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Advances in the form-finding of structural membranes

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Abstract

The present contribution presents the application of the newly developed isogeometric B-Rep analysis to the form-finding and structural analysis of structural membranes. Therefore, the necessary basis of the model description is briefly outlined. The result of this approach is the possibility to perform mechanically accurate form-finding (and follow-up analyses) directly within a CAD-environment on a full NURBS-based CAD-model.
Selected benchmark examples show the accuracy and robustness of the developed method, assessed against analytical solutions if applicable. Two selected application examples that are entirely treated within the augmented CAD-environment highlight the potential of the method: Using one common model not only facilitates geometrical analysis and preparation of the model, but also allows for smooth and close interaction between design and analysis, which is of crucial importance in membrane design.

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Keywords: structural membranes; form-finding; isogeometric B-Rep analysis (IBRA); minimal surface; CAD-CAE-interaction

1. Introduction and Motivation

Structural membranes provide minimal use of material combined with an attractive and impressive language of shapes. These shapes are directly mechanically motivated: based on the boundary conditions and the prescribed prestress field, form-finding analysis is used to determine the shape of equilibrium which allows the membrane to act in pure tension.

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This mechanical background usually leads to an iterative design procedure, where the mechanical form-finding and the modification of boundary conditions and prestress conditions as design handles mutually interact until a solution that is desirable from a structural as well as from an aesthetical point of view is found.

Classically the architectural part of the design of structural membranes is realized within a CAD (*computer-aided design*) environment, whereas the form-finding and subsequent analyses are performed within a CAE-environment (*computer-aided engineering*), usually a finite element-code. The separation of these models requires considerable amounts of time and is obviously rather error-prone.

Recently, the *isogeometric B-Rep analysis* (IBRA) has been proposed as a consequent generalization of the *isogeometric analysis* (IGA) with the aim of directly using the CAD-model – enriched by mechanical information – for the analysis of structures [3].

In the present contribution, the form-finding of structural membranes with IBRA will be presented, focusing on the form-finding directly on the basis of CAD models, and the potential of the applied method.

The contribution is outlined as follows: Section 2 gives a brief introduction to the *isogeometric B-Rep analysis* and sketches the development of an IBRA membrane and a cable element. Section 3 presents benchmark examples for the form-finding and points out the accuracy and robustness of the developed method. In Section 4 a prototypic full-scale application and a sculptural inflation example are presented in order to demonstrate the capability and potential of the derived methods for structural membranes. Finally Section 5 gives a conclusion and an outlook on future work.

2. Introduction to the isogeometric B-Rep analysis and the development of a membrane and a cable element

The *isogeometric B-Rep analysis* (IBRA), introduced by Breitenberger *et al.* in [3] is a relatively recent development in the field of finite element analysis: Its main objective is to allow for structural analyses on full CAD geometries, *i.e.* in general trimmed and coupled NURBS-surfaces, without the need for creating a separate analysis mesh. Hence the separation between a design model and a model for analysis shall be overcome since the gap between those is simply omitted.

This section introduces the *isogeometric B-Rep analysis* with some of its fundamentals as well as the elements required for structural membranes, *i.e.* mainly a prestressed membrane and cable element.

2.1. B-Rep description in CAD

In today's CAD systems, the *Boundary-Representation* (B-Rep) is a technique used to describe arbitrary geometrical entities by their boundaries. Following the B-Rep approach, for a three dimensional object a set of adjacent bounded surface elements called faces describes the "skin" of the object and thus the object itself. These faces at their turn are bounded by sets of edges which are curves lying on the surface of the faces. Several edges meet in points that are called vertices.

The B-Rep approach intrinsically incorporates trimmed surfaces, since the trimming curves become part of the boundaries in inner resp. outer trimming loops, see Figure 1.

As basis functions for the B-Rep description, commonly non-uniform rational B-splines (NURBS) are used [5]. The NURBS-based B-Rep description of geometries has become the standard for geometry description in modern CAD-systems.

