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Modeling and managing uncertainties in rock-fall hazards

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The assessment of rock-fall hazards is subject to significant uncertainty, which is not fully considered in general practice and research. This paper reviews and classifies the various sources of the uncertainty. Taking a generic framework for risk assessment as source, a probabilistic model is presented that consistently combines the different types of uncertainties, in order to obtain a unified estimate of rock-fall risk. An important aspect of the model is that it allows for incorporating all available information, including physical and empirical models, observations and expert knowledge, by means of Bayesian updating. Detailed formulations are developed for various types of information. Finally, two examples considering rock-fall risk on roads, with and without protection structures, illustrate the application of the probabilistic modeling framework to practical problems.

Keywords: Bayesian analysis; model updating; reliability; risk assessment; rock-fall; uncertainties

Introduction

In mountainous regions, infrastructure facilities and, to a lesser degree, industry and housing are commonly exposed to rock-fall hazards. Transportation infrastructure, such as roads and railways, often has to pass through potentially hazardous areas. Consequently, in mountainous regions, highly trafficked roads are exposed to rock-fall. This became evident in Switzerland on 31 May 2006, when a block-fall event on the Gotthard-Autobahn (highway A2) killed two people and led to the closure of the road for an entire month. Notably, this road is the main north-south transit road in Switzerland, and any closure of this road affects the transportation of people and goods through Europe and is associated with large societal costs.

The responsible authorities address rock-fall risk through a variety of different measures, including land-use planning, appropriate choice of locations and routes for infrastructure systems, controlled release of rock masses, temporal closure of critical road sections and passive protection measures, such as galleries and flexible nets. Considering the significant costs (and risks) associated with these mitigation measures, it is crucial that decisions regarding the measures are made based on a scientifically sound assessment of the risks. This, in turn, necessitates a proper assessment of the uncertainties involved in the modeling of the frequency and intensity of detached rocks, the possible trajectories of falling rocks and the performance characteristics of the mitigation measures. In this paper, we present a framework for such an uncertainty modeling, based on a recently formulated general risk assessment formulation for civil and environmental systems.

As for most gravitational natural hazards, rock-fall events are highly site-specific phenomena, with rates of occurrence and consequences varying with time. Frequently, useful historical data is not available, and while phenomenological (physical) models are helpful in understanding the relevant processes, currently they do not capture the stochastic nature of these processes in a satisfying manner. For these reasons, rock-fall modeling is associated with large uncertainties, which have been considered in the past. It has been realized that the stochastic nature of rock-fall can only be captured by describing the release of rock mass in terms of probabilities or frequencies, typically using a power-law to describe the relation between frequency and rock volume (Hovius 1997, Hungr *et al.* 1999, Dussauge-Peisser *et al.* 2002). However, the uncertainty associated with this model is not generally quantified. In this paper, we will demonstrate how this model uncertainty can be included in the analysis. The uncertainty in the falling process is addressed by available simulation programs by means of a simple Monte-Carlo algorithm, whereby uncertain parameters are modeled by Normal or uniform distributions (Stephens 1998, Guzetti *et al.* 2002). However, to our knowledge, no procedure for consistently integrating this uncertainty with a description of the stochastic nature of rock detachment is provided in the literature. Furthermore, few

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publications deal with the quantitative analysis of the uncertainty related to the performance of the protection measures. Exceptions include a reliability analysis for protection galleries as presented in Schubert *et al.* (2005) and a reliability analysis for a flexible net as described in Roth (2002). Finally, the uncertainty related to the consequences of rock-fall should be included in the analysis (e.g. by the probability of vehicles and people being present during a rock-fall event, in a similar way as described for avalanches by Wilhelm (1997), or using Bayesian networks as outlined in Straub (2005)).

Several publications have presented generic risk assessment procedures for rock-fall hazards (Guzzetti *et al.* 2003, Baillifard *et al.* 2003, Budetta 2004). These methods provide integral procedures for estimating rock-fall risk that, to some extent, account for the uncertainties as described above. Due to the similarities between rock-fall and landslide hazards, it is also worth noting that for the latter phenomena a number of risk assessment procedures have been proposed (an overview is provided by Aleotti and Chowdhury (1998), Dai *et al.* (2002) and Fell *et al.* (2005)). While differences in the characteristics of rock-fall and landslide prohibit the direct transfer of the uncertainty modeling from one field to the other, many aspects of the risk management problem are identical. In the end, a risk management framework must encompass all natural hazards in an integral manner.

In this paper, we aim to extend the existing risk assessment procedures for rock-fall using an advanced (Bayesian) approach to uncertainty modeling. We will show that this approach ensures a mathematically rigorous risk assessment and leads to optimal decisions regarding risk mitigation actions, based on all available information. The framework will be presented in a general form, but its implementation will be illustrated by the two examples that conclude this paper.

Risk assessment framework

In this section, a generic framework for the assessment of natural hazard risks based on Faber *et al.* (2007) is presented and adapted to the case of rock-fall hazards. The proposed framework is not an entirely novel concept, rather it formalises the way rock-fall hazard assessments have been carried out in the past. It provides an overview of all involved processes and aspects and ensures a systematic and scientifically sound treatment of the uncertainties involved at the different levels of analysis. The ultimate goal is the computation (and optimisation) of the risk, which is defined as the expected consequences, following the utility-theory by Von Neumann and Morgenstern (1945) that is commonly accepted as the rationale for making optimal decision under uncertainty.

While some might question the need for a novel terminology, we feel that this facilitates a mathematically rigorous approach to the computation of the risk. Existing terms, such as vulnerability and hazard, do not have unique definitions in the literature (Roberts *et al.* 2007), and using these terms can lead to misunderstandings, in particular in an interdisciplinary context (the structural engineer designing a protection structure might use different definitions than the geologist). An additional advantage of the framework employed here is that it has been developed for and is applicable to any type of hazard on any type of engineering or civil system.

The suggested framework is shown in Figure 1. It is distinguished between the three main components *system exposure*, *system resistance* and *system robustness*, which in the case of rock-fall hazards are illustrated in Figure 2. It is then distinguished between *direct consequences* or *indirect consequences*, depending on where in the system they occur. Although not directly part of the risk assessment (but of the risk management), *actions* are also considered, i.e. potential measures influencing the risk. The application of the different elements of the framework for rock-fall risk assessment is treated in the following sections.

