

# THE CONTRIBUTION OF GPS MEASUREMENTS TO EARTH ROTATION STUDIES

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**ABSTRACT.** In the last years the contribution of GPS to the study of Earth rotation has been steadily growing. This is, on one hand, due to the increasing density and coverage of the global GPS network, on the other hand due to the maturing of the technique, i.e., the improvements in processing strategies and modeling. Today, the GPS Earth rotation parameters contribute significantly to the IERS Bulletin A and B series, to the determination of nutation amplitudes for periods below 20 days, and to the monitoring and study of high-frequency variations in Earth rotation.

## 1. INTRODUCTION

The basis for the estimation of Earth rotation parameters (ERPs) using the GPS is the data collected by the global network of the International GPS Service (IGS). In the last few years this network has been steadily growing and by now consists of more than 120 sites. These sites are quite well-distributed and provide a stable, dense reference frame for the determination of ERPs. This circumstance, together with the fact that several satellites are observed simultaneously from each site leads to a wealth of observations (around 250'000 per day assuming a 3-minute sampling of the data) present in neither VLBI nor SLR nor DORIS. GPS is therefore a tool well-suited to monitor the Earth's rotation, especially the high-frequency part, where the geometry and coverage given by the observations is very essential.

Because the orbits of the GPS satellites have to be estimated together with the ERPs, there are limitations to what can be achieved with GPS. Due to the well-known, full correlation between the orbital elements defining the orientation of the orbital planes in space (e.g. right ascension of the ascending node  $\Omega$ , inclination  $i$ , and argument of latitude  $u$ ) on one side and the ERP components UT1 and nutation in longitude and obliquity on the other side, the GPS has no direct access to the inertial frame (UT1 and the nutation offsets) and the quality of UT1 rate and nutation rate estimates is limited by the modeling problems caused by the non-gravitational forces acting on the satellites. In addition, the revolution period of the GPS satellites of one sidereal day coincides with the tidal periods of 12 and 24 hours. Modeling errors in the orbits may therefore propagate into amplitude estimates for, e.g., ocean tide terms at these periods.

Despite these weaknesses this paper will show the important and strong role that GPS plays

in the field of Earth rotation studies nowadays.

## 2. CONTRIBUTION OF GPS TO THE IERS SERIES

At present there are seven Analysis Centers (AC) of the IGS that are routinely computing ERPs from the data of the global GPS network. The results of these seven centers are combined by the IGS Analysis Center Coordinator (Jan Kouba) to generate the official IGS Earth rotation parameter series [Kouba *et al.*, 1998]. Two ERP products are delivered to the IERS by the AC Coordinator: the *rapid* ERP values, which are available one day after the observations and the *final* values, which are produced with a delay of about 10 days. These values are taking part in the generation of the Bulletin A and Bulletin B ERP series of the IERS. As an example Figure 1 shows the standard deviations of the individual contributions with respect to the combined Bulletin A solution for the x- and y-pole coordinates over the last one and a half years as they are published each week by the IERS Subbureau. We see that both, the rapid and final