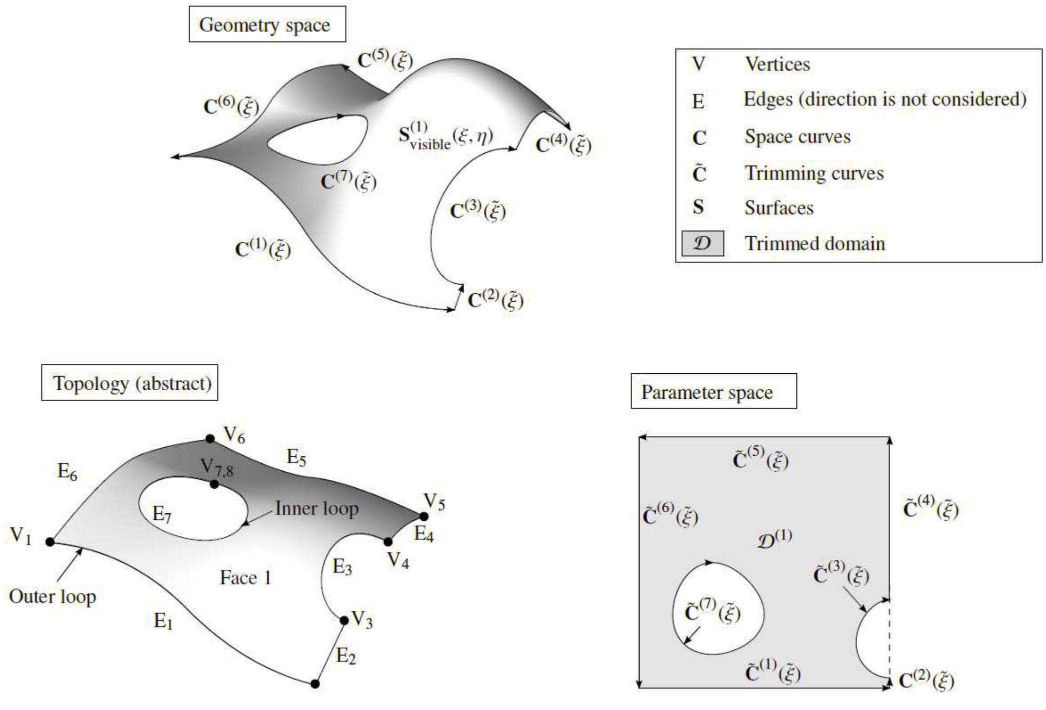

Fig. 1. B-Rep description of a free form surface with a trimmed hole [1,2].

2.2. The isogeometric B-Rep analysis (IBRA) as consequent generalization of the isogeometric analysis (IGA)

The *isogeometric B-Rep analysis* (IBRA) [3] is a generalization of the *isogeometric analysis* (IGA) introduced by Hughes and co-workers [8]. The main concept in IGA is to use the same basis functions for the computation of the discretized system that are used for the geometry representation in CAD, *i.e.* commonly NURBS, see above.

While IGA in its pure form is restricted to complete patches, IBRA at its turn refers to the full B-Rep description from the CAD system, *i.e.* including trimmed patches (see Fig. 1). Moreover, the concept of IBRA permits enforcing various conditions to the B-Rep entities, *e.g.*, coupling or support conditions [3] along the trimming edges or mechanical entities like a geometrically non-linear beam element [9] or the membrane and cable elements [1] applied in the sequel. Thus the pure geometrical CAD model can be augmented to an analysis suitable model.

Besides the ease of setting up a computation model in accordance with the design-model it can be stated that – at least in general – the NURBS representation that is used within IGA/IBRA is very well suited for free form geometries as they are dealt within architectural membranes' design. Ultimately this allows to use a very limited number of DOFs in order to already obtain very good results, cf. [3,8], which helps to substantially increase the speed of computation.

2.3. The Analysis in CAD as framework for the integration of structural analyses in a CAD environment

The IBRA serves as mechanical framework for the *analysis in CAD* (AiCAD)-approach which follows the philosophy of basing the structural analysis directly on the CAD model, thus omitting the need for a separate, specialized model [1-3,6], and providing a smooth design-through analysis workflow.

2.4. Formulation of a prestressed membrane element and of an embedded B-Rep edge cable element

Referring to the B-Rep description, the membrane element [1] is formulated as a NURBS discretized element according to classical membrane mechanics, including prestress and a prestress projection scheme as presented in [11]. In order to reflect the geometrical trimming operation, an integration procedure that takes this operation into account is chosen, referred to as *Nested Jacobian Approach* [3].

