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Computational Aspects of Risk-Based Inspection Planning

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Abstract: *The significant computational efforts required to compute risk-based inspection plans founded on the Bayesian decision theory has hindered their application in the past. In this article, a computationally efficient method for the calculation of risk-based inspection (RBI) plans is presented, which overcomes the problem through the use of a generic approach. After an introduction in RBI planning, focus is set on the computational aspects of the methodology. The derivation of inspection plans through interpolation in databases with predefined generic inspection plans is demonstrated and the accuracy of the methodology is investigated. Finally, an overview is given on some recent applications of the generic approach in practice, including the implementation in efficient software tools.*

1 INTRODUCTION

Engineering systems are ideally designed to ensure an economically efficient operation throughout the anticipated service life in compliance with given requirements and acceptance criteria. Such acceptance criteria are typically related to the safety of personnel and risk to the environment. Deterioration processes such as fatigue crack growth and corrosion will always be present to some degree, and depending on the adapted design philosophy in terms of degradation allowance and protective measures, the deterioration processes may reduce the performance of the system beyond what is acceptable. To ensure that the given acceptance criteria are fulfilled throughout the service life of the engineering systems it may thus be necessary to control the development of deterioration and, if required, to install corrective maintenance measures. In most practical applications, inspection is the most relevant and effective means of deterioration control.

During the last 15–20 years, reliability-based and risk-based approaches have been developed for the planning of inspections, for example, in Thoft-Christensen and Sørensen (1987), Fujita et al. (1989), Madsen et al. (1989), Sørensen et al. (1991), and Moan et al. (2000). These approaches are based on the Bayesian decision theory to minimize overall service life costs including direct and implied costs of failures, repairs, and inspections. In contrast to earlier approaches introduced in Yang and Trapp (1974, 1975), they are based on structural reliability analysis (SRA) for the evaluation of the probabilities and are thus much more flexible in regard to the applied limit state functions (LSFs) describing the deterioration and the inspection performance. However, the application of risk-based inspection (RBI) planning has been limited in the past due to the significant numerical effort required by these methods. This motivated the development of a generic approach to RBI, as first proposed in Faber et al. (2000). A comprehensive documentation of the approach is provided in Straub (2004). Due to its computational efficiency, the generic approach not only allows for a wider application of RBI in practice, but also provides the means for inspection planning of systems, see Straub and Faber (2005a), and has recently been implemented by the industry as reported in Faber et al. (2005), Chakrabarti et al. (2005), and Goyet et al. (2004).

After a summary of the general RBI methodology, the generic approach to RBI is introduced following Straub (2004). The article thereafter concentrates on the computational aspects of the methodology, both in the derivation of the generic inspection plans and the calculation of specific inspection plans for individual

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structural elements. It is shown how the accuracy of the calculations can be evaluated and the trade-off between computational speed and accuracy is discussed. These aspects are of particular interest because the lack of computational efficiency has hindered a widespread application of RBI in the past. Therefore, the article concludes with an overview on recent applications of the generic approach, which demonstrate its practical relevance.

The current article is focused on the general RBI methodology, however, for completeness the probabilistic deterioration modeling and inspection performance modeling underlying the presented examples are provided in the Appendix. The example is a typical welded tubular joint in an offshore structure subject to fatigue, which is modeled by a two-dimensional fracture mechanics-based crack growth model (FM model); therein the stress intensity factors are evaluated using the parametric models from Newman and Raju (1981).

2 CONTEXT

The RBI procedures presented in this article must be embedded in a general maintenance strategy; in Goyet et al. (2002), where a general strategy for inspection optimization is described, they are termed *Detailed RBI* to emphasize that they form only one step in the total asset integrity management process. This process comprises a general, more qualitative analysis, a detailed analysis for the most critical parts of the system, and an implementation strategy. This general strategy, which is indispensable for any practical application, is not the subject of this article. Here it is only pointed out that the methodology presented in the following addresses only identified deterioration and failure modes. The identification of the potential failure modes and locations must be performed at an earlier stage during a qualitative risk analysis procedure. Especially the problem of so-called gross errors must be covered by such procedures. The identified potential locations of deterioration failures are termed *hot spots* in this article.

Although the present article concentrates on structures subject to fatigue, the generic approach is equally applicable to steel structures subject to corrosion, as described in Straub (2004).

3 RBI PLANNING

Traditionally, RBI is performed for individual hot spots separately. This section is thus limited to RBI for single hot spots; for the inclusion of system effects, the reader is referred to Straub and Faber (2005a).

RBI is based on the preposterior analysis from the Bayesian decision theory as presented, for example, in Raiffa and Schlaifer (1961) or Benjamin and Cornell (1970). Based on probabilistic models of both the deterioration and the inspection performance, it aims at minimizing the total expected cost of an inspection and maintenance plan, $\mathrm{E}[C_\mathrm{T}]$, which is the sum of the expected costs of failure, inspection, and repair (Equation (1)).

$$\mathrm{E}\left[C_\mathrm{T}\right] = \mathrm{E}\left[C_\mathrm{F}\right] + \mathrm{E}\left[C_\mathrm{I}\right] + \mathrm{E}\left[C_\mathrm{R}\right] \tag{1}$$

The different expected costs are evaluated considering the different branches in the decision tree, illustrated in Figure 1. All branches in the decision tree are characterized by their branching probability of occurrence and the associated costs. Note that the failure events are modeled as terminal events; no reconstruction after the failure event is taken into account. In principle, it is possible to extend the model by including the rebuilding of the element or structure after failure. These so-called renewal models are studied in the literature, see, for example, Streicher and Rackwitz (2004). Because in general the service life is assumed to be finite and the reliability of structural elements is high, detailed modeling of the behavior after failure will change the final results only slightly, if at all, as can be seen from Kübler and Faber (2002). The applied simplification is thus reasonable, especially because all the events and actions after failure can be included in the expected consequences of failure.

Each branch in the decision tree represents an intersection of different decisions and random outcomes. When several inspections are planned, the total number of these branches becomes prohibitive for the full evaluation of the decision tree, because the calculations of the branching probabilities are computationally demanding. Therefore, all published approaches use a simplified decision tree to limit the number of branches that must be computed. As an example, consider the simplified decision tree from Straub (2004) shown in Figure 2. The decision tree is simplified by differentiating only between repair and no-repair events after the inspection and by representing the repaired hot spot by a model identical to a new hot spot (where the hot spot after repair is stochastically independent of the hot spot before repair). The branching probabilities in this decision tree are then given by the probability of failure given no-repair p_F and the probability of repair after the inspections p_R. The probability of repair is evaluated by calculating the probabilities of the different possible inspection outcomes (as a function of the considered inspection technique) and by relating the repair decision to the outcome by means of a decision rule d. Typical decision rules are, for example, that every defect indicated by an inspection is repaired or that every defect with a measured size s larger than some limit s_R is repaired. A LSF for the repair event R is then defined by combining the decision rule d

