GOCE GRAVITY FIELD MODEL DERIVED FROM ORBIT AND GRADIOMETRY DATA APPLYING THE TIME-WISE METHOD

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ABSTRACT

A first GOCE gravity field model based on two months of GOCE orbit and gradiometry data has been computed applying the time-wise method. The paper gives an overview of the software system, and discusses the key features of the solution strategy. The resulting global gravity field model, resolved complete to degree/order 224, is GOCE-only in a strict sense, i.e., no a priori gravity field information entered the solution. Realistic stochastic models for both the orbit and gradiometer observations have been included. Thus, the coefficient error information, provided as full variance-covariance matrix, reflects the true error behaviour of the solution. The resulting GOCE model is assessed and validated against state-of-the art gravity field models. Since the model is independent of any gravity prior information, it can be used to assess the additional information content of GOCE, and can be combined with complementary satellite and terrestrial gravity field information.

1. INTRODUCTION

The dedicated satellite gravity mission GOCE (Gravity field and steady-state Ocean Circulation Explorer; [6]), the first Earth Explorer Core Mission, in the context of ESA's Living Planet programme, strives for a high-accuracy, high-resolution global model of the Earth's static gravity field. GOCE is based on a sensor fusion concept: satellite-to-satellite tracking in the high-low mode (hl-SST) using GPS, and satellite gravity gradiometry (SGG). While the low frequencies are mostly derived from hl-SST, the details of the gravity field are obtained from the analysis of SGG.

The scientific data processing (Level 1b to Level 2) is performed by the “European GOCE Gravity Consortium” (EGG-C), a consortium of 10 European university and research institutes, in the framework of the ESA-funded project “ GOCE High-Level Processing Facility” (HPF; [20]). In the frame of this contract, the “Sub-processing Facility (SPF) 6000”, a co-operation of TU Graz, TU München, University of Bonn, and Austrian Academy of Sciences, is responsible for the processing of a spherical harmonic Earth’s gravity model and the corresponding full variance-covariance matrix from the precise GOCE orbit and SGG data, and the production of quick-look gravity products in parallel to the GOCE mission for a fast system diagnosis.

The mathematical model for the parameterization of the Earth's gravity field is based on a series expansion of spherical harmonics. The model presented in this paper has a resolution complete to degree and order 224, which requires solving for more than 50,000 unknown spherical harmonic coefficients. The determination of these coefficients from the complementary hl-SST and SGG data sets is a demanding numerical and computational task, and efficient solution strategies have to be applied to solve the corresponding large normal equation systems. During the last decade, several approaches have been developed to perform this task (e.g., [5], [9], [14], [15]). In [15], [16], the rigorous solution of the large normal equation matrix by means of a parallel processing strategy implemented on a Linux-PC cluster was proposed.

2. ARCHITECTURAL DESIGN

Fig. 1 shows the architectural design, the main components and the product flow through the SPF6000 software system. It is conceived in a highly modular manner that allows the investigation of specific aspects of gravity modelling such as filtering, numerical stability and optimum regularization, complementary relations of SST and SGG and their optimum weighting.

The software system is composed of two main components: the Quick-Look Gravity Field Analysis (QL-GFA), and the Core Solver (CS), which will be briefly described in the following.

Proc. ‘ESA Living Planet Symposium’, Bergen, Norway
28 June – 2 July 2010 (ESA SP-686, December 2010)

2.1. Quick-Look Gravity Field Analysis (QL-GFA)

This stand-alone software system performs the computation of fast approximate gravity field solutions based on SGG and hl-SST data, for the purpose to derive a fast diagnosis of the GOCE system performance and of the Level 1b and Level 2 input data in parallel to the mission with short latencies. These gravity field products are input to ESA's calibration/validation activities in the frame of the GOCE mission control.

While post-processing high-precision gravity field solutions are processed also by two other consortia within HPF ([5], [14]), the continuous quick-look processing in parallel to the mission is a unique additional feature of the SPF6000 processing system.

A detailed description of the architecture design and functionality of the QL-GFA processor can be found in [18], and first operational results are provided in [12].