System exposure

The system exposure describes the probability of occurrence of the potential hazards in the considered system. The hazard is generally described by its location and one or several physical parameters representative of its damaging potential. For rock-fall hazards, the relevant parameter is typically the volume of detached rock v or its mass m. The exposure is uncertain, and the rock-fall volume is modeled as a random variable V (we utilise upper case to denote random variables and lower case to denote deterministic variables, including realizations of random variables). This random variable is typically described by its annual exceedance frequency $H_v(v) = \mathrm{E}[\mathrm{N}^+(v)]$, with $N^+(v)$ the number of rocks per year larger than v, and E[ ] the expectation operation. Alternatively, rock-fall volume can be described by the annual exceedance probability $P_V(v) = \Pr[\mathrm{N}^+(v) \geq 1]$. Under the common assumption that rock-fall follows a Poisson process, the conditions of which are discussed later, the probability $\Pr[N^+(v) = 0]$ is described by the Poisson

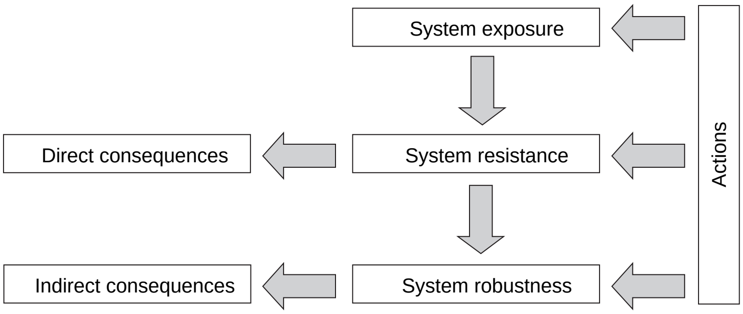

Figure 1. A framework for engineering risk assessment (Faber *et al.* 2007).

distribution with parameter H_V(v) and argument zero. It follows that $P_V(v)$ and $H_V(v)$ are related by

$$P_V(v) = 1 - e^{-H_V(v)}. \quad (1)$$

The use of $P_V(v)$ is appropriate for sites where only extreme events are relevant. This is the case when protection structures are installed or planned and the probability of two damaging events per year can be neglected. On the other hand, $H_V(v)$ is appropriate for situations where each single rock-fall event has consequences, e.g. in the case of rock-fall exposure to an unprotected road link.

Both $P_V(v)$ and $H_V(v)$ have a reference period of one year. Acceptable probabilities of failure events in the built environment are typically expressed as annual risks (e.g. Eurocode Basis of Design (2002), Annex B, makes recommendations for acceptable reliability in terms of reliability indexes β for a reference period of one year). Therefore, risk assessments should intend to express all probabilities and risks as annual values (Rackwitz 2000).

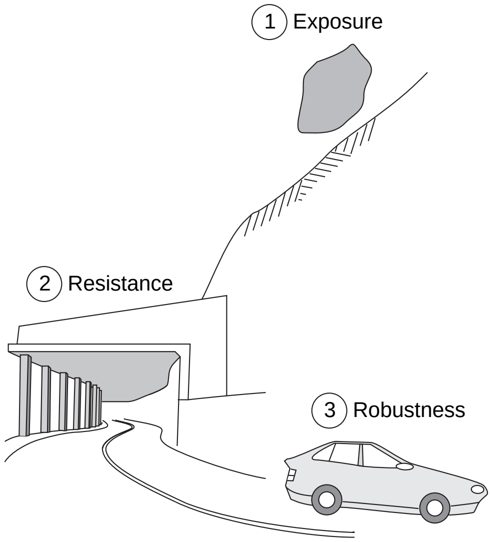

Figure 2. Illustration of system exposure, system resistance and system robustness for the case of rock-fall hazards.

To compute risk, it is beneficial to work with the probability density function (PDF) of the maximum annual rock-fall event, $f_V(v)$, or the annual frequency density h_V(v). These are defined as

$$f_V(v) = -\frac{d}{dv}P_V(v), \quad (2)$$

$$h_V(v) = -\frac{d}{dv}H_V(v). \quad (3)$$

In some instances, the rock-fall activity during an alternative time period ΔT is of interest. If it is reasonable to assume that rock-fall has a constant mean rate of occurrence (i.e. if it is a homogeneous Poisson process), one can make use of the fact that the frequency is proportional to the considered time period: $H_V^{\Delta T}(v) = (\Delta V/1 \text{ year})H_V(v)$ and $h_V^{\Delta T}(v) = (\Delta V/1 \text{ year})h_V(v)$. From the above equations it follows that the PDF of the maximum rock-fall volume for any time period ΔT can be computed by

$$f_V^{\Delta T}(v) = e^{-H_V^{\Delta T}(v)}h_V^{\Delta T}(v). \quad (4)$$

The above (and all common probabilistic rock-fall models) is based on the assumption of rock-fall as a Poisson process, the conditions for which are (a) that the number of occurrences in a given time interval is independent of the number of occurrences in a previous time interval (memorylessness), and (b) that the probability of more than one event in a small time interval is order of magnitudes lower than the probability of one event (Benjamin and Cornell 1970). In the special case of a homogeneous Poisson process, the additional assumption (c) is that the mean rate of rock-fall is constant with time. While these conditions are not generally fulfilled, they represent a reasonable approximation to the real situation for many decision problems, as will be demonstrated in this paper. Note that the assumption of a Poisson process is generally not reasonable for landslides. For this reason, the uncertainty modeling, and consequently the risk assessment, is different for landslide hazards.

System resistance

The system resistance includes all intermediate processes and elements that may modify (stop, reduce, but also accelerate) the characteristics of the hazard within the system. Generically, the resistance is described as the probability of one or several damage or failure events F_i (e.g. the impact of a rock on a road or a building), dependent on the type and magnitude of the exposure V. Generically, this can be represented by a conditional probability $\mathrm{PR}(F_i|v)$. The system resistance is typically modeled by a rock-fall simulation that computes the probability of the rock hitting at certain locations (e.g. the road) conditional on a rock volume. If protection structures are present, their performance must be included in the analysis, and in this case F_i is the event of failure of the protection system.

System robustness

The robustness describes how the system reacts to a damaging or failure event F_i. To model the system robustness, it is necessary to study the possible scenarios K_j following an initial event F_i. As an example, the event F_i is a rock falling on a road, one scenario K_j could be road closure but no accident, another scenario is a collision accident followed by road-closure. Such scenarios are best represented by event trees (Benjamin and Cornell 1970, DeGroot 1970) or Bayesian networks (Straub 2005). To ensure consistency in the calculation of the risk, the definition of the various scenarios K_j must be such that either the scenarios are mutually exclusive or that the consequences associated with different scenarios are additive.

Consequences

Consequences are distinguished between direct consequences C_{D} and indirect consequences C_{ID}. The former are the physical damages associated with the system resistance, the latter may comprise physical as well as economical, social or ecological damage, and are sometimes referred to as follow-up consequences. For the case of rock-fall, direct consequences, for example, are the cost of repairing damage to the road and protection structures. Indirect consequences, for example, are administrative costs, societal cost of road closure and injuries and fatalities sustained. The distinction between direct and indirect consequences provides information on the system characteristics. That is, if the contribution of the indirect consequences to the total risk is small, the system can be called robust (Baker *et al.* 2007).

Consequences are often expressed in monetary terms, requiring the quantification of the 'value of life' (see Rackwitz 2006). In principle, other value systems may be used (e.g. the multi-attribute utility theory of Keeney and Raiffa (1976)), but any optimization of decisions must be based on a trade-off between the different attributes, thereby implicitly assigning a value to life.

Actions

The aim of risk management is the assessment of cost-optimal mitigation actions. To facilitate optimization of actions, uncertainty models should be formulated as a function of possible actions. Actions can be applied on all three levels in the system. For example, for rock-fall on roads, the risk can be reduced by (1) setting anchors to increase the stability of the rock mass, thus reducing exposure occurrence probability; (2) constructing protection systems such as galleries or flexible nets, thereby increasing the resistance of the system; and (3) improving visibility for drivers on the endangered road section, reducing the probability of drivers crashing into rocks lying on the road, thus increasing the robustness of the system.

The classification of a specific process in the presented categories is ambiguous. As an example, when the focus is on a protection structure, the exposure may be considered as being the impact energy on that structure. In this case, the process of falling is included in the exposure model. The framework is intended as a support to structure the problem and not as a strictly prescribed, unique model of natural hazard risks. Thus, the ambiguousness is not crucial if the definitions are applied consistently within a specific project.

