0, \qquad \forall\, r\ \in R^w, \quad w\ \in W \tag{53}$$

$$aer_{a,r}^w -\ paer_a + \underline{paer_a}(1 - y_{a,r}^w) \leq 0, \qquad \forall\, r\ \in R^w, \quad w\ \in W \tag{54}$$

$$aer_{a,r}^{w} - paer_a + \overline{paer_a}(1 - y_{a,r}^{w}) \geq 0, \qquad \forall\, r \in R^w, \quad w \in W \tag{55}$$

$$x_a \in \{0|1\}, \qquad \forall a \in A \tag{41}$$

$$y_{a,r}^{w} \in \{0|1\}, \qquad \forall\, r \in R^w, \quad w \in W, \quad a \in A \tag{42}$$

$$l_a \leq d_a, \qquad \forall a \in A \tag{43}$$

$$0 \leq aer_{a,r}^{w} \leq R_{max}, \qquad \forall\, r \in R^w, \quad w \in W, \quad a \in A \tag{44}$$

$$0 \leq paer_a \leq R_{max}, \qquad \forall\, r \in R^w, \quad w \in W, \quad a \in A \tag{45}$$

The formulation above reflects the mixed integer linear program of the set-covering location model with wireless charging system design. Both previously nonlinear terms, the objective function as well as the constraints for determining the added energy are expressed as linear terms by equations (48) – (51) and (52) – (55).

4 Model application

Preliminary considerations for the application and analysis framework, in particular the definition of the baseline network and the computational modelling and optimization framework that are valid for both models are described in section 4.1. The model-specific scenarios and results of the application of the flow-capturing location model with stochastic user equilibrium to a discrete road network are presented in section 4.2. In addition, the model-specific scenarios and results of the application of the set-covering location model with charging system design are given in section 4.3.

4.1 Application and analysis framework

In the following section, a framework for testing and analyzing both models is developed in order to demonstrate validity and solution quality of both models. The following three steps are conducted for the model application:

1. Definition of baseline network
2. Computational modelling and optimization framework
3. Model-specific application and analysis
    a. Scenario design
    b. Evaluation of results

First, the baseline transportation network that is used for both models is defined. Second, an overview on the applied computational modelling and optimization framework is given. Third, in line with the building blocks of both model formulations, a set of scenarios to be analyzed is created in order to present a proof of concept, give practical insights into the capability and ensure traceability of both models. The scenarios determine which input parameters and sets are to be varied for the numerical experiments and which outputs are to be analyzed in detail. Furthermore, numerical experiments are conducted to give a practical example of how the applied model performs and to give guidance on the expected outcomes for further prospective model applications. Subsequently, for both models, the respective scenarios, input parameters and the analyses of the models' results are presented differentiating between vehicle, system and location analyses.

4.1.1 Definition of baseline network

The experiments are performed using the NGUYEN AND DUPUIS [1984] network as a benchmark network. Possible facilities for dynamic wireless charging infrastructure are assumed to be located at the centroid of each link. The network, including nodes, directed links and the possible locations of the possible dynamic wireless charging facilities are presented in Fig. 4.1.

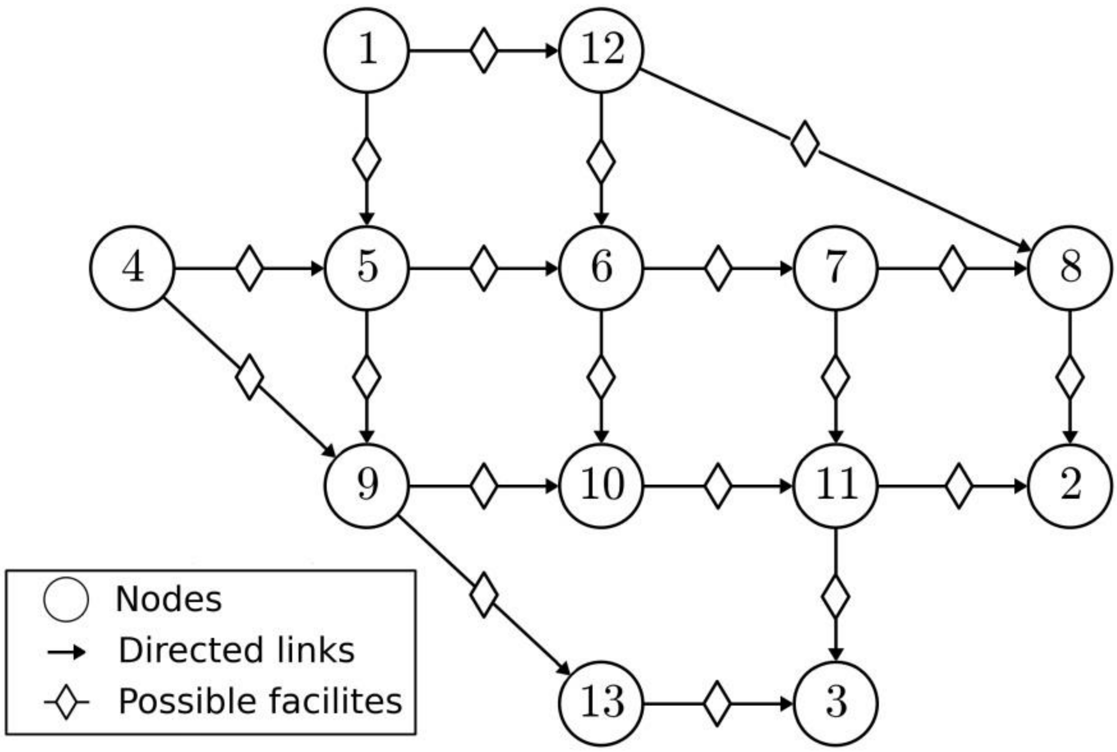

Fig. 4.1 Nguyen-Dupuis network with possible wireless charging facilities

In addition to defining the network and the potential facilities that are to be analyzed, the developed models require further link properties as input parameters. Link distances must be defined for both models and furthermore, in order to investigate the flow-capturing location model with stochastic user equilibrium additional link properties such as free-flow travel time and link capacity must be included as well. These parameter settings are listed in Tab. 4.1. Free-flow travel times and link capacity are taken from XU ET AL. [2011].

Link: Nodes	Free-flow travel time [min]	Link distance [km]	Link Capacity [vehicles/h]
1: 1-5	7	14	300
2: 1-12	9	18	200
3: 4-5	9	18	200
4: 4-9	12	24	200
5: 5-6	3	6	350
6: 5-9	9	18	400
7: 6-7	5	10	500
8: 6-10	13	26	250
9: 7-8	5	10	250
10: 7-11	9	18	300
11: 8-2	9	18	500
12: 9-10	10	20	550
13: 9-13	9	18	200
14: 10-11	6	12	400
15: 11-2	9	18	300
16: 11-3	8	16	300
17: 12-6	7	14	200
18: 12-8	14	28	300
19: 13-3	11	22	200

Tab. 4.1 Input parameters for Nguyen-Dupuis network: Free-flow travel time, link distance and link capacity

Moreover, for both models the feasible route sets have to be defined. Instead of choosing to assign traffic flow to all possibly feasible routes within the network, a k shortest path approach is chosen and respectively, two and three shortest paths ($k = 2$ or $k = 3$) are used as basis paths for route choice. For further details on finding the k shortest paths, please refer to YEN [1971] and EPPSTEIN [1998]. Offering this variation of feasible route sets is considered sufficient for conducting the reference study for the reason that the objective of the model application is to illustrate the model performance. Nevertheless, it should be mentioned that the choice of feasible route sets varies according to the use case investigated. In addition, when applying the flow-capturing location model with equilibrium constraints, it is necessary to prescribe the origin-destination (OD) demand to be assigned to the network beforehand. OD demand is assumed to account for 25% of the OD demand suggested by NGUYEN AND DUPUIS [1984]. The reduction of the OD demand is justified as the focus of this study is not to compute the equilibrium state in a congested network but to determine the effects on route choice and travel times due to the availability of charging infrastructure on the network. OD demand and feasible route sets are presented in Tab. 4.2.

OD pair	OD demand	Paths ($k = 2$)	Paths ($k = 3$)
1 – 2	100	2-18-11	2-18-11
		1-5-7-9-11	1-5-7-9-11
			1-5-7-10-15
1 – 3	200	1-6-13-19	1-6-13-19
		1-5-7-10-16	1-5-7-10-16
			1-5-8-14-16
4 – 2	150	3-5-7-10-15	4-12-14-15
		3-5-7-9-11	3-5-7-9-11
			3-5-7-10-15
4 – 3	50	4-13-19	4-13-19
		3-5-7-10-16	3-5-7-10-16
			4-12-14-16

Tab. 4.2 OD demand and feasible routes sets

4.1.2 Computational modelling and optimization framework

Returning to the fully linearized model formulations given before, solutions methods to solve linear problems, which are already implemented in software optimization packages, can be applied. The computational programming was performed in a Python 2.7 environment, and the Python application programming interface (API) of CPLEX 12.6 was employed to solve the set of numerical experiments.

CPLEX is chosen as solver as it provides the appropriate flexibility for solving the two formulated mixed integer linear problems. In addition, because CPLEX offers a Python API for solving the mathematical model, this approach offers the possibility to manage the mathematical program within a Python environment. Not only is Python particularly advantageous for creating a comprehensive optimization and analysis framework, but also considering the available packages and libraries for data analysis and visualization, for instance NetworkX for creating and manipulating networks [HAGBERG ET AL., 2008] and matplotlib for data analysis and visualization [HUNTER, 2007]. Both sets of experiments were conducted on a 64-bit Windows 8 Server 2012 R2 operating system, an Intel(R) Xeon(R) CPU E5-2640 v3 @ 2.60GHz and 128 GB RAM.

4.2 Flow-capturing location model with stochastic user equilibrium

The following subsections contain first, an overview over the designed scenarios and model inputs (4.2.1), followed by the evaluation of results (4.2.2) and the discussion of results (4.2.3).

4.2.1 Scenario design

Considering the model formulation of the flow-capturing model with stochastic user equilibrium, the model outputs can be analyzed to answer further specifics of the research question. First, how much flow is captured when assuming different vehicle setups? Taking the model formulation into account, a difference in vehicle setup can be achieved by assuming different ranges that the vehicle is able to cover. Hence, the model input for the range of the electric vehicles (EV) is varied to generate the inputs for the according scenarios.

Second, how much flow is captured when assuming different system setups for the charging infrastructure and the transportation network? Subsequently, by varying the parameter for sensitivity towards the availability of charging facilities β and the set of feasible routes R^w , the effects of different system setups can be investigated.

Third, at which candidate facility locations are wireless charging facilities placed and what are the resulting equilibrium traffic flow patterns and travel times? These effects are to be investigated in order to determine if the formulated model behaves as expected.

For all analyses, the experiments are performed using the NGUYEN AND DUPUIS [1984] network as a benchmark network (see Fig. 4.1). For each experiment, it is assumed that every link in the network can accommodate a wireless charging facility. However, note that the model formulation allows more practical considerations regarding the suitability of links for accommodating such facilities depending on the structural conditions of the specific link. Possible facilities for dynamic wireless charging infrastructure are assumed to be located at the centroid of each link. Furthermore, it is assumed that all of the vehicles start their trips with a fully charged battery, hence at each node of trip origin access to a stationary charging facility must exist. Furthermore, the MNL parameter α (scaling parameter for travel time) is kept constant at α = 0.1 because the effects of the variation of β which is related to the charging infrastructure are to be analyzed. In addition, the BPR travel time function is used to calculate the travel time. The piecewise linearization is implemented by applying SOS Type 2 using up to 15 breakpoints in the partition scheme when linearizing the logarithmic functions.

To summarize, the flow-capturing location model with stochastic user equilibrium is tested on the presented 19-link transportation network under varying number of facilities to be located, varying sensitivity to availability of charging stations (β), varying feasible routes and varying EVR (electric vehicle range) for determining the percentage of captured flow and locations of charging stations. Tab. 4.3 presents the parameters which are to be varied for model application.

Building block	**Parameter**
Charging system	Number of wireless charging facilities to be located, p
	Positive scaling parameter for the availability of charging facilities, β
Vehicle	EVR
Transportation network	R^w, Set of all feasible routes r of OD pair $w \in W$

Tab. 4.3 Varied parameters for scenario development: FCLM with stochastic user equilibrium

The model application with all parameter variations would lead to a large set of experiments to be conducted. Hence, when creating the parameter settings, a prioritization according to the specific analyses is taken into consideration. The scenarios with the concrete parameter settings for the respective analyses are derived in sections 4.2.1.1, 4.2.1.2 and 4.2.1.3.

4.2.1.1 Vehicle analysis

The change of EVR is investigated in order to determine how much flow is captured when different vehicles setups are taken into consideration. This would imply that the vehicles can be equipped with different sets of batteries depending on the average range that is to be achieved. For determining the boundaries for the EVR, network specifics were taken into consideration. The lower boundary for the EVR is set to 40 km, as a lower range does not allow for the creation of the set of candidate facility locations capable of recharging EV flows so that the EV can traverse the segment ($\Psi^{w}_{i,j,r}$). The upper boundary for the EV range is set to 80 km because the longest total trip length that is considered falls below 80 km as well. Consequently, the impact of different EV ranges of 40 km, 50 km, 60 km, 70 km and 80 km on the model solution is investigated. It should be noted that electric vehicles already available in the market today can exceed the assumed range. Nevertheless, the range which is assumed for the vehicle analysis is justified from a theoretical standpoint as the aim of the numerical study is to present a proof of concept of the model and second, from a practical standpoint when electric vehicles with dynamic charging capabilities are introduced a reduction in battery size and range can be expected. Furthermore, the number of wireless charging facilities to be located is varied between 0 and 19 since the transportation network consists of a total of 19 links with candidate facility locations for wireless charging infrastructure. The parameter for the availability of charging facilities is set to $\beta = 0.8$ to account for a high sensitivity of EV drivers towards the availability of a charging facility on their respective route. In addition, the set of feasible routes is based on the $k = 2$ scenario. The overview over the input parameters for the vehicles analysis is given in Tab. 4.4.