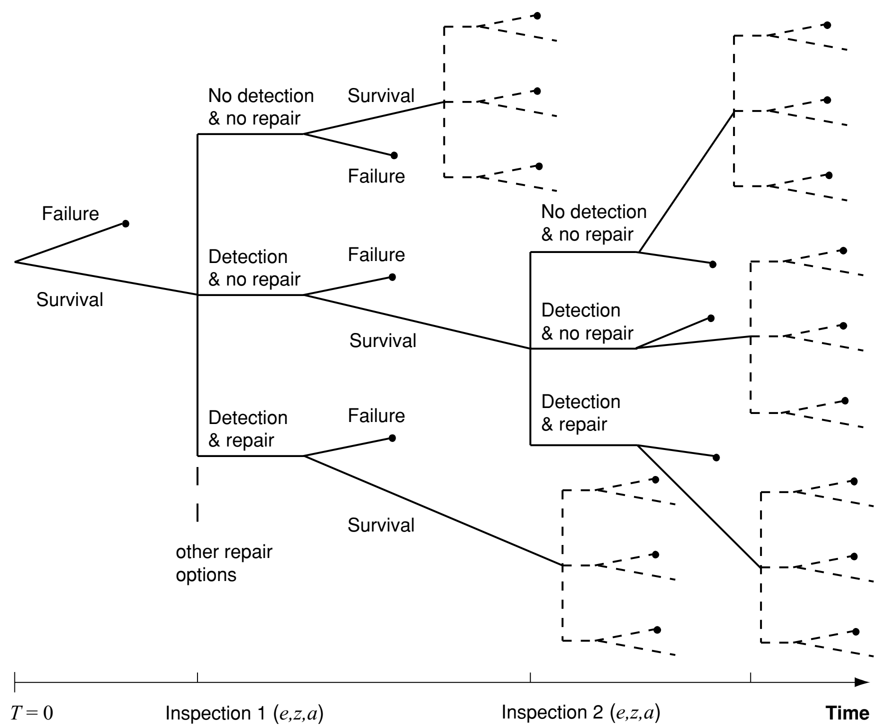

Fig. 1. The general decision tree for risk-based inspection planning.

with the inspection performance model. These calculations are described in detail in previous publications, see, for example, Fujita et al. (1989), Faber (2002), or Straub (2004).

Based on the probabilities and the (monetary) consequences related to the individual events, the expected costs are calculated as net present values following the decision tree in Figure 2. As an example, the expected cost of failure is given for a particular inspection strategy $\underline{\mathbf{e}}$ as

$$E\left[C_F\left(\underline{\mathbf{e}}, d, T_{SL}\right)\right] = \sum_{t=1}^{T_{SL}} \left[\left(1 - p_R\left(\underline{\mathbf{e}}, d, t - 1\text{ yr}\right)\right) \frac{c_1}{(1+r)^t} \right]$$
$$c_1 = \left(p_F\left(\underline{\mathbf{e}}, d, t\right) - p_F\left(\underline{\mathbf{e}}, d, t - 1\text{yr}\right)\right) c_F + \left(p_R\left(\underline{\mathbf{e}}, d, t\right) - p_R\left(\underline{\mathbf{e}}, d, t - 1\text{yr}\right)\right) E\left[C_F\left(\underline{\mathbf{e}}, d, T_{SL} - t\right)\right] \tag{2}$$

where $\underline{\mathbf{e}}$ is a vector describing the inspection times and qualities, d is the decision rule on the repair actions, T_{SL} is the anticipated service life of the structure given in years, c_F is the cost of failure, and r is the real rate of interest. The notations $p_F(\underline{\mathbf{e}}, d, t)$ and $p_R(\underline{\mathbf{e}}, d, t)$ are introduced to express that both the cumulative probability of failure and the cumulative probability of repair at time t are conditional on no-repair at all inspections; they are thus a function of $\underline{\mathbf{e}}$ and d. The calculation of $p_F(\underline{\mathbf{e}}, d, t)$ and $p_R(\underline{\mathbf{e}}, d, t)$ for the presented example is described in the Appendix. The first additive term in c_1 in Equation (2) represents the cost of failure at time t without a repair, the second corresponds to the expected cost of failure over the years following a repair at time t (in the anticipated remaining service life period $T_{SL}^{\text{new}} = T_{SL} - t$). Because of this latter term, Equation (2) is recursive.

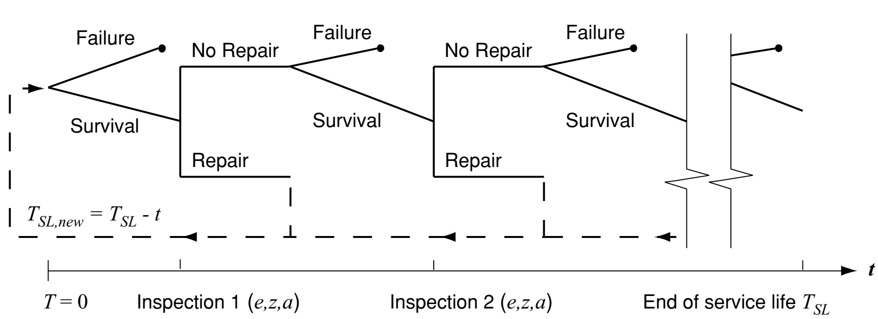

Fig. 2. The simplified decision tree for risk-based inspection planning.

Alternatively, the classical renewal models may be applied, for example, Streicher and Rackwitz (2004). The calculation of the expected cost of inspections and repairs is in analogy to Equation (2), the interested reader is referred to Straub (2004) for details.

3.1 Optimization of inspection strategies

An inspection strategy is defined by $\underline{\mathbf{e}}$ and d; the aim of RBI is thus the identification of the set of $\underline{\mathbf{e}}$ and d that yields the minimum expected total cost, that is,

$$\min_{\underline{\mathbf{e}},d} E[C_T(\underline{\mathbf{e}}, d, T_{SL})]$$
$$\text{subject to} \quad \Delta p_F(\underline{\mathbf{e}}, d, t) \leq \Delta p_F^{max}, \quad t = 0, \ldots, T_{SL} \quad (3)$$

where Δp_F^{max} is the acceptable probability of failure per year and must be determined in accordance with the relevant code and the preferences of the owner or operator of the structure, see Straub and Faber (2005b). $\Delta p_F(\underline{\mathbf{e}}, d, t)$ is the annual probability of failure (also known as the failure rate), defined as

$$\Delta p_F(\underline{\mathbf{e}}, d, t) = \frac{p_F(\underline{\mathbf{e}}, d, t) - p_F(\underline{\mathbf{e}}, d, t - \Delta t)}{\Delta t\,(1 - p_F(\underline{\mathbf{e}}, d, t - \Delta t))} \quad (4)$$

The decision rule d to apply is often prescribed by the operator of the structure, otherwise the number of feasible decision rules is in general limited and the identification of the optimal d is straightforward. The number of possible $\underline{\mathbf{e}}$ on the other hand is very large and thus prohibitive for the direct evaluation of Equation (3). For this reason, the *threshold approach* and the *equidistant approach* to inspection planning were introduced, as described in Faber et al. (2000). In the equidistant approach, the number of possible $\underline{\mathbf{e}}$ is limited by prescribing that the time between subsequent inspections is a constant; the inspection times are then fully described by the length of the inspection interval ΔT_{Insp}, respectively the total number of inspections n_{Insp} (for given service life). When e denotes the type of inspection to perform, the general optimization problem can be rewritten as

$$\min_{e,\Delta T_{Insp},d} E[C_T(e, \Delta T_{Insp}, d, T_{SL})]$$
$$\text{subject to} \quad \Delta p_F(e, \Delta T_{Insp}, d, t) \leq \Delta p_F^{max}, \quad t = 0, \ldots, T_{SL} \quad (5)$$

Applying the threshold approach, inspections are planned in the year before the annual probability of failure exceeds a threshold Δp_F^T, as illustrated in Figure 3. The obtained inspection intervals increase with time. This is commonly observed for many deterioration phenomena and is explained by the decrease in uncertainty after each inspection, which is accounted for by the Bayesian updating of the probabilities following an inspection.

