THE PERTURBATION OF THE ORBITAL ELEMENTS OF GPS SATELLITES THROUGH DIRECT RADIATION PRESSURE AND Y-BIAS

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ABSTRACT

A direct radiation pressure p_0 and a constant p_2, so-called *y-bias*, are estimated for each GPS satellite and each *arc of one to three days* on top of the a priori radiation pressure model defined by the *IERS Standards* in the daily analyses performed at the *CODE Analysis Center of the IGS*.

In this article we are analyzing the influence of the two estimated parameters on the orbital elements using the methods of special perturbations. We develop approximated expressions for the perturbations in the Keplerian elements which allow us to study the perturbations analytically. We are in particular interested in the correlations of these parameters with the length of day (*UT1-UTC drift*) estimates and the so-called *pseudo-stochastic parameters* which are also solved for routinely at *CODE*. Using the IGS data stemming from a time interval of 46 days in 1994 we show that the theoretical results are confirmed in practice.

INTRODUCTION

CODE, Center for Orbit Determination in Europe, is one of at present seven Analysis Centers of the *IGS, the International GPS Service for Geodynamics*. *CODE* is formed as a joint venture of the Astronomical Institute of the University of Bern (AIUB), the Swiss Federal Office of Topography (L+T), the German Institute for Applied Geodesy (IfAG), and the French Institut Géographique National (IGN). *CODE* is located at the AIUB in Bern.

Since 21 June 1992 ephemerides for all GPS satellites are available and daily values for the earth orientation parameters (x and y of the pole position, and drift values for *UT1-UTC*) are solved for and made available to the scientific community by *CODE*. For test purposes the drifts $\Delta\dot{\psi}$ and $\Delta\dot{\epsilon}$ in the nutation corrections are determined for test purposes. These values are *not (yet)* made available to the community.

Other parameters like station coordinates and their velocities are estimated by combining the daily solutions. So far, they were produced for the annual solutions of the *IERS, the International Earth Rotation Service.*

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The *CODE* products are analysed and compared to other solutions stemming from *GPS* or from other space techniques like *VLBI* and *SLR*. Whereas in general the *CODE* results compare well to all the other results there is a slight problem in the *UT1-UTC* drift rates as determined by *CODE*. This problem is not obvious if we just compare our length-of-day estimates to those established by VLBI – there clearly all the signatures of a physical nature are present. Figure 1 shows our length-of-day estimates (after removal of the tidal terms up to 35 days).

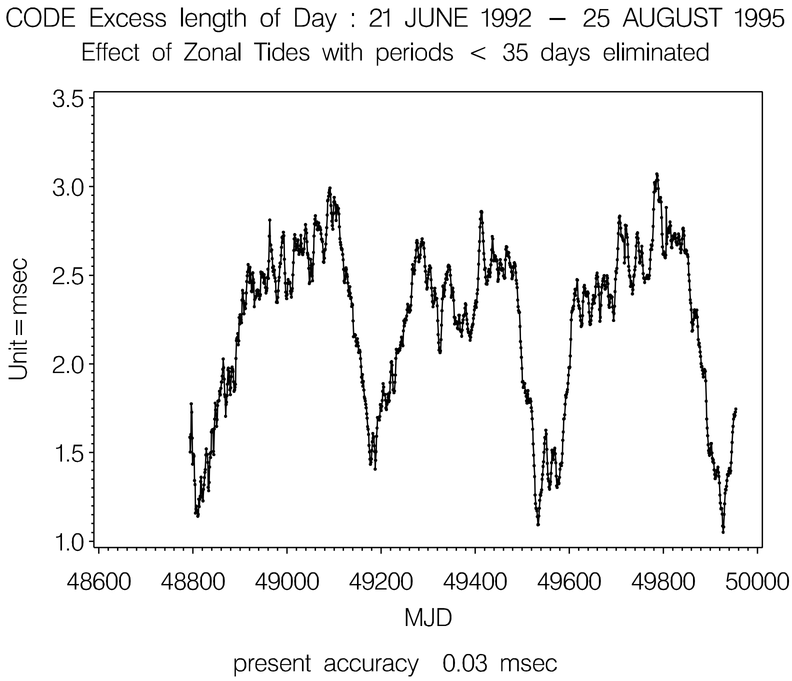

Figure 1. *CODE* length-of-day estimates after removal of tidal terms.

That there is a small bias with respect to the *VLBI* becomes obvious if we sum up our daily *UT1-UTC* drifts and subtract the corresponding *VLBI UT1-UTC* series from our series. Figure 2 shows the result. There is a net drift between the two series. There also seems to be a net change of the drift around May 1995.

