Turbulence and Aeroacoustics
Research team of the Centre Acoustique
LMFA, École Centrale de Lyon & UMR CNRS 5509

Int'l summer school on non-normal and nonlinear effects in aero- and thermoacoustics 17th May - 18th May 2010, TU München

Non-linear aspects in noise generation and propagation

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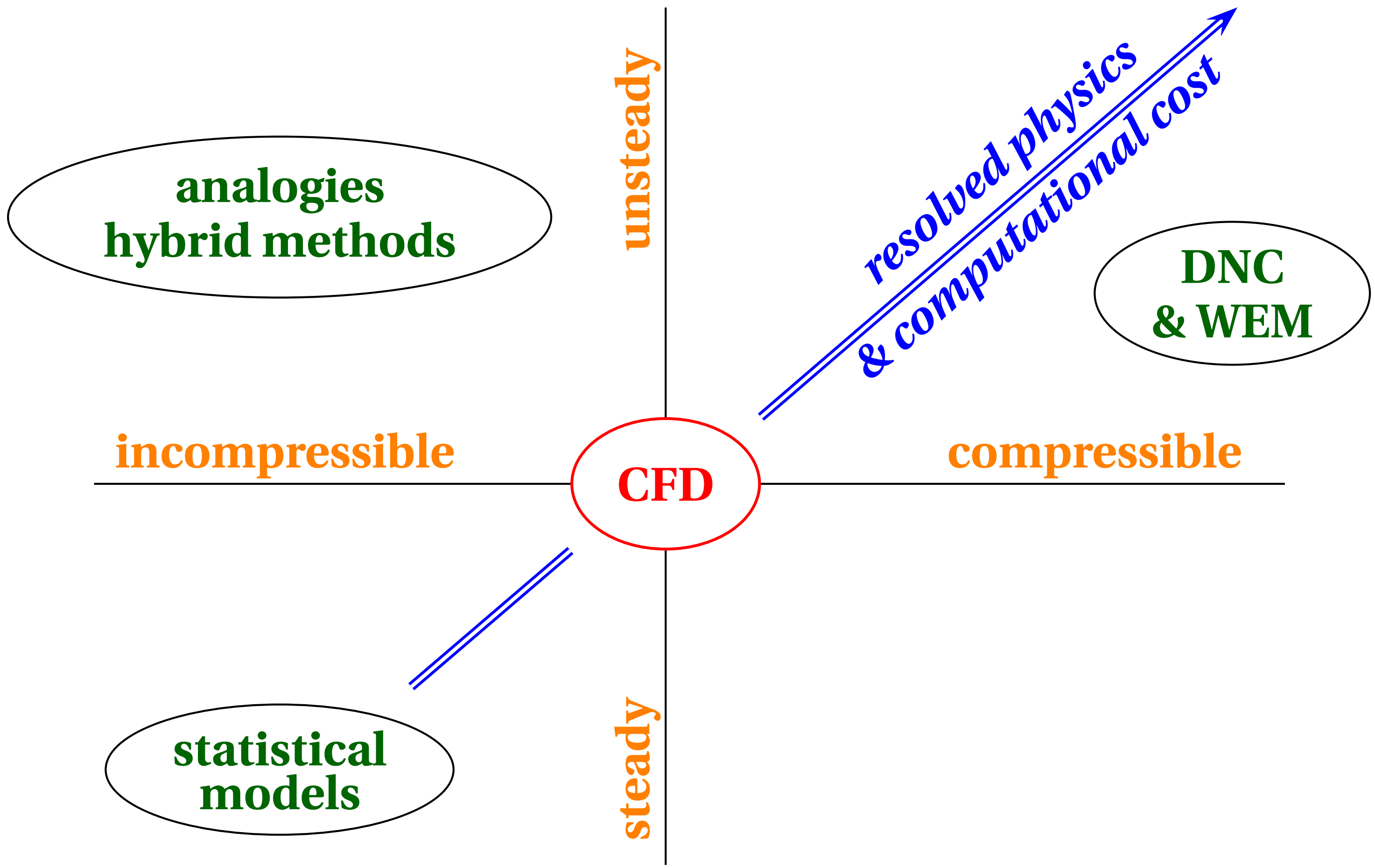
Outline of the talk

- **Aerodynamic noise**
  - Direct computation of aerodynamic noise
  - Lighthill's theory of aerodynamic noise
  - Mean flow effects
  - Model problem – vortex pairing in a mixing layer
  - Physics of subsonic jet noise
  - Two short examples of supersonic flow noise
- **Long range propagation in Earth's atmosphere**
  - Mechanisms of sound absorption
  - First simulations of long-range infrasound propagation
- **Some references**

- **Different modelling levels in aeroacoustics**



DNC = Direct Noise Computation

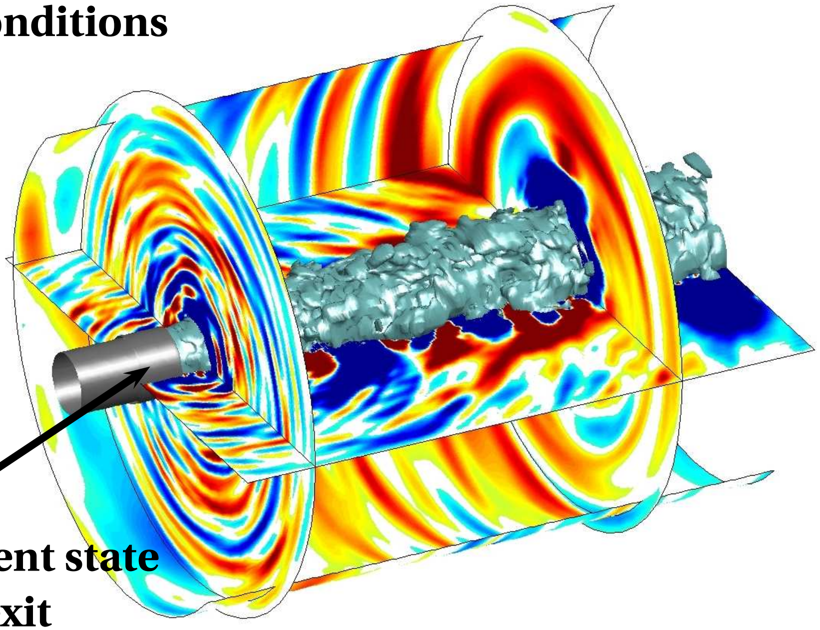
WEM = Wave Extrapolation Methods

Direct computation of aerodynamic noise

- **High fidelity flow/noise simulation in a physically and numerically controlled environment**

  - **Snapshot of vorticity ω in the flow and of the fluctuating pressure field p' outside**

non-reflecting boundary conditions



artificial turbulent state at nozzle exit to mimic turbulent BL

$\delta_\theta|_{\text{num}} \sim \delta_\theta|_{\text{exp}}$

Barré, Ph.D. Thesis, ECLyon (2006)

⇝ disparity of scales

$M = 0.9 \quad Re_D = 4 \times 10^5 \quad T_j = T_\infty$

$\delta_\theta / D = 2.5 \times 10^{-2}$

turbulent field : $u'/u_j \simeq 0.16$

radiated acoustic field :

$\lambda_a \sim D \quad p'_a \sim 70$ Pa

$\lambda_a \sim 10^2 \delta_\theta \qquad u'_a \sim 3 \times 10^{-4} u'$

An error of 1% on the aerodynamic pressure field yields an error of 100% on the acoustic field !

Non-reflecting outflow boundary conditions

- **Many contributions and methods for non-reflecting BC**

  - **based on characteristics**

    - Thompson, Giles, Poinsot & Lele, Serterhenn, ...

  - **based on the Linearized Euler equations**

    - radiation BC (Bayliss & Turkel, Tam & Webb)
    - Perfectly Matched Layer (Hu)

Not sufficient for **silent outflow BC**
in aeroacoustics

  - **sponge or buffer zone**

    (or Energy Transfer and Annihilation)

    Key-point for Direct Noise Computation !

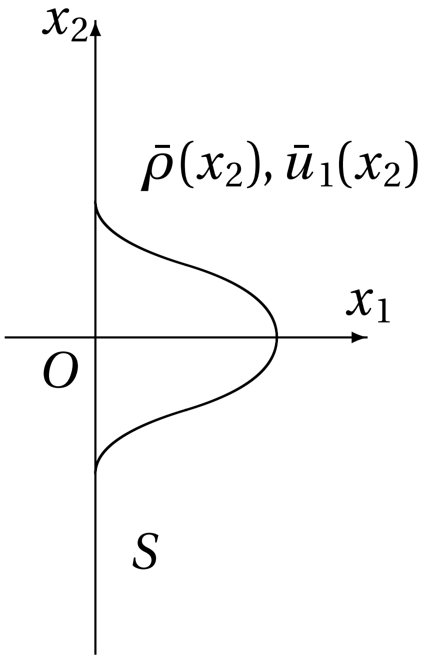
Engquist & Majda (1977)
Hedstrom (1979)
Rudy & Strikwerda (1980)
Bayliss & Turkel (1982)
Thompson (1987, 1990)
Giles (1990)
Poinsot & Lele (1992)
Colonius, Lele & Moin (1993)
Tam & Webb (1993)
Ta'asan & Nark (1995)
Collino (1996)
Tam & Dong (1996)
Hu (1996, 2001)
Sesterhenn (2001)
Bogey & Bailly (2002)
Edgar & Visbal (2003)
Colonius (2004)
Hu (2005, 2008)

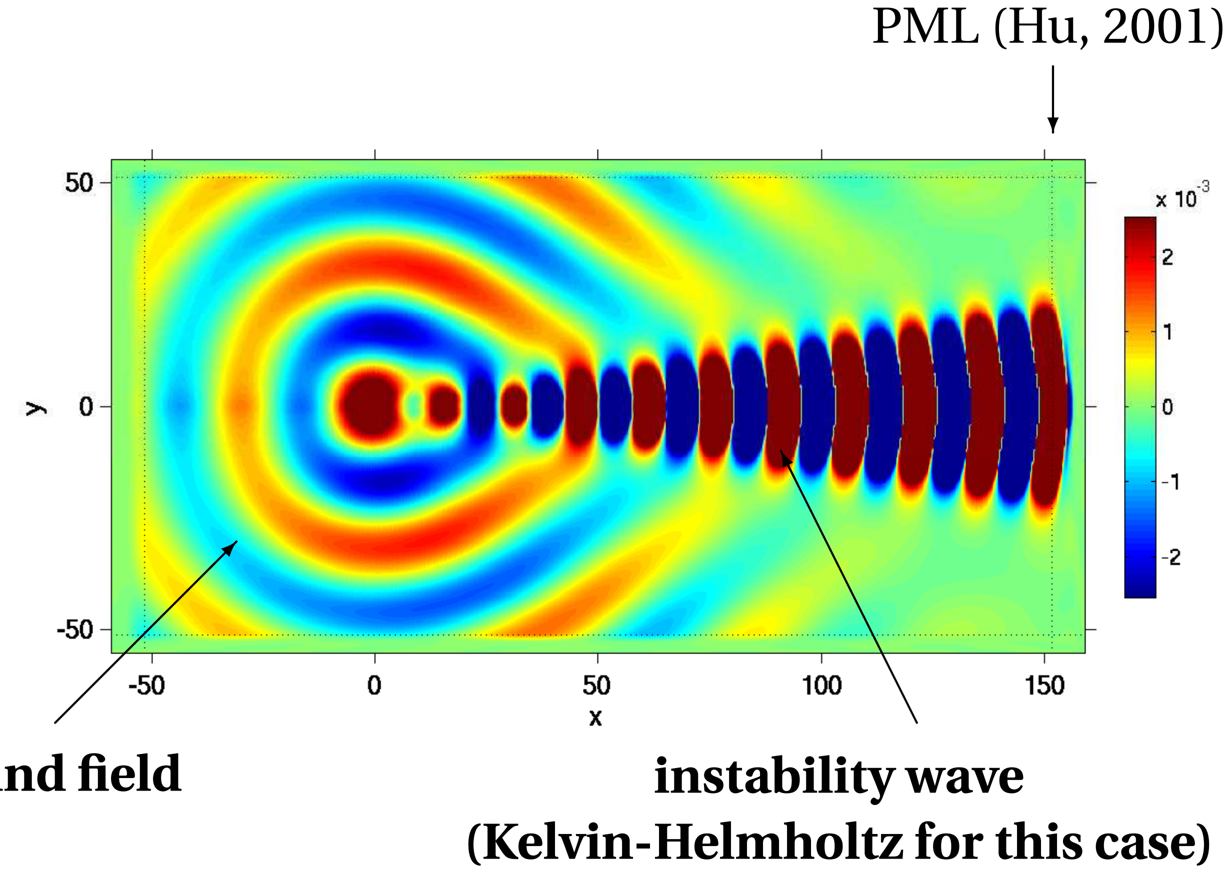
not an exhaustive list !

Non-reflecting outflow boundary conditions

- **Radiation and refraction of sound waves through a 2-D shear layer**

  (4th CAA workshop, NASA CP-2004-212954)





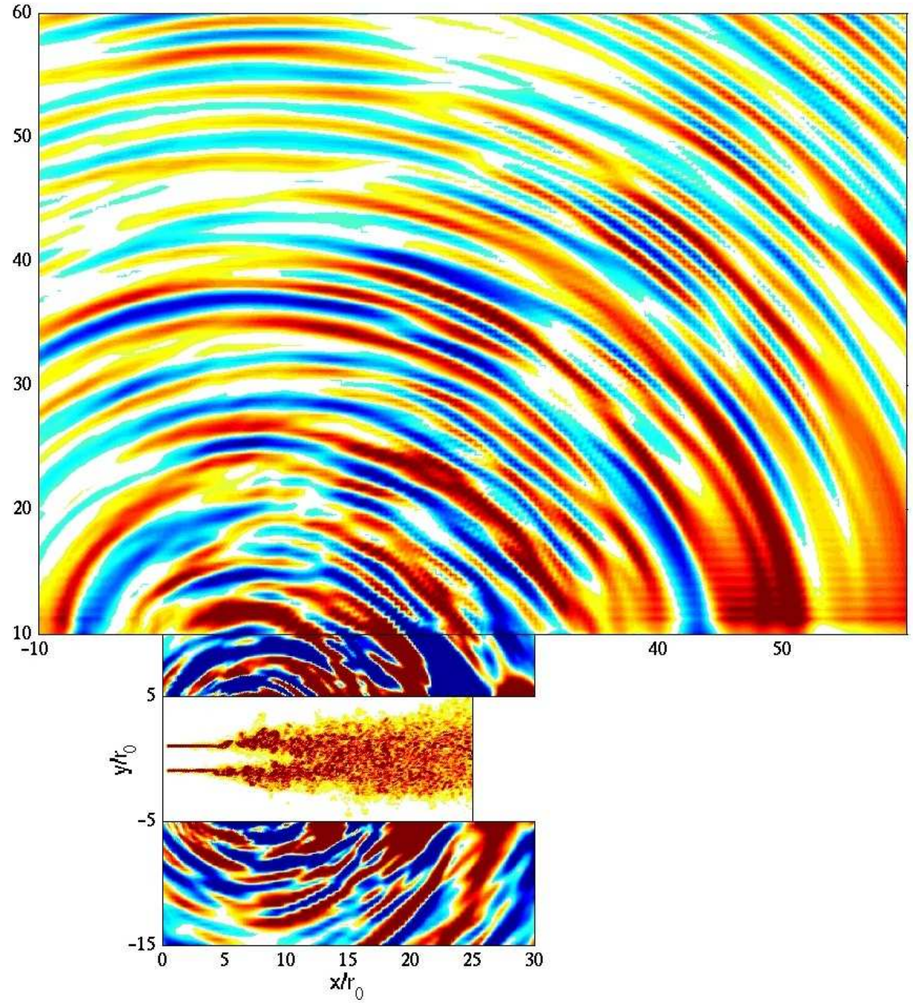
Linearized Euler's Equations

Thomas Emmert - 2004 - Diplomarbeit Technische Universität München - ECL

Linearized Euler Equations as WEM

- **DNC of a high Re_D subsonic circular jet**

  Extrapolated pressure field with LEE (Bogey, 2005)

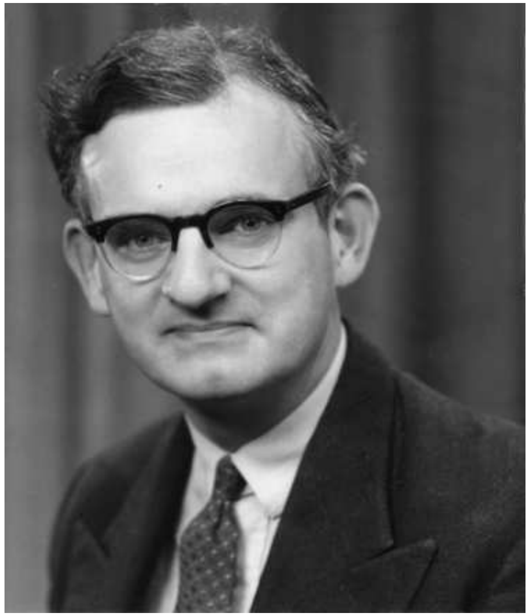


Lighthill's theory of aerodynamic noise (1952)

- **Formulation :** The simplest wave equation from the conservation of mass and Navier-Stokes equations :

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial \left(\rho u_i u_j\right)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \qquad (2)$$



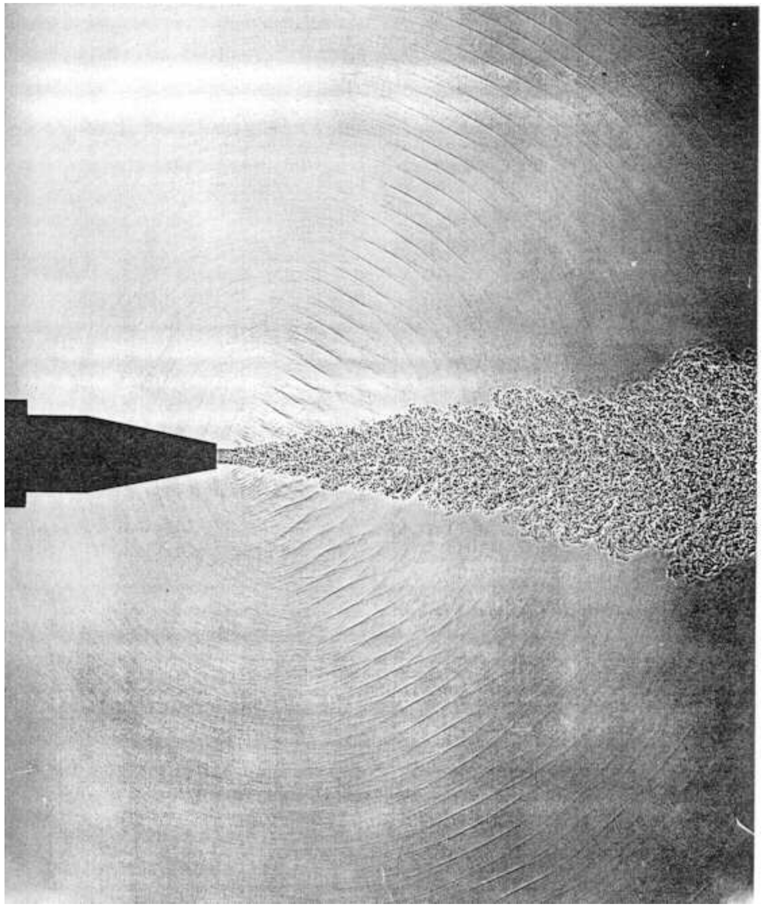
Sir James Lighthill (1924-1998)

$$\frac{\partial}{\partial t}(1) - \frac{\partial}{\partial x_i}(2) \qquad \text{and} \qquad c_\infty^2 \nabla^2 \rho = c_\infty^2 \frac{\partial^2}{\partial x_i \partial x_j}(\rho \delta_{ij})$$

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \qquad \text{with} \qquad T_{ij} = \rho u_i u_j + \left(p - c_\infty^2 \rho\right) \delta_{ij} - \tau_{ij}$$

Lighthill's tensor

Interpretation



In a uniform medium at rest ρ_∞, p_∞, c_∞

$$\frac{\partial^2 \rho'}{\partial t^2} - c_\infty^2 \nabla^2 \rho' = 0$$

$$\frac{\partial^2 \rho'}{\partial t^2} - c_\infty^2 \nabla^2 \rho' = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

source volume of turbulence

Forcing term $\quad \dfrac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$

equivalent distribution of noise sources

Retarded-time solution of Lighthill's equation

$$\rho(\boldsymbol{x},t) = \frac{1}{4\pi c_\infty^4}\int_V \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j}\left(\boldsymbol{y}, t - \frac{r}{c_\infty}\right)\frac{d\boldsymbol{y}}{r}$$

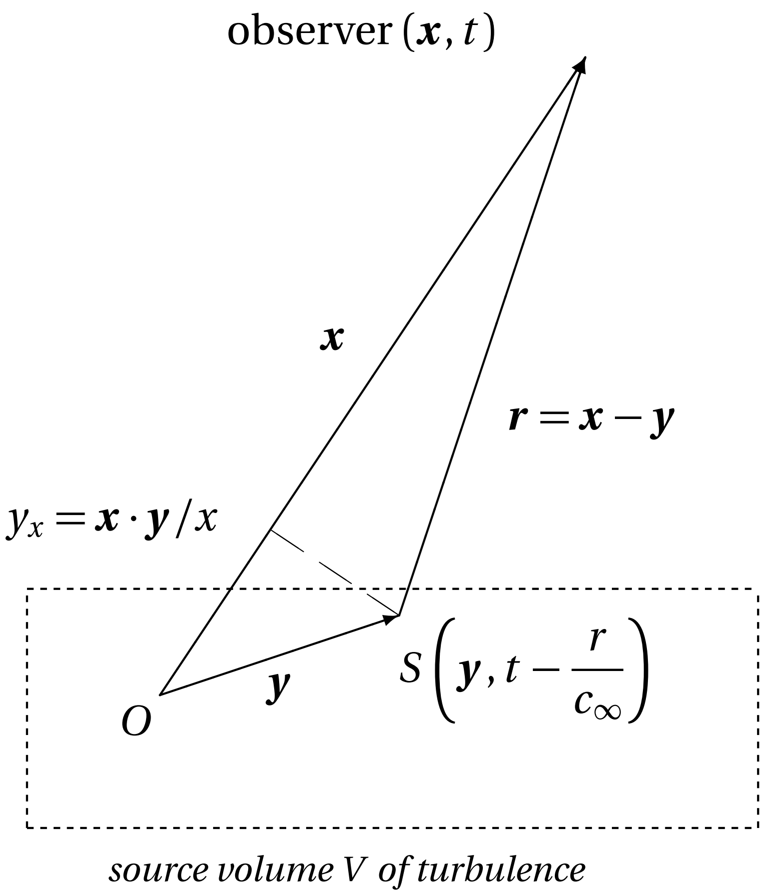
By using

$$r = \left|\boldsymbol{x}-\boldsymbol{y}\right| \simeq x - \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{x} + \mathcal{O}\left(\frac{y^2}{x}\right) \qquad x \gg y$$

$$\frac{\partial}{\partial y_i} \rightsquigarrow -\frac{1}{c_\infty}\frac{x_i}{x}\frac{\partial}{\partial t} \qquad x \gg y$$

$$\rho'(\boldsymbol{x},t) \simeq \frac{1}{4\pi c_\infty^4 x}\frac{x_i x_j}{x^2}\int_V \frac{\partial^2 T_{ij}}{\partial t^2}\left(\boldsymbol{y}, t - \frac{r}{c_\infty}\right) d\boldsymbol{y}$$

in the far-field approximation



- **Some remarks about these subtle integral formulations**
  Crighton (1975), Ffowcs Williams (1992)

  - The integral solution is a convolution product. **In free space :**

$$\rho' = G * \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} = \frac{\partial^2 G}{\partial x_i \partial x_j} * T_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}\left(G * T_{ij}\right)$$

  - May we neglect the retarded time differences in the integral solutions ?

$$t - \frac{r}{c_\infty} = t - \frac{|\boldsymbol{x}-\boldsymbol{y}|}{c_\infty} \simeq t - \frac{x}{c_\infty} + \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{x c_\infty} + \cdots \qquad \frac{\boldsymbol{x}\cdot\boldsymbol{y}}{x c_\infty} \sim \frac{L}{c_\infty}$$

$$\frac{\text{difference in time emission}}{\text{turbulent time}} \sim \frac{L/c_\infty}{L/u'} \sim \mathrm{M}_t \qquad \text{turbulent Mach number}$$

Yes if $\mathrm{M}_t \ll 1$, compact sources

Some remarks (cont'd)

- Viscous effects are very weak noise sources. For relatively low Mach numbers and flows nearly isentropic, $p' - c_\infty^2 \rho' = (p_\infty / c_v)\, s'$ for a perfect gas,

$$T_{ij} \simeq \bar{\rho}\, u_i u_j \simeq \rho_\infty u_i u_j$$

...but acoustic - mean flow interactions are definitively lost for propagation!