Building block	**Parameter**	**Unit**	**Value**
Charging system	Number of wireless charging facilities to be located, p	[ ]	0-19
	Positive scaling parameter for the availability of charging facilities, β	[ ]	0.8
Vehicle	EVR	[km]	40, 50, 60, 70, 80
Transportation network	R^{w}, Set of all feasible routes r of OD pair $w \in W$	[ ]	$k = 2$

Tab. 4.4 Input parameters for vehicle analysis: variation of EVR

4.2.1.2 System analysis

To determine the percentage of captured flow with varying system setups, two scenarios for the system analysis are developed and analyzed. First, the scaling parameter β (where β is the scaling parameter for the availability of charging facilities) and second, the number of feasible routes is varied in order to determine if the model behaves as expected.

Variation of scaling parameter β for the availability of charging facilities

First, the impact of β on route choice and the model solution are investigated. Through the incorporation of β into the MNL, which dictates the route choice, the sensitivity of EV drivers towards the availability of a charging facility on their respective route is included into the model formulation. A system analysis is conducted for different values of β varying between 0.0 and 1.0 (in steps of 0.1) to determine if the model is truly capable of incorporating the changes in traffic flow towards routes which host a wireless charging facility. It is expected that a higher value of β leads to higher links flows on links which are equipped with a wireless charging facility. Given the baseline network and the feasible routes, the range of the EVs is set to 50 km to ensure that electric vehicles have to charge at a wireless charging station in order to complete their trips. Furthermore, to illustrate the general effects of a variation of β, it is deemed sufficient to perform the experiments with the set of feasible routes based on the $k = 2$ scenario. As higher values of wireless charging facilities to be located might lead to a fully captured flow and consequently the availability of a charging facility would not influence route choice anymore. In order to investigate the effects of placed facilities on route choice, it is necessary to perform the model application with small numbers of facilities to be located. Consequently, two different scenarios are investigated, one in which only one facility is to be located ($p = 1$) and the other where two facilities are to be located ($p = 2$). An overview of the input parameters for the system analysis is shown in Tab. 4.5.

Building block	Parameter	Unit	Value
Charging system	Number of wireless charging facilities to be located, p	[ ]	1, 2
	Positive scaling parameter for the availability of charging facilities, β	[ ]	0.0-1.0
Vehicle	EVR	[km]	50
Transportation network	R^w, Set of all feasible routes r of OD pair $w \in W$	[ ]	k = 2

Tab. 4.5 Input parameters for system analysis: variation of β

<u>Variation of feasible routes</u>

The impact of a varying number of facilities to be located on the percentage of captured flow is analyzed for the $k = 2$ and $k = 3$ scenarios in order to determine the effects of varying the feasible routes r of OD pair w. Generally, an increase in feasible routes should involve an increased number of facilities which are to be located. The range is kept constant at 50 km for all numerical experiments in order to ensure comparability and ensure that the effects of the varied parameters on the objective value can be captured. Furthermore, the parameter for the availability of charging facilities, β, is set to 0.3, 0.5 and 0.8, respectively in order to determine and investigate the expected changes in traffic flow. The input parameters for the number of wireless charging facilities to be located, the EVR and the scaling parameters for the $k = 2$ scenario are given in Tab. 4.6.

Building block	**Parameter**	**Unit**	**Value**
Charging system	Number of wireless charging facilities to be located, p	[ ]	0-19
	Positive scaling parameter for the availability of charging facilities, β	[ ]	0.3, 0.5, 0.8
Vehicle	EVR	[km]	50
Transportation network	R^w, Set of all feasible routes r of OD pair $w \in W$	[ ]	k = 2

Tab. 4.6 Input parameters for system analysis: variation of feasible routes $k = 2$

Similarly, the input parameters for the $k = 3$ scenario are given Tab. 4.7.

Building block	**Parameter**	**Unit**	**Value**
Charging system	Number of wireless charging facilities to be located, p	[ ]	0-19
	Positive scaling parameter for the availability of charging facilities, β	[ ]	0.3, 0.5, 0.8
Vehicle	EVR	[km]	50
Transportation network	R^w, Set of all feasible routes r of OD pair $w \in W$	[ ]	k = 3

Tab. 4.7 Input parameters for system analysis: variation of feasible routes $k = 3$

4.2.1.3 Location analysis

Two scenarios for the location analysis are developed in order to investigate which links are equipped with wireless charging facilities and to analyze the resulting traffic flow patterns on each link. Furthermore, the resulting changes in travel times compared to the free flow travel times are to be analyzed.

Flow analysis

The flow analysis is comprised of investigating the resulting traffic flow patterns when the number of facilities to be located is varied. The number of wireless charging facilities to be located is varied between 0 and 19. β is varied between 0.3 and 0.8 in order to depict a lower and upper sensitivity scenario. Furthermore, to give an insight into the general model behavior, an analysis of the $k = 2$ scenario is included. The parameter setting for the flow analysis are presented in Tab. 4.8.

Building block	Parameter	Unit	Value
Charging system	Number of wireless charging facilities to be located, p	[ ]	0-19
	Positive scaling parameter for the availability of charging facilities, β	[ ]	0.3, 0.8
Vehicle	EVR	[km]	50
Transportation network	R^w, Set of all feasible routes r of OD pair $w \in W$	[ ]	k = 2

Tab. 4.8 Input parameters for location analysis: flow analysis

Travel times analysis

The travel time analysis deals with the impact of the number of facilities that are located on the percentage increase of travel time on each link compared to the free-flow travel time. The parameter settings are set to those used for the flow analysis (see Tab. 4.8). However, in order to determine the effects of placed facilities on the link travel times, it is deemed sufficient to conduct the model application for the travel time analysis with $\beta = 0.8$. Tab. 4.9 depicts an overview over the parameter settings.

Building block	Parameter	Unit	Value
Charging system	Number of wireless charging facilities to be located, p	[ ]	0-19
	Positive scaling parameter for the availability of charging facilities, β	[ ]	0.8
Vehicle	EV range, EVR	[km]	50
Transportation network	R^w, Set of all feasible routes r of OD pair $w \in W$	[ ]	k = 2

Tab. 4.9 Input parameters for location analysis: travel time analysis

4.2.2 Evaluation of results

Contents of this section were previously published in *Riemann, R., Wang, D. Z. W. and Busch, F., 2015. Optimal location of wireless charging facilities for electric vehicles: Flow-capturing location model with stochastic user equilibrium. Transportation Research Part C: Emerging Technologies, 58, pp.1–12.*

The results of the model application for the flow-capturing location model with stochastic user equilibrium are concerned with the analysis of the results in relation to a vehicle-specific, system-specific and location-specific analysis and presented in sections 4.2.2.1 – 4.2.2.3. The according parameters are varied as defined in the scenario design.

4.2.2.1 Vehicle analysis

Fig. 4.2 presents the percentage of captured flow (model objective) under varying EVR. The results show that the range has a great impact on the number of facilities that must be located in order of capture the entire flow. To achieve a full coverage and with the lowest range of 40 km, five facilities on the network are required whereas only four facilities are required for the ranges of 50 km and 60 km, respectively. Three facilities are required to capture the total flow with the range of 70 km. However, the range of 70 km requires no charging facility at all in order to capture more than 80% of all flow on the network. In comparison, the lowest range of 40 km would require at least three charging facilities to capture around 80% of the flow as well. When the range is set to 80 km, no additional wireless charging facility is required in order to complete the trips.

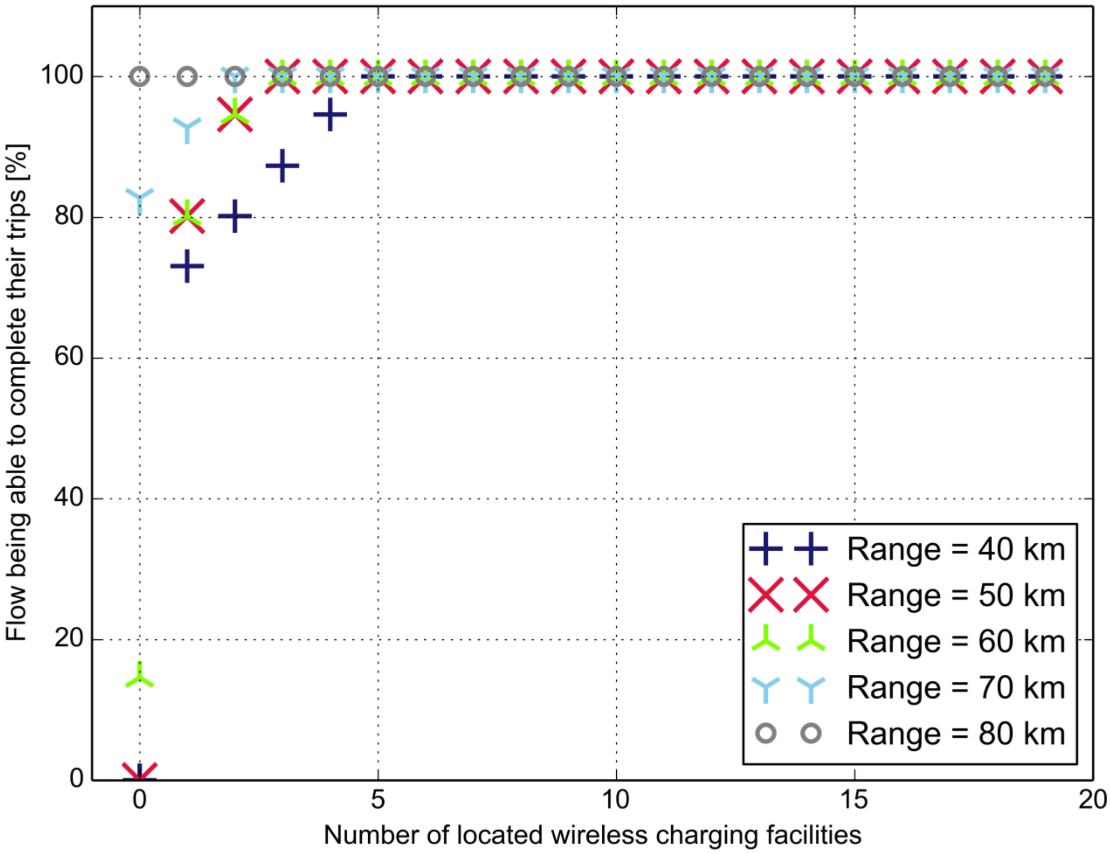

Fig. 4.2 Number of located wireless charging facilities and percentage of flow being able to complete trips

4.2.2.2 System analysis

<u>Variation of scaling parameter β for the availability of charging facilities</u>

Fig. 4.3 depicts the resultant percentage of total flow captured under varying values of β and varying numbers of facilities to be located. For all settings of β when only one facility is

located, less flow is captured in comparison to the experiments when two facilities are available. Furthermore, the results show that in both cases when only facility, respectively two facilities are to be located, higher β-values (with a constant sensitivity to travel time α) lead to a higher percentage of captured flow.

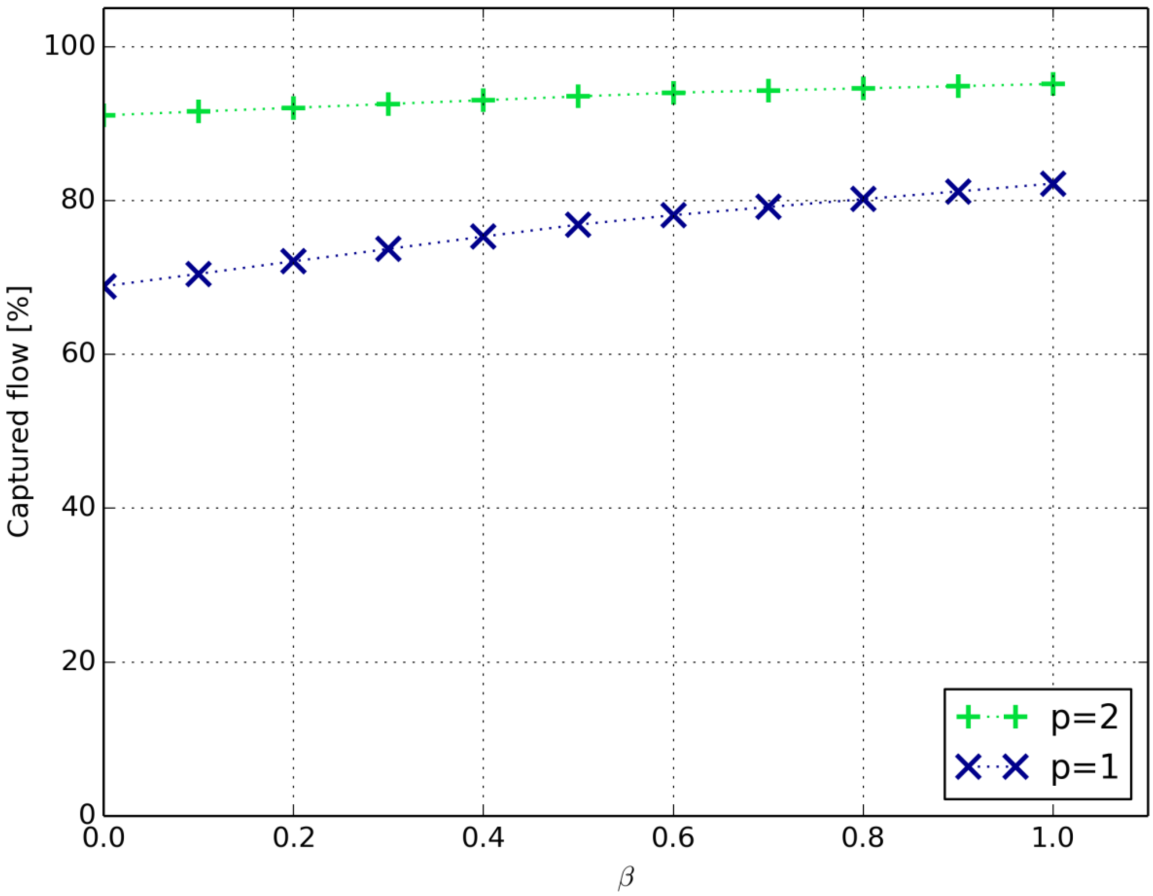

Fig. 4.3 Impact of β on captured flow

<u>Variation of feasible routes</u>

Fig. 4.4 illustrates that as the number of wireless charging facilities rises, the percentage of captured flow increases steadily until the entire traffic flow is captured. Again, the percentage of captured flow increases with higher values of β. When one charging facility is built more than 70% of flow is captured for any parameter setting of β (0.3, 0.5 and 0.8). In addition, the entire traffic flow will be captured when three charging facilities are built for any parameter setting of β. Likewise, when the number of possible routes is increased ($k = 3$) the number of wireless charging facilities which need to be installed to capture the total traffic flow on the network also increases (see Fig. 4.5). Again, the value of the β parameter influences the total amount of captured flow when one, two, three or four facilities are located. However, when five facilities are located the total amount of flow can be captured for each parameter setting. In comparison to the $k = 2$ scenario, for the $k = 3$ scenario, the

number of charging facilities required to capture 100% of the flow increases from three to five facilities.