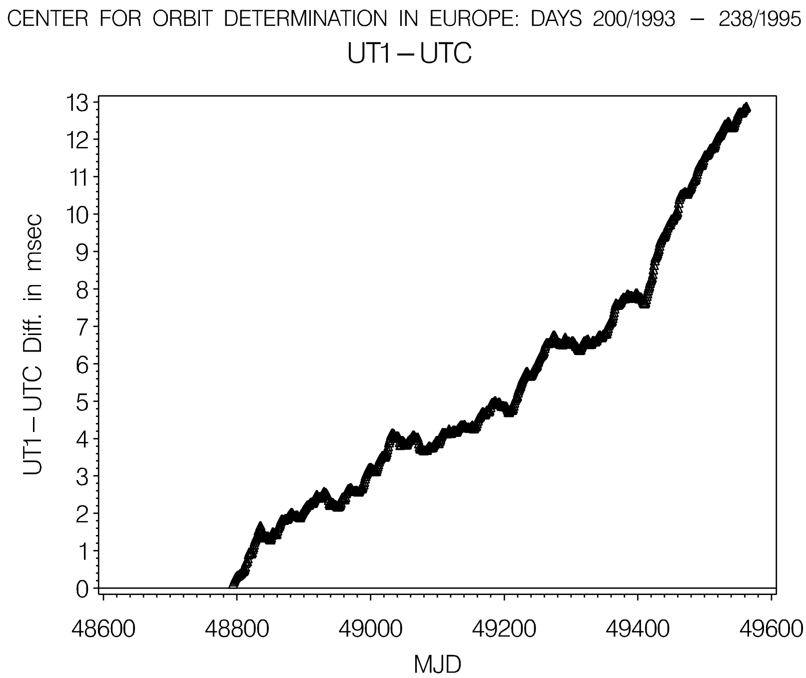

Figure 2. *CODE UT1-UTC* series minus corresponding *VLBI* series (actually the IERS Bulletin A series).

It is well known that satellite geodesy *not having direct access to direction observations in the celestial reference frame* is not able to determine *UT1-UTC* itself because of the correlation with the right ascensions of the ascending nodes of the GPS satellites which have to be solved for in each arc.

Could we assume that the force field acting on GPS satellites is completely known, we would be in a perfect position to solve for the *UT1-UTC drift*, however. This statement is *almost* true in the case of GPS satellites: The earth's gravity field may be assumed known for these high orbiting satellites, third body perturbations and tidal effects do not pose problems. On the other hand, we have to solve for radiation pressure parameters for each arc and each satellite. Should these parameters generate (among other effects) a net rotation of the orbital system, a correlation with the *UT1-UTC drift estimates* would be possible. This aspect is studied on a theoretical basis in the next section, empirical evidence is presented in the subsequent section.

PERTURBATIONS DUE TO RADIATION PRESSURE: ORDERS OF MAGNITUDE

In this section we study the influence of the radiation pressure model used at *CODE* on the estimated orbits using the method of special perturbations. For that purpose we first have to develop analytical expressions for the perturbing forces, then we have to present (simplified) perturbation equations. We assume in particular that GPS orbits are almost circular. Finally we will develop and discuss the effects and possible correlations of radiation pressure estimates with earth rotation parameters.

Analytical Expressions for the Radiation Pressure

Figure 3 shows the projection on a unit sphere of the geocentric positions of the satellite and the sun at a particular instant of time t. For the description of the positions of satellite and sun we introduce two coordinate systems, one which is described by the axes x_1, x_2 and x_3 in Figure 3 *and* the other which is described by the axes y_1, y_2 and y_3. The latter system is rotating with the satellite, it may be generated from the first coordinate system by a rotation around the $x_3 = y_3$ axis with the argument of latitude u of the satellite as the rotation angle.

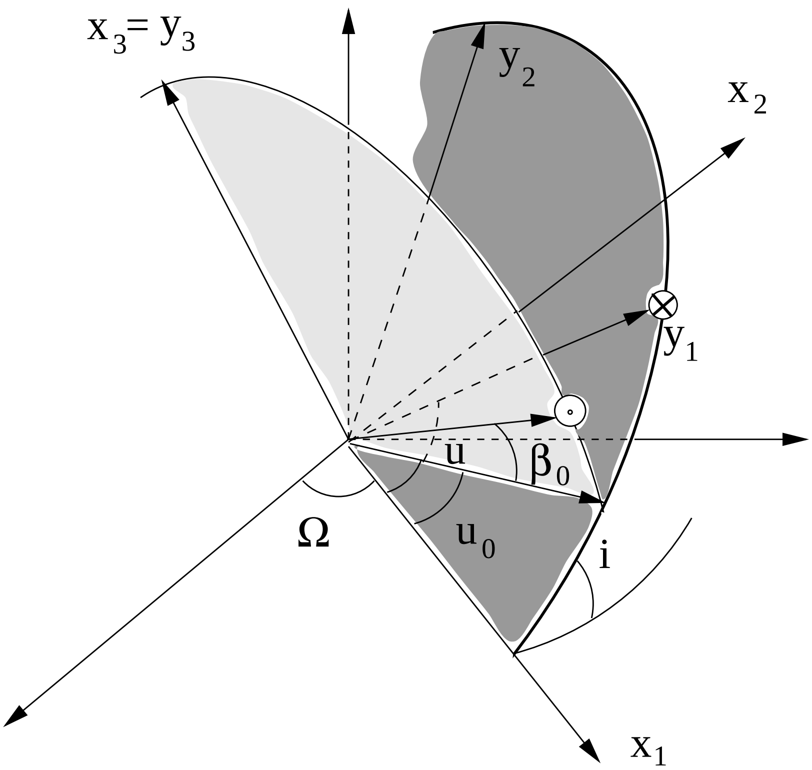

Figure 3. Positions of satellite and sun. u and u_0 are the arguments of latitude of satellite and sun, β_0 is the elevation of the sun above the orbital plane; Ω is the right ascension of the ascending node of the satellite's orbital plane.