For the threshold approach, the optimization problem is written as

$$\min_{e,\Delta p_F^T,d} E\left[C_T\left(e, \Delta p_F^T, d, T_{SL}\right)\right]$$
$$\text{subject to} \quad \Delta p_F^T \leq \Delta p_F^{max} \quad (6)$$

The threshold approach has the advantage that the constraint is formulated directly on the optimization parameter. This facilitates the interpretation of the optimization results because it is immediately recognizable if an inspection plan complies with the requirements in terms of the acceptable annual probability of failure Δp_F^{max}.

The optimization with both the threshold approach and the equidistant approach is carried out by evaluating the total expected costs for different values of the

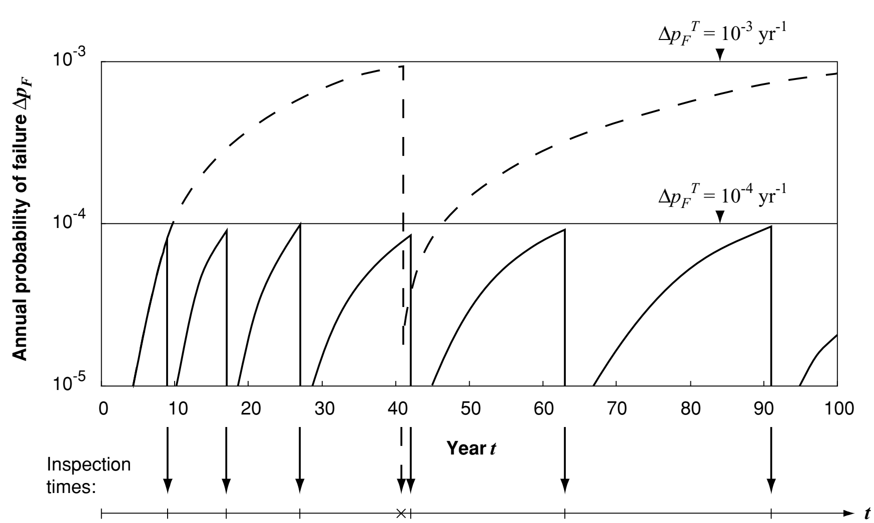

Fig. 3. Determination of the inspection times as a function of the threshold on the annual probability of failure for the hot spot as described by the probabilistic model in the Appendix.

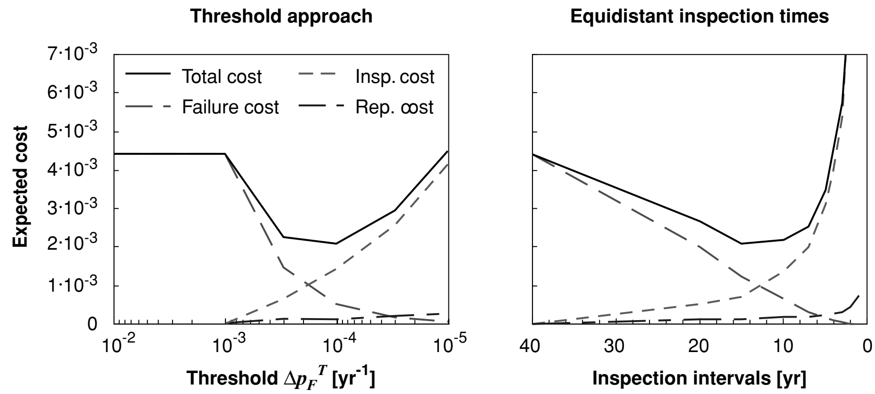

Fig. 4. Optimization of inspection efforts: Comparing the threshold and the equidistant approach. The cost model is $c_F = 1$, $c_{Insp} = 10^{-3}$, $c_R = 10^{-2}$, $r = 0.05$; the deterioration and inspection modeling is provided in the Appendix.

optimization parameters. For the threshold approach, the obtained solution is only an approximation because the threshold is a continuous variable, yet the accuracy is in general sufficient for any practical purpose. Figure 4 shows the optimization using both approaches; it is observed that the calculated minimal expected costs are approximately the same for both approaches.

4 COMPUTATIONAL ASPECTS OF RBI

The first approaches to RBI, Yang and Trapp (1974, 1975), were computationally very efficient because the applied deterioration models and inspection performance models led to closed form solutions for the reliability updated with the inspection outcomes. Analytical solutions for the probabilities, which are required to compute the expected costs, see Equation (2), were thus available. Unfortunately, these approaches have limited value in practical applications due to the lack of flexibility in regard to the applied models for describing deterioration and inspection performance. It was the introduction of reliability updating using SRA, see, for example, Madsen (1987), that provided the means for overcoming these limitations and motivated the development of RBI procedures as presented in the previous section.

Reliability updating using SRA, that is, FORM/SORM, is commonly subject to convergence problems which prevent the automation of the calculation, as discussed in Sindel and Rackwitz (1998) where it is noted that "optimization even in the specialized form required by FORM/SORM can require some skill, experience, and insider knowledge." This implies that even for the specialist the computations can become very time-consuming. As a consequence, the authors do not advocate the use of FORM/SORM for the evaluation of inspection plans in an industrial context. Nevertheless, FORM/SORM is indispensable for some of the other tasks involved in RBI, such as the calibration of the crack growth model to the SN fatigue model as outlined in the Appendix; additionally it is very useful for verification purposes. For the computation of the inspection plans, however, the use of Monte Carlo simulation (MCS) is suggested.

4.1 Application of crude MCS

The use of MCS for the calculation of the inspection plan is advocated because MCS is the only reliability evaluation technique where the accuracy of the result is not dependent on the starting values (which are required for every numerical evaluation). It thus allows for a fully automated calculation procedure. Although MCS requires a large number of evaluations of the LSFs and consequently is time-consuming, it is by far the most economical technique with regard to the man-days required to establish the inspection plans. As computational power is much cheaper than man power, this is economically relevant. The direct evaluation of inspection plans for particular hot spots in a structure is impractical (in Straub, 2004 it is reported that the calculation of an inspection plan may take up to one day on a standard personal computer with a Pentium 4 processor); however, MCS is highly efficient in the derivation of generic inspection plans that are the basis of the generic approach to RBI as presented in the subsequent section. Although MCS has major limitations, these are not crucial in the calculations of the inspection plans:

(1) MCS is not suited for reliability updating with equality constraints. However, as presented above, the events considered in the inspection planning phase (no-detection, respectively a measured defect smaller than a certain size) are all inequality constraints.

(2) MCS is not suited for the calculation of very low probabilities, but because probabilities of failure considered in the inspection planning are typically in the range of 10^{-3}–10^{-5} per year, the accuracy of MCS is in general acceptable, as shown below.