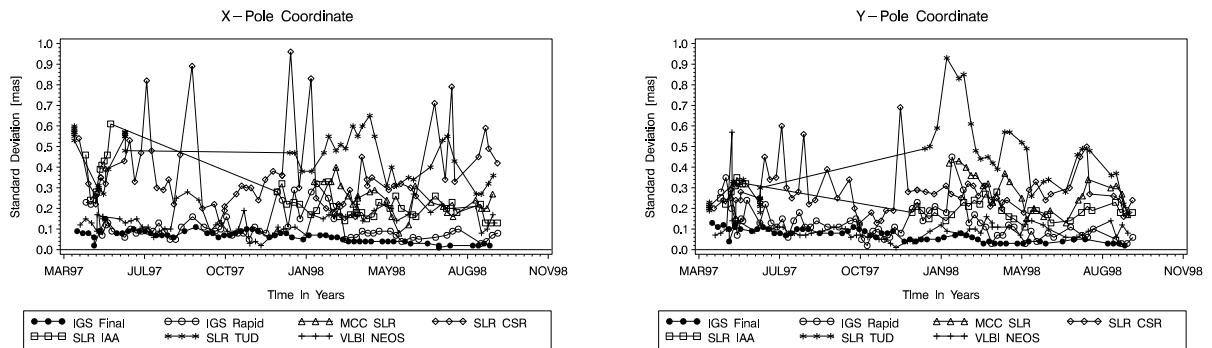


Figure 1: Standard deviations of the individual polar motion series with respect to the combined Bulletin A series.

IGS solutions, have reached a precision of below 0.1 mas and clearly dominate the Bulletin A combination. A similar behavior may be seen when comparing the Bulletin B solutions. The precision of the NEOS- and the IGS-series is almost the same in this case, however. One of the reasons for the strong weight of the IGS ERP solutions in Bulletin A is also the timeliness of the rapid results. It should be mentioned, that Bulletin A also benefits from the Length of Day (LOD) estimates stemming from GPS solutions, although these LOD values contain biases that heavily depending on the exact orbit model used by each individual AC. The GPS LOD values are used in particular to bridge the gaps between the weekly UT1 determinations from VLBI. More about the use of GPS LOD may be found in the contribution by [McCarthy, this volume].

## 3. ESTIMATION OF NUTATION

When using satellite methods, Earth rotation parameters and orbital elements have to be estimated simultaneously. It is well-known, however, that not all ERP components may be determined due to the perfect correlation between the orbital elements  $\Omega$  (right ascension of ascending node),  $i$  (inclination w.r.t. the equator), and  $u_0$  (argument of latitude) and the ERP components UT1,  $\Delta\epsilon$  (nutation in obliquity), and  $\Delta\psi$  (nutation in longitude). The following formulas (see [Rothacher *et al.*, 1998a] for a detailed discussion) give the relationship between

these two sets of parameters:

$$\Delta(\text{UT1}-\text{UTC}) = -(\Delta\Omega + \cos i \cdot \Delta u_0)/\rho \quad (1)$$

$$\delta\Delta\epsilon = \cos\Omega \cdot \Delta i + \sin i \sin\Omega \cdot \Delta u_0 \quad (2)$$

$$\delta\Delta\psi \cdot \sin\epsilon_0 = -\sin\Omega \cdot \Delta i + \sin i \cos\Omega \cdot \Delta u_0 \quad (3)$$

where  $\rho \approx 1.0027379$ ,  $\Delta\Omega$ ,  $\Delta i$ , and  $\Delta u_0$  are changes in the orbital elements  $\Omega$ ,  $i$ , and  $u_0$ , and  $\Delta(\text{UT1}-\text{UTC})$ ,  $\delta\Delta\epsilon$ , and  $\delta\Delta\psi$  the corresponding changes in UT1 and the nutation offsets  $\Delta\epsilon$  and  $\Delta\psi$ , and  $\epsilon$  is the obliquity of the ecliptic.

Although UT1 and nutation offsets are not directly accessible to satellite space-geodetic techniques, satellite observations have been used since a long time to determine UT1-*rates*. There is, however, no fundamental difference between estimating UT1-rates (or LOD) and estimating nutation rates  $\Delta\dot{\epsilon}$  and  $\Delta\dot{\psi}$  as we can see when taking the time derivative of Equation (1):

$$(\text{UT1}-\text{UTC}) = -\text{LOD} = -(\dot{\Omega} + \cos i \cdot \dot{u}_0)/\rho \quad (4)$$

$$\Delta\dot{\epsilon} = \cos\Omega \cdot \dot{i} + \sin i \sin\Omega \cdot \dot{u}_0 \quad (5)$$

$$\Delta\dot{\psi} \cdot \sin\epsilon_0 = -\sin\Omega \cdot \dot{i} + \sin i \cos\Omega \cdot \dot{u}_0 \quad (6)$$

