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A FINITE ELEMENT FOR BULK REACTING POROUS SOUND ABSORBERS

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INTRODUCTION

The realistic simulation of sound absorbing materials in a finite element calculation of a coupled structure - fluid system is a complex matter. There exist two main approaches in the literature to consider the interaction between the fluid and the structure. The first one is to model the physical behavior of the absorbing liner by a special finite volume element which implements the equation of motion of the absorbing material. While this may be the physically more correct approach, it suffers the drawback, that often additional degrees of freedom have to be introduced and that the measurement of the involved material parameters is complicated.

The second approach is to consider essentially the effects of the absorbing liner on the fluid in form of a boundary condition. The effects on the structure are neglected apart from the fluid pressure, which acts as a normal force on the structure, and an optional mass coating on the structure. This has the advantage that the parameters describing the boundary condition can be determined by measuring the effect of the absorbing liner on the fluid with microphones.

The most commonly used boundary condition is the normal incidence impedance boundary condition [1]. As the name indicates, it is valid for sound incidence normal to the absorbing surface. For other angles of incidence it leads to correct results only if the absorbing material is point reacting. For bulk reacting materials the boundary condition according to Bliss [2] is better suited. This boundary condition contains the impedance boundary condition as well as a correction term which accounts for the effect of bulk reaction. If it is further extended by a fluid-structure coupling term, it can be used for the development of an new finite element for the realistic simulation of sound absorbing layers on the fluid-structure interface.

DERIVATION OF THE FINITE ELEMENT

Starting point is the weak formulation of the wave propagation equation after the transformation to the frequency domain:

$$\int_V \nabla^T w \nabla p \, dV - \underbrace{i\omega\rho_0 \int_S w \underline{v}_f^T \underline{n} \, dS}_{a} - \frac{\omega^2}{c_0^2} \int_V w \, p \, dV = 0, \quad \forall w \in C_0 \tag{1}$$

where w is any C_0-continous test function, p is the fluid pressure, $\underline{v}_f$ is the velocity of air particles, $\underline{n}$ is the unit vector normal to the fluid surface, c_0 is the speed of sound, ρ_0 is the density of air, ω is the angular frequency, V the fluid volume, S the fluid surface and $\nabla^T := (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ is the nabla operator. The first and the last term of (1) will lead to the well known 'mass' matrix and 'stiffness' matrix[1] of the fluid after the Galerkin discretisation. The middle term (a) is an integral over the fluid boundary and is used to introduce the boundary condition according to Bliss into the equation of motion:

$$p + B\Delta_s p = Z \underline{v}_f^T \underline{n} \qquad (2)$$

The boundary condition is valid for thin layers of rigid porous sound absorbing material on a rigid wall. Here Z is the normal incidence impedance, $\underline{v}_f$ is the fluid velocity, and B is the bulk reaction coefficient. The operator $\Delta_s := \nabla^T (\underline{E} - \underline{n}\underline{n}^T)^2 \nabla$ is the surface Laplacian. If the wall, where the absorbing layer is fastened, is moving, it is reasonable to assume, that the absorbing layer is moving with the same velocity. Therefore replacing the fluid velocity by the difference between fluid $\underline{v}_f$ and structure velocity ($i\omega\underline{u}_s$) and solving for $\underline{v}_f$, we get:

$$\underline{v}_f^T \underline{n} = \frac{1}{Z}\left(p + B\Delta_s p\right) + i\omega\, \underline{u}_s^T \underline{n} \qquad (3)$$

substituting (3) for $\underline{v}_f^T \underline{n}$ in the middle term of (1) yields:

$$a = i\omega\rho_0 \int_S \frac{1}{Z} w\, p\, dS + \underbrace{i\omega\rho_0 \int_S \frac{B}{Z} w\, \Delta_s p\, dS}_{b} - \omega^2 \rho_0 \int_S w\, \underline{u}_s^T \underline{n}\, dS \qquad (4)$$

Again the middle term (b) of eq.(4) needs a closer look. It contains second derivatives of the pressure and therefore can not be approximated with C_0-continous element shape functions directly. This problem is solved by integrating (b) by parts:

$$b = -i\omega\rho_0 \int_S \nabla_s^T \left(\frac{B}{Z} w\right) \nabla_s p\; dS + i\omega\rho_0 \oint_C \frac{B}{Z} w\, \nabla_s^T p\; \underline{n}_C\; dC \qquad (5)$$

