Bandwidth Efficient and Rate-Matched Low-Density Parity-Check Coded Modulation

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Outline

- Motivation: Bandwidth-Limited Communication
- Contribution: A New Coded Modulation Scheme

- Enabeling Technology: Fixed-Length Distribution Matching

Bandwidth-Limited Communication

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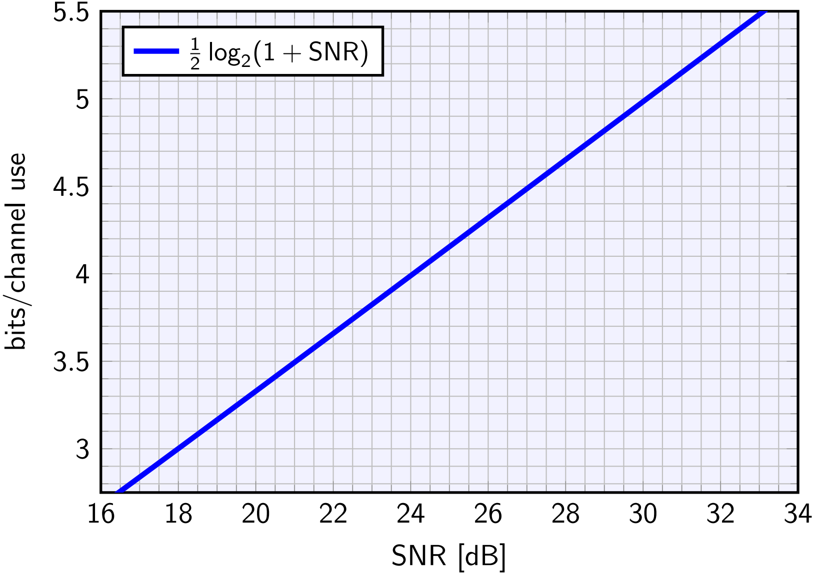


Higher Order-Modulation

- **Equidistant 2^m-ASK constellation:**

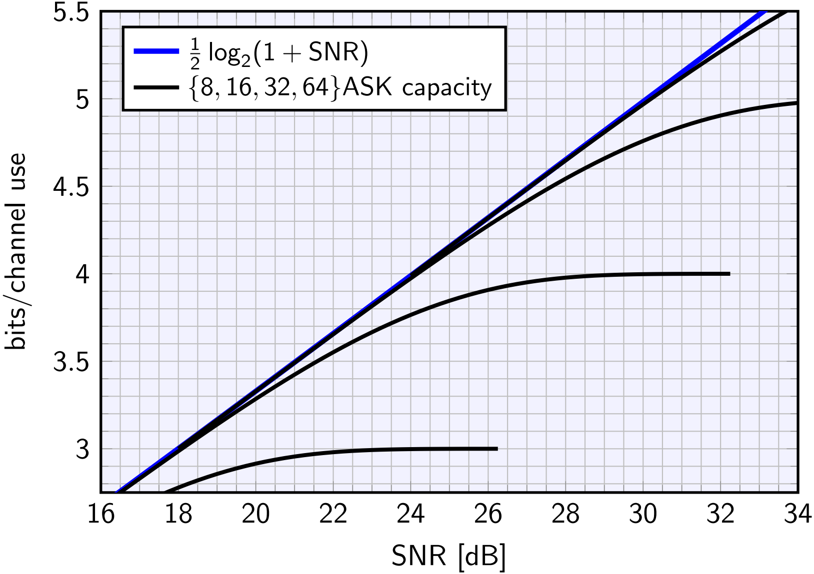
$$\mathcal{X} = \{\pm 1, \pm 3, \ldots, \pm(2^m - 1)\}.$$

- **I/O-relation:**

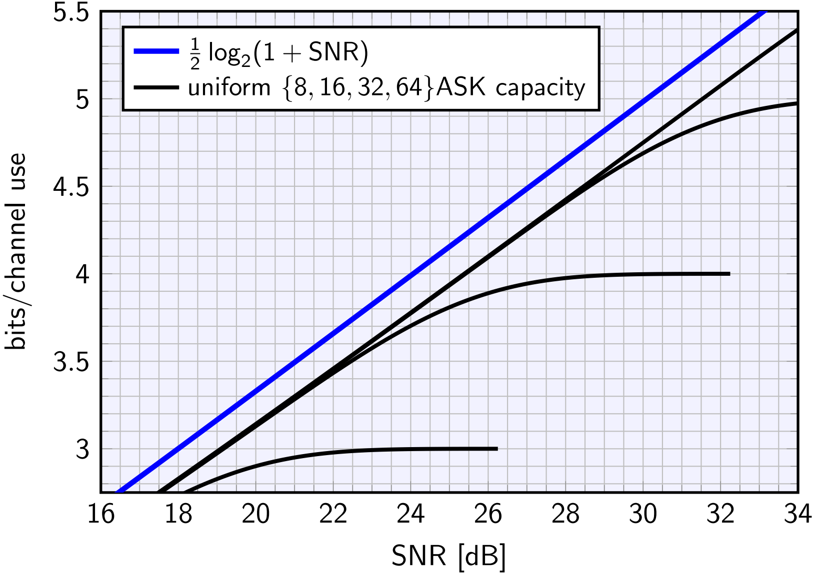
$$Y = \Delta \cdot X + Z.$$

  - Noise Z is zero mean, variance one.
  - Input X with distribution P_X on $\mathcal{X}$.
  - Δ scales the constellation $\mathcal{X}$.

