Discrete Signaling for Non-Coherent, Single-Antenna, Rayleigh Block-Fading Channels

Marcin Pikus[12], Gerhard Krammer[2], and Georg Böcherer[2]

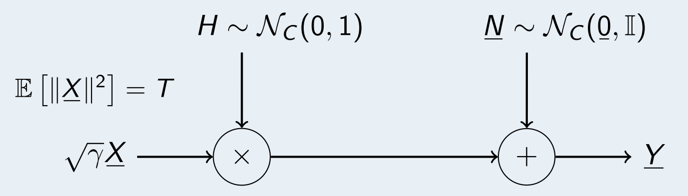
[1]Huawei Technologies Duesseldorf GmbH, Germany [2]Institute for Communications Engineering, Technical University of Munich, Germany

Rayleigh Block Fading Channel (RBFC)

- Used in modeling of wireless channels — it captures the uncertainty of wireless channels and fadings correlation-in-time

---

- The fading coefficient H is constant within a block of T symbols and changes independently between blocks
- The parameter T represents the channel coherence time
- The fading coefficient H is unknown to the transmitter and receiver
- The receiver has to estimate the channel state and the data from the received symbols



$$[Y_1,\dots,Y_T]^\dagger = H\cdot\sqrt{\gamma}\cdot[X_1,\dots,X_T]^\dagger + [N_1,\dots,N_T]^\dagger \quad (1)$$

$$\underline{Y} = H\cdot\sqrt{\gamma}\underline{X} + \underline{N} \quad (2)$$

Signaling in the Literature

CSI Capacity When the receiver **knows** H, Gaussian IID input signal $\underline{X} \sim \mathcal{N}_{\mathbb{C}}(\underline{0},\mathbb{I})$ achieves the capacity. This rate upper-bounds all non-coherent rates.

Gaussian IID When the receiver **does not know** H, Gaussian IID input signal $\underline{X} \sim \mathcal{N}_{\mathbb{C}}(\underline{0},\mathbb{I})$ in not capacity-achieving. However it performs well for large T [1].

Pilot schemes The transmitter inserts a fixed pilot symbol inside each fading block. At the receiver the pilot symbol is used to estimate the fading H and then the estimate $\hat{H}$ is used to decode the data [2].

USTM Unitary Space-Time Modulation transmits in each fading block a vector $\underline{\Phi}$ uniformly distributed on the T-dimensional complex sphere with radius $\sqrt{T}$ [3].

Experimental Signaling

Discrete Product Form is the proposed discrete signaling scheme inspired by the capacity achieving distribution for RBFC [4].

On-Off USTM is an extension of the USTM. The scheme transmits either a vector $\underline{\Phi}$ as in USTM or a vector of 0's. We optimize the probability P_0 of transmitting the $\underline{0}$-symbol. This scheme is an isotropicaly (continuous) counterpart of DPF.

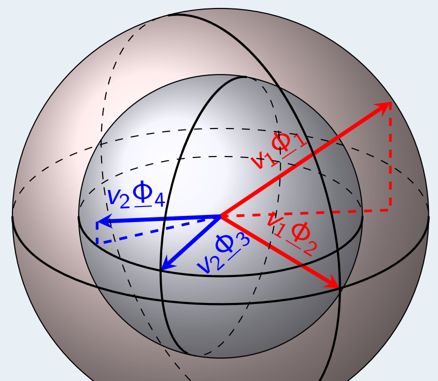
On-Off Gaussian is an extension of Gaussian IID signaling. The scheme transmits either a vector of Gaussian IID RVs or a vector of 0's. We optimize the probability P_0 of transmitting the $\underline{0}$-symbol.

The Capacity-Achieving Coding Scheme

The capacity-achieving signaling over non-coherent RBFC (also called **product form**) reads as

$$\underline{X} = V\underline{\Phi} \quad (3)$$

- V and $\underline{\Phi}$ are independent RVs
- $\underline{\Phi}$ is uniformly distributed on the complex T-dimensional sphere with radius $\sqrt{T}$
- V is discrete real non-negative RV



Discretization

1. Choose spheres radius, i.e., choose RV V

$$V = \begin{cases} 0, & \text{with probability } 1-P_1 \\ v_1 = \sqrt{\frac{1}{P_1}}, & \text{with probability } P_1 \end{cases} \quad (4)$$

2. Sample the spheres by choosing the following points

$$\underline{\Theta} = [\Theta_1,\dots,\Theta_T] \quad (5)$$

where Θ_i are IID uniformly distributed RVs on the QPSK alphabet $\mathcal{A}_4 = \{1,-1,j,-j\}$.