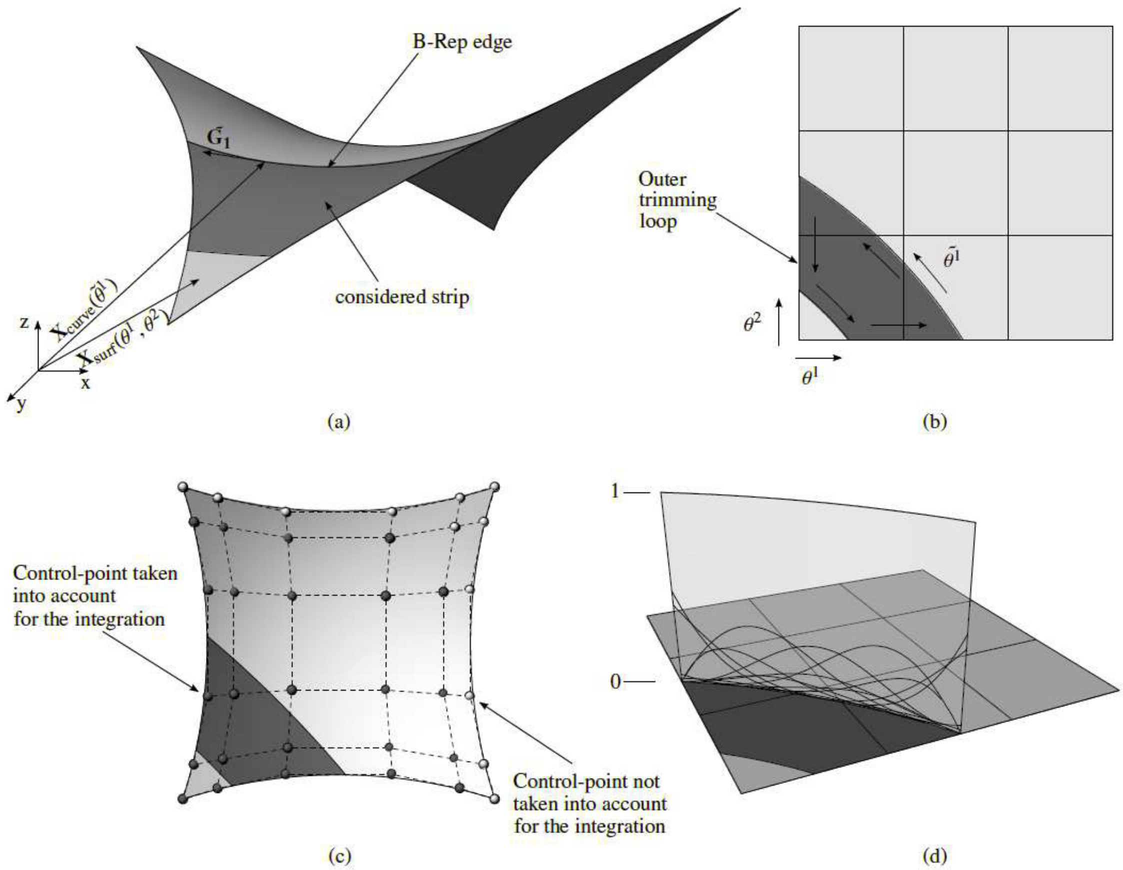

Fig. 2. B-Rep edge cable element in geometry and parameter space, and corresponding integration procedure [1].

As illustrated in Figure 2, the cable element at its turn makes use of the B-Rep edges: Since these are part of the trimmed NURBS patch, they can also share the patch's basis functions and extract these along the trimming curve. Following standard cable mechanics, an integrated cable (applied at outer or inner edges) can thus be formulated. Its contribution to the membrane's mechanical behavior is taken into account during the integration, which assembles the cable's contribution to the control points of the membrane (see Fig. 2) [1,3].

3. Benchmark examples for the form-finding of structural membranes with IBRA

Selected benchmark examples for the form-finding with IBRA are presented in order to underline the accuracy and robustness of the developed elements and the applied framework. As form-finding approach, the Updated Reference Strategy (URS) [7] has been applied in the present examples.