In general, the accuracy of the MCS is determined only by the number of simulations n_{MC} and the (unknown) true probabilities p; the accuracy is approximately proportional to $\sqrt{n_{\mathrm{MC}}/\mathrm{E}[p_{\mathrm{MC}}]}$ for small p's and large n_{MC} 's, with $\mathrm{E}[p_{\mathrm{MC}}]$ being the Monte Carlo estimate of the true probability. For inspection planning, it is proposed to assess the calculation accuracy through the probability of prescribing the inspection in a wrong year, denoted by $P(W)$. Following the threshold approach, an inspection is planned when the (calculated) annual probability of failure Δp_{F} exceeds the threshold $\Delta p_{\mathrm{F}}^{\mathrm{T}}$; a failure in the determination of Δp_{F} can consequently lead to the inspection being planned in a "wrong" year. As the inspection times are a main outcome of the procedure, $P(W)$ appears to be a reasonable indicator for the calculation accuracy. Following Straub (2004), $P(W)$ is calculated as

$$P(W) \approx \frac{1}{\varepsilon_{\Delta p_{\mathrm{F}}}} \int_{\Delta p_{\mathrm{F}}^{\mathrm{T}}-\varepsilon_{\Delta p_{\mathrm{F}}}}^{\Delta p_{\mathrm{F}}^{\mathrm{T}}} \begin{bmatrix} \left(1 - F_{\Delta p_{\mathrm{F}}}\left(\Delta p_{\mathrm{F}}^{\mathrm{T}} \mid \Delta p_{\mathrm{F}}, \sqrt{\Delta p_{\mathrm{F}}/n_{\mathrm{MC}}}\right)\right) + \\ F_{\Delta p_{\mathrm{F}}}\left(\Delta p_{\mathrm{F}}^{\mathrm{T}} \mid \Delta p_{\mathrm{F}} + \varepsilon_{\Delta p_{\mathrm{F}}}, \sqrt{\left(\Delta p_{\mathrm{F}} + \varepsilon_{\Delta p_{\mathrm{F}}}\right)/n_{\mathrm{MC}}}\right) \end{bmatrix} \mathrm{d}\Delta p_{\mathrm{F}} \tag{7}$$

where $F_{\Delta p_{\mathrm{F}}}(\Delta p_{\mathrm{F}}|\mu, \sigma)$ is the probability distribution function of the Monte Carlo estimate following a normal distribution with mean μ and standard deviation σ. $\varepsilon_{\Delta p_{\mathrm{F}}}$ is the annual increase in Δp_{F} around the threshold $\Delta p_{\mathrm{F}}^{\mathrm{T}}$. $\varepsilon_{\Delta p_{\mathrm{F}}}$ has a strong influence, because an error in the determination of the inspection time is more likely when the increase in the annual failure probability is smaller. The calculations are more accurate for high-probability thresholds, due to the dependency of the accuracy on $\Delta p_{\mathrm{F}}^{\mathrm{T}}$, although this effect is partly compensated by the fact that $\varepsilon_{\Delta p_{\mathrm{F}}}$ is larger for thresholds with low probabilities. Figure 5 gives an example of the calculated $P(W)$ for a typical case.

When assessing probabilities using MCS, it is to be noted that the number of simulations used for calculating the probabilities decreases with each additional conditioning event, because the considered subset of the event space becomes smaller with each additional constraint. As the calculated probabilities $p_{\mathrm{F}}(\underline{\mathbf{e}}, d, t)$ and $p_{\mathrm{R}}(\underline{\mathbf{e}}, d, t)$ are conditional on no-failure in past years and on no-repair at previous inspections, the number of simulations is decreasing with increasing t and after each considered inspection. For this reason, the accuracy of the calculations is not constant; this effect is illustrated in Figure 6 which shows the annual probability of failure for the considered hot spot calculated three times with different starting values for the simulations. On the other hand, the evaluated expected costs are net present values, taking into account the real rate of interest. For this reason, future events have less effect on the expected costs than events in the present; the decrease in calculation accuracy is thus in parallel to a decrease in the importance of the calculated probabilities on the final result. Based on these considerations, it has been found for past applications that $n_{\mathrm{MC}} = 2 \times 10^6$ simulations is a reasonable choice, but clearly such a value will always depend on the applied thresholds, in accordance with Figure 5. The same n_{MC} should be used for all applied thresholds, as the n_{MC}-simulated realizations of the deterioration process can be reused for assessing the probabilities for all thresholds. The n_{MC} realizations of the deterioration process are thereby temporarily stored and the inspection plans are evaluated for different thresholds and inspection techniques using the same n_{MC} deterioration histories from the storage. Because the computation time required for simulating the deterioration process is much higher than that required to simulate the inspections, at least for the considered fatigue deterioration model, there is little gain in reducing n_{MC} for higher values of the threshold $\Delta p_{\mathrm{F}}^{\mathrm{T}}$.

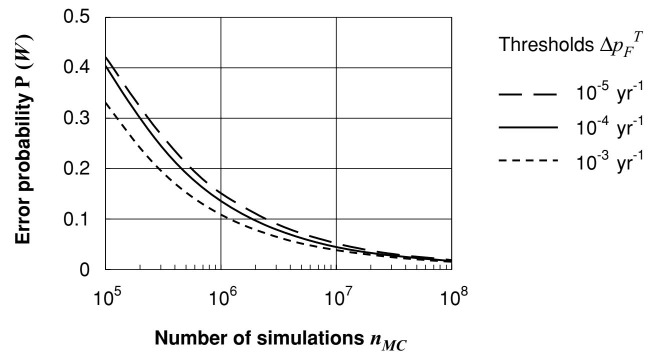

Fig. 5. Assessing the accuracy of MCS for inspection planning: Probability of predicting the required inspections in the wrong year for the example hot spot from the Appendix.

5 GENERIC APPROACH TO RBI

The core of the generic approach to RBI is the prefabrication of inspection plans for generic hot spots which are representative for the particular hot spots in the considered structures. These prefabricated plans are termed *generic inspection plans*. The inspection plans for the individual hot spots in a structure are then obtained from the generic inspection plans through an interpolation procedure. All hot spots for which the model is valid

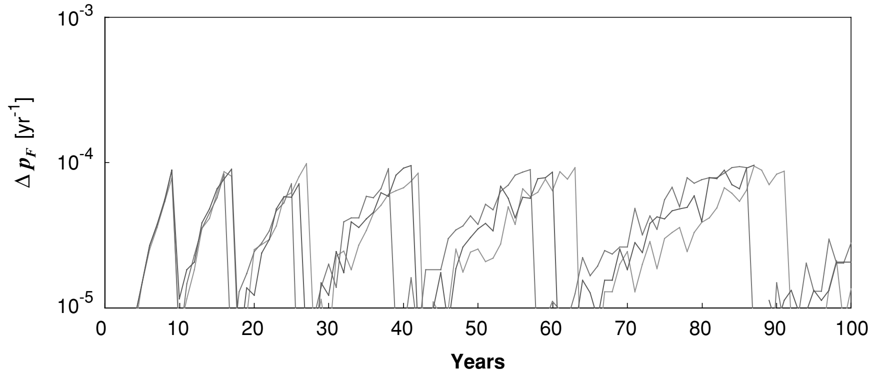

Fig. 6. Annual probabilities of failure for an inspection plan with threshold $\Delta p_F^T = 10^{-4}\text{yr}^{-1}$ for the example provided in the Appendix. Three different calculations (using different seed values for the simulation) each with 2×10^6 simulations at $t = 0$.

are fully described by the so-called generic parameters. These are the input parameters to the model that vary from one hot spot to the next and which are indicators of the relevant deterioration mechanism. For the example investigated in this article, the generic parameters are:

(1) the calculated design fatigue life T_{FL} (respectively the dimensionless fatigue design factor FDF),
(2) the applied SN curve (which is representative for the detail type and the environment),
(3) the wall thickness at the hot spot,
(4) the number of stress cycles per year,
(5) the uncertainty on the calculated equivalent stress ranges (which is a function of the accurateness of the load modeling and the structural modeling),
(6) the shape factor of the Weibull distribution, representing the stress ranges,
(7) the inspection type.

