

DYNAMIC OCEAN TOPOGRAPHY FROM GOCE - SOME PREPARATORY ATTEMPTS

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ABSTRACT

Tests of dynamic ocean topography (DOT) estimation have been carried out in anticipation of the availability of GOCE gravity models. Mean ocean surface models of recent years, based on data of satellite radar altimetry missions are thereby combined with high resolution geoid and gravity gradient data from GRACE. Both data sets have been made spectrally consistent, on the one hand by filtering in the spectral domain the geoid and constructing a spherical harmonic representation of the ocean surface and on the other hand by applying identical filters to geoid and sea surface heights sampled along individual tracks. Both approaches are accompanied by error propagation using the variance-covariance matrix of the gravity field coefficients and the error covariance function of the altimeter data. In a second step the DOT is converted to surface velocities under the assumption of geostrophic balance; also these computations are accompanied by rigorous error propagation. Finally, data assimilation is carried out of DOT data with varying degrees into a finite element ocean model employing the method of ensemble based Kalman filtering.

1. INTRODUCTION

In the studies by (Wunsch and Gaposchkin, 1980) and (Ganachaud et al., 1997) in situ hydrographic data and a circulation model derived from altimetry and geoid information are combined to derive a global estimate of the absolute oceanic general circulation. The realization of this concept has been one of the basic motivations of the current gravity field satellite missions CHAMP, GRACE and GOCE.

The dynamic ocean topography derived from altimetric observation and a geoid model can be assimilated into an ocean model. The dynamically consistent ocean model assimilation is based on a combined geodetic-oceanographic cost function. The model input and output are iteratively compared to assess the consistency of each of the data sets and the estimated state of the ocean.

The basic relation to compute the steady-state dynamic ocean topography (DOT) H is very simple:

$$H = h - N \quad (1)$$

where h is the mean sea surface height (SSH) relative to a reference ellipsoid and N is the geoid height referring to the same ellipsoid. However the problem is complicated by the fact that two quantities are subtracted with different spectral representation and different spectral content. Geoid heights are usually provided as a truncated spherical harmonic series, i.e. in a band-limited global spectral representation on a sphere, while the altimetric measurements are given as weighted mean values over the footprint of the radar signal along the ground track of the spacecraft.

The GRACE geoid models available today have a spatial resolution of about 170 km and a geoid accuracy of few centimeters. With GOCE the spatial resolution is expected to increase to about 100 km with a geoid accuracy of 1-2 cm. Altimeter satellites allow measurement of very precise, regular and quasi-global sea surface heights. The altimetry data are collected for many years over the repeated tracks of several missions. Actually we consider measurements from 6 satellites over a range of 17 years. A multi-mission cross-calibration of all these altimeter systems (Bosch and Savcenko, 2007) provides an estimate of the radial accuracy which is in the order of 2 -3 cm.

The resolution of a geoid model is expressed by the maximum harmonic degree of its harmonic representation. The spectrum is truncated usually where the signal to noise ratio exceeds unity. For all degrees (and orders) less or equal to L one has the coefficients of the model and their error variances (the commission error). The signal for degrees above L is not modeled here, but it is identified as the omitted signal or omission error.

The SSH data contain information with higher spatial resolution than is included in the geoid model. These short scale features will contain both dynamic topography and geoid features and must be removed by filtering, to make sure that the computed DOT is consistent with the geoid field. Furthermore in (Losch et al., 2002) is shown that the omission error can leak in the commission error of the filtered signal when different base functions are involved. Therefore it is attempted here to find a common representation for geoid and SSH and to obtain a consistent resolution for the data applying the same low-pass filter.

Spectral consistency is not only required between altimetry and ocean model but also between the estimated DOT and the ocean circulation model and it is

really important to understand how the spatial resolution of an ocean model is defined.

2. GEODETIC DYNAMIC OCEAN TOPOGRAPHY

The altimetric data considered in the following computations are the measurements of the missions Topex-Poseidon, Jason1, Envisat and GFO for the year 2004. The geoid heights are derived from the ITG03S gravity field (Bonn University).

