Seitz, F.; Krügel, M.: Inverse model approach for vertical load deformations in consideration of crustal inhomogeneities.

In: Drewes, H. (eds.) Geodetic Reference Frames, IAG Symposia, Vol. 134, 23-29, Springer, ISBN (Print) 978-3-642-00859-7, ISBN (Online) 978-3-642-00860-3, ISSN 0939-9585, 2009.

DOI: 10.1007/978-3-642-00860-3_4

In: GEODETIC REFERENCE FRAMES
H. Drewes (Ed.), IAG Symposia, Vol. 134,
23-29, Springer, Berlin, 2009.

Inverse model approach for vertical load deformations in consideration of crustal inhomogeneities

Florian Seitz (1) and Manuela Krügel (2)

(1) Earth Oriented Space Science and Technology (ESPACE), Technische Universität München, Arcisstr. 21, D-80333 Munich, Germany

(2) Deutsches Geodätisches Forschungsinstitut (DGFI), Alfons-Goppel-Str. 11, D-80539 Munich, Germany

e-mail: seitz@dgfi.badw.de, kruegel@dgfi.badw.de

Abstract. Mass redistributions in various components of the Earth system exert time-variable surface loads on the solid Earth. The resulting variations of the Earth's geometry are reflected by vertical and horizontal displacements of geodetic markers. Usually the effect of loading on crustal deformation is computed by means of a weighting function which is based on site-independent load Love numbers (Green's function). But as the Earth's crust is composed of heterogeneous material, the adequateness of a site-independent approach deserves a review. We propose a procedure for the computation of vertical crustal deformations in which the Green's function is substituted by a site-dependent exponential function. Its parameters are estimated by means of least-squares adjustment using time series of globally distributed GPS sites. On the basis of the crustal model Crust2.0 regions are predefined for which identical parameters are determined. Pressure fields of atmosphere, continental hydrosphere and oceans are considered as forcing. In order to validate the numerical results, model time series from both the traditional and the site-dependent approach are compared with GPS observations. Explicit improvement is achieved in regions which are covered well with observations and feature strong pressure variability. However in regions like Africa and Antarctica parameter estimation is difficult due to the sparse distribution of GPS sites.

Keywords. Inverse model, vertical site displacements, surface loading, Green's function, GPS

1 Introduction

A multitude of geophysical processes in the Earth system entail continuous changes of the Earth's geometry, its rotation and its gravity field. Modern space geodetic techniques allow for the observation and documentation of these variations with an accuracy on the mm-level. In order to interpret the observations geophysically, the connection between specific signal components in the geodetic time series and individual processes has to be ascertained. Since the geodetic observations reflect the integral effect of all causative forces, a separation is in general not possible without independent model approaches. But vice versa theoretical models can be supported and improved by entering geodetic observations which in this way contribute to the gain of knowledge about interactions in the Earth system. Such inverse models which use time series of geodetic parameters as an input for the estimation of unknown model parameters are in contrast to forward models. The latter are geared towards a simulation of geodetic time series using predefined and (during the course of the model run) invariable numerical model parameters.

On seasonal to interannual time scales the largest effects on the geodetic parameters are due to the redistribution and motion of mass elements within the atmosphere and the oceans. Fluctuations of rotation and gravity field of the Earth which are caused by mass variations in these subsystems are widely known from analyses based on assimilating models and reanalysis data (e.g., Brzezinski et al. 2002, Chao and Au 1991, Chen et al. 2000, Gross et al 2003; Seitz 2005). Since the transfer functions between the mass variations and the consequent direct effects on Earth rotation and gravity field are reasonably well-known (Lambeck 1980), their contributions to the respective geodetic parameters can be assessed from suitable forward models.

However the dynamics of the Earth's crust and mantle as well as geophysical processes within the Earth's interior are far less understood. Mass redistributions on the Earth surface exert time-variable loads which lead to variations of the Earth's geometry. These are reflected by site displacements and cause mass variations within the solid Earth. The latter influence both the rotational dynamics and the gravity potential of the Earth. Consequently, expedient information about loading effects is required for an advanced interpretation of geodetic observations. The largest displacements emerge in continental regions where vertical deformations are up to a few centimetres. They have a significant effect on geodetic observations (Schuh et al. 2004). Horizontal displacements do not exceed several millimeters and are thus inferior (Rabbel and Zschau 1985, Sun et al. 1995). They shall not be discussed in the present paper.