Because all these parameters are obtained from standard fatigue evaluation procedures, the RBI can, in principle, be performed without specialist knowledge once the generic inspection plans are available.

The computational efficiency of the generic approach is founded in the replacement of the demanding probability evaluations by an interpolation of the probabilities, which are calculated previously and stored in the generic database. Whereas the full computation of inspection plans using simulation takes several hours on standard computers, the interpolation of inspection plans requires only a few seconds. Although the calculations of the generic inspection plans are still demanding, these are performed at a previous stage; the establishment of inspection plans for particular structures from the generic database on the other hand is very efficient and can be integrated in the daily asset integrity management procedures of the owner or operator of the structure.

The relation between the required inspection times and the design fatigue life T_{FL} presented in Figure 7 exemplifies the influence of the generic parameters: The inspection times are obtained as a function of the

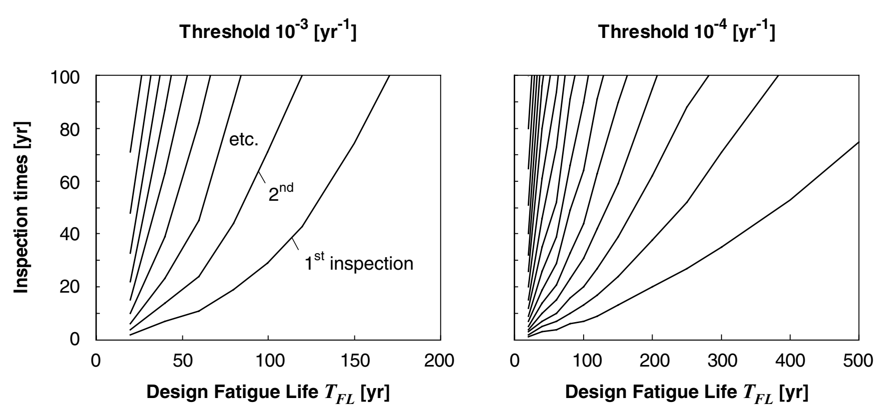

Fig. 7. Inspection times as a function of the design fatigue life T_{FL} for two different thresholds Δp_F^T on the annual probability of failure, calculated from the models given in the Appendix.

calculated T_{FL} for different thresholds $\Delta p_{\mathrm{F}}^{\mathrm{T}}$ on the annual probability of failure. Such relationships can be established for all generic parameters. Note that the results in Figure 7 are based on particular (fixed) values of all other (generic) parameters. In general, the inspection plans are a function of several generic parameters and the determination of particular inspection plans thus requires the use of multidimensional interpolation; these (computational) aspects are treated in the subsequent section.

The generic inspection plans are calculated for various thresholds or alternatively for various equidistant inspection intervals, as outlined previously. Each inspection plan consists of the required inspection times and the different probabilities of failure and inspection outcomes (respectively repair events), corresponding to the branches in the decision tree shown in Figure 2. The cost model (i.e., the cost of failure c_{F}, the cost of inspections c_{I}, the cost of repair c_{R} as well as the real rate of interest r) is not included in the generic inspection plans. When performing inspection planning for a particular hot spot in a structure, the expected costs are calculated for the particular cost model representing this hot spot. Because the evaluation of the expected cost is straightforward given that the probabilities of the different branches in the decision tree are known, see Equation (2), this does not increase the computation time significantly.

Figure 8 illustrates the principles of the generic approach to RBI. In the context of inspection and maintenance planning, the structure is represented in terms of the (previously identified) hot spots. Each hot spot is characterized by its particular values of the generic parameters, which together with the cost model are stored in a database. This database is then combined with the generic database containing the precalculated generic inspection plans, that is, the inspection times and the probabilities given no-repair at all inspections. Applying the decision model presented previously (i.e., the decision tree in Figure 2, respectively the formulas for the expected costs such as Equation (2)) and interpolating the inspection costs as well as the inspection times, inspection plans are efficiently obtained for all hot spots in the structure.

As indicated in Figure 8, instead of an application to RBI planning, the generic approach is equally suitable for reliability-based inspection planning, where costs are not considered and where the goal is the determination of the required inspection times to comply with given risk acceptance criteria, in accordance with Figure 3.

6 EVALUATION OF THE INSPECTION PLANS BY INTERPOLATION

The framework for the multidimensional interpolation is illustrated in Figure 9 for the case of two generic parameters; for the general case with n generic parameters the interpolation is performed correspondingly in a n-dimensional space.

The generic representations shown in Figure 9 are the values of the generic parameters for which generic inspection plans are calculated; they are the supporting

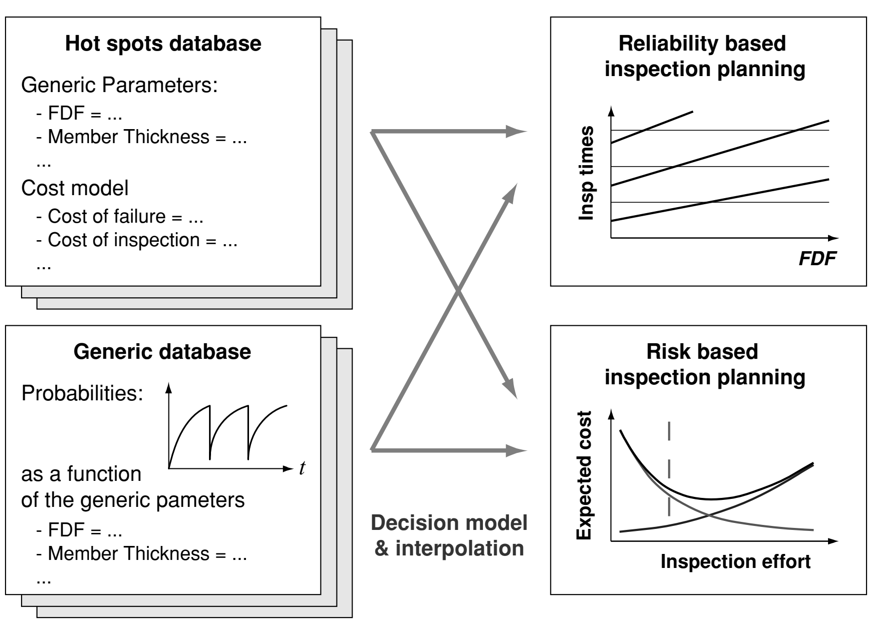

Fig. 8. Scheme of the generic approach to RBI.