2.2. Core Solver (CS)

This software component delivers a rigorous ultimate-precision solution of the very large normal equation systems applying parallel processing strategies. The Core Solver is composed of the *Final Solver*, taking the full normal equation matrix into account, and the *Tuning Machine*, being based on the method of preconditioned conjugate gradients, which verifies and tunes the involved software components of the Final Solver in many respects. Concerning the hl-SST processing, the energy integral approach is applied.

The *Tuning Machine* consists of two main modules:

- *pcgma* (pre-conditioned conjugate gradient adjustment): It acts as a stand-alone gravity field solver, and is used to verify and tune the involved software components of the Core Solver, e.g., to derive optimum regularization and weighting parameters.
- *Data analysis tool*: The data inspection and filter design tool is used to verify external and internal products, and to define the filter coefficients which will be used in the Final Solver.

The *Final Solver* consists of the following main modules:

- *SST processor*: The information content of the SST data is exploited by making use of the precise GOCE orbit expressed in terms of position and velocity information including quality description. The software can process both kinematic and reduced-dynamic orbits. The principle of energy conservation is applied in an inertial reference frame ([1]). Favourable features of this approach are a strictly linear observation model as well as the fact that gravity functionals are processed. In contrast to QL-GFA, which performs a block-diagonal approximation, the CS SST processor exploits the information content of the full normal equation matrix.
- *SGG processor*: Given the precise GOCE orbit, the calibrated gravity gradients defined in the Gradiometer Reference Frame (GRF) are directly related to the unknown geopotential coefficients resulting in the linear observation model for all relevant tensor components, allowing to exploit the high degree of precision and resolution of the data. The complications arising from the coloured noise of the gradiometer are managed by a recursive filter

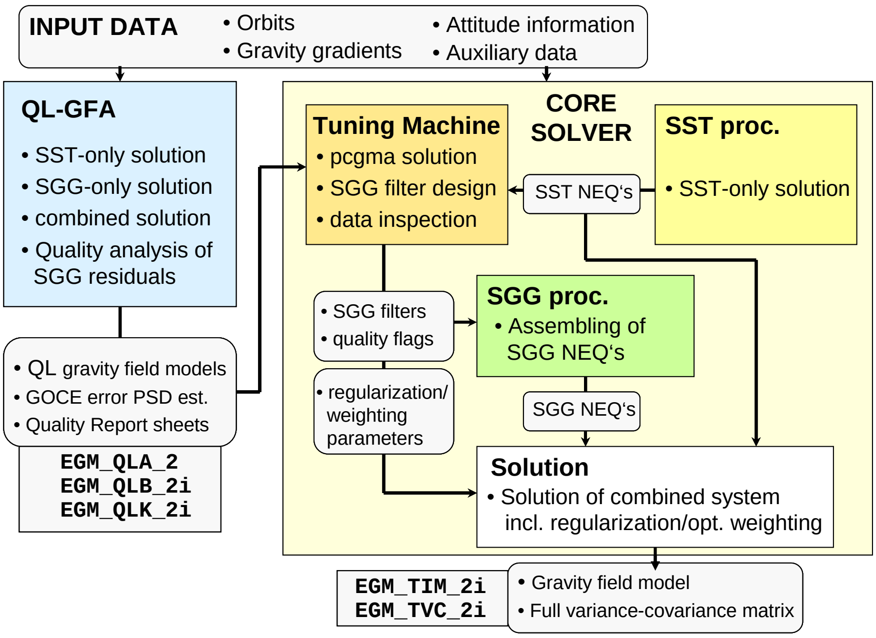

Figure 1. Software architecture and product flow

procedure in time domain ([21], [22]). The SGG processor assembles the full normal equations applying parallel processing on a Linux-PC-Cluster.

- *Solver*: The mathematical models for SGG and SST data are combined to the overall mathematical model by means of superposition of the normal equations, applying an optimum weighting of the individual data types. The solution is processed applying a parallelized Cholesky reduction. The ill-posedness of the normal equations due to the polar gaps is managed by optimized regularization techniques ([13]).