(will be illustrated later)

- **Crudest approximation for jet noise scaling**

In the far field and for $M_t \leq 1$ (compact sources)

$$\rho'(\boldsymbol{x},t) \simeq \frac{1}{4\pi c_\infty^4 x}\frac{x_i x_j}{x^2}\int_V \frac{\partial^2 T_{ij}}{\partial t^2}\left(\boldsymbol{y}, t-\frac{x}{c_\infty}\right) d\boldsymbol{y}$$

$$\sim \frac{1}{c_\infty^4 x}\frac{\rho_j U_j^2}{(D/U_j)^2} D^3 \qquad \left\{\begin{array}{l}\text{jet nozzle diameter } D\\ \text{jet exit velocity } U_j\end{array}\right.$$

Hence,

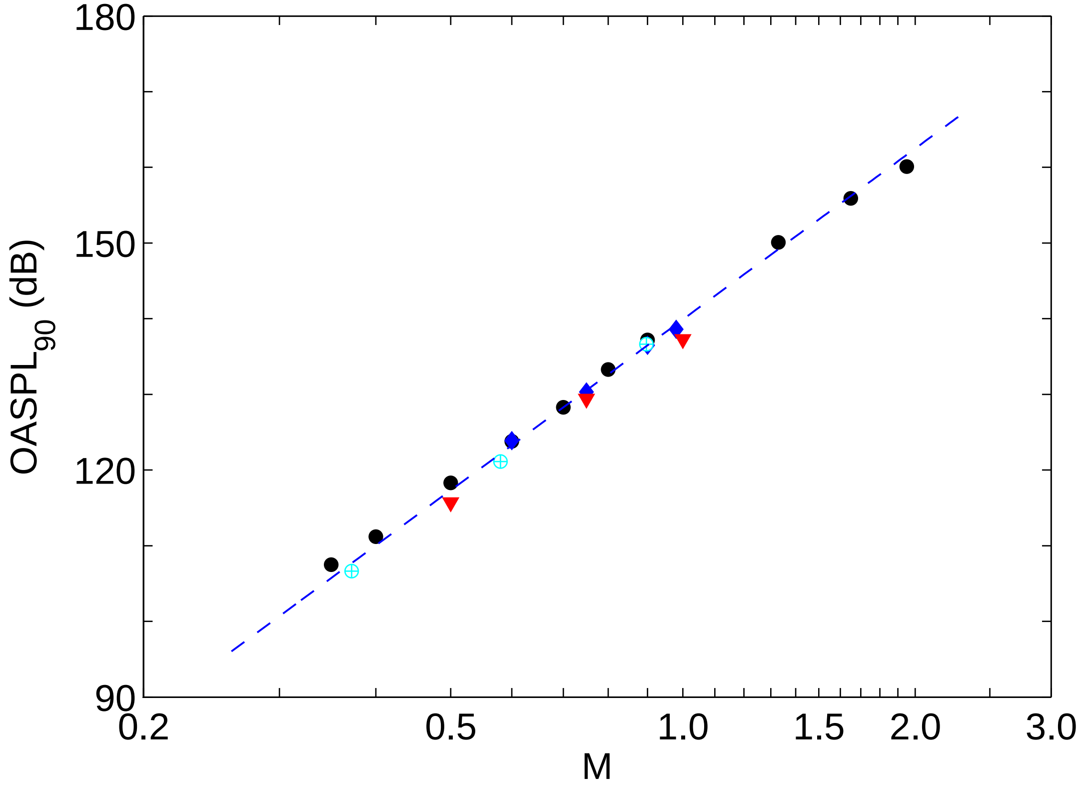
$$W \sim \frac{\rho_j}{\rho_\infty}\frac{U_j^5}{c_\infty^5} A\rho_j U_j^3 \qquad (A=\pi D^2/4)$$

Lighthill's eigth power law (1952)

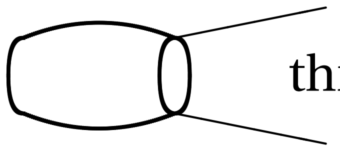
$$\overline{p'^2}\Big|_{\theta=90^o} = K\rho_\infty^2 c_\infty^4 \frac{A}{r^2}\left(\frac{\rho_j}{\rho_\infty}\right)^2 M^{7.5} \qquad K \simeq 1.9\times 10^{-6}$$

Jet noise scaling – acoustic efficiency η

$$\eta = \frac{W_{\text{acoustic}}}{W_{\text{mechanical}}} \simeq 1.2 \times 10^{-4} \, (\rho_j/\rho_\infty) \, \mathrm{M}^5$$



$$W_{\text{acoustic}} \sim A u_j^8$$



$$\text{thrust} = A \rho_j u_j^2$$

$$W_{\text{mechanical}} = A \rho_j u_j^3/2$$

♦ Bogey *et al.* (2007), • Tanna (1977), ⊕ Lush (1971), ▼ QinetiQ 1983 NTF data, ■ MolloChristensen (1964)

Free-field loss-less data scaled to a nozzle exit area A of 1 m^2, $\boldsymbol{T_j/T_\infty = 1}$

Other formulations derived from Lighthill's analogy

- **Vortex sound theory**

  Powell (1964), Howe (1975), Möhring (1978), Yates (1978) ...

  Reformulation of Lighthill's equation to emphasize the role of vorticity in the production of sound

  For incompressible flows, $\nabla \cdot \boldsymbol{u} = 0$, at low Mach number :

$$\nabla \cdot \nabla \cdot (\boldsymbol{u}\boldsymbol{u}) = \nabla \cdot (\boldsymbol{\omega} \times \boldsymbol{u}) + \nabla^2 \left(\frac{\boldsymbol{u}^2}{2} \right) \qquad \boldsymbol{L} = \boldsymbol{\omega} \times \boldsymbol{u} \quad \text{Lamb's vector}$$

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \nabla^2 \rho \simeq \rho_\infty \nabla \cdot (\boldsymbol{\omega} \times \boldsymbol{u})$$

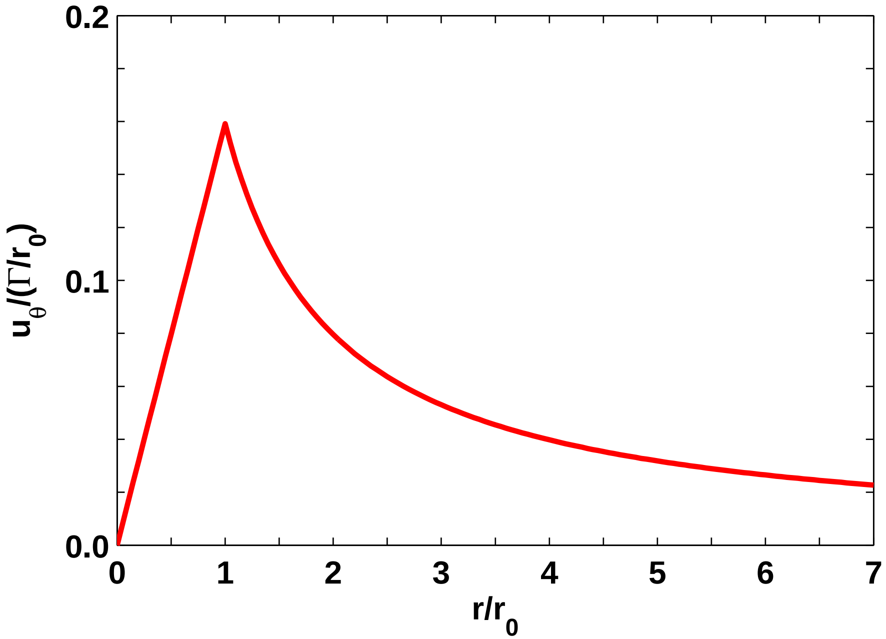
- Use of a wave operator including all **mean flow effects** to **remove the linear propagative part** from Lighthill's tensor

Vortex sound : co-rotating vortices

- **Rankine's vortex : circular patch of uniform vorticity**

  $\boldsymbol{\omega} = \omega \boldsymbol{z}$, $\omega = \omega_0 = 2v_0/r_0$ if $r \leq r_0$ and $\omega = 0$ else

$$\begin{cases} u_\theta(r) = v_0 \dfrac{r}{r_0} & r \leq r_0 \\ u_\theta(r) = v_0 \dfrac{r_0}{r} & r > r_0 \end{cases}$$



$$\Gamma = \pi r_0^2 \omega_0 = 2\pi v_0 r_0 \neq 0 \qquad v_0 = \frac{\Gamma}{r_0} \frac{1}{2\pi}$$

pressure field

$$\begin{cases} p = p_\infty - \dfrac{\rho v_0^2}{2} \left(2 - \dfrac{r^2}{r_0^2} \right) & r \leq r_0 \\ p = p_\infty - \dfrac{\rho v_0^2}{2} \dfrac{r_0^2}{r^2} & r > r_0 \end{cases}$$

Vortex sound : co-rotating vortices

- **Rectilinear vortex filament**

  (irrotational 2-D incompressible flow)

  - Velocity potential ϕ and stream function ψ

$$\phi = \frac{\Gamma\theta}{2\pi} \qquad \psi = -\frac{\Gamma}{2\pi}\ln(r)$$

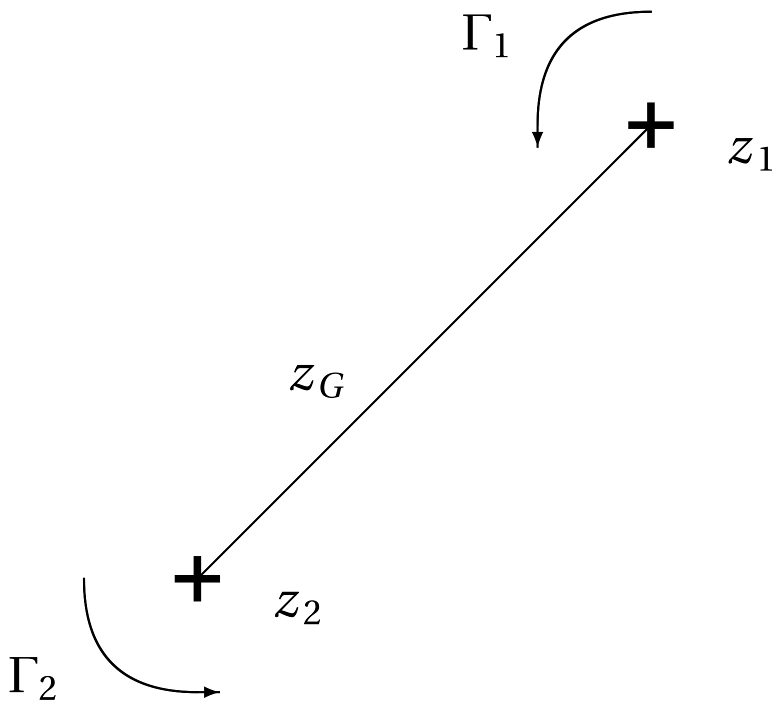
$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\partial\phi}{\partial r} = 0 \qquad u_\theta = -\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = \frac{\Gamma}{2\pi r}$$

  - Complex potential $f = \phi + i\psi$, $z = x + iy$

$$\frac{df}{dz} = u_x - i u_y = (u_r - i u_\theta)e^{-i\theta}$$

$$f = -i\frac{\Gamma}{2\pi}\ln z \qquad \frac{df}{dz} = -i\frac{\Gamma}{2\pi r}e^{-i\theta} \qquad u_r = 0 \qquad u_\theta = \frac{\Gamma}{2\pi r}$$

Co-rotating vortices



- velocity at z_1 induced by Γ_2

$$u_1 - i v_1 = -i \frac{\Gamma_2}{2\pi} \frac{1}{z_1 - z_2}$$

- velocity at z_2 induced by Γ_1

$$u_2 - i v_2 = -i \frac{\Gamma_1}{2\pi} \frac{1}{z_2 - z_1}$$

$$\rightsquigarrow \quad \Gamma_1(u_1 - i v_1) = -\Gamma_2(u_2 - i v_2)$$

center of the system :

$$z_G = \frac{\Gamma_1 z_1 + \Gamma_2 z_2}{\Gamma_1 + \Gamma_2} \qquad \frac{dz_G^\star}{dt} = \frac{\Gamma_1}{\Gamma_1 + \Gamma_2} \frac{dz_1^\star}{dt} + \frac{\Gamma_2}{\Gamma_1 + \Gamma_2} \frac{dz_2^\star}{dt} = 0$$

Co-rotating vortices

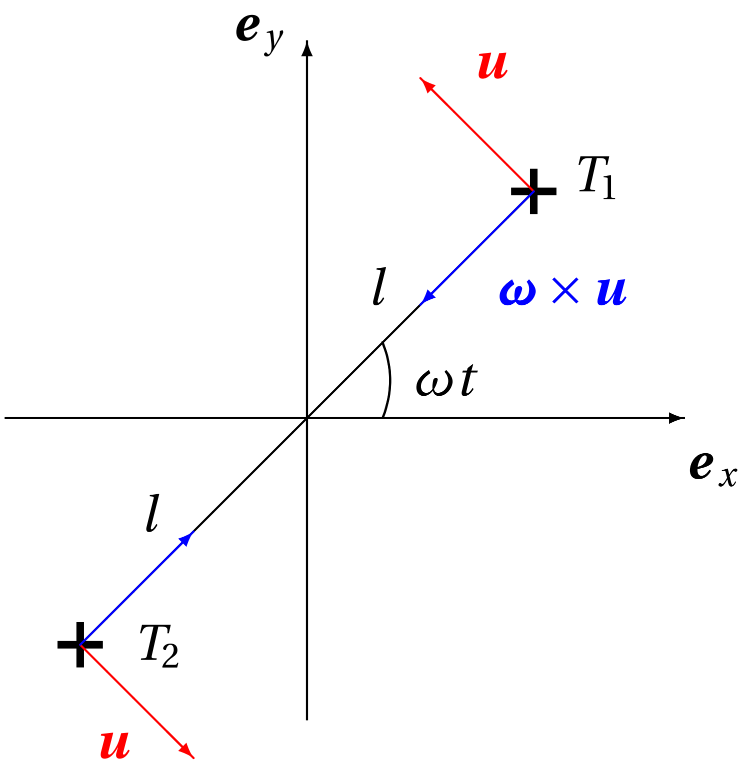
$$z - z_G = \frac{\Gamma_1}{\Gamma_1 + \Gamma_2}(z - z_1) + \frac{\Gamma_2}{\Gamma_1 + \Gamma_2}(z - z_2)$$

$$z_1 - z_G = \frac{\Gamma_2}{\Gamma_1 + \Gamma_2}(z_1 - z_2) \qquad \omega = \frac{|dz_1/dt|}{|z_1 - z_G|} = \frac{\Gamma_1 + \Gamma_2}{2\pi}\frac{1}{|z_1 - z_2|^2}$$

The distance $|z_1 - z_2|$ remains constant (no velocity along $z_1 - z_2$) and therefore the angular frequency ω is also constant.

- if $\Gamma_1 = \Gamma_2 = \Gamma$, $z_G = (z_1 + z_2)/2$, $\omega = \dfrac{\Gamma}{4\pi l^2}$
- if $\Gamma_1 = -\Gamma_2$, z_g at infinity, uniform velocity of $|z_1 - z_2|$ at $U = \Gamma/(\pi l)$

Co-rotating vortices – Powell (1964)



- **Noise radiated by spinning vortices ?**

$$\rho'(\boldsymbol{x}, t) = \frac{\rho_\infty}{4\pi c_\infty^2 x} \int_V \frac{\partial^2 u_i u_j}{\partial y_i y_j} \left(\boldsymbol{y}, t - \frac{r}{c_\infty} \right) d\boldsymbol{y}$$

- local vorticity
- incompressible flow, $\nabla . \boldsymbol{u} = 0$ at low Mach number $M = U/c_\infty$, $U \sim \Gamma/(4\pi l)$

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \nabla^2 \rho \simeq \rho_\infty \nabla \cdot \nabla \cdot (\boldsymbol{u}\,\boldsymbol{u})$$

$$\simeq \rho_\infty \nabla \cdot (\boldsymbol{\omega} \times \boldsymbol{u}) + \rho_\infty \nabla^2 \left(\frac{\boldsymbol{u}^2}{2} \right)$$

Vortex sound : co-rotating vortices

- Integral solution in the far field $x \gg y$

$$\rho'(\mathbf{x},t) \simeq \frac{\rho_\infty}{4\pi c_\infty^2 x}\left[-\frac{1}{c_\infty}\frac{x_i}{x}\underline{\frac{\partial}{\partial t}\int_V (\boldsymbol{\omega}\times\mathbf{u})_i(\mathbf{y},t^\star)d\mathbf{y}} + \frac{1}{c_\infty^2}\frac{\partial^2}{\partial t^2}\int_V \frac{\mathbf{u}^2}{2}(\mathbf{y},t^\star)d\mathbf{y}\right]$$

$\rightsquigarrow$ Leading order-term in low Mach number flow $\sim \mathcal{O}(M^4)$ vs. $\mathcal{O}(M^6)$

- Expansion of the retarded time variation $t^\star = t - \frac{r}{c_\infty} \simeq t - \frac{x}{c_\infty} + \frac{\mathbf{x}\cdot\mathbf{y}}{x c_\infty}$

$$\underline{} = \frac{\partial}{\partial t}\int_V (\boldsymbol{\omega}\times\mathbf{u})_i\left(\mathbf{y},t-\frac{x}{c_\infty}\right)d\mathbf{y} + \frac{1}{c_\infty}\frac{x_j}{x}\frac{\partial^2}{\partial t^2}\int_V y_j(\boldsymbol{\omega}\times\mathbf{u})_i\left(\mathbf{y},t-\frac{x}{c_\infty}\right)d\mathbf{y}$$

The first term is zero by applying the divergence theorem

$$\boldsymbol{\omega}\times\mathbf{u} = \nabla\cdot\left[\nabla\cdot(\mathbf{u}\mathbf{u}) - \nabla\left(\frac{\mathbf{u}^2}{2}\right)\right]$$

Vortex sound : co-rotating vortices

- Integral solution in the far field $x \gg y$

$$\rho'(\boldsymbol{x},t) \simeq -\frac{\rho_\infty}{4\pi c_\infty^4 x}\frac{x_i x_j}{x^2}\frac{\partial^2}{\partial t^2}\int_V y_j(\boldsymbol{\omega}\times\boldsymbol{u})_i\left(\boldsymbol{y},t-\frac{x}{c_\infty}\right)d\boldsymbol{y}$$

- 2-D problem

$$G_{2D}(x_1,x_2) = \int_{-\infty}^{+\infty} G_{3D}(x_1,x_2,x_3)\,dx_3$$

$$x = \sqrt{r^2+z^2} \quad r^2 = x_1^2 + x_2^2 \quad x_1 = r\cos\theta, x_2 = r\sin\theta, x_3 = z$$

$$\rho'(\boldsymbol{x},t) \simeq -\frac{\rho_\infty}{4\pi c_\infty^4}\int_{-\infty}^{+\infty}\frac{x_i x_j}{(r^2+z^2)^{3/2}}\frac{\partial^2}{\partial t^2}\left\{\int_V y_j(\boldsymbol{\omega}\times\boldsymbol{u})_i(\boldsymbol{y},t^\star)d\boldsymbol{y}\right\}dz$$

$$t^\star = t - \sqrt{r^2+z^2}/c_\infty \quad i,j = 1,2$$

Vortex sound : co-rotating vortices

- Co-rotating vortices

$$(T_1) \qquad \boldsymbol{\omega} \times \boldsymbol{u} = \Gamma \boldsymbol{e}_z \times \frac{\Gamma}{4\pi l} \boldsymbol{e}_\theta = -\frac{\Gamma^2}{4\pi l} \boldsymbol{e}_r$$

$$\sum_{T_1, T_2} \omega \times \boldsymbol{u} = -\frac{\Gamma^2}{4\pi l} \left| \begin{array}{c} \cos(\omega t) \\ \sin(\omega t) \end{array} \right. -\frac{\Gamma^2}{4\pi l} \left| \begin{array}{c} -\cos(\omega t) \\ -\sin(\omega t) \end{array} \right. = 0$$

