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A Tutoring System for Fourier-Transforms in Electrical Engineering

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ABSTRACT

The Fourier-Transform, a basic discipline in electronic engineering, has two aspects, a mathematical one and a technical one. With the use of standard Fourier-Pairs and Fourier-Transform rules, thought patterns are supplied for the understanding of signals in the context of technical systems.
As an educational aid a tutoring system has been developed covering the technical range of the topic Fourier-Transform. The knowledge base and the inference module of this system are implemented using the computer algebra system MATHEMATICA. With each implemented rule additional information is stored to explain the steps taken by the system. After describing the components of the tutoring system, two applications in teaching will be demonstrated.

1. Introduction

System-Theory is a basic discipline in electronic engineering and is applied in topics like controlling technical systems, information and communication technology, circuit analysis, filter design, speech and image processing, etc.[1]. Part of the System-Theory is the Fourier-Transform, which describes the relation between a signal $x(t)$ and its spectrum $X(\omega)$ by the following pair of equations:

Fourier-Transform (analysis)	$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt = FT\{x(t)\}$
Inverse Fourier-Transform (synthesis)	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t} d\omega = IFT\{X(\omega)\}$

The Fourier-Transform has two different aspects: a mathematical one and a technical one [2].

- Solving the Fourier-Integrals, finding rules for inverting the Fourier-Transform or the connection of common attributes between signal and spectrum is the mathematical range. For solving the Fourier-Integrals computer algebra systems offer a useful tool.
- In the technical range the Fourier-Theory supplies thought patterns for the understanding of signals in the context of technical systems. Reference books covering the technical aspect of the Fourier-Theory solve problems not by the mathematical approach but by using already known (standard) Fourier-Pairs and Fourier-Transform rules [1].

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As an educational aid for electrical engineering students, a tutoring system based on the technical aspect of the Fourier-Theory was developed by using the computer algebra system MATHEMATICA [3].

In the following sections the architecture of the tutoring system is presented and two applications are illustrated.

2. Components of the tutoring system

The block diagram of the tutoring system, which corresponds to the general architecture of an expert system, is illustrated in Fig. 1 [4].

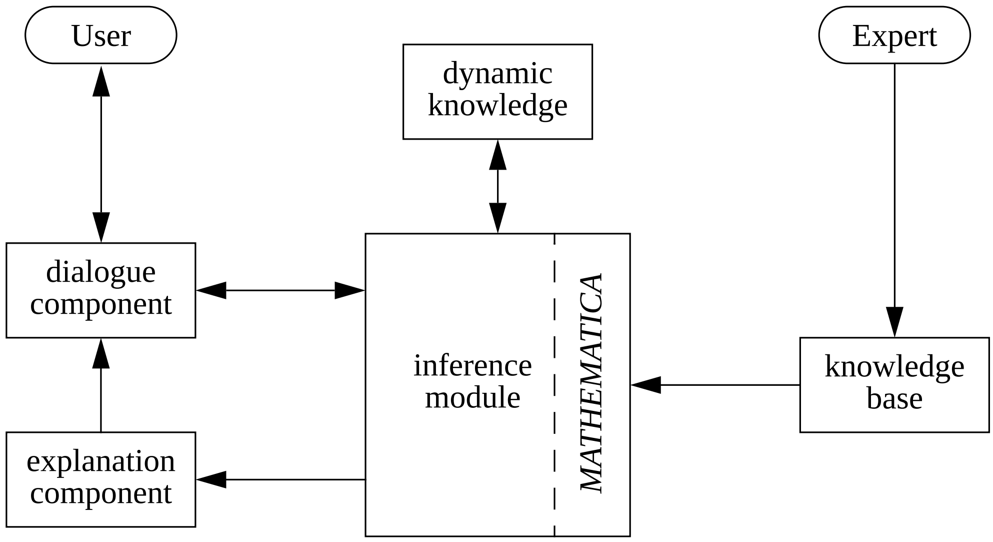

Figure 1: Block diagram of the tutoring system

The three components knowledge base, inference module and dynamic knowledge are necessary for task solving. The dialogue component handles the interaction between user and system, the explanation component justifies the steps within the solution process.

2.1 Knowledge base

The knowledge base contains the expert knowledge available to the system. This knowledge is stored in the form of facts and rules and allows the system to act as an expert [5].

In our tutoring system the knowledge is represented by production rules

IF "condition" THEN "action"

in MATHEMATICA syntax covering general mathematical transformations as well as standard Fourier-Pairs and Fourier-Transform rules.
Altogether, 176 rules are implemented in the knowledge base. The production rules relating to the Fourier-Theory cover the textbook written by Oppenheim and Willsky[1].