Most approaches for the conversion of surface loads into deformations of the solid Earth follow a simple theory based on load Love numbers which are associated with a site-independent weighting function (Green's function) (Farrell 1972, Moritz and Mueller 1987). As the load Love numbers are global values, this procedure does not account for inhomogeneities of crustal material. However a distinct relation between regional characteristics of the crust and the Earth's reaction on surface loads might be expected. Consequently this approach appears to be in need of improvement. We discuss an alternative method in which the common Green's function is substituted by a site-dependent weighting function considering different densities of crustal material. In our inverse model approch unknown parameters of the site-dependent function are estimated by least-squares adjustment.

2 Green's function and site-dependent weighting function

Commonly the vertical displacement of the Earth's elastic body at a position $P(\varphi_P, \lambda_P)$ which is caused by surface loads $q(\varphi_Q, \lambda_Q)$ (given in units of [kg/m^2]) is estimated according to the well-known formulation (Moritz and Mueller 1987)

$$d_r(P) = \frac{R^3}{M} \iint_{\sigma_Q} q_Q \sum_{n=0}^{\infty} h'_n P_n(\cos\psi_{PQ})\, \mathrm{d}\sigma_Q \,. \qquad (1)$$

In this equation R stands for the mean radius of the (spherical) Earth, M for its total mass and h'_n denotes the degree n load Love numbers. The spherical distance between P and the location $Q(\varphi_Q, \lambda_Q)$ of an individual point mass is given by ψ_{PQ} which is the argument of the degree n Legendre-Polynomial $P_n(\cos\psi_{PQ})$. More compact Eq. (1) can be written as

$$d_r\,(P) = R^2 \iint_{\sigma_Q} q_Q G\,(\psi_{PQ})\; \mathrm{d}\sigma_Q \,, \qquad (2)$$

where the abbreviation

$$G\,(\psi_{PQ}) = \frac{R}{M} \sum_{n=0}^{\infty} h'_n P_n\,(\cos\psi_{PQ}) \qquad (3)$$

is the Green's function for the vertical displacement (Farrell 1972). Function $G(\psi_{PQ})$ works as a weighting operator which relates an individual (point) load to the associated deformation of the solid Earth according to the spherical distance. Figure 1 shows the Green's function for continental crust as computed from load Love numbers up to degree $n = 2000$ which are based on the Preliminary Reference Earth Model (PREM) (Dziewonski and Anderson 1991, Scherneck 1990). The strong variability of the curve reflects the truncation error.

Since the load Love numbers are global mean values, this weighting function is not capable

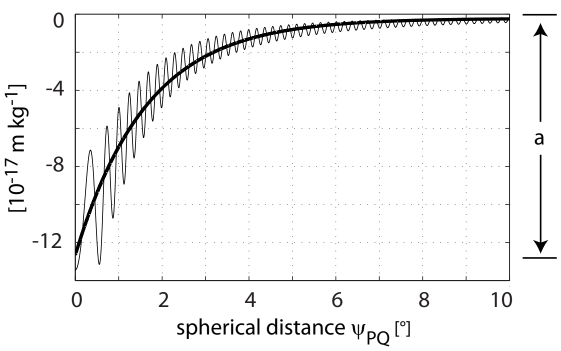

Fig 1. Green's function for continental crust based on PREM up to degree 2000 (thin line) and exponential function for the parameters $a = -12.5$ and $b = -35$ (thick line).

of regarding local crustal inhomogeneities in the model. In order to overcome this deficiency we propose a site-dependent weighting function in our study. The function $G(\psi_{PQ})$ displayed in Figure 1 is substituted by a simple exponential function of the general form

$$F(\psi_{PQ}) := 10^{-17} a\, e^{-b\, \psi_{PQ}}. \qquad (4)$$

The choice of different numerical values for the parameters a and b allows to account for regional discrepancies of crustal densities. The parameter a which has the unit [m/kg] provides a measure of the vertical deformation of a cell on the Earth's surface if loaded by a certain mass. The parameter b determines the decay of the curve, i.e., the effect of a load on neighboring cells as a function of their spherical distance. Figure 2 illustrates the principle of the approach.