In order to compute a consistent geodetic DOT, first of all, some obvious preliminary corrections must be applied to the data. Geoid and SSH must refer to a common reference system, be defined in the same tide system and expressed using the same coordinates system. Two different filter techniques are applied, one regarding the data globally distributed (“global” approach) and a second method with the geoid and altimetric data along the altimetric satellite tracks (“profile” approach).

2.1. Global approach

The global approach tries to solve the problem of the spectral inconsistency, expanding the altimetric sea surface into the land areas. In this way geoid and sea surface have the same global spectral representation and can be processed in a compatible form. The mean sea surface is “extended” by filling the land areas with the geoid heights derived from a gravity model (for example EGM2008, see (Pavlis et al., 2008)). Then, using spherical harmonic analysis and synthesis in an iterative process, the land-ocean transition is smoothed, as described in (Gruber, 2000) and (Wang, 2007). After each iteration, the original mean sea surface is kept unchanged in the ocean areas while on land the “mixed” surface (computed by spherical harmonic synthesis) is adopted. After a certain number of iterations, the step between land and ocean is nicely smoothed, (Albertella and Rummel, 2009). At the end of the iterative procedure geoid and sea surface have the same representation in term of spherical harmonic functions and they can be spectrally filtered, up to a selected harmonic degree. Then the DOT is computed subtracting a geoid model from altimetry. In this way the DOT is defined over the entire earth, even so the values on land have no physical meaning. The DOT can now be written in a spherical harmonic expansion:

$$H(P) = \sum_{l=0}^L \sum_{m=0}^l \left(C_{lm}^H \cos m\lambda_p + S_{lm}^H \sin m\lambda_p \right) \bar{P}_{lm}(\cos \vartheta_p) \quad (2)$$

Here (ϑ_p, λ_p) are the spherical coordinates of the point P , C_{lm}^H and S_{lm}^H are the normalized spherical

harmonic coefficients, L is the highest degree considered in the spherical harmonic expansion, $\bar{P}_{lm}(\cos \vartheta)$ are the fully normalized associated Legendre functions of degree l and order m . The optimal filter is a Gauss filter because it corresponds to a Gaussian function in both the space and spectral domain. The formula is defined in (Jekeli, 1981) and has been modified in (Wahr et al., 1998). It can be computed by recursive formulas:

$$W_0 = 1, \quad W_1 = \left[\frac{1 + e^{-2b}}{1 - e^{-2b}} - \frac{1}{b} \right], \dots$$

$$W_l = -\frac{2l+1}{b} W_{l-1} + W_{l-2}$$

where $b = \frac{\ln(2)}{1 - \cos \alpha}$. The radius α is empirically related to the harmonic degree, (Zenner, 2006). The filtered DOT is obtained multiplying the harmonic coefficients C_{lm}^H, S_{lm}^H by the coefficients W_l . Its resolution is now established by the maximum harmonic degree considered in the filtering.

2.2. Profile approach

The profile approach was developed to avoid the problems related to the gridding of the data, maintaining as long as possible the high resolution data along the measurement profiles, (Bosch and Savcenko, 2010). Also in this case, the procedure ensures that sea surface heights and geoid undulations are spectrally consistent before they are subtracted to estimate the DOT. The profile approach is based on the following strategy:

1. A Gauss type filter with a length of 250 km is applied to the spherical harmonics of the gravity field removing the artificial, meridional stripings typical of the GRACE satellite-only gravity fields.
2. The smoothed geoid is evaluated along the altimeter profiles. In order to achieve spectrally consistent sea surface heights the altimetry profiles are smoothed in the same way (i.e. applying the same Gauss type filter in the one-dimensional space domain).
3. Unfortunately, the one-dimensional along track filter is not perfectly equivalent to the two dimensional filtering in the space or spectral domain.
4. To correct for these systematic distortions, two versions of an ultra high resolution geoid (EGM08) are evaluated at the altimeter profiles, a spectrally smoothed and an unsmoothed version. The latter is smoothed by the 1-D along track filter and subtracted from

the first. The result is taken to correct for the systematic distortion identified under point 3.

The approach proves to be very efficient and even accounts for data gaps and the coastal transition zone where missing data complicates the application of any filter procedure.

In Fig.1 the DOTs filtered up to degree 60, computed following the profile and the global approach are shown. The two surfaces show in general a good agreement, but along the coastlines the differences can reach 0.4 m.