3.1. Benchmark for the accuracy of the form-finding approach

In order to test the accuracy of the applied method and the developed elements, the form-finding of a catenoid, illustrated in Fig. 3, has been chosen for two particular characteristics: (i) As one of the first minimal surfaces to be discovered, a closed mathematical solution for the surface area A_{catenoid} of the catenoid exists, which is given as

$$A_{\text{catenoid}} = r_0^2\pi \cdot \sinh\left(\frac{H}{r_0}\right) + r_0\,\pi H\,. \tag{1}$$

Here H is the height of the catenoid and r_0 is the minimal radius along the z-coordinate which is given as

$$R = r\left(z = \pm H/2\right) = r_0 \cdot \cosh\left(\frac{H}{2r_0}\right) \tag{2}$$

for the examined configuration of a catenoid with two equal radii $R = R\left(z = \pm H/2\right)$ at the top and the bottom. Note that Eq. (2) cannot be solved analytically for r_0, but has to be solved for a given pair of parameters R and H. For a catenoid of height H=1 and radius R=1, the minimal radius r_0 is numerically determined to be r_0 = 0.552434124, the corresponding surface area is A_{catenoid} = 5.99179697580228.

(ii) Following the cosh-shape of a catenary, the shape of a catenoid cannot be represented exactly with common basis functions, including NURBS. Hence the catenoid is well suited for refinement studies as the correct solution can only be approached, but not reached.

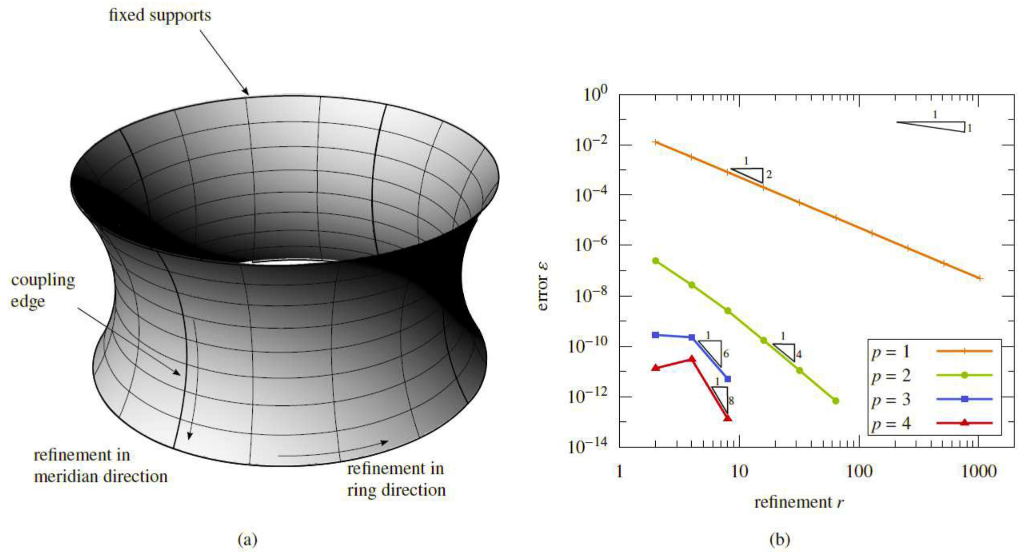

Fig. 3. Surface area for the form-finding of a stable catenoid with uniform prestress: (a) problem configuration, (b) log-log-error plot [1].

As refinement strategy, by raising the polynomial degree p of the basis functions starting from p = 1 (for this case, a “classical” bi-linear FEM-solution is obtained) up to p = 4 and the insertion of additional knots (referred to as refinement r) is taken, see Fig. 3. Note that the refinement is only applied in the meridian direction, along the ring direction a polynomial degree of p = 2 is already sufficient for the exact approximation of a circle and therefore further refinement doesn’t affect the quality of the result.

As can be seen in Figure 3, the error ε, measured as the relative deviation from the analytical solution, decreases for all polynomial degrees, ultimately to machine precision at around 10^{-14}. Furthermore the convergence rate obtained from the log-log-error plot (Fig. 3(b)) reveals the correct formulation and implementation of the elements since the convergence rate corresponds to the best possible solution. For further discussion on this aspect, see [1].

