

Correlation Transmission Line Matrix (CTLM) Modeling of Stochastic Electromagnetic Fields

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Abstract—In this work we introduce the Correlation Transmission Line Matrix (CTLM) method for time-domain computation of the auto- and cross correlation functions (ACFs and CCFs) of stationary stochastic electromagnetic fields. These ACFs and CCFs are computed from the Johns matrices, i.e. the discrete-time TLM Green’s functions and are directly related to the EMI power spectra.

I. INTRODUCTION

Future computing and communication systems will exhibit high data bandwidth demands of up to 100 Gb/s [1]. On the one hand, switching operations in such broadband circuits will generate radiated electromagnetic interference (EMI), and on the other hand, due to the high bandwidth and the low power levels these circuits also will be very sensitive to EMI. Therefore coexistence of systems with ubiquitous electromagnetic noise and reliable operation requires a careful circuit and system design accounting for signal integrity (SI).

Advanced computer aided design techniques accounting for electromagnetic compatibility (EMC) and SI will be crucial for reducing time-to-market. Integrated circuits are main sources of radiated EMI in electronic circuits and systems. Therefore the EMI radiated from the integrated circuits has to be assessed [2]–[4]. Also interconnects in printed circuit boards can be sources of radiated EMI [5].

In a stochastic EM field numerical amplitudes cannot be specified for the field values. A complete description of the stochastic EM field with Gaussian amplitude probability distribution can be given by specifying all auto correlation functions of the field amplitudes and the cross correlation functions of each pair of field variables [6]. The auto and cross correlation functions of the field variables in the observation points OP_1 and OP_2 in Fig. 1 can be computed if the auto and cross correlation functions of the sources $S_1 \dots S_5$ are known.

The transmission line matrix (TLM) method is an efficient time- and space discrete numerical method for modeling of complex electromagnetic structures [7], [8]. In this work, we introduce the Correlation Transmission Line Matrix (CTLM) method for time-domain computation of the auto- and cross correlation functions (ACFs and CCFs) of stationary stochastic electromagnetic fields. These ACFs and CCFs are computed from the Johns matrices, i.e. the discrete-time TLM Green’s functions and are directly related to the EMI power spectra. By convolution of the Johns matrices second order discrete Green’s functions - relating the correlation functions of pairs

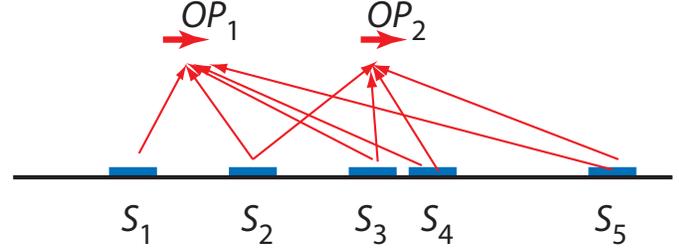


Fig. 1. Stochastic sources $S_1 \dots S_5$ and observation points OP_1 and OP_2 .

of observation points to the correlation functions of all pairs of source points - are introduced.

II. STOCHASTIC EM FIELDS

The stochastic electric field with the time-windowed amplitude spectrum $\mathbf{E}_T(\mathbf{x}_a, \omega)$ is represented by the dyadic [6]

$$\underline{\underline{\Gamma}}_E(\mathbf{x}_a, \mathbf{x}_b, \omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \langle \mathbf{E}_T(\mathbf{x}_a, \omega) \mathbf{E}_T^\dagger(\mathbf{x}_b, \omega) \rangle, \quad (1)$$

where the subscript T denotes the amplitude spectrum of the field, time-windowed by a rectangular window covering the time interval $[-T, T]$, and the excitation current density with the time-windowed amplitude spectrum $\mathbf{J}_T(\mathbf{x}_a, \omega)$ is represented by the dyadic

$$\underline{\underline{\Gamma}}_J(\mathbf{x}_a, \mathbf{x}_b, \omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \langle \mathbf{J}_T(\mathbf{x}_a, \omega) \mathbf{J}_T^\dagger(\mathbf{x}_b, \omega) \rangle. \quad (2)$$