3. FROM DOT TO GEOSTROPHIC VELOCITIES

The knowledge of the DOT allows studying the absolute circulation of the ocean and determining the associated geostrophic surface currents. The gradient of the DOT is directly related to the geostrophic currents. The geostrophic equations express the equilibrium between the horizontal pressure gradients and the Coriolis accelerations. The vertical component of the geostrophic equations expresses the equilibrium between the vertical pressure gradient and gravity. In (Elema, 1993) the formulas for the surface water velocities of the ocean circulation, in longitude and in latitude direction are derived as:

$$\begin{aligned} u_s &= \frac{g}{f} \frac{1}{R} \frac{\partial H}{\partial \vartheta} \\ v_s &= \frac{g}{f} \frac{1}{R \sin \vartheta} \frac{\partial H}{\partial \lambda} \end{aligned} \quad (3)$$

Here g is the gravitational acceleration, $f = 2\Omega \cos \vartheta$ is the Coriolis term, Ω is the angular velocity of the earth and R is the radius of the spherical earth. Differentiating Eq. 2 respect to λ and ϑ and combining the result with the geostrophic Eq. 3, the surface velocities in longitude and in latitude direction as function of the harmonic coefficients C_{lm}^H, S_{lm}^H are obtained as:

$$\begin{aligned} u_s &= \frac{g}{fR} \sum_{l=0}^L \sum_{m=0}^l (C_{lm}^H \cos m\lambda_p + S_{lm}^H \sin m\lambda_p) \bar{P}_l(\cos \vartheta_p) \\ v_s &= \frac{g}{fR \sin \vartheta} \sum_{l=0}^L \sum_{m=0}^l m (-C_{lm}^H \sin m\lambda_p + \\ &\quad + S_{lm}^H \cos m\lambda_p) \bar{P}_l(\cos \vartheta_p) \end{aligned} \quad (4)$$

The direction of the surface current vector is $A = \tan^{-1} \left(\frac{u_s}{v_s} \right)$ and its length is $v = \sqrt{u_s^2 + v_s^2}$.

Fig. 2 shows the components of the surface velocity in longitude and in latitude direction computed from the DOT filtered up degree and order 60. The eastward component is the dominant one. If a coarser filter is applied, more details in the DOT and in the velocities fields are visible. In Fig. 3 the results in a regional area, obtained with two Gaussian filters are shown. The distortions close to the land-ocean transitions are the weak point in the DOT computation by the global approach. DOT, mean geostrophic velocities and the corresponding streamlines for different resolutions are shown in the area $\varphi = [-10^\circ, -160^\circ]$ $\lambda = [260^\circ, 350^\circ]$. Two Gaussian filters are considered: up to $L = 60$ and up to $L = 120$.

4. ERROR PROPAGATION

Recent models of the geopotential gravity field are available together with the complete statistical information. The gravity model ITG03S, computed from GRACE data by University of Bonn (Mayer-Guerr, T., 2006), is provided with its full variance-covariance matrix up degree and order 180. From that matrix the full error propagation to the geoid undulation considering various maximum degrees has been performed. In Fig. 4 the covariances of the geoid height at the point $P = (0^\circ, 0^\circ)$ are shown considering the full variance covariance matrix of the ITG03S gravity model up degree and order 150 are shown. The covariances are not isotropic in longitude and in latitude directions. A stripping effect is visible in longitude as well as in latitudinal direction.

The goal of the GOCE mission is the determination of the stationary part of the gravity field to a high degree of accuracy and spatial resolution. The expectation is a significant error reduction at high degree and orders.

The knowledge of the full stochastic model of the geoid undulation, together the error information of the sea surface height (in first approximation altimetry can be considered uncorrelated) represents the complete statistical information of the absolute sea surface topography. That information can improve the accuracy of the assimilation procedure of the observed DOT in an ocean model.

5. DATA ASSIMILATION EXPERIMENTS

The data assimilation experiment is performed using the Finite-Element Ocean circulation Model (FEOM), (Wang et al., 2008) configured on a global almost regular triangular mesh with the spatial resolution of 1.5° . There are 24 unevenly spaced levels in the vertical direction. FEOM solves the standard set of hydrostatic ocean dynamic primitive equations using continuous linear representations for the horizontal velocity, surface elevation, temperature and salinity.

