GOCE GRAVITY FIELD ANALYSIS IN THE FRAMEWORK OF HPF: OPERATIONAL SOFTWARE SYSTEM AND SIMULATION RESULTS

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ABSTRACT

In the framework of the ESA-funded project “GOCE High-level Processing Facility” (HPF), an operational hardware and software system for the scientific processing (Level 1b to Level 2) of GOCE data has been set up by the European GOCE Gravity Consortium EGG-C. One key component of this software system is the processing of a spherical harmonic Earth’s gravity field model and the corresponding full variance-covariance matrix from the precise GOCE orbit and satellite gravity gradiometry (SGG) data. In parallel to two other HPF teams, this key component is performed by the “Sub-processing Facility (SPF) 6000”. The second main task of SPF6000 is the production of quick-look gravity field products in parallel to the GOCE mission for system diagnosis purposes. The paper gives an overview of the operational software system. On the basis of a numerical case study, which is based on the data of an ESA GOCE end-to-end simulation, the processing architecture is presented, and several aspects of the involved functional and stochastic models are addressed.

1. INTRODUCTION

The dedicated satellite gravity mission GOCE (Gravity field and steady-state Ocean Circulation Explorer; [6]), the first Earth Explorer Core Mission, in the context of ESA's Living Planet programme, strives for a high-accuracy, high-resolution global model of the Earth's static gravity field. GOCE is based on a sensor fusion concept: satellite-to-satellite tracking in the high-low mode (hl-SST) using GPS, and satellite gravity gradiometry (SGG). The GOCE mission, when successfully completed, will provide a huge data set consisting of several hundred million orbit data plus very precise gravity gradiometry data, which contains abundant information about the gravity field of the Earth on a near-global scale, from very low (derived mostly from hl-SST) to high (derived mostly from SGG) frequencies.

The scientific data processing (Level 1b to Level 2) is performed by the “European GOCE Gravity Consortium” (EGG-C), a consortium of 10 European university and research institutes, in the framework of the ESA-funded project “ GOCE High-Level Processing Facility” (HPF; [23]). In the frame of this contract, the “Sub-processing Facility (SPF) 6000”, a co-operation of TU Graz, Austrian Academy of Sciences, University of Bonn, and TU Munich, under the lead of TU Graz, is responsible for the processing of a spherical harmonic Earth’s gravity field model and the corresponding full variance-covariance matrix from the precise GOCE orbit and SGG data, and the production of quick-look gravity field products in parallel to the GOCE mission for the purpose of a fast system diagnosis.

The mathematical model for the parameterization of the Earth's gravity field is based on a series expansion into spherical harmonics. In the case of a model resolution complete to degree and order 250, this yields approximately 63000 unknown spherical harmonic coefficients. The determination of these coefficients from the complementary hl-SST and SGG data sets is a demanding numerical and computational task, and therefore efficient solution strategies are required to solve the corresponding large normal equation systems. During the last decade, several approaches have been developed to perform this task (e.g., [22], [24], [8], [16], [14]). In [16], [17], the rigorous solution of the large normal equation matrix by means of a parallel processing strategy implemented on a Linux-PC cluster was proposed. While direct methods perform an epoch-wise processing of the gravity field observations, the semianalytic approach considers the observations along a satellite track as a time-series ([22], [16], [29], [19]).

Proc. ‘The 3rd International GOCE User Workshop’, ESA-ESRIN, Frascati, Italy,
6–8 November 2006 (ESA SP-627, January 2007)

The present paper outlines the architectural design of the operational software system, the processing strategies for the computation of a high-accuracy, high-resolution spherical harmonic model of the static Earth's gravity field, including a quality description in terms of a full variance/covariance matrix. On the basis of a numerical case study, which is based on the data of an ESA GOCE end-to-end simulation, the key components of the processing architecture are presented, and several aspects of the involved functional and stochastic models are addressed.

2. ARCHITECTURAL DESIGN

Fig. 1 shows the architectural design, the main components and the product flow through the SPF6000 software system. The software system for time-wise gravity field processing is conceived in a highly modular manner that allows the investigation of specific aspects of gravity modelling such as filtering, numerical stability and optimum regularization, complementary relations of SST and SGG and their optimum weighting.