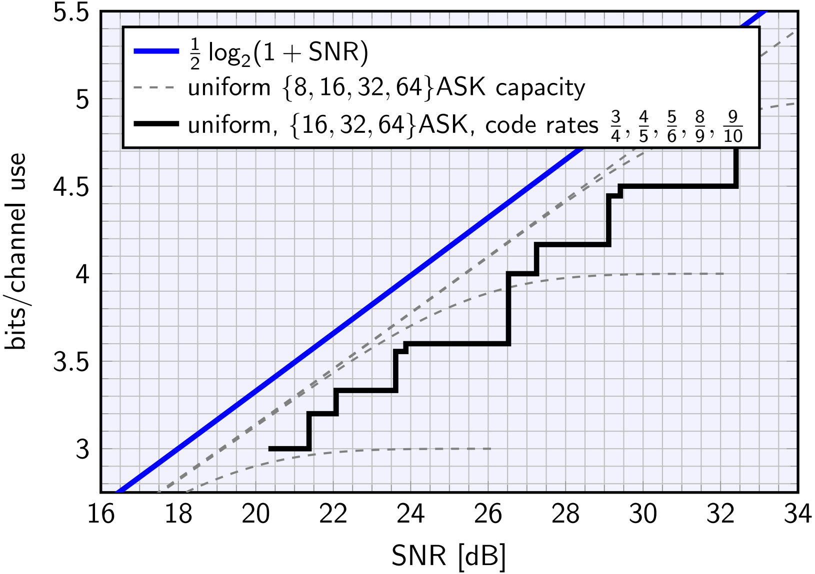
Bandwidth-Limited Communication



Bandwidth-Limited Communication



DVB-S2 LDPC Codes, Uniform Input, FER$\leq 10^{-3}$



Probabilistic Amplitude Shaping

ASK Capacity

- **ASK capacity:**

$$\mathsf{C}_{\mathsf{ask}}(\mathsf{P}) = \max_{\Delta, P_X :\ E[|\Delta X|^2] \leq \mathsf{P}} \mathsf{I}(X; \Delta X + Z).$$

- Capacity-achieving distribution P_{X^*}.

$\Rightarrow$ Construct a codebook with entries $\sim P_{X^*}$.

Capacity-Achieving Distribution P_{X^*}

- P_{X^*} is **symmetric**:

$$P_{X^*}(x) = P_{X^*}(-x).$$

- P_{X^*} **factorizes:**

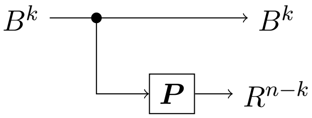
$$P_{X^*}(x) = P_S(\mathrm{sign}(x)) \cdot P_A(|x|)$$

where $A := |X|$ and $S := \mathrm{sign}(X)$.

- The sign S is **uniform**:

$$P_S(-1) = P_S(1) = \frac{1}{2}.$$

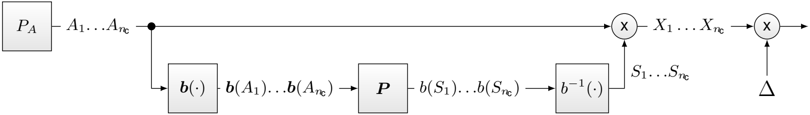
Shaping and Error Correction



- Binary systematic $k \times n$ generator matrix $\mathbf{G} = [\mathbf{I}|\mathbf{P}]$.
- Binary information B^k arbitrarily distributed.
- Uniform check bit assumption:

$$R_1, R_2, \ldots \text{ iid} \sim \text{Bernoulli}(1/2).$$

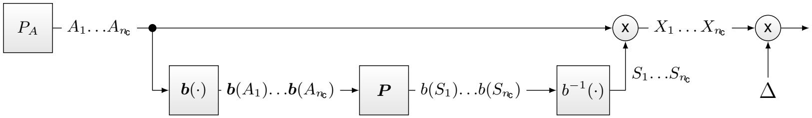
Probabilistic Amplitude Shaping



- n_c number of channel uses.
- Represent amplitudes by $(m-1)$ bits: $A \mapsto \mathbf{b}(A)$.
- Represent signs by 1 bit: $S \mapsto b(S)$.
- Multiply amplitude labels by parity matrix $\mathbf{P}$ to get sign labels.
- Code rate $c = (m-1)/m$.