The experiments conducted here use the local SEIK filter algorithm [implemented within PDAF, (Pham et

al., 1998), (Pham, 2001), (Nerger et al., 2006), (Nerger et al., 2005)] to update the full model state, consisting of temperature, salinity, sea surface height (SSH) and velocity fields. During the assimilation, the analysis of the full ocean field is carried out for each water column of the model separately. Update of the each column depends only on observations within a specified influence region. In addition, observational increments are weighted depending on their distance from the point at which the assimilation is made. For weighting a 5th order piecewise rational function is used that goes to 0 at 900 km, (Gaspari and Cohn, 1999). This function is compactly supported and approximates well a Gaussian function with the length scale of 246.5 km. Tuning of the observational radius was done for the global ocean model to be as large as possible in order to include as many observations as possible and still obtain reasonably accurate results. The large sets of experiments have indicated that the observational radius should be 900 km. Experiments were repeated for DOT data up to degree 120 and 150. In these cases, a 5th order piecewise rational function was chosen that goes to 0 at 450 km and 360 km respectively, approximating Gaussians with length scales of 123 and 98.6 km, respectively.

In the analysis, a diagonal observation error covariance matrix is used with an error variance of 9 cm^2 for DOT up to degree 60, 9.6 cm^2 for DOT up to degree 120 and 84.6 cm^2 for DOT up to degree 150. The error variance includes both the error of altimetry and gravity data, mapping errors as well as the effects of cross-correlations introduced by interpolation (Janjic and Cohn, 2006). The 13 ensemble members are used for assimilation. The initial field and initial error covariance was the same as in (Skachko et al., 2008). The 12 EOFs and the initial state estimate were used to generate the initial ensemble of 13 model states by second-order exact sampling (Pham, 2001).

The same dynamical ocean topography (DOT) data are assimilated every 10 days for 15 months. Mean dynamical ocean topography obtained by averaging every 10 day outputs over the last 12 months is compared to data. In Fig. 5 the difference between observed data and mean dynamical topography calculated are shown. The differences with the product of the analysis (left panel) are smaller than the forecast (right panel). The more problematic areas are the coastlines and the boundary of the Antarctic region.

6. CONCLUSION AND PERSPECTIVES

In this work the high resolution DOT determination has been studied. The geodetic observations (sea surface heights and geoid heights) are combined and consistently filtered, taking into account the different representation and resolution of the data. The two developed approaches (profile and global) give results

with good, but not perfect, agreement, in particular around the land-ocean transition, that is one of the critical points of the problem. The geodetic DOT is assimilated into the FE ocean model employing a sequential Kalman filtering approach.

The impact of different spatial resolutions (or different Gauss filters) on DOT estimates were studied also in terms of geostrophic velocities. Applying a narrower filter (for example up to degree 120 or 150) small scale features become visible. The nature of these features must be investigated to understand if they are numerical effects, distortions of altimetry, of the geoid surface or if they are real ocean features. This point is of particular importance in view of high resolution DOT models to be derived from GOCE.

The modification of the observational error covariance by different weighting functions affects the accuracy of the results in the assimilation process. For this reason the full error propagation must be completed and improved introducing the correlations of the altimetric data. Finally the spectral consistency between the geodetic DOT and the ocean circulation model must be understood.