The coordinates $\boldsymbol{r}_x$ of the satellite position $\boldsymbol{r}$ and the coordinates $\boldsymbol{e}_{s,x}$ of the unit vector $\boldsymbol{e}$ pointing from the satellite to the sun may be expressed as follows in the coordinate system

$\{x_1, x_2, x_3\}$:

$$\boldsymbol{r}_x = r \cdot \begin{pmatrix} \cos u \\ \sin u \\ 0 \end{pmatrix} \tag{1}$$

$$\boldsymbol{e}_{s,x} = \begin{pmatrix} \cos u_0 \cdot \cos\beta_0 \\ \sin u_0 \cdot \cos\beta_0 \\ \sin\beta_0 \end{pmatrix} \tag{2}$$

where we assume that the direction *satellite - sun* may be approximated by the direction *center of earth - sun.*

The perturbation equations given in the next section ask for the radial, the tangential, and the out of plane components $\boldsymbol{R}$, $\boldsymbol{S}$, and $\boldsymbol{W}$ of the perturbing forces. We thus have to express the two position vectors in the coordinate system $\{y_1,\ y_2,\ y_3\}$:

$$\boldsymbol{r}_y = \boldsymbol{R}_3(u) \cdot \boldsymbol{r}_x = r \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

$$\boldsymbol{e}_{s,y} = \boldsymbol{R}_3(u) \cdot \boldsymbol{e}_{s,x} = \begin{pmatrix} \cos(u - u_0) \cdot \cos\beta_0 \\ -\sin(u - u_0) \cdot \cos\beta_0 \\ \sin\beta_0 \end{pmatrix} \tag{4}$$

where $\boldsymbol{R}_3(u)$ stands for a rotation matrix describing a particular rotation around axis 3 and angle u.

At CODE the acceleration $\boldsymbol{a}_{drp}$ due to the solar radiation pressure is modeled as (Beutler et al., 1994):

$$\boldsymbol{a}_{drp} = \boldsymbol{a}_{Rock} - p_0 \cdot \boldsymbol{e}_s + p_2 \cdot \boldsymbol{e}_y \tag{5}$$

where

$\boldsymbol{a}_{Rock}$ is the standard model Rock4 for Block *I*, the standard model Rock42 for Block *II* satellites (Fliegel et al., 1992),

$\boldsymbol{e}_s$ is the unit vector *satellite - sun*, in our approximation the unit vector *center of earth - sun*,

$\boldsymbol{e}_y$ is the unit vector pointing along the spacecraft's solar panels axis (Fliegel et al.,1992):

$$\boldsymbol{e}_y = -\frac{\boldsymbol{e}_s \times \boldsymbol{r}}{\mid \boldsymbol{e}_s \times \boldsymbol{r} \mid} \tag{6}$$

p_0 and p_2 are the direct radiation pressure parameters estimated for each satellite and each arc.

It should be pointed out that – in a first approximation – the vector $\boldsymbol{a}_{Rock}$ is parallel to the vector $p_0 \cdot \boldsymbol{e}_s$. We therefore do *not* analyse the term stemming from the Rock4/42 models separately. By forming the vector product in the coordinate system $\{y_1, y_2, y_3\}$ we obtain:

$$\begin{aligned}
\boldsymbol{a}_{drp,y} &\doteq \boldsymbol{a}_{Rock,y} + \boldsymbol{a}_{0,y} + \boldsymbol{a}_{2,y} \\
&= \boldsymbol{a}_{Rock,y} + p_0 \cdot \begin{pmatrix} -\cos(u-u_0)\cdot\cos\beta_0 \\ \sin(u-u_0)\cdot\cos\beta_0 \\ -\sin\beta_0 \end{pmatrix} - \\
&\quad - \frac{p_2}{\sqrt{\sin^2\beta_0 + \cos^2\beta_0\cdot\sin^2(u-u_0)}} \cdot \begin{pmatrix} 0 \\ \sin\beta_0 \\ \cos\beta_0\cdot\sin(u-u_0) \end{pmatrix}
\end{aligned} \tag{7}$$

It is of interest to write down the equation (7) for the cases $\beta_0 \to \frac{\pi}{2}$ (corresponding to the case when the sun is in the zenith of the orbital plane) and $\beta_0 \to 0$ (corresponding to the case where the the sun lies in the orbital plane). In the subsequent expressions we neglect all terms of first or higher order in the small angle β_0 resp. $\beta_0 - \frac{\pi}{2}$.

(a) $\beta_0 \to \frac{\pi}{2}$**:**

$$\begin{aligned}
\boldsymbol{a}_{drp,y} &\doteq \boldsymbol{a}_{Rock,y} + \boldsymbol{a}_{0,y} + \boldsymbol{a}_{2,y} \\
&= \boldsymbol{a}_{Rock,y} + p_0 \cdot \begin{pmatrix} 0 \\ 0 \\ -\mathrm{sign}(\beta_0) \end{pmatrix} - \\
&\quad -p_2 \cdot \begin{pmatrix} 0 \\ \mathrm{sign}(\beta_0) \\ 0 \end{pmatrix}
\end{aligned} \tag{8}$$

(b) $\beta_0 \to 0$**:**

$$\begin{aligned}
\boldsymbol{a}_{drp,y} &\doteq \boldsymbol{a}_{Rock,y} + \boldsymbol{a}_{0,y} + \boldsymbol{a}_{2,y} \\
&= \boldsymbol{a}_{Rock,y} + p_0 \cdot \begin{pmatrix} -\cos(u-u_0) \\ \sin(u-u_0) \\ 0 \end{pmatrix} - \\
&\quad - \frac{p_2}{\sqrt{\beta_0^2 + \sin^2(u-u_0)}} \cdot \begin{pmatrix} 0 \\ \beta_0 \\ \sin(u-u_0) \end{pmatrix}
\end{aligned} \tag{9}$$

In the first case ($\beta_0 \to \frac{\pi}{2}$) the perturbation parallel to the direction *sun - satellite* is a *constant* out of plane acceleration. In the second case ($\beta_0 \to 0$) this perturbation lies in the orbital plane and oscillates. Both, the R and S components oscillate with a period equal to the revolution period of the satellite.