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Additional information is stored with each production rule:

- the number of the rule for identification.
- the assignment of the rule to one out of the areas
  general mathematical transformation,
  Fourier-Transform rule, or
  standard Fourier-Pair.
- the description of the rule in textual form.
- an integer number corresponding to the difficulty a human expert may have by applying the rule.

Table. 1 gives an example of an implemented rule and its additional information. The syntax used for (Inverse) Fourier-Transform is $FT[x[t], t, w]$ ($IFT[X[w], w, t]$), in which $x[t]$ ($X[w]$) denotes the signal (spectrum), t the time and w the frequency variable. For a better understanding, the rule is not represented in MATHEMATICA language.

Table 1: Implemented rule and its additional information

production rule:	IF ($FT[\delta[t-\tau], t, w]$ is element of *expression*) AND ($\tau \neq f[t]$) THEN replace $FT[\delta[t-\tau], t, w]$ by $Exp[-jw\tau]$ in *expression*
number:	418
group:	Standard Fourier-Pair
description:	Fourier-Transform of a (shifted) Dirac-Impulse: $FT[\delta[t-\tau], t, w] = Exp[-jw\tau]$
level of difficulty:	1

2.2 Inference module

The inference module of the tutoring system has two areas of responsibility:

- Deduction of new, task-oriented knowledge by use of the static knowledge base:
  The inference module transfers an actual problem into a modified problem by making use of the applicable rules stored in the knowledge base. Therefore in connection with expert systems the inference module is sometimes called the problem-solving component due to its capability to apply the given rules to a problem [4].
  Normally, each applicable rule is used by the inference for deduction, but sometimes it might be necessary to use only a certain selection. The supervision of this selection process is done by meta-knowledge, i.e. information the system has about its own knowledge. When using meta-knowledge, the system is looking at its own operations from the viewpoint of an outside observer.
  In our tutoring system the deduction process is carried out using MATHEMATICA, the communication between MATHEMATICA and the remaining components of the tutoring system is done by using MATHLINK [6].

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- Search strategy:
  The search strategy determines the processing order of the deduced problems. What kind of strategy is used depends on the purpose [7]. For searching some path from a given task to a goal search algorithms such as depth-first search, breath-first search, beam search or best search are used. By applying search strategies like the British Museum procedure or dynamic programming the optimal path is determined. Furthermore, there are a lot of algorithms used in games like chess.
  The search strategies used in our tutoring system are the breadth-first search, the depth-first search and a modified British Museum procedure.

2.3 Dynamic knowledge

The dynamic knowledge is the task-oriented knowledge generated by applying the rules in the knowledge base to a given task. Frames are used to present this knowledge. A frame is a data-structure for representing a stereotype attached with several kinds of information [8]. In the tutoring system we use two kinds of frames:

- Nodes:
  A node describes a task-specific problem (the task itself or one of the deduced problems). Furthermore, information is stored like the rules which led to this problem or the rules which can be applied to the problem.
- Edges:
  An edge connects a pair of nodes and contains the information about the rule which was used to transfer the problem represented by one node into the new problem represented by the second node.

These two kinds of frames are used to generate a directed graph containing the task-oriented knowledge.

2.4 Explanation component

The explanation module is the heart of any tutoring system. Beyond simply solving a given task, a tutor (or an expert) is also capable of explaining the reasoning that led to the solution. The explanation consists typically of an identification of the steps in the solution process and a justification for each step [5].

The information available by the explanation component of our tutoring system is a combination of the additional rule information stored in the knowledge base and the task-oriented knowledge generated by solving a task.

2.5 Dialogue component

The dialogue component receives information from the user and translates it into a form acceptable to the remainder of the system or obtains information from the system and converts it into a form that can be understood by the user.

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3. Application of our tutoring system

In the following we describe two application examples for the tutoring system.
At first, the system appears in the role of a teacher, who is solving a task defined by the user.
After that, based on the same principle, the system acts as a proof-reader verifying steps contained in the user's solution.

3.1 Working out solutions

To carry out the job of a teacher, the tutoring system needs the capability to

- find the solution of a task defined by the user.
- explain the solution method.
- suggest different methods for solving.
- evaluate the effectiveness of the different methods of solution.

In connection with our tutoring system, a task presented out of the topic Fourier-Transform is a mathematically formulated problem containing the Fourier-instructions $FT[x[t], t, w]$ and/or $IFT[X[w], w, t]$. A solution of a task is defined as a deduced problem including no more of these Fourier-instructions.