Observer at $x_1 = r\cos\theta, x_2 = r\sin\theta$

$$x_j y_j \, x_i(\boldsymbol{\omega} \times \boldsymbol{u})_i = \Gamma^2/(4\pi l)$$

$$\{[r\cos\theta \; l\cos(\omega t) + r\sin\theta \; l\sin(\omega t)]\,[-r\cos\theta\cos(\omega t) - r\sin\theta\sin(\omega t)]$$

$$+ \; [-r\cos\theta \; l\cos(\omega t) - r\sin\theta \; l\sin(\omega t)]\,[r\cos\theta\cos(\omega t) + r\sin\theta\sin(\omega t)]\}$$

$$x_j y_j \, x_i(\boldsymbol{\omega} \times \boldsymbol{u})_i = -\frac{\Gamma^2}{4\pi l} \, r^2 l \, 2\cos^2(\theta - \omega t) = -\frac{\Gamma^2 r^2}{4\pi} [1 + \cos(2\theta - 2\omega t)]$$

Vortex sound : co-rotating vortices

- Sound radiated by spinning vortices

$$\rho'(\boldsymbol{x},t) \simeq -\frac{\rho_\infty}{4\pi c_\infty^4}\frac{\Gamma^2 r^2}{4\pi}4\omega^2 \int_{-\infty}^{+\infty} \cos\left(2\theta - 2\omega t^\star\right)\frac{dz}{(r^2+z^2)^{3/2}}$$

$$t^\star = t - \sqrt{r^2+z^2}/c_\infty$$

Using the method of stationary phase

$$\frac{1}{r^2}\int_{-\infty}^{+\infty} e^{ir\psi(\tilde{z})}\frac{d\tilde{z}}{(1+\tilde{z}^2)^{3/2}} \simeq \frac{1}{r^2}\, e^{i\pi/4}\sqrt{\frac{2\pi}{r\, 2\omega/c_\infty}} \qquad \text{as } r \to \infty$$

$$\psi = \frac{2\omega}{c_\infty}\sqrt{1+\tilde{z}^2}$$

$$\frac{\partial\psi}{\partial z} = \frac{2\omega}{c_\infty}\frac{\tilde{z}}{\sqrt{1+\tilde{z}^2}} \qquad \frac{\partial\psi}{\partial\tilde{z}} = 0 \text{ in } \tilde{z} = 0$$

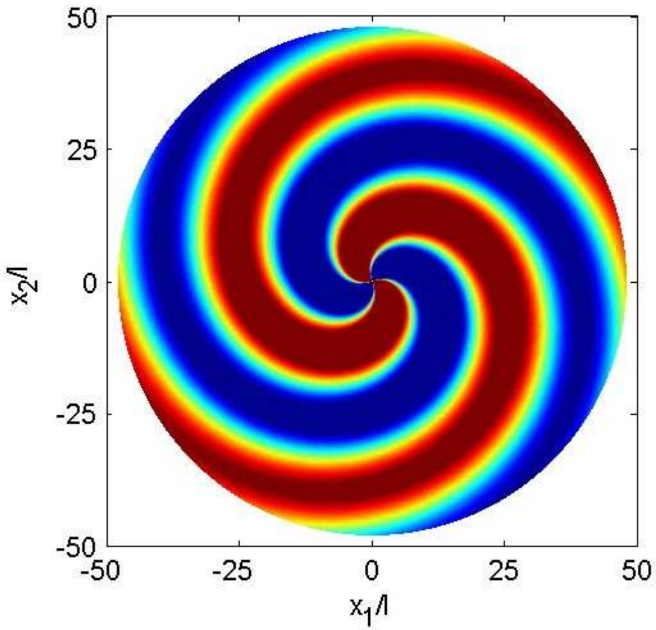
$$\frac{\partial^2\psi}{\partial\tilde{z}^2} = \frac{2\omega}{c_\infty}\frac{1}{\sqrt{1+\tilde{z}^2}} - \frac{2\omega}{c_\infty}\frac{\tilde{z}^2}{(1+\tilde{z}^2)^{3/2}} \quad \text{and thus} \quad \left.\frac{\partial^2\psi}{\partial\tilde{z}^2}\right|_{\tilde{z}=0} = \frac{2\omega}{c_\infty} > 0$$

- Sound radiated by spinning vortices

  Powell (1963) - Howe (1998)

$$\rho'(r,\theta,t) \simeq -\rho_\infty \, 4\sqrt{\pi}\,\sqrt{\frac{l}{r}}\, M^{7/2}\cos\left[2\theta - 2\omega\left(t - \frac{r}{c_\infty}\right) + \frac{\pi}{4}\right]$$

  - $1/\sqrt{r}$ decay of the pressure
  - power radiated per unit length in z, $W \sim M^7$ ($\rightsquigarrow$ 2-D turbulence)



Method of the stationary phase (Kelvin, 1887)

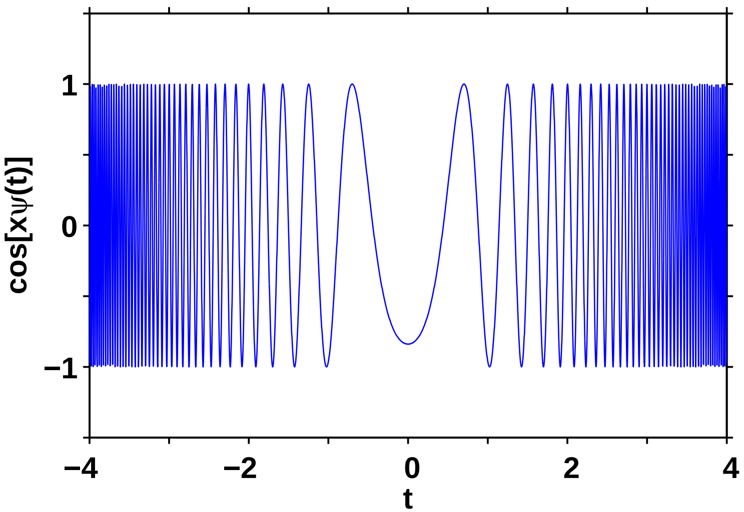
- **Asymptotic behavior of integrals**

$$I(x) = \int_a^b f(t) e^{ix\psi(t)} dt \qquad \text{as } x \to \infty, \quad (a, b, \psi) \text{ reals}$$

  - Easiest case : only one stationary point $t^\star$, $\psi'(t^\star) = 0$, $a < t^\star < b$

$$I(x) = \int_a^b f(t) e^{ix\psi(t)} dt \simeq f(t^\star) \sqrt{\frac{2\pi}{x|\psi''(t^\star)|}} \, e^{i(x|\psi(t^\star)| \pm \pi/4)} \quad \text{as } x \to \infty$$

with the sign $\pm$ according as $\psi''(t^\star) > 0$ or $\psi''(t^\star) < 0$



$e^{ix\cosh t} \qquad \psi(t) = \cosh(t)$

stationary point at $t^\star = 0$

Method of the stationary phase (Kelvin, 1887)

- **Asymptotic behavior of integrals**

  - **Example :**

$$H_0^{(1)}(x) = \frac{1}{i\pi}\int_{-\infty}^{+\infty} e^{ix\cosh t}\,dt \qquad \begin{cases} \psi(t) = \cosh(t), \psi'(t) = \sinh(t), t^\star = 0 \\ \psi''(t) = \cosh(t), \psi''(t^\star) = 1 > 0 \end{cases}$$

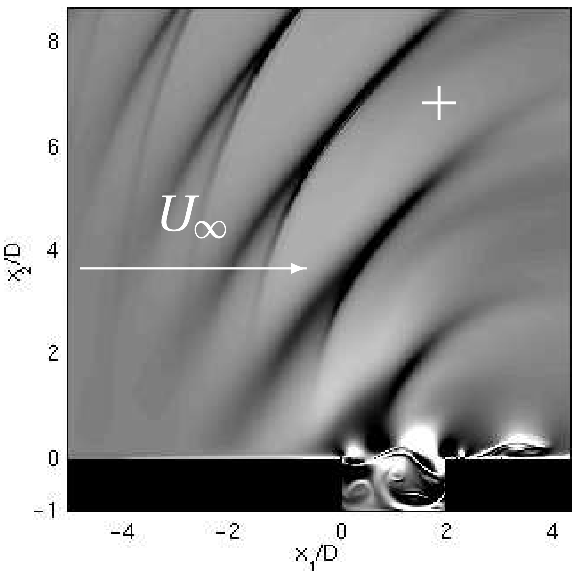
$$H_0^{(1)}(kr) \simeq \sqrt{\frac{2}{\pi kr}}\, e^{i(kr-\pi/4)} \qquad \text{as } kr \to \infty$$

  - **Extension**

    - stationary point at an end point, $t^\star = a$ for instance, half contribution $\rightsquigarrow$ 1/2 factor
    - several stationary points : summation of their contributions

Mean flow effects – Quiz

- **Flow past a cavity : observer in a uniform medium U_∞, ρ_∞, c_∞**
  How to interpret Lighthill's equation ?



Gloerfelt *et al.*, 2003, *J. Sound Vib.*, 266.

Introducing the following (arbitrary) decomposition :

$$\rho \equiv \rho_\infty + \rho' \qquad u_i \equiv U_\infty \delta_{1i} + u'_i$$

$$T_{ij} = \rho u_i u_j = (\rho_\infty + \rho')(U_\infty \delta_{1i} + u'_i)(U_\infty \delta_{1j} + u'_j)$$

Lighthill's equation writes :

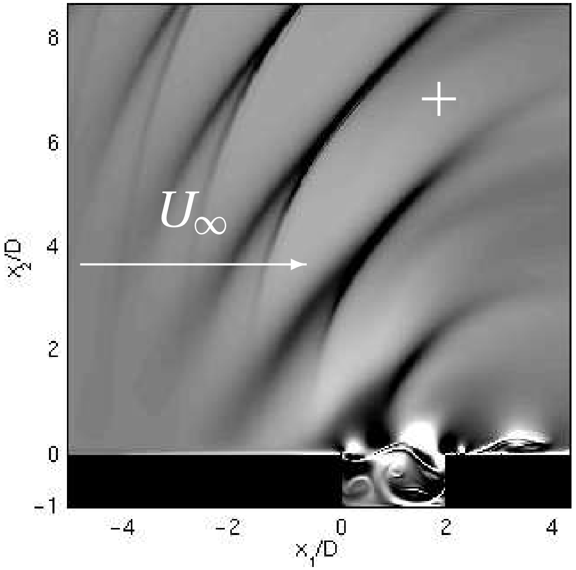
$$\frac{\partial^2 \rho'}{\partial t^2} - c_\infty^2 \nabla^2 \rho' = \frac{\partial^2}{\partial x_i \partial x_j}(\rho u'_i u'_j) + 2U_\infty \frac{\partial^2}{\partial x_1 \partial x_j}(\rho u'_j) + U_\infty^2 \frac{\partial^2 \rho'}{\partial x_1^2}$$

$$= \frac{\partial^2}{\partial x_i \partial x_j}(\rho u'_i u'_j) - 2U_\infty \frac{\partial^2 \rho'}{\partial t \partial x_1} - U_\infty^2 \frac{\partial^2 \rho'}{\partial x_1^2}$$

by using the conservation of mass $\quad \dfrac{\partial \rho'}{\partial t} + U_\infty \dfrac{\partial \rho'}{\partial x_1} + \dfrac{\partial}{\partial x_j}(\rho u'_j) = 0$

Mean flow effects – Quiz

- **How to interpret Lighthill's equation ?**



Gloerfelt *et al.*, 2003, *J. Sound Vib.*, 266.

In a uniform medium U_∞, ρ_∞, c_∞, sound is governed by the convected wave equation :

$$\left(\frac{\partial}{\partial t}+U_\infty\frac{\partial}{\partial x_1}\right)^2\rho' - c_\infty^2\nabla^2\rho' = 0$$

$$\left(\frac{\partial}{\partial t}+U_\infty\frac{\partial}{\partial x_1}\right)^2\rho' \equiv \frac{\partial^2\rho'}{\partial t^2}+2U_\infty\frac{\partial^2\rho'}{\partial t\partial x_1}+U_\infty^2\frac{\partial^2\rho'}{\partial x_1^2}$$

Lighthill

$$\frac{\partial^2\rho'}{\partial t^2} - c_\infty^2\nabla^2\rho' = \frac{\partial^2}{\partial x_i\partial x_j}(\rho u_i' u_j') - 2U_\infty\frac{\partial^2\rho'}{\partial t\partial x_1} - U_\infty^2\frac{\partial^2\rho'}{\partial x_1^2}$$

- **Mean flow - acoustic interactions are included in T_{ij}**
- **Aerodynamic noise source term $\equiv$ non-linear part of T_{ij}**

Lighthill's theory of aerodynamic noise

- **How to interpret Lighthill's equation ?**

  - Mean flow effects are contained in the linear **compressible** part of the Lighthill tensor T_{ij}.

$$T_{ij} - \bar{T}_{ij} = T^f_{ij} + T^l_{ij} = \rho u'_i u'_j + \rho \bar{u}_i u'_j + \rho u'_i \bar{u}_j$$

  - For pratical applications, we often intend to get an estimate of the radiated noise from an **incompressible** turbulent computation which is less expensive.

In practice

$$\begin{cases} \boldsymbol{u} = \bar{\boldsymbol{u}} + \boldsymbol{u}'_{\text{aero}} & \text{mean flow interactions are lost} \\ \boldsymbol{u} = \bar{\boldsymbol{u}} + \boldsymbol{u}'_{\text{aero + acous}} & \text{mean flow interactions are in } T^l_{ij} \end{cases}$$

Lighthill's theory of aerodynamic noise

- **How to interpret Lighthill's equation ?**

  Mean flow effects must be removed from the source term T_{ij} and must be recovered by taking into account by a more complete wave operator in the second step – *i.e.* the acoustic propagation.

  - **Replace the D'Alembertian $\partial^2/\partial t^2 - c_\infty^2 \nabla^2$ by the Linearized Euler Equations**

  - **Acoustic Perturbation Equations (APE)**, sound propagation in an irrotational mean flow (extension of vortex sound theory)

$$\begin{cases} \partial_t \rho' + \partial_{x_i}(\rho' \bar{u}_i + \bar{\rho} u_i') = 0 \\ \partial_t u_i' + \partial_{x_j}(\bar{u}_i u_j') + \partial_{x_i}(p'/\bar{\rho}) = \boldsymbol{q}_m \qquad \boldsymbol{q}_m \simeq -(\boldsymbol{\omega} \times \boldsymbol{u})' \\ \partial_t p' - \bar{c}^2 \partial_t \rho' = 0 \end{cases}$$

Ref. Howe (1998), Möhring (1999)
Ewert & Schröder, 2003, *J. Comput. Phys.*

The Linearized Euler Equations

- **Small perturbations arround a steady mean flow** $(\bar{\rho}, \bar{\boldsymbol{u}}, \bar{p})$
  (perfect gas, no gravity)

$$\left\{ \begin{array}{l} \partial_t \rho' + \nabla \cdot (\rho' \bar{\boldsymbol{u}} + \bar{\rho} \boldsymbol{u}') = 0 \\ \partial_t (\bar{\rho} \boldsymbol{u}') + \nabla \cdot (\bar{\rho} \bar{\boldsymbol{u}} \boldsymbol{u}') + \nabla p' + (\bar{\rho} \boldsymbol{u}' + \rho' \bar{\boldsymbol{u}}) \cdot \nabla \bar{\boldsymbol{u}} = 0 \\ \partial_t p' + \nabla \cdot [p' \bar{\boldsymbol{u}} + \gamma \bar{p} \boldsymbol{u}'] + (\gamma - 1) p' \nabla \cdot \bar{\boldsymbol{u}} - (\gamma - 1) \boldsymbol{u}' \cdot \nabla \bar{p} = 0 \end{array} \right.$$

► Acoustic propagation in the presence of a flow (atmosphere, ocean, turbulent flow, ...) is governed by the **Linearized Euler Equations (LEE)**

► In the general case, this system cannot be reduced exactly to a **single wave equation**.

Ref. Blokhintzev (1946)
Pridmore-Brown (1958), Lilley (1972), Goldstein (1976, 2001, 2003)

The Linearized Euler Equations

- **For a parallel base flow $\bar{u}_i = \bar{u}_1(x_2, x_3)\delta_{1i}$, $\bar{\rho} = \bar{\rho}(x_2, x_3)$** (and thus $\bar{p}$ constant), the LEE can be recasted into a wave equation based on the pressure,

$$\mathcal{L}_0 [p'] = 0$$

$$\mathcal{L}_0 \equiv \frac{\bar{D}}{\bar{D}t}\left[\frac{\bar{D}^2}{\bar{D}t^2} - \nabla\cdot\left(\bar{c}^2\nabla\right)\right] + 2\bar{c}^2\frac{\partial \bar{u}_1}{\partial x_i}\frac{\partial^2}{\partial x_1 \partial x_i} \qquad i = 2,3 \quad \bar{D} \equiv \partial_t + \bar{u}_1\partial_{x_1}$$

- From the (exact) Navier-Stokes equations, we can also form an inhomogeneous wave equation based on $\mathcal{L} \rightarrow \mathcal{L}_0$ at leading order,

$$\mathcal{L}[p'] = \Lambda \qquad \text{Lilley (1972)}$$

$$\frac{d}{dt}\left\{\frac{d^2\pi}{dt^2} - \frac{\partial}{\partial x_i}\left(c^2\frac{\partial \pi}{\partial x_i}\right)\right\} + 2\frac{\partial u_i}{\partial x_j}\frac{\partial}{\partial x_i}\left(c^2\frac{\partial \pi}{\partial x_j}\right) = -2\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_k}\frac{\partial u_k}{\partial x_i} \qquad \pi = \ln p \quad \pi' \simeq (1/\gamma)p'/p_0$$

$$+2\frac{\partial u_i}{\partial x_j}\frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_i}\right) + \frac{d^2}{dt^2}\left(\frac{1}{c_p}\frac{ds}{dt}\right) - \frac{d}{dt}\left\{\frac{\partial}{\partial x_i}\left(\frac{1}{\rho}\frac{\partial \tau_{ij}}{\partial x_j}\right)\right\}$$

Hybrid method using source terms in LEE

- Identification of the **source term** $\boldsymbol{S} = [0, S_1, S_2, S_3, 0]^t$ associated with LEE, (unidirectional sheared mean flow)

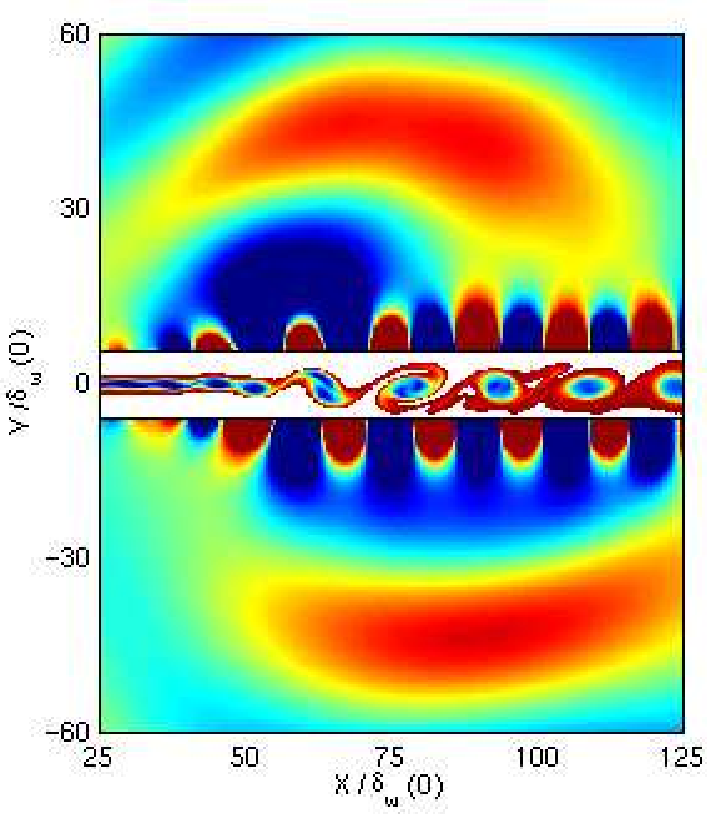
$$S_i = S_i^f - \overline{S_i^f} = -\bar{\rho}\frac{\partial u_i' u_j'}{\partial x_j} + \bar{\rho}\overline{\frac{\partial u_i' u_j'}{\partial x_j}} \qquad \text{(non-linear)}$$

At least for a sheared mean flow, an hybrid method based on LEE can be proposed : generalization to an arbitrary mean flow (open question !)