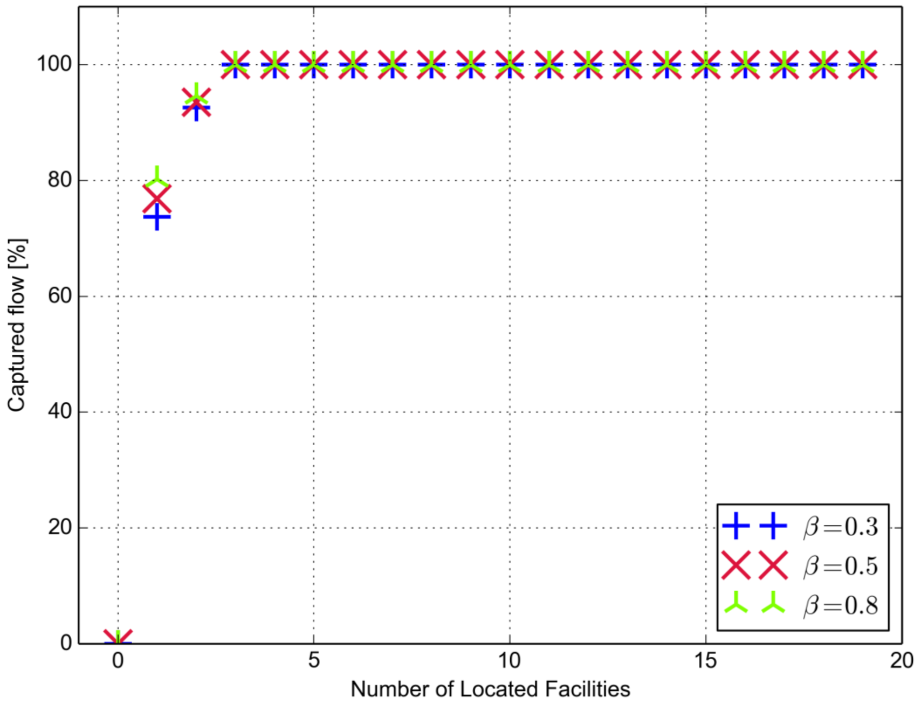

Fig. 4.4 Percentage of captured flow depending on number of facilities to be located p ($k = 2$)

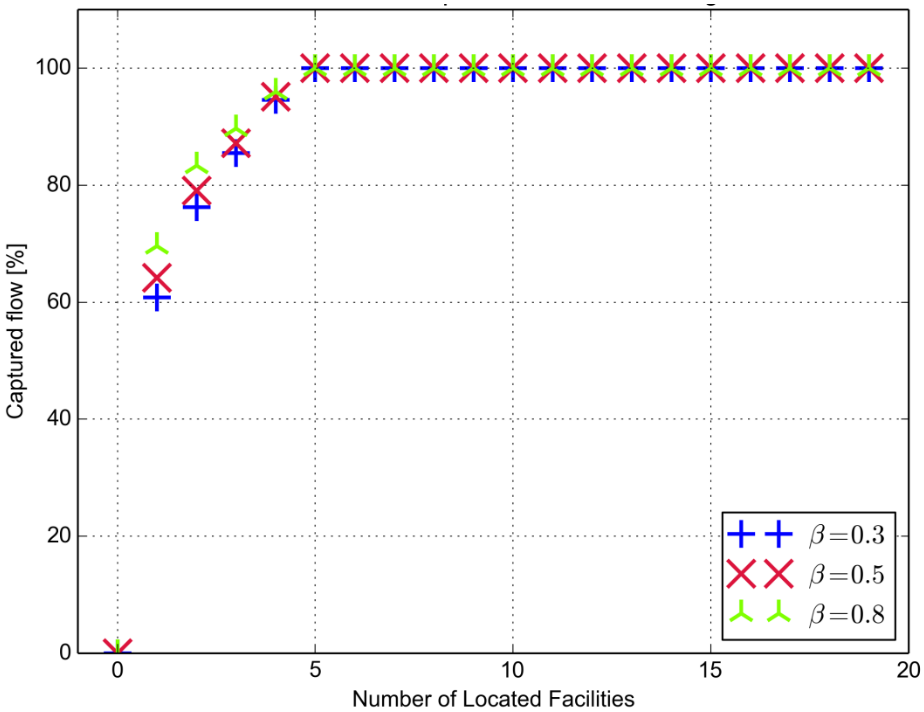

Fig. 4.5 Percentage of captured flow depending on number of facilities to be located p ($k = 3$)

4.2.2.3 Location analysis

Flow analysis

Fig. 4.6 depicts the traffic flow pattern on the network when zero wireless charging facilities are to be located. The results show equal amounts of link traffic flow h_a for both parameter settings of β that determines the utility of the availability of a wireless charging facility for the EV drivers.

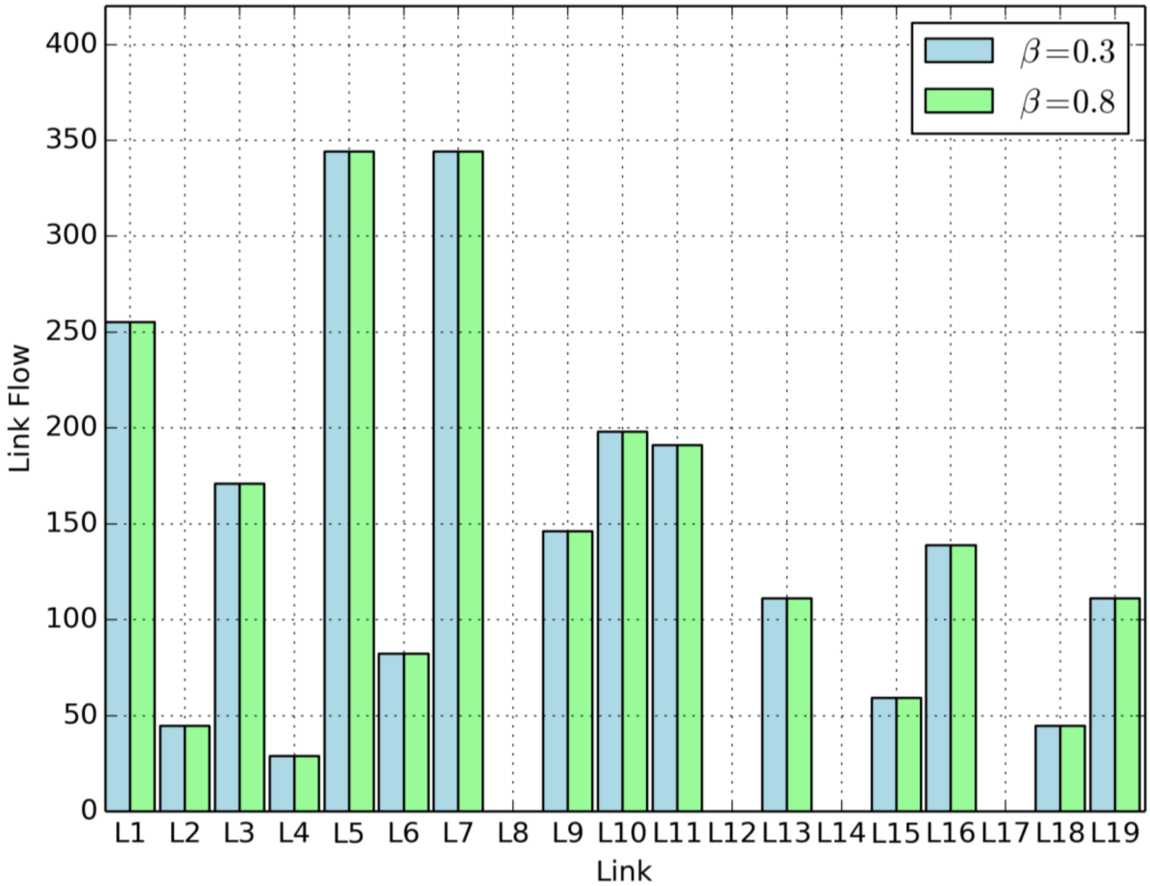

Fig. 4.6 Link flow h_a $(p = 0, k = 2)$

In addition, Fig. 4.7 illustrates the traffic flow pattern on the network when one wireless charging facility is to be located. The charging facility is located on link 5, which shows the highest value of link flow h_a besides link 7. Moreover, the results illustrate that the distribution of link flow is significantly influenced by the scaling parameter β. The traffic flow on link 5 increases when the parameter is set higher and vice versa. When comparing the two scenarios for the two different scaling parameters, the traffic flows differ from each other on the majority of links and only link 15 exhibits equal link flows.

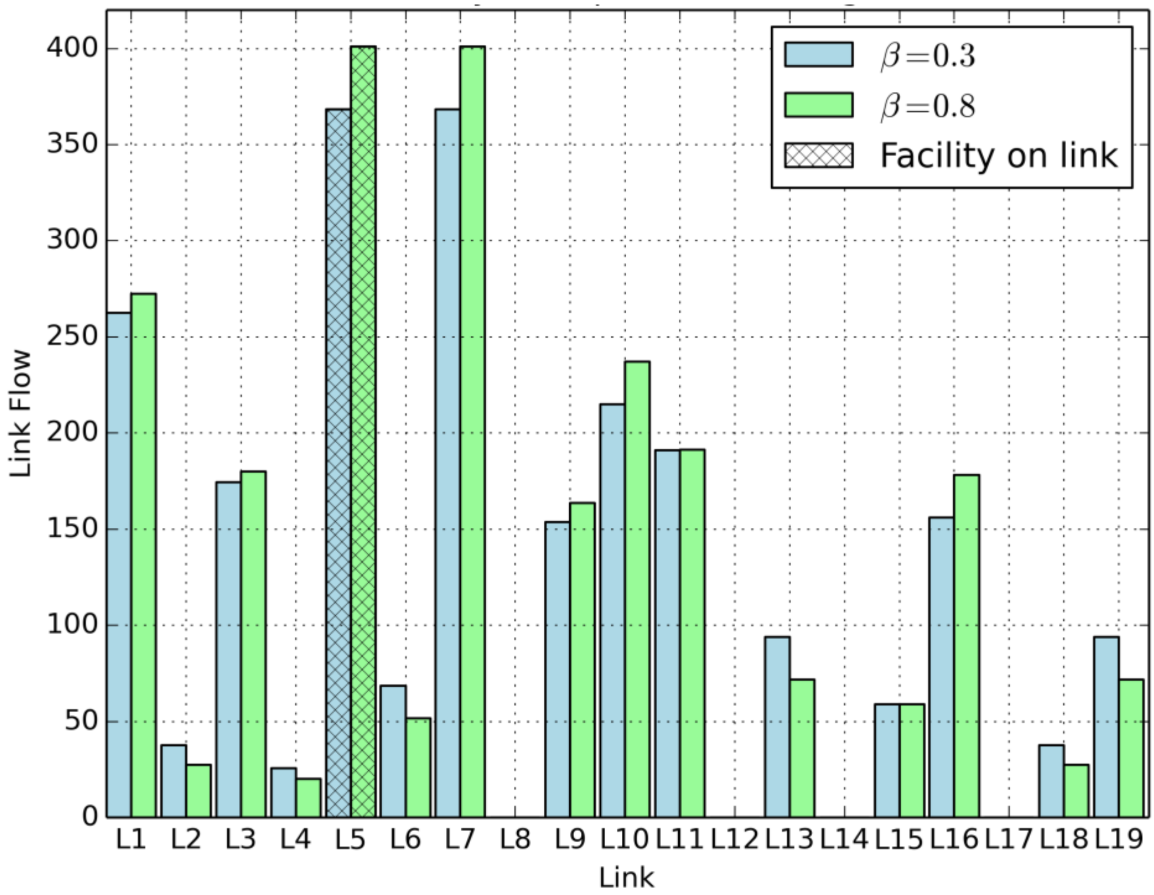

Fig. 4.7 Link flow h_a ($p = 1$, $k = 2$)

Additionally, the link flows h_a for two and three wireless charging facilities ($p = 2$, $p = 3$) to be located are presented in Fig. 4.8 and Fig. 4.9. The second link to be equipped with a facility following link 5 is link 13 (Fig. 4.8). As compared to the scenario where only one facility is located, the results shows that the amount of traffic flow h_a on some links, for example links 3, 10 and 19, is equal even under different values of the scaling parameter β.