We get the **Discrete Product Form** scheme

$$\underline{X} = \begin{cases} \underline{0}, & \text{with probability } 1-P_1 \\ \sqrt{\frac{1}{P_1}}\underline{\Theta} & \text{with probability } P_1, \end{cases} \quad (6)$$

and P_1 is chosen to maximize the mutual information.

Computing the Mutual Information

- The following expansion will be used to compute the mutual information

$$I(\underline{Y};\underline{X}) = h(\underline{Y}) - h(\underline{Y}|\underline{X}) \quad (7)$$

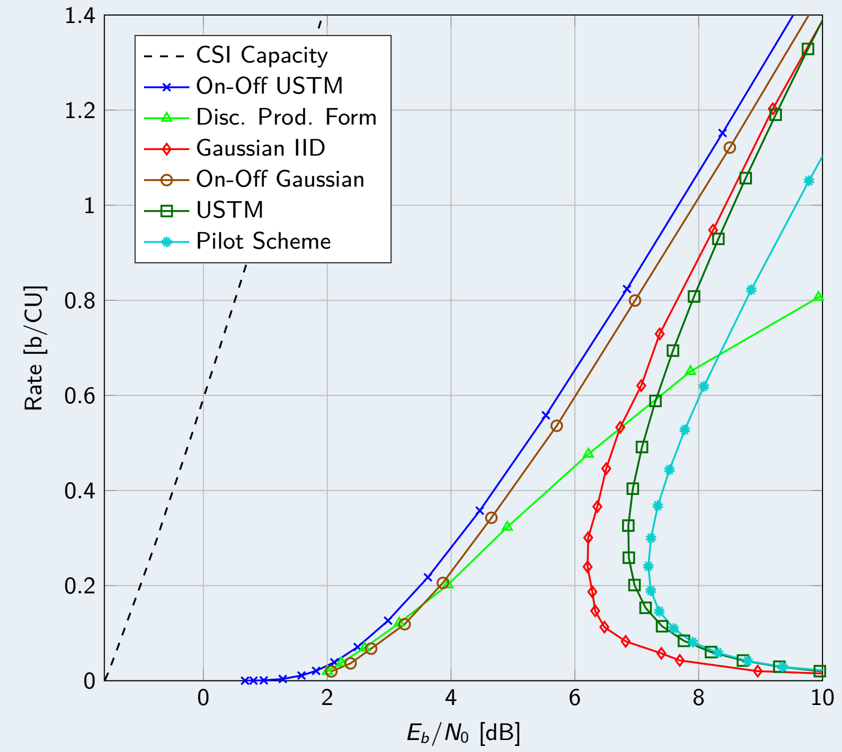
- The RV $\underline{Y}|\{\underline{X}=\underline{x}\}$ is a zero-mean circular-symmetric Gaussian RV. The covariance matrix equals to

$$\mathbb{E}\left[\underline{Y}\underline{Y}^\dagger \middle| \underline{X}=\underline{x}\right] = \mathbb{I}_{n_B} + \gamma\underline{x}\underline{x}^\dagger \quad (8)$$

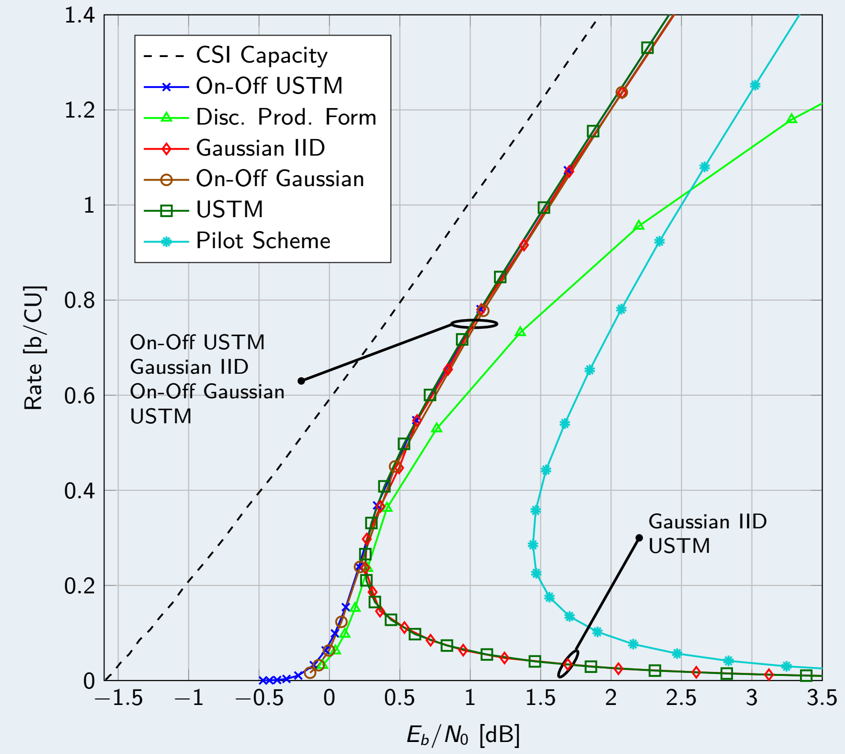
- Using the log-det formula, the entropy $h(\underline{Y}|\underline{X})$ is

$$h(\underline{Y}|\underline{X}) = \mathbb{E}_{\underline{X}}\left[h(\underline{Y}|\underline{X}=\underline{X})\right] = T\log(\pi e) + P_1\log\left(1+\gamma\frac{T}{P_1}\right).$$

Results: Rate vs Eb/N0



Information rates for block length $T=2$.