Ref. Bogey, Bailly & Juvé, 2002, *AIAA Journal*, 40(2) & AIAA Paper 2001-2255
Bogey, Gloerfelt, Bailly, 2003, *AIAA Journal*, 41(8)
Bailly & Bogey, 2004, *IJCFD*, 18(6)
Bailly, Bogey & Candel, 2010, *Int. J. Aerocoustics*

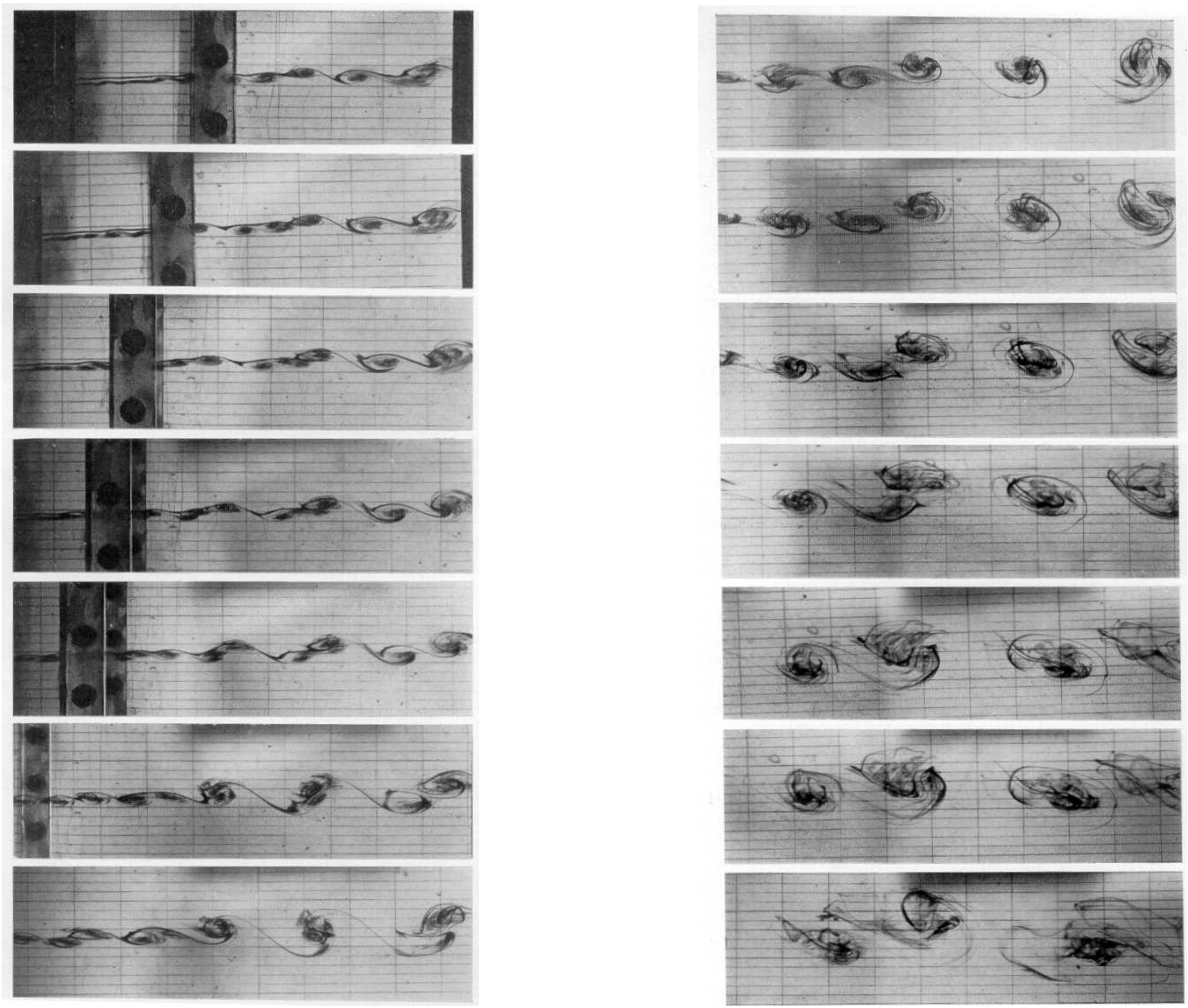
Colonius, Lele & Moin, 1997, *J. Fluid Mech.*
Goldstein, 2001, 2003, 2005, *J. Fluid Mech.*

Noise generated by vortex pairings in a mixing layer



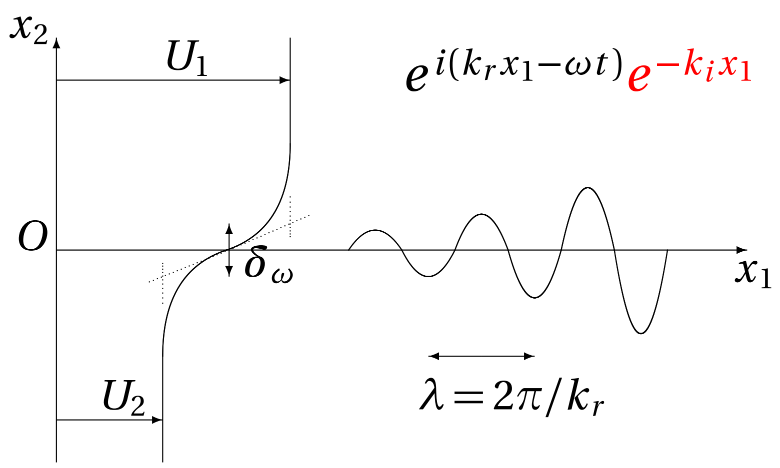
Vortex pairings in a plane mixing layer

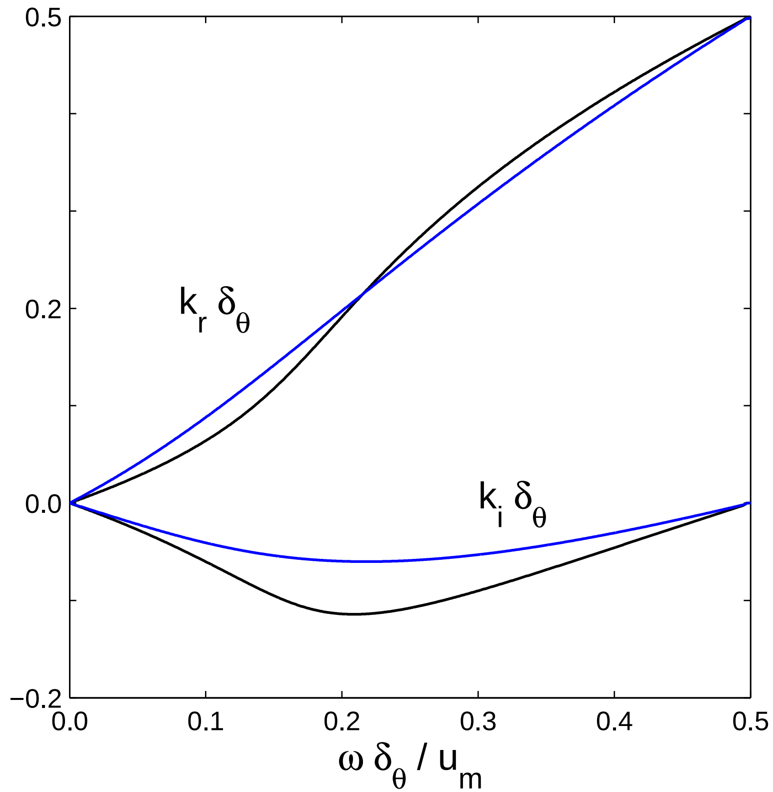
Winant & Browand, J.F.M. (1974)



Vortex pairings in a plane mixing layer

- **Configuration** $u_1(x_2)/u_m = 1 + R_u \tanh[x_2/(2\delta_\theta)]$ $\delta_\omega = 4\delta_\theta$



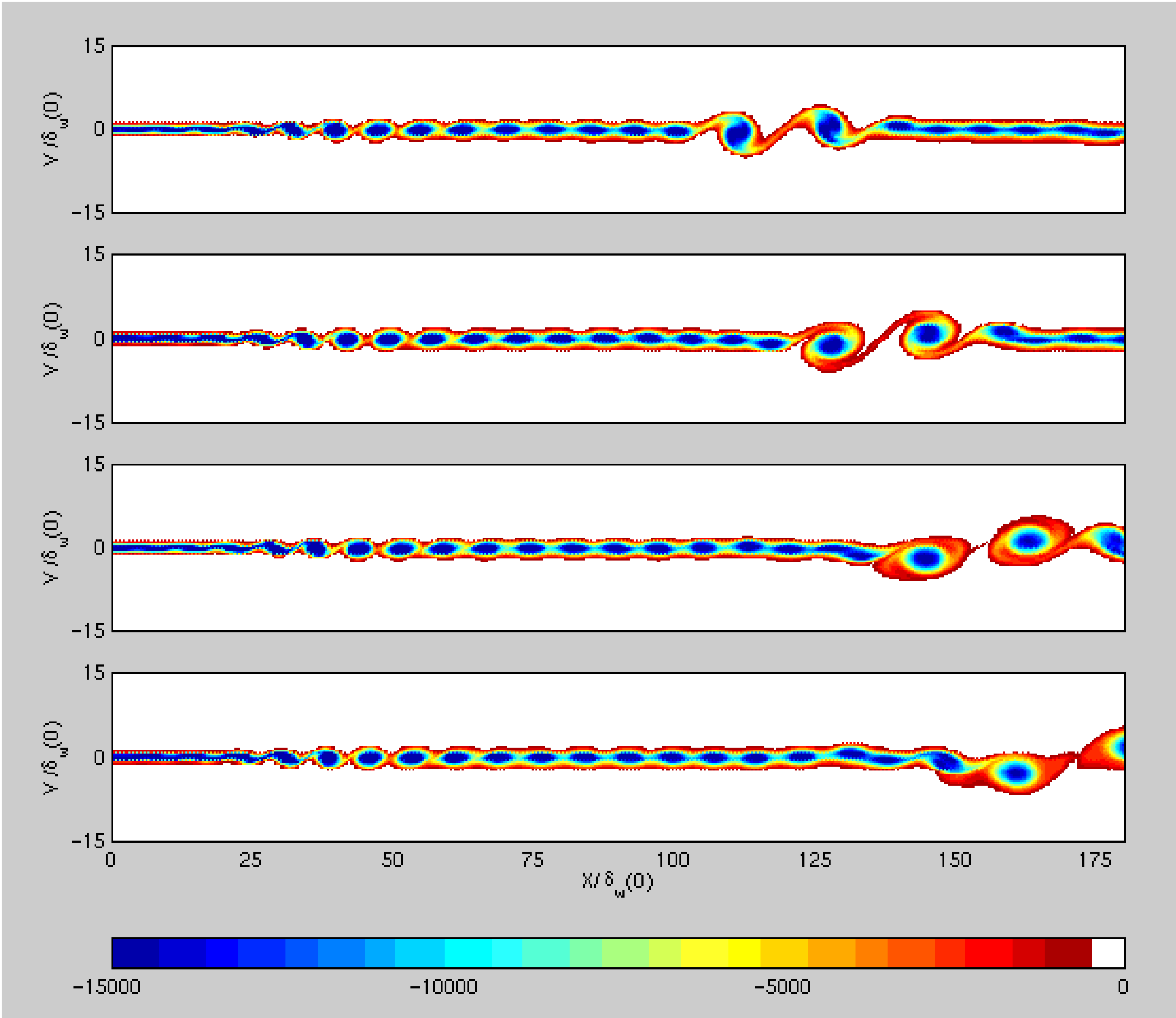


$$u_m = \frac{U_1 + U_2}{2} \qquad R_u = \frac{\Delta U}{2u_m} = \frac{U_1 - U_2}{2u_m}$$

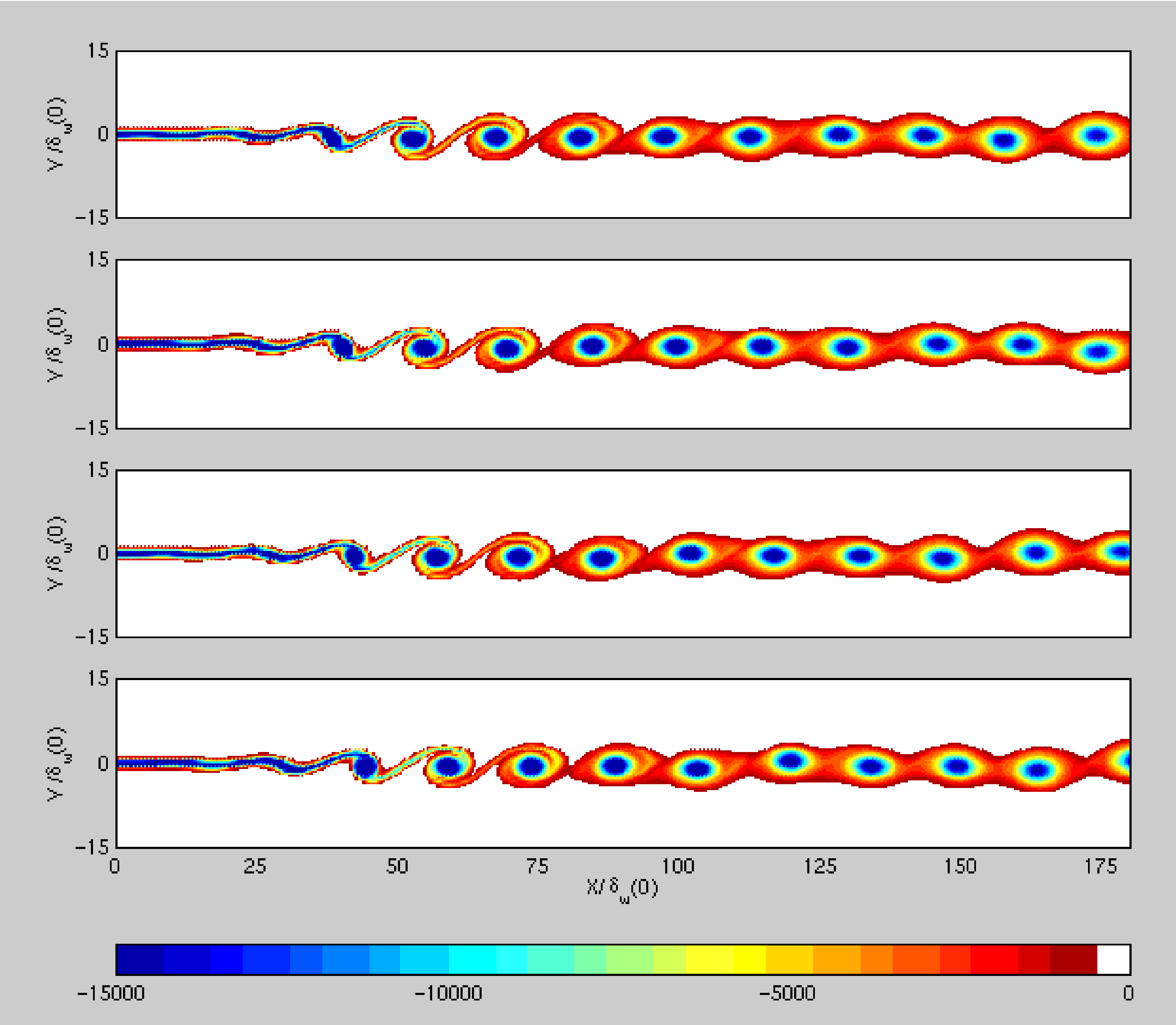
$U_1 = 40$ m.s^{-1} & $U_2 = 160$ m.s^{-1} ($R_u = 0.6$, $u_m = 100$ m.s^{-1})

initial vorticity thickness $\delta_{\omega(0)}$, $\mathrm{Re}_\omega = \dfrac{\delta_{\omega(0)}\Delta U}{\nu} = 12800$

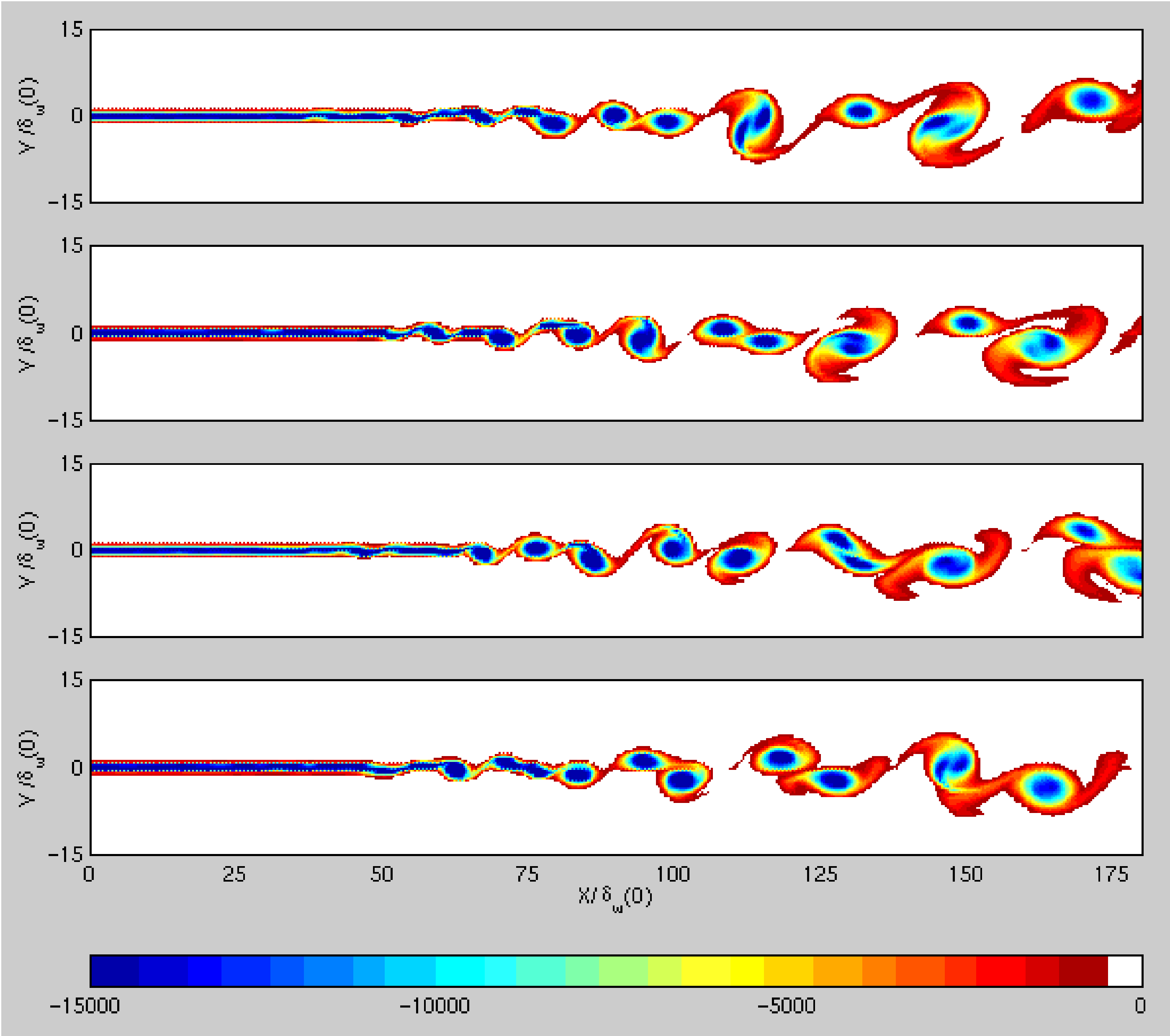
Forcing at f_0



Forcing at $f_0/2$

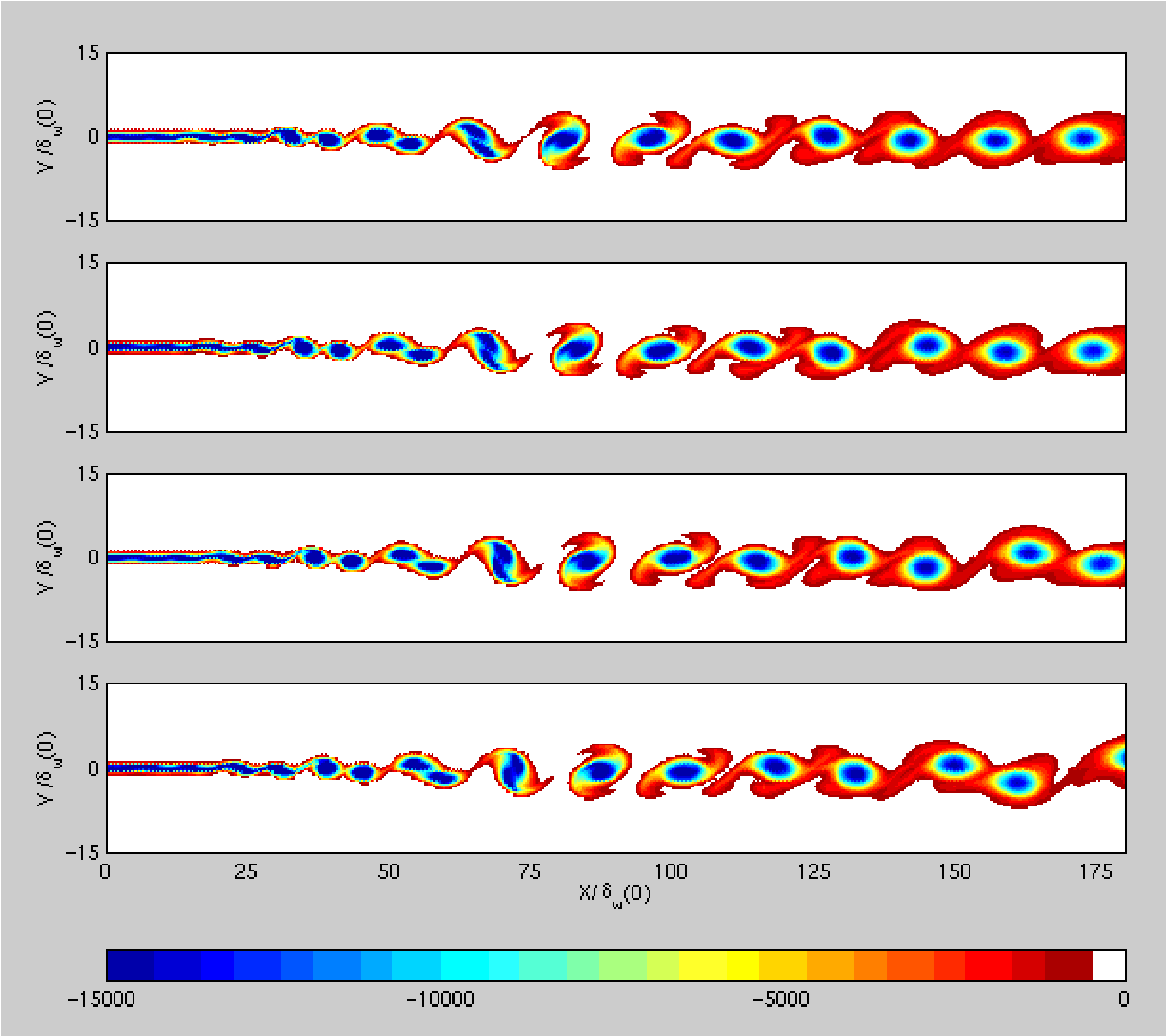


Forcing with broadband noise



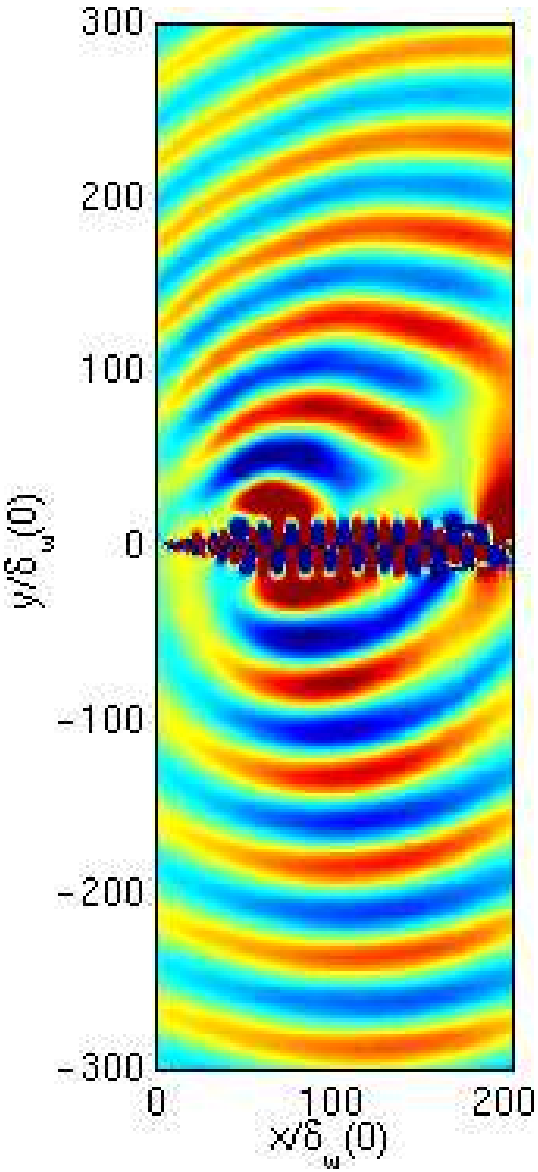
Forcing at f_0 and $f_0/2$

vortex pairing locations are fixed around $75\delta_{\omega(0)}$



Noise generated by a mixing layer

- **Dilatation field $\Theta = \nabla \cdot \boldsymbol{u}$ on the whole domain**



