p(t)MOR and Applications for Moving Loads

Maria Cruz Varona*, Matthias Geuss, Boris Lohmann

Dynamic systems with time-varying parameters arise in numerous industrial applications, e.g. in structural dynamics or systems with moving loads. A spatial discretization of such systems often leads to high-dimensional linear parameter-varying models, which need to be reduced in order to enable a fast simulation. In the following we present time-varying parametric model order reduction (p(t)MOR) based on matrix interpolation and apply this novel framework to a system with moving load.

### Parametric Model Order Reduction

**High-dimensional parametric system:**

\[
E(p)x(t) = A(p)x(t) + B(p)u(t), \quad p \in \mathbb{D} \subset \mathbb{R}^d
\]

\[
y(t) = C(p)x(t)
\]

**Projective pMOR:**

Choose appropriate projection matrices \(V(p), W(p) \in \mathbb{R}^{n \times n}\) to approximate the state-vector by \(x(t) \approx V(p)x(t)\).

**Reduced Order Model:**

\[
E_r(p)\dot{\hat{x}}_r(t) = A_r\hat{x}_r(t) + B_ru(t), \quad \hat{x}_r(t) = C_rx(t)
\]

### Time-Dependent Parametric Model Order Reduction

**High-dimensional linear parameter-varying system (LPV):**

\[
E(p)x(t) = A(p)x(t) + B(p)u(t), \quad p(t) \in \mathbb{D} \subset \mathbb{R}^d
\]

\[
y(t) = C(p)x(t)
\]

**Projective p(t)MOR:**

Analogously, we aim to approximate the state-vector by \(x(t) \approx V(p(t))x(t)\).

**Reduced Order Model:**

\[
E_r(p(t))\dot{\hat{x}}_r(t) = A_r\hat{x}_r(t) + B_ru(t), \quad \hat{x}_r(t) = C_rx(t)
\]

### Application for Systems with Moving Loads

**Systems with Moving Loads:**

- position of the acting load varies with time
- varying load position can be regarded as a time-dependent parameter of the system
  - Linear parameter-varying system

**Considered example:**

- Timoshenko beam with moving load
- Finite Element Discretization
- LPV system with parameter-dependent input matrix \(B(p(t))\)

**Relevance of \(V(p(t))\):**

- Parameter-varying state transformation of the original model shows the importance of the consideration of \(V(p(t))\)

### Current Work and Outlook

**Model Reduction:**

- p(t)MOR by Matrix Interpolation applied
- Order of locally reduced systems: \(n = 20\)

**Further Study:**

- Interpretation of the new matrix \(\hat{A}_{new,r}\)
- Remedial actions against unstable interpolated systems
- Application of p(t)MOR to generalized linear parameter-varying systems

**References**