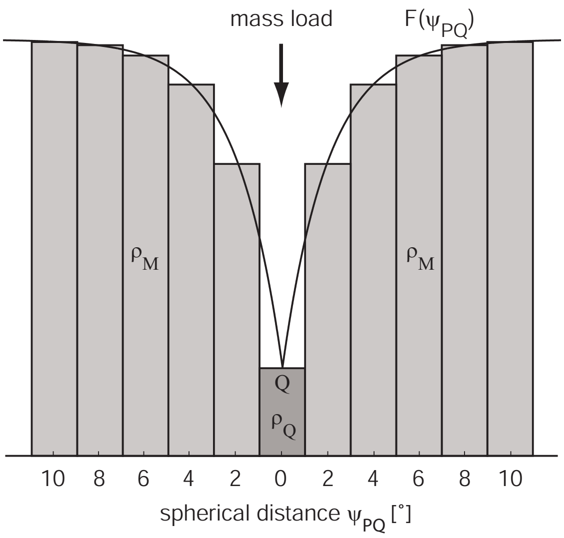

Fig 2. Principle of the site-dependent approach. Parameter a is related to the density ρ_Q of the loaded grid cell, parameter b is related to the mean density ρ_M of the surrounding cells.

In a first approximation parameter a shall be related to the density of the actually loaded grid cell whereas parameter b is associated with the (mean) density of surrounding cells. Consequently the two parameters are considered to be independent. The values for which the function $F(\psi_{PQ})$ fits best to the displayed Green's function are $a = -12.5$ and $b = -35$ (Figure 1, thick black line).

The numerical values of the parameters a and b are estimated for cells of 2° x 2° using geodetic observations of vertical site displacements from globally distributed GPS permanent stations. In order to reduce the amount of different parameters, cells have to be identified for which similar rheological properties can be assumed.

3 Classification of grid cells

Clusters of grid cells for which the same parameters are applied are determined on the basis of the model Crust2.0 (Bassin et al. 2000) which provides global information on crustal material, thickness and density for 2° x 2° blocks. Figure 3 displays the mean density of the crust composed of soft and hard sediments, upper, mean and lower mantle. Density values range from 2.60 to 2.95 g/cm^3. However cells with mean densities lower than 2.7 g/cm^3 appear exclusively in oceanic regions and at some places inside Antarctica. Since these areas are not covered by GPS stations well enough (see below), some simplifications are necessary. We assume that the entire sea floor consists of material with consistent density for which the mean value of the Pacific Ocean is applied. The same holds for Antarctica where the mean density of the whole continent is applied for all Antarctic cells. The largest density values appear in Africa which are not found in other regions of the world. Due to the adverse distribution of GPS sites no parameters can be estimated here, too. Similar to Antarctica the mean density of the continent is applied for

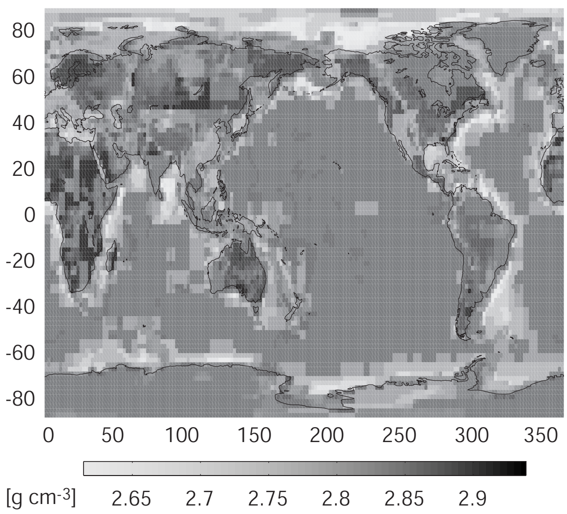

Fig 3. Map of the mean density of the crust composed of soft and hard sediments, upper, mean and lower mantle from the model Crust2.0.