- Wave fronts coming from pairing location with a wavelength corresponding to frequency $f_p = f_0/2$

$$\lambda_{f_p} \simeq 51\delta_\omega(0)$$

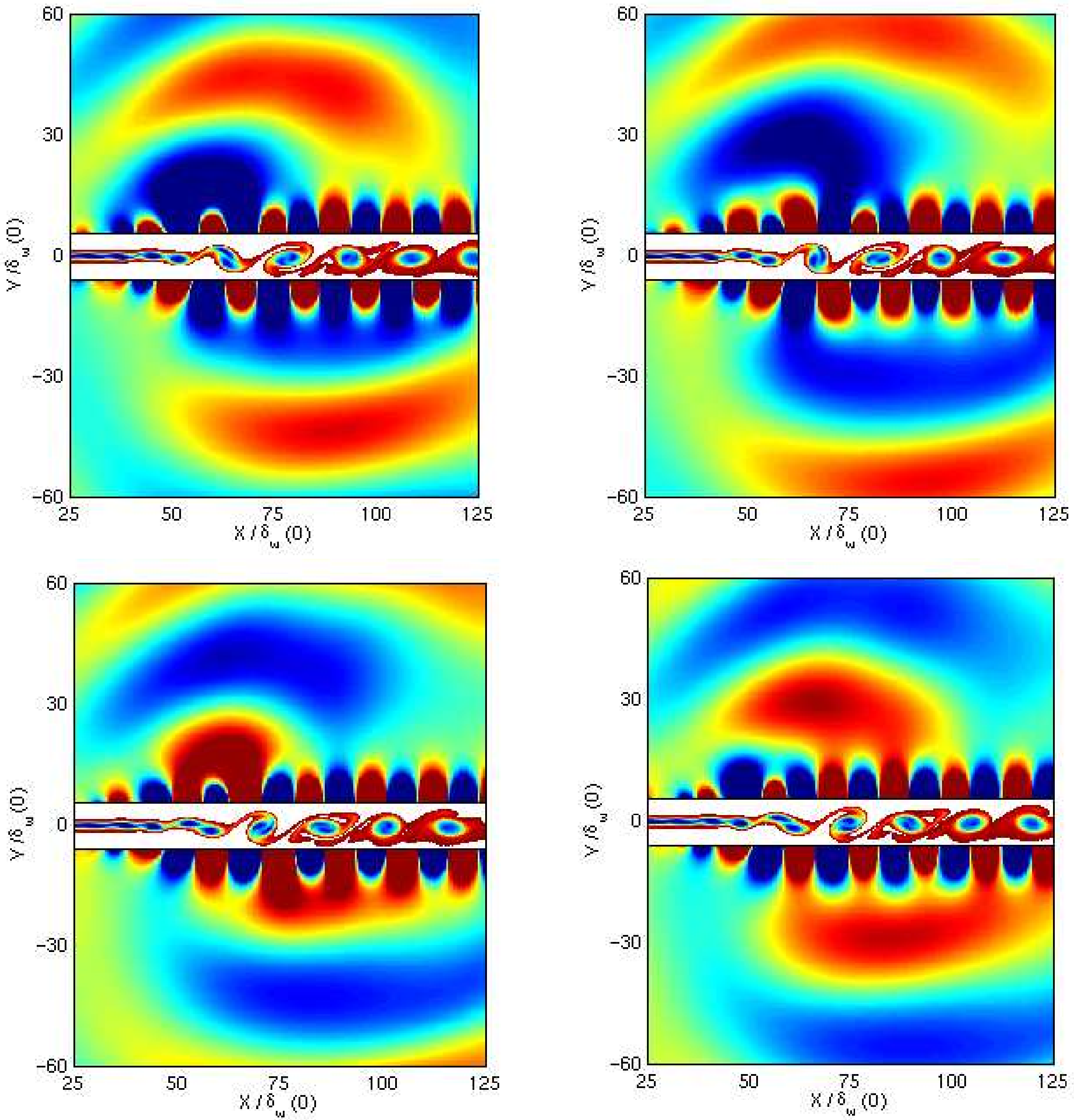
- Convection effects (particularly in the upper rapid flow)

- Noise generation by pairing process : double spiral pattern, similar to quadrupolar structure described in work of Powell (1964) and Mitchell (1995) on co-rotative vortices

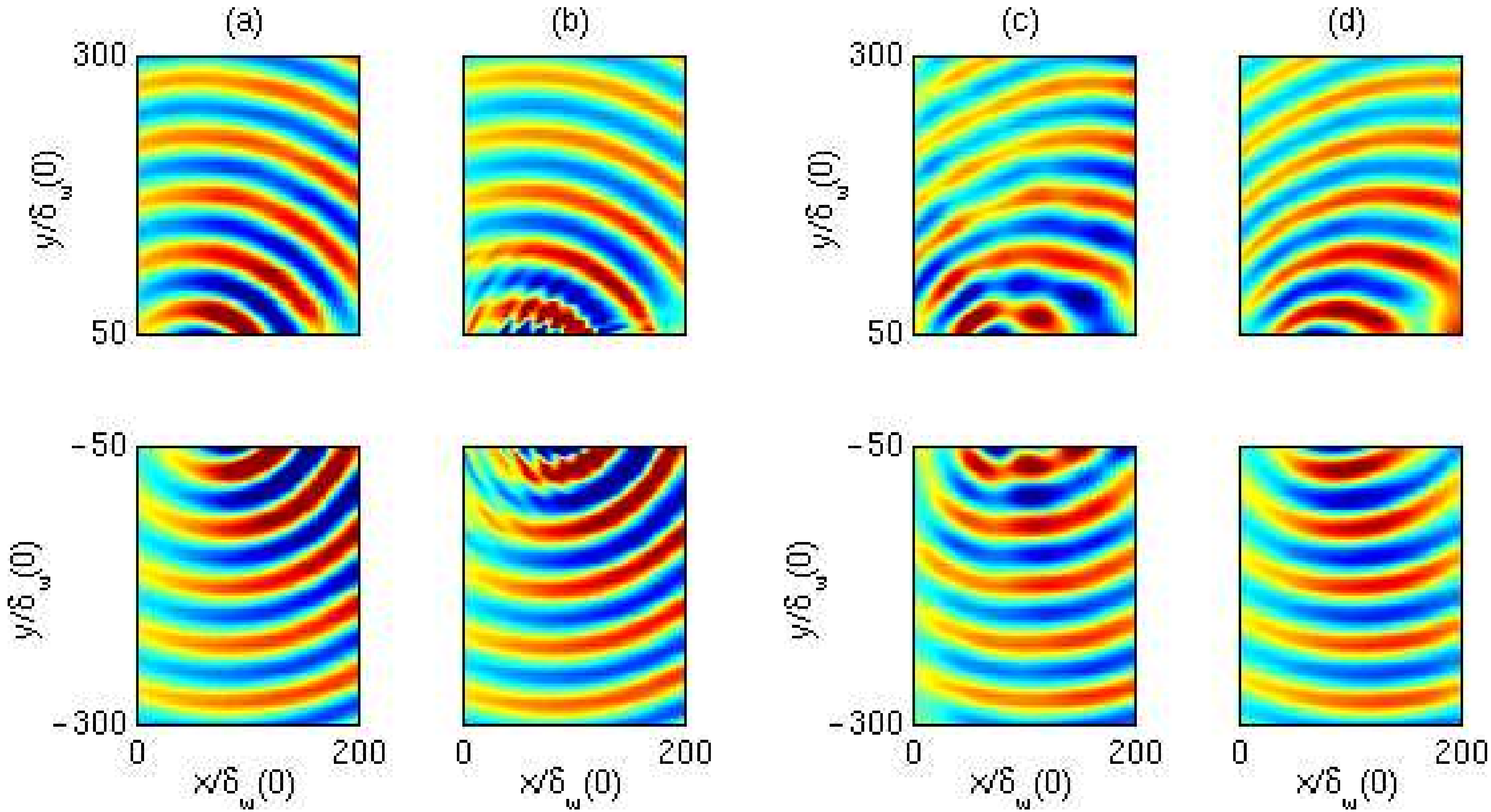
Noise generated by a mixing layer

t $\quad$ $t + T_p/4$

$t + T_p/2$ $\quad$ $t + 3T_p/4$



Noise generated by a mixing layer

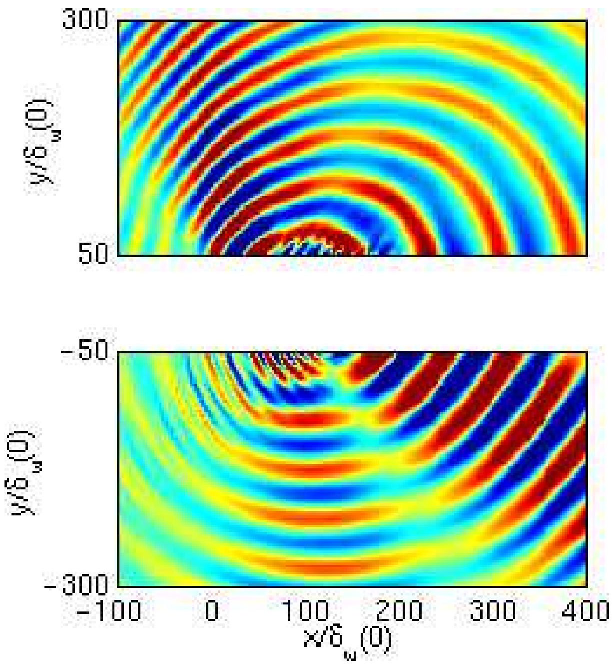


(a) T^f_{ij} (b) LEE without mean flow (c) $T^t_{ij} = T^f_{ij} + T^l_{ij}$ (d) DNC

⇝ **acoustic - mean flow interactions included in the linear part of Lighthill's tensor**

Noise generated by a mixing layer

- **Inappropriate formulation of the source term in LEE**



$\boldsymbol{S}^t$ introduced as source term into LEE instead of $\boldsymbol{S} = \boldsymbol{S}^f$

⇝ overestimation of refraction effects

- It remains one problem with the linearized Euler equations, as for any exact formulation in the time domain taking into account the presence of a mean flow

physical coupling   acoustic waves / instability waves

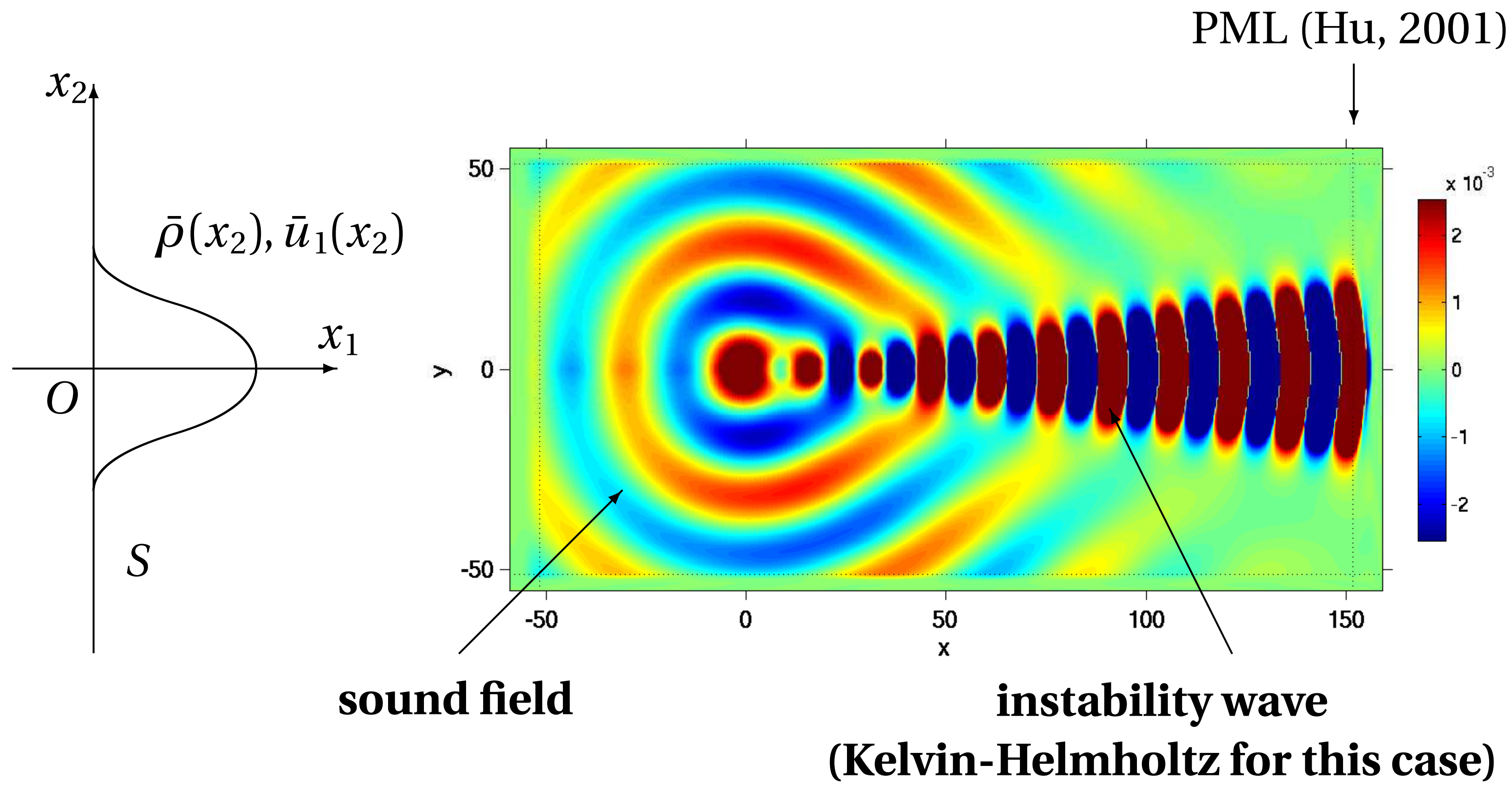
« Simplified » formulation of LEE by removing a gradient term associated with refraction effects

Hybrid method using source terms in LEE

- **Analogy based on LEE not so well-posed for noise generation**

  Radiation and refraction of sound waves through a 2-D shear layer

  (4th CAA workshop, NASA CP-2004-212954)



Thomas Emmert - 2004 - Diplomarbeit Technische Universtät München - ECL

Hybrid method using source terms in LEE

- $\mathscr{L}_0\,[p']=0 \qquad \mathscr{L}_0 \sim \text{LEE}$

  - generalization of the **Rayleigh equation** (1880) for a compressible perturbation

  - three families of **instability waves** (including Kelvin-Helmholtz) which can overwhelm the acoustic solution or/and generate noise (dominant noise mechanism for supersonic jets)

    Tam & Burton, 1984, *J. Fluid Mech.*
    Tam & Hu, 1989, *J. Fluid Mech.*
    Tam, 1995, *Annu. Rev. Fluid Mech.*

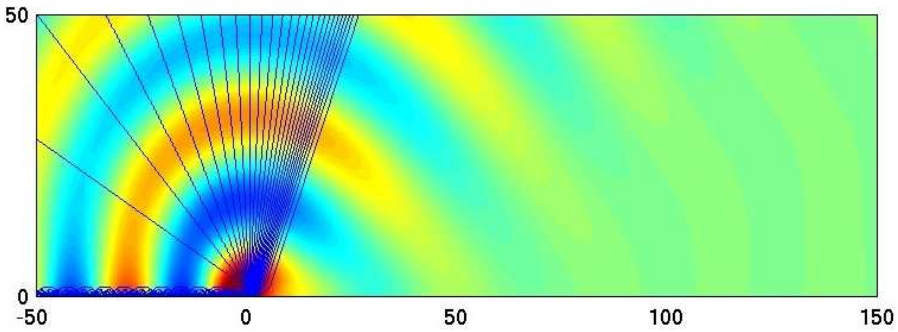
  - acoustic and instability waves are coupled except for the high-frequency limit : **geometrical acoustics** (ray tracing)

    Candel, 1977, *J. Fluid Mech.*

  - interesting numerical test case in the proceedings of the 4th CAA workshop

    Agarwal, Morris & Mani, 2004, *AIAA Journal*, 42(1)
    4th CAA workshop, NASA CP-2004-212954

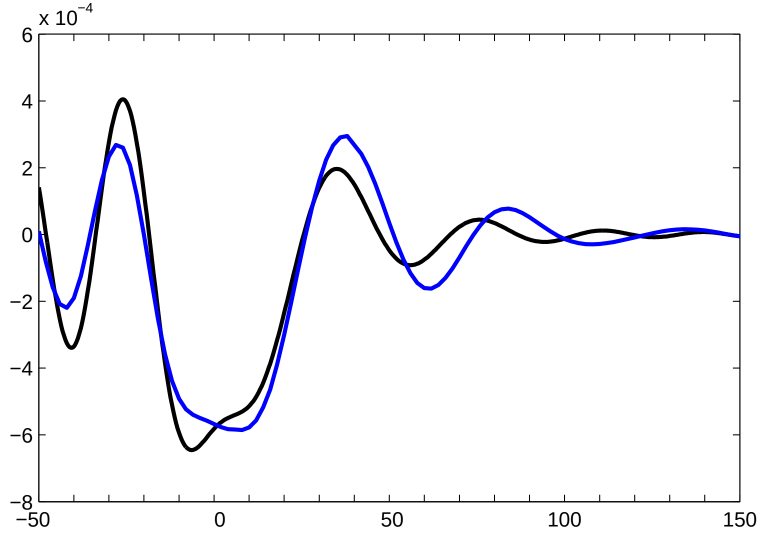
- **« Simplified » formulation of LEE**
  by removing a gradient term associated to refraction effects



$\mathrm{M}_j = 0.756 \quad T_j = 600\ K$

$\mathrm{St} = f_0 2b/u_j \simeq 0.085$

Bailly & Bogey, NASA CP-2004-212954.



pressure along the line $y = 15b$

analytical solution

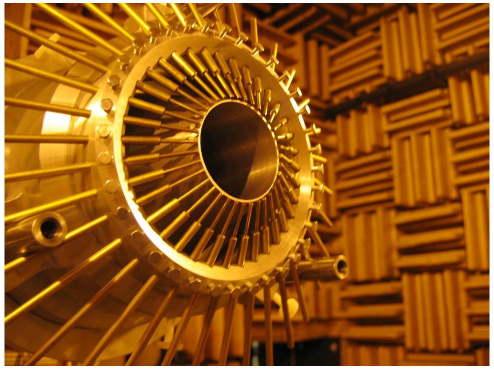
Agarwal, Morris & Mani (*AIAA Journal*, 2004)

- LEE approximation not valid for low frequency applications
- $\neq$ usual high-frequency approximation given by geometrical acoustics

Motivations

- **Physics-based predictions for real jets, *i.e.* dual, hot, with co-flow, shock-cells and noise reduction devices : shape optimization, variable geometry chevrons or fluidic actuators**

Chevrons tested on 777-300ER
(GE90- 115B engines)

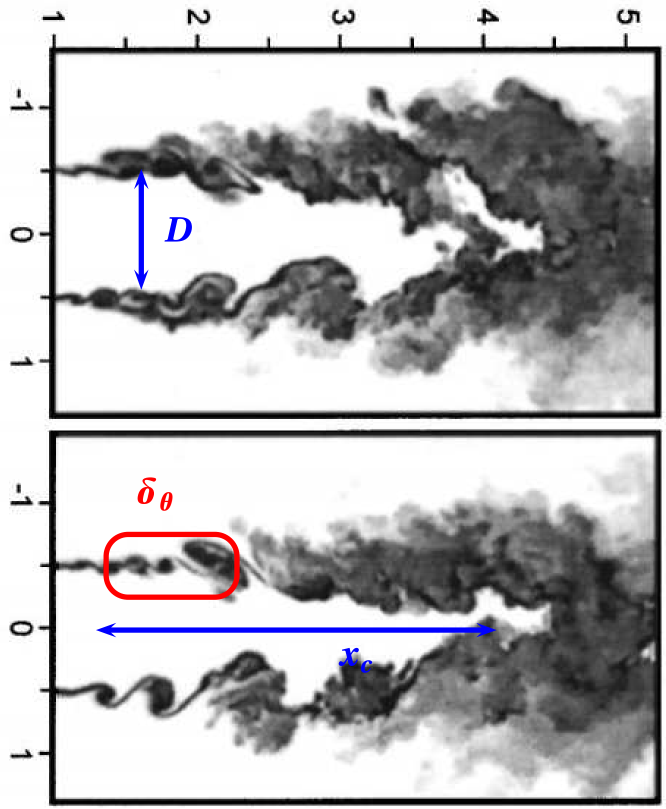


Castelain *et al.*
***AIAA Journal*, 2008, 45(5)**

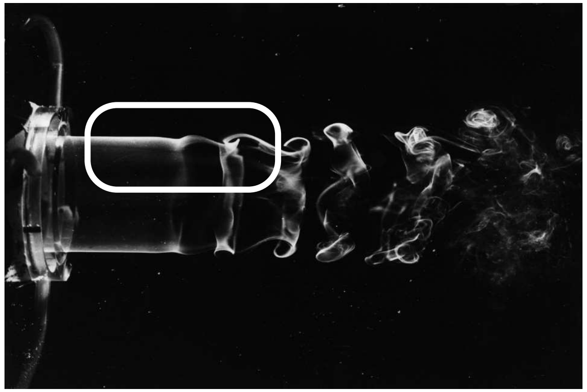
chevron nozzle (sawtooth nozzle)
Trent 800 engine - Boeing 777-200 ER
net engine benefit of ∼ 2.5 EPNdB
at takeoff certification conditions (GEAE)

- **providing reliable predictions and reference solutions**
- **understanding of jet noise mechanisms**
- **giving insight for flow control and noise reduction**