Information rates for block length $T=50$.

Computing $h(\underline{Y})$

- The following formula can be used to compute $h(\underline{Y})$

$$h(\underline{Y}) = \mathbb{E}_{\underline{Y}}[-\log p(\underline{Y})]. \quad (9)$$

- The knowledge of $p(\underline{y})$ is needed to evaluate (9). However, a direct computation fails due to exponential complexity growth in block length T

$$p(\underline{y}) = \sum_{\underline{x}} P(\underline{x})p(\underline{y}|\underline{x}) \quad (10)$$

- Instead, we use the fact that $\underline{Y}$ conditioned on V and H is a vector of IID RVs. First expand

$$p(\underline{y}) = \Pr(V=0)\underbrace{p(\underline{y}|V=0)}_{\sim\mathcal{N}_{\mathbb{C}}(\underline{0},\mathbb{I})} + \Pr(V=v_1)p(\underline{y}|V=v_1) \quad (11)$$

- Then the second term

$$\begin{aligned} p(\underline{y}|v_1) &= \mathbb{E}_H\left[p(\underline{y}|v_1,H)\right] \\ &= \mathbb{E}_H\left[\prod_{i=1}^{T} p(y_i|v_1,H)\right] \\ &= \mathbb{E}_H\left[\prod_{i=1}^{T}\sum_{\theta_i\in\mathcal{A}_4} P(\theta_i)p(y_i|\theta_i,v_1,H)\right] \\ &= \mathbb{E}_H\left[\prod_{i=1}^{T}\sum_{\theta_i\in\mathcal{A}_4}\frac{1}{4}\cdot\frac{1}{\pi}e^{-|y_i-H\sqrt{\gamma}\theta_i v_1|^2}\right] \\ &= \mathbb{E}_H\left[f\left(\underline{y},v_1,H\right)\right] \end{aligned} \quad (12)$$

- The complexity of computing $f\left(\underline{y},v_1,H\right)$ grows linearly with respect to the block size T. To evaluate the expectation in (12) we apply MC averaging.

Discussion/Conclusions

Gap to continuous dist.: The proposed Discrete Product Form (light green) achieves similar performance to its isotropical counterpart (On-Off USTM, blue) up to 0.4 b/CU and saturates approx at 1 b/CU and 2 b/CU for block lengths $T=2$ and $T=50$, respectively.

Using the $\underline{0}$-symbol: Compare the performance of the corresponding schemes: USTM (dark green) and On-Off USTM (blue) or Gaussian IID (red) and On-Off Gaussian (brown). Introducing a non-zero probability mass at the $\underline{0}$-symbol significantly improves the performance. Also for higher SNR, especially with small T.

Using on-a-sphere signal: For $T=2$ observe the gap between On-Off Gaussian (brown) and On-Off USTM (blue). Both signals are on-off schemes so the gain is due to the on-a-sphere structure of the On-Off USTM during th on-cycle. For low rates, also Discrete Product Form (light green) is better than On-Off Gaussian for the same reason.

References

[1] F. Rusek, A. Lozano, and N. Jindal, "Mutual information of IID complex Gaussian signals on block Rayleigh-faded channels," *IEEE Trans. Inf. Theory*, vol. 58, no. 1, pp. 331–340, Jan. 2012.

[2] B. Hassibi and B. Hochwald, "How much training is needed in multiple-antenna wireless links?" *IEEE Trans. Inf. Theory*, vol. 49, no. 4, pp. 951–963, Apr. 2003.

[3] B. Hochwald and T. Marzetta, "Unitary space-time modulation for multiple-antenna communications in Rayleigh flat fading," *IEEE Trans. Inf. Theory*, vol. 46, no. 2, pp. 543–564, Mar. 2000.

[4] M. Pikus, G. Kramer, and G. Bocherer, "Discrete signaling for non-coherent, single-antenna, rayleigh block-fading channels," *IEEE Communications Letters*, vol. PP, no. 99, pp. 1–1, 2016.

Institute for Communications Engineering

HUAWEI

Unterstützt von / Supported by Alexander von Humboldt Stiftung/Foundation

Technische Universität München