Separation Principle in Event-triggered Interconnected Networked Control Systems


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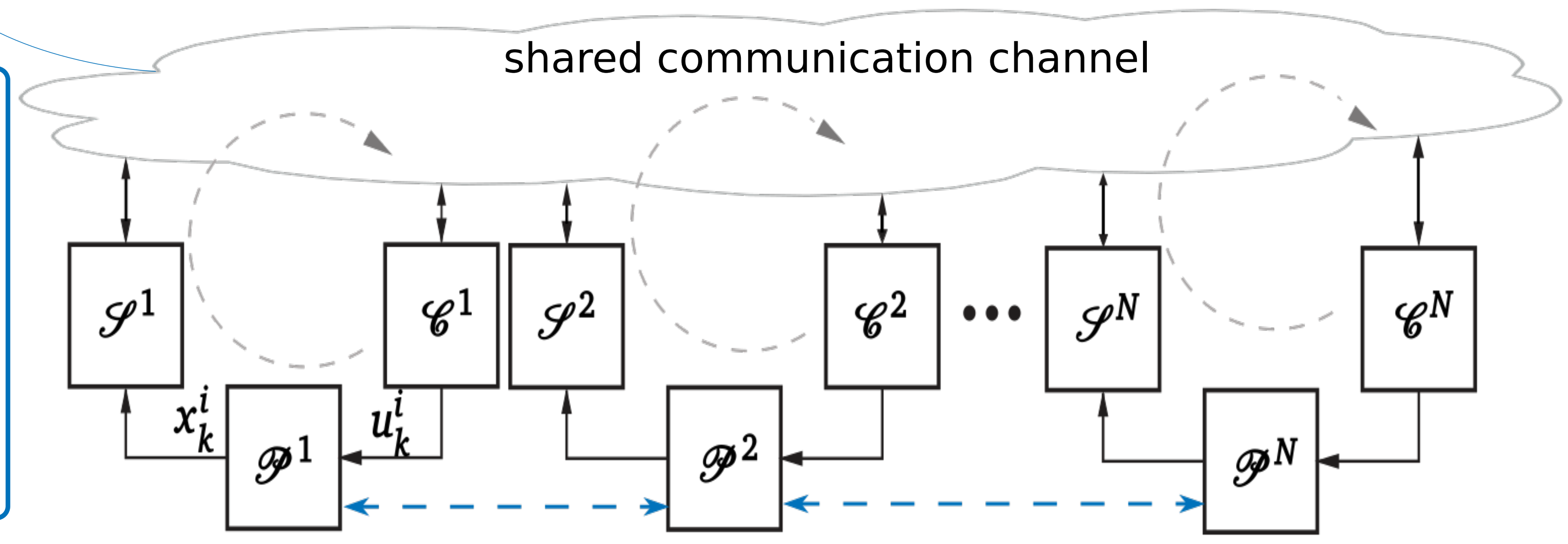
Motivation

The design of Large-scale Networked Control Systems needs to consider some important aspects, e.g:

- Efficient use of resources (eg. communication constraints)
- Maximize control performance
- Distributed information structure



System Model

Consider the physically interconnected multidimensional system

$$x_{k+1}^i = A_i x_k^i + B_i u_k^i + \sum_{j \in \mathcal{N}_i} A_{ij} x_k^j + w_k^i$$

$\mathcal{N}_i$ represents the direct neighbors of *i*-th node

$w_k^i \sim \mathcal{N}(0, \Sigma_w^i)$ i.i.d. and independent of initial state x_0^i

Event-trigger & Network manager

$$\delta_k^i = \begin{cases} 1 & \text{try to update } x_k^i \\ 0 & \text{use estimate} \end{cases} \qquad q_k^i = \begin{cases} 1 & x_k^i \text{ sent through the channel} \\ 0 & x_k^i \text{ blocked} \end{cases}$$

Information structure

$$z_k^i = \begin{cases} x_k^i & \delta_k^i = 1 \wedge q_k^i = 1, \\ \emptyset & \text{otherwise.} \end{cases}$$

At every time-step the information available at subsystem *i*

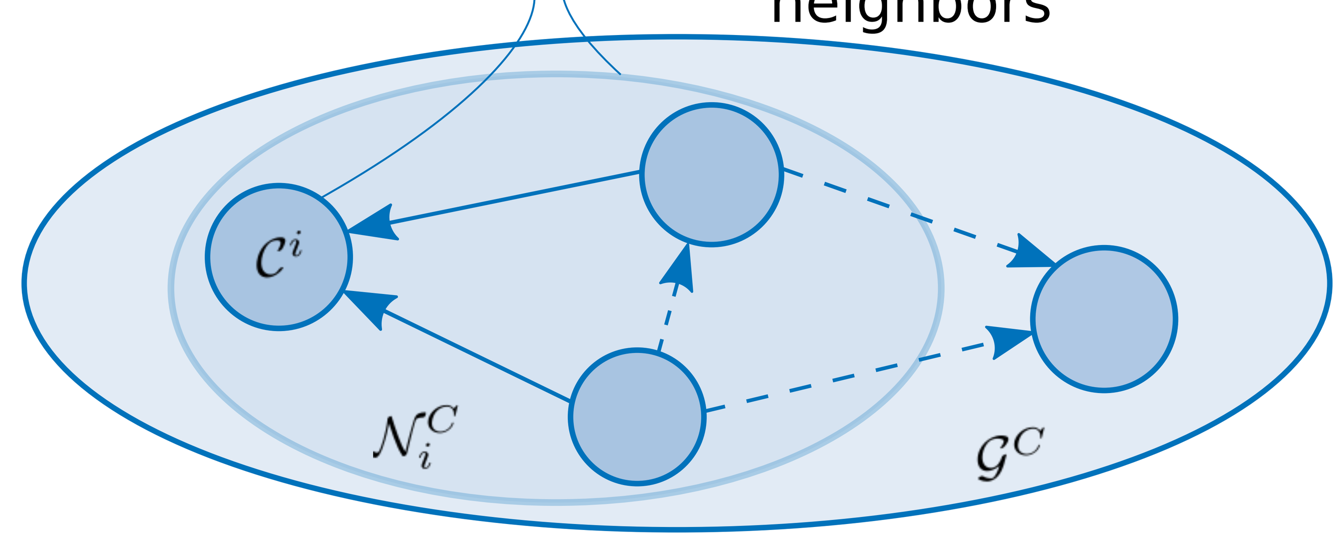
$$\mathcal{I}_k^i = \mathcal{I}_{k-1}^i \cup \{z_k^i\} \cup \{z_k^j\}, \quad \forall j \in \mathcal{N}_i^C$$