Data transfer between SPF6000 and the central HPF data repository CPF (Central Processing Facility) is managed via automated interfaces. At SPF6000, the data are stored on a central access local data server.

The software system is composed of two main components: the Quick-Look Gravity Field Analysis (QL-GFA), and the Core Solver (CS), which will be briefly described in the following.

2.1 Quick-Look Gravity Field Analysis (QL-GFA)

This stand-alone software system performs the computation of fast approximate gravity field solutions based on SGG and hl-SST data, for the purpose to derive a fast diagnosis of the GOCE system performance and of the Level 1b input data in parallel to the mission with short latencies. These gravity field products are input to ESA's calibration/validation activities in the frame of the GOCE mission control.

Key tasks of QL-GFA are:

- Check of SGG and hl-SST input data in parallel to the mission and analysis of partial / incomplete SGG and hl-SST data sets.
- Computation of quick-look gravity field models (SGG only, SST only, combined SST+SGG) aiming at a fast analysis of the information content of the input data on the level of the gravity field solution. Additionally, quick-look gravity solutions are statistically tested against reference gravity models.
- Estimation of the gradiometer error PSD (power spectral density) from the residuals of a SGG-only gravity field analysis, and application of previously defined statistical hypothesis test strategies in time and frequency domain ([10]).
- Production of Diagnosis Report Sheets: All these system diagnosis products are reported by means of a standardized Diagnosis Report Sheet.

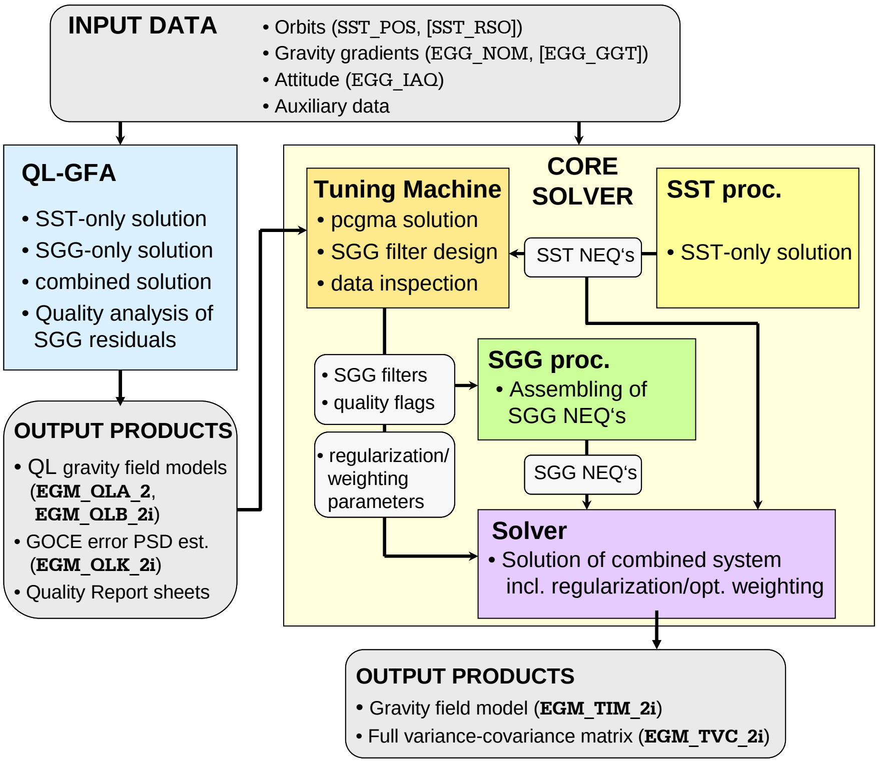

Fig. 1. Software architecture and product flow

QL-GFA solutions complete to degree/order 250 can be processed within the order of one to two hours on a standard PC. The efficiency and speed of QL-GFA is founded mainly on the application of FFT techniques, the assumption of block-diagonality of the normal equation matrix, and also on a simplified filter strategy in the spectral domain to cope with the coloured noise characteristics of the gradiometer. Deviations from this assumption are incorporated by means of an iterative procedure ([19], [21], [18]).