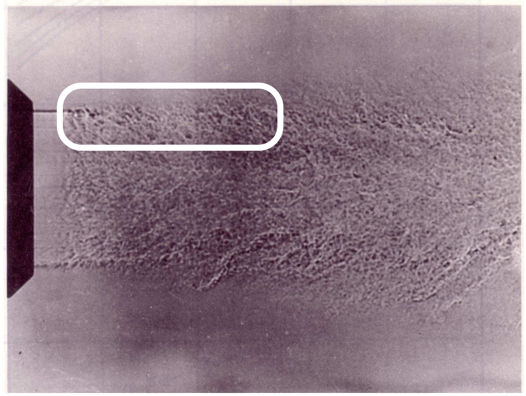
- **Reynolds number $\mathrm{Re}_D = u_j D/\nu$ and characteristic length scales**



Meyer, Dutton & Lucht (2001)
$\mathrm{Re}_D = 2.3 \times 10^4$
(two snapshots by PLIF)



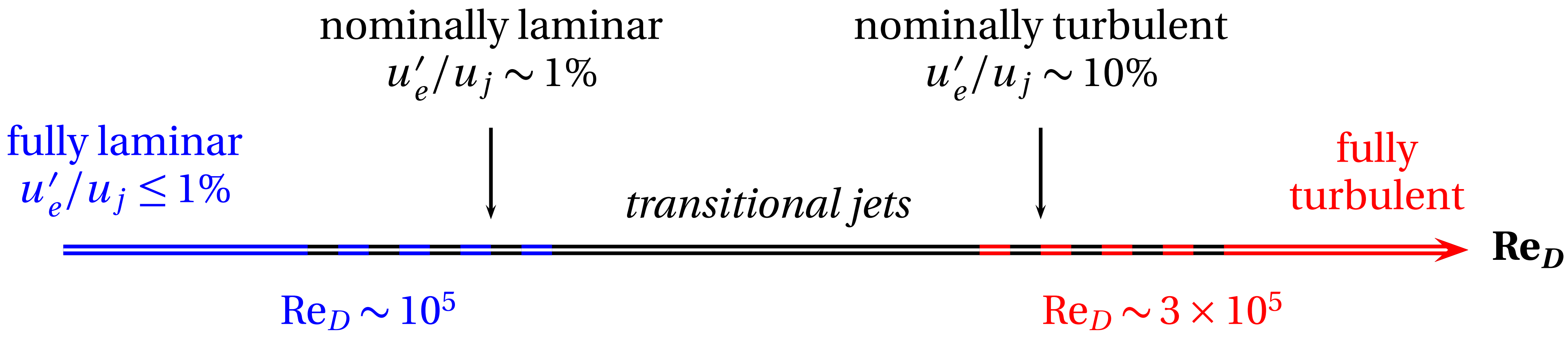
Kurima, Kasagi & Hirata (1983)
$\mathrm{Re}_D \simeq 5.6 \times 10^3$



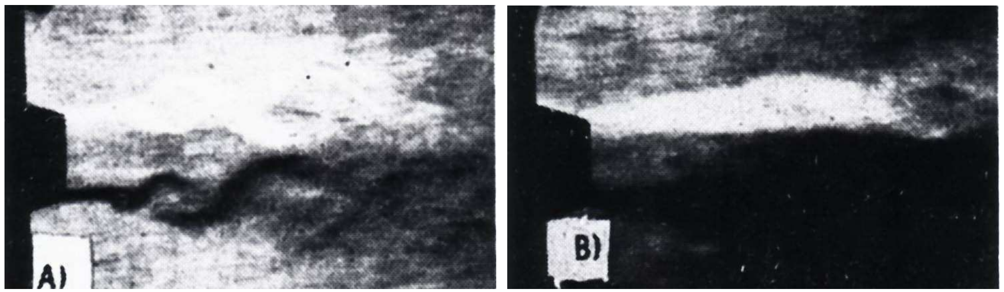
Mollo-Christensen (*MIT*, 1963)
$\mathrm{Re}_D = 4.6 \times 10^5$

- most unstable (receptive) frequency of the initial shear-layer $\mathrm{St}_{\delta_\theta} = f_0 \delta_\theta / U_j \simeq 0.012$
- preferred mode or jet column mode
  $\mathrm{St}_D = f D / U_j \simeq 0.2 - 0.5$
  (passage frequency of large-scale structures at the end of the potential core)

- **Transitional jets for $\mathrm{Re}_D \leq 3 \times 10^5$**

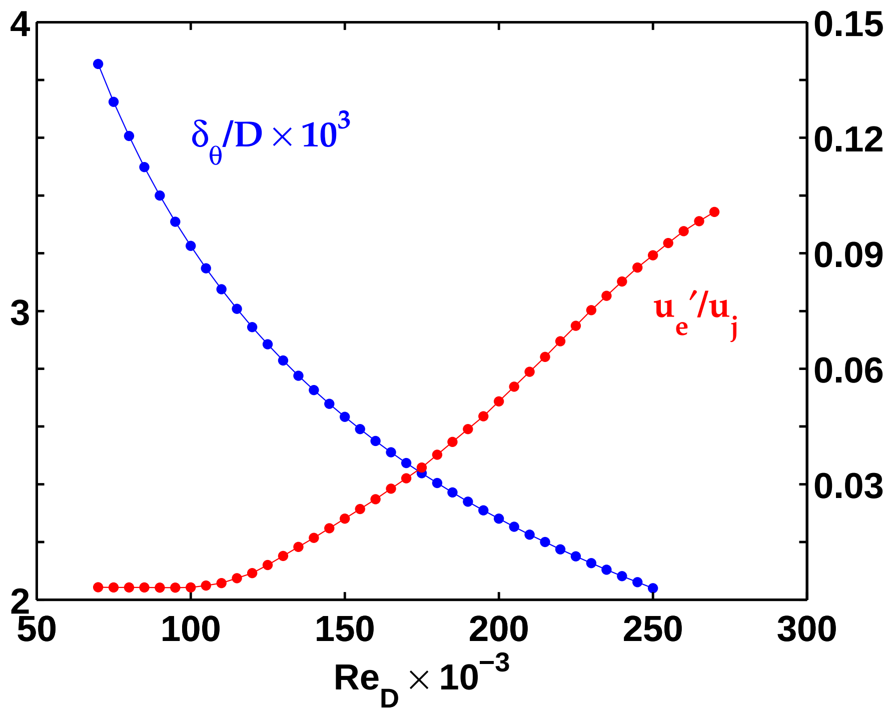


  - **for $\mathrm{Re}_D \leq 3 \times 10^5$, nominally laminar $\longrightarrow$ nominally turbulent by tripping with a laminar mean velocity profile, $u'_e/u_j \sim 10\%$ and $\delta_\theta \nearrow$**



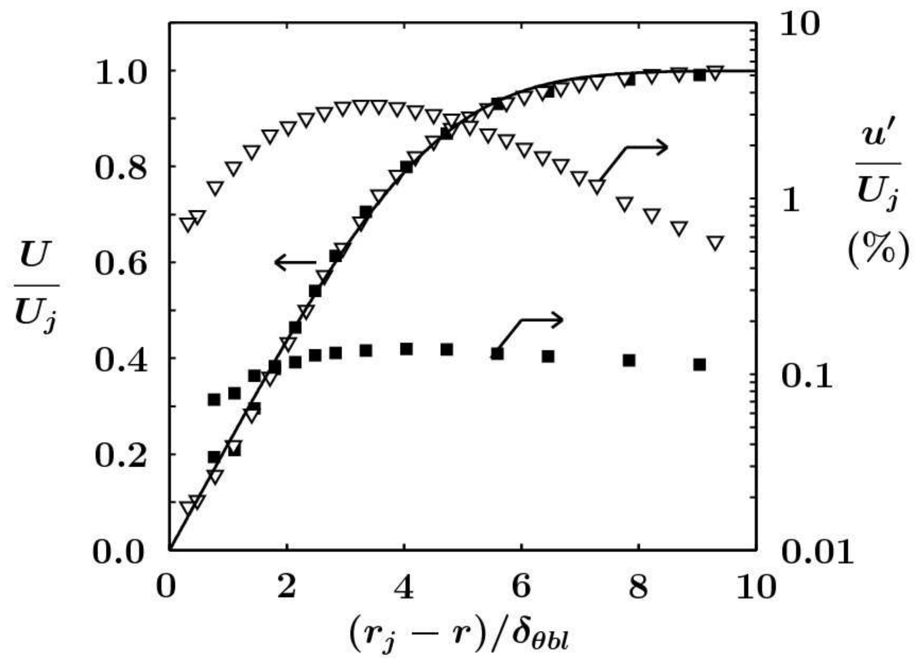
Hill, Jenkins & Gilbert, *AIAA Journal* (1976)
(A) laminar / (B) turbulent exit boundary layer
$\mathrm{Re}_D \simeq 3.4 \times 10^4$

- **Transitional jets**



$$\delta_\theta/D \simeq 1.02/\sqrt{Re_D}$$

Zaman, *AIAA Journal* (1985)



■ $Re_D = 6.7 \times 10^4$ ▽ $Re_D = 1.3 \times 10^5$

—— **Blasius profile**

Fleury (Ph.D. Thesis ECLyon, 2006)

Subsonic jet flow

- **Consequences on jet noise**

  - **to reproduce the physics of jet flows, the LES should display the same initial conditions as experiments (usually not fully available)**
  - **need for considering initially turbulent jets at high Reynolds number to prevent any form of pairing noise like in real jets : less noisy jets are indeed observed for a natural smooth development of their turbulent boundary layer**

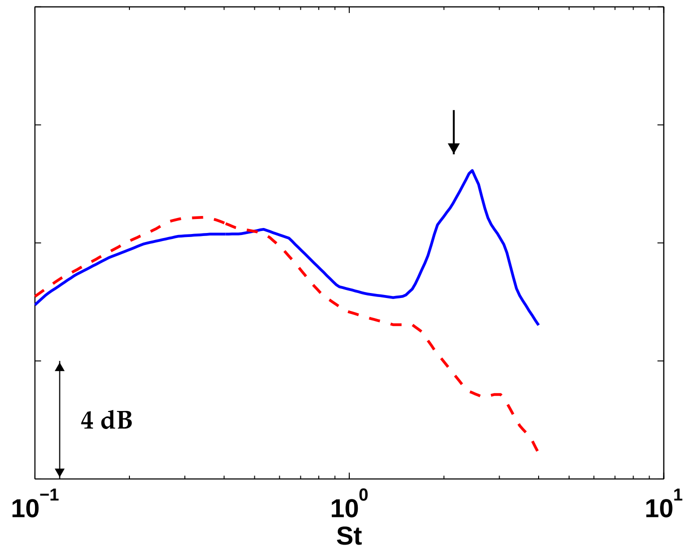
Expts	Zaman, 1985, *AIAA Journal* & *J. Fluid Mech.*
	Bridges & Hussain, 1987, *J. Sound Vib.*
	Raman, Zaman & Rice, 1989, *Phys. Fluids A*

$\mathrm{Re}_D = 1.3 \times 10^5$

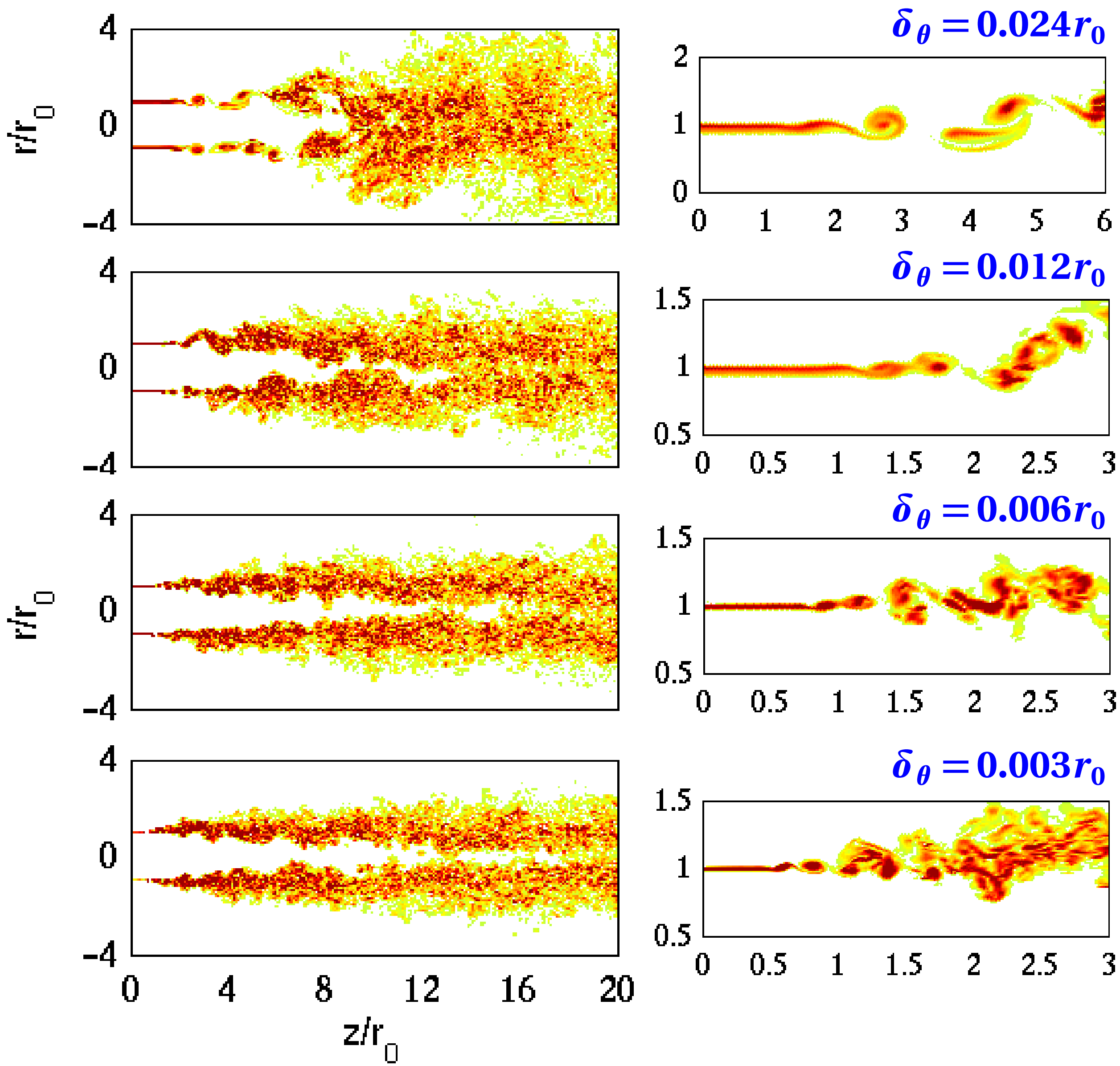
laminar jet ($u'_e/u_j \simeq 0.03\%$, $\delta_\theta/D \simeq 2.8 \times 10^{-3}$)

tripped jet ($u'_e/u_j \simeq 0.09\%$, $\delta_\theta/D \simeq 9 \times 10^{-2}$)

far field pressure spectra at $\theta = 90^o$



- **Jet development – shear layer transition (snapshots of vorticity)**



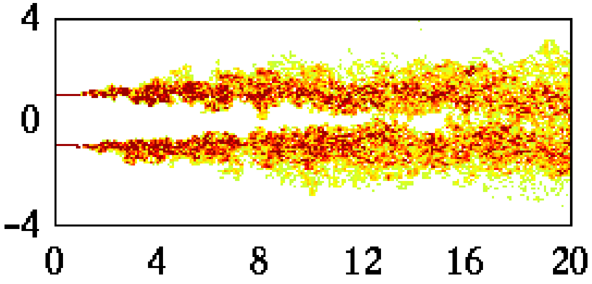
- **Smaller shear-layer thickness results in delayed jet development and longer potential core**

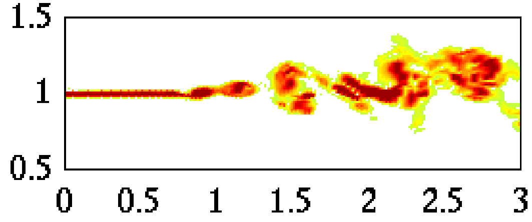
- **All transitions are characterized by shear-layer rolling-up and a first stage of strong vortex pairings**

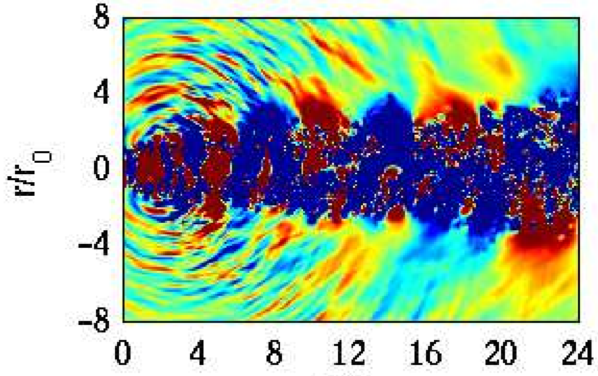
Sensitivity to inlet noise

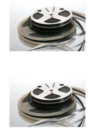
- **Jet development – shear layer transition ($\delta_\theta = 0.006 r_0$)**

no inlet noise

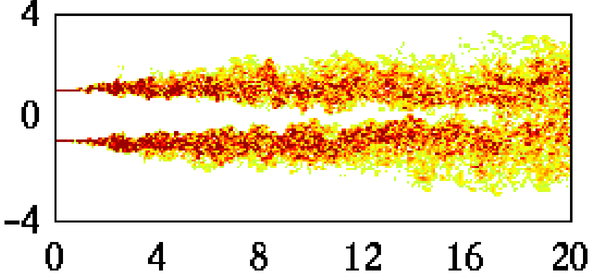


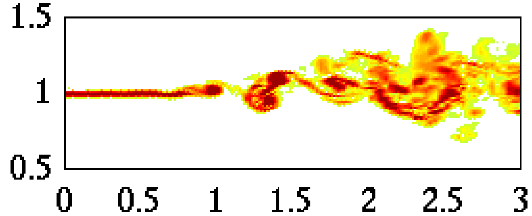


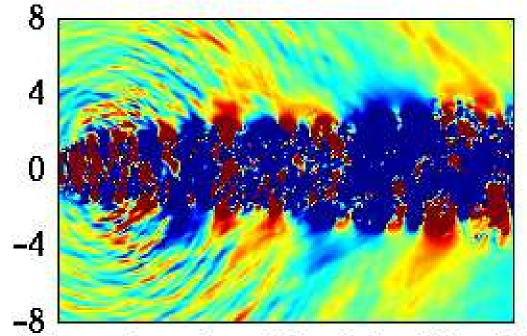




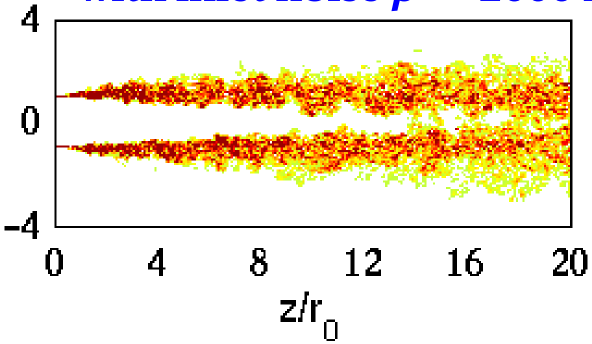
with inlet noise $p' = 250$ Pa

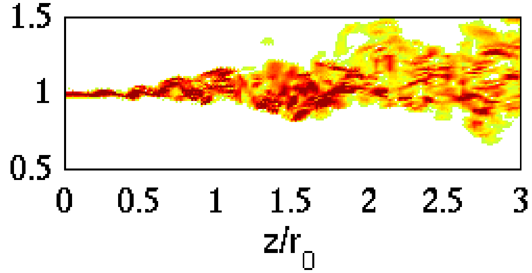


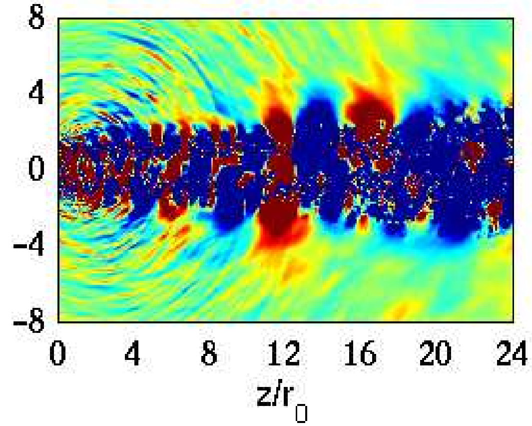




with inlet noise $p' = 2000$ Pa

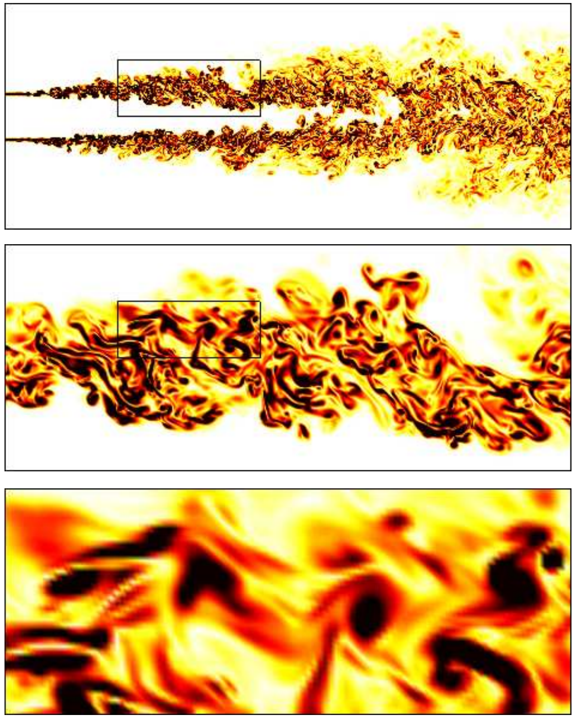






Large Eddy Simulation of turbulent jets

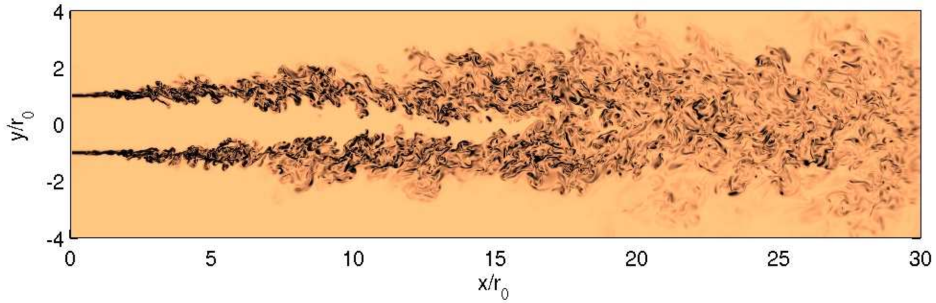
- **Direct Numerical Simulation of the turbulent development of a round jet at Reynolds number 11,000 and Mach number M=0.9**