The curve-integral along the boundary C of the fluid surface is zero by definition, because a closed surface has no boundary. Collecting all remaining terms together we get the following weak formulation of the wave propagation equation for a fluid inside a bulk reacting moving absorbing boundary:

$$\int_V \nabla^T w \nabla p\, dV - i\omega\rho_0 \left(\int_S \frac{1}{Z} w\, p\, dS - \int_S \nabla_s^T \left(\frac{B}{Z} w\right) \nabla_s p\, dS \right) - \omega^2 \left(\frac{1}{c_0^2} \int_V w\, p\, dV - \rho_0 \int_S w\, \underline{u}_s^T \underline{n}\, dS \right) = 0, \qquad (6)$$

$$\forall\, w \in C_0$$

Now, setting $p(x,y,z) = \sum_i N_i^f(x,y,z)\, p_i$ and $\underline{u}(x,y,z) = \sum_j \underline{N}_j^s(x,y,z)\, u_j$, where N^f and $\underline{N}^s$ are the shape functions for the fluid pressure and displacements of the structure respectively and performing the galerkin discretization we get a matrix equation for the fluid part of the coupled equations of motion of the structure - fluid system:

$$\left(-\omega^2 \left[\underline{A}, \underline{M}\right] + i\omega \left[\underline{O}, \underline{D}^Z + \underline{D}^B\right] + \left[\underline{O}, \underline{K}\right]\right) \begin{bmatrix} u_j \\ p_i \end{bmatrix} = 0 \qquad (7)$$

where u_j are the displacements of the structural nodes and p_i the sound pressures at the nodes of the fluid. If the normal incidence impedance Z and the bulk reaction coefficient B are assumed to

[1] These matrices don't really have the units of mass and stiffness. They are called 'mass-' and 'stiffness' matrix in imitation of the corresponding structural matrices.

be constant over the entire finite element, then these two parameters can be pulled outside the integrals, and we get the following matrices:

$$\underline{M} = \left[m_{i,j}\right] = \frac{1}{\rho_0 c_0^2}\int_V N_i^f N_j^f dV \qquad \underline{K} = \left[k_{i,j}\right] = \frac{1}{\rho_0}\int_V \nabla^T N_i^f \nabla N_j^f dV$$

$$\underline{D}^Z = \left[d_{i,j}^Z\right] = \frac{1}{Z}\int_S N_i^f N_j^f dS \qquad \underline{D}^B = \left[d_{i,j}^B\right] = -\frac{B}{Z}\int_S \nabla_s^T N_i^f \ \nabla_s N_j^f \, dS \tag{8}$$

$$\underline{A} = \left[a_{i,j}\right] = \int_S N_i^f \underline{N}_j^{sT} \underline{n}_s dS \qquad \underline{O} = \text{Matrix of zeros}$$

where M and K are the mass matrix and the stiffness matrix of the fluid, $\underline{D}^Z$ is responsible for the normal incidence impedance behavior of the fluid boundary, $\underline{D}^B$ is the bulk reaction correction term and A describes the coupling between fluid surface nodes and structural nodes.

Implementation. The matrices M and K can be calculated with almost every FE-program. The other three matrices $\underline{D}^Z$, $\underline{D}^B$ and A result from the coupling equation. The most straight forward finite element implementation of these matrices would be a triangular or square coupling element with fluid nodes on the one side and coincident structure nodes on the other side. This approach was not followed, because

1. it would lead to a loss of flexibility, because many coupled fluid-structure problems use meshes with non-coincident nodes at the interface and
2. there are FE-codes available, which are capable of computing the coupling matrix A for non-coincident meshes.

The remaining two damping matrices $\underline{D}^Z$ and $\underline{D}^B$ only depend on fluid surface nodes. The finite elements they result from could be called 'fluid surface elements', accordingly. A triangular and a quadrilateral isoparametric fluid surface element was implemented using linear i.e. C_0-continous shape functions and a Gauss integration of order 3 for the quadrilateral element. The triangular element is simple enough to do an exact integration.

VERIFICATION

A finite element model of an impedance tube was used to investigate the correctness of the derived fluid surface element. It consists of a 3-dimensional model of a rectangular fluid volume (fig. 1) which is harmonically exited on one end by a piston (spring-mass-system) which is moving under the influence of a dynamic force with constant amplitude.