QL-GFA will be applied at two stages: Quick-Look-A (QL-A) is applied to Level 1b preliminary orbits (accuracy ~10 m) and the Level 1b gravity gradients. The main purpose at this stage is a rough check of the SGG time series, with special concern on the testing of the SGG error PSD. For QL-A, consecutive gravity field solutions will be available in a daily interval. They will be generated with a latency of 4 hours after arrival of all required input data. The achievable accuracy is mainly dependent on the correct (internal) calibration of the Level 1b gradients.

Quick-Look-B (QL-B) is applied after the availability of the Level 2 rapid science orbit solution (accuracy in the decimetre range) and the calibrated gravity gradients. In this phase, the corresponding SST and SGG time series are checked on the level of the Earth's gravity field, also testing the gradiometer error model. For QL-B, consecutive gravity field solutions will be available in a weekly interval. The maximum degree and order for the QL-GFA gravity field models will be optimized with respect to the global coverage of the input data.

2.2 Core Solver (CS)

This software component will deliver a rigorous ultimate-precision solution of the very large normal equation systems applying parallel processing strategies. The Core Solver is composed of the Final Solver, taking the full normal equation matrix into account, and the Tuning Machine, being based on the method of preconditioned conjugate gradients, which will verify and tune the involved software components of the Final Solver in many respects. Concerning the hl-SST processing, the energy integral approach is applied.

The objective is to compute a high-accuracy, high-resolution spherical harmonic model including a quality description of the static Earth's gravity field from GOCE SGG and SST observations. The parameterization of the model will be complete at least up to degree and order 200, and a resolution up to degree and order 250 is envisaged, depending on the actual accuracy of the SGG observations. Additionally, a quality description in terms of a full variance-covariance matrix will be provided.

The *Tuning Machine*, whose development, implementation and integration is completely in the responsibility of the HPF work package partner University of Bonn, consists of two main modules:

- *pcgma* (pre-conditioned conjugate gradient adjustment): It acts as a stand-alone solution strategy, and is used to verify and tune the involved software components of the Core Solver in many respects, e.g., to derive optimum regularization and weighting parameters.
- *Data analysis tool*: The data inspection and filter design tool is used to verify external and internal products, and to define the filter coefficients which will be used in the Final Solver ([27]).

The *Final Solver* consists of the following main modules:

- *SST processor*: The information content of the SST data is exploited by making use of the precise GOCE orbit expressed in terms of position and velocity information including quality description. The software can process both kinematic and reduced-dynamic orbits. The principle of energy conservation is applied ([7], [1], [2]). Favourable features of this approach are a strictly linear observation model as well as the fact that gravity functionals are processed. In contrast to QL-GFA, which performs a block-diagonal approximation, the CS SST processor exploits the information content of the full normal equation matrix.
- *SGG processor*: Given the precise GOCE orbit, the calibrated gravity gradients defined in the Gradiometer Reference Frame (GRF) are directly related to the unknown potential coefficients resulting in the linear observation model for all relevant tensor components, allowing to exploit the high degree of precision and resolution of the data. The complications arising from the coloured noise of the gradiometer are managed by a recursive filter procedure in time domain ([24], [25], [26], [28], [16]). The SGG processor assembles the full normal equations applying parallel processing on a Linux-PC-Cluster.
- *Solver*: The mathematical models for SGG and SST data are combined to the overall mathematical model by means of superposition of the normal equations, applying an optimum weighting of the individual data types. The solution will be processed applying a parallelized Cholesky reduction. The ill-posedness of the normal equations due to the polar gaps is managed by optimized regularization techniques ([12], [13]). Together with the GOCE gravity field model coefficients, a statistical error description in terms of the full variance-covariance matrix is processed.

Chronologically, the first processing steps will be performed by QL-GFA. Table 1 gives an overview of the resulting official output products. Additionally, several internal products (residuals, flags, regularization and weighting parameters) are generated.