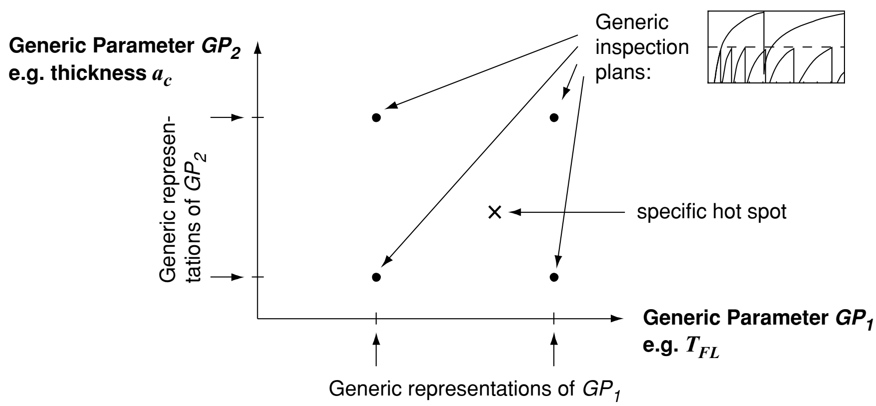

Fig. 9. Illustration of the definitions and the interpolation framework in the generic approach to RBI.

points of the interpolation. For many generic parameters more than two generic representations are required. As an example consider the design fatigue life T_{FL}. To represent the nonlinear relation between T_{FL} and inspection times shown in Figure 7, several generic representations are required. However, because a multidimensional linear interpolation is applied, only the two closest generic representations are taken into account when calculating the inspection plans for a particular hot spot.

The applied multidimensional linear interpolation is the simplest applicable algorithm. For the interpolation of the expected costs, an additional logarithmic transformation of the costs is advocated; see Straub (2004) for details. More sophisticated interpolation methods are abandoned because of the empirical basis of the interpolation. It is believed that the most stable results are achieved by a simple scheme that is thoroughly tested to assure that enough interpolation points (generic representations) are available. Higher order interpolations result in a less transparent procedure, which would make it difficult to decide on the required generic representations.

Using this simple interpolation algorithm, the accuracy of the approach is mainly dependent on the choice of the generic representations. Similar to the choice of the number of simulations n_{MC}, a balance between accuracy and computational speed must be identified. Each additional generic representation will increase the number of required generic inspection plans and thus the required computation time in establishing the generic database. The choice must be based on sensitivity analyses as presented in Figure 7, where the inspection plans are calculated as a function of varying one parameter at a time. Figure 10 shows the determination of the generic representations for the parameter wall thickness a_c. It compares the inspection times directly evaluated for several values of a_c with those obtained by an interpolation from the chosen generic representations.

Because the final generic database contains several generic parameters, which are varied simultaneously, a

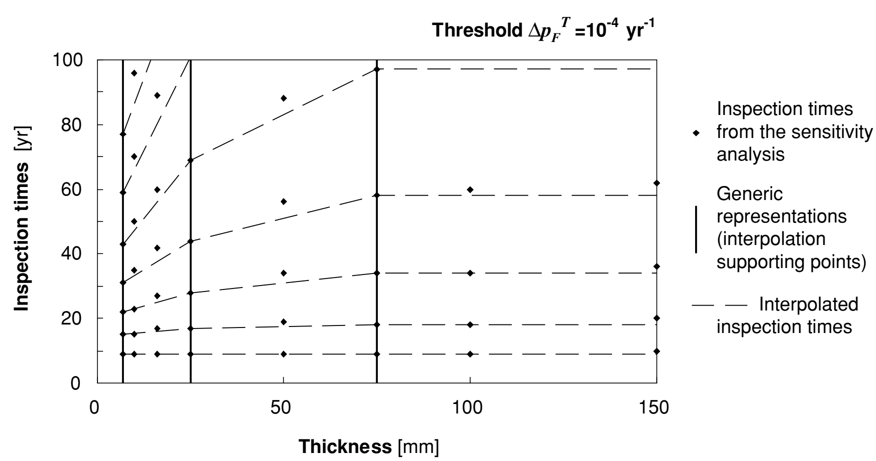

Fig. 10. Determination of the generic representations for the parameter wall thickness a_c for the numerical example provided in the Appendix.

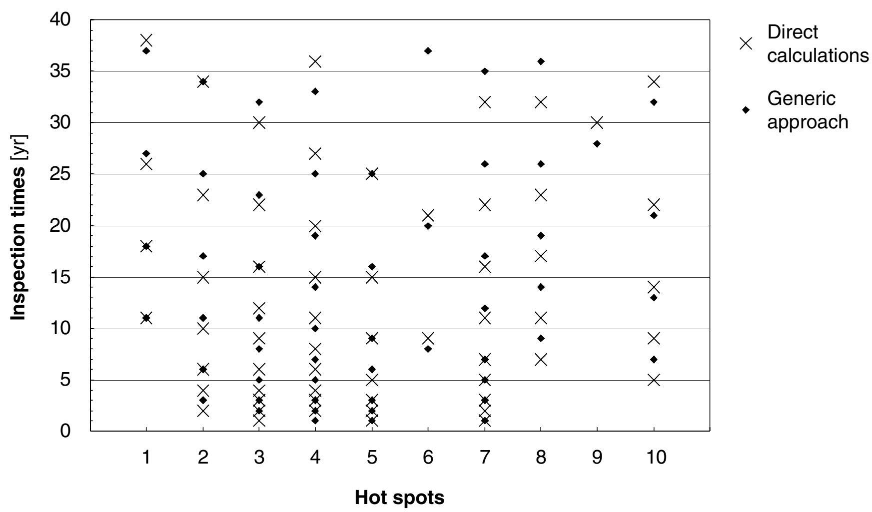

Fig. 11. Comparing inspection plans obtained with the generic approach to directly calculated inspection plans for a representative set of hot spots. The calculations are performed with the models given in the Appendix and the values of the generic parameters stated in Table 1.

final verification of the accuracy is required. For this purpose, inspection plans are calculated for a selection of hot spots directly; these are then compared to the inspection plans obtained using the generic approach. Such a comparison is shown in Figure 11. The generic parameters corresponding to the different cases are presented in Table 1.

When comparing with the inspection plans resulting from direct calculations (Figure 11), it must be borne in mind that these plans also are not exact solutions, but that they are subject to the scatter arising from the inaccuracy of the simulation procedure as discussed earlier. On the other hand, this scatter is reduced for the inspection plans from the generic approach. These plans are based on a set of inspection plans, each determined with n_{MC} simulations, and the total number of simulations is thus much larger for the interpolated inspection plans. For small deviations between the results from the directly calculated inspection plans and those derived through the generic approach, it is therefore not known whether these are due to the calculation accuracy of the directly calculated inspection plan or the imprecision of the generic interpolation scheme.

7 INDUSTRIAL APPLICATION OF THE GENERIC APPROACH TO RBI

Several industrial projects have been carried out in the last 2 years based on the generic approach to RBI. Some

Table 1
Different hot spots for the verification of the generic database

	Generic parameters: (corresponding to the model provided in the Appendix, see Table A1)					
Hot spot	T_{FL} *(yr)*	ν *(10^6 yr^{-1})*	CoV_{B_S}	$\lambda_{\Delta S}$	a_c *(mm)*	*DoB*
1	200	2	0.30	0.70	10	0.2
2	135	4	0.35	0.90	40	0.3
3	83	6	0.33	1.10	20	0.4
4	64	8	0.30	1.30	13	0.5
5	175	5	0.40	1.50	50	0.6
6	302	1	0.35	1.00	30	0.7
7	79	3	0.35	1.00	8	0.8
8	160	1	0.30	0.60	12	0
9	400	2	0.30	0.80	45	0.4
10	300	7	0.35	1.20	10	0.5

of these projects are described in Faber et al. (2005) and Chakrabarti et al. (2005), both related to offshore jacket structures, as well as in Goyet et al. (2004) related to floating production, storage, and offloading units.