In the first case ($\beta_0 \to \frac{\pi}{2}$) the perturbation due to the *y-bias* is a constant along-track acceleration (the sign depending on the position of the sun). In the second case ($\beta_0 \to 0$) this perturbation consists of an *along-track* and an *out-of-plane* component. The out-of-plane component is constant in absolute value, but changes sign for $u = u_0$ and $u = u_0 + \pi$. The along-track component is zero everywhere *except* near $u = u_0$ and $u = u_0 + \pi$, where the acceleration reaches p_2 in absolute value. Obviously we have in this case a strong correlation with the pseudo-stochastic pulses.

Approximate Perturbation Equations for Almost Circular Orbits

In (Beutler,1991) we find the perturbation equations for the Keplerian elements semimajor axis a, numerical eccentricity e, inclination i, right ascension of the ascending node Ω, argument of perigee ω, and mean anomaly σ at time t_0:

$$\dot{a} = \frac{2}{n \cdot \sqrt{1-e^2}} \cdot \left(e \cdot \sin v \ \cdot \boldsymbol{R} + \frac{p}{r} \cdot \boldsymbol{S}\ \right) \tag{10}$$

$$\dot{e} = \frac{\sqrt{1-e^2}}{n \cdot a} \cdot \left(\sin v \ \cdot \boldsymbol{R} + (\cos v + \cos E) \cdot \boldsymbol{S}\right) \tag{11}$$

$$i^{(\cdot)} = \frac{r \cdot \cos(\omega + v)}{n \cdot a^2 \cdot \sqrt{1-e^2}} \cdot \boldsymbol{W} \tag{12}$$

$$\dot{\Omega} = \frac{r \cdot \sin(\omega + v)}{n \cdot a^2 \cdot \sqrt{1-e^2} \cdot \sin i} \cdot \boldsymbol{W} \tag{13}$$

$$\dot{\omega} = \frac{\sqrt{1-e^2}}{e \cdot n \cdot a} \cdot \left(-\cos v \cdot \boldsymbol{R} + \left(1 + \frac{r}{p}\right) \cdot \sin v \cdot \boldsymbol{S}\right) - \cos i \cdot \dot{\Omega} \tag{14}$$

$$\dot{\sigma} = \frac{1-e^2}{e \cdot n \cdot a} \cdot \left(\left(\cos v - 2 \cdot e \cdot \frac{r}{p}\right) \cdot \boldsymbol{R} - \left(1 + \frac{r}{p}\right) \cdot \sin v \cdot \boldsymbol{S}\right) + \frac{3}{2} \cdot \frac{n}{a} \cdot (t - t_0) \cdot \dot{a} \tag{15}$$

where r is the length of the radius vector at time t, n is the mean motion of the satellite, E is the eccentric anomaly, v the true anomaly, and $p = a \cdot (1 - e^2)$ the parameter of the ellipse. $\boldsymbol{R}$, $\boldsymbol{S}$, and $\boldsymbol{W}$ are the accelerations in radial, tangential (actually normal to $\boldsymbol{R}$ in the orbital plane), and out-of-plane directions.

GPS orbits are low eccentricity orbits. If we neglect all terms of the first and higher orders in the eccentricity e (circular orbit), these equations may be considerably simplified. We may formally assume the perigee to coincide with the ascending node at time t_0, which allows us to use the approximation $v = E = u$, where $u = n \cdot (t - t_0)$. Moreover we do not need to consider the equation for the argument of perigee, in this case. We include, instead, an equation for the argument of perigee u_0 at time t_0 by adding the equations for the argument of perigee ω and for the mean anomaly σ at time t_0. The resulting perturbation equations are:

$$\dot{a} = \frac{2}{n} \cdot \boldsymbol{S} \tag{16}$$

$$\dot{e} = \frac{1}{n \cdot a} \cdot (\sin u \cdot \boldsymbol{R} + 2 \cdot \cos u \cdot \boldsymbol{S}) \tag{17}$$

$$i^{(\cdot)} = \frac{\cos u}{n \cdot a} \cdot \boldsymbol{W} \tag{18}$$

$$\dot{\Omega} = \frac{\sin u}{n \cdot a \cdot \sin i} \cdot \boldsymbol{W} \tag{19}$$

$$\dot{u}_0 = -\frac{2}{n \cdot a} \cdot \boldsymbol{R} - \cos i \cdot \dot{\Omega} + \frac{3}{2} \cdot \frac{n}{a} \cdot (t - t_0) \cdot \dot{a} \tag{20}$$