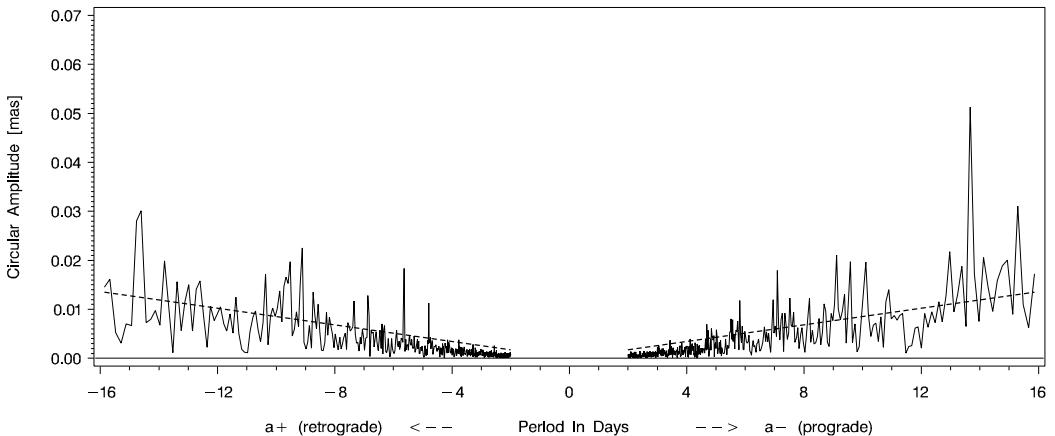
In the Keplerian approximation (two-body problem with spherically symmetric mass distribution) the orbital elements  $\Omega$ ,  $i$ , and  $u_0$  are integration constants and are therefore not changing with time. This means that there is no problem to estimate nutation rates and UT1-rates in principle. This is also true for perturbed orbital elements as long as the perturbing forces are exactly known. Critical are, however, unmodeled perturbations (e.g. from solar radiation pressure forces), because they lead to changes in the orbital elements and thus to systematic errors in LOD,  $\Delta\dot{\epsilon}$ , and  $\Delta\dot{\psi}$ .

The Center for Orbit Determination in Europe (CODE) — a cooperation of the Astronomical Institute, University of Berne, Switzerland, the Federal Office of Topography in Berne, Switzerland, the Federal Agency of Cartography and Geodesy in Frankfurt, Germany, and the Institut Géographique National in Paris, France — started with the estimation of nutation rates in the global GPS solutions in March 1994. Today, a series of about 4 years of  $\Delta\dot{\epsilon}$  and  $\Delta\dot{\psi}$  estimates is available from CODE.

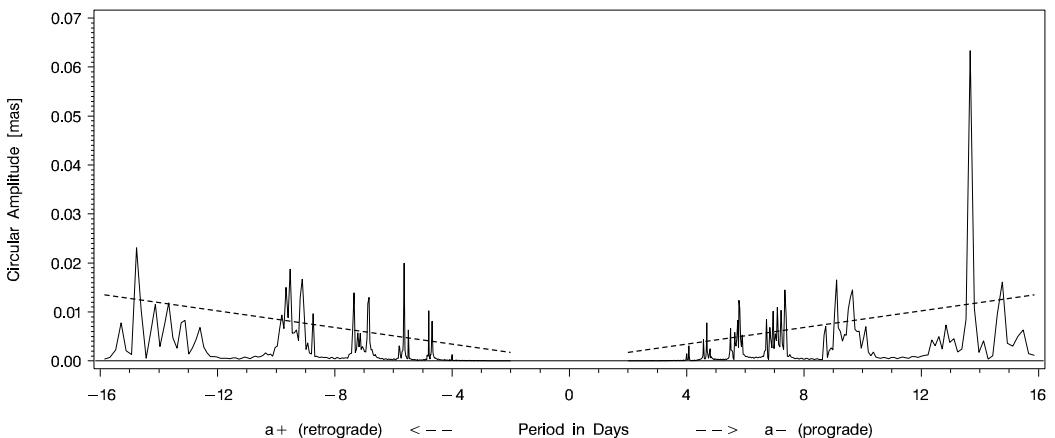
To get a first insight into the information contained in this series, a spectral analysis was performed using the nutation rates, which are rate corrections relative to the IAU 1980 model. The high frequency part of the spectrum obtained is given in Figure 2a. The *rate amplitudes* were thereby converted to actual amplitudes and transformed from  $\Delta\epsilon$  and  $\Delta\psi \cdot \sin\epsilon_0$  to *circular nutation* amplitudes  $a^+$  and  $a^-$  (see [Rothacher *et al.*, 1998a] or [Herring *et al.*, 1994] for a definition of circular amplitudes). Figure 2b, for comparison, shows the spectrum derived from the differences between the IAU 1980 and the IERS 1996 nutation model [McCarthy, 1996]. Many of the deficiencies of the IAU 1980 model visible in Figure 2b, discovered by VLBI about a decade ago, are clearly seen by GPS, as well.