Table 1: Output products of QL-GFA

Identifier	**Product description**
EGM_QLA_2	QL gravity field solution from SGG-only, based on Level 1b data
EGM_QLB_2i EGM_QST_2i EGM_QSG_2i EGM_QCO_2i EGM_QQR_2i	QL solutions based on Level 2 data: SST-only gravity field model SGG-only gravity field model combined SST+SGG grav. model Quality Report Sheet
EGM_QLK_2i	GOCE error PSD estimate

In the Core Solver processing, the SST and SGG normal equations are assembled separately. The SST normal equations (and other internal products) are transferred to the Tuning Machine and the Final Solver. In the Tuning Machine, the SGG normal equations are set-up using a sparse matrix scheme ([3]), and gravity field solutions are computed applying the pcgma algorithm ([4]). The residuals of the adjustment are analyzed by the Data Inspection tool, and filter coefficients, regularization and weighting parameters are derived, which are provided to the Final Solver. Here, the full SGG normal equations are assembled, and optimally combined with the SST normal equations. Finally, the gravity field solution and the full inverse of the normal equation matrix are computed rigorously. The final output products of the Core solver processing are summarized in Table 2. Also here, several internal output products, such as residuals, flags, regularization and weighting parameters, etc., are produced.

Table 2: Output products of the Core Solver

Identifier	**Product description**
EGM_TIM_2i	time-wise gravity field solution: coefficients
EGM_TVC_2i	corresponding full variance-covariance matrix

3. NUMERICAL CASE STUDY

The operability of the software system shall be demonstrated by a numerical case.

3.1 Test data sets

The numerical case study is based on the data of an ESA GOCE end-to-end simulation ([5]). This test configuration was also used during the official ESA Acceptance Review 2 for the testing of the final operational software (at the end of the development phase) in the framework of the HPF. The test data sets consist of:

- *Gravity gradients*: 60 days of 1 Hz rate simulated gravity gradients defined in the GRF, based on the gravity model EGM96 ([11]) complete to degree/order 360, superimposed by colored noise (cf. Fig. 2).
- *Orbit*: The gradients are defined along an orbit with GOCE characteristics (inclination i = 96.5°, eccentricity $e < 2 \cdot 10^{-3}$, mean altitude ~240 km). The orbit positions (and velocities) were generated by orbit integration, based on the gravity model EGM96, complete to degree/order 200, and including a full external force model and drag free and attitude control (DFAC) simulation.
- *Attitude*: The orientation of the satellite body axes (and hence the GRF) with respect to the inertial frame is given in terms of quaternions, which are computed from a combination of star tracker and gradiometer information. Correspondingly, they include attitude biases and noise ([15]), related to the star tracker and gradiometer inaccuracies modelled in the end-to-end simulation.

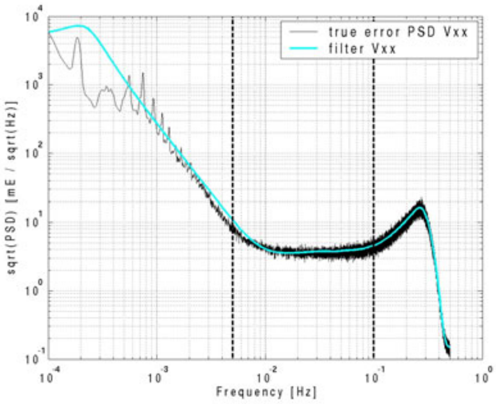

Fig. 2. GOCE error PSD and digital filter model

3.2 Results: QL-GFA

In this paper, mainly the results from the Core Solver processing shall be presented. The results of the QL-GFA, based on the test configuration described in section 3.1, are presented in a separate paper ([20]).

3.3 Results: Core Solver

In the following, the results of the main Core Solver components, i.e., SST processing, Tuning Machine, SGG processing, and Final Solver, will be presented.

SST processing

The SST processing based on the energy integral method was applied to kinematic orbits. A numerical differentiation procedure based on the Newton-Gregory method ([1], [2]) was applied to the kinematic orbit positions, which results in orbit velocities (in the inertial frame, [1]) representing, after application of accelerometry to cope with the non-conservative forces, the basic pseudo-observations. The noise level of the derived velocities is in the order of $1 \cdot 10^{-4}$ m/s. The SST normal equations are set-up complete to degree/order 90, which turned out to be sufficient to finally obtain a smooth combined SST+SGG solution. The [red] dotted curve in Fig. 3 shows the resulting SST-only solution in terms of the degree error median

$$\sigma_l = median_m \left\{ \left| \overline{R}_{lm}^{(est)} - \overline{R}_{lm}^{(EGM)} \right| \right\} \tag{1}$$

where $\overline{R}_{lm} = \left\{ \overline{C}_{lm}; \overline{S}_{lm} \right\}$ are the fully normalized spherical harmonic coefficients, *(est)* denotes the estimated quantities, and *(EGM)* refers to the reference model EGM96. The small spectral leakage effect, which is mainly visible at the upper limit of resolution of the parameter model at $l_{max} = 90$, results from the fact that the orbit contains gravity field signal complete to degree/order 200. Finally, the SST normal equations are transferred to the Tuning Machine and the Final Solver.