Let us assume that the initial values for these elements at time t_0 are a_0, e_0, i_0, Ω_0, and u_{00}. The influence of the perturbation terms due to radiation pressure at time t are computed by the equations $\delta a \doteq a(t) - a_0$, $\delta e \doteq e(t) - e_0$, $\delta i \doteq i(t) - i_0$, $\delta\Omega \doteq \Omega(t) - \Omega_0$, and $\delta u_0 \doteq u_0(t) - u_{00}$. Usually the dominant perturbation term is that in along track direction, i.e. the error in the argument of latitude $u(t)$ at time t:

$$\delta u(t) = \delta u_0(t) + \delta n \cdot (t - t_0) \tag{21}$$

where, using Kepler's third law, we have

$$\delta n = -\frac{3}{2} \cdot \frac{n}{a} \cdot \delta a \tag{22}$$

The solutions of the above simplified perturbation equations formally may be written as follows:

$$\delta a(t) = \frac{2}{n} \cdot \int_{t_0}^{t} \boldsymbol{S} \cdot dt' \tag{23}$$

$$\delta e(t) = \frac{1}{n \cdot a} \cdot \int_{t_0}^{t} (\sin u \cdot \boldsymbol{R} + 2 \cdot \cos u \cdot \boldsymbol{S}) \cdot dt' \tag{24}$$

$$\delta i(t) = \frac{1}{n \cdot a} \cdot \int_{t_0}^{t} \cos u \cdot \boldsymbol{W} \cdot dt' \tag{25}$$

$$\delta\Omega(t) = \frac{1}{n \cdot a \cdot \sin i} \cdot \int_{t_0}^{t} \sin u \cdot \boldsymbol{W} \cdot dt' \tag{26}$$

$$\begin{aligned} \delta u_0(t) = & -\frac{2}{n \cdot a} \cdot \int_{t_0}^{t} \boldsymbol{R} \cdot dt' - \cos i \cdot \delta\Omega(t) \\ & + \frac{3}{2} \cdot \frac{n}{a} \cdot (t - t_0) \cdot \delta a - \frac{3}{2} \cdot \frac{n}{a} \cdot \int_{t_0}^{t} \delta a(t') \cdot dt' \end{aligned} \tag{27}$$

This last equation allows us to write the equation for the argument of latitude in the more explicit form

$$\delta u(t) = -\frac{2}{n \cdot a} \cdot \int_{t_0}^{t} \boldsymbol{R} \cdot dt' - \cos i \cdot \delta\Omega(t) - \frac{3}{2} \cdot \frac{n}{a} \cdot \int_{t_0}^{t} \delta a(t') \cdot dt' \tag{28}$$

Solution of the Perturbation Equations for Radiation Pressure Terms

Let us deal separately with the terms p_0 and p_2, and with the cases when the sun lies in the orbital plane ($\beta_0 = 0$) resp. when the sun is in the zenith of the orbital plane ($\beta_0 = \frac{\pi}{2}$).

Perturbations due to p_0, Sun in Zenith of Orbital Plane ($\beta_0 = \frac{\pi}{2}$)

The perturbing acceleration reads as

$$\boldsymbol{a}_{0,y} = \begin{pmatrix} \boldsymbol{R} \\ \boldsymbol{S} \\ \boldsymbol{W} \end{pmatrix} = p_0 \cdot \begin{pmatrix} 0 \\ 0 \\ -\mathrm{sign}(\beta_0) \end{pmatrix} \tag{29}$$

We easily see that there are only short period perturbations in the orbital elements i and Ω. The result of the integration is easily verified to be:

$$\begin{aligned} \delta\Omega(t) &= \frac{p_0 \cdot \text{sign}(\beta_0)}{n^2 \cdot a \cdot \sin i} \cdot [\cos u - 1] \\ \delta i(t) &= -\frac{p_0 \cdot \text{sign}(\beta_0)}{n^2 \cdot a} \cdot \sin u \end{aligned} \tag{30}$$

The perturbations are purely periodical where the period is equal to the revolution period of the satellite. Obviously, a small error in the parameters p_0 will *not* contaminate the estimation of earth rotation parameters. This statement is no longer valid if we are looking for a sub-diurnal resolution of these terms. In this case, a correlation of the perturbation in Ω with length-of-day estimates is very well possible. For GPS satellites (semimajor axis $a \approx 26500 km$, revolution period $U \approx 43'200 s$, $p_0 \approx 10^{-6} m/s^2$) we have:

$$\begin{aligned} &\delta\Omega(t) \approx 449 \text{ mas} \cdot \text{sign}(\beta_0) \cdot [\cos u - 1] \\ &\delta i(t) \approx -368 \text{ mas} \cdot \text{sign}(\beta_0) \cdot \sin u \end{aligned} \tag{31}$$

or, in meters:

$$\begin{aligned} &a \cdot \delta\Omega(t) \approx 58 \text{ m} \cdot \text{sign}(\beta_0) \cdot [\cos u - 1] \\ &a \cdot \delta i(t) \approx -47 \text{ m} \cdot \text{sign}(\beta_0) \cdot \sin u \end{aligned} \tag{32}$$

The periodic variations thus are of a considerable size.

Perturbations due to p_0, Sun in Orbital Plane ($\beta_0 = 0$)

The perturbing acceleration reads as

$$\boldsymbol{a}_{0,y} = \begin{pmatrix} \boldsymbol{R} \\ \boldsymbol{S} \\ \boldsymbol{W} \end{pmatrix} = p_0 \cdot \begin{pmatrix} -\cos(u - u_0) \\ \sin(u - u_0) \\ 0 \end{pmatrix} \tag{33}$$

In this case there obviously is *no* out-of-plane component, and no correlation with earth rotation parameters has to be expected. On the other hand there are strong $\boldsymbol{R}$ and $\boldsymbol{S}$ components. One easily verifies that these accelerations generate *periodic perturbations* (period equal to orbital period, amplitudes comparable to those in the previous section) in a, u_0, and $u(t)$. The *prominent* perturbation feature, however, is the perturbation in the eccentricity e, where we have perturbations proportional to $\sin u_0$. Because u_0 has an annual period (revolution of the sun around the earth (or vice-versa)) the period of this perturbation will be roughly one year, too. Its amplitude is of the order of a few kilometers (Beutler, 1992).