The search strategy within the inference module of the tutoring system is a modified British Museum procedure based on breadth-first search.
Using the breadth-first search would result in detecting some path between the task and solution because the search process stops if the solution is reached the first time. Therefore, the tutoring system would not be able to suggest alternative methods of solving the task.
On the other hand, the Museum procedure determines all possible paths, in which the optimal path is included. This means, in comparison to the breadth-first search, the search process stops if no more new problems are deducible by applying the rules within the knowledge base. Unfortunately, this procedure has a combinatorial problem; even if the task is quite simple, the corresponding solution graph is increasing exponentially and may exceed the memory of the computer.
The modified British Museum procedure implemented in the tutoring system avoids this combinatorial problem. By using meta-knowledge during the search process the search itself is supervised and controlled. This means that possible solution paths are not traced any longer if their effectiveness is too small in comparison to the optimal path already detected.

The effectiveness of a solution path is determined by an overall path difficulty. This overall difficulty is calculated by summing up the difficulties of the used rules within this path. The optimal path (relative to its effectiveness) is classified as the path with the minimum overall difficulty. If different paths exist with the same overall difficulty, the shorter paths are preferred.

Using the tutoring system as a teacher means that the user defines a task, the system works on this problem and stores all information within the dynamic knowledge. After the search is finished, the solution graph is available as task-oriented knowledge.

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For an example, Fig. 2 shows the generated solution graph for the task $FT[\delta[2 \cdot t + 1], t, w]$ (no meta-knowledge was necessary for limitation of the graph). The rectangles and the corresponding expressions represent the task and the deduced problems, the edges represent the rules transferring one node to another.