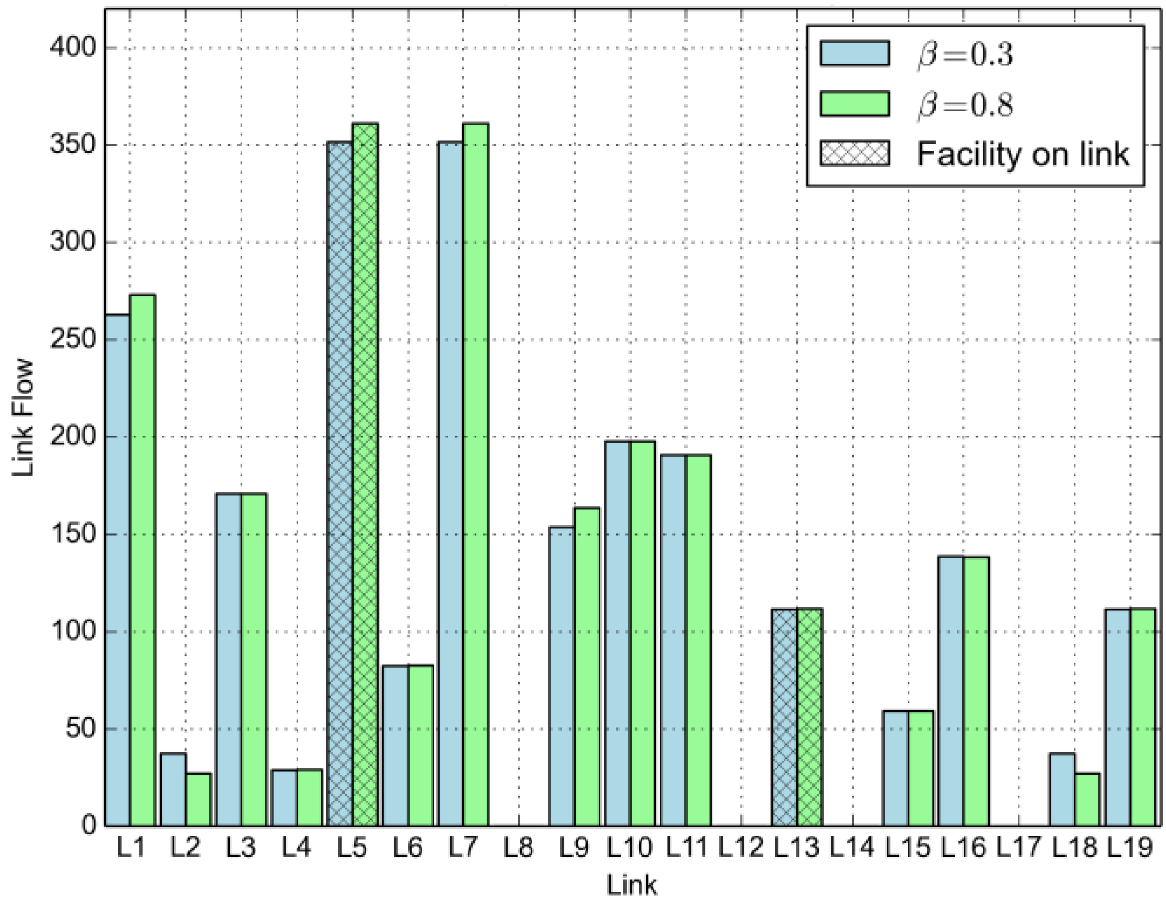

Fig. 4.8 Link flow h_a ($p = 2$, $k = 2$)

As presented in Fig. 4.4, all of the traffic flow is captured when three wireless charging facilities are located ($p = 3$). The corresponding facilities are located on links 5, 13 and 18 as illustrated in Fig. 4.9. When three charging facilities are placed on the network, it is observable that the resulting traffic flow patterns are equal for the two scenarios with $\beta = 0.3$ and $\beta = 0.8$, respectively. Furthermore, the resulting link traffic flow when three facilities are located is equal to the link traffic flow when zero facility is located (see Fig. 4.6).

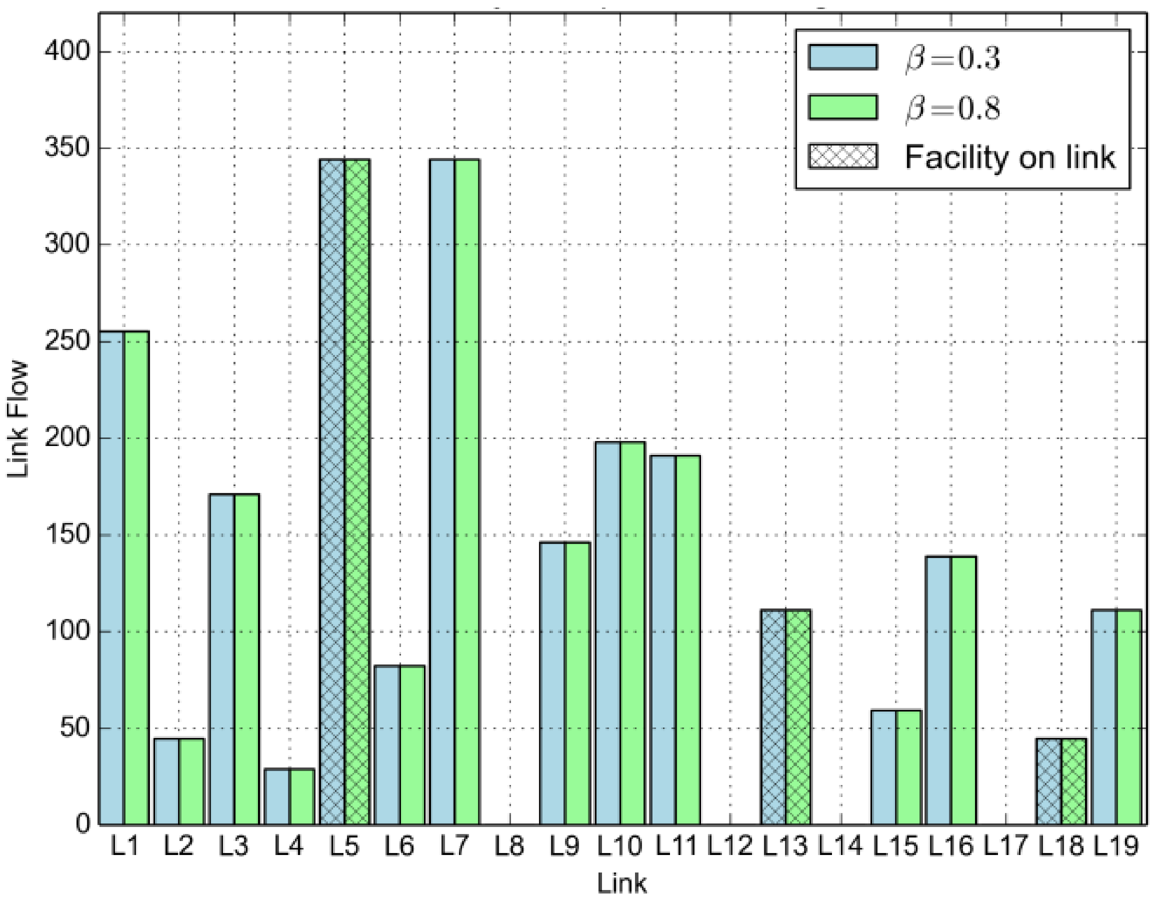

Fig. 4.9 Link flow h_a ($p = 3$, $k = 2$)

It shall be noted that for the remaining scenarios when four to nineteen facilities ($p = 4$ to $p = 19$) are to be located, the resulting flows on each link are equal to the scenario when $p = 3$. Subsequently, the remaining results for four to nineteen facilities to be located are not presented in this section.

Travel time analysis

Tab. 4.10 illustrates the results of the travel time analysis and depicts the increase in travel time t_a compared to free flow travel times t_a^0 when different numbers of wireless charging facilities are to be located, namely zero to three facilities. When no facility is located zero flow is captured and when three facilities are located all flow is captured (see Fig. 4.4). The results show that the first facility is placed at link 5 and subsequently, a strong increase in travel time of 18.30% compared to the free flow travel time is observable. Furthermore, travel time on link 3 increases from 6.90% to 9.10%. When two or three facilities are located the average travel time on link 5 decreases from 18.30% to 11.80% and to 9.60%, respectively and in addition, the travel time on link 3 decreases again to 6.90% from 9.10%. Travel times on link 13 which hosts the second wireless charging facility show a decrease of travel time when one facility is located (from 1.40% to 0.30%) in comparison to the scenarios when zero, two or three facilities are located (1.40%). Furthermore, the third link which is equipped with a wireless charging facility and where only a small amount of link flow is present (link 18, see Fig. 4.6-4.9) does not show any increase in travel time.

	Facility placed at link?				Increase in travel time			
Link	p=0	p=1	p=2	p=3	p=0	p=1	p=2	p=3
1	×	×	×	×	6.00%	7.20%	7.30%	6.00%
2	×	×	×	×	0.10%	0.00%	0.00%	0.10%
3	×	×	×	×	6.90%	9.10%	6.90%	6.90%
4	×	×	×	×	0.00%	0.00%	0.00%	0.00%
5	×	✓	✓	✓	9.60%	18.30%	11.80%	9.60%
6	×	×	×	×	0.00%	0.00%	0.00%	0.00%
7	×	×	×	×	2.50%	4.40%	3.10%	2.50%
8	×	×	×	×	0.00%	0.00%	0.00%	0.00%
9	×	×	×	×	1.50%	2.60%	2.60%	1.40%
10	×	×	×	×	3.20%	4.70%	3.20%	3.20%
11	×	×	×	×	0.60%	0.60%	0.50%	0.60%
12	×	×	×	×	0.00%	0.00%	0.00%	0.00%
13	×	×	✓	✓	1.40%	0.30%	1.40%	1.40%
14	×	×	×	×	0.00%	0.00%	0.00%	0.00%
15	×	×	×	×	0.00%	0.00%	0.00%	0.00%
16	×	×	×	×	0.90%	1.70%	0.90%	0.90%
17	×	×	×	×	0.00%	0.00%	0.00%	0.00%
18	×	×	×	✓	0.00%	0.00%	0.00%	0.00%
19	×	×	×	×	1.40%	0.30%	1.40%	1.40%

Tab. 4.10 Comparison of increase in link travel time t_a compared to free-flow travel time t_a^0 $(k = 2)$

4.2.3 Discussion of results

The results of the vehicle, system and location analysis presented in sections 4.2.2.1 – 4.2.2.3, are discussed in subsections 4.2.3.1. – 4.2.3.3.

4.2.3.1 Vehicle analysis

The variation of EVR showed that with an increasing range the number of facilities to capture the total flow decreases. Hence, the model results are in line with the expected outcome. It should be noted that trips can be completed (hence the flow is considered captured) due to the input assumption that at the nodes of trip origin access to a stationary charging facility must exist. When assuming that vehicles with a range of 80 km are fully charged at the beginning of their trips and with the input that no trip exceeds a length of 80 km, no further wireless charging facility is required during the trips as the vehicles can complete their trips with their initial range.

4.2.3.2 System analysis

Variation of scaling parameter β for the availability of charging facilities

As expected and intended by the formulated model, a higher β-value (with a constant sensitivity to travel time α) leads to a higher percentage of captured flow as EV drivers are set to be more sensitive towards the availability of a charging station, hence they are more likely to choose a route with a charging station. Furthermore, the results show that it is crucial to consider the setting of the scaling parameters when there are not enough charging stations available to capture all charging demand.

Variation of feasible routes

The observation of the results for the variation of feasible routes indicates that the model behaves as expected since more available routes require more facilities to be located in order to capture the total flow of electric vehicles. Furthermore, the results show, that at least for this scenario, an increase of the β-value does not influence the number of facilities required to capture the total flow.

4.2.3.3 Location analysis

Flow analysis

The results of the flow analysis illustrate the location of the facilities and the resulting flow pattern assigned to the links of the network. As the model seeks to locate facilities in order to maximize the captured flow, it stands to reason that link 5, as the link with the maximum amount of flow passing through, constitutes the first link to be equipped with a wireless charging facility. Moreover, the results show that the distribution of link flow is significantly influenced by the scaling parameter β that determines the utility of the availability of a wireless charging facility for the EV drivers. The traffic flow on link 5 increases when the parameter is set higher and vice versa.

It is observable that while the first facility is located at the link with the highest values of links flows, the subsequent facilities however are not located at links with second or third highest values. This is due to the fact that the model considers the complete vehicle routes and not only links when determining if flows are captured or not.

Furthermore, when a sufficient number of charging facilities is placed on the network and all flow can be captured, all routes are equally favourable in terms of the availability of charging infrastructure. Hence, in this case the number of facilities to be located and the scaling parameter β do not impact route choice which in return results in equal traffic flow patterns for the two scenarios with $\beta = 0.3$ and $\beta = 0.8$, respectively.