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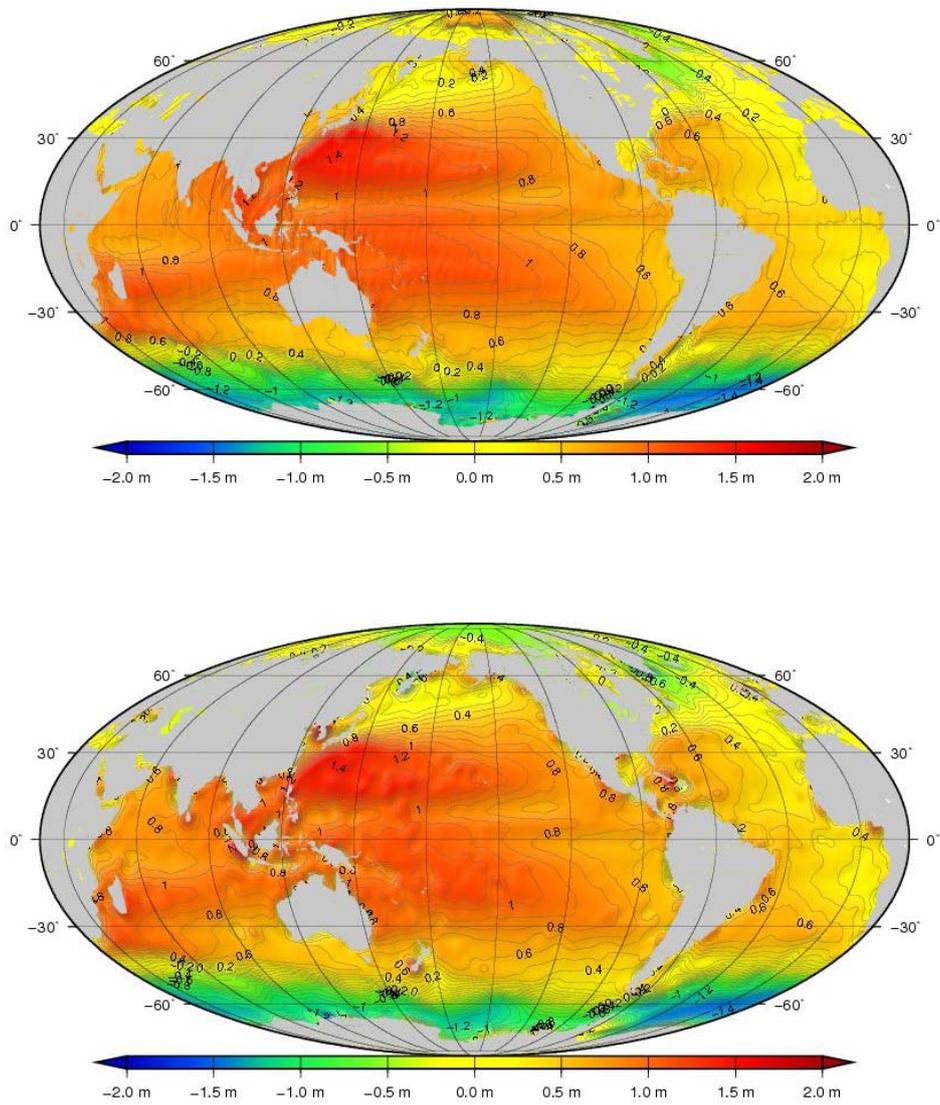


Figure 1. DOT computed with the “profile” approach (top) and with the “global” approach (bottom). Both DOTs are filtered up degree 60, which corresponds to a radius of 241.667 km in the space domain. Units are meters.