Perturbations due to p_2, Sun in Zenith of Orbital Plane ($\beta_0 = \frac{\pi}{2}$)

The perturbing acceleration reads as

$$\boldsymbol{a}_{2,y} = \begin{pmatrix} \boldsymbol{R} \\ \boldsymbol{S} \\ \boldsymbol{W} \end{pmatrix} = -p_2 \cdot \begin{pmatrix} 0 \\ \text{sign}(\beta_0) \\ 0 \end{pmatrix} \tag{34}$$

We merely have a constant along-track-component $\boldsymbol{S}$ of the perturbing acceleration. Periodic variations (period=revolution period, amplitudes comparable to those in Ω and i given above) in e will be one consequence.

The *prominent* perturbation will be that in a and in $u(t)$, however. We include these results *although a correlation with UT1-UTC is not expected in this case.* The quadrature of the perturbation equation in the semi-major axis a may be performed without problems:

$$\delta a(t) = -\frac{2}{n} \cdot p_2 \cdot (t - t_0) \cdot \text{sign}(\beta_0) \tag{35}$$

This perturbation in a produces a very pronounced perturbation in along-track direction:

$$a \cdot \delta u(t) \approx \frac{3}{2} \cdot p_2 \cdot (t - t_0)^2 \cdot \text{sign}(\beta_0) \tag{36}$$

where we have left out all periodic perturbations.

Typically the *y-bias* p_2 is of the order of a few 10^{-10} m/s^2. Assuming a value of $p_2 \approx 5 \cdot 10^{-10}$ m/s^2 this gives rise to an along-track-effect of about 6 meters in along-track-direction. Theoretically there might be a correlation of an orbit error in along-track direction $\boldsymbol{S}$ with the UT1–UTC drift. This correlation would be for low inclination satellites. Because GPS satellites have inclination of 55^{o} and because with 3 days our arc-length is usually quite long we do *not* expect a significant contribution in this case.

Perturbations due to p_2, Sun in Orbital Plane ($\beta_0 = 0$)

The perturbing acceleration reads as

$$\boldsymbol{a}_{2,y} = \begin{pmatrix} \boldsymbol{R} \\ \boldsymbol{S} \\ \boldsymbol{W} \end{pmatrix} = -p_2 \cdot \begin{pmatrix} 0 \\ \beta_0 / \sqrt{\beta_0^2 + \sin^2(u - u_0)} \\ \text{sign}(\sin(u - u_0)) \end{pmatrix} \tag{37}$$

Let us start our considerations by looking at the along-track component $\boldsymbol{S}$. This component is very small everywhere but in the immediate vicinity of these instants of time where $u(t) \approx u_0$ and $u(t) \approx u_0 + \pi$. For $u = u_0$ and $u = u_0 + \pi$ the $\boldsymbol{S}$-component reaches its maximum value (which is equal to $\mid p_2 \mid$). The total effect thus actually corresponds to a velocity change at the time when $u = u_0$ resp. $u = u_0 + \pi$. Keeping in mind that p_2 is of the order of a few 10^{-10} m/s^2 which acts only for a very short period of time, we conclude that the along track effect is very small when compared to the same effect when the sun is in the zenith of the orbital plane (compare previous section). We will not further consider this effect.

The out-of-plane component *will* in general produce a net rotation of the orbital plane! This becomes apparent if we introduce the above $\boldsymbol{R}$ acceleration into the perturbation equation for Ω:

$$\delta\Omega(t) = -\frac{p_2}{n \cdot a \cdot \sin i} \cdot \int_{t_0}^{t} \sin u \cdot \text{sign}(\sin(u - u_0)) \cdot dt' \tag{38}$$

The worst case obviously results for $u_0 = 0$ or $u_0 = \pi$ (which will be the case in spring and fall). Let us give the result for $u_0 = 0$:

$$\delta\Omega(t) = -\frac{p_2}{n \cdot a \cdot \sin i} \cdot \int_{t_0}^{t} \mid \sin u \mid \cdot dt' \tag{39}$$

It is thus easy to verify that the net rotation after one revolution is:

$$\delta\Omega(t_0 + U) = -\frac{4 \cdot p_2}{n^2 \cdot a \cdot \sin i} \tag{40}$$

For a typical y-bias of $p_2 \approx 5 \cdot 10^{-10}$ m/s^2 a rotation about 1.8 mas per day is the result. This seems to be a very small value, but it is not negligible. From Figure 2 we conclude that we are looking for an effect of about 0.22 mas/day ! The conclusion is thus clear: a correlation of the $y-biases$ with the drift parameter in *UT1-UTC* cannot be excluded. Let us look for empirical evidence in the next section.

RESULTS FROM GLOBAL IGS DATA PROCESSING USING DIFFERENT ORBIT STRATEGIES

The IGS Data and Orbit Strategies Used

To study the correlation between the y-bias and the *UT1-UTC drift* 46 days of data from the global IGS network were processed using different options (see below). The time period from day 245 to day 290 in 1994 was selected because it contains both, a period where *no* satellites were eclipsed (at the beginning of the series) and a period with 8 satellites (PRN 1, 6, 7, 18, 26, 28, 29, and 31) going through the earth shadow once per revolution (from the middle to the end of the series). The data from about 40 IGS sites were included. Overlapping 3-days solutions — as in the routine *CODE* analysis — were produced for the period mentioned above using the following orbit estimation strategies:

Strategy A: No pseudo-stochastic pulses were set up, one y-bias and one direct radiation pressure parameter estimated on top of the Rock 4/42 models.