Travel time analysis

The results of the travel time analysis show that the traffic flow pattern and the resulting travel times are equal for the “zero flow captured” ($p = 0$) scenario and the “all flow captured” ($p = 3$). These results occur due to the reason that when no facility is located, the assignment of the traffic flow on each link depends solely on travel time. Similarly, when three facilities are placed and subsequently all traffic flow is captured by the respective charging facilities, the traffic assignment solely depends on travel time as well as the availability of charging stations will not have an impact on route choice anymore.

However, changes in travel times are observable when analyzing the model results for one or two wireless charging facilities to be located ($p = 1$ or $p = 2$). When one facility is located on link 5, travel times strongly increase on this link as more flow is passing through the link in order to be intercepted by a charging facility. As the equilibrium flow pattern depends on both travel time and charging station availability, the percentage increase in link travel time gets more balanced and decreases again on most links when two facilities are located, as additional routes with a charging station become available. When analyzing the “in between scenarios” the sometimes counteracting effects of the different utilities for travel time and charging station availability must be taken into consideration. More specifically, the effects of minimizing travel times and the availability of charging stations somewhat counteract, as a high scaling parameter for the availability of charging stations will influence the electric vehicles in choosing routes, which may demonstrate higher route travel times, yet enable the charging of batteries on these longer routes. To summarize, the results show that the location of wireless charging facilities significantly influences traffic flow patterns and the resulting travel times.

4.3 Set-covering location model with charging system design

The following subsections are comprised of an overview over the designed scenarios and model inputs (4.3.1), followed by the evaluation of results (4.3.2) and the discussion of results (4.3.3).

4.3.1 Scenario design

Taking the model formulation of the set-covering location model with charging system design into consideration, the model outputs can be analyzed to answer further specifics of the research question. First, what is the minimum length of wireless charging facilities and their locations when assuming different vehicle setups? In this context, different vehicle setups can be accounted for by setting different ranges that the vehicle is able to cover, as well as different average speed and average power consumptions.

Second, what is the minimum length of wireless charging facilities and their locations when assuming different system setups for the charging infrastructure and the transportation network? Consequently, the variation of the power level received by the vehicle from the dynamic charging segment on link a allows for the investigation of different system setups.

Third, which candidate facility locations are equipped with wireless charging properties and what percentage of the transportation network must be equipped with wireless charging facilities in order to capture all electric vehicles?

The set of experiments for the set-covering location model with charging system design is conducted using the NGUYEN AND DUPUIS [1984] network as benchmark example (Fig. 4.1). Moreover, it is assumed that every link is capable of hosting a dynamic wireless charging facility. All vehicles are assumed to start their trips with a fully charged battery, which implies that at each node of trip origin, electric vehicles must have access to a charging facility. In order to ensure comparability, the scenarios are analyzed for equal feasible routes r of OD pair w. It is assumed that all k-3-routes (see Tab. 4.2) are feasible route sets; hence, all possible k-3-routes are traversable and must be covered. The parameters to be varied for the model application are given in Tab. 4.11.

Building block	Parameter
Charging system	Power level received by vehicle from dynamic charging segment on link a, PL_a
Vehicle	EV power consumption, PC
	Maximum electric range of EVs, R_{max}
	Average vehicle speed on link a, V_a

Tab. 4.11 Varied parameters for scenario development: set-covering location model with charging system design

Similarly, the model outputs are analyzed and it is investigated if the results are in line with the expected model behavior. When creating the parameter settings for the specific analyses, a prioritization according to the effects which are to be analyzed is taken into consideration. The scenarios with the concrete parameter setting which are derived for the respective analyses are presented in sections 4.3.1.1, 4.3.1.2 and 4.3.1.3.

4.3.1.1 Vehicle analysis

The impact of different vehicle setups on the total length of wireless charging segments to be installed is investigated. For the set-covering location model with charging system design, the different vehicle setups can be modelled by varying the parameters of the range, the speed and the power consumption of the electric vehicles which are assigned to the network. The default value of 13 kWh/100km for the average power consumption is based on a reported average power consumption of an electric vehicle available in the market [BMW, 2016]. Moreover, the input parameters for the power levels received by the vehicles are based on the reported power levels which can be achieved by dynamic wireless charging test beds as presented in section 2.2.2.

<u>Variation of range</u>

First, the variation of the available range of electric vehicles is investigated in order to determine the total length of wireless charging facilities that have to be built. The considerations concerning the boundaries for the range are according to the considerations set out in section 4.2.2.1. However, the maximum parameter setting for the range is increased to 100 km as the formulation of the set-covering model with wireless charging system design accounts for a 20% range buffer. For analyzing the effects of the variation of the range, the model is solved with the following input parameters: power levels are varied between 50 kW, 100 kW, 150 kW and 200 kW, EV power consumption is set to the default value of 13 kWh/100km and the range is analyzed between 40 km and 100 km with intervals of 10 km. In addition, in order to create a lower and higher average speed scenario, the average speed is set to 50 km/h and to 100 km/h. It should be noted that equal values of average speed are assumed for all links in the network in order to ensure direct comparability of different speed levels. Nevertheless, theoretically the model formulation allows for the consideration of different average speeds on each link. An overview over the input values is given in Tab. 4.12

Building block	**Parameter**	**Unit**	**Value**
Charging system	Power level received by vehicle from dynamic charging segment on link a, PL_a	[kW]	50, 100, 150, 200
Vehicle	EV power consumption, PC	[kWh/100km]	13
	Maximum electric range of EVs, R_{max}	[km]	40-100
	Average vehicle speed on link a, V_a	[km/h]	50, 100

Tab. 4.12 Input parameters for vehicle analysis: variation of R_{max}

<u>Variation of speed</u>

Next, the variation of average speeds between 20 km/h and 160 km/h with intervals of 10 km/h in the network is analyzed. The input parameters for solving the optimization model are depicted in Tab. 4.13. Again, the EV power consumption is set to the default value of 13 kWh/100km, EV ranges are varied between 40 km and 60 km and power levels which can be received by the vehicles from the dynamic charging infrastructure are set to 50 kW, 100 kW, 150 kW and 200 kW.

Building block	**Parameter**	**Unit**	**Value**
Charging system	Power level received by vehicle from dynamic charging segment on link a, PL_a	[kW]	50, 100, 150, 200
Vehicle	EV power consumption, PC	[kWh/100km]	13
	Maximum electric range of EVs, R_{max}	[km]	40, 60
	Average vehicle speed on link a, V_a	[km/h]	20-160

Tab. 4.13 Input parameters for vehicle analysis: variation of V_a

Variation of power consumption

The third analysis which is concerned with the electric vehicle investigates the effects of a variation in power consumption on the overall lengths of wireless charging segments which need to be installed within the transportation network. Tab. 4.14 presents the input parameters which are used to solve the set-covering model. Again power levels are fixed to 50 kW, 100 kW, 150 kW and 200 kW, EV ranges are classified into two categories of 40 km and 60 km and in order to ensure comparability the average speed in the network is set to 50 km/h. Furthermore, the model is solved for average power consumptions between 5 and 20 kW/100km with intervals of 1 kW/100km.

Building block	Parameter	Unit	Value
Charging system	Power level received by vehicle from dynamic charging segment on link a, PL_a	[kW]	50, 100, 150, 200
Vehicle	EV power consumption, PC	[kWh/100km]	5-20
	Maximum electric range of EVs, R_{max}	[km]	40, 60
	Average vehicle speed on link a, V_a	[km/h]	50

Tab. 4.14 Input parameters for vehicle analysis: variation of PC

4.3.1.2 System analysis

For the system analysis, the power levels are varied between 20 kW and 200 kW in intervals of 10 kW and the maximal EV ranges are set to 40 km, 50 km, 60 km, 70 km and 80 km. The EV power consumption and the average speed are set to the default values of 13 kWh/100km. For the experiments, the two average speeds of 50 km/h and 100 km/h are chosen to investigate differences in the objective values (see Tab. 4.15).

Building block	Parameter	Unit	Value
Charging system	Power level received by vehicle from dynamic charging segment on link a, PL_a	[kW]	20-200
Vehicle	EV power consumption, PC	[kWh/100km]	13
	Maximum electric range of EVs, R_{max}	[km]	40, 50, 60, 70, 80
	Average vehicle speed on link a, V_a	[km/h]	50, 100

Tab. 4.15 Input parameters for system analysis: variation of PL_a

4.3.1.3 Location analysis

The location analysis is comprised of investigating the locations and lengths of dynamic wireless charging infrastructure that must be installed at each link in the network. The range is set to 40 km and varying power levels of 50 kW and 200 kW are assumed. Furthermore, EV power consumption is set to the default value and for comparability of the lower speed scenario, the average vehicle speed is set to 50 km/h. For both sets of experiments, the percentage of link length which would need to be equipped with a wireless charging facility is investigated. The parameters are summarized in Tab. 4.16.

Building block	Parameter	Unit	Value
Charging system	Power level received by vehicle from dynamic charging segment on link a, PL_a	[kW]	50, 200
Vehicle	EV power consumption, PC	[kWh/100km]	13
	Maximum electric range of EVs, R_{max}	[km]	40
	Average vehicle speed on link a, V_a	[km/h]	50

Tab. 4.16 Input parameters for location analysis: variation of PL_a

4.3.2 Evaluation of results

The results of the model application of the set-covering location model with charging system design are concerned with the analysis of the results in relation to a vehicle-specific, system-specific and location-specific analysis. The parameter variations are defined in the scenario design serve as basis for the model application.

4.3.2.1 Results of vehicle analysis

<u>Variation of range</u>

The length of wireless charging segments depending on varying EV ranges R_{max} and power levels PL_a are depicted in Fig. 4.10 and Fig. 4.11. First, Fig. 4.10 shows that an increase in the power level results in a linear decrease in the total length of wireless charging segments which have to be installed in order to cover all routes. However, with an increasing range of electric vehicles the differences in the total length of wireless charging segments which have to be installed decreases as well.

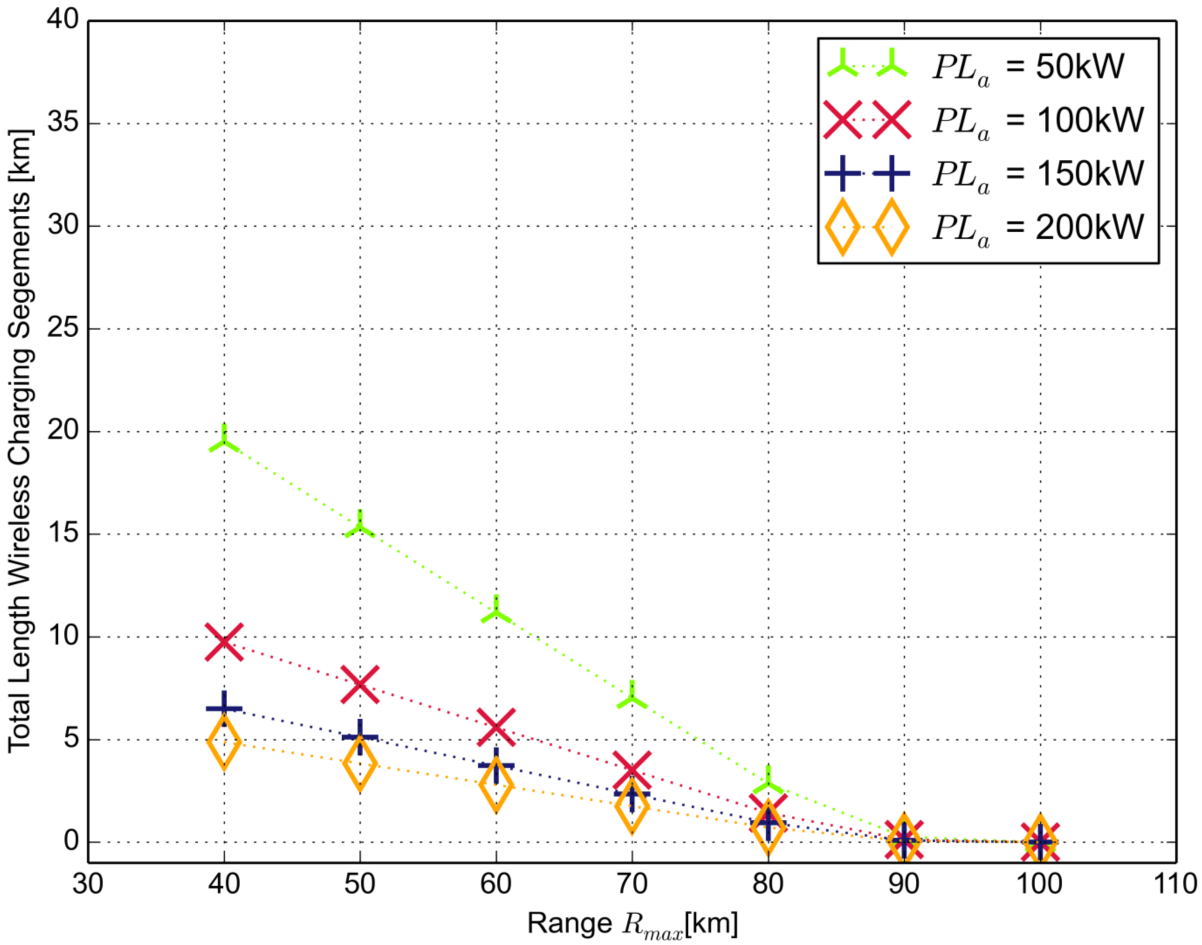

Fig. 4.10 Total length of wireless charging segments depending on Range R_{max} and power level PL_a (V_a = 50km/h)

When the experiment is conducted with the higher average speed $V_a = 100$ km/h (Fig. 4.11) the overall trends of a smaller total length of wireless charging segments with increasing range and increasing power levels can be observed as well. Nevertheless, when the average speed V_a in the network increases, the magnitude of total length of wireless charging segments which needs to be installed doubles.