3.2. Benchmarks for the robustness of the form-finding approach with the developed elements

Based on the form-finding of the catenoid in the previous section, another well-known characteristic of that shape is exploited: Depending on the ratio between the radii R and the height H of the catenoid, two different minimal surface solutions might be obtained: below a certain critical height H_{critical}, the catenoid develops continuously, above that limit the obtained shape suddenly changes towards a so-called two-disk-solution, *i.e.* two filled circles with a constant surface area of $A_{\text{two-disk}} = 2\pi$. The critical height can be determined analytically to be $H_{\text{critical}}/R = 1.32548684$.

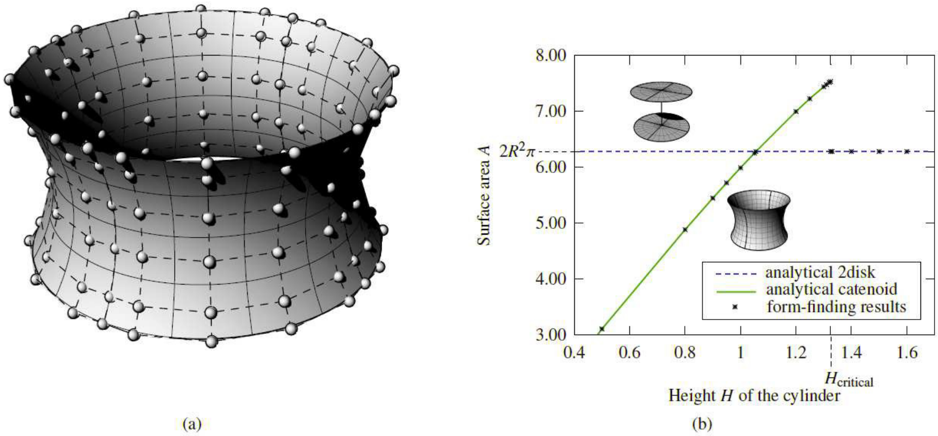

Fig. 4. Form-finding of a catenoid (a) Problem discretization, (b) limit determination for radii $R = 1$ and varying height H [1].

The development of the surface area with varying height H and the mentioned limit height are represented in Fig. 4. The form-finding results plotted in the graph in Fig. 4(b) show the sharp reproduction of that limit as well as the mentioned resulting shapes below and above that limit. This example provides a good benchmark for the accuracy of the chosen form-finding approach.

Along with the example of the burst catenoid above the critical limit, the form-finding of Costa's minimal surface, illustrated in Figs. 5 and 6, highlights the robustness of the developed elements: Although in the collapsed version Fig. 5(right), the elements are degenerated to a line, no convergence problems are observed up to that point. For the applied NURBS-based elements of high continuity, this degeneration is even more challenging, as can be seen in Fig. 6(right).

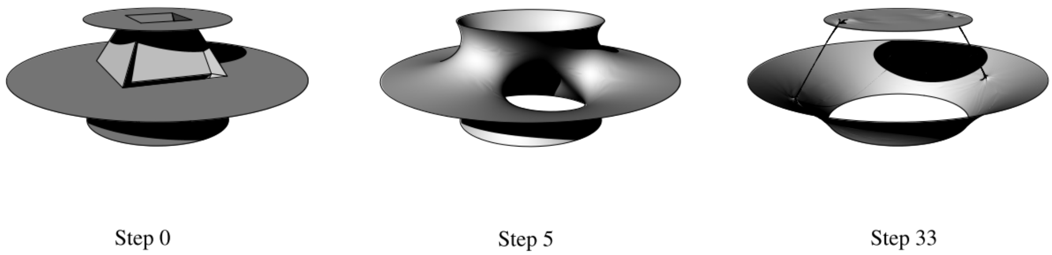

Fig. 5. Form-finding of Costa's minimal surface. Starting configuration, intermediate solution, and "collapsed" final solution (left to right) [4].