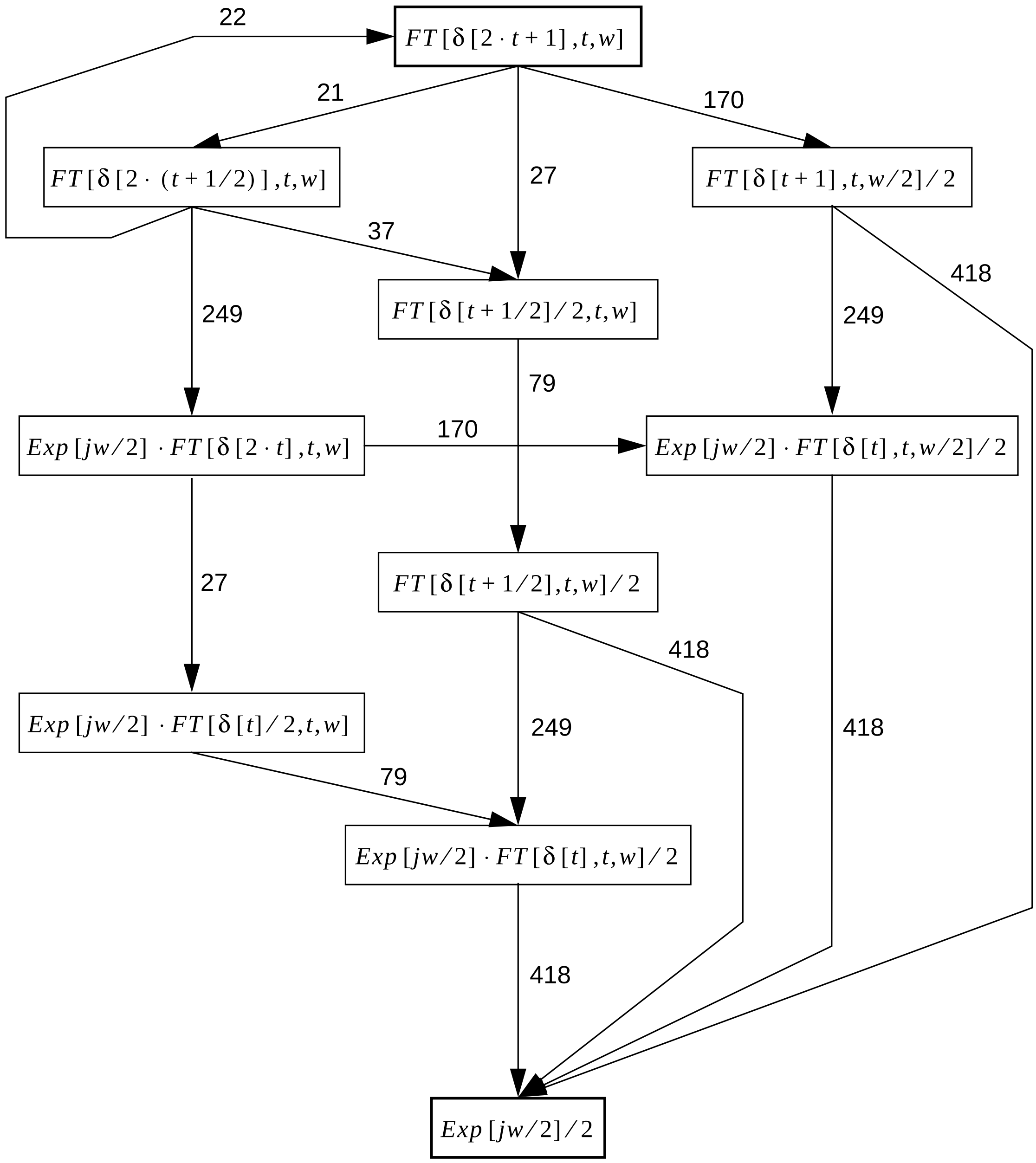

Figure 2: Solution graph generated by solving the task $FT[\delta[2 \cdot t + 1], t, w]$; next to the edges the number of the used rule is given.

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The user is free to choose between the following options:

- display the solution for the task:
  By choosing this option the system displays the task and the solution to this task. If the user solved the task by himself, he is able to compare his solution with the solution determined by the system.
- display and explain the various methods of solution:
  The system displays the information of each path inside the solution graph between the task and the solution. The path information exists of
  - the deduced problems,
  - the used rules and their additional information,
  - the overall difficulty of the path.

  The user is informed about the different methods of solving this task, each step of the paths is explained. Furthermore, he might compare his method of solution with the paths offered by the system.
- detect, display and explain the optimal method of solution:
  The system displays the same information as above, but only for the optimal path. Table. 2 shows for the graph obtained by solving the task $FT[\delta[2 \cdot t + 1], t, w]$ the information about the best path. The overall difficulty of this path is 3, altogether eight different paths are detected with an overall difficulty up to 5.

Table 2: Information about the best method of solving the task

Information about the optimal method of solution (overall difficulty: 3):	
Starting with the task: $FT[\delta[2 \cdot t + 1], t, w]$	
By using the rule:	number: 170 group: Fourier-Transform rule description: Scaling a signal by a real number a : $FT[x[a \cdot t], t, w] = (1/Abs[a]) \cdot FT[x[t], t, w/a]$ level of difficulty: 2
Deduced problem: $FT[\delta[t + 1], t, w/2]/2$	
By using the rule:	number: 418 group: Standard Fourier-Pair description: Fourier-Transform of a (shifted) Dirac-Impulse: $FT[\delta[t - \tau], t, w] = Exp[-jw\tau]$ level of difficulty: 1
Solution: $Exp[jw/2]/2$	

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3.2 Checking a given expression

Checking a given solution (or steps of a solution) calculated by the user means examining the correctness of expressions like

$$term1 = term2 .$$

The expression is correct if the subtraction of these two terms is identical to zero.
Therefore, the subtraction expression is defined as the task the tutoring system has to solve. The search strategy used by the inference module is set to the breadth-first search until the first deduced problem is detected containing no Fourier-instructions. At this point the search strategy is changed to the depth-first search starting at that deduced problem. By switching over to the depth-first search, the verification process needs less computing time.

If the following rule applications fail in generating a deduced problem identical to zero, the analysed expression is incorrect.
Regardless of the correctness, the decision is justified by the tutoring system.

4. Further work

4.1 The user specifies the method of solution

At the moment the tutoring system plays the role of a teacher by solving a task defined by the user and displaying additional information concerning the task.

A further application is taking away the search strategy from the inference module and handing it over to the user.

By modifying the dialogue component the user will be able to inform the system about the rule he is going to apply next, the system generates a deduced problem by executing the rule on the actual problem.
Therefore the user plays an active part in solving a task, however, without making any calculations by hand. After finishing the task, the student receives information concerning the effectiveness of his path within the solution graph in comparison to the optimal path detected by the tutoring system.

4.2 Other topics out of the System-Theory

The production rules stored in the knowledge base cover the Fourier-Transform rules and standard Fourier-Pairs.

By changing or adding rules and their accompanying information within the knowledge base, the area of application will be extended to other linear transforms such as Laplace-Transform or z-Transform. The remaining components of the tutoring system need not be modified.

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