In the Core Solver processing, the SST and SGG normal equations are assembled separately. The SST normal equations (and other internal products) are transferred to the Tuning Machine and the Final Solver. In the Tuning Machine, the SGG normal equations are set-up using a sparse matrix scheme ([3]), and gravity field solutions are computed applying the pcgma algorithm ([4]). The residuals of the adjustment are analyzed by the Data Inspection tool, and filter coefficients, regularization and weighting parameters are derived, which are provided to the Final Solver. Here, the full SGG normal equations are assembled, and optimally combined with the SST normal equations. Finally, the gravity field solution and the full inverse of the normal equation matrix are computed rigorously. The final output products are GOCE gravity field model coefficients, and a statistical error description in terms of the full variance-covariance matrix. Additionally, internal products, such as normal equations of the individual components, residuals, flags, regularization and weighting parameters, etc., are produced. The file names of the official SPF6000 output products are:

- Gravity field coefficients:
  EGM_GOC_2__20091101T000000_20100111T000000_0001.IDF
- Full variance-covariance matrix:
  EGM_GVC_2__20091101T000000_20100111T000000_0001.IDF

Remark: The HPF-internal identifiers for these products are EGG_TIM_2I and EGG_TVC_2I (cf. Fig. 1).

3. DATA SETS

The first GOCE gravity field model is based on the data period from 01-11-2009 to 11-01-2010, thus covering slightly more than one full orbit repeat cycle of 61 days. The following key products have been used (product identifiers according to [8]):

- Orbits: SST_PSO_2I (sub-products: SST_R
- PKI_2I [kinematic orbits], SST_PCV_2I [variance-covariance information of orbit positions])
- Gradients: EGG_NOM_2
- Attitude: EGG_IAQ_2C (corresponds to columns 56 to 59 of EGG_NOM_2; [8])

Additionally, models for temporal gravity field reduction (ephemeris of Sun and Moon, ocean tide models, correction coefficients for non-tidal temporal variation signals), and for Earth's rotation (AUX_IERS) have been applied.

The gradients are processed in the original GRF. Thus, the base functions have to be rotated to this frame. The transformation from the Earth-fixed reference frame, in which the spherical harmonic base functions are originally computed, to the inertial frame is computed by in-house routines (i.e., the respective orbit sub-product SST_PRM is not used). As a second step, the rotation from this inertial to the target GRF is performed by using the quaternion information provided in EGG_IAQ_2C.

Fig. 2 shows (smoothed) gradiometer error power spectral densities (PSD) of the three main diagonal gradiometer components V_{XX}, V_{YY} and V_{ZZ}, as well as the trace V_{XX}+ V_{YY}+ V_{ZZ}. Evidently, the trace is slightly above 20 mE/sqrt(Hz) in the measurement bandwidth (MBW) from 5 to 100 mHz (black dashed lines), i.e. by more than a factor of 2 larger than the original specification. Additionally, the performance of the V_{ZZ} component (blue curve), which is the most important component for gravity field recovery, performs worse than the other two main diagonal components V_{XX} and V_{YY}.

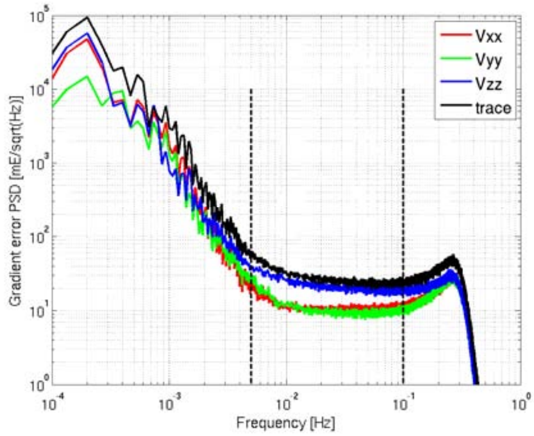

Figure 2. Gradiometer error PSDs

4. RESULTS

4.1. SST processing

In principle, two orbit types are available. Kinematic orbits (SST_PKI), which are purely geometrical orbit solutions based on the GPS observations without including any gravitational and non-gravitational force models, and reduced-dynamic orbits (SST_PRD). For the computation of the latter the gravity model EIGEN-5S ([7]) has been used, and thus they are constrained very tightly to this prior gravity field model.