The excited electric field $\mathbf{E}(\mathbf{x}, \omega)$ is related to the source current distribution $\mathbf{J}(\mathbf{x}', \omega)$ by

$$\mathbf{E}(\mathbf{x}, \omega) = \int_V \mathbf{G}_{EJ}(\mathbf{x} - \mathbf{x}', \omega) \mathbf{J}(\mathbf{x}', \omega) d^3x', \quad (3)$$

where $\mathbf{G}_{EJ}(\mathbf{x} - \mathbf{x}', \omega)$ is the Green’s dyadic relating the excited electric field $\mathbf{E}(\mathbf{x}, \omega)$ to $\mathbf{J}(\mathbf{x}, \omega)$ and the integration is extended over the whole volume V where $\mathbf{J}(\mathbf{x}, \omega)$ is non-vanishing [6], [9]. From (3), (2), and (1) we obtain

$$\begin{aligned} \underline{\underline{\Gamma}}_E(\mathbf{x}_a, \mathbf{x}_b, \omega) &= \iint_V \mathbf{G}_{EJ}(\mathbf{x}_a - \mathbf{x}'_a) \\ &\times \underline{\underline{\Gamma}}_J(\mathbf{x}'_a, \mathbf{x}'_b, \omega) \mathbf{G}_{EJ}^\dagger(\mathbf{x}_b - \mathbf{x}'_b) d^3x'_a d^3x'_b. \end{aligned} \quad (4)$$

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III. DIGITAL SIGNAL PROCESSING

A. Time-Discrete Signals

The discrete-time signal function $\mathbf{s}[n]$ is related to the continuous-time signal function $s(t)$ of length T_0 by

$$\mathbf{s}[n] = s(n\Delta t), \quad \text{with } T_0 = N\Delta t, \quad (5)$$

where Δt is the sampling interval. If a real-valued discrete time sequence $x_j[n]$ is applied to a linear time-invariant system with the impulse response $h_{ij}[n]$, the output sequence $y_i[n]$ is obtained by discrete convolution of the input signal $x_j[n]$ with the impulse response $h_{ij}[n]$ as [10, p. 64]

$$y_i[n] = h_{ij}[n] * x_j[n] \equiv \sum_{m=-\infty}^{\infty} h_{ij}[n-m]x_j[m], \quad (6)$$

where the symbol $*$ denotes the convolution operation.

B. Discrete-Time Correlation Functions

The discrete-time correlation function $\mathbf{c}_{ij}[n, n+m]$ of two real-valued discrete time sequences $x_i[n]$ and $x_j[n]$ is defined as [10, p. 65]

$$\mathbf{c}_{ij}^x[n, n+m] = \langle x_i[n]x_j[n+m] \rangle, \quad (7)$$

where $\langle \cdot \rangle$ denotes the ensemble average. If $\mathbf{s}_i[n]$ and $\mathbf{s}_j[n]$ are stationary ergodic processes, $\mathbf{c}_{ij}[n, n+m]$ is independent from n and the ensemble average is identical with the time average and we can write

$$\begin{aligned} \mathbf{c}_{ij}^x[m] &= \langle x_i[n]x_j[n+m] \rangle \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N x_1[n]x_2[n+m]. \end{aligned} \quad (8)$$

From (6) and (7) we obtain

$$\begin{aligned} \langle y_p[n]y_q[n+m] \rangle &= \\ &= \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} h_{pr}[k] \langle x_r[n-k]x_s[n+m-l] \rangle h_{qs}[l], \end{aligned} \quad (9)$$

$$\mathbf{c}_{pq}^y[m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} h_{pr}[k] \mathbf{c}_{rs}^x[m+k-l] h_{qs}[l]. \quad (10)$$