The risk is defined as the expected damage (the consequence for the system) per reference period, which, as discussed earlier, is generally one year. Based on the above definitions, the risk R is, in generic format, obtained by

$$R = \int_V \sum_F C_T(v, F_i)\mathrm{Pr}(F_i|v)h_V(v)dv. \qquad (5)$$

The total consequences as a function of v and F_i, $C_T(v,\ F_i)$, are given as the sum of the direct consequences $C_{\mathrm{D}}(v,\ F_i)$ and the weighted sum of the indirect consequences $C_{\mathrm{ID}}(v,\ F_i,\ K_j)$ by

$$C_T(v, F_i) = C_{\mathrm{D}}(v, F_i) + \sum_K C_{\mathrm{ID}}(v, F_i, K_j)\mathrm{Pr}(K_j|v, F_i). \qquad (6)$$

Equations (5) and (6) state that the risk is an expected value obtained by integration and summation over all uncertain factors V, F, K. In the above formulation, it is assumed that the exposure V is described in a continuous state space, whereas the resistance F and the robustness K are represented by discrete events. It is noted that the above equation also applies if one is interested only in the risk related to fatalities and/or injuries, in which case $C_D(v, F_i)$ is typically zero and $C_{ID}(v,F_i,K_j)$ is the number of people killed or injured during a particular scenario F_i,K_j.

By assessing the influence of various actions on the risk and comparing this with the cost the actions, optimal actions can be determined in accordance with the principles of Bayesian decision theory (Benjamin and Cornell 1970).

Modeling uncertainty in rock-fall hazards

Engineering risk assessment is generally based on a Bayesian interpretation of probabilities (Faber and Stewart 2003). Within this framework, it is useful to distinguish two fundamentally different types of uncertainties, namely epistemic and aleatory uncertainties. This distinction has been considered for risk assessment of technical systems (Apostolakis 1990, Helton and Burmaster 1996), and increasingly for natural hazards (Hall 2003, Apel *et al.* 2004, Straub and Der Kiureghian 2007), but has been discussed also for general geological applications by Mann (1993). Aleatory uncertainties are interpreted as random uncertainties, which, for a given model, are inherent to the considered process; epistemic uncertainties are related to our incomplete knowledge of the process, often because of limited data.

Rock-fall is generally considered an inherently uncertain process, i.e. it is not possible to deterministically predict the time and the extent of the next event. However, it is possible to describe rock-fall using a probabilistic model, describing the frequency with which a rock of a certain volume or larger is detached, $H_V(v)$. As the assessment of rock-fall is based on little data and simplified models, the probabilistic model is subject to uncertainty itself, which can be represented by modeling the parameters of $H_V(v)$ as random variables. In this case, we write $H_V(v|\boldsymbol{\theta})$ to indicate that the model is defined conditional on the values of its parameters $\boldsymbol{\theta}$. This epistemic uncertainty on $\boldsymbol{\theta}$ can be depicted by credible intervals (the Bayesian equivalent to confidence intervals) on the exceedance frequency curve, as demonstrated in Figure 3.

The distinction between these two fundamental types of uncertainty is relevant because aleatory uncertainty cannot be reduced for a given model. In contrast, epistemic uncertainty *can* be reduced by collecting additional information. For this reason, a clear identification of the epistemic uncertainties in

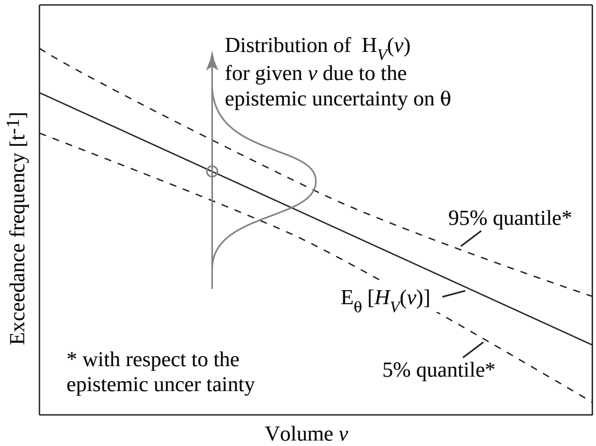

Figure 3. Exceedance frequency, illustrating the difference between epistemic and aleatory uncertainty.

the analysis is crucial, as these may be reduced at a later stage. Furthermore, neglecting epistemic uncertainty, as typically occurs, can lead to strong underestimation of the risk (e.g. Coles *et al.* 2003).

Uncertainties in rock-fall exposure

As with most natural hazards, the uncertainties related to the system exposure are large for rock-fall hazards. In the literature, this uncertainty is generally represented by an exceedance frequency as illustrated in Figure 3, yet without consideration of the epistemic uncertainty. Instead, it is (implicitly) assumed that the frequency of an event with a certain rock volume is a deterministic value, implying that if the site were observed over a sufficiently long period, the exact predicted frequency of rocks would be experienced. Clearly, this is not the case; instead, the predicted frequency is a best estimate of the true rate of occurrence.

In the literature, various methods are proposed for identifying the exceedance frequency at a specific site. This includes (a) the analysis of historical datasets (Hungr *et al.* 1999, Dussauge-Peisser 2002), (b) empirical models which describe rock-fall exposure as a function of different indicators (observable parameters), such as topography and geology (Budetta 2004, Baillifard *et al.* 2004), (c) phenomenological (mechanical) models (Duzgun *et al.* 2003, Jimenez-Rodriguez *et al.* 2006), and (d) expert opinion (Schubert *et al.* 2005). All these methods are useful in a particular context. While methods (a) and (b) are generally more appropriate for the analysis of larger areas with less accuracy, (c) and (d) are more suited for the detailed analysis of a specific site. Using an example, we will demonstrate that a proper uncertainty modeling allows combining the different models within a single multi-scale model in a consistent manner.

Large-scale models (a and b) are generally based on statistical methods. Consequently, it is mathematically convenient to express the exceedance frequency in a parametric format. Traditionally, a power-law has been applied to describe the relation between rock volume V and exceedance frequency:

$$H_V(v|\boldsymbol{\theta}) = av^{-b}. \tag{7}$$

The parameters of the model are $\boldsymbol{\theta} = [a,b]^T$. The epistemic uncertainty is included in the analysis by modeling $\boldsymbol{\theta}$ as a random vector. With the PDF of $\boldsymbol{\theta}$, $f_\Theta(\boldsymbol{\theta})$, the unconditional exceedance frequency is computed as

$$H_V(v) = \int_{\boldsymbol{\Theta}} f_{\boldsymbol{\Theta}}(\boldsymbol{\theta}) H_V(v|\boldsymbol{\theta})\, d\boldsymbol{\theta}. \tag{8}$$

There are various sources for epistemic uncertainties in large-scale models, preventing an exact prediction of the exceedance frequency for a particular site as listed in the following:

Statistical uncertainty. The parameters of the large-scale models are derived empirically from data-sets. Due to the limited size of these data sets, the estimated parameters are subject to statistical uncertainty.

Measurement uncertainty. Measurements and recordings of the geological properties are typically subject to uncertainty, and observations of rock-fall events are often incomplete and biased and must rely on local experts. As an example, rocks on a road will generally be reported and documented, but frequently those that miss the road may not be noted.