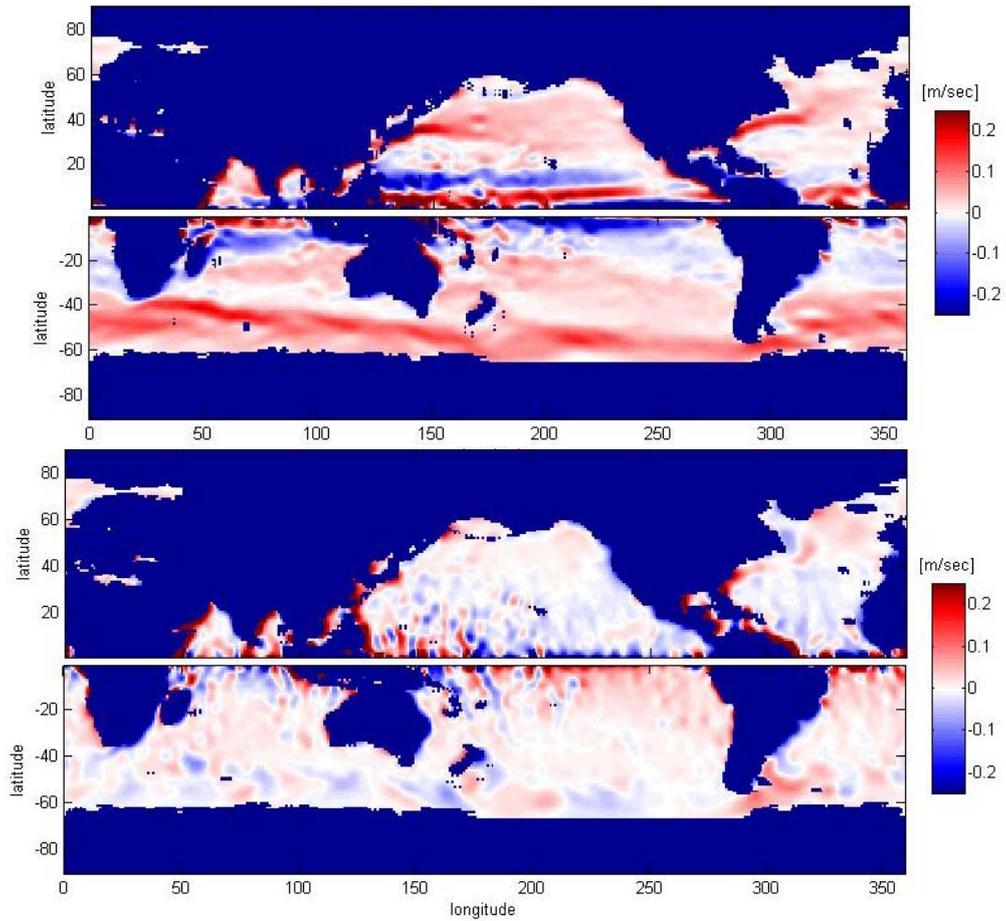


Figure 2. Eastward component (upper panel) and northward component (lower panel) of the surface water velocity derived from the geodetic DOT filtered up degree and order 60. Units are m/s.

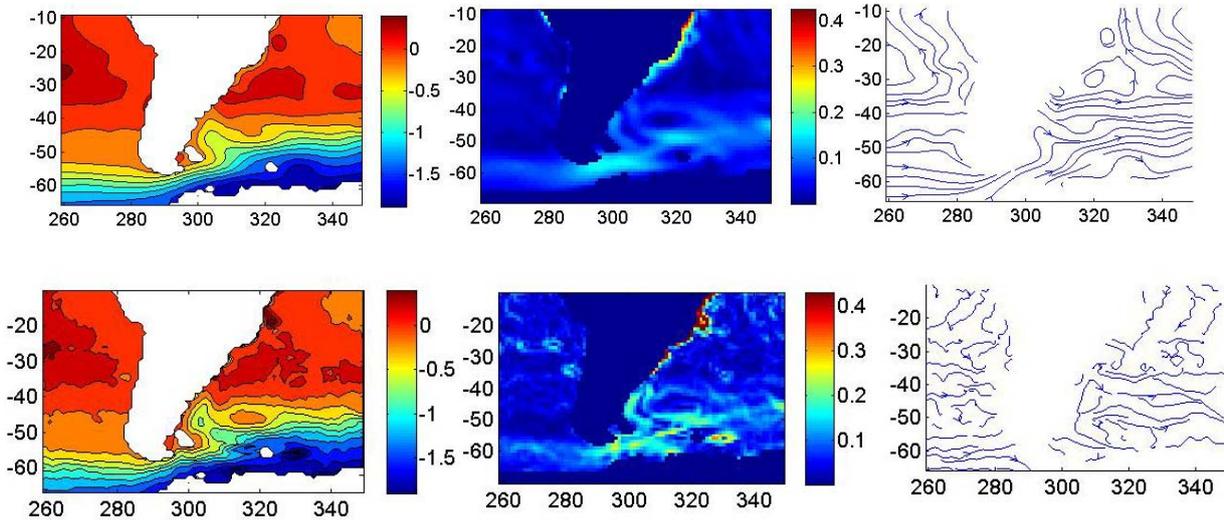


Figure 3. DOT (left), mean geostrophic velocities (middle) and the corresponding streamlines (right) for different resolutions, in the area $\varphi = [-10^\circ, -60^\circ]$ $\lambda = [260^\circ, 350^\circ]$. Two Gaussian filters are considered: in the upper panel up $L = 60$ and in the lower one up $L = 120$. Units are meters for the DOT and meters/sec for the velocities.