Fig. 3 shows the results when applying the energy integral method to these two orbit types (resolution degree/order 100). The green curve shows the performance of the kinematic orbit solution in terms of the degree error median

$$\sigma_l = median_m \left\{ \left| \overline{R}_{lm}^{(est)} - \overline{R}_{lm}^{(EGM)} \right| \right\} \qquad (1)$$

where $\overline{R}_{lm} = \left\{ \overline{C}_{lm}; \overline{S}_{lm} \right\}$ are the fully normalized spherical harmonic coefficients, *(est)* denotes the estimated quantities, and *(EGM)* refers to the reference model, which is in this case EIGEN-5C ([7]). Correspondingly, the blue curve shows the substantially better performance when using the reduced-dynamic orbits. This result is not surprising, because this orbit type is heavily biased towards the GRACE prior model. Therefore, since our philosophy is to compute a GOCE-only solution, in spite of the lower performance we are using kinematic orbits for the combined solution.

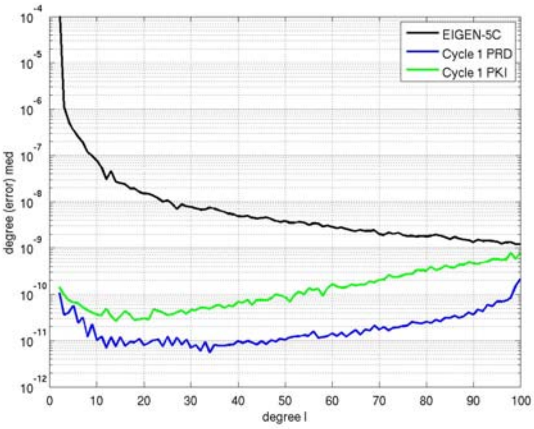

Figure 3. Degree medians of SST-only solutions

Additionally, it should be emphasized that for the computation of the SST-solutions, the variance information of the kinematic orbit positions is used as stochastic model, because detailed analyses have shown that the orbit errors are latitude-dependent, which is related to the GPS configuration. Thus, by including this error information we achieve a SST normal equation system which reflects the true error behaviour. This is important also for an optimum weighting when combined with the SGG normal equations later on.

4.2. Tuning Machine

As discussed in chapter 2, the key tasks of the Tuning Machine are:

- Gravity gradient analysis and outlier detection
- Stochastic modelling of gradients and filter design
- Tuning of regularization and weighting parameters
- Definition of baseline configuration for final solution

Concerning outlier detection in the frame of gravity field processing, the main advantage is that with the Tuning Machine we have a full fledged iterative gravity field solver in place. Thus, outlier detection can not only be applied to the gravity field signals, as it is done in the frame of the pre-processing in SPF3000 ([2]), but also to the residuals of the gravity field adjustment, which have a much smaller amplitude, so that outliers become more distinct.

Another key element of the SPF6000 processing is the correct stochastic modelling of the gradiometer errors (cf. Fig. 2). Digital filters are used to set-up the variance-covariance information of the gradient observations ([21], [22]). Technically, this is done by applying these filters to the full observation equation, i.e., both to the observations and the columns of the design matrix. Thus, the gradiometer error information is introduced as the metrics of the normal equation system. Thus, the full spectral range of the gravity gradients enters the gravity field solution, but they are properly weighted according to their spectral behavior.

The red curve in Fig. 4 shows the error PSD of the gradiometer component V_{ZZ} as it was estimated from the residuals of a previous gravity field adjustment. Filter models of different complexity have been fit to this error PSD ([23]). The most obvious features are the peaks at frequencies of *k* per orbit revolution, with integer *k*, mostly below the MBW. They can either be modeled peak by peak (green curve), or as some average (orange and blue curves). All these different filter models have been applied to compute gravity field models. The final validation of this multitude of different gravity field model results revealed that a less complex filter model (blue curve) is sufficient for all three gravity gradient components V_{XX}, V_{YY} and V_{ZZ}. Key advantages of this filter model are that it has a relatively short filter order of 52 (and thus is computationally efficient), and a short warm-up time of only 2000 seconds.