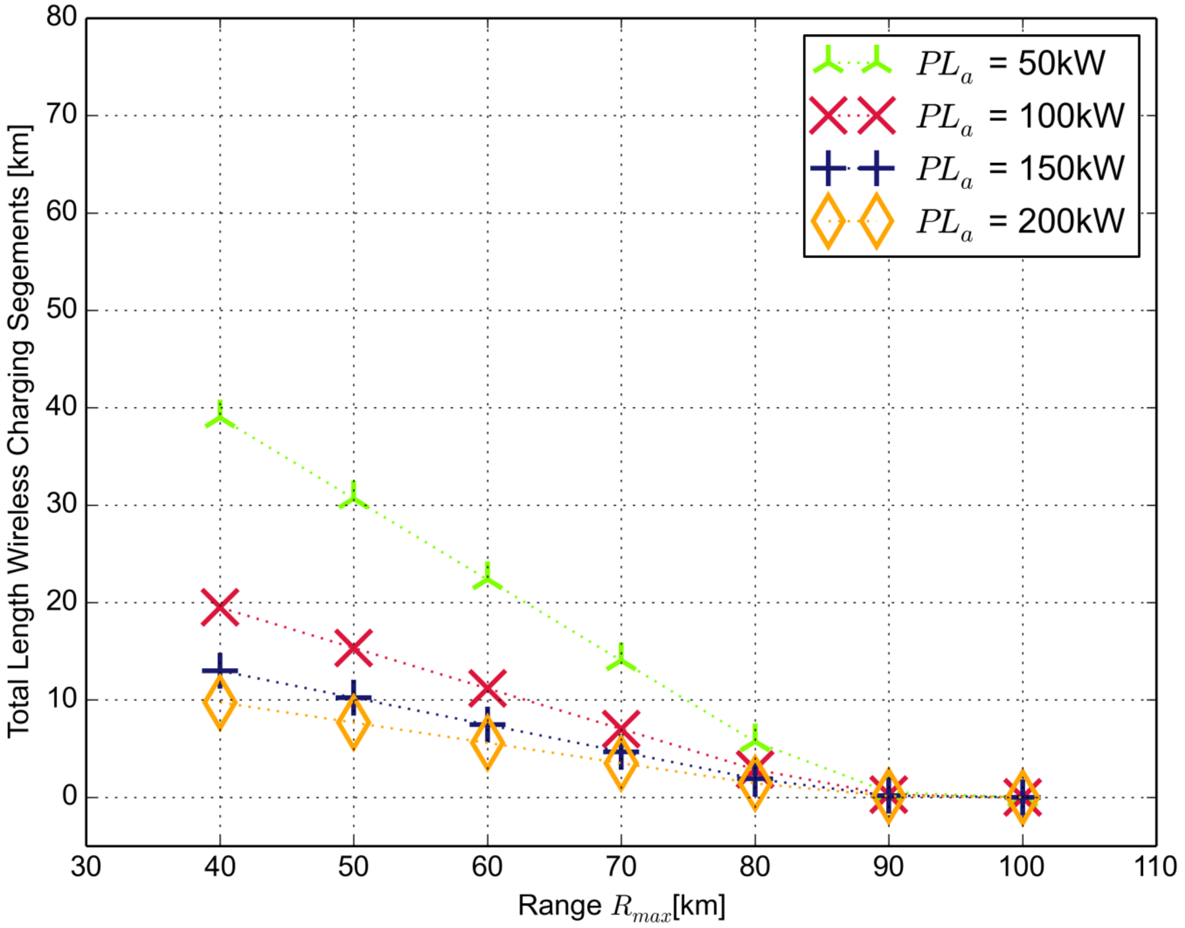

Fig. 4.11 Total length of wireless charging segments depending on Range R_{max} and power level PL_a (V_a = 100km/h)

The differences in range as well as in average speed have a significant impact on the overall length of wireless charging segments. The higher the range and the lower the average speed, the lower the total length of the dynamic wireless charging infrastructure segments. However, when the optimization model is solved under the constraints of lower average speeds such as 50 km/h, the lengths of the wireless charging segments for the infrastructure significantly decrease as compared to the scenarios with higher average speeds. Both scenarios show that only ranges of at least 90 km allow the vehicles to complete their trips without falling below 20% of the starting range.

Variation of speed

The results for the scenario with a range R_{max} of 40 km which are depicted in Fig. 4.12 show that depending on the power level PL_a, the total length of wireless charging segments linearly increases. In addition, the higher the average speed V_a, the more links must be equipped with longer segments of wireless charging facilities.

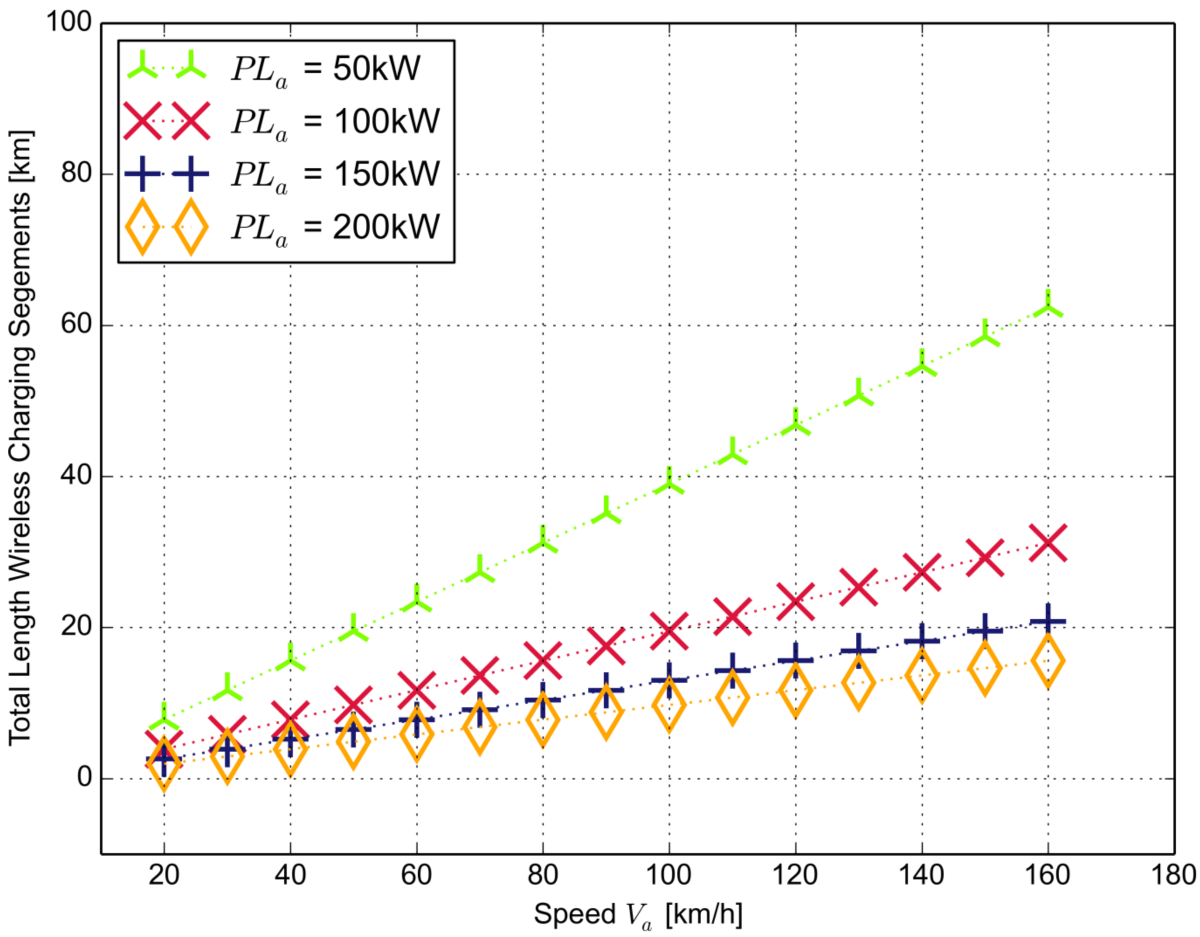

Fig. 4.12 Total length of wireless charging segments depending on speed V_a and power level PL_a (R_{max} = 40km)

In Fig. 4.13 the results for the scenario with a range R_{max} of 60 km are depicted. The same trends influencing the increase in total required length of charging segments as in the scenario with a maximum range of 40 km are observable. The results of both scenarios which were conducted show that besides the power level that can be received by the vehicle from the roadway, the average speed has a significant impact on the length of wireless charging segments as well. Furthermore, the differences between the power levels PL_a of 50 kW, 100 kW, 150 kW and 200 kW at each speed level are not constant.

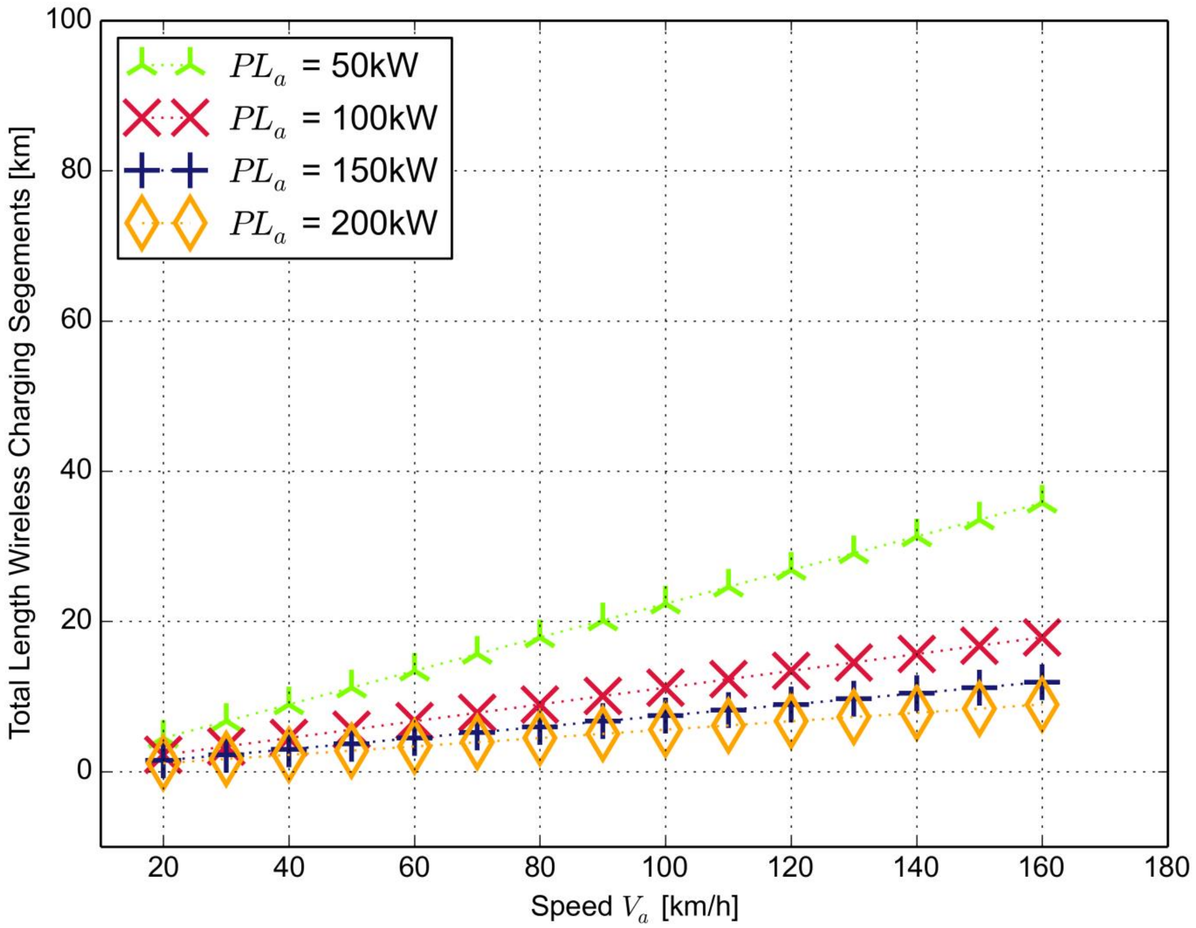

Fig. 4.13 Total length of wireless charging segments depending on speed V_a and power level PL_a (R_{max} = 60km)

<u>Variation of power consumption</u>

Fig. 4.14 depicts the influence of different average power consumption of EVs PC and various power levels PL_a on the objective value for a vehicle range $R_{max} = 40$ km. The total length of wireless charging segments that must be installed linearly increases with increasing power consumption. In addition, higher power levels lead to a decrease in total length of for wireless charging segments.