- $P_{X_i} = P_{X^*}$!

Rate Compatibility



- Control rate $\mathsf{H}(A)$ via P_A.
- Control power $\mathsf{E}[|\Delta X|^2]$ via Δ.

Enabeling Technology: Distribution Matching

Emulating discrete memoryless source P_A:

- B_i iid Bernoulli(1/2).
- $\bar{A}_i$ approximately iid $\sim P_A$ in the sense of

$$\frac{\mathrm{D}(P_{\bar{A}^n} \| P_A^n)}{n} \overset{n\to\infty}{\to} 0, \qquad \frac{k}{n} \overset{n\to\infty}{\to} \mathrm{H}(A)$$

- invertible: B^k can be recovered from $\bar{A}^n$ with zero error.

Constant Composition Codes

The empirical distribution of a output sequence $\mathbf{c}$ of length n is

$$P_{\mathsf{A},\mathbf{c}}(a) := \frac{n_a(\mathbf{c})}{n},$$

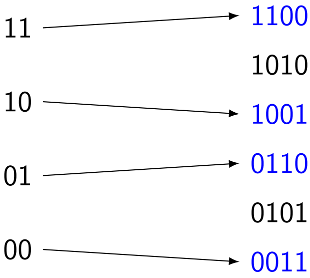
- $n_a(\mathbf{c}) = |\{i : c_i = a\}|$ number of occurences of symbol a in $\mathbf{c}$
- $P_{\mathsf{A},\mathbf{c}}$ is the *type* of $\mathbf{c}$
- $\mathcal{T}^n_{P_{\mathsf{A}}}$ is set of all n-type P_{A} sequences
- Codebook $\mathcal{C}_{\text{ccdm}} \subseteq \mathcal{A}^n$ is called a *constant composition code* if all codewords are of the same type, i.e. $\mathcal{C}_{\text{ccdm}} \subseteq \mathcal{T}^n_{P_{\mathsf{A}}}$

Constant Composition Distribution Matching

Idea

Use codewords of the same type

- Choose sequence length of the matcher output n
- Choose n-type approximation $P_{\bar{\mathsf{A}}}$ of the target distribution P_{A}
- Choose $m = \left\lfloor \log_2 \left| \mathcal{T}^n_{P_{\bar{\mathsf{A}}}} \right| \right\rfloor$
- Construct a unique mapping $\{0,1\}^m \to \mathcal{T}^n_{P_{\bar{\mathsf{A}}}}$



Bit-Metric Decoding (1)

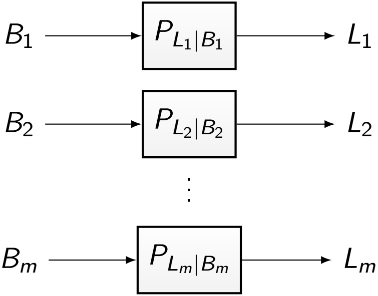
- $\text{label}(X) = b(S)\mathbf{b}(A) = B_1 \cdots B_m$.
- Demapper soft-decision:

$$L_i = \log \frac{P_{B_i|Y}(0|Y)}{P_{B_i|Y}(1|Y)}, \quad i = 1, \ldots, m.$$

- No iterative demapping!

Bit-Metric Decoding (2)

- To the receiver, the channel appears as



Achievable Rate (2)

- Achievable rate:[1]

$$\mathsf{R}_{\mathsf{BMD}} = \mathsf{H}(\mathbf{B}) - \sum_{i=1}^{m} \mathsf{H}(B_i|L_i)$$

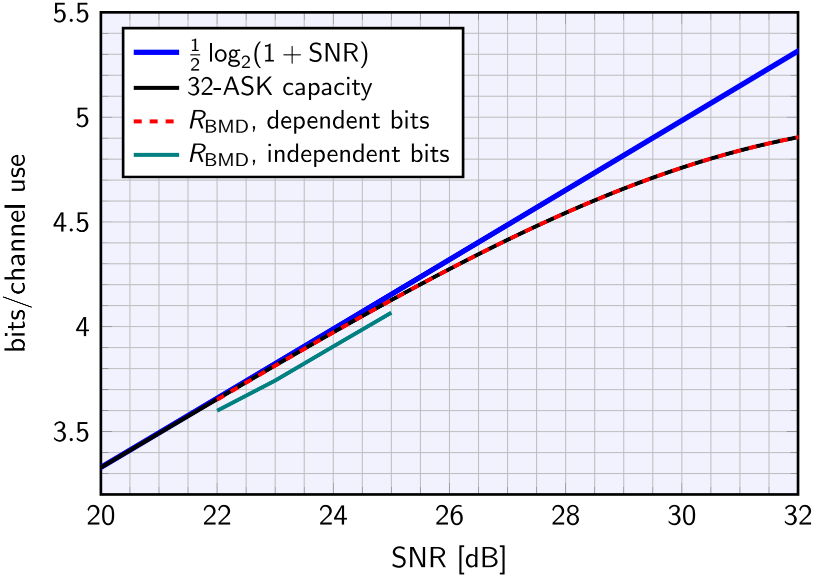
$$= \underbrace{\sum_{i=1}^{m} \mathsf{I}(B_i; L_i)}_{(\star)} - \mathsf{D}(P_{\mathbf{B}} \| \textstyle\prod_i P_{B_i}).$$