Because of the novelty of the approach, in these projects the main stress was laid on the technical aspects of the generic approach to RBI as well as its documentation. However, the experiences gained from these projects show that the great advantage of the generic approach is that the main focus can be (and must be) put on the integral asset integrity management procedure; the "detailed" RBI for the most critical locations (the hot spots) in the structure then represents only one step in this general procedure. Because of the computational efficiency of the generic approach, the engineering efforts required for the detailed RBI are now balanced with the importance of this task in the general procedure.

In past industrial projects, the generic RBI was implemented through the use of an Excel spreadsheet that serves both as the generic database and as an interface for the calculation of inspection plans for particular hot spots. The generic inspection plans are stored in hidden worksheets in the spreadsheet; a database scheme for these inspection plans is proposed in Straub (2004). Visual Basic for Application is used for the computation of the routines that perform the evaluation of the expected costs, the interpolation of the inspection plans for the specific hot spots and the presentation of the final results. Although this software layout is not optimized for computational speed, it ensures greatest flexibility for both the designer and the user of the database. All the options of Excel are available for the user to illustrate the results and full compatibility with the software applied for reporting purposes is ensured. If desired, it is furthermore possible to uncover the database and the codes to the user and in this way provide full transparency.

8 OUTLOOK

The generic approach, due to its computational efficiency, provides the means to consistently account for system effects related to the performance of the structure and the inspections as described in Straub and Faber (2005a). Such additional considerations take into account the functional and statistical dependencies between the individual locations of deterioration (the hot spots). For large structures, it is necessary to address these aspects to determine optimal inspection strategies.

In addition, future research and development efforts should be directed toward the formulation and the improvement of probabilistic deterioration models as well as the models describing inspection performance. For fatigue, such models exist, but these can be improved through the evaluation of in-service inspection results. For this reason, it is of major importance that the general asset integrity management procedure ensures that inspection data is systematically stored. This observation is even more relevant for other deterioration mechanisms, such as various corrosion phenomena, for which only very crude probabilistic models have been published so far.

9 CONCLUSIONS

The article presents an overview of the recently developed generic approach to RBI with a special focus on the computational aspects. The generic approach facilitates the automated derivation of inspection plans as a function of the most influential parameters for all hot spots in the structure. In doing so, it represents a highly efficient tool which can be integrated in the general inspection and maintenance activities by the owners and operators of structures.

The involved probability calculations make the computation of inspection plans a demanding task. In the presented generic approach, these calculations are preestablished and the results are stored in a database which then forms the basis for the inspection planning of specific structures. It is advocated to use crude MCS for establishing the generic database, whereby the probability calculations can be automated and the operations involving man-hours are reduced. The increased computational time required by MCS is of minor importance in this phase and it has been shown that the limitations of MCS are not crucial for this type of application.

The generic RBI has been implemented in industrial projects. It is emphasized that the computational efficiency of the generic RBI allows for focusing on the entire inspection and maintenance strategy instead of on the technical aspects of the method. Once the generic database is established, the RBI can be performed by engineers who are not necessarily experts in probabilistic modeling fatigue crack growth and reliability analysis. This greatly enhances the applicability of the method in practice.

If there are very few hot spots in a structure with unique characteristics, it may be more efficient to calculate inspection plans for the critical hot spots directly. However, such a case is rare, as RBI planning is normally undertaken by operators with large and/or many structures. Furthermore, even when there are only few hot spots to analyze it may be beneficial to apply a generic approach, as this facilitates the study of possible changes in the input parameters and thus an optimization of mitigation actions.

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APPENDIX—PROBABILISTIC DETERIORATION, INSPECTION QUALITY, AND COST MODELING

The calculation of the conditional probabilities of failure and repair as a function of time, $p_F(\underline{\mathbf{e}}, d, t)$ and $p_R(\underline{\mathbf{e}}, d, t)$, requires the formulation of appropriate LSFs for the event of failure due to the deterioration process and for the inspection and repair events. The example considered in this article is a typical welded joint in a tubular offshore structure subject to fatigue. Failure of such an element can be described by the SN LSF (Equation (A1)).

$$g_{SN}(t) = \Delta - D_{tot}(t) \qquad \text{(A1)}$$

where Δ is the failure criteria and $D_{tot}(t)$ is the accumulated damage at time t, which is a function of the stress ranges and the SN curve representing the type of detail and environment. In many instances analytical solutions for $D_{tot}(t)$ exist, for example, for the fairly typical situation when the stress ranges can be modeled by a Weibull distribution.

Because the updating of fatigue reliability after an inspection requires that the LSF describing fatigue performance contains the same (geometric) parameters as the inspection performance model, an FM model must be used. Therefore an FM model is calibrated to the SN model, in such a way that the evaluated reliability is the same for the two models, as shown in Figure A.1.

The LSF describing ductile fatigue failure using the FM approach is

$$g_{FM}(t) = N_I + N_P - \nu t \tag{A2}$$

where ν is the stress cycle rate, N_I is the number of cycles to crack initiation, and N_P is the number of cycles it takes for the crack to grow from the initial crack size to the failure size. N_I is determined according to Lassen (1997) as

$$N_I = N_{I_0} \frac{(150\ Nmm^{-2})^{m_1}}{\Delta S_e^{m_1}} \tag{A3}$$

where ΔS_e is the equivalent stress range, m_1 is the slope parameter of the SN curve, and N_{I_0} is the number of cycles to initiation at the normalizing mean stress range equal to 150 Nmm^{-2}. N_{I_0} is correlated to the crack propagation parameter C_P.

N_P is calculated by solving a coupled differential equation based on Paris' law (Equation (A4)). This model is based on the assumption of a semielliptical crack shape with crack depth a and length $2c$.

$$\begin{aligned} \frac{\mathrm{d}a}{\mathrm{d}N} &= C_{P,\,a}\,(\Delta K_a\,(\Delta S_e, a, c, \mathrm{DoB})\,Y)^{m_{FM}} \\ \frac{\mathrm{d}c}{\mathrm{d}N} &= C_{P,\,c}\,(\Delta K_c\,(\Delta S_e, a, c, \mathrm{DoB})\,Y)^{m_{FM}} \end{aligned} \tag{A4}$$

N is the number of cycles, $C_{P,a}$, $C_{P,c}$, and m_{FM} are (empirical) parameters describing the crack growth. ΔK_a (ΔS_e, a, c, DoB) and ΔK_c (ΔS_e, a, c, DoB) are the stress intensity factor ranges in the a and c direction and are calculated using the empirical solutions from Newman and Raju (1981) as a function of the equivalent stress range ΔS_e and the degree of bending DoB. DoB is the fraction of the stresses at the hot spot which are due to bending (in the undamaged condition). Y is the model uncertainty related to the stress intensity factors. The boundary conditions for the solution of Equation (A4) are given by the initial crack size as $a(N = N_I) = a_0$, $c(N = N_I) = c_0$. The value of the parameters given in Table A.1 are applied in the examples in this article, if not stated otherwise.

The value of ΔS_e is calculated from the design fatigue life T_{FL} by setting all parameter values in the SN model equal to their design values and by determining the stress range for which the fatigue life becomes equal to T_{FL}.

