CHAMP Gravity Field Recovery with the Energy Balance Approach: First Results

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Summary. Using the principle of energy conservation has been considered for gravity field determination from satellite observations since the early satellite era, see e.g. O'Keefe (1957), Bjerhammar (1968), Reigber (1969) or Ilk (1983). CHAMP is the first satellite to which the energy balance approach can be usefully applied, now that near-continuous orbit tracking by GPS is available, aided by accelerometry. Simulation studies show the feasibility of the approach. One concern is the sensitivity to velocity errors. As a next step CHAMP's Rapid Science Orbits (RSO) are used. Their error level is sufficiently low to demonstrate the feasibility of the energy balance approach with real data. The accelerometer data (ACC), used for modeling the non-conservative forces, give rise to further concern.

Key words: gravity field, energy integral, accelerometer bias

1 Energy Integral

The concept is based on the total energy E of a satellite, which is assumed to be a constant of motion. According to Landau and Lifschitz (1976) it reads for a unit mass

$$E = \frac{1}{2}\,\dot{\boldsymbol{r}}^2 \,-\, \frac{1}{2}\,(\boldsymbol{\Omega} \times \boldsymbol{r})^2 \,+\, U, \tag{1}$$

where $\boldsymbol{\Omega}$ is the rotation vector of the earth and $\boldsymbol{r}$ and $\dot{\boldsymbol{r}}$ are position and velocity of the satellite in an earth fixed frame. Equation (1) is also referred to as energy integral. For the potential energy U the sign convention of physics is employed (opposite to geodetic convention). In order to derive from (1) the gravitational potential of the earth, U must be corrected for the tidal potential of third bodies.

In reality, however, E is not constant in time, due to non-conservative forces like air drag or solar radiation pressure. In the case of CHAMP these forces are measured by accelerometry and the respective loss of energy can be determined by integration of the measured accelerations $\boldsymbol{a}$ along the orbit. Equation (1) can thus be rearranged to

$$V \;=\; \frac{1}{2}\,\dot{\boldsymbol{r}}^2 \;-\; \frac{1}{2}\,(\boldsymbol{\Omega}\times\boldsymbol{r})^2 \;-\; V_t \;-\; \int_{\boldsymbol{r}} \boldsymbol{a}\cdot \mathrm{d}\boldsymbol{r} \;-\; E, \tag{2}$$

where V is the gravitational potential of the earth (now with the geodetic sign convention) and V_t is the tidal potential. V can be determined up to the unknown constant E. Subtraction of a normal field gives the disturbing potential T as time series along the orbit. In the sequel T is used as pseudo observation, with $\boldsymbol{r}$,$\dot{\boldsymbol{r}}$ and $\boldsymbol{a}$ being measured.

2 Analysis Method

A semi-analytical approach, proposed by Sneeuw (2000), was used for gravity field analysis. Under the assumption of constant orbit height and inclination, potential coefficients of different spherical harmonic (SH) order m are uncorrelated and the normal equation matrix shows a block-diagonal structure. Therefore the inversion of the matrix can be performed very efficiently. Errors due to the applied approximations can be minimized by iteration.

The potential, up to SH-degree L, along the orbit can be expressed as

$$V(u,\Lambda) = \sum_{m=-L}^{L}\sum_{k=-L}^{L} A_{mk}\, e^{i(ku+m\Lambda)} \tag{3}$$

$$A_{mk} = \sum_{l=\max(|m|,|k|)}^{L} \frac{GM}{R}\left(\frac{R}{r}\right)^{l+1} F_{lmk}(I)\, K_{lm}, \tag{4}$$

where u is the argument of latitude, Λ the longitude of the ascending node and the lumped coefficients A_{mk} are computed from inclination function F_{lmk} and the potential coefficients K_{lm}. From equation (3) the A_{mk} can be determined by a 2D Fourier-transformation. For this purpose the disturbing potential along the orbit is generated in a regular grid on the surface of a torus. This is the proper space domain for a 2D Fourier-series (see Sneeuw, 2001).