The series of nutation rates from GPS can be used to determine the amplitudes of nutation terms with short periods. A simple variance-covariance analysis shows that, when using the nutation rates as pseudo-observations in a least squares algorithm to determine nutation amplitudes, the errors of the estimated amplitudes will grow linearly with the period of the terms involved. This means that the nutation rate estimates from GPS cannot be used to derive any nutation amplitudes for long periods, but only terms with periods below approximately 20 days.

Nutation coefficients were determined from the GPS series for 34 selected nutation periods. Figure 3 shows the differences between the nutation coefficients from GPS and the coefficients of the IERS 1996 model. The shaded area represents the  $2\sigma$  error bars of the coefficients derived from GPS and shows the increase of these uncertainties with the nutation period. With a few



(a) Spectrum of nutation corrections from GPS relative to the IAU80 model



(b) Spectrum of differences between the IERS96 and the IAU80 model

Figure 2: Spectrum of circular nutation amplitudes at low periods generated from (a) the GPS series of nutation rates converted to actual nutation amplitudes and (b) the differences between the IERS96 and the IAU80 model. The straight lines indicate the  $1-\sigma$  uncertainties of the amplitude estimates.

exceptions all the nutation coefficients agree with the IERS 1996 model at the level of twice the formal uncertainties (shaded area). The median agreement between the GPS results and the IERS 1996 model over all the 136 coefficients amounts to about  $10 \mu\text{as}$ . No major deviations from the IERS 1996 model can be detected.

Let us have a closer look at the most interesting nutation period below 20 days, i.e., the 13.66-day period. Figure 4 shows the comparison of various VLBI and LLR results given in the literature and the GPS results, with the very recent model by Souchay/Kinoshita 1997.2 [Herring, 1997]. Apart from the IAU 1980 model, the IERS 1996 model and the GPS results, the comparison includes the coefficients from [McCarthy and Luzum, 1991] (combined analysis of 10 years of VLBI and about 20 years of LLR data), [Herring et al., 1991] (9 years of VLBI data), [Charlot et al., 1995] (16 years of VLBI and 24 years of LLR data), and [Souchay et al., 1995] (14 years of VLBI data). We see that the GPS results are in very good agreement with the recent model by Souchay and Kinoshita, much better than most of the VLBI/LLR results of the past. This shows that GPS may give a significant contribution to nutation in the high