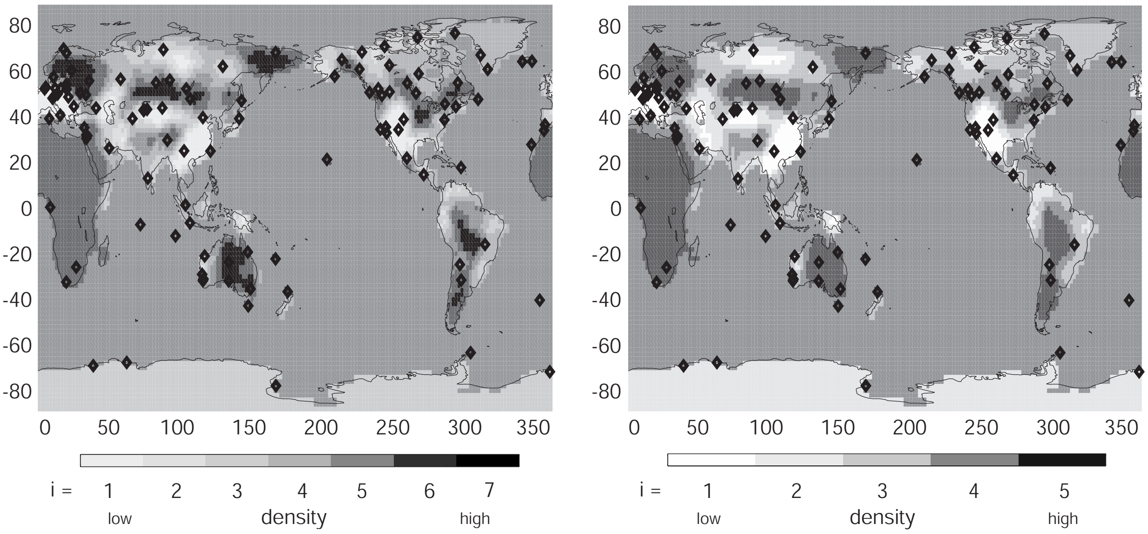

Fig 4. Spatial distribution of the parameters a_i (left) and b_i (right). Low indices correspond to regions with low densities (cf. Figure 3). Black diamonds indicate the locations of GPS sites used in this study.

the African cells. The remaining density values which range between 2.7 and 2.9 g/cm^3 are discretised in steps of 0.01 g/cm^3.

It is assumed that loads are effective on grid cells within a radius of 10°. In order to decide which parameter b_i is effective for a certain cell, the mean density values of the cells within a circle of 10° around this cell are computed. This leads to a smoothing of the density values which implies that there are less parameters b_i than a_i. First numerical experiments showed that each parameter a_i and b_i should be determined from at least 20 stations in order to minimise the influence of measurement errors on the parameter estimation. Taking everything into account, seven different parameters a_i and five different parameters b_i are applied. Their spatial distribution is displayed in Figure 4.

4 Least-squares adjustment of the parameters a_i and b_i

The numerical values of the parameters are estimated by least-squares adjustment. The observation equation reads

$$d_r\left(P\right) = \sum_{\substack{k=1\\ \psi_{PQ_k}<10^\circ}}^{N} 10^{-17} q_{Q_k} A_{Q_k} \left(a_{ik}\, e^{b_{ik}\psi_{PQ_k}}\right) \quad (5)$$

where A_{Q_k} is the area of the loaded grid cell k. Observations of vertical site displacements on the left hand side of the equation are taken from weekly solutions of the global GPS station network of the International GNSS Service (IGS). These station position time series are based on homogeneously combined data sets as they were prepared for the ITRF2005 computation. They are available since 1996. Up to now the IGS processing strategy does not account for non-tidal loading effects. Consequently the time series contain the variations due to atmospheric, hydrological and non-tidal oceanic mass redistributions. In order to ensure that the interpretability of the station position time series is not limited by slight discrepancies with respect to the geodetic datum of individual IGS solutions, the weekly solutions are transformed to a combined multi-year solution applying a seven parameter similarity-transformation (Meisel et al. 2005). The resulting station position time series are subsequently reduced by mean and trend (which is performed separately for individual sections if there are discontinuities in the time series). In order to include at least two complete annual cycles, only stations are regarded for which more than two years of observations are available. Altogether we consider 105 globally distributed GPS stations (Figure 4, black diamonds) with a total of 29500 observations in our study.

The model is forced by variations of surface mass loads which are exerted on the solid

Earth by atmosphere, continental hydrosphere and oceans. For the atmosphere and the oceans respective fields $q(\varphi_Q, \lambda_Q)$ are computed from a consistent combination of atmospheric surface pressure variations from the National Centers for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) (Kalnay et al. 1996) and oceanic bottom pressure variations from the constrained version kf049f of the global ocean circulation model ECCO (Fukumori 2002). Outputs of both models are provided in daily intervals, spatial resolutions are 2.5° x 2.5° for NCEP/NCAR (globally) and 1° x 1° for ECCO (between 70° N/S; densification of the grid around the equator). Since atmospheric pressure forcing is not taken into account by ECCO, an inverse barometric correction is applied to the NCEP/NCAR fields, i.e. air pressure is set to zero over the oceans. Pressure variations $p(\varphi_Q, \lambda_Q)$ [Pa] are converted into mass loads according to the equation $q(\varphi_Q, \lambda_Q)=p(\varphi_Q, \lambda_Q)/g$ where g is the gravitational acceleration. Variations of continental hydrology are taken from the Land Dynamics Model (LaD; version *Euphrates*) (Milly and Shmakin 2002). LaD data comprehends monthly values of global water and groundwater storage [mm] as well as snow loads per 1° x 1° grid cell. The forcing fields of all applied models are adapted spatially to the resolution of the 2° x 2° grid of Crust2.0 and temporally to the weekly resolution of the GPS observations.