3 Simulation Studies and Error Budget

In order to test the feasibility of the energy approach, simulation studies were carried out. From an *a priori* gravity field (EGM96s was used) an orbit was generated by numerical integration. Simulated positions and velocities along the orbit were then used to compute the disturbing potential according to the equations given in section 1. In order to decrease the approximation error induced by the analytic assumptions (see section 2) the disturbing potential was reduced to a constant orbit height using the radial gradient $\partial T/\partial r$. The time series of pseudo observations T was then used for estimation of potential coefficients. In principle it should be possible to recover the *a priori* field

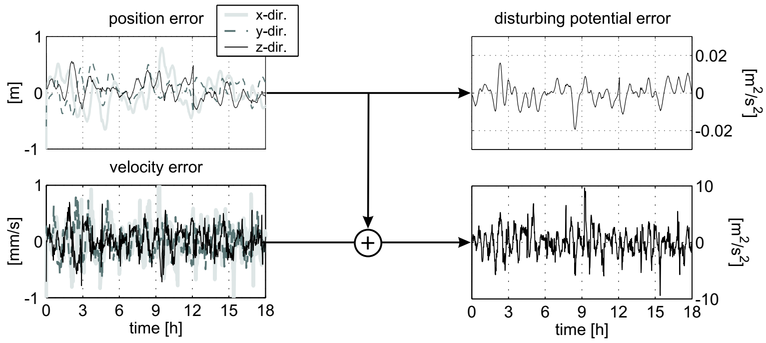

Fig. 1. Error simulation for orbit data with RSO-quality. Empirical position and velocity errors (left) are used to estimate the accuracy of the potential, computed from the energy integral (upper right: using position errors only; lower right: using both position and velocity errors)

without significant errors. In our simulation the difference between the recovered field and the *a priori* field shows RMS values per latitude at the level of 1–2 cm in geoid height at satellite altitude. This is not sufficient for high precision gravity field determination but is far below the error level estimated for the RSO data used in section 4. The errors are due to simple interpolation and gridding algorithms.

The error budget of the CHAMP RSO data was estimated in comparison to a precise reduced dynamic orbit, POD (Precise Orbit Determination). Position and velocity differences between POD and RSO were used as empirical error estimate for the RSO. These errors were added to the simulated orbit to generate noisy time series. Again potential coefficients were estimated. Figure 1 shows the errors in position and velocity as well as the propagated error in the disturbing potential along the orbit. The simulation shows that the energy integral is highly sensitive to velocity errors, while position errors are of low influence. The RMS of the error in the disturbing potential is at the level of 20 cm at orbit altitude. This shows that one cannot expect to determine the gravity field to an accuracy below the decimeter level, when using data of RSO-quality.

4 Gravity Field Recovery Using Real Data

Eleven days of CHAMP RSO and ACC data, as provided by GFZ, have been used as a first step in applying the energy balance approach to real data. The RSO is given in an earth fixed frame (Conventional Terrestrial System CTS), while the ACC measurements are given in an instrument fixed frame but can be transformed to space fixed frame (Conventional Inertial System CIS) using the provided attitude data. As shown in section 3, velocity errors are most critical for the method and errors at the decimeter level (at orbit height) can be expected when using the RSO data.

4.1 Non-conservative Forces

Disregarding the accelerometer measurements in the energy integral (2) leads to a drift of about -700 m^2 s^{-2} per day. This is the amount of energy dissipation caused by non-conservative forces. The value corresponds to a descent of CHAMP of about 80 m per day. The energy dissipation reads in discrete form

$$\int_{\boldsymbol{r}} \boldsymbol{a} \cdot \mathrm{d}\boldsymbol{r} \approx \sum \Delta \boldsymbol{r}'_e \cdot (\boldsymbol{R}^s_e \cdot \boldsymbol{a}_s), \tag{5}$$

with $\Delta \boldsymbol{r}_e$ being the distance between two epochs in CTS, $\boldsymbol{a}_s$ the accelerations in CIS and $\boldsymbol{R}^s_e$ the transformation matrix from CIS to CTS.

In order to model the non-conservative forces correctly, special attention must be given to the ACC data. Here the largest contribution arises from the along-track component. The other components (cross-track and radial) enter only via the order of misalignment between the velocity vector and the space craft body system. Forces which act orthogonal to the orbit give no contribution. If the ACC data is biased, the integration in (2) leads to a drift of the dissipative energy and — in consequence — also in the derived disturbing potential. The energy integral is not constant in that case. Still, this is not a drawback of the energy method — in contrary, the condition of energy conservation can be used to estimate the ACC bias.