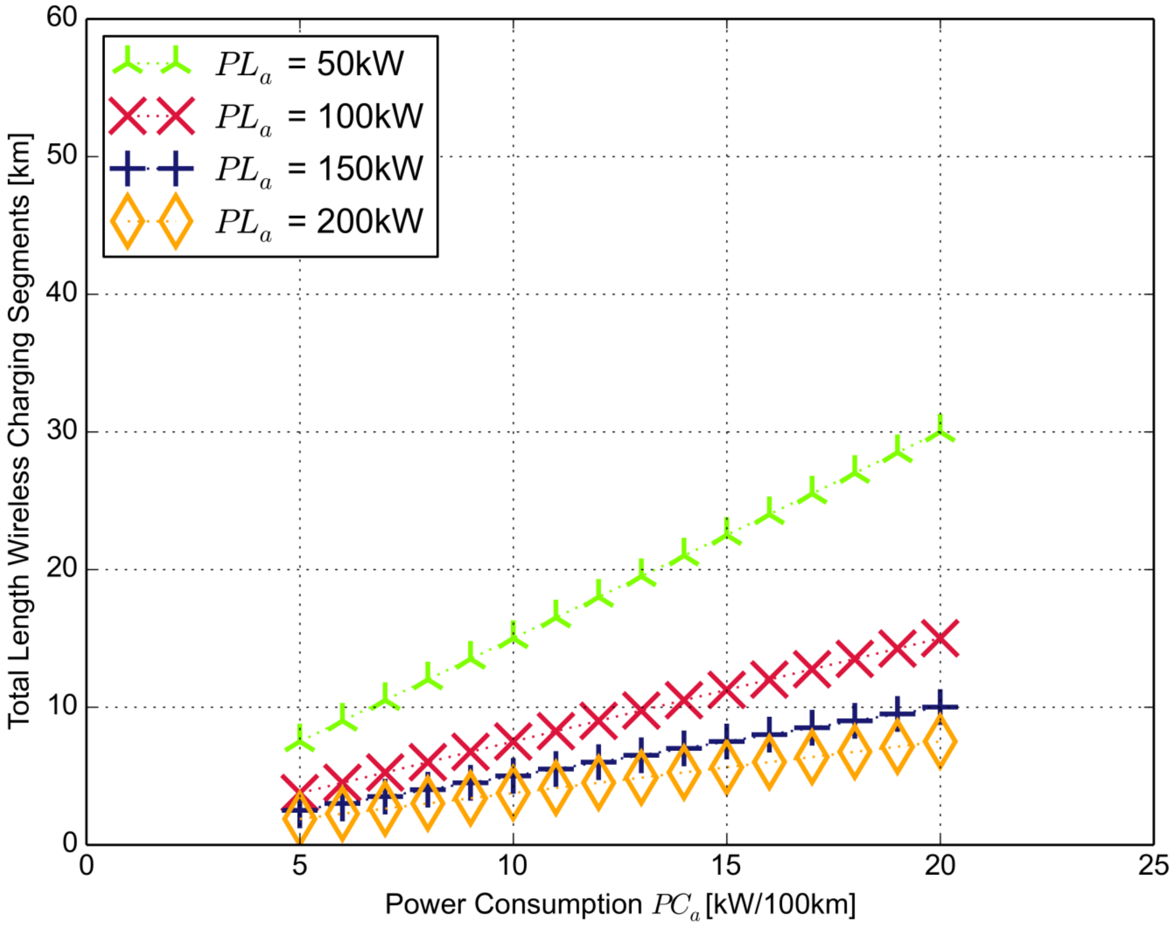

Fig. 4.14 Total length of wireless charging segments depending on power consumption PC and power level PL_a (R_{max} = 40km)

The according analysis for the scenario with a range of 60 km is illustrated in Fig. 4.15. Again, the analysis and comparison of both sets of experiments yields to results with equal trends. The analysis shows that depending on power consumption and power level the length of wireless charging segments which need to be installed to cover all demand drastically changes. It can be observed, that the power consumption linearly influences the overall length of wireless charging segments necessary to equip the network with sufficient wireless charging infrastructure to cover all energy demand.

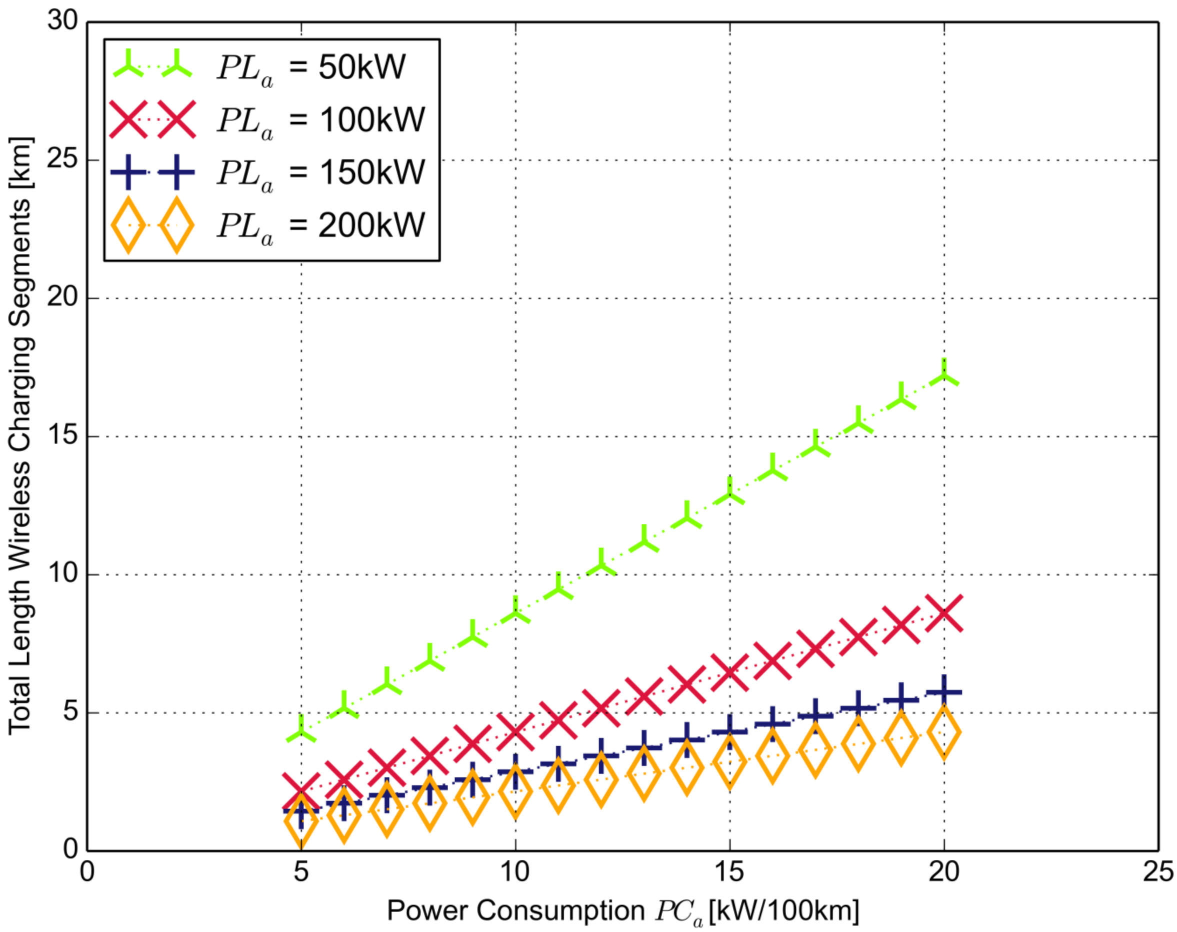

Fig. 4.15 Total length of wireless charging segments depending on power consumption PC and power level PL_a (R_{max} = 60km)

4.3.2.2 System analysis

The variation of the power which can be received by the vehicles PL_a in relation to the length of wireless charging segments which must be installed to cover all charging demand is depicted in Fig. 4.16 and Fig. 4.17. For the scenario with with an EV range of R_{max} = 40 km one can obtain from Fig. 4.16 that the total length of wireless charging segments that must be installed is more than 50% lower when comparing power levels of 20 kW and 50 kW.

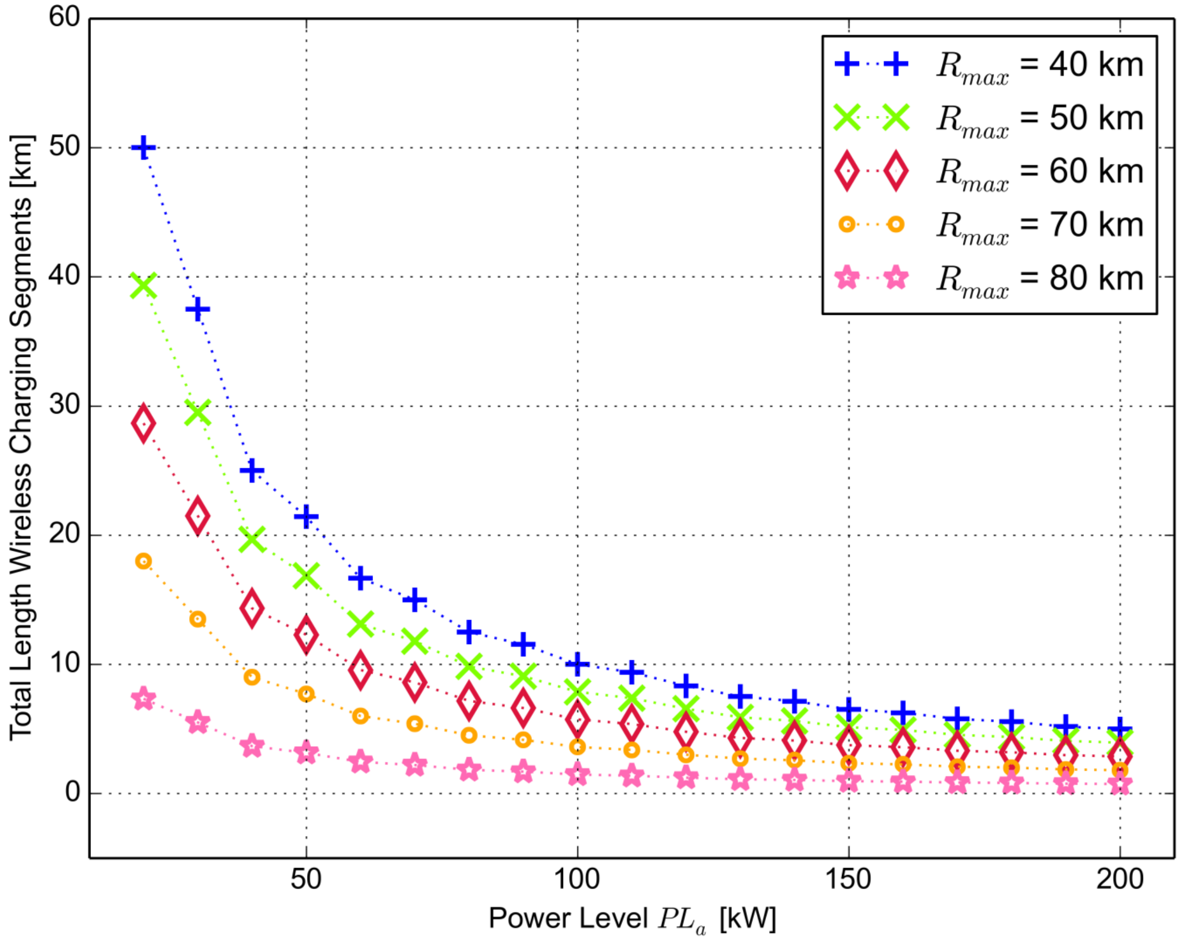

Fig. 4.16 Total length of wireless charging segments depending on power level PL_a and Range R_{max} (V_a = 50km/h)

For the analysis with an average speed of V_a = 100km/h, there is as well an observable increase in total length of wireless charging segments which have to be installed within the network. Again, with higher EV ranges the overall length of dynamic charging infrastructure segments decreases as less infrastructure needs to be installed in order to cover all charging demand. Especially when assessing the lower levels between 20 kW and 50 kW of power that vehicles can pick up from the roadway, the total length of wireless charging segments is significantly influenced.

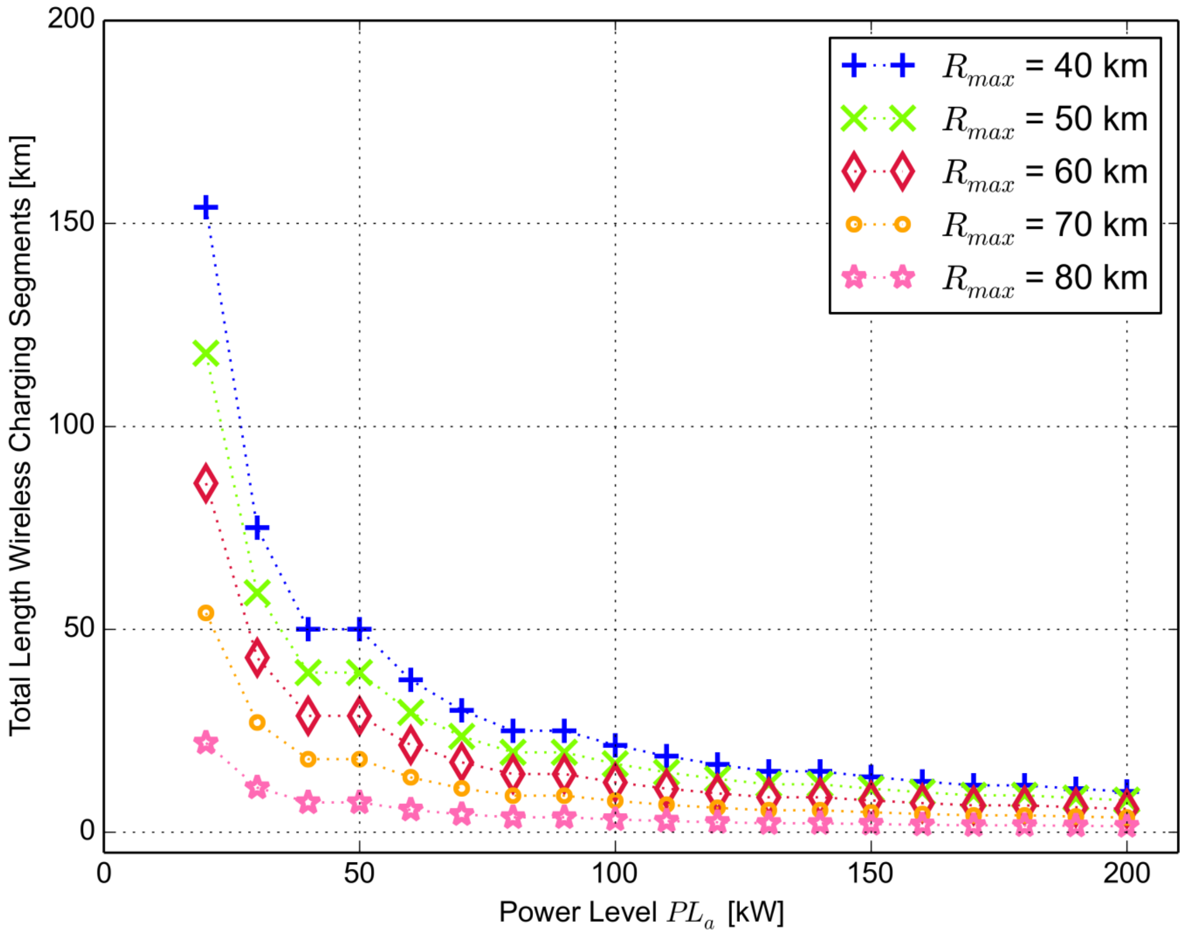

Fig. 4.17 Total length of wireless charging segments depending on power level PL_a and Range R_{max} (V_a =100km/h)