Model uncertainty. Extrapolation of the statistical models to areas other than those for which observations are available leads to additional uncertainty, as the geological and topographical characteristics will be different for these areas. GIS-based models will take into account some of these parameters, but the omitted parameters will lead to an uncertainty on the model predictions.

Model uncertainty. Although the power-law is commonly assumed, it has not been justified by phenomenological considerations. Thus, that the parametrical model accurately represents the actual behavior is not ensured.

Spatial variability. Rock-fall frequency varies in space. The observations represent an average over an area and the resulting parameter values, therefore, do not reflect the variations from the average.

Temporal variability. Rock-fall frequency varies in time. When working with annual frequencies, the seasonal changes do not affect the analysis, but the frequency may change over the years or may be dependent on extreme events (e.g. earthquakes). However, in certain instances, e.g. when temporal closure of the road is considered as a risk reduction measure, seasonal variations must be explicitly addressed by the analysis.

How can these uncertainties be quantified? Statistical uncertainty can be quantified by using standard statistical methods such as Bayesian analysis (see, e.g. Coles 2001). Measurement uncertainty can generally be estimated when the data collection method is known. Unfortunately, no simple analytical method is available for estimating model uncertainties. A solution is to rely on expert opinion, i.e. to ask experts about their confidence in the models. It is also possible to compare the model with observations that have not been used in the calibration of the model (model validation) or to compare different models. Furthermore, it is possible to include additional parameters in the formulation of the exceedance frequency (see Equation (7)). The model uncertainties are then reduced while the statistical uncertainties increase, but the latter can then be estimated analytically. Coles *et al.* (2003) demonstrated this for the analysis of rainfall data. The spatial and temporal variability of rock-fall can be analyzed quantitatively if data is available in sufficiently small scale (a data-set showing the spatial distribution of rock-falls is presented in Dussauge-Peisser *et al.* (2002)). Spatial variability can be described by the spatial correlation of the relevant characteristics (e.g. rock-fall frequency). In most practical cases, however, a simplified approach is favorable, whereby smaller areas are determined within which the spatial variability can be neglected. Temporal (typically seasonal) variability can be described by time-dependent parameters $\boldsymbol{\theta}$ in the exceedance frequency model, corresponding to the assumption of rock-fall following an inhomogeneous Poisson process.

For small-scale models, the application of the power-law is not always justified, in particular if different mechanisms are underlying the detachment of smaller and larger rocks. In such cases, it might be more appropriate to utilize a non-parametric model in which the rock volume is divided into a discrete number of intervals (e.g. 10–50 m^3) and the model gives the annual frequency of rocks for the different volume ranges.

Bayesian analysis and updating

For the modeling of rock-fall exposure, Bayesian analysis is particularly useful, as it facilitates the consistent combination of different information in a single model. This is because the probabilistic model can be updated when new information becomes available. Consider the case where rock-fall exposure at a particular location is expressed by the model $H_V(v|\theta)$ with uncertain parameters $\boldsymbol{\theta}$. When new information becomes available (denoted by $\mathbf{z}$), the probability distribution of the uncertain parameters can be updated using Bayes' theorem, which in its general form can be written as

$$f_{\Theta}(\boldsymbol{\theta}|\mathbf{z}) \propto L(\boldsymbol{\theta}|\mathbf{z}) f_{\Theta}(\boldsymbol{\theta}), \tag{9}$$

where $f_{\Theta}(\boldsymbol{\theta})$ is the prior probabilistic model, $f_{\Theta}(\boldsymbol{\theta}|\mathbf{z})$ is the updated model and $L(\boldsymbol{\theta}|\mathbf{z})$ is the likelihood function, which describes the new information. The proportionality constant is obtained from the fact that integration of $f_{\Theta}(\boldsymbol{\theta}|\mathbf{z})$ over the entire domain of θ must yield one. The likelihood function is proportional to the probability of the observed information given the parameters $\boldsymbol{\theta}$, i.e.

$$L(\boldsymbol{\theta}|\mathbf{z}) \propto \Pr(\mathbf{z}|\boldsymbol{\theta}). \tag{10}$$

To demonstrate the derivation of the likelihood function, consider the case where the available information is a set of observed detached rocks $i = 1 \ldots n$ for a specific mountain slope, which are described by their volume v_i and the time-period ΔT_z during which they occurred. Only rocks with a volume larger than v_{th} have been recorded (th = threshold). We make the following simplifying assumptions: (a) that the rock-fall follows a homogeneous Poisson process as discussed earlier, and (b) that the observations are free of error (i.e. all rocks are recorded). These assumptions hold under particular circumstances only, yet they are a reasonable approximation to many real situations and they are suitable for illustrative purposes. Under these assumptions, the probability of observing exactly n rocks with a volume larger than v_{th} is given by the Poisson distribution with parameter $H_V(v_{th}|\theta)\Delta T_z$ as

$$\Pr(n|\boldsymbol{\theta}) = \frac{[H_V(v_{th}|\boldsymbol{\theta})\Delta T_z]^n}{n!} \exp[-H_V(v_{th}|\boldsymbol{\theta})\Delta T_z]. \tag{11}$$

Given that a rock with volume larger than v_{th} has detached, the likelihood of its volume being v_i is proportional to $h_V(v_i|\boldsymbol{\theta})/H_V(v_{th}|\boldsymbol{\theta})$ for $v_i \leq v_{th}$. As all observations are assumed independent events, the likelihood function is obtained by multiplying these terms. The likelihood function representing the observation of n rocks with volumes $v_1 \ldots v_n$ on the considered mountain area is then:

$$L(\boldsymbol{\theta}|\mathbf{z}) \propto \exp[-H_V(v_{th}|\boldsymbol{\theta})\Delta T_z] \prod_{i=1}^{n} h_V(v_i|\boldsymbol{\theta}), \tag{12}$$

where $h_V(v_i|\boldsymbol{\theta})$ is the annual frequency density of V according to Equation (3). Note that the observations apparently must relate to the frequency density and not the probability density because we cannot observe only the largest rock that has fallen during a certain period, rather the observed rocks may all be from the same time period.

Uncertainties in rock-fall trajectory

Once a rock is released, its trajectory is mainly determined by the topography, its mode of motion (free fall, rolling bouncing or sliding), and the characteristics of the surfaces of the rock and the ground. All these factors contribute to the uncertainty in the prediction of the trajectory. Existing numerical tools model this uncertainty by means of crude Monte Carlo simulation (MCS); an overview is provided by Guzzetti *et al.* (2002). There exist two- or three-dimensional models and differences in the physical representation of the rock – the so called lumped mass approach represents the rock by a single mass point, neglecting the geometry of the stone. The rigid body approach models the stone by idealized geometries (e.g. cylinders, spheres or a cuboidal shape; Ettlin 2006) with varying physical and material properties. Hybrid models combine a lumped mass approach to simulate the free fall with a rigid body approach to simulate the contact with the ground surface. Finally, different models are used to simulate the impact of the rock on the ground (Dorren 2003), a simple approach being the use of coefficients of restitution (Stevens 1998). The impact is the most intricate part of the falling process, and its modeling is associated with large uncertainties. The modeling cannot account for the variability in the ground material (particularly in zones covered with vegetation) and the local geometry of the ground and the rock. These uncertainties are inherent to the model and, therefore, can be considered as aleatory. In addition, there is an epistemic uncertainty because of the limited basis for estimating the model parameters (see e.g. Robotham *et al.* (1995), Azzoni *et al.* (1995) and Chau *et al.* (2002) for an estimation of coefficients of restitution). Additional epistemic uncertainty is due to the simplified modeling of the slope profile at the impact location. In many applications, the profile surface in the models is generated from a digital elevation model (DEM) with limited resolution and between the points provided by the DEM, the terrain is assumed linear. If the model is two-dimensional, the reduction to a single plane is an additional source of epistemic uncertainty.