4.2 Estimation of Accelerometer Bias

Using the ACC data (as shown in figure 2) in equation (2), leads to a drift in the potential of about -2290 m^2 s^{-2} per day. This drift is assumed to be

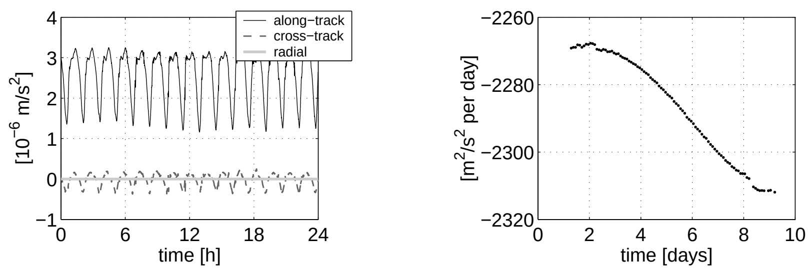

Fig. 2. Left: ACC data over one day after preprocessing. The cross-track component has been reduced for its mean value, which seems reasonable considering the satellite's attitude. The radial component, which is known to be of poor quality was not used at all. **Right**: Slope of the drift in the energy integral, caused by a biased ACC along-track component (plotted at normal points in 2 hours interval)

caused by a bias in the ACC data. It can be determined at cross-over points of the satellite tracks. The derived potential values at those points (reduced to constant orbit altitude) are expected to be equal. For biased accelerations the energy integral does not hold and the values at identical points differ by

$$\Delta T = \sum \Delta \boldsymbol{r}'_e \cdot (\boldsymbol{R}^s_e \cdot \boldsymbol{b}), \tag{6}$$

where $\boldsymbol{b}$ is the bias. Equation (6) is the mathematical model for the estimation of $\boldsymbol{b}$ from the potential differences at cross-over points. In our computations we have only estimated the along-track component of the bias. A mean value of $-3.4 \cdot 10^{-6}$m s^{-2} was derived for it.

The slope s of the drift is given by $s = \Delta T/\Delta t$, where ΔT is the potential difference at the cross-over points and Δt is the time span between the corresponding epochs. The slope (see figure 2, right) reveals variations in the drift of the order of $\pm\, 20$ m^2 s^{-2}. This corresponds to a temporal variation of the bias. For the used period the bias differs between -3.54 m s^{-2} and -3.43 m s^{-2}. A polynomial of 9th order was used to model the bias. The variation of the slope indicates periodic variations in the bias.

4.3 Results

After estimation of the along-track bias, the energy integral was recomputed (a scale factor of 0.8 was used for all components) and a gravity field determined from the derived disturbing potential. No correlations between the pseudo observations were modeled in the estimation process. The field was compared to the GRIM5-C1 potential model. The coefficients $C_{2,1}$ and $S_{2,1}$ could not be estimated to a sufficient accuracy, which might be due to different reference frame definitions in the two fields. Both coefficients were disregarded in the following comparison. Figure 3 shows the difference in geoid height between the two fields at orbit height. The latitude-RMS shows

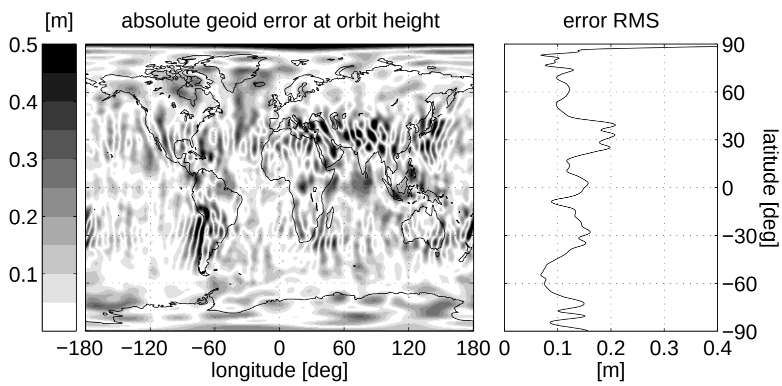

Fig. 3. Empirical geoid error at satellite altitude from comparison to GRIM5-C1.

values of 10–20 cm, which corresponds to the accuracy one can expect from RSO data (see section 3). The structure of the empirical error shows correlations to orbit tracks. This again indicates that the errors are to a large extent caused by orbit errors. It is worthwhile noticing, that in case of poor orbit quality a similar error structure is a well known phenomenon in geoids based on satellite altimetry.

5 Summary

The energy balance approach, proposed since the early days of satellite geodesy, could be applied for the first time with real data. A gravity field has been computed from 11 days of CHAMP RSO and ACC data. Comparison to GRIM5-C1 shows an accuracy of 10–20 cm in geoid height at satellite altitude. This corresponds to the expected accuracy level obtained from an error simulation. The energy integral is highly sensitive to velocity errors. This shows that a very precise orbit is necessary to recover the gravity field to a reasonable quality.

Furthermore it has been shown, that the semi-analytical torus approach can be successfully applied in gravity field recovery and that the energy balance approach is useful for accelerometer calibration.

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