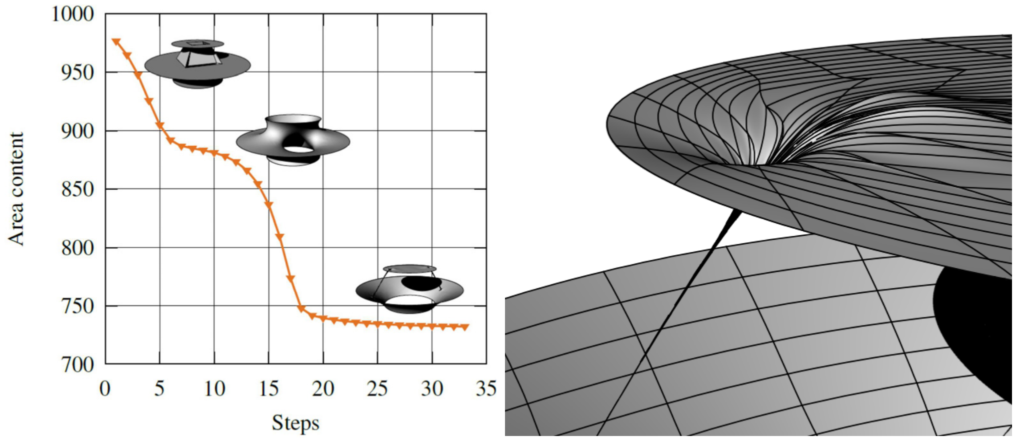

Fig. 6. Form-finding of Costa's minimal surface: Evolution of surface area throughout form-finding steps (left), distorted elements (right) [4].

Together these examples underline the accuracy and robustness of the developed elements in combination with the Updated Reference Strategy.

4. Prototypic applications for the form-finding and structural analysis of tensile membranes with IBRA

Two more complete applications shall highlight the ease of modelling and the potential of the presented methods: The form-finding and structural analysis of a prototypic five-point sail and the inflation of a cluster of bubbles.

4.1. Form-finding and structural analysis of a prototypic five-point sail

The full-scale prototypic example of a five-point sail is illustrated in Figure 7. The geometry is modeled as a trimmed multi-patch NURBS geometry (top). Along the edges – partially trimming edges – the formulated prestressed B-Rep edge cables are applied. Fixed supports are applied on the vertices as well as on arbitrary positions in the patches (central support at the top left). The two patches are coupled along their common edge (see [3,10] for the coupling approach). The structure is form-found with the prestress in the membrane and the edge cables (middle). On the form-found structure – which is still a complete CAD-geometry, *i.e.* a trimmed NURBS geometry – a snow load is applied and a non-linear analysis is carried out (bottom).

Hence the design and analysis of the treated structure are entirely integrated in the developed design-through-analysis workflow realized with the AiCAD-approach.