The outcome of a two-dimensional rock-fall model is presented in Figure 4. In this example, the relevant numerical result that will be utilized for risk assessment is the PDF of the energy of the rocks when reaching the road. This distribution should be evaluated conditional on the rock volume, $f_E(e|v)$, for different values of v. This can then be combined with the distribution representing the rock detachment. Available rock-fall analysis software typically allow entering the detached volume as a Normal distributed random variable, but because the volume of detached rocks is generally not Normal distributed, results obtained with this assumption cannot be used for risk assessment directly.

MCS in existing rock-fall trajectory analysis tools accounts only for the aleatory uncertainty. However, while it is important to be aware of the additional epistemic uncertainty associated with these models, for most practical applications, the error associated with neglecting this uncertainty is tolerable. This is because in the analysis of rock-fall trajectories, unlike in the modeling of rock-fall exposure, the probability of extreme events is of less importance, and the middle range of the distribution is less affected by the epistemic uncertainties.

Uncertainty in the performance of protection structures

Protection structures, such as flexible nets or fixed galleries, can stop the rocks, but their capacity is limited. This capacity, denoted by R, can be quantified in terms of the amount of energy that the structure can absorb. R depends on the type of structure, but also on the characteristics of the rock beyond the impact energy. The uncertainty in the capacity is considered by modeling R as a random variable, represented by its PDF conditional on the rock volume, $f_R(r|v)$. Hereby, the velocity of the rock at the impact is determined as a function of the energy and the volume. $f_R(r|v)$ should include both epistemic and aleatory uncertainty related to the structural capacity. Structural reliability analysis can be used to evaluate $f_R(r|v)$ for a given type of structure (Schubert *et al.* 2005). Alternatively, for standard protection systems, $f_R(r|v)$ can also be estimated from tests. However, because of their cost, the number of tests is often limited, therefore test results should be combined with a reliability analysis to obtain a probabilistic estimate of the capacity.

Uncertainties in rock-fall robustness

The robustness of the system is accounted for by estimating the expected consequences for a given failure event F_i, according to the second term of Equation (6). As an example, the expected number of fatalities and injuries is evaluated by multiplying the probability that a number of people are present at the location at the time of a rock-fall with the probability that somebody is killed or injured by the rock. These probabilities represent aleatory uncertainties. There is an uncertainty as to the values of these probabilities, which is of an epistemic nature (it could be reduced by collecting additional data), but because only the

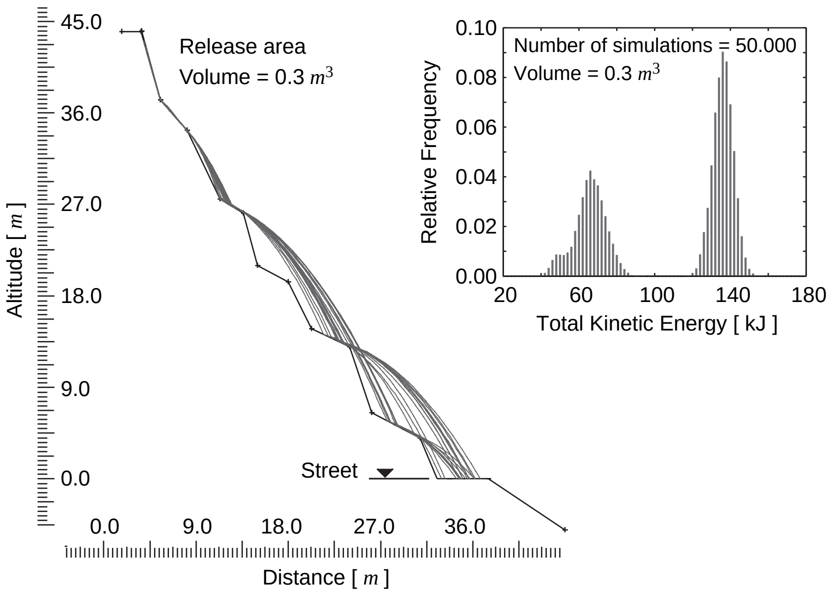

Figure 4. Illustration of the rock-fall trajectory modelling.

expected number of fatalities and injuries enters the computation, the computed risk generally will not be very sensitive to these epistemic uncertainties. In most instances, they can be neglected, as is done in practice.

An important part of system robustness modeling is the assessment of so-called 'user costs', representing the socio-economical costs inflicted by the temporary disuse of the considered system, typically a transportation link. The user costs as assessed by road authorities exhibit large differences (Nash 2003). However, it must not be concluded that these differences are due to epistemic uncertainty; rather, they are caused by different model assumptions. Therefore, this problem must be addressed by the decision maker, who must determine the model assumptions that represent his preferences.

Example – risk due to of rock-fall on an unprotected road

This example demonstrates the use of Bayesian updating for reducing model uncertainty. Consider the case of an unprotected road that is exposed to rock-fall events. In the first step, a parameter-based approach to determine rock-fall frequencies is applied with the parameters being geological and hydrological characteristics and topography. Let us assume that the resulting model can be described by an annual exceedance frequency $H_V(v)$ according to Equation (7) (power law), whose parameters are given in Table 1 (the parameters are assumed to be statistically independent *a-priori*). The model is shown in Figure 5. This model is hypothetical, but it represents a typical outcome of a parametric model, such as those referenced earlier, combined with an estimation of the epistemic model uncertainty. This uncertainty is large, reflecting the generic nature of such models; it is illustrated in Figure 5 by the 5% and the 95% quantile of $H_V(v)$. Note that the unconditional exceedance frequency shown in Figure 5 is computed according to Equation (8) and is not equal to the curve obtained by using the mean values of the parameter. For this reason, the model is not linear in the log–log scale.

In a second step, expert opinion is considered. Assume that the expert, upon looking at the assumptions in the parameter-based model, concludes that the model is over-estimating the rock-fall frequency because of local geological properties that are not included in the model (e.g. the orientation of the bedding is favorable). The geologist states that in his opinion the frequency is half that predicted by the model. Bayesian updating can be used to account for this statement, but this requires that the uncertainty in the geologist's statement is quantified. The statement of the geologist, which we denote by x hereafter, is related to rocks with a volume of $0.1\,\mathrm{m}^3$ or more. Therefore, this statement means that the geologist's best estimate for $x = H_V(0.1\,\mathrm{m}^3)$ is only 50% of the one given by the model, i.e. $\mu_X = 5.5$ year^{-1}, according to Figure 5. The uncertainty related to that statement is estimated to 50%, i.e. x having a coefficient of variation (c.o.v.) of 0.5 and x is represented by a Lognormal distribution. The prior model obtained from the parameter-based approach can then be updated with this information using Bayes' theorem (Equation (9)). Here, the likelihood function is

$$L(\boldsymbol{\theta}|x) = f_X[H_V(0.1\mathrm{m}^3|\boldsymbol{\theta})], \qquad (13)$$

with $f_X[x]$ the Lognormal PDF with mean μ_X and standard deviation $\sigma_X = 0.5\,\mu_X$.