4.3.2.3 Location analysis

The results depicted in Tab. 4.17 and 4.18 illustrate the percentage share of wireless charging segments which must be installed at each link to ensure a full coverage of the charging demand. Tab. 4.17 illustrates that a total of approximately 5.95% of the total length of the transportation network would have to be equipped with wireless charging infrastructure in order to ensure that all demand is covered when the defined scenario is taken as input for the model application. However, the percentage share of wireless charging segment per link differs largely between the links. The model results show that 26% of the length of link 5 would have to be equipped with wireless charging segments while 7.43%, 8.67%, 13%, 7.8%, 7.22%, 17.33%, 10.11%, 9.75%, 6.50% and 13% are required for links 1, 3, 11, 12, 13, 14, 15, 16, 18 and 19 in order to cover all charging demand.

Link number	Link length [km]	Length wireless charging segment [km]	Percentage of wireless charging segment per link [%]
1	14	1.04	7.43
2	18	0.00	0.00
3	18	1.56	8.67
4	24	0.00	0.00
5	6	1.56	26.00
6	18	0.00	0.00
7	10	0.00	0.00
8	26	0.00	0.00
9	10	0.00	0.00
10	18	0.00	0.00
11	18	2.34	13.00
12	20	1.56	7.80
13	18	1.30	7.22
14	12	2.08	17.33
15	18	1.82	10.11
16	16	1.56	9.75
17	14	0.00	0.00
18	28	1.82	6.50
19	22	2.86	13.00
Total	**328**	**19.50**	**5.95**

Tab. 4.17 Lengths of wireless charging segments (PL_a = 50 kW)

When taking the same boundary conditions into account but increasing the power level to 200 kW, approximately 1.5% of the total length of the network would have to be equipped with wireless charging segments in order to cover all charging demand (see Tab. 4.18). Furthermore, the analysis shows that wireless charging segments must be installed at links 2, 5, 10, 11, 12, 13, 14 and 15. While 10.83% of link 5 must host wireless charging segments and thereby constitutes the upper level, the lower level of required percentage of charging segments is observable on link 15 with 0.33%.

Link number	Link length [km]	Length wireless charging segment [km]	Percentage of wireless charging segment per link [%]
1	14	0.00	0.00
2	18	0.58	3.22
3	18	0.00	0.00
4	24	0.00	0.00
5	6	0.65	10.83
6	18	0.00	0.00
7	10	0.00	0.00
8	26	0.00	0.00
9	10	0.00	0.00
10	18	0.52	2.89
11	18	0.45	2.50
12	20	0.39	1.95
13	18	1.30	7.22
14	12	0.91	7.58
15	18	0.06	0.33
16	16	0.00	0.00
17	14	0.00	0.00
18	28	0.00	0.00
19	22	0.00	0.00
Total	**328**	**4.86**	**1.48**

Tab. 4.18 Lengths of wireless charging segments (PL_a = 200 kW)

To summarize, when installing wireless charging segments with power levels of 50 kW and 200kW, eleven and eight links would have to be equipped with wireless charging infrastructure. However, when increasing the power levels, the total length of wireless charging segments which must be installed significantly decreases from 5.95% to 1.48% of the total network length.

4.3.3 Discussion of results

The impact of different vehicle setups on the total length of wireless charging segments to be installed is investigated and the results of the vehicle, system and location analysis are discussed in subsections 4.3.3.1. – 4.3.3.3.

4.3.3.1 Vehicle analysis

Variation of range

Higher ranges result in a decrease in total length of wireless charging segments which have to be installed. In addition, higher average speed results in an increase in total length of wireless charging segments. Hence, the model behaviour is in line with the expected behaviour. Even though the longest trip in the network falls below 80 km, ranges of at least 90 km are necessary to ensure that all vehicles can complete their trips without recharging. This effect occurs due to the modelling assumption for the range constraints which ensure that for all vehicles in the network the available range never falls below 20%. Hence, even with ranges of 80 km, vehicles would have to recharge to ensure that the range never falls below safety margin.

Variation of speed

Lower speeds allow the vehicles to capture more energy from the roadway. Consequently, the length of dynamic charging segments which have to be installed decreases with lower speed levels. Furthermore, the results imply that the category of the roads (e.g. urban road or motorway) and their average speeds must be taken into consideration when planning the locations of dynamic wireless charging infrastructure. In urban road scenarios where the typical average speed is below 50 km/h and especially under the constraint of lower EV ranges (e.g. 40 km) the overall lengths of wireless charging segments are drastically lower as compared to scenarios where the average speed is above 100 km/h.

Variation of power consumption

When vehicles are modelled with a higher average power consumption, the total length of wireless charging segments which must be installed to ensure that all flow is captured, increases. Subsequently, the model results confirm the expected outcome when analyzing the variation of power consumption.

4.3.3.2 System analysis

More specifically, the observable increase in total length of wireless charging segments with lower levels of power received is not surprising as this causes the vehicles to receive less energy from one charging infrastructure and consequently more charging segments must be installed. Furthermore, with higher EV ranges the overall length of dynamic charging infrastructure decrease as less infrastructure must be installed in order to cover all charging demand. To conclude, the system analysis showed as well that the model results are in line with the expected model behavior.

4.3.3.3 Location analysis

The analysis showed that the model is capable of determining the percentage of the transportation network that would have to be equipped with wireless charging facilities in order to cover all charging demand of electric vehicles. Furthermore, the link-based location of the required facilities can be deducted from the model results as well. The observed results are in line with the expected model results. First, the total length of wireless charging segments is varying depending on the power levels and second, higher power levels result in a lower total length of installed wireless charging segments.

5 Conclusion

The following concluding section is comprised of a review of the research work (section 5.1), an overview over the key findings, contributions and limitations (section 5.2) and suggestions for future research (section 5.3).

5.1 Review

First, the relevant literature for modelling and optimizing facility locations in discrete networks in the context of charging infrastructure for electric vehicles is presented. The review showed that the placement of charging infrastructure for electric vehicles can be modelled with various different inputs and assumptions and a comprehensive approach is required when modelling the placement of charging stations for electric vehicles. Furthermore, it showed that previous research did not consider modelling optimal placement of wireless charging infrastructure and traffic assignment. Moreover, little peer-reviewed research has been conducted for the optimal placement of wireless charging infrastructure and system design.

Subsequently, two different modelling approaches were developed to optimally locate charging facilities with wireless power transfer capabilities, the flow-capturing location model with stochastic user equilibrium and the set-covering location model with charging system design. The first model uses the flow-covering location problem as a basis but is extended with the capability to take vehicle range and wireless charging infrastructure into consideration. In addition, simultaneous traffic assignment following stochastic user equilibrium (SUE) is considered. The second model is an extension of the set-covering model that seeks to cover all charging demand in the network and refines the charging system design and additionally taking vehicle speed, range and power consumption into account.

When revisiting the research question: “How can the location of dynamic wireless charging infrastructure be optimized on a discrete road network?

1. What percentage of charging demand of electric vehicles can be covered with a predetermined number of facilities?
2. What is the minimum number of facilities and their locations that results in a full coverage of charging demand from electric vehicles?”

it can be concluded that the results of the flow-capturing location model with stochastic user equilibrium answers the first specification, and the second specification is answered by the results of the set-covering model with wireless charging system design. Furthermore, when an optimization problem is formulated as a mixed integer linear problem, existing algorithms can be used for solving the model. Hence, as both models are initially formulated as mixed integer nonlinear problems and this research seeks to solve the models with readily available solution methods implemented in standard software packages, both models are linearized. Finally, both models and their applications are discussed and an overview of the lessons learned is presented.

5.2 Contributions and limitations

Contributions

First, previous flow-capturing and flow-refueling location problems discussed in the literature review usually classify link traffic flow on the network as an exogenously given input parameter, which is determined by the assignment of the origin-destination (OD) demand to the shortest paths. These shortest paths are determined using standard shortest path algorithms such as Dijkstra's algorithm. This leads to the route choice behavior of the network users only being influenced by the trip distance but not by travel times which can change with the assigned traffic flow. Hence, the proposed model incorporates both the captured traffic flow and a route choice model, which takes the availability of wireless charging facilities and travel time into account. The change in route choices is modelled by extending the standard flow-capturing models with the consideration of stochastic user equilibrium. In addition, the proposed model simultaneously considers the assignment of traffic flow and the optimal locations of wireless charging infrastructure. Subsequently, this approach allows for the modelling of the interactions between the optimally placed wireless charging infrastructure and actual traffic flow on the routes which in return influences the travel times in the network. Furthermore, depending on the setting of the scaling parameters, the attractiveness or the utility of the availability of a charging facility as well as the travel times is influenced.

Secondly, the set-covering location model with charging system design seeks to minimize the total length of the wireless charging system that must to be installed to cover all charging demand in the network. It encompasses the design of the wireless charging infrastructure, the setup of the electric vehicles and the transportation network. The solution of the model results in the charging facilities' location plan and the according length of the wireless charging segments that are to be installed.

In comparison to the flow-capturing location model with equilibrium constraints, the set-covering location model features a model detailed view on the charging system design and incorporates a more realistic recharging logic. The vehicles do not fully recharge at each available facility but can only recharge as much as the charging system design allows. Subsequently, the proposed model depicts a flow-based set-covering location model that incorporates infrastructural and vehicular constraints as well as charging system design constraints.

Limitations

Models generally depict a restricted view of the reality and the degree of modelling accuracy especially in relation to the modelling of cost is limited and only implicitly incorporated by the number of facilities which must be located to serve the charging demand. The objective functions and consequently, the parameters and constraints of both models could be extended by including cost terms. This could be achieved by incorporating different types of costs that arise for the design, installation, operation and maintenance of dynamic wireless charging infrastructure. Furthermore, both model formulation assume that all vehicles in the network are electric vehicles, hence, different types of users within the road network and different electric vehicle types are not considered. Consequently, this simplification does not allow to model other potential means of transport that could influence route choice, travel times and average speed in the network. Moreover, the presented model formulations do not consider the capabilities of the power system and detailed technical properties of the dynamic wireless charging system.

Taking the model formulations into consideration, the nature of the formulated facility location problems includes binary constraints (does the location for the proposed charging link exist yet?), subsequently, this approach will lead to a mixed integer model formulation. It is worthwhile to explore further possibilities of either formulating the models as linear problems which could lead to enhanced solution efficiency and possible applicability to bigger networks.

5.3 Future research

This research laid out the basic model framework and the proposed models are tested on a benchmark network. However, in future research, it is worthwhile testing the model framework on real networks and implement real boundary conditions to optimally locate charging infrastructure in a real city. While the models are applicable to any network, one has to ensure that abstractions of real networks are made where necessary. Second, it must be kept in mind that due to the large number of integers the solving times may increase for large networks. To circumvent this limitation, the application of different solution strategies such a heuristics could be taken into consideration when bigger networks are to be analyzed. Furthermore, the models proposed could be integrated into traffic management and planning tools in order to account for charging infrastructure in future applications.

The location model, the prediction of charging behavior and the spatial analysis of possible locations play a crucial role in the different approaches. Therefore, in future research more comprehensive approaches taking the aforementioned variables and limitations into account are worthwhile investigating. In addition, the field of modelling frameworks for the placement of charging infrastructure for electric vehicles is an active field of research and it can be expected that more sophisticated and comprehensive approaches will be developed in future research. To conclude, further advances on location optimization of charging infrastructure are worthy areas of future research, as electric vehicles will only become more present in our transportation system if well-planned and well-designed charging infrastructure is implemented.

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Abbreviations

AC	Alternating Current
AC-PC FRLM	Arc-Cover Path-Cover Flow Refueling Location Model
API	Application Programming Interface
BPR	Bureau of Public Roads
DC	Direct Current
EV	Electric Vehicle
EVR	Electric Vehicle Range
FCLM	Flow-Capturing Location Model
FCLP	Flow-Capturing Location Problem
FRLM	Flow-Refueling Location Model
MCLP	Maximal Covering Location Problem
MILP	Mixed Integer Linear Program
MINLP	Mixed Integer Nonlinear Program
MIQP	Mixed Integer Quadratic Program
MNL	Multinomial Logit
OD	Origin-Destination
OLEV	Online Electric Vehicle
RLT	Reformulation-Linearization Technique
SCLP	Set-Covering Location Problem
SOS	Special Ordered Set
TR	Technology Readiness
TRL	Technology Readiness Level

Figures

Tables