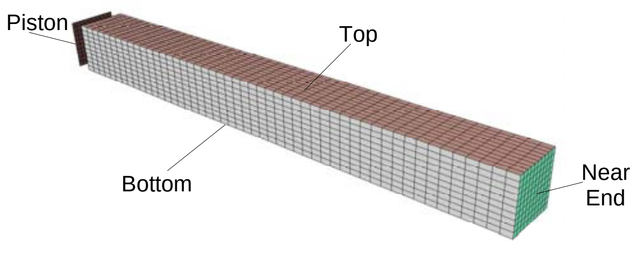


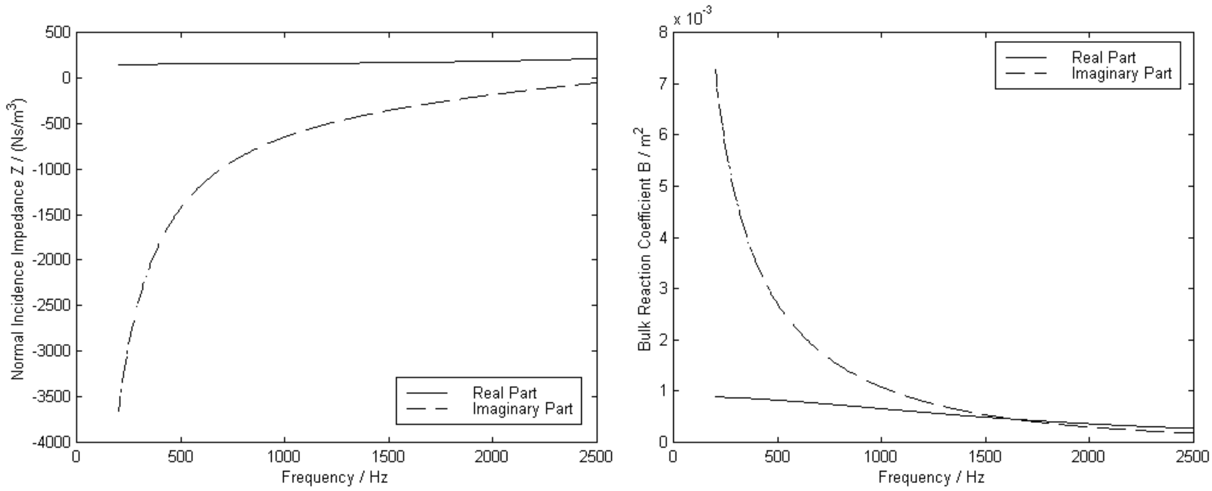
***Fig. 1**: Finite Element model of the impedance tube*

Boundary Conditions. First the ability of the fluid surface element to model the normal impedance boundary condition was examined. For this purpose the boundary of the fluid volume opposite to the piston was equipped with a layer of fluid surface elements. Various boundary conditions were studied by variation of the parameter Z while B was set to zero:

- The rigid wall (or natural) boundary condition. The normal velocity of the fluid is zero at the surface. This is equivalent to solving eq. (1) without the middle term (a). One can achieve the same effect by choosing $Z \to \infty$ (respectively a very big number).
- The 'open tube' boundary condition. The sound pressure is zero at the near end. The classical way of accomplishing that, is to choose shape functions that fulfil this boundary condition automatically. In the finite element context this means that the pressure nodes at the fluid surface are constrained to zero, and the matrix equation (7) is reduced by the surface nodes. This can be simulated by choosing $Z \to 0$ (respectively a very small number). The effect is that of a penalty term in eq. (4) which forces the sound pressures at the boundary to zero.
- The 'adapted' boundary condition occurs when the wall impedance Z equals the characteristic impedance $\rho_0 c$ of the fluid. In this case plane waves will not be reflected from the near end of the tube but fully absorbed, resulting in propagating waves.

All these boundary conditions can be modeled with high accuracy by choosing the normal incidence impedance of the fluid surface elements properly.

In a next step the top and bottom side of the tube was supplied with a bulk reacting porous sound absorber (fig. 1). Its normal incidence impedance and bulk reaction coefficient were derived from the theory of Bliss and are shown in figure 2. Then the frequency response functions for the sound pressures at the piston and at the opposite end were calculated (fig. 3b). For comparison in figure 3(a) the same calculation was done but without considering the bulk reaction effects by setting B to zero. The additional damping of the sound waves caused by the bulk reaction effects is quite significant especially for high frequencies. In figure 4 the magnitude of the simulated sound pressure field on the middle axis of the tube is depicted for a frequency of 600 Hz. The calculated field corresponds very well with the theoretical results obtained by Bliss.