5 Results

As approximate values for the unknowns we introduce the aforementioned values $a = -12.5$ and $b = -35$ which result from the fit of function $F(\psi_{PQ})$ to the above Green's function. After five iterations convergence is reached. The adjusted parameters are given together with their standard deviations in Table 1. The values agree quite well with the approximate values (especially a_1 to a_5 and b_1 to b_4) and in principle the results match the expectations: For cells with low densities the deformation is stronger and the function is steeper than for cells with higher densities. However this statement is not valid for all parameters (for example $a_3 < a_2$) and the values of a_7 and b_5 are not highly significant.

Figure 5 shows the model time series for three stations on different continents and the respective GPS observations. The dotted lines are the curves that follow from the Green's function approach applying identical forcing. Correlation coefficients between the results of the site-dependent approach and the GPS time series are also provided. The respective correlations between the Green's function approach and the observations are almost identical since the overall patterns of the model curves (e.g. spikes) are largey imposed by the applied geophysical forcing. The curves differ mainly with respect to their amplitudes for which the correlation coefficient is less sensitive.

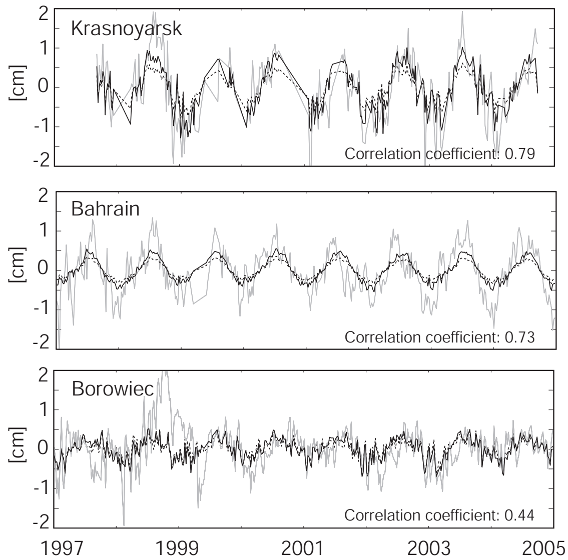

Fig 5. Model time series for three stations on different continents (black), respective GPS observations (grey) and corresponding correlation coefficients. Dotted lines are the results from the Green's function approach.

Table 1. Adjusted values and standard deviations of the parameters a_i and b_i.

	value	σ	density	
a1	-11.4	0.8	low	strong deformation
a2	-10.2	1.3		
a3	-17.0	0.9		
a4	-7.8	0.7		
a5	-8.5	0.7		
a6	-3.2	0.2		
a7	-0.3	0.2	high	slight deformation
b1	-37.4	3.0	low	steep function
b2	-30.3	3.1		
b3	-29.2	2.5		
b4	-27.6	2.4		
b5	-4.1	2.1	high	flat function

Analyses of the RMS differences reveal that in general the proposed approach is more effective in explaining the observations: For none of the stations in our study the agreement between observations and model results deteriorates when the site-dependent weighting functions are applied instead of the Green's function. The largest discrepancies between both methods (RMS differences up to 2 mm) are apparent in regions where high annual vertical displacements are observed, e.g., in Siberia, the arctic regions of North America and the Gulf region. But on the other hand there are regions where the improvement is marginal, e.g., Europe and SE-Asia.

6 Discussion

Compared to the Green's function approach significant improvement is achieved for some regions. But on the other hand the inverse model approach suffers some deficiencies which require critical discussion. Least-squares adjustment of the unknown model parameters improved the model in the sense of an optimal reproduction of the geodetic observations. However among the prerequisites for good results are adequate quality and global availability of both the geophysical forcing and the geodetic time series. Indeed the adverse distribution of the GPS sites is one of the crucial points. As mentioned above there are large regions for which either no parameters can be estimated (e.g., Africa) or the estimates are not significant because stations are far away from the respective cells or contain errors which are sometimes larger than the signal itself (e.g., the erroneous pattern in the time series of Borowiec around 1999, see Figure 5). Apart from (more or less randomly distributed) measurement errors, the observations feature systematic errors from applied reductions (e.g., troposphere correction) which could easily lead to a systematic falsification of the estimated parameters. In order to improve the data situation and densify the network, an integration of regional GPS networks is advisable.