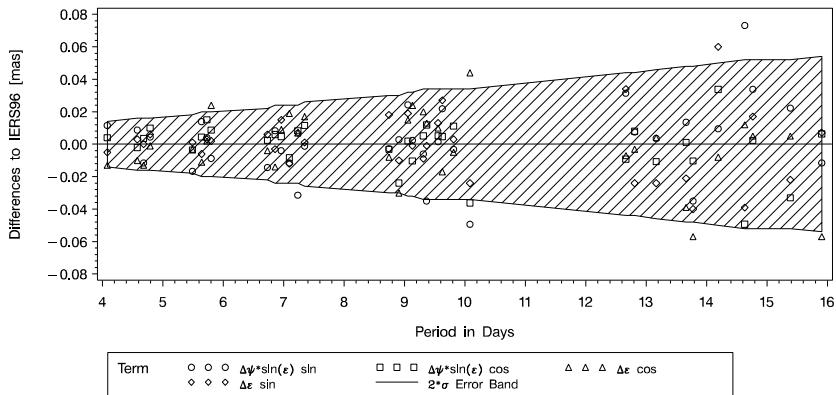


Figure 3: Nutation corrections relative to the IERS96 model for 34 periods estimated from the rate series obtained from GPS data. The shaded area represents the 95% confidence interval ( $2\sigma$ ).

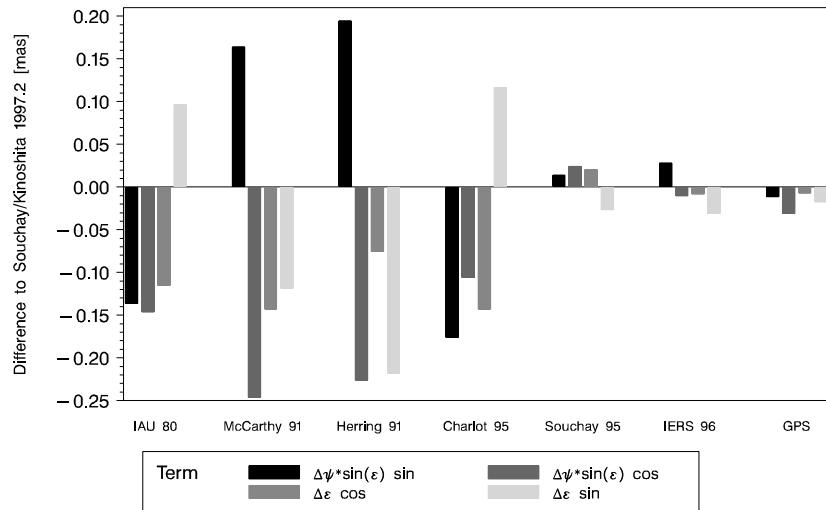


Figure 4: Comparison of the 13.66-day nutation coefficients from different sources with the most recent model by Souchay/Kinoshita 1997.2 .

frequency range of the spectrum (periods below 20 days). The long term behavior is, however, reserved to VLBI and LLR.

Many more details about the analysis of the nutation rate series from GPS and the estimation of nutation coefficients may be found in [Rothacher *et al.*, 1998a].

#### 4. HIGH-FREQUENCY EARTH ROTATION

The potential of GPS to monitor high-frequency Earth rotation has already been demonstrated during intensive campaigns like CONT94, CONT96, etc. Today we are, however, in a much better position to assess the importance of GPS in this field. This is mainly due to the fact that a *long, continuous* series of 2-hourly ERPs is now available from the CODE AC to study the details of sub-daily variations in Earth rotation. CODE started in March 1996 with the routine

generation of ERP solutions with a time resolution of 2 hours and — due to a reprocessing effort (from March 1996 back to January 1995) — the series now covers about 3.5 years. The series is not very homogeneous, because a lot of changes and improvements in the processing strategies and modeling have taken place in the course of the 3.5 years. The most recent strategy used at CODE involves an elevation cut-off angle of 10 degrees, an elevation-dependent weighting of the observations, 3-day satellite arcs with the estimation of 5 radiation pressure parameters [Rothacher *et al.*, 1998b]. Due to the fact that the exactly diurnal retrograde terms in polar motion are fully correlated with the orientation of the orbital planes in space, these terms have to be constrained to zero.