(a) (b)

Fig. 2: *Frequency characteristic of the employed sound absorbing material. (a) Normal Incidence Impedance, (b) Bulk Reaction Coefficient.*

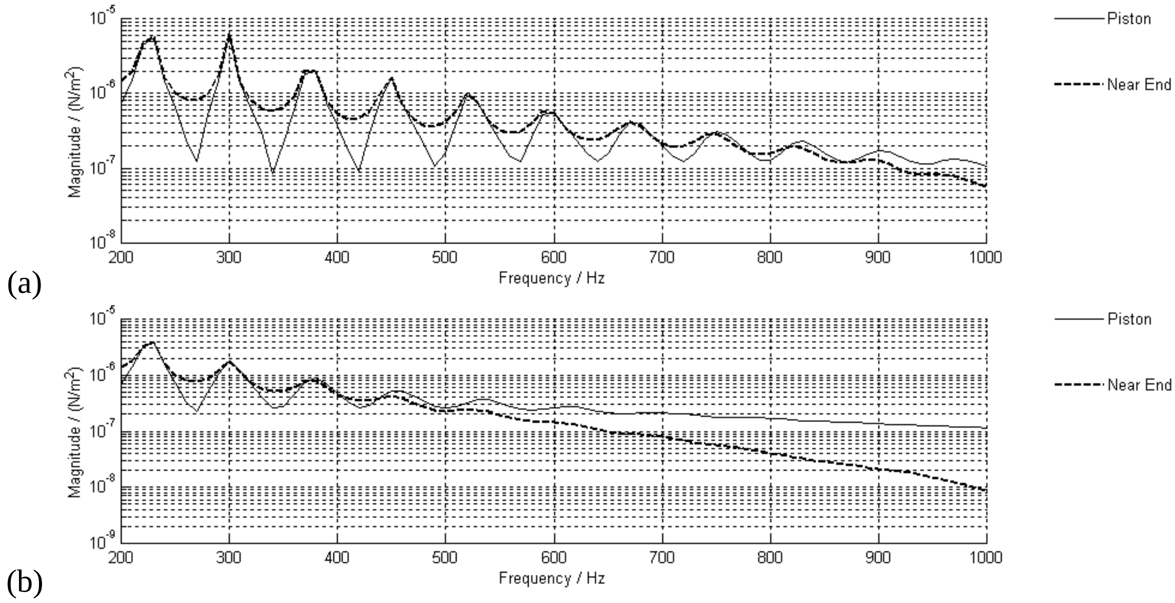


Fig. 3: *Calculation of the sound pressure without (a) and with (b) consideration of bulk reaction effects. The bulk reaction accounts for a considerable damping of the sound wave.*

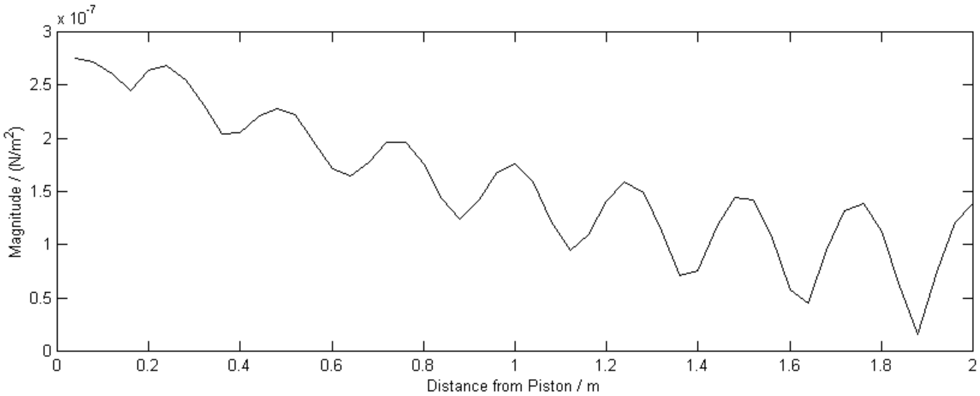