Using a rock-fall simulation program, it is determined that, of all rocks detached, only $\lambda = 50\%$ actually land on the road, independent of the volume (this represents the system resistance). The epistemic uncertainty related to this model is neglected. The persons responsible for freeing the road from all rocks are now consulted. They state that during the past five years they removed on average three to four

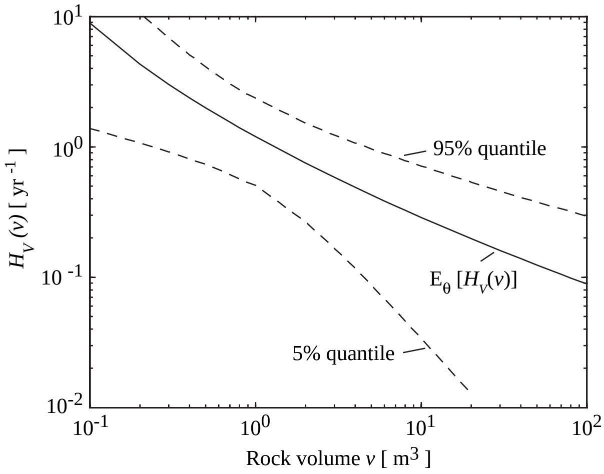

Figure 5. Exceedance frequency as obtained using the parameters in Table 1. The through line is the unconditional model, the dashed lines represent the 5% and 95% quantiles with respect to the uncertainty in the parameters, respectively.

Table 1. Parameters describing the annual exceedance frequency in example 1.

Parameter	Mean	Standard deviation	Distribution
A	1.2	0.6	Lognormal
B	0.7	0.3	Lognormal

rocks larger than $0.1\,m^3$ from the road. They remember only one rock that was larger than $1\,m^3$. This information, which we denote by y, is used to update the model again. The observation time is $\Delta T_y = 5$ year, the number of rocks smaller than $1\,m^3$ during that period is approximated by 17 ($\approx 3.5\,\text{year}^{-1} \cdot 5\,\text{year}^{-1}$). The likelihood function for this observation is obtained by noting that, if the rock-fall events follow a Poisson process, the number of observed rocks is described by a Poisson distribution. It can then be shown that

$$L(\boldsymbol{\theta}|y) \propto [H_V(0.1\text{m}^3|\boldsymbol{\theta}) - H_V(1\text{m}^3|\boldsymbol{\theta})]^{17} \\ H_V(1\text{m}^3|\boldsymbol{\theta}) \exp[-H_V(0.1\text{m}^3|\boldsymbol{\theta})\Delta T_y \lambda]. \quad (14)$$

The exceedance frequency model can be updated with this information. To this end, Bayes' theorem is applied again, with the earlier computed posterior distribution as the new prior distribution. The final posterior distribution is

$$f_q(\mathbf{q}|x, y) \propto L(\mathbf{q}|y) L(\mathbf{q}|x) f_q(\mathbf{q}). \quad (15)$$

Figure 6 shows the unconditional models of $H_V(v)$: (a) the original model, (b) after consideration of the geologist's expertise and (c) after updating of the model with the observations of the road maintenance personnel. Note that the final model (c) is close to a straight line in the log–log-plot of Figure 6. This indicates a reduction of the epistemic uncertainty when the model is updated with the additional observations because without epistemic uncertainty, $H_V(v)$ would be linear in a log–log-scale.

With the final exceedance frequency model, the risk for this road segment can now be evaluated by considering system robustness. This includes the probability of a traffic accident because of the rocks, the cost of road closures and traffic delay, the cost of removing the rocks as well as the additional risk to road maintenance personnel while removing them. For the sake of the example, consider the case where fatalities are the only relevant consequence. The expected number of fatalities as a function of rock volume can be estimated as a function of the daily traffic volume, accounting for the event of a direct hit of a vehicle or a pedestrian and the event of an accident caused by a rock lying on the road. For the considered example, the expected number of fatalities is estimated as

$$\begin{array}{ll} C_T(v) = 0 & v \leq 0.1\text{m}^3 \\ C_T(v) = 0.01 & 0.1\text{m}^3 < v \leq 1\text{m}^3. \\ C_T(v) = 0.03 & 1\text{m}^3 < v \end{array}$$

The risk, i.e. the expected number of fatalities per year, is then calculated as

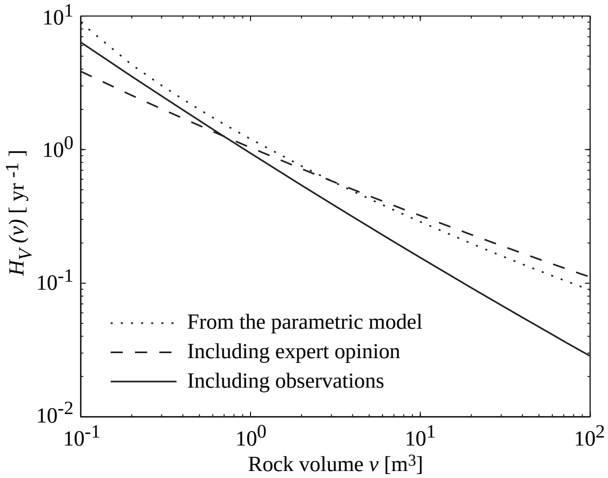

Figure 6. Exceedance frequency as obtained (a) using a parameter-based model; (b) updated the model with geologist's expertise; (c) updated using observations from road maintenance personnel.

$$R = \int_V C_T(v) h_V(v) dv \\ = 0.01 \cdot [H_V(0.1\text{m}^3) - H_V(1\text{m}^3)] + 0.03 \cdot H_V(1\text{m}^3) \quad . \quad (16)$$

This risk is computed as 0.085 fatalities per year. It is possible to reduce the risk, e.g. by installing rock-fall protection or by partial closure of the road. The effect of these actions can be investigated by updating the models again, taking into account the effectiveness of the measure. In the next example, a detailed study of the effectiveness of a protection measure is performed.

Example – reliability of a rock-fall protection gallery

In this study, a rock-fall protection gallery in the Swiss Alps, built in 1975 is investigated. The structure is part of a main transit route through the Alps, and has been subjected to several rock-fall events in recent years. In the 1970s, when many of these structures were built in the Swiss Alps, no analytical tools were available for modeling rock detachment or for modeling the failing process. Decision-making was based purely on engineering judgment. For the considered structure, the geologist had defined a release zone and a 'maximum' stone volume. To design the protection structure, a free fall of the stone was assumed and the corresponding energy was calculated.

This example demonstrates the reliability assessment for this protection structure. The goal of this analysis is to establish a rational basis for determining the need to strengthen the structure. A similar analysis can also be performed for a new structure to optimize the design. Figure 7 shows a representative cross section of the considered area, with the identified release zones and the gallery. The gallery, which connects two tunnels, has a length of 217 m.