To give an example of the quality of the 2-hourly ERP estimates from GPS, Figure 5 shows the comparison of 5 days of GPS estimates with the sub-daily ERP model by Ray (see [McCarthy, 1996] or [Chao *et al.*, 1996]), which was derived from TOPEX/Poseidon altimetry data. The

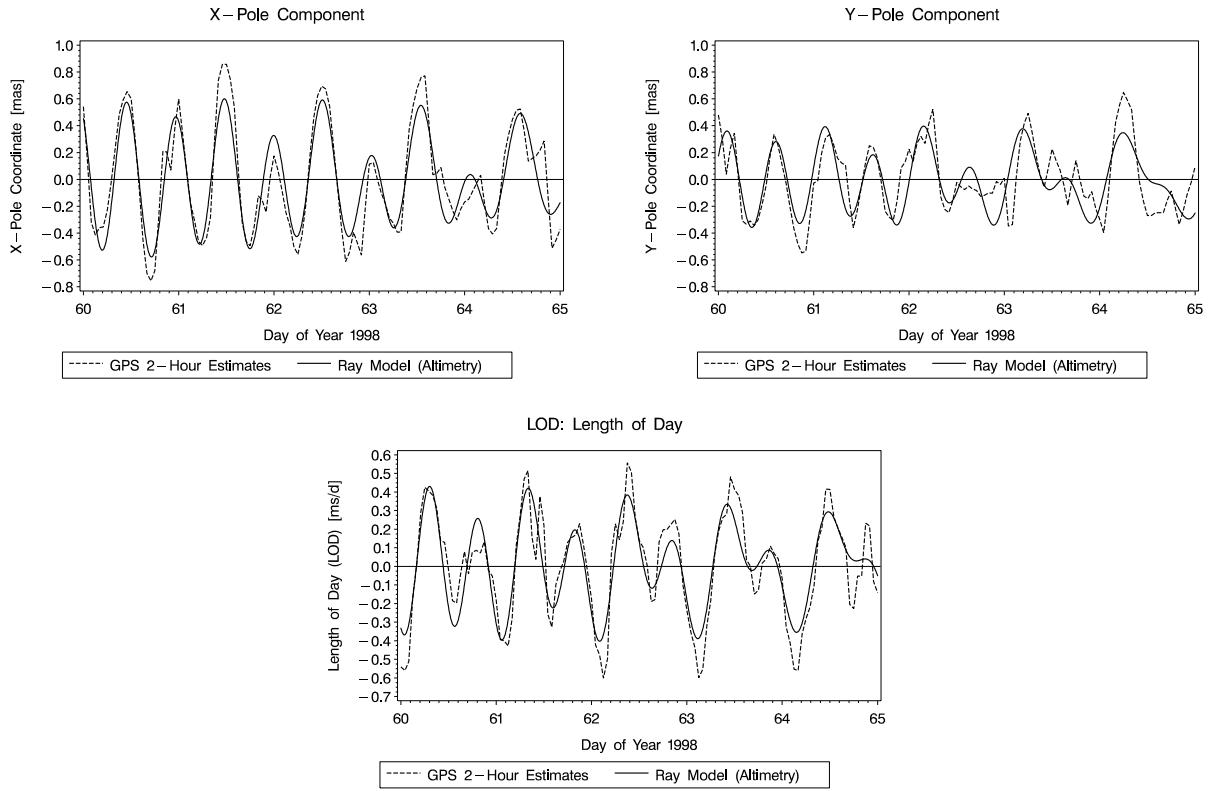


Figure 5: High-frequency variations in Earth rotation from GPS compared to the Ray model derived from altimetry data.

rms difference between the GPS estimates and the Ray model is about 0.25 mas and 0.20 ms/day for polar motion and LOD, respectively.

Because a continuous series of ERPs is at our disposal, we may easily generate amplitude spectra for the interesting diurnal and semi-diurnal frequency bands. As an example, the UT1 amplitude spectra for the diurnal and semi-diurnal bands are given in Figure 6. The very low noise level of the spectra not only enables us to clearly see all the major tidal terms, it should also allow the detection of possible non-tidal signals, if such signals exist with amplitudes larger than the noise level present in the spectra.