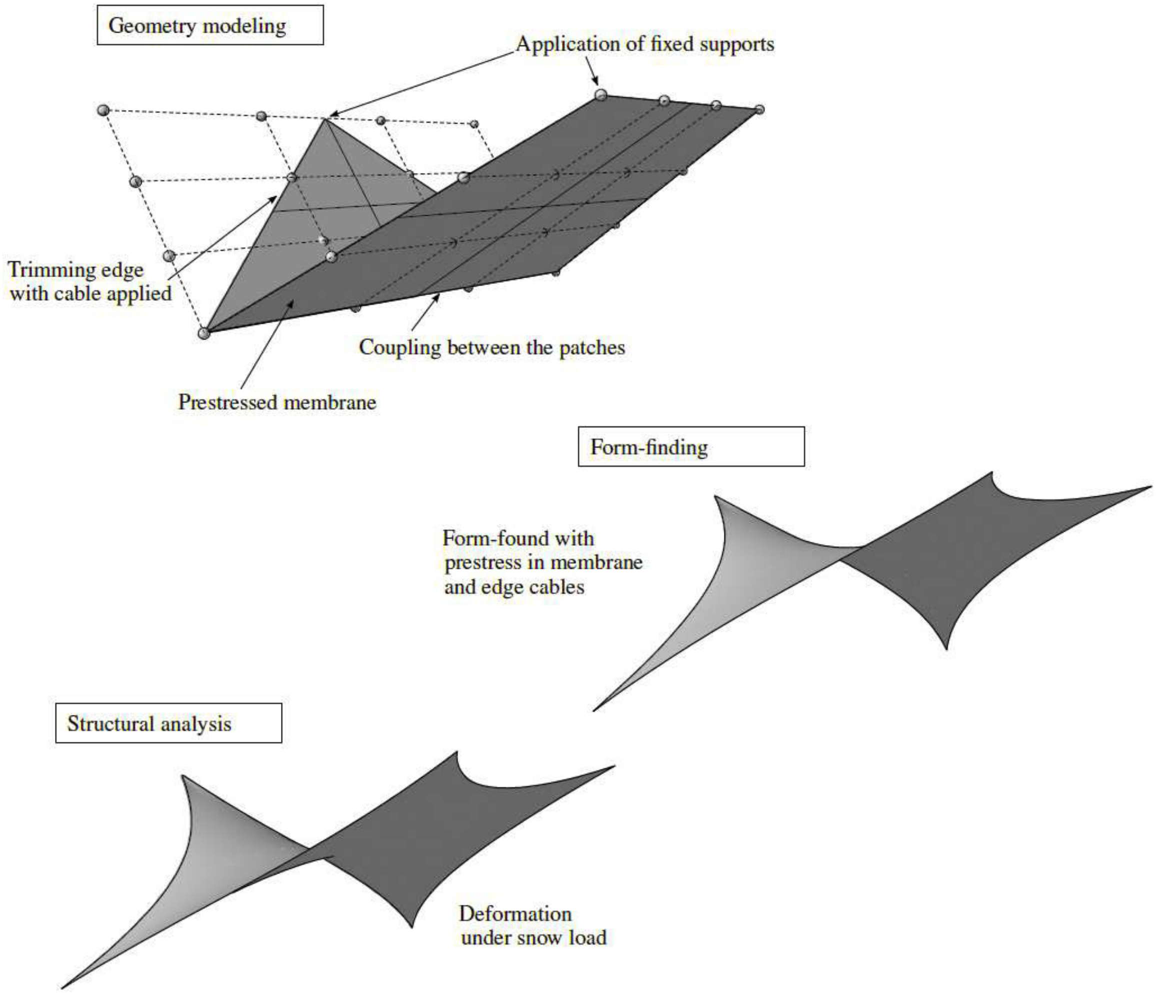

Fig. 7. Full-scale prototypic example of a five-point sail: Form-finding and structural analysis realized with the AiCAD-approach.

As illustrated in Figure 7, the result of each analysis is again a CAD-geometry, which means it can directly be used within design applications. This is of special interest for the result of the form-finding analysis.

4.2. Inflation of a cluster of soap-bubbles

Probably the most important potential of the method lies – besides interesting characteristics in terms of speed and accuracy of the computation – in the power of integrating design and analysis within one single model. The example of the soap bubble-cluster, illustrated in Figure 8, shall underline this idea.

The soap bubble cluster has been created the following manner:

- A single soap bubble is inflated by applying constant internal pressure on an initially flat circular membrane.
- Once the bubble has reached the desired size/stress level, the obtained geometry – still an intact NURBS based CAD geometry – is manipulated in the CAD-environment: The bubble is copied two times, the resulting bubbles are then intersected.
- Mechanically this intersection corresponds to a displacement-coupling along the common intersection lines.

- The resulting bubble cluster is then inflated with increasing varying pressure, until ultimately the shape in Fig. 8, right, is revealed.