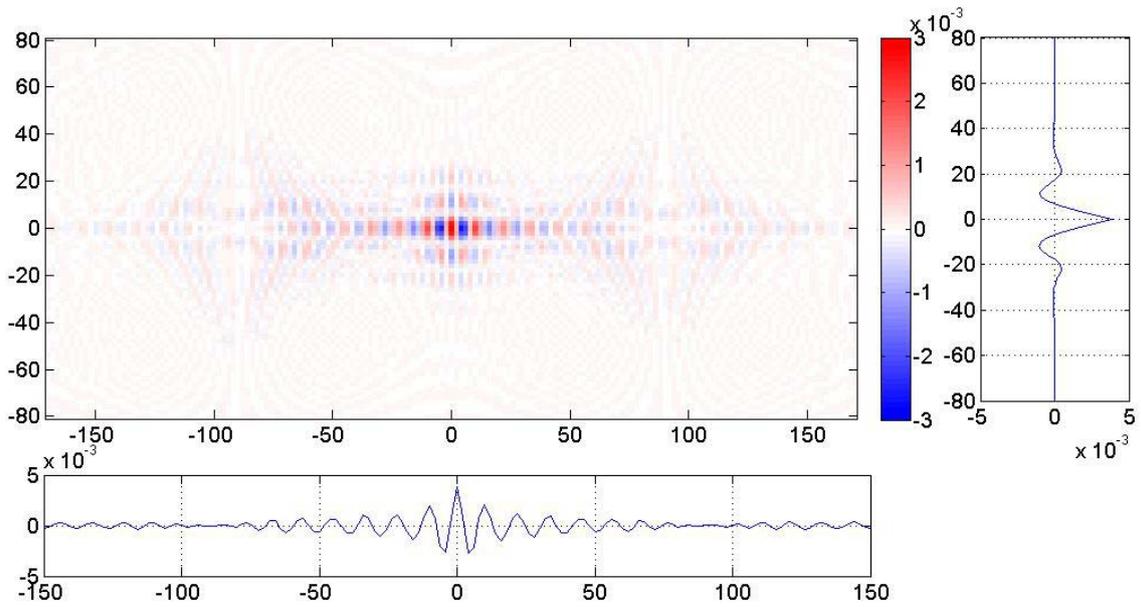


Figure 4. Geographical distributions of the covariances of the geoid height at the point $P = (0^\circ, 0^\circ)$ computed considering the full variance covariance matrix of the ITG03 gravity model up degree and order 150. Sections along longitude (bottom) and along latitude (right) are shown. Units are $m \times m$. According to the cumulative error for GRACE, the value in the origin is ~ 7 cm.

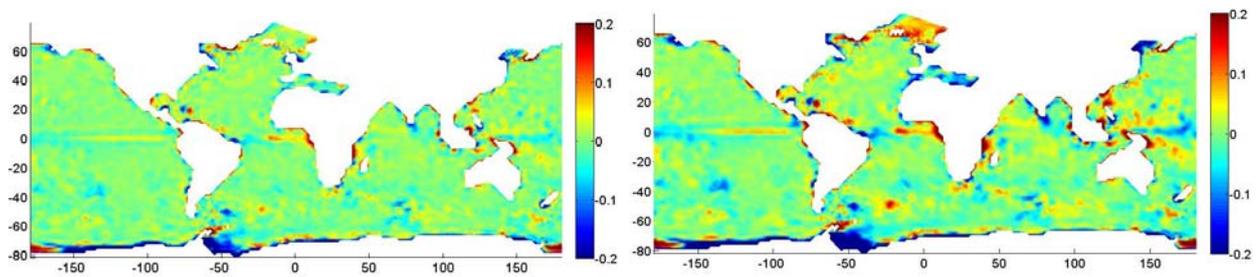


Figure 5. DOT filtered up to degree 60 minus mean dynamical ocean topography obtained by averaging analysis (left) and forecast (right). The biggest differences are located along the coastline and in particular on the boundary of the Antarctic region. As expected the differences with the forecast are larger than the differences with the result of the analysis. Units are meters.