- $(\star)$ known as “BICM capacity”, “pragmatic capacity”,...[2]

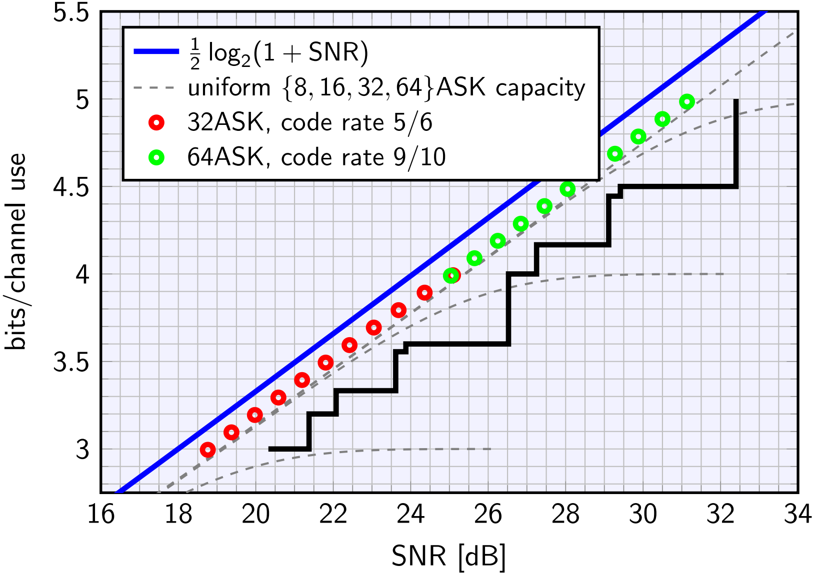
[1] G Böcherer Achievable Rates for Shaped Bit-Metric Decoding, http://arxiv.org/abs/1410.8075.

[2] Guillen i Fabregas, Martinez Bit-interleaved coded modulation with shaping, ITW 2010.

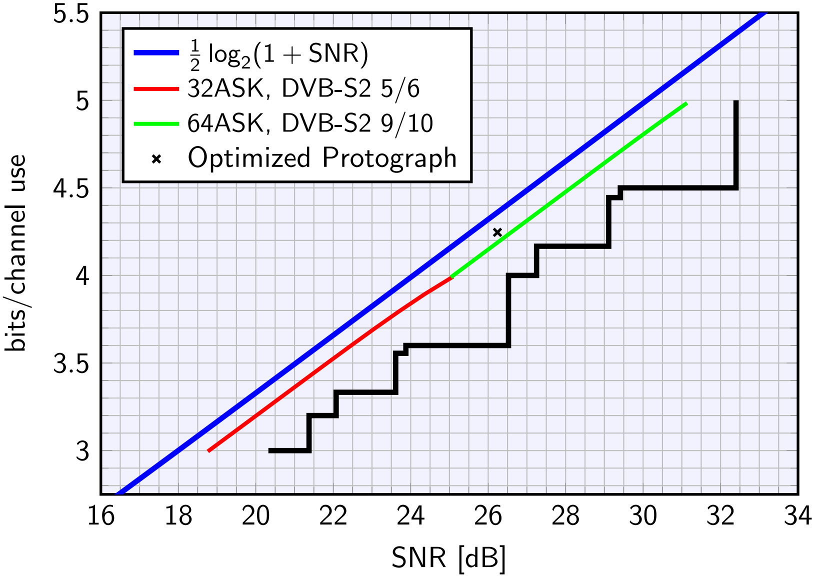
Achievable Rate (3)



Probabilistic Amplitude Shaping with DVB-S2 Codes FER $\leq 10^{-3}$



FER 10^{-3} within 0.69 dB of Capacity



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- P.Schulte, G Böcherer: Constant Composition Distribution Matching, http://arxiv.org/abs/1503.05133.
- G Böcherer Achievable Rates for Shaped Bit-Metric Decoding, http://arxiv.org/abs/1410.8075.
- F. Steiner, G Böcherer, G. Liva Protograph-Based LDPC Code Design for Bit-Metric Decoding, ISIT 2015.