Since no reliable data and no models are available for this site, rock-fall frequency is modeled based on expert judgment. The results of a geological investigation, describing the lithology together with an estimate of the frequency of rock-fall events, are available for the site. The relevant release areas comprise of limestone. It is estimated that rocks with volume $v < 0.5\,\mathrm{m}^3$ are detached with high frequency, rocks with a volume of $0.5\,\mathrm{m}^3 < v < 5\,\mathrm{m}^3$ with a moderate frequency, rocks with a volume of $5\,\mathrm{m}^3 < v < 10\,\mathrm{m}^3$ with a small frequency, and larger volumes detach only with a very small frequency. The geologist also provides a quantitative interpretation of the frequencies. An event is considered to have a high frequency if it occurs daily to monthly, a moderate frequency is understood by the geologist as a return period between 1 and 20 years, a small frequency as a return period between 20 and 100 years, and a very small frequency as a return period of more than 100 years. These frequencies correspond to rock-fall events within the entire length of the gallery. From this information, the models in Table 2 are derived. To account for the uncertainty in the estimates, a distribution is assigned to the occurrence frequencies of the four categories in accordance with

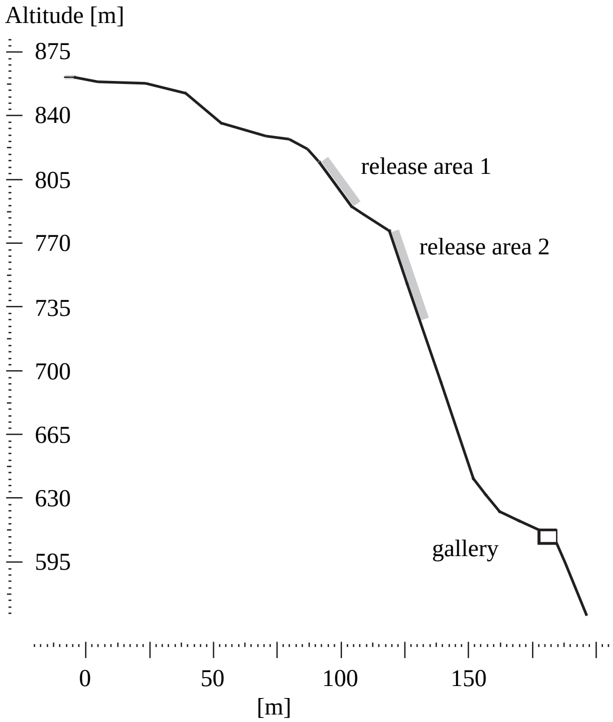

Figure 7. Profile of the slope in the considered example.

the ranges stated by the geologist. For the quantitative analysis, it is assumed that the uncertainty can be characterized by Lognormal distributions.

We assume that the exceedance frequency can be adequately represented by the power-law (Equation (7)). To estimate the parameters $\theta = [a,b]^{\mathrm{T}}$, the likelihood function corresponding to the information provided in Table 2 is established as:

$$L(\boldsymbol{\theta}|\mathbf{z}) \propto \prod_{i=1}^{4} f_Z[H_V(v_i|\boldsymbol{\theta})|\lambda_i, \zeta_i], \tag{17}$$

where $f_Z[H_V(v_i|\boldsymbol{\theta})|\lambda_i, \zeta_i]$ denotes the Lognormal PDF with argument $H_V(v_i|\theta)$ and parameters λ_i and ζ_i.

Here, as an alternative to the Bayesian analysis, maximum likelihood estimation (MLE) is applied to determine the parameters $\boldsymbol{\theta}_{\mathrm{MLE}}$, whereby a parameter estimate is obtained by maximizing the log-likelihood function:

$$\boldsymbol{\theta}_{\mathrm{MLE}} = \operatorname{argmax}\{\ln[L(\boldsymbol{\theta}|\mathbf{z})]\}. \tag{18}$$

The full distribution of the parameters $f(\boldsymbol{\theta})$ can be approximated by a multivariate Normal distribution with mean value $\boldsymbol{\theta}_{\mathrm{MLE}}$ and a covariance matrix equal to the inverse of the Hessian matrix of the log-likelihood function evaluated at $\boldsymbol{\theta}_{\mathrm{MLE}}$ (Lindley 1965). The resulting probabilistic model of the parameters $\boldsymbol{\theta}$ is provided in Table 3. The resulting exceedance frequency is shown in Figure 8.

To model the falling process, a two-dimensional rock-fall trajectory model is applied, with the representative profile shown in Figure 7. The RocFall (2001) software, based on a two dimensional model with a lumped mass approach, is used. This software allows modeling all input parameters as Normal distributed random variables and performs a MC simulation to determine the distribution of trajectories and associated velocities, impact energies and run-off distances. The analysis was repeated for different values of the rock volume v in the range from zero to 1000 m^3, even though these models are not realistic for large volumes. To account for the variability in the geometry over the length of the slope, the X and Y co-ordinates of the slope are modeled as Normal distributed parameters (Stevens 1998). Since the investigated gallery connects two roadway tunnel segments, rock-falls outside the range of the gallery have no consequences for the road. The distribution of detached rocks in the two release zones (Figure 7) is modeled by a uniform distribution. Additional details on the modeling, such as the coefficients of restitutions, can be found in Schubert *et al.* (2005). The outcome of the MC simulations is the distribution of impact energy on the gallery for given volumes, $f_{\mathrm{E}}(e|v)$, in a similar way as illustrated in Figure 4. Together with the distribution of the

Table 2. Summary of the geologist's expert judgment characterized by Lognormal distributions of the frequency for the different rock volume ranges.

	Volume range (m^3)	Representative volume (m^3)	Estimated frequency for range i:Z_i ($year^{-1}$) (LN distributed)			
			Mean	c.o.v.	Parameters	
Rock-fall	0.0–0.5	0.2	4.0	0.5	$\lambda_1 = 1.28$	$\zeta_1 = 0.47$
Rock-fall–block-fall	0.5–5.0	1.5	0.25	0.8	$\lambda_2 = 1.63$	$\zeta_2 = 0.70$
Block-fall	5.0–10.0	7	0.025	0.8	$\lambda_3 = -3.93$	$\zeta_3 = 0.70$
Block-fall–landslide	10–1000	100	0.004	0.8	$\lambda_4 = -5.$	$\zeta_4 = 0.70$

annual maximum rock volume $f_V(v)$ according to Equation (2), $f_E(e|v)$ represents the load acting on the structure.

The resistance of the structure is modeled by a mechanical model for punching failure, as presented in Schubert *et al.* (2005). For most galleries of this type, punching failure is the predominant failure mode. It is worth noting that in the original design, the punching failure mode was not considered, since it was not included in codes and standards at the time. The gallery roof is built of reinforced concrete and has a protective cushion layer on top of the concrete structure, a compound of sand and gravel, which mitigates so-called hard impacts and reduces the energy transmitted between the rock and the concrete. The concrete slab has a thickness of 50 cm and the cushion layer has an average thickness of 75 cm. At the impact of the rock mass, the protective cushion layer is plastically deformed, thus dissipating energy of the impact, and distributes the load to a larger area of the concrete slab. According to Bucher (1997), the influence of the dynamic impact on the stiffness of the structure can be neglected. Hence, it is possible to separate analysis of the structural behavior from the analysis of the impact. A procedure for the calculation of static equivalent loads from the dynamic structural analysis of the impact has been formulated and verified in tests by Montani (1996). With this equivalent load approach, $\Pr(F|e,\ v)$, the conditional probability of a gallery failure due to punching can be calculated for given deterministic values of the energy e and the volume v of the rock, using structural reliability analysis (SRA) (Madsen *et al.* 1986). This accounts for all uncertainties in the modeling of the gallery performance. The details of the probabilistic modeling are given in Schubert *et al.* (2005).