Tuning Machine (TM)

One main task of the TM is the approximation of an appropriate SGG digital filter model to introduce the correct metrics to the SGG normal equation system ([24], [25], [16], [28]). Fig. 2 shows the error characteristics of the gravity gradient tensor component V_{XX} (black curve) in terms of an error PSD, and the corresponding filter model using a cascaded ARMA filter with an effective filter order of 52 (light [blue] curve). The other main diagonal tensor components V_{YY} and V_{ZZ} (not shown) have similar error characteristics. The corresponding cascaded filter models have an effective filter order of 42 (V_{YY}) and 32 (V_{ZZ}).

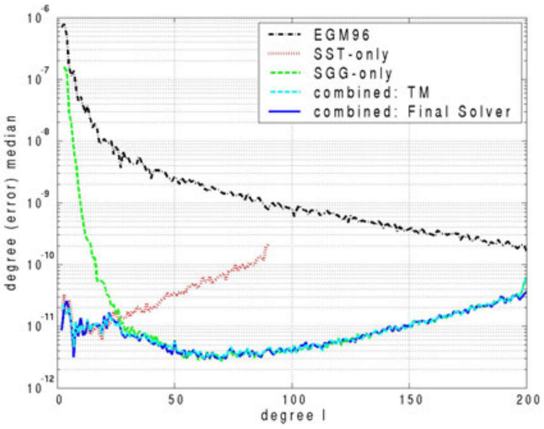

Fig. 3. Degree error median of diverse GOCE gravity field solutions

A combined gravity field solution, based on the SST normal equation complete to degree/order 90 described above, and SGG normal equations complete to degree/order 200, was computed by the pcgma. The dashed light [blue] curve in Fig. 3 shows the results in terms of the degree error median.

SGG processing

The full SGG normal equations were assembled on a Linux-PC-Cluster, which was installed under the umbrella of the initiative “Scientific Supercomputing” at TU Graz. The key parameters of this Beowulf cluster are: 54 Dual-Xeon 2.6GHz PCs with 1-2 GB RAM, GigaBit-Ethernet connection, performance 210 GFlops.

The final goal of this simulation was an optimum gravity field solution complete to degree/order 200. Since the signal content of the SGG input data is degree/order 360, a spectral leakage effect due to the non-parameterized signals from degree 201 to 360 has to be expected. In [25], it was shown that the spectral leakage effect mainly affects the coefficients in the spectral region close to the upper limit of resolution (in the present case 200). Therefore, in order to reduce the effect, SGG normal equations complete to degree/order 204 are assembled, and the final solution is truncated at degree 200, thus eliminating the coefficients of degree 201 to 204, which absorb most of the unresolved high-frequency signals. From a theoretical point of view, this strategy is not strictly correct, because there are correlations among the coefficients below and above the cut-off degree 200, which are disregarded when truncating the normal equation system. However, numerical studies have revealed that the error resulting from this procedure is negligible. During the assembling of the SGG normal equations, the digital filter model derived by the Tuning Machine was applied to the observations and the columns of the design matrix ([24], [25], [16], [28]).

Final Solver

After the assembling of the SST (D/O 90) and the SGG (D/O 204) normal equations, they are superposed and solved by a rigorous parallel solver. The memory size of the upper triangle of the normal equations (double precision arithmetics) is about 6.5 GBytes for the D/O 204 system.

An optimum weighting based on variance-component estimation ([9]) among the individual normal equation systems was applied. The optimum weighting factor was computed by the Tuning Machine.

The Spherical Cap Regularization Approach (SCRA; [12], [13]), a regularization technique which is dedicated to the specific problem of the non-polar orbit configuration of the GOCE satellite was applied.

The main idea of the SCRA is the filling of the polar gaps, where no observations are available, with an artificial signal, which shall be described analytically. The main advantage of this method is that it is spatially restricted to the problem areas of the polar gaps. Unlike standard regularization techniques, such as Kaula or Tikhonov regularization, which act on the parameters to be estimated in spectral domain (harmonic coefficients), the SCRA acts almost purely in the space domain, and thus represents an optimum strategy for the reduction of the polar gap problem.