×4

×16

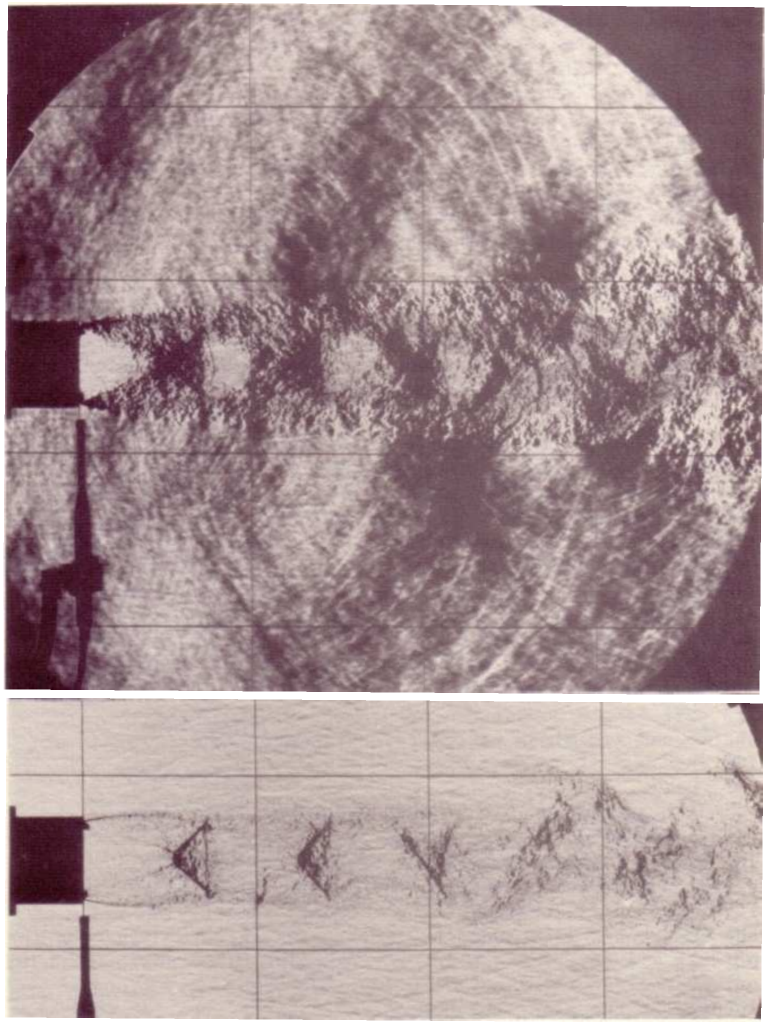
vorticity norm in the plane $z=0$
full jet in top view $0 \le x \le 27.5 r_0$
(116 pts bottom figure)



NEC SX-8 cluster at HLRS center in Stuttgart, Germany
212 GFlops, 250 GB of memory, 30,000 CPU hours

$n_x \times n_y \times n_z = 2010 \times 613 \times 613 \simeq 755 \times 10^6$ **pts**
$k_c \eta \simeq 1.5$ k_c **grid cut-off wavenumber**
$\Delta_x = \Delta y = \Delta z = r_0/68$
simulation time $T = 3000 r_0/u_j$**, 295,000 iterations**
$\delta_\theta = 0.01 \times r_0$

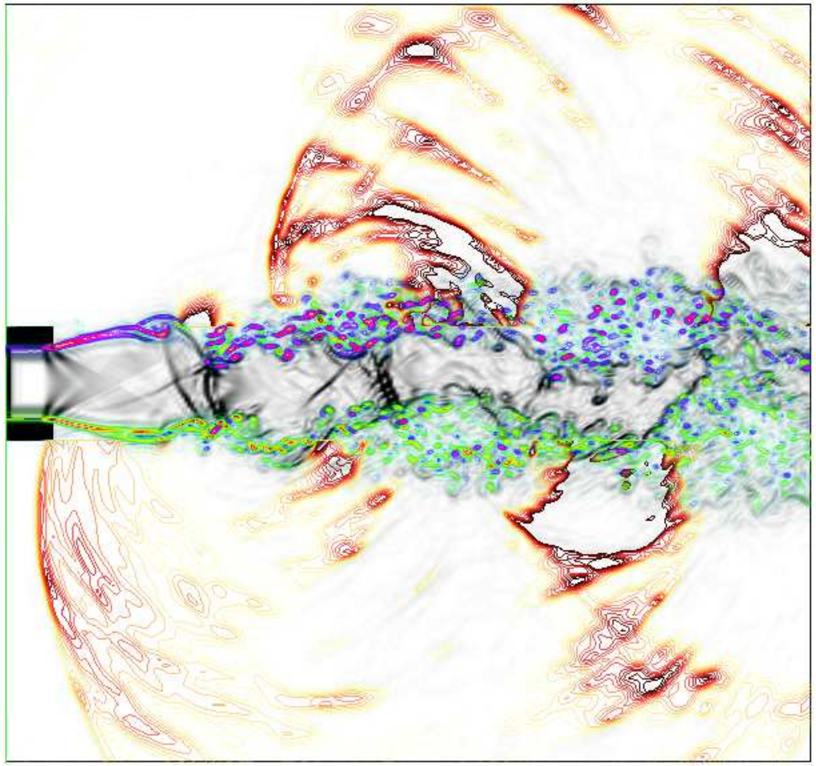
Supersonic jet noise



$p_R/p_\infty = 2.48$, $D = 5.76$ cm

$p_e/p_\infty = 2.48$, $M_j = 1.67$

Westley & Wooley, Prog. Astro. Aero., 43, 1976



Computation of the generation of screech tones in an underexpanded supersonic jet

$M_j = 1.55$ & $Re_h = 6 \times 10^4$

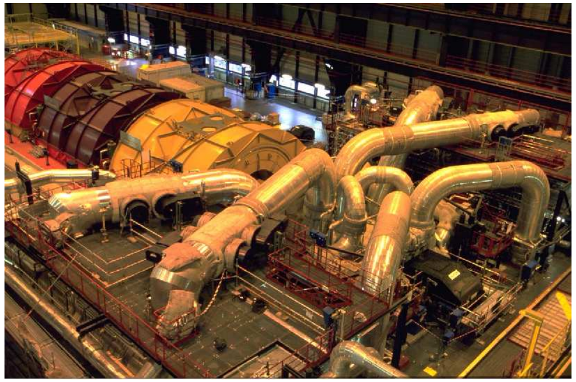
$p_e/p_\infty = 2.09$

Berland, Bogey & Bailly, *Phys. Fluids*, 19, 2007

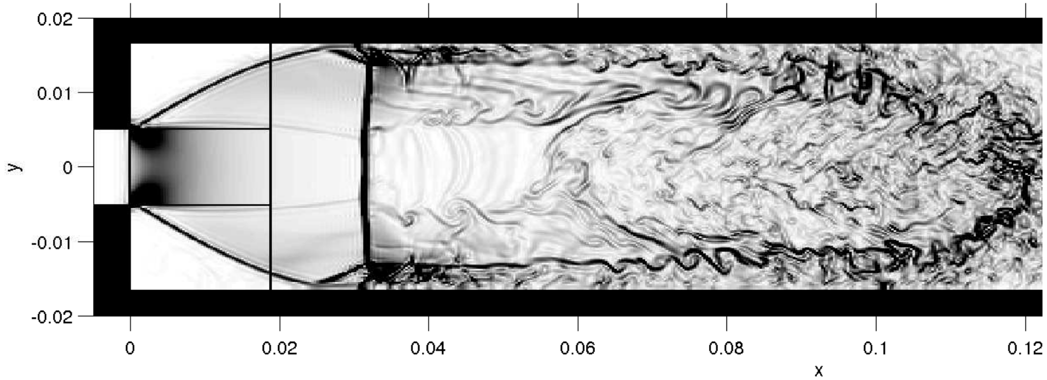
Transonic ducted flow

- **Sudden expansion of transonic flow**

Noise generated by valves in pipe systems of power plants

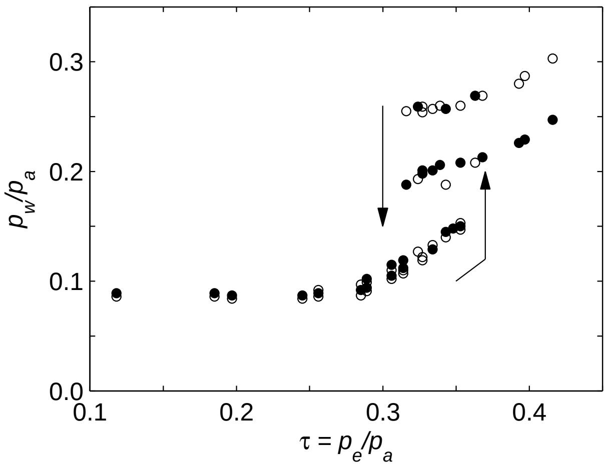


$\tau = 0.31$

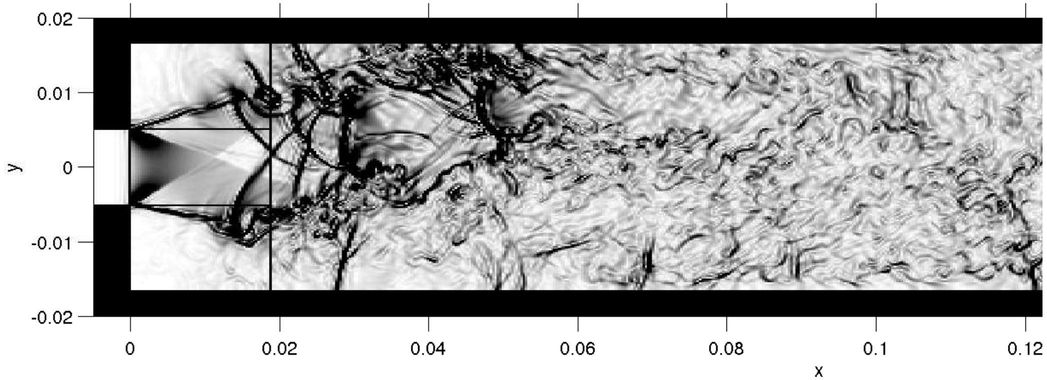


symmetrical & oscillating

hysterisis / pressure ratio τ



$\tau = 0.32$



asymmetrical & non oscillating

Emmert, Lafon (LaMSID, EDF & CNRS) & Bailly, 2009, *Phys. Fluids*, 21

Outline of the talk

- **Aerodynamic noise**
  - Direct computation of aerodynamic noise
  - Lighthill's theory of aerodynamic noise
  - Mean flow effects
  - Model problem – vortex pairing in a mixing layer
  - Physics of subsonic jet noise
  - Two short examples of supersonic flow noise
- **Long range propagation in Earth's atmosphere**
  - Mechanisms of sound absorption
  - First simulations of long-range infrasound propagation
- **Some references**

Mechanisms of sound absorption

- **Sound absorption**

  - **What is included in Navier-Stokes equations**
    classical low-frequency approximation
    Stokes-Kirchhoff equation & solution

  - **Navier-Stokes equations including vibrational relaxation effects**

- **Linearized Navier-Stokes equations**

  Assumptions : perfect gas, propagation in a homogeneous medium at rest

  Linearization of Navier-Stokes equations leads to

$$
\begin{aligned}
\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \boldsymbol{u}' &= 0 \\
\rho_0 \frac{\partial \boldsymbol{u}'}{\partial t} &= -\nabla p' + \nabla \cdot \boldsymbol{\tau}' \\
p' &= c_0^2 \rho' + \frac{p_0}{c_v} s' \\
\rho_0 T_0 \frac{\partial s'}{\partial t} &= \lambda_{\mathrm{th}} \nabla^2 T'
\end{aligned}
$$

  where

$$
\nabla \cdot \boldsymbol{\tau}' = \mu \nabla^2 \boldsymbol{u}' + \left(\frac{\mu}{3} + \mu_b \right) \nabla \left(\nabla \cdot \boldsymbol{u}' \right) = \mu \nabla^2 \boldsymbol{u}' + \mu_s \nabla \left(\nabla \cdot \boldsymbol{u}' \right)
$$

$$
\mu_s = \mu/3 + \mu_b \qquad \text{(just a notation)}
$$

Thermo-viscous effects in sound propagation

- **Remarks about the viscous stress tensor**

  - **Stokes's hypothesis (1845)**

$$\begin{aligned}\boldsymbol{\tau} &= \mu\left[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^t\right] + \lambda_2(\nabla\cdot\boldsymbol{u})\boldsymbol{I}\\ &= \mu\left[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^t - \frac{2}{3}(\nabla\cdot\boldsymbol{u})\boldsymbol{I}\right] + \left(\lambda_2 + \frac{2}{3}\mu\right)(\nabla\cdot\boldsymbol{u})\boldsymbol{I}\\ &= 2\mu\boldsymbol{S}^D + \mu_b\boldsymbol{S}^I\end{aligned}$$

μ **shear viscosity** $\mu_b = \lambda_2 + (2/3)\mu$ **bulk viscosity**

λ_2 second viscosity (μ & λ_2 equivalent to Lamé coefficients in elasticity)

Stokes's hypothesis $\mu_b = 0$

  - The bulk viscosity can be deduced experimentally from absorption of sound

$\mu_b \equiv 0$ for monoatomic gases, $\mu_b = \mu_b(T) \geq 0$

Ref. Karim, S.M. & Rosenhead, L., 1952, *Rev. Modern Phys.*, 24(2), 108-116.
Lighthill (1956) Landau & Lifchitz (1989), Pierce (1994)

Thermo-viscous effects in sound propagation

Classical low-frequency approximation

After some algebra,

$$\frac{\partial^2 p'}{\partial t^2} - c_0^2 \nabla^2 p' - \left(\frac{\mu_s}{\rho_0} + \frac{(\gamma - 1)\lambda_{\text{th}}}{\rho_0 c_p}\right) \underline{\nabla^2 \left(\frac{\partial p'}{\partial t}\right)} = \frac{(\gamma-1)\lambda_{\text{th}}}{\rho_0}\left(\frac{\lambda_{\text{th}}}{c_p} - \frac{\mu_s}{\rho_0}\right)\nabla^4 T'$$

high-order term $\sim (\lambda_{\text{th}}, \mu)^2 \omega^4$

Solution for a progressive plane wave

$$\begin{aligned} p(\boldsymbol{x}) &= \hat{p}_0 \exp[i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)] \qquad \text{with} \qquad \boldsymbol{k} = \boldsymbol{k}_r + i\boldsymbol{k}_i \\ &= \hat{p}_0 \exp(-\boldsymbol{k}_i \cdot \boldsymbol{x}) \exp[i(\boldsymbol{k}_r \cdot \boldsymbol{x} - \omega t)] \end{aligned}$$

By introducing

$$l_v = \frac{\mu_s}{\rho_0 c_0} \qquad l_{\text{th}} = \frac{\lambda_{\text{th}}}{c_p \rho_0 c_0} \qquad \text{and} \qquad l = l_v + (\gamma - 1) l_{\text{th}}$$

or equivalently $\tau_v = l_v / c_0$ and $\tau_{\text{th}} = l_{\text{th}} / c_0$

$\omega \tau_v \sim$ Stokes number

- **Classical low-frequency approximation**

  2nd order equation for k

$$k_r/k_0 \simeq 1 - \frac{3(k_0 l)^2}{8} \qquad k_i/k_0 \simeq \frac{k_0 l}{2} \qquad k_0 = \frac{\omega}{c_0}$$

which yields for the phase velocity

$$v_\varphi = \omega/k_r \simeq c_0 \left(1 + \frac{3(k_0 l)^2}{8}\right)$$

Low frequency approximation or « classical absorption »

$$\boldsymbol{k \simeq k_0 + i(l/2)k_0^2 \quad \rightsquigarrow \quad \text{attenuation coefficient } \alpha \sim k_0^2 \sim \omega^2}$$

$$\alpha = \frac{\omega^2 \mu}{2\rho_0 c_0^3} \left(\frac{4}{3} + \frac{\mu_b}{\mu} + \frac{\gamma - 1}{\mathrm{Pr}}\right)$$

- **Stokes-Kirchhoff equation**

  After some tedious calculations, 6th-order differential equation for the temperature perturbation (exact)

$$\frac{\partial^3 T'}{\partial t^3} - \left[c_0^2 \frac{\partial}{\partial t} + (l_v + \gamma l_{\mathrm{th}}) c_0 \frac{\partial^2}{\partial t^2} \right] \nabla^2 T' + \left[c_0^3 l_{\mathrm{th}} + c_0^2 l_v \gamma l_{\mathrm{th}} \frac{\partial}{\partial t} \right] \nabla^4 T' = 0$$

Solution for a progressive plane wave, Stokes-Kirchhoff equation (1868)

$$1 - \left[1 - i k_0 (l_v + \gamma l_{\mathrm{th}}) \right] (k/k_0)^2 - (i k_0 l_{\mathrm{th}} + k_0 l_v \gamma k_0 l_{\mathrm{th}})(k/k_0)^4 = 0$$

- classical low-frequency approximation is retrieved
- « bi-squared equation » ⇝ analytical expression
- see expts of Greenspan for monoatomic gases, 1956, *J. Acoust. Soc. Am.*

Kirchhoff's investigation in Rayleigh (1945, vol. II, art. 348)

or $$\frac{d^2X}{dt^2}\left\{1+\frac{P}{S}\sqrt{\left(\frac{\mu}{2n\rho}\right)}\right\}+\frac{P}{S}\sqrt{\left(\frac{n\mu}{\rho}\right)}\frac{dX}{dt}=a^2\frac{d^2X}{dx^2} \quad \text{....(10).}$$

The velocity of sound is approximately

$$a\left\{1-\tfrac{1}{2}\frac{P}{S}\sqrt{\left(\frac{\mu}{2n\rho}\right)}\right\} \quad \text{.....................(11),}$$

or in the case of a circular tube of radius r,

$$a\left\{1-\frac{1}{r}\sqrt{\left(\frac{\mu}{2n\rho}\right)}\right\} \quad \text{.....................(12).}$$

The result expressed in (12) was first obtained by Helmholtz.

348[1]. In the investigation of Kirchhoff[2], to which we now proceed, account is taken not only of viscosity but of the equally important effects arising from the generation of heat and its communication by conduction to and from the solid walls of a narrow tube.

The square of the motion being neglected, the "equation of continuity" (3) § 237 is

$$\frac{ds}{dt}+\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0 \quad \text{.................. (1);}$$

so that the dynamical equations (13) § 345 may be written in the form

$$\frac{du}{dt}+\frac{1}{\rho_0}\frac{dp}{dx}=\frac{\mu}{\rho_0}\nabla^2u+\frac{\mu}{3\rho_0}\frac{d^2s}{dx\,dt} \quad \text{.............. (2).}$$

The thermal questions involved have already been considered in § 247. By equation (4)

$$\frac{d\theta}{dt}=\beta\frac{ds}{dt}+\nu\nabla^2\theta \quad \text{.................... (3),}$$

where ν is a constant representing the thermometric conductivity.

By (3) § 247

$$p/\rho_0=b^2(1+s+\alpha\theta) \quad \text{.....................(4),}$$

in which b denotes Newton's value of the velocity of sound, viz. $\sqrt{(p_0/\rho_0)}$. If we denote Laplace's value for the velocity by a,

$$a^2/b^2=\gamma=1+\alpha\beta \quad \text{.......................(5),}$$

so that $$\beta=(a^2-b^2)/b^2\alpha \quad \text{....................... (6).}$$

[1] This and the following §§ appear for the first time in the second edition. The first edition closed with § 348, there devoted to the question of dynamical similarity.

[2] *Pogg. Ann.* vol. cxxxiv., p. 177, 1868.

It will simplify the equations if we introduce a new symbol θ' in place of θ, connected with it by the relation $\theta'=\theta/\beta$. Thus (3) becomes

$$\frac{d\theta'}{dt}-\frac{ds}{dt}=\nu\nabla^2\theta \quad \text{.......................(7),}$$

and the typical equation (2) may be written

$$\frac{du}{dt}+b^2\frac{ds}{dx}+(a^2-b^2)\frac{d\theta'}{dx}=\mu'\nabla^2u-\mu''\frac{d^2s}{dx\,dt} \quad \text{....... (8),}$$

where μ' is equal to μ/ρ_0. μ'' represents a second constant, whose value according to Stokes' theory is $\frac{1}{3}\mu'$. This relation is in accordance with Maxwell's kinetic theory, which on the introduction of more special suppositions further gives

$$\nu=\tfrac{5}{2}\mu' \quad \text{.............................(9).}$$

In any case μ', μ'', ν may be regarded as being of the same order of magnitude.