From the entire sub-daily GPS series a set of 57 tides in polar motion and 41 tides in UT1

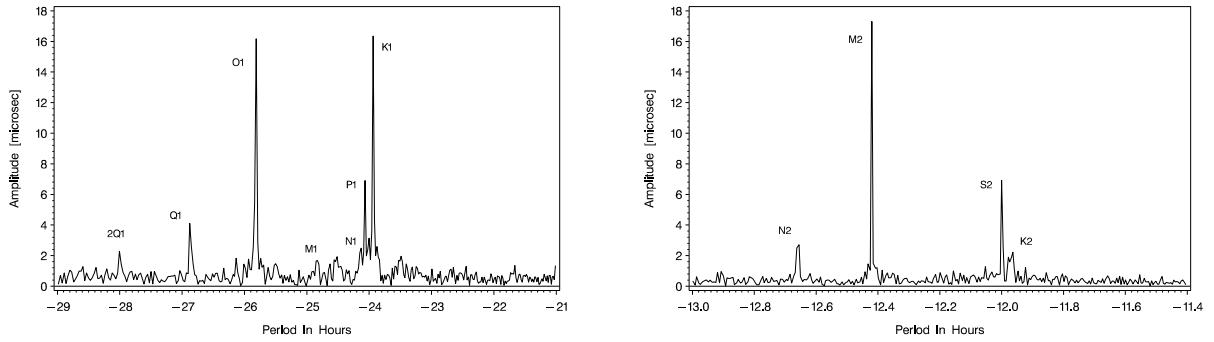


Figure 6: Amplitude Spectra of diurnal and semi-diurnal UT1 variations.

were estimated (for more details see [*Rothacher et al.*, 1998c]). The results of a comparison of these tidal amplitudes with those from three other space techniques, namely VLBI [*Gipson*, 1996], SLR [*Eanes*, 1998, private communication], and altimetry [*Chao et al.*, 1996], are given in Tables 1 and 2. We see that the tidal amplitudes of all these different techniques agree with

Table 1: RMS differences in  $\mu\text{s}$  of tidal UT1 amplitudes derived from different space techniques.

Technique	GPS	VLBI	SLR	Altim.
GPS		1.0	1.1	0.9
VLBI	1.0		1.2	1.1
SLR	1.1	1.2		1.2
Altim.	0.9	1.1	1.2	

Table 2: RMS differences in  $\mu\text{as}$  of tidal polar motion amplitudes derived from different space techniques.

Technique	GPS	VLBI	SLR	Altim.
GPS		9.4	12.0	9.1
VLBI	9.4		9.6	6.5
SLR	12.0	9.6		9.0
Altim.	9.1	6.5	9.0	

each other on the level of  $1 \mu\text{s}$  in UT1 and  $10 \mu\text{as}$  in polar motion. With the 3.5-year series of sub-daily ERPs from GPS, it is therefore possible to obtain results comparable to those of the other space techniques. Of special interest is finally the study of the residual spectra of high frequency Earth rotation after the removal of the known tidal components. First residual spectra and first analyses thereof may be found in [*Rothacher et al.*, 1998c]. There the 2-hourly

ERP series from CODE is studied in much more detail.

## 5. CONCLUSIONS

In the last few years GPS has become one of the major techniques to monitor Earth rotation. When combining different techniques to generate the Bulletin A and B series, GPS plays a very important role today. The LOD values from GPS are now included in the generation of UT1 series, mainly because of the timeliness of the GPS results.

Because there is no fundamental difference between the estimation of UT1 rates (or LOD) and nutation rates, it has been recognized that GPS may contribute to the determination of nutation amplitudes for terms with periods below about 20 days. The long term behavior, however, is only accessible to VLBI and LLR. The comparison of the GPS results with the very recent model by Souchay and Kinoshita (1997.2) shows an agreement of  $10 \mu\text{as}$  (median) for the estimated nutation coefficients. GPS thus allows an independent verification of present-day theoretical nutation models and VLBI results at the high frequency end of the spectrum.