The normal equation system, extended by the regularization part, reads

$$\left[A^T PA + \alpha(\Phi,\Phi)\right]x = A^T Py + \alpha(\Phi,g) \qquad (2)$$

where A^TPA and A^TPy are the (unregularized) normal equation matrix and the right-hand side, x is the parameter vector, and α is the regularization parameter.

The regularization matrix $R=(\Phi,\Phi)$ is the inner product of the spherical harmonic base functions $\Phi=\{\Phi_{lm}\}$, evaluated on the domain that is restricted to a subset of the sphere Σ, i.e. Σ_S, which in the present case consists of two spherical caps arranged symmetrically at the poles. The additional term of the right-hand side of the normal equation system is expressed analytically by the inner product of the base functions Φ and the stabilizing function g, which is defined on the northern and southern polar cap.

Since one of the main goals is to compute a GOCE-only solution, i.e., no a-priori gravity field information shall be introduced, the choice of the stabilizing function is a critical issue. In order to fulfil this requirement, the following strategy was introduced: An independent SST-only solution complete to degree/order 50 was computed based on the orbit data described in section 3.1. Due to the lower cut-off degree, such a solution is only slightly affected by the polar gap problem. This solution was then used to compute the stabilizing function in the polar gap regions. The spectral leakage effect inherent in this low-degree SST-only solution was a-priori estimated to be in the order of 2 m.

Finally, the large combined SST+SGG normal equation system, complete to degree/order 204, is solved rigorously, and afterwards truncated at degree/order 200 in order to reduce spectral leakage. Fig. 4 shows the coefficient deviations from the reference gravity field model EGM96, as well as the corresponding standard deviations (square root of diagonal elements of the variance-covariance matrix). Evidently, the absolute errors and the statistical error estimates are quite consistent, except of the (near-)zonals, whose accuracy is slightly overestimated.

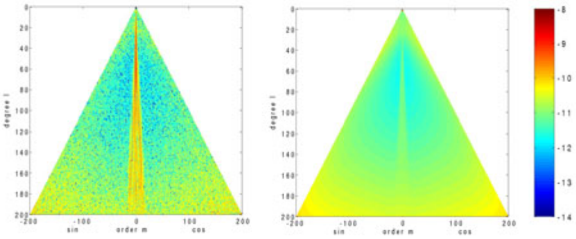

Fig. 4. Coefficient deviations from EGM96 (left) and standard deviations (right) of the combined SST+SGG solution. Scaled in $\log_{10}(|\ldots|)$.

The corresponding degree error median of this solution is displayed as [blue] solid curve in Fig. 3. Evidently, it is stabilized in the low-degree range mainly by the SST component ([red] dotted curve), and dominated by SGG ([green] dashed curve) from about degree 25 onwards.

The combined solutions processed by the Tuning Machine (light [blue] dashed curve) and the Final solver ([blue] solid curve) show a very good agreement. The fact that two independent methods and implementations obtain practically identical results supports the conclusion that the remaining coefficient errors are due to the noise of the input data, but are not produced by insufficiencies of the processing algorithms. The main differences between the TM and the final solution are visible in the very high degrees. They are due to the truncation strategy of the Final Solver. Since the Tuning Machine solution has been parameterized to degree 200 und no truncation strategy was applied, the SGG signals of degrees 201-360 leak into the solution, while most of this effect was reduced by the truncation strategy applied in the Final Solver solution.

Based on the coefficient estimates of the combined solution of the Final Solver (Fig. 4, left), cumulative geoid height errors at degree/order 200 have been processed, and are displayed in Fig. 5.

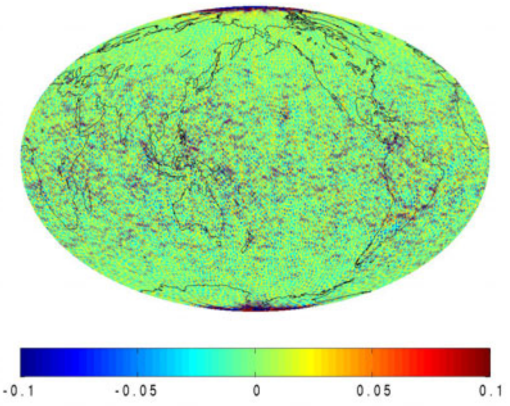

Fig. 5. Cumulative geoid height errors [m] at degree 200, based on the combined SST+SGG solution.