Additional tuning processes have been performed concerning the optimum regularization and weighting based on variance component estimation ([10]). In total, for the purpose of tuning the final solution more than 30 full-fledged combined gravity field models, including the SST normal equations described in the previous section, and different versions of filter models, regularization and weighting parameters have been processed. A more detailed discussion on the refinement of the stochastic model for this GOCE solution is given in [23].

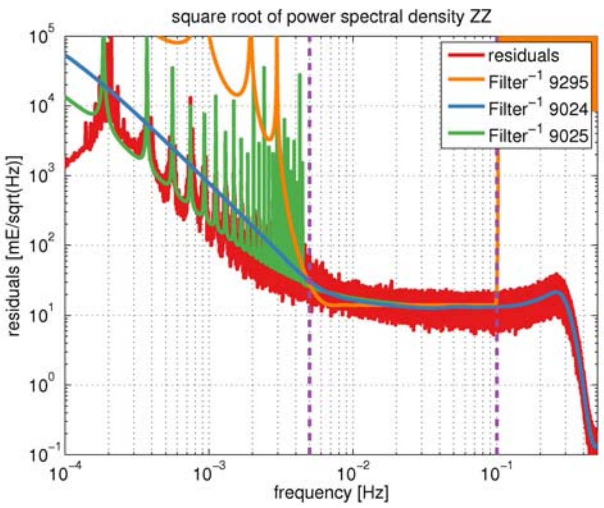

Figure 4. Different SGG filter models

4.3. Final Solver

After SST processing and diverse tuning steps, the full SGG normal equations complete to degree/order 224 have been assembled on a Linux-PC-Cluster ([17]), using only the three main diagonal components V_{XX}, V_{YY} and V_{ZZ}. Finally, the combined solution was processed by addition of the SST and SGG normal equations, and applying regularization and optimum relative weighting.

Special emphasis has to be given to constraining the combined normal equation system. Two different approaches have been investigated. The sun-synchronous orbit of GOCE and the resulting polar gaps with a opening half-angle of 6.5° lead to an oscillation of the solution in the polar regions. Correspondingly, the zonal and near-zonal coefficients can be estimated only with reduced accuracy. The Spherical Cap Regularization Approach (SCRA, [13]), a regularization technique which is dedicated to the specific problem of the non-polar orbit configuration of the GOCE satellite, was applied. The main idea is the filling of the polar gaps, where no observations are available, with an artificial signal, which shall be described analytically. The main advantage of this method is that it is spatially restricted to the problem areas of the polar gaps. Unlike standard regularization techniques, such as Kaula or Tikhonov regularization, which act on the parameters to be estimated in the spectral domain (harmonic coefficients), the SCRA acts almost purely in the space domain. Since one of the main goals is to compute a GOCE-only solution, i.e., no a-priori gravity field information shall be introduced, the choice of the stabilizing function in the poles is a critical issue. In order to fulfil this requirement, the following strategy was introduced: An independent SST-only solution complete to degree/order 50 was computed based on the kinematic orbit data described in chapter 3. Due to the lower cut-off degree, such a solution is only slightly affected by the polar gap problem. This solution was then used to compute the stabilizing function in the polar gap regions. The spectral leakage effect inherent in this low-degree SST-only solution was a-priori estimated to be in the order of 2 m. Since we are filling the polar caps again with GOCE information, this solution can be considered as unconstrained.

As a second approach, Kaula regularization was applied, but only to selected groups of coefficients. The first group involves all zonal and near-zonal coefficients, which are affected by the polar gap, according to the rule of thumb given by [24]. Second, Kaula regularization was also applied to coefficients with degrees larger than 170 in order to improve the signal-to-noise ratio in the very high degrees. It is important to emphasize that we do not use any reference gravity field model for our solution, but estimate the gravity field coefficients from the scratch. Therefore, the solution is Kaula constrained towards a zero model, but not towards a reference gravity model.