Strategy B: Pseudo-stochastic pulses were set up in **R** and **S** direction for the eclipsing satellites only. The two pulses per revolution were set up about 45 minutes after the shadow exit of the satellite. Y-bias and direct pressure parameters as in the strategy A.

Strategy C: Pseudo-stochastic pulses were set up in the **R** and **S** direction for *all* satellites once per revolution (at 0:00 h and 12:00 h).

Strategy D: Pulses were set up as in solution C, but for all the three components (**R,S,W**).

Strategy E: Same solution as C, but the y-bias parameters were *constrained* using an a priori sigma of $10^{-10} m/s^2$ to the values determined from 2 years of y-bias results.

Strategy F: Same solution as C, but instead of estimating a y-bias for each satellite (and arc) *a constant along track acceleration* was determined. The same constraints were applied to the along-track acceleration as in the strategy E to the y-bias.

In all these solutions 12 troposphere parameters per day and site were set up and 8 orbital elements (6 Keplerian elements and the parameters p_0 and p_2 (see eqn. (5)) or, for solution of type F, a constant along-track acceleration) were determined. The x- and y-pole coordinates, and UT1-UTC were estimated exactly in the same way as in the routine processing CODE (Rothacher et al., 1995).

UT1-UTC Results and Comparison to the Rotation of the Satellite Nodes

The UT1-UTC drifts of the middle days of the 3-days solutions were summed up to give GPS-derived UT1-UTC series. The series resulting from strategies A through F are shown in Figure 4. The figure clearly indicates that the orbit estimation strategy has an important impact on the *UT1-UTC* drift. Strategies E and F exhibit the smallest drifts, at least for this time period.

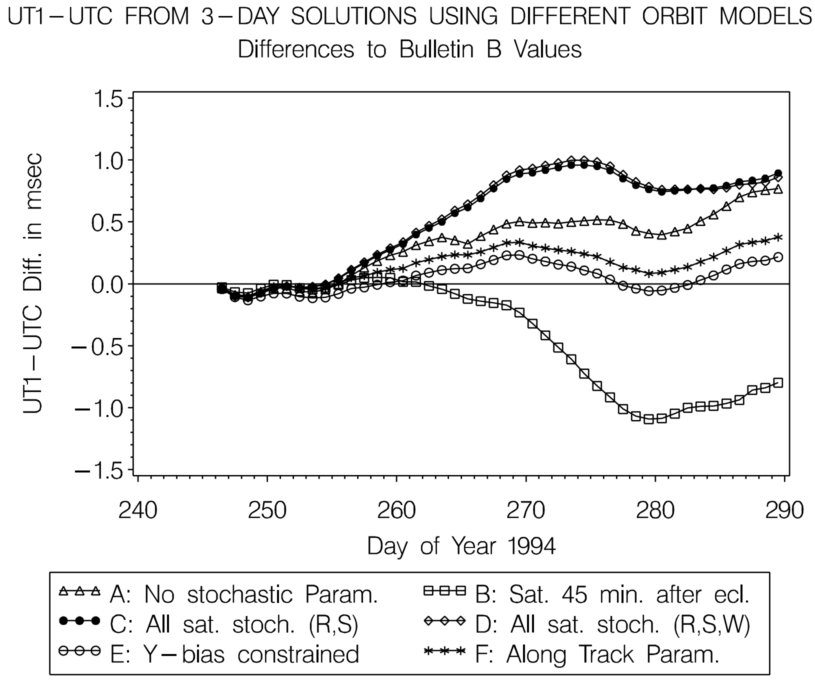

Figure 4. UT1-UTC series estimated from global 3-days solutions using different orbit estimation strategies

To see whether the perturbations due to p_2 may explain the UT1-UTC series in Figure 4), the perturbation eqn. (27) was used to compute the rotation of the ascending node Ω due to p_2 for each individual satellite. The mean value of these daily rotations over all satellites should then reflect a mean rotation of the reference frame that may be absorbed by a drift in UT1-UTC. The actual computation was performed by numerically integrating eqn. (27) using eqn. (7) for the out-of-plane perturbing force $\boldsymbol{W}$ and the y-bias values that were saved for each individual solution. The resulting mean rotation of the nodes is shown in Figure 5 for all strategies except F. Because the y-bias was fixed to the same a priori values as in solution E, the rotation of the nodes would be very similar to the ones of type E.