Table 3. Moments of the bi-normal distribution of the parameters of $H_V(v|\boldsymbol{\theta})$.

Parameter	Mean	Covariance
$\boldsymbol{\theta} = \begin{bmatrix} a \\ b \end{bmatrix}$	$\begin{bmatrix} 0.302 \\ 1.21 \end{bmatrix}$	$\begin{bmatrix} 9.23\ 10^{-3} & 3.59\ 10^{-3} \\ 3.59\ 10^{-3} & 1.63\ 10^{-2} \end{bmatrix}$

By noting that $\Pr(F|e,\ v) = \Pr(R \leq e|v) = F_R(e|v)$, we can assess the distribution of the gallery capacity in terms of the impact energy for given volumes v as

$$f_R(e|v) = \frac{d\Pr(F|e,v)}{de}. \quad (19)$$

Figure 9 shows $f_R(e|v)$ for given rock-fall volumes of $v = 0.5$ m^3 and $v = 15$ m^3 together with the PDF of the rock-fall energy given these volumes $f_E(e|v)$. Hereby the marginal annual probability of the rock-fall volume exceeding 0.5 m^3 is equal to $1 - F_V(v = 0.5\ \mathrm{m}^3) = 0.72\ \mathrm{year}^{-1}$ and that exceeding 15 m^3 is $1 - \mathrm{FV}(\mathrm{v} = 15\ \mathrm{m}^3) = 3.5 \cdot 10^{-4}\ \mathrm{year}^{-1}$. The large variance of $f_R(e|v)$ depicted in Figure 9 reflects the large uncertainties in the modeling of the punching failure mode.

The total annual failure probability $\Pr(F)$ is then assessed by calculating the expected value of the

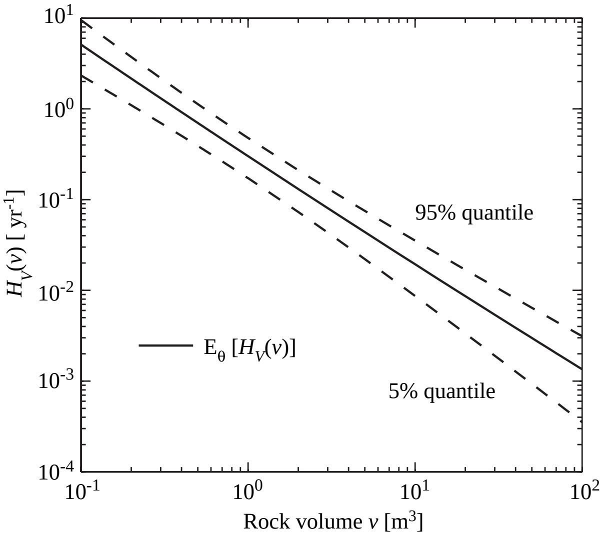

Figure 8. Exceedance frequency H_V and the 5% and 95% quantiles with respect to the epistemic uncertainty represented by $f(\boldsymbol{\theta})$.

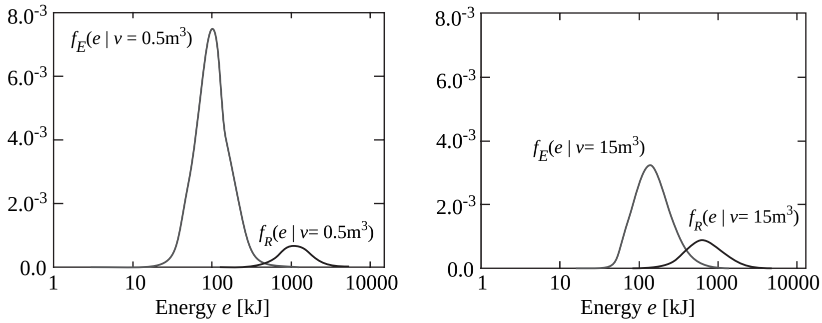

Figure 9. Probability density functions for the maximal annual energy and the capacity of the gallery given the rock-fall volumes ($v = 0.5$ m^3 (left) and $v = 15$ m^3 (right)).

probability of failure with respect to the energy and the volume of the rocks:

$$\Pr(F) = \int_0^\infty \int_0^\infty \Pr(F|e, v) f_E(e|v) f_V(v)\, de\, dv. \tag{20}$$

The annual probability of failure of the protection structure, calculated from Equation (20), is $\Pr(F) = 6.3 \cdot 10^{-3}$ year^{-1}. It is observed that the original design load (a design energy of 1717 kJ) has an annual exceedance probability of only $3.4 \cdot 10^{-4}$ year^{-1}, a highly conservative assumption. However, as punching failure was not considered in the original design, the structure almost certainly fails at that design load. This explains why the probability of failure is higher than the probability of occurrence of the design load.

The decision maker, in this case the road authorities, must now decide if the probability of failure of the protection structure is acceptable. To this end, the system robustness and the consequences associated with these events must be evaluated to compute the risk associated with a failure of the protection gallery. These risks must then be compared with the cost of improving the capacity of the structure. If a strengthening of the structure is found not to be economical, then the fatality risk for people using the road must be compared with societal acceptance criteria to determine acceptability of road safety (Rackwitz 2000).

Concluding remarks

Rock-fall is a highly site-specific phenomenon, which involves many parameters that vary in space and time. It is not possible to determine all these parameters, and rock-fall assessment must therefore be based on simplified physical models, empirical models and expert judgment. Inevitably, the resulting rock-fall estimates are subject to large uncertainties. To ensure rational decision-making, it is essential that these uncertainties be explicitly addressed by the models within a single framework to ensure consistency. In this paper, we outlined a framework that can deal with the various sources of uncertainty. Existing procedures for rock-fall assessment typically account for some of the uncertainties, but in particular, epistemic uncertainties (i.e. uncertainties related to limited knowledge) are commonly neglected. The first example demonstrates how the explicit modeling of epistemic uncertainties facilitates the combination of different models and observations using Bayesian analysis into a single rock-fall model. Such an approach enables a consistent multi-scale approach to modeling rock-fall hazards, which makes maximum use of all available information from different sources and at different degrees of detailing.

In this paper, the focus is on the representation of uncertainties in the modeling of rock-fall hazard and the performance of protection structures. Ultimately, the models presented here serve to identify the optimal set of actions, which is the one that maximizes the expected utility while ensuring compliance with relevant risk acceptance criteria. As indicated by the second example presented, the uncertainty models, combined with mathematical tools such as structural reliability analysis, facilitate such an optimization. However, for certain risk mitigation actions, more refined hazard models will be required. As an example, when considering organizational measures, such as temporary closure of roads or monitoring of rock-fall activity, the model must address the temporal variability of the rock-fall activity.

The examples presented in this paper are based on certain assumptions that may not hold for all applications. In particular, approximating rock-falls by a Poisson process will not always be a reasonable assumption because several rocks can be detached during a single event. A more realistic modeling may include the distribution of the number of detached rocks, conditional on the event of detachment.

However, the suitability of any assumption depends strongly on the decision problem, i.e. on the considered risk reduction measures. Different models are thus required for different problems and the framework presented in this paper should be seen as a guideline that must be adjusted to the specific decision problem at hand. In this context, it is highly relevant that the engineer in charge understands the assumptions underlying the applied models. We hope that this paper contributes to this understanding.

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