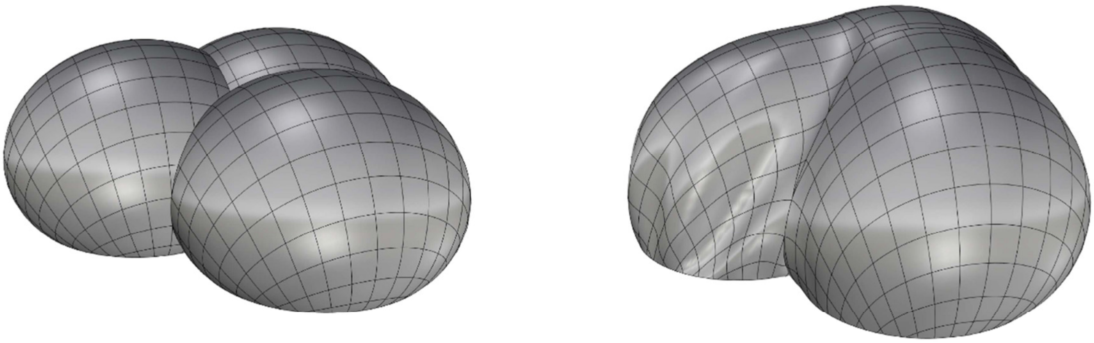

Fig. 8. Inflation of a cluster of soap-bubbles, inflated with constant pressure up to a certain level (left) followed by varying levels (right).

Although it does not represent a realistic mechanical application, this example highlights the advantage of a common model for design and analysis: Not only the one-time conversion is omitted – thus omitting an important source of error and inconsistencies – but also the advantages of both "worlds" come to use. Although still the cluster of three individual bubbles in Fig. 8(left) could easily be "designed", *i.e.* geometrically modeled, the generation of more complex shapes by a physical approach as the one illustrated in Fig. 8(right) with its wrinkles could hardly be modeled without the mechanical analysis behind.

5. Conclusions and outlook

In the present contribution the application of the *isogeometric B-Rep analysis* to the form-finding of structural membranes has been presented. Several benchmark examples are used to discuss the accuracy and robustness of the developed method. The application examples – a prototypic architectural membrane and a more sculptural example of a soap-bubble cluster – highlight the potential of the presented approach through the full integration of the analysis and the design.

These advantages are mainly related to the continuous surface description of the membrane's shape that stays intact throughout all design steps. Having the CAD-geometry of all steps available, geometric operations and investigations may be performed within the specialized CAD-environments. This could be of interest when it comes to curvature analyses, the investigation of ponds or also for the separation into individual strips as a first step towards the cutting pattern generation.

Thinking in the direction of design cycle for architectural membranes, the development of a cutting pattern generation for these strips certainly is the next step, which is part of current research activities at the authors' Chair [12]. Further development of the method in terms of the form-finding might include the integration of stress adaption for the case of anisotropic prestress ratios as presented in [11].

Note: All structural computations in this contribution have been performed in the FE-code CARAT++, continuously developed at the Chair of Structural Analysis at the Technische Universität München. The integration of CAD and analysis has been realized as an in-house developed Rhinoceros®-plugin.

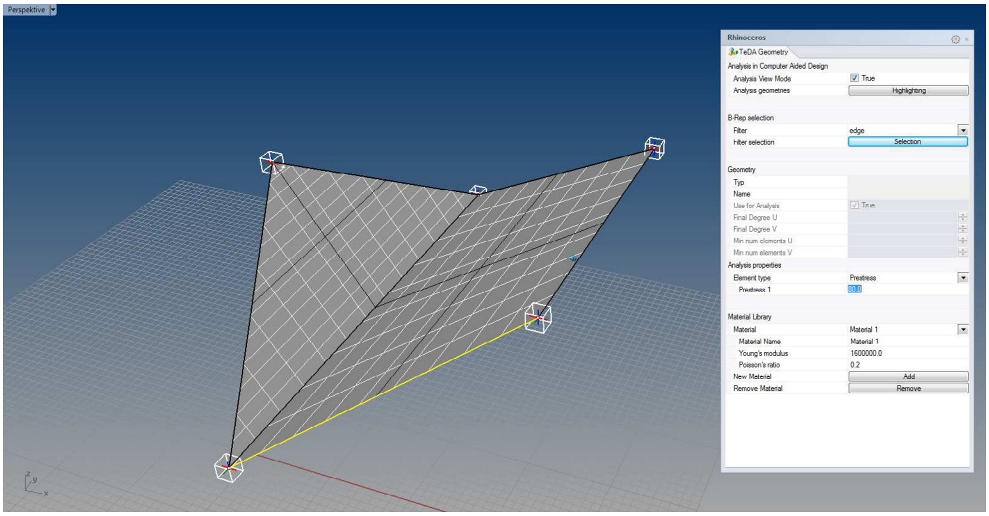

Fig. 9. Preprocessing of the five-point sail within the Rhinoceros-plugin to integrate design and analysis [2].

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