Beside the problems on the observation side, there are some open questions regarding the approach itself. In this paper we propose a very simple model in order to deal with a low number of parameters. But further analysis is required on both the suitability of the functional model and the criteria for the clustering of cells. In particular the following points should be discussed:

- is it justified to compute the mean density of all layers in a cell or should the averaging be restricted to a certain number of top layers?
- how should the thickness of the crust be regarded?
- which other criteria besides the density should be considered (e.g., material, structure and age of crust; plate boundaries)?

Even though further investigation and refinement of the model is necessary, the study shows that a site-dependent approach for the computation of load deformations is advisable. The results reveal that not all cells should be treated equally. Some of the parameters depart significantly from the approximate values from the Green's function which entails (at least in some regions) a considerable improvement of the model results.

Acknowledgements

This project was supported by DFG grant DR 143/12-1. The authors thank Barbara Meisel (DGFI) for the preparation of the GPS time series. NCEP/NCAR reanalysis data was provided by the NOAA-CIRES Climate Diagnostics Center, ECCO data is a contribution of the Consortium for Estimating the Circulation and Climate of the Ocean funded by the National Oceanographic Partnership Program. We thank R. Gross and H. Schuh for their quick and helpful reviews of the manuscript.

References

Bassin C, Laske G, Masters G (2000): The current limits of resolution for surface wave tomography in North America. EOS Trans AGU 81: F897

Brzezinski A, Bizouard C, Petrov SD (2002): Influence of the atmosphere on earth rotation: What can be learned from the recent atmospheric angular momentum estimates?. Surveys in Geophysics 23: 33-69

Chao BF, Au AY (1991): Temporal variation of Earth's zonal gravitational field caused by atmospheric mass redistribution: 1980-1988. J Geophys Res 96(B4): 6569-6575

Chen JL, Wilson CR, Chao BF, et al. (2000): Hydrological and oceanic excitations to polar motion and length-of-day variations. Geophys J Int 141: 149-156

Dziewonski AM, Anderson DL (1981): Preliminary Reference Earth model (PREM). Phys. Earth Planet. Int. 25:297-356

Farrell W (1972): Deformation of the Earth by surface loads. Rev Geophys Space Phys 10: 761-797

Fukumori I (2002): A partitioned Kalman filter and smoother. Mon. Weather Rev. 130: 1370-1383

Gross RS, Fukimori I, Menemenlis D (2003): Atmospheric and oceanic excitation of the Earth's wobbles during 1980-2000. J Geophys Res 108: doi:10.1029/2002JB002143

Kalnay E, Kanamitsu M, Kistler R, et al. (1996): The NCEP/NCAR 40-Year Reanalysis Project. Bull. Amer. Meteor. Soc. 77: 437-471

Lambeck K (1980): The Earth's Variable Rotation: Geophysical Causes and Consequences. Cambridge University Press, New York

Meisel B, Angermann D, Krügel M, et al. (2005): Refined approaches for terrestrial reference frame computations. Advances in Space Research 36: 350-357

Moritz H, Mueller II (1987): Earth Rotation: Theory and Observation. Ungar Publishing Company, New York

Milly PC, Shmakin AB (2002): Global modeling of land water and energy balances. Part I: The land dynamics (LaD) model. Journal of Hydrometeorology 3(3): 283-299

Rabbel W, Zschau J (1985): Static deformations and gravity changes at the Earth's surface due to atmospheric loading. J Geophysics 56: 81-99

Schuh H, Estermann G, Crétaux JF, van Dam TM, Bergé-Nguyen M (2004): Investigation of hydrological and atmospheric loading by space geodetic techniques. In: Hwang C, Shum CK, Li JC (eds) Satellite Altimetry for Geodesy, Geophysics and Oceanography, IAG Symposia 126, Springer, Berlin, pp 123-132

Scherneck HG (1990): Loading Green's functions for a continental shield with a Q-structure for the mantle and density constraints from the geoid. Bulletin d'Information Marées Terrestres 108: 7757-7792

Seitz F (2005): Atmospheric and oceanic influences on polar motion - Numerical results from two independent model combinations. Artificial Satellites 40(3): 199-215

Sun HP, Ducarme B, Dehant V (1995): Effect of the atmospheric pressure on surface displacements. J Geodesy 70: 131-139