Fig. 5 demonstrates the effect of constraining the solution. It shows degree medians of the deviations of the GOCE solutions from EIGEN-5C. Evidently, compared to the unconstrained solution (red curve), the constrained Kaula solution (blue curve) shows significantly lower energy in the very high degrees.

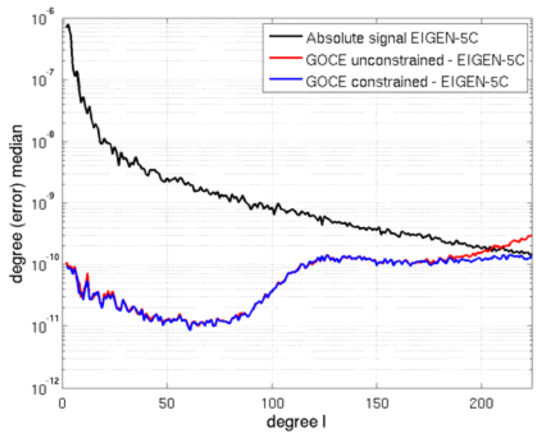

Figure 5. Degree medians: constrained vs. unconstrained solution

In order to show the impact of these constraints more lucidly, Fig. 6 illustrates the GOCE redundancy factors for these two gravity models. It expresses the "GOCE-onlyness" of the solution, i.e. to which extent GOCE information was used for the estimation of specific harmonic coefficients. The unconstrained solution (left) is a pure GOCE solution (all values are '1'), because kinematic orbits have been used for the low degrees, and the polar caps have been filled with GOCE information. Also the constrained solution is a GOCE-only solution, because no external gravity field information was included. However, the constraints towards zero are visible for (near-)zonal coefficients, and are gradually increasing with growing degree, starting from degree/order 170.

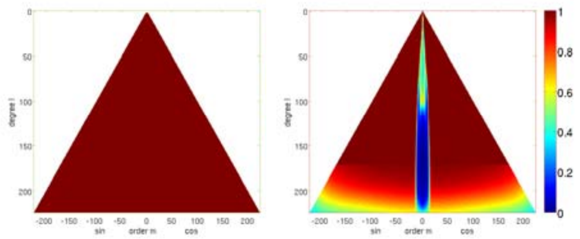

Figure 6. "GOCE-onlyness" of solutions: (left) unconstrained; (right) constrained solution

Eventually, due to the improved signal-to-noise ratio, the constrained solution has been selected as the official and final gravity field model.

5. SOLUTION VALIDATION

The solution has been internally validated by different strategies. Here, we present mainly the validation against state-of-the-art external global gravity field models. In the first step, we compare with external combined gravity field models, and in the second step with GRACE-only models.

Based on the coefficient estimates of the combined solution as described in chapter 4, cumulative gravity anomaly deviations at degree/order 200 from the combined gravity field models EIGEN-5C (Fig. 7) and EGM2008 ([19]; Fig. 8) have been processed.

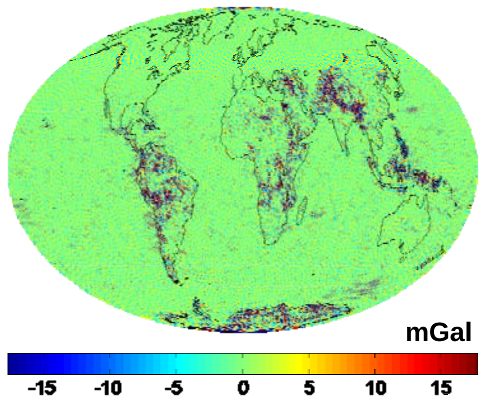

Figure 7. Gravity anomaly deviations (D/O 200) of the GOCE solution from EIGEN-5C

Large deviations appear in regions where the terrestrial gravity data are known to be of low accuracy, such as South America, Africa, Himalaya, or Antarctica. As it will be shown later on, in these regions GOCE confirms rather GRACE-only than combined models.