We define the *correlation transfer function*

$$k_{pq,rs}^y[l] = \sum_{k=-\infty}^{\infty} h_{pr}[k] h_{qs}[l+k] \quad (11)$$

and can write (10) as

$$\mathbf{c}_{pq}^y[m] = \sum_{l=-\infty}^{\infty} k_{pq,rs}^y[l] \mathbf{c}_{rs}^x[m-l]. \quad (12)$$

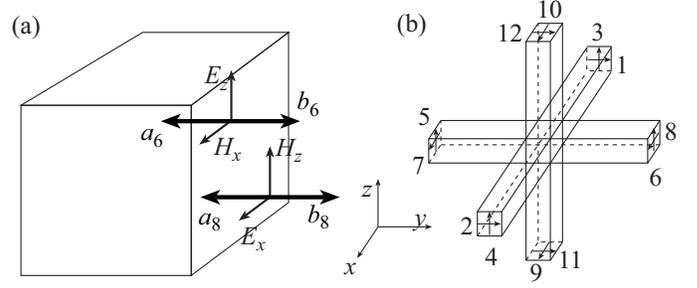


Fig. 2. Schematic of the TLM cell: a) Space cell with samples of the tangential electric and magnetic field values and wave pulse amplitudes, b) TLM node.

IV. TLM - A DISCRETE SCHEME OF ELECTROMAGNETISM

In the TLM-method, the electromagnetic field is modeled by wave pulses propagating on a Cartesian mesh of transmission lines [11]–[13]. In 3D-TLM with symmetric condensed node, the mesh node is modeled by a twelve-port with the scattering matrix. Figure 2a) exemplifies the assignment of incident and scattered wave amplitudes to the electromagnetic field amplitudes, and Fig. 2b) shows the schematic representation of a symmetric condensed TLM node. To account for the two transverse polarizations, a pair of transmission lines is assigned to every branch of the mesh. In a compact formulation of the TLM scheme we summarize all $12N$ incident wave pulses in the vector $\mathbf{a}[k]$ and all $12N$ scattered wave pulses in the vector $\mathbf{b}[k]$. The argument k enumerates the discrete time step. We can formulate the TLM scheme in the compact Hilbert space notation [14]–[16]:

$$\mathbf{b}[k+1] = \mathbf{S} \mathbf{a}[k], \quad (13a)$$

$$\mathbf{a}[k] = \mathbf{\Gamma} \mathbf{b}[k], \quad (13b)$$

where the scattering matrix \mathbf{S} describes the instantaneous scattering of the wave pulses in the TLM node and $\mathbf{\Gamma}$ describes the connection of the TLM nodes with the adjacent TLM nodes. The TLM scheme as formulated in (13a) and (13b) is a compact representation of discrete electrodynamics.

V. DISCRETE TLM GREEN'S FUNCTIONS

The response to a wave pulse incident on the boundary of a certain spatial domain may be represented by discrete TLM Green's functions. On the other hand, the response to a localized electromagnetic excitation at the boundary of a certain spatial domain may be calculated directly from Maxwell's equations and be represented by analytic TLM Green's functions. For low frequencies and small wave numbers, the analytic TLM Green's functions coincide with the discrete TLM Green's functions [14], [17]–[19]. Applying the analytic TLM Green's functions in the absorbing boundary condition at the boundary to the open half-space reduces the computational effort considerably when compared with the application of the discrete TLM Green's functions [20].

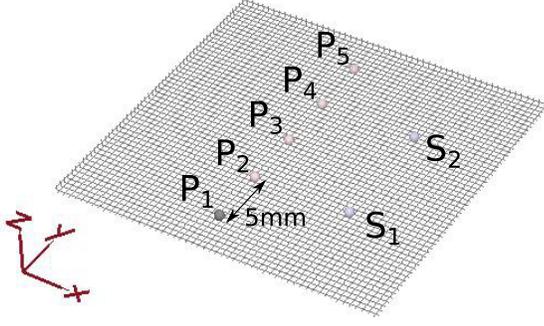


Fig. 3. Arrangement of source and observation points.