We will now, following Kirchhoff closely, introduce the supposition that the variables u, v, w, s, θ' are functions of the time on account only of the factor e^{ht}, where h is a constant to be afterwards taken as imaginary. Differentiations with respect to t are then represented by the insertion of the factor h, and the equations become

$$du/dx+dv/dy+dw/dz+hs=0 \quad \text{........... (10),}$$

$$\left.\begin{aligned} hu-\mu'\nabla^2u&=-dP/dx\\ hv-\mu'\nabla^2v&=-dP/dy\\ hw-\mu'\nabla^2w&=-dP/dz\end{aligned}\right\} \quad \text{..................(11),}$$

$$P=(b^2+h\mu'')\,s+(a^2-b^2)\,\theta' \quad \text{..............(12),}$$

$$s=\theta'-(\nu/h)\,\nabla^2\theta' \quad \text{.....................(13).}$$

By (13), if s be eliminated, (12) and (10) become

$$P=(a^2+h\mu'')\,\theta'-\frac{\nu}{h}(b^2+h\mu'')\nabla^2\theta' \quad \text{......... (14),}$$

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}+h\theta'-\nu\nabla^2\theta'=0 \quad \text{........... (15).}$$

By differentiation of equations (11) with respect to x, y, z, with subsequent addition and use of (14), (15), we find as the equation in θ'

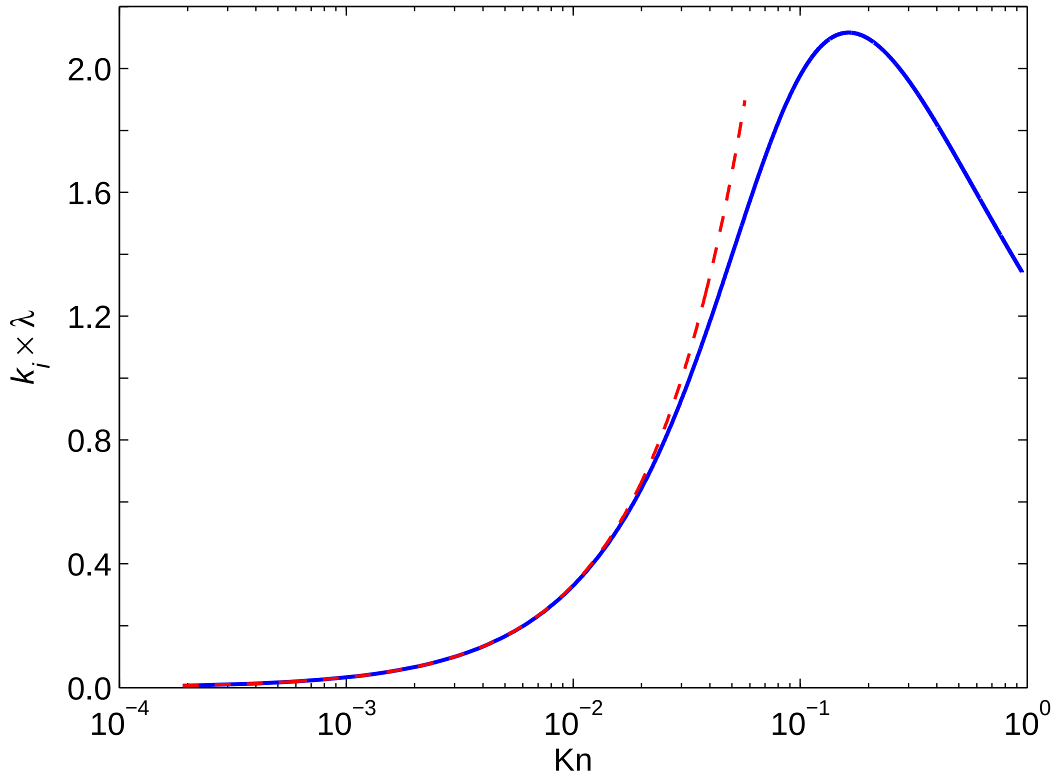
$$h^2\theta'-\{a^2+h(\mu'+\mu''+\nu)\}\nabla^2\theta'+\frac{\nu}{h}\{b^2+h(\mu'+\mu'')\}\nabla^4\theta'=0 \quad \text{...(16).}$$

Thermo-viscous effects in sound propagation

- **Solution of Stokes-Kirchhoff equation**

  Air $T_0 = 20^o$ C, data from Sutherland & Bass (2004, 2006)

Absorption coefficient $k_i \lambda$ vs. Kundsen number $\mathrm{Kn} = \sqrt{\gamma\pi/2}\,(\nu/c_0)/\lambda$



exact solution — low-frequency approximation

(Knudsen number $\mathrm{Kn} \sim \nu f / c_0^2$)

Gas in thermal non-equilibrium state

Stress tensor $\boldsymbol{\sigma}$ in Navier-Stokes equations

$$\sigma_{ij} = (-p + \mu_b \nabla \cdot \boldsymbol{u})\delta_{ij} + 2\mu S_{ij}^D$$

- Def. of thermodynamic pressure $p = (\gamma - 1)\rho e \implies$ **$\mu_b \sim$ rotational modes**

  Thermal equilibrium : one temperature T for the translational-rotational internal energy $\implies$ $\boldsymbol{\mu_b \simeq 0.6\, \mu(T)}$

$\gamma = 5/3$ for **monoatomic gas** (3 dof)

$\mu_b = 0 \rightsquigarrow \boldsymbol{e} = \boldsymbol{e}_{\text{tr}}$

$\gamma = 7/5$ for **diatomic gas** (5 dof)

$\mu_b \neq 0 \rightsquigarrow \boldsymbol{e} = \boldsymbol{e}_{\text{tr}} + \boldsymbol{e}_{\text{rot}}$

Ref. Laplace, P.S., 1816
Herzfeld, K. F. & Rice, F. O., 1928, *Phys. Rev.*
Greenspan, M., 1959, *J. Acoust. Soc. Am.*
Hanson, F. B., Morse, T. F. & Sirovich, L., 1969, *Phys. Fluids*
Pierce, A.D., 1978, *J. Sound Vib.*
Alig, A., 1997, *Thermochimica Acta*

Gas in thermal non-equilibrium state

Near equilibrium state (vibration temperature $T_\beta \rightarrow T$)

(energy transfer due to the passage of a sound wave, $T = \bar{T} + T'$)

$$\rho e = \rho e_{\mathrm{tr}} + \rho e_{\mathrm{rot}} + \rho e_{\mathrm{vib}} = \frac{p}{\gamma - 1} + \rho r \sum_\beta X_\beta T_\beta^\star e^{-T_\beta^\star / T_\beta}$$

$$\partial_t(\rho T_\beta) + \nabla \cdot (\rho \boldsymbol{u} T_\beta) = \rho (T - T_\beta)/\tau_\beta$$

$\rightsquigarrow$ relaxation equation at the lowest order for each specie β,

Landau-Teller equation (1936)

$\boldsymbol{\tau_\beta = 1/f_\beta}$ associated relaxation time or frequency $f_\beta = f_\beta(p, \mu, X_\beta)$, $\tau_{\mathrm{tr}} < \tau_{\mathrm{rot}} \ll \tau_{\mathrm{vib}}$

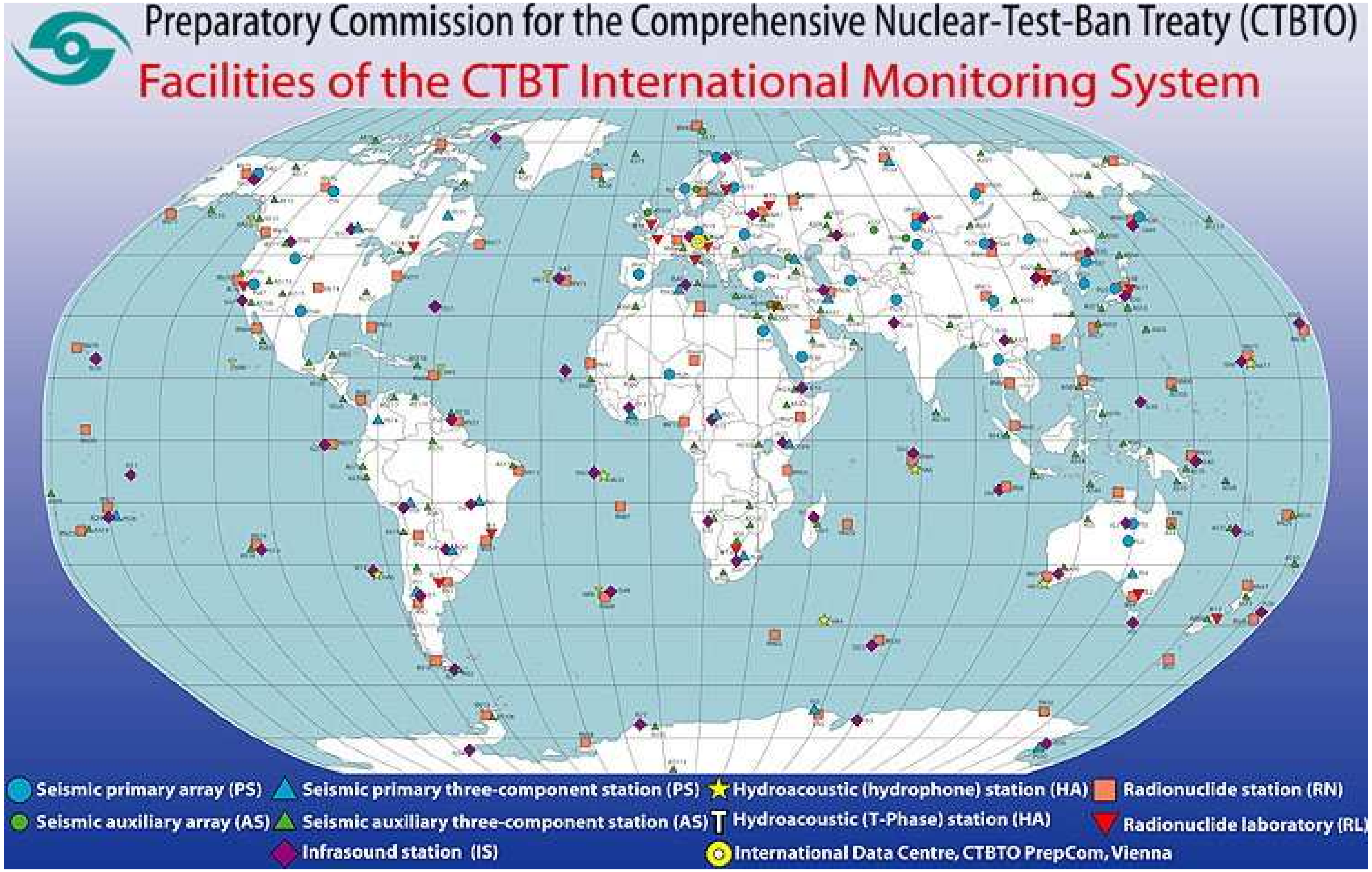
$T_\beta^\star$ temperature associated with molecular vibration at equilibrium

X_β mole fraction

$$\boldsymbol{T_\beta \longrightarrow T \qquad T \simeq T_\beta < T_\beta^\star, \qquad e^{-T_\beta^\star/T_\beta} \simeq 0}$$

$$\boldsymbol{T \simeq T_\beta > T_\beta^\star, \qquad e^{-T_\beta^\star/T_\beta} \simeq 1} \qquad \textbf{excited state}$$

- **Comprehensive Nuclear-Test-Ban Treaty (CTBT)** http://www-dase.cea.fr

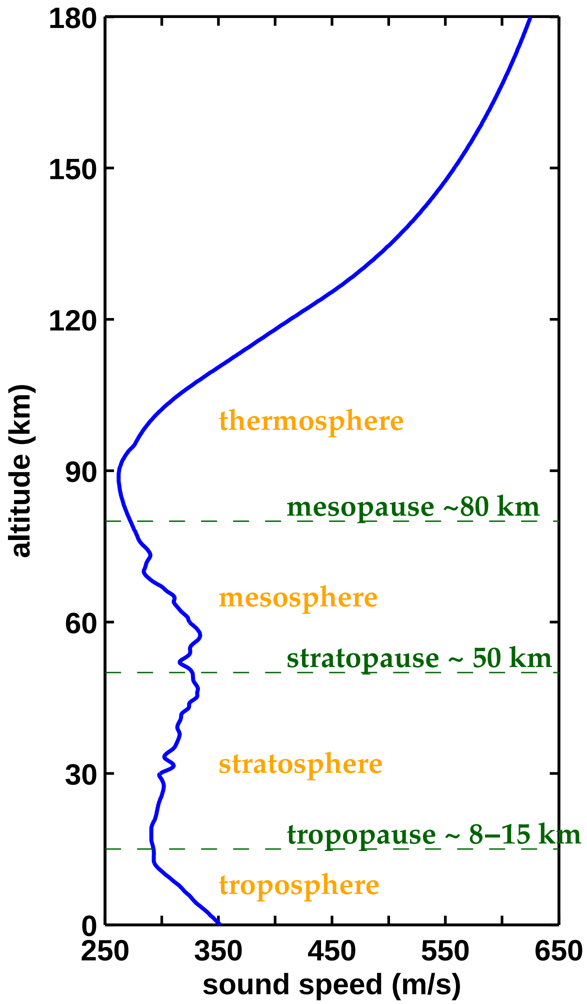


► **Generation of infrasound signals :** supersonic aircraft, space shuttle, rockets, meteorite, active volcano, industrial or military explosions, bombings, ...

Reference case for atmospheric propagation

Data provided by CEA - Gainville, O., 2007, Ph.D. ECLyon

Los Alamos National Laboratory, Sandia National Laboratories & CEA, White Sands Missile Range, 1987

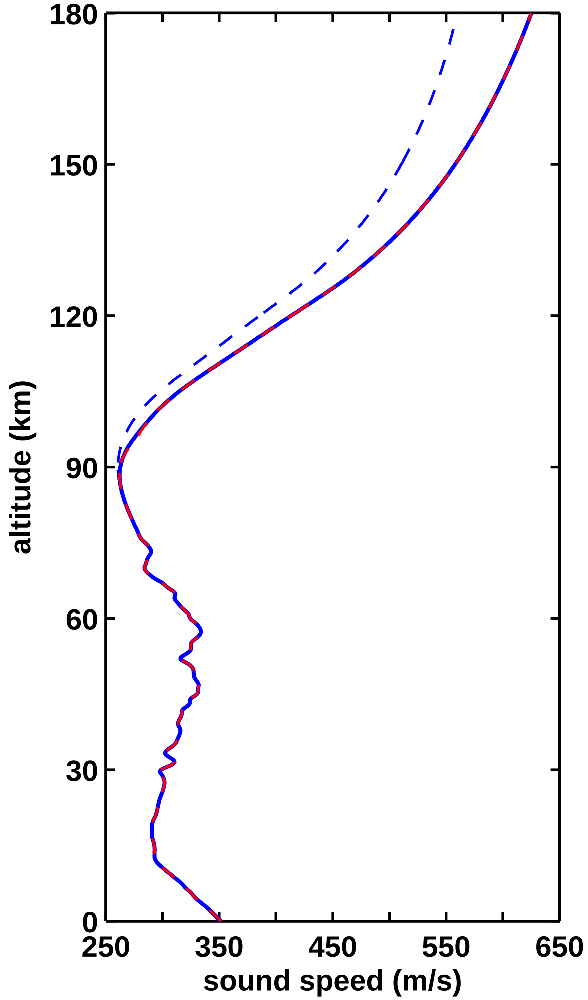


Sound speed profile

► sound waves are refracted towards the axis of the two sound channels

Reed J.W., Church, H.W. & Huck, T.W., SAND–87-2978C, 1987

Mean flow profiles at high altitude



Classical approach for computing atmospheric profiles

- measured temperature profile $T(z)$
- atmosphere model $\rightsquigarrow$ mean molecular weight $M = M(z)$

  Bass, H. E., Sutherland, L. C., Zuckerwar, A. J., Blackstock, D. T. & Hester, D. M., 1995, 1996, *J. Acoust. Soc. of Am.*
- hydrostatic equation (valid for $z \leq 200$ km)

$$\ln\left(p/p_0\right) = -\frac{g}{R}\int_0^z \frac{M}{T}\,dz$$

- perfect gas $\rho = p/(rT)$

—— $c(z)$ with $M = M(z)$

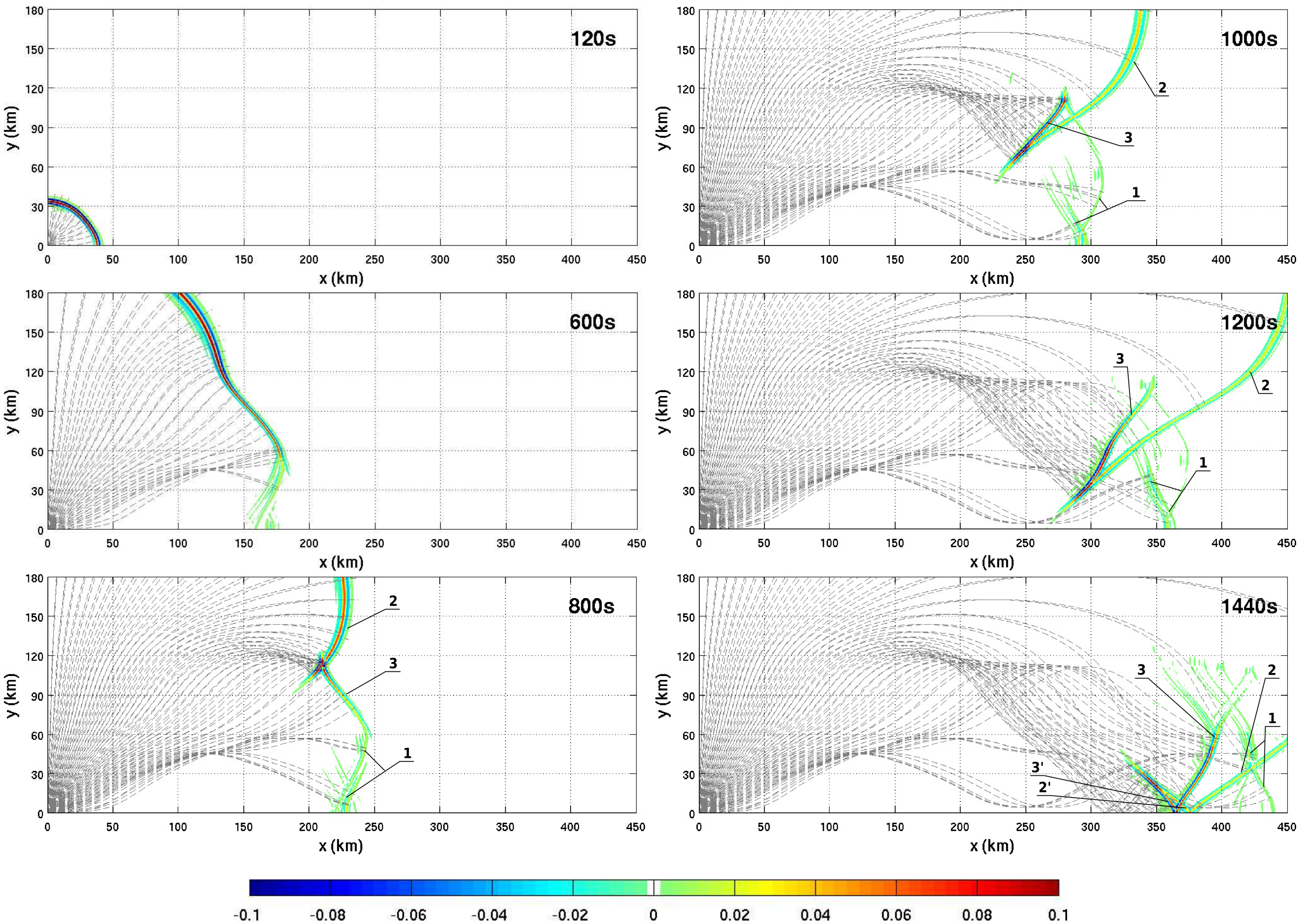
- - - $c(z)$ with $M_0 = M(z=0)$

} **from measured (in part) temperature profile $T(z)$**

« Misty Picture » experiment

- **Time history of the normalized pressure field $p'/\sqrt{\bar{\rho}}$**

  (LEE, CFL $= 0.8$, $\Delta = 300$ m, $\sigma_d = 0.1$, 1515×606 pts, - - - superimposed ray-tracing)



1 - stratospheric waves

2 - thermospheric waves refracted between 120 and 180 km

2' - ground reflexion of waves (2)

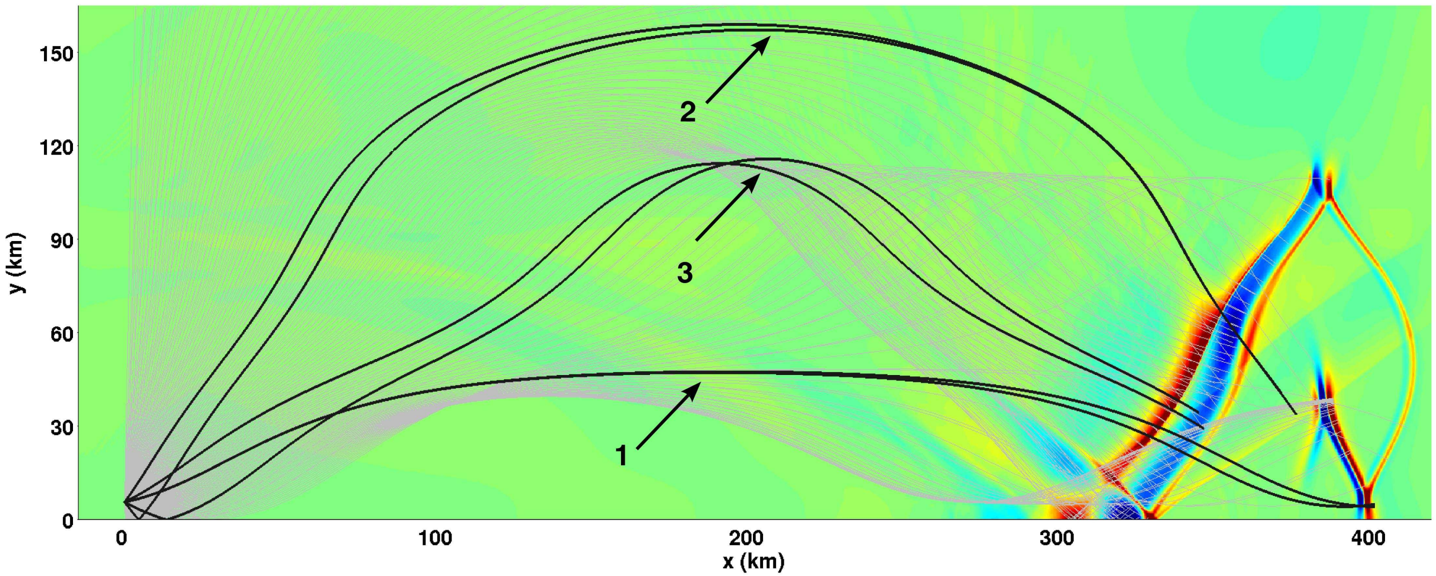
3 - thermospheric waves refracted around 115 km

3' - ground reflexion of waves (3)

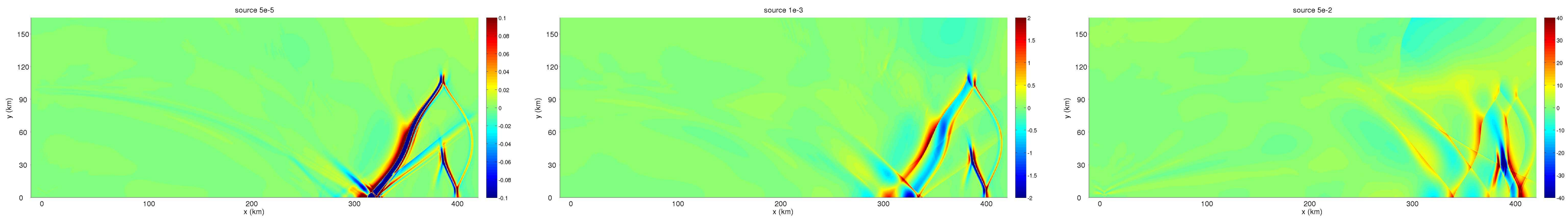
« Misty Picture » experiment

- **Nonlinear effects through the source amplitude on the time signature**

  Marsden *et al.*, 2008, 13th Long-Range Sound Propagation (LRSP)



source amplitude $A/\sqrt{\bar{p}}$ from 5×10^{-5} to 5×10^{-2}



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  - Two short examples of supersonic flow noise
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  - Mechanisms of sound absorption
  - First simulations of long-range infrasound propagation
- **Some references**

Textbooks

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