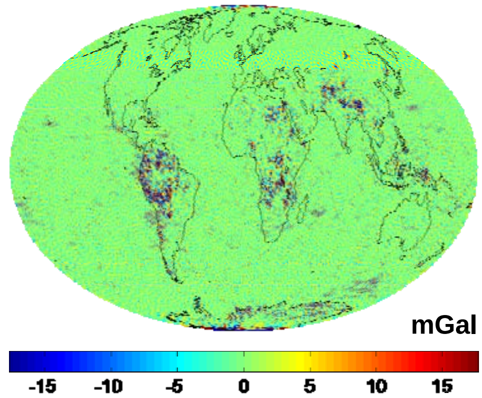

Figure 8. Gravity anomaly deviations (D/O 200) of the GOCE solution from EGM2008

This fact becomes also visible in Fig. 9, where the degree median deviations of GOCE from EIGEN-5C (blue) and from ITG-GRACE2010s ([11]; red) are compared with the error estimates of these two gravity field models (cyan and green curve, respectively). Concerning EIGEN-5C, the characteristic feature starting at degree 100 is also visible in the formal error estimates, indicating that in this spectral region GOCE shows an improved performance w.r.t. EIGEN-5C on a global scale. From Fig. 9 it can also be deduced that the coefficient differences to the Bonn GRACE-only model ITG_GRACE2010s and the corresponding error estimates show a cross-over at degree 150. Beyond, the two-months GOCE solution starts to become superior. In the low degrees, as it has to be expected the low-low-SST concept of GRACE is superior to GOCE.

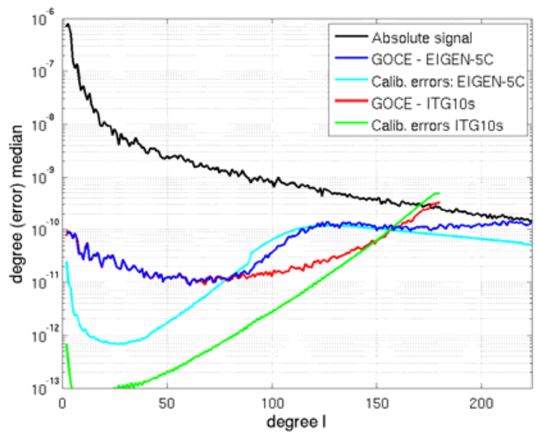

Figure 9. Comparison of GOCE with external combined and GRACE-only gravity field models

This feature can also be seen very nicely when analyzing global gravity anomaly deviations from the ITG-GRACE2010s model. Fig. 10 shows cumulative gravity anomaly differences up to degree/order 150, which turn out to be below the mGal level. In conclusion, the GOCE-only model, which is based on a data period of 2 months, and the GRACE-only model, which is based on almost 8 years of GRACE data, show great consistency. Again it should be emphasized that GOCE confirms rather GRACE than the combined gravity field models EIGEN-5C and EGM2008 in this spectral window.

However, if the deviations of the GOCE-only model from the ITG-GRACE model are compared for higher degrees, the superior performance of GOCE becomes evident. Fig. 11 shows these deviations for the spectral window of degrees 171 to 180. Evidently, typical error structures, as they are known from GRACE, appear. Thus, it can be concluded that based on this two months solution, GOCE becomes superior to GRACE at degree 150, and beyond. It goes without saying that this cross-over will decrease significantly when using longer GOCE data periods in the future.

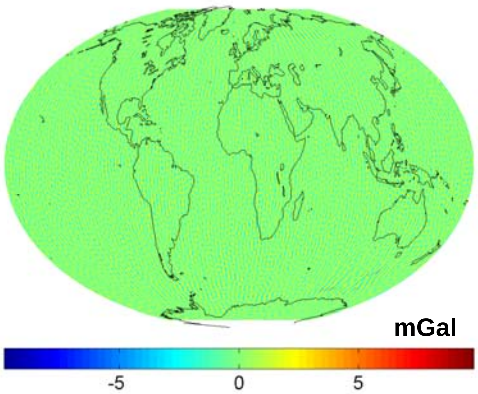

Figure 10. Gravity anomaly deviations up to D/O 150 of the GOCE-only solution from ITG-GRACE2010s