The inspection performance is modeled by means of probability of indication (PoI) models. The PoI corresponds to the probability of detection (PoD) where additionally the probability of a false error is included, that is, the probability that a defect is indicated where none is present, see also Straub and Faber (2003). The applied PoI model is dependent on the crack depth a and given as

$$\begin{aligned} \mathrm{PoI}\,(a) &= \mathrm{PoD}\,(a) + (1 - \mathrm{PoD}\,(a))\,\mathrm{PFI} \\ \mathrm{PoD}\,(a) &= \frac{\exp\,(\alpha_D + \beta_D \ln\,(a))}{1 + \exp\,(\alpha_D + \beta_D \ln\,(a))} \end{aligned} \tag{A5}$$

The parameters of the PoI are given as $\alpha_D = 0.63$, $\beta_D = 1.16$, and PFI $= 0.138$. The event of indication of a crack with depth a at the inspection is described by a LSF (Equation (A6)) in accordance with Hong (1997).

$$g_I(t) = Z - \Phi^{-1}\,(\mathrm{PoI}\,(a(t))) \tag{A6}$$

Z is a standard normal distributed random variable and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal distribution function.

A repair is performed when a defect is indicated and measured as being deeper than 1 mm. This is the applied decision rule d, see Straub (2004) for other possible rules. The event of repair therefore requires the definition of an additional measurement limit state, defined as

$$g_M(t) = a_R - a(t) - \varepsilon_m \tag{A7}$$

with a_R being the repair criterion (here $a_R = 1$ mm) and ε_m the measurement uncertainty, modeled as normal distributed random variable with mean $\mu_{\varepsilon_m} = 0$ mm and standard deviation $\sigma_{\varepsilon_m} = 0.5$ mm.

The probabilities $p_F(\underline{\mathbf{e}}, d, t)$ and $p_R(\underline{\mathbf{e}}, d, t)$ are now computed from the LSFs using crude MCS (and SRA for validation purposes). The probability of repair at the first inspection is given as

$$\begin{aligned} & p_R(\underline{\mathbf{e}}, d, t = t_{\mathrm{Insp\_1}}) \\ &= \mathrm{P}(g_I(t_{\mathrm{Insp}}) \\ &\leq 0 \cap g_M(t_{\mathrm{Insp}}) \leq 0) \end{aligned} \tag{A8}$$

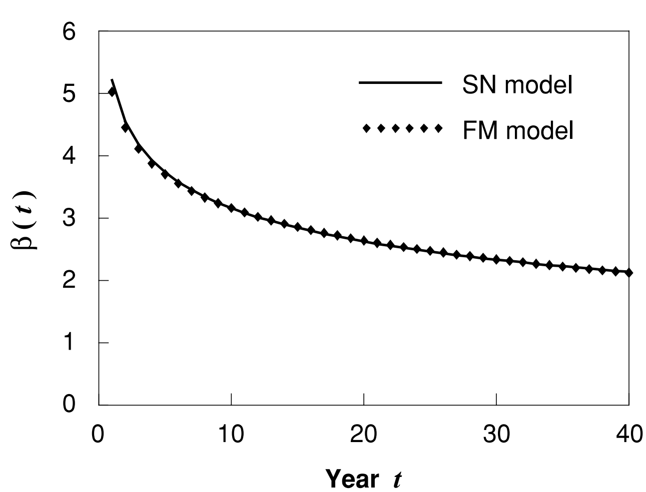

Fig. A.1. The reliability of the considered fatigue example as evaluated using the SN and the FM approach.

Table A.1
Parameters of the FM model

Parameter	*Dimension*		*Distribution*	*Mean*	*CoV*
ν^*	yr^{-1}	Stress cycle rate	Deterministic	3×10^6 (10^6–10^7)	
$T_{\mathrm{FL}}{}^*$	yr	Design fatigue life	Deterministic	120 (20–1,000)	
$B_{\mathrm{S}}{}^*$	–	Uncertainty on the stress ranges	Lognormal	1.03 (0.5–1.2)	0.25 (0.05–0.60)
$\lambda_{\Delta \mathrm{S}}{}^*$	–	Shape factor of the stress range distribution (Weibull)	Deterministic	0.9 (0.5–1.5)	
$a_c{}^*$	mm	Wall thickness	Deterministic	16 (7–150)	
$\Delta S_{\mathrm{e}}{}^\dagger$	Nmm^{-2}	Equivalent stress ranges	Deterministic	26.1 (for $T_{\mathrm{FL}} = 120$ yr)	
DoB*	–	Degree of bending	Deterministic	0–0.99	
$\ln C_{P,a}{}^\ddagger$		Parameter in Paris' law	Normal	-29.61§	0.77
m_{FM}		Parameter in Paris' law	Calculated from	$\ln C_{P,a} = -15.84 - 3.34 m_{FM}$	
$\ln C_{P,c}$	–	Parameter in Paris' law	Calculated from	$C_{P,c} = 0.9^{m_{\mathrm{FM}}} C_{P,a}$	
Y	–	Model uncertainty	Lognormal	$1.0^{\S}$	0.1
$N_{\mathrm{I}_0}{}^\ddagger$	–	Number of cycles to crack initiation at the normalizing stress range	Weibull	145×10^3	50×10^3
m_1	–	Slope parameter from the SN curve	Deterministic	3	
a_0	mm	Initial crack depth	Deterministic	0.1	
a_0/c_0	–	Initial crack ratio	Deterministic	0.2	

*Generic parameters (the values in parentheses show the considered range).
†Calculated as a function of the design fatigue life.
‡$\ln C_{P,a}$ and N_{I_0} are correlated with correlation factor $\rho_{\ln C_{P,a}, N_{\mathrm{I}_0}} = -0.50$.
§Obtained from the calibration of the FM model to the SN model.

The probability of failure in the first t years given no repair at previous inspections is calculated from

$$p_{\mathrm{F}}(\underline{\mathbf{e}}, d, t) = \frac{\mathrm{P}\left(g_{\mathrm{FM}}(t) \le 0 \cap \left(\bigcap_{i=1}^{n_{\mathrm{Insp}}(t-1\mathrm{yr})} (g_I(t_i) > 0 \cup g_{\mathrm{M}}(t_i) > 0)\right)\right)}{\mathrm{P}\left(\bigcap_{i=1}^{n_{\mathrm{Insp}}(t-1\mathrm{yr})} (g_{\mathrm{I}}(t_i) > 0 \cup g_{\mathrm{M}}(t_i) > 0)\right)} \tag{A9}$$

$n_{\mathrm{Insp}}(t - 1\ \mathrm{yr})$ is the number of inspections that are planned in the interval $[0, t - 1\ \mathrm{yr}]$ and t_i are the times of inspections, as prescribed by the inspection vector $\underline{\mathbf{e}}$. The conditional probability of repair after an inspection in year t is evaluated accordingly as

$$p_{\mathrm{R}}(\underline{\mathbf{e}}, d, t) = \frac{\mathrm{P}\left((g_{\mathrm{I}}(t) \le 0 \cap g_{\mathrm{M}}(t) \le 0) \cap \left(\bigcap_{i=1}^{n_{\mathrm{Insp}}(t-1\mathrm{yr})} (g_{\mathrm{I}}(t_i) > 0 \cup g_{\mathrm{M}}(t_i) > 0)\right)\right)}{\mathrm{P}\left(\bigcap_{i=1}^{n_{\mathrm{Insp}}(t-1\mathrm{yr})} (g_{\mathrm{I}}(t_i) > 0 \cup g_{\mathrm{M}}(t_i) > 0)\right)} \tag{A10}$$