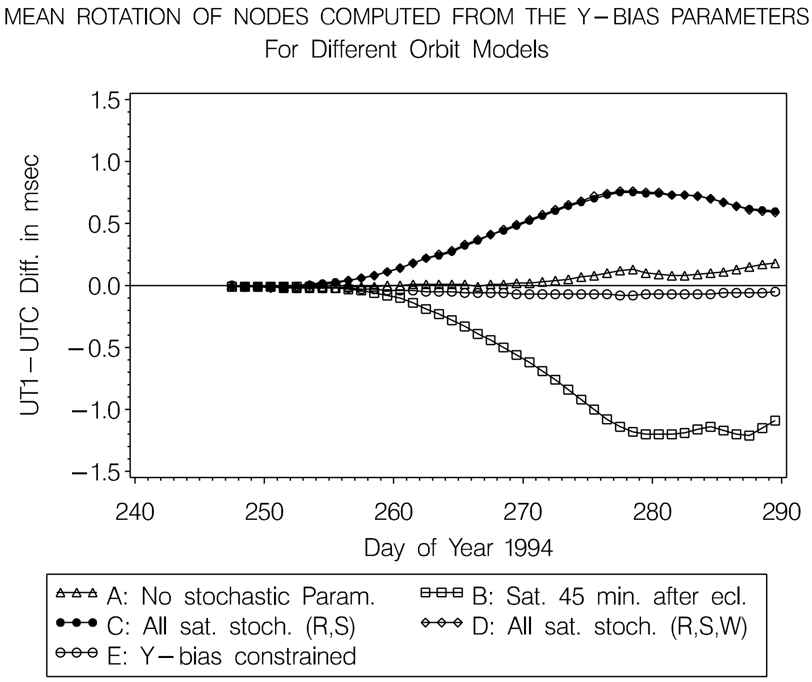

Figure 5. Mean rotation of the nodes computed from the *y-bias* parameters estimated using different orbit strategies

Comparing Figures 4 and 5 we see the high correlation between the UT1-UTC series and the mean rotations of the nodes. This shows that the estimated y-bias plays indeed a key role in the UT1-UTC series originating from GPS. Although most of the features visible in the UT1-UTC series may be explained by the correlation with the y-bias estimate discussed above, there still remain residual drifts in UT1–UTC when compared to VLBI. The remaining drift will have to be studied in future.

Orbit Quality of the Strategies Used

From Figures 4 and 5 we see that constraining the y-biases (solution type E) or estimating a constant along-track acceleration (solution type F) instead of a y-bias improves the stability of the UT1-UTC estimates. It is therefore important to check whether the orbit quality is the same in these two cases compared to e.g. solution type C. Figure 6 summarizes the rms errors obtained by fitting a 7-days arc through the 7 daily solutions of GPS week 765 using the radiation pressure model described in (Beutler et al., 1994).

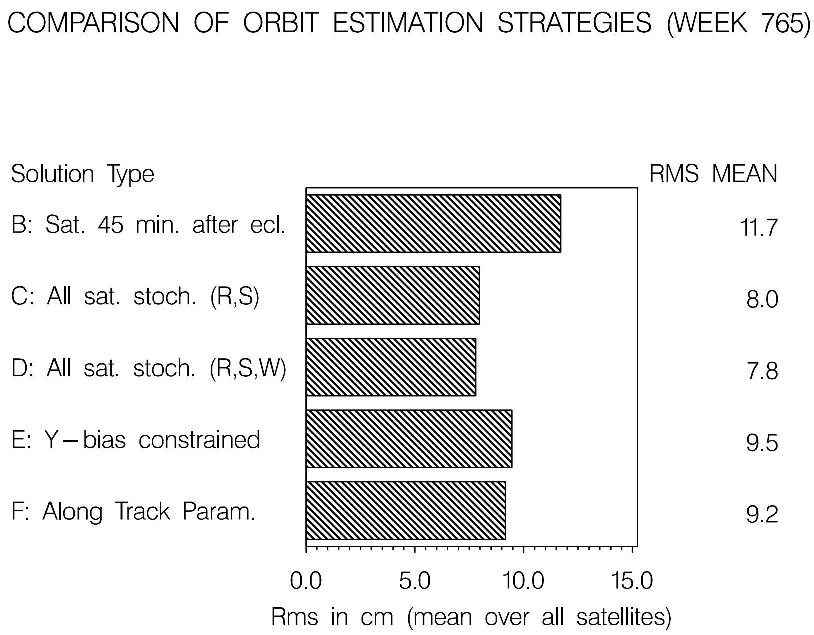

Figure 6. Orbit quality assessment by fitting a 7-days arc through the 7 daily solutions

The small degradation of the orbit quality for solutions E and F with respect to solutions types C and D indicates that there remain some unmodeled biases in the orbits if the y-biases are heavily constrained or not estimated at all. The estimation of a constant along-track acceleration is *not* sufficient to get down to the consistency level obtained with strategies C and D. It is interesting to note that the estimation of pseudo-stochastic pulses in the $\boldsymbol{W}$ direction does not significantly change the UT1-UTC behaviour.

SUMMARY AND CONCLUSIONS

In the first part of this article we developed the perturbations due to the radiation pressure parameters p_0 and p_2 using special perturbation theory. We showed that the nature for these perturbations is quite different for the cases when the sun is in the zenith of the orbital plane resp. lies in the orbital plane. We have shown in particular that a net rotation of the orbital plane around the axis pointing to the celestial pole will result due to a constant y-bias p_2 in the case where the sun lies in the orbital plane. Such rotations are *potentially dangerous* because they are very closely correlated with the estimates for the *drifts in UT1-UTC*.

We found evidence that our theoretical considerations are relevant in practice: Figures 4 and 5 underline this statement. Since June 1995 solution of Type *C* is the official solution of the *CODE Analysis Center*. We are in the process of studying solutions of type **E** in order to reduce the bias in our *UT1-UTC drift* estimates. Our concern at present is a degradation of the orbit quality as seen in Figure 6.

An alternative which is believed to be superior consists of removing the Rock models completely and to replace them by the new radiation pressure model developed in (Beutler et al., 1994). An analysis of the kind presented in this paper will have to be performed for this new model, too.

We should state in conclusion, however, that length-of-day estimates made by GPS will always suffer from potential correlations with orbit parameters which have to be estimated. This situation could be changed radically if direction observations of highest quality (of the order of a few mas) were available for GPS satellites.

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