Objective

minimize $\quad \frac{1}{N}\sum_{k=1}^{N} J_i$

communication constraints $\quad \sum_{i=1}^{N} \delta_k^i \leq c$

local control objective $\quad J_i = \frac{1}{T}\mathrm{E}\left[\sum_{k=0}^{T-1} x_k^{i,\top} Q_i x_k^i + u_k^{i,\top} R_i u_k^i\right]$



Separation Principle

The control is said to have a ***dual effect*** when, in addition to its effect on the state of the system, it affects the uncertainty of the system's state

$$x_k^i - \mathrm{E}[x_k^i | \mathcal{I}_k^i] = M(u_k^i)$$

The ***separation property*** holds if the closed-loop optimal control depends on the data only through its estimates

$u_k^{i*} = \gamma_k^i(x_1, \dots x_N)$ deterministic

$u_k^{i*} = \gamma_k^i(\mathrm{E}[x_k^1|\mathcal{I}_k^1], \dots \mathrm{E}[x_k^N|\mathcal{I}_k^N])$ stochastic case

Separation imply independent design of controller and estimator → *tractable* problem.

Result

$\mathcal{G}$ DAG physical interconnection

$\mathcal{G}^C$ control network

if $(i \to j) \in \mathcal{G}$ and $(j \to i) \notin \mathcal{G}^C$ then

$$x_k^i - \mathrm{E}[x_k^i | \mathcal{I}_k^i] = M_k(x_0^i, W_i^{k-1})$$

Stability

uniform network manager

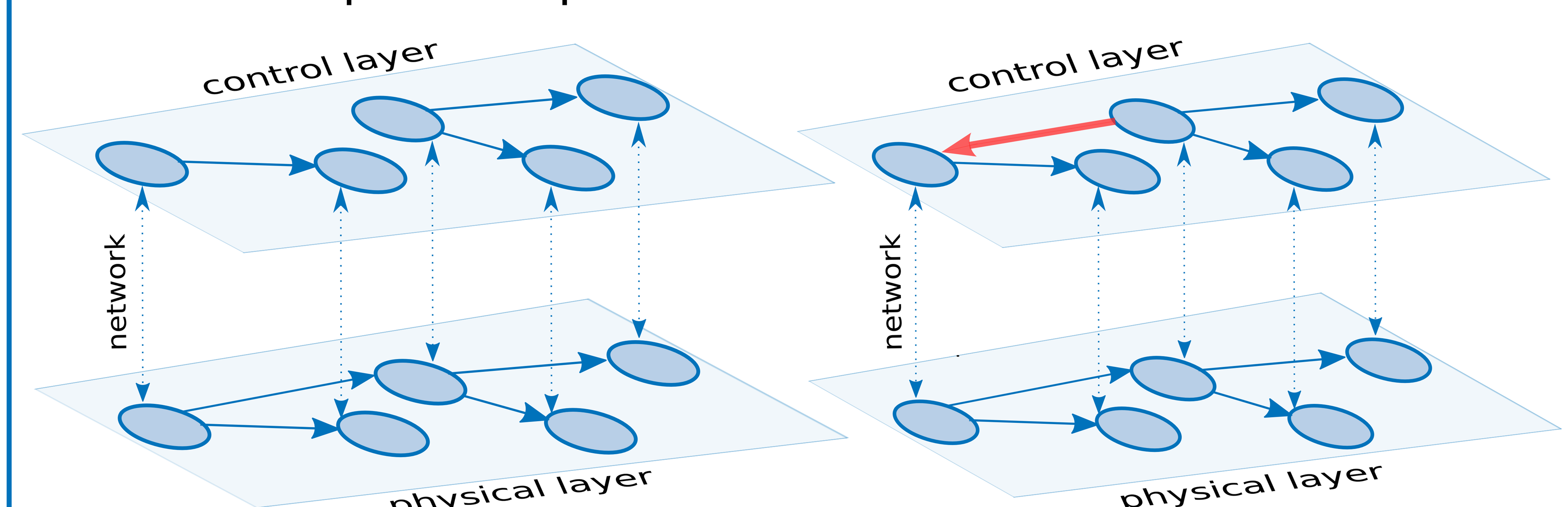
$\frac{c}{N} > 1 - \frac{1}{\|A^i\|_2^2}, \forall i$

$\delta_k^i = 1$ if $\|e_k^i\| > M^i$

→ *ergodicity*

Example

With an appropriate design of the control graph we can prove separation



separation — no separation

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References

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