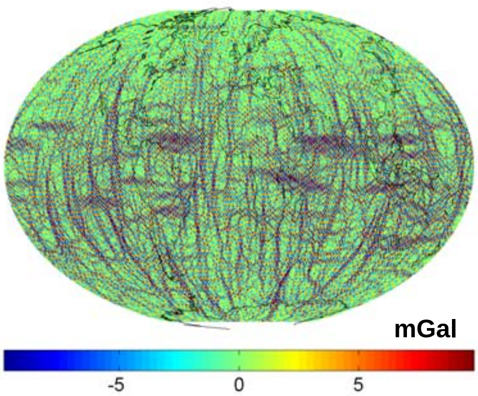

Figure 11. Gravity anomaly deviations for degree range 171-180 of the GOCE-only solution from ITG-GRACE2010s

Together with the coefficient solution, also a full variance-covariance matrix complete to degree/order 224 was output of this processing. In order to prove the plausibility of this matrix, rigorous covariance propagation was performed to propagate the coefficient errors to geoid height errors on a global grid. Fig. 12 shows the specific error structure of this field.

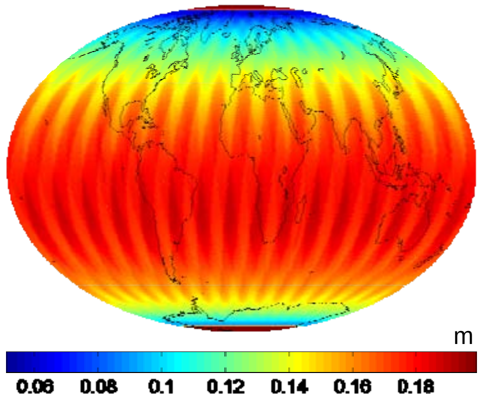

Figure 12. Geoid height standard deviations [m] at degree 224

The zonal band structure with larger errors in the equatorial regions is due to the fact that a larger number of observations is measured at high latitudes, due to the meridian convergence, and thus the convergence of the satellite's ground tracks. The asymmetry with respect to the equator and larger standard deviations in the southern hemisphere result from the orbit configuration, because the average satellite altitude is higher in this region, leading to a slightly increased attenuation of the gravity field signals at satellite height. The longitudinal striping structure is an expression of the data distribution (orbit ground tracks), because with a data volume of 71 days slightly more than one full repeat cycle of 61 days was included in the solution. Also the significantly degraded performance in the polar cap areas, where no observations are available, is correctly expressed by the variance-covariance information.

The actually achieved gravity field accuracy of this 2-months GOCE-only solution is estimated to be 8 cm in terms of geoid height, and 2.5 mGal in terms of gravity anomalies, at degree/order 200. A projection to the full nominal mission period (~18 months) yields predictions of 2.7 cm / 0.9 mGal.

6. CONCLUSIONS

The key philosophy for the processing of the first GOCE global gravity field model in the frame of SPF6000 is to produce a GOCE-only model in a rigorous sense, i.e., no external gravity field information has been used, neither as reference model, nor for constraining the solution. Correspondingly, the SST part is based only on GPS observations (kinematic orbits).

The final model was constrained by Kaula regularization applied to (near-)zonal coefficients and degrees >170. Since both for SST and SGG a realistic stochastic model was used, the variance-covariance matrix indeed reflects the true error behavior of the coefficient solution.

From a user's point of view, since this solution is completely independent of any gravity field information other than GOCE, it can be used for an *independent* comparison with other satellite-only models (such as those derived from GRACE), terrestrial gravity data, or satellite altimetry, and the added value compared to any existing gravity field data or (combined) gravity field models can be evaluated. It can also be used for combination with complementary gravity field information (GRACE, terrestrial data, satellite altimetry) on the level of normal equations. Since in the low degrees the solution is based solely on 2 months of kinematic GOCE orbits, but no external (GRACE) information, it is not competitive with GRACE models in the low degrees.

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