The discrete Green's function for TLM can be written as $G[n_i, k; n_j, k']$ and relates the wave pulses $a[n_i, k']$ incident on boundary port n_i and time k' to the wave pulses $b[n_j, k]$ scattered from boundary port n_j and time k [18], [21]. We can write

$$b_i[k] = \sum_{n_j \in B} \sum_{k'=-\infty}^{\infty} G_{i,j}[k-k'] a_j[k'], \quad (14)$$

where B is a set $\{n_1, n_2, \dots, n_N\}$ of N boundary nodes. For stationary stochastic electromagnetic fields we can introduce the following auto- and cross correlation functions of the wave amplitudes:

$$c_{ij}^a[m] = \langle a_i[n] a_j[n+m] \rangle, \quad (15a)$$

$$c_{ij}^b[m] = \langle b_i[n] b_j[n+m] \rangle. \quad (15b)$$

We introduce the *Correlation Green's Function* (CGF) $K_{ij;pq}[k]$ for the TLM wave amplitude correlation functions

$$K_{ij;pq}[k] = \sum_{l=-\infty}^{\infty} G_{i,p}[l] G_{j,q}[l+k]. \quad (16)$$

With (15a), (15b), and (16), we obtain in analogy to (10) the relation

$$c_{ij}^b[m] = \sum_{n_r, n_s \in B} \sum_{l=-\infty}^{\infty} K_{ij;rs}[l] c_{rs}^a[m-l], \quad (17)$$

relating the auto- and cross correlation functions $c_{ij}^b[m]$ of the wave amplitudes scattered from the boundary to the auto- and cross correlation functions $c_{rs}^a[m]$ incident to the boundary.

VI. NUMERICAL EXAMPLE

In this numerical two-dimensional example, correlation data according to (17) are obtained for five observation points receiving EMI from two sources. This arrangement is depicted in Fig. 3. Impulse responses at the observation points P_1, \dots, P_5 are computed for excitation at the source points S_1 and S_2 using the TLM-based MEFiSto electromagnetic full wave solver. The impulse excitation is band limited to avoid spurious solutions. Three cases are considered: correlated in-phase, correlated antiphase, and uncorrelated sources. The computed autocorrelation functions at the observation points,

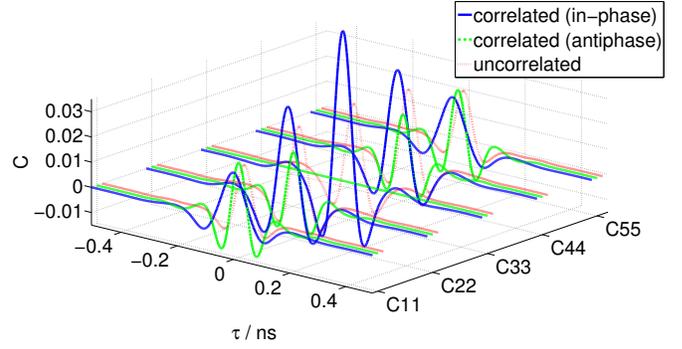


Fig. 4. Time domain autocorrelations of the observation points for a two-source excitation with correlated in-phase, correlated antiphase, and uncorrelated sources.

also a measure for the power density at the respective points, are graphed in Fig. 4. At the central point P_3 , we obtain a power maximum for correlated in-phase sources and zero power for correlated antiphase sources, and pronounced directivity patterns for either case. For the uncorrelated case, this pattern is vanishing. Arbitrary degrees of source correlations may be considered with this methodology. The information obtained in this manner for the propagation of the correlation information are available over a broad bandwidth.

VII. CONCLUSION

In this work we introduced and applied the Correlation Transmission Line Matrix (CTLTM) method for direct computation of the auto and cross correlation functions of stationary stochastic electromagnetic fields. We used *Correlation Green's Functions* (CGFs) to compute these correlations and gave a numerical example to demonstrate this method. The computation is time-efficient since the correlation depends only on the time delay of the correlation function and not on the time.

VIII. ACKNOWLEDGMENT

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