Due to the dense global network and the wealth of observations, GPS is extremely well-suited to monitor the high-frequency variations in the Earth's rotation. The sub-daily ERP series from CODE with a time resolution of 2 hours is presently the only *long and continuous* series available. This series, which covers a time interval of about 3.5 years by now, allows the estimation of ocean tide amplitudes with a quality comparable to VLBI, SLR, and altimetry, namely  $1 \mu\text{s}$  in UT1 and  $10 \mu\text{as}$  in polar motion. The residual spectra show signals that may be of non-tidal origin or artifacts of the orbit modeling. An analysis of the series using wavelet methods might give more insight into possible signals in the series caused by atmospheric or oceanic normal modes.

## REFERENCES

- Chao, B.F., D.R. Ray, J.M. Gipson, G.D. Egbert, C. Ma, 1996, Diurnal/semidiurnal polar motion excited by oceanic tidal angular momentum, *J. Geophys. Res.*, **101**(B9), 20151–20163.  
Gipson, J.M., 1996, Very long baseline interferometry determination of neglected tidal terms in high-frequency Earth orientation variation, *J. Geophys. Res.*, **101**(B12), 28051–28064.  
Charlot, P., O.J. Sovers, J.G. Williams, X.X. Newhall, 1995, Precession and Nutation From Joint Analysis of Radio Interferometric and Lunar Laser Ranging Observations, *Astron. J.*, **109**, 1, 418–427.  
Herring, T.A., B.A. Buffett, P.M. Mathews, I.I. Shapiro, 1991, Forced Nutations of the Earth: Influence of Inner Core Dynamics, 3. Very Long Interferometry Data Analysis, *J. Geophys. Res.*, **96**(B5), 4745–4754.  
Herring, T.A., and D. Dong, 1994, Measurement of Diurnal and Semidiurnal Rotational Variations and Tidal Parameters of Earth, *J. Geophys. Res.*, **99**(B9), 18051–18071.  
Herring, T.A., 1997, Analysis of most recent VLBI nutation offset series available at [ftp://gemini.gsfc.nasa.gov/pub/solutions/1083\\_aug97/eop\\_nutation.1083c](ftp://gemini.gsfc.nasa.gov/pub/solutions/1083_aug97/eop_nutation.1083c) (private communication).  
Kouba, J., and Y. Mireault, 1998, 1997 Analysis Coordinator Report, *IGS 1997 Annual Report*, editors J.F. Zumberge and R.E. Neilan, IGS Central Bureau, Jet Propulsion Laboratory, Pasadena, California U.S.A. (in press).  
McCarthy, D.D., and B.J. Luzum, 1991, Observations of Luni-Solar and Free Core Nutation, *Astron. J.*, **102**, 5, 1889–1895.  
McCarthy, D.D., 1996, IERS Conventions (1996), *IERS Technical Note 21*, Observatoire de Paris, Paris, July 1996.

- Rothacher, M., G. Beutler, T.A. Herring and R. Weber, 1998, Estimation of Nutation Using the Global Positioning System, *J. Geophys. Res.*, accepted in October 1998.
- Rothacher, M., T. A. Springer, S. Schaer, G. Beutler, D. Ineichen, U. Wild, A. Wiget, C. Boucher, S. Botton and H. Seeger, 1998, Annual Report 1997 of the CODE Processing Center of the IGS, *IGS 1997 Annual Report*, editors J.F. Zumberge and R.E. Neilan, IGS Central Bureau, Jet Propulsion Laboratory, Pasadena, California U.S.A. (in press).
- Rothacher, M., G. Beutler, R. Weber and J. Hefty, 1998, High-Frequency Earth Rotation Variations From Three Years of Global Positioning System Data, *Habilitation (2nd Ph.D.)*, University Press, University of Berne.
- Souchay, J., M. Feissel, C. Bizouard, N. Capitaine, M. Bougeard, 1995, Precession and Nutation for a Non-Rigid Earth: Comparison Between Theory and VLBI Observations, *Astron. Astrophys.*, **299**, 277–287.