Table 3 summarizes the standard deviations of the geoidal heights σ_N and corresponding gravity anomalies $\sigma_{\Delta g}$ in the latitudinal region -83.5° < φ < 83.5°, which is covered by GOCE observations, of the Tuning Machine and the Final Solver solution.

Table 3 Cumulative geoidal height and gravity anomaly errors of the combined SST+SGG solutions in the latitudinal range $|\varphi| < 83.5°$

$\|\varphi\| < 83.5°$	σ_N [cm]	$\sigma_{\Delta g}$ [mGal]
Tuning Machine	3.29	0.93
Final Solver	2.93	0.81

Also here, the slight improvement (in the order of 10-15 %) of the Final Solver solution can be explained mainly by the truncation strategy to reduce spectral leakage. Globally, i.e. also including the polar regions, the maximum geoid error of the Final Solver solution is N_{max} = 2.49 m. This is very consistent with the a priori estimate related to the SCRA using the degree 50 SST-only model as stabilizing function at the poles. Of course, the solution could be further improved by using a more precise gravity field information in the polar regions, but this would be in conflict with the requirement to process a rigorous GOCE-only gravity field model. It should be emphasized, that an even slightly better performance, without applying any prior gravity field information, can be obtained applying the second order SCRA ([13]).

Together with the coefficient solution, also a full variance-covariance matrix, complete to degree/order 200, was output of this processing. In order to prove the plausibility of this matrix, a rigorous covariance propagation was performed to propagate the coefficient errors to geoid height errors on a global grid. Fig. 6 shows the specific error structure of this field.

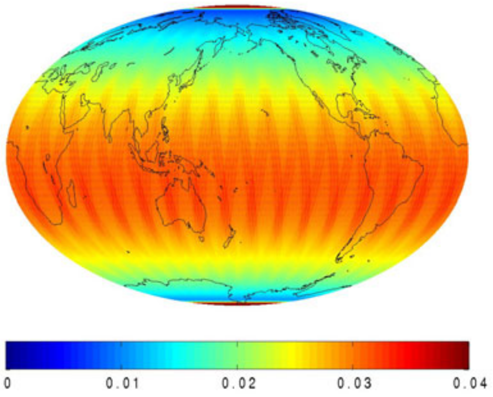

Fig. 6. Geoid height standard deviations [m] at degree 200, propagated from the full variance-covariance matrix of the gravity field coefficients.

The zonal band structure with larger errors in the equatorial regions is due to the fact that a larger number of observations are performed at high latitudes, due to the meridian convergence, and thus the convergence of the satellite's ground tracks. The asymmetry with respect to the equator (larger standard deviations) in the southern hemisphere results from the orbit configuration, because the average satellite altitude in the present test configuration is higher in this region, leading to a slightly increased attenuation of the gravity field signals at satellite height. The longitudinal striping structure is an expression of the data distribution (orbit ground tracks) and the stochastic behaviour of the SGG data (introduced by the filter, cf. Fig. 2).

Compared with the amplitude of absolute geoid height errors (Fig. 5), their statistical error estimates (Fig. 6) match quite well, proving consistency of this numerical closed-loop case study.

4. SUMMARY AND CONCLUSIONS

In this paper the architectural design of the SPF6000 is described. The software is now fully implemented, and the hardware and software system is integrated. Based on the official HPF Acceptance Test scenario, the data flow through the SPF6000 and the interplay of the system modules is described, and the main output products are presented as an example for a multitude of test scenarios which have been processed to validate the software system extensively. In conclusion, SPF6000 is now ready for operation.

5. ACKNOWLEDGEMENTS

This study was performed in the framework of the ESA-project GOCE High-level Processing Facility (Main Contract No. 18308/04/NL/MM). It was supported by the Austrian Space Application Programme initiative by the FFG (contracts no. ASAP-CO-008/03 and ASAP-WV-211/05). Funding was also provided by the GOCE-GRAND project in the frame of the German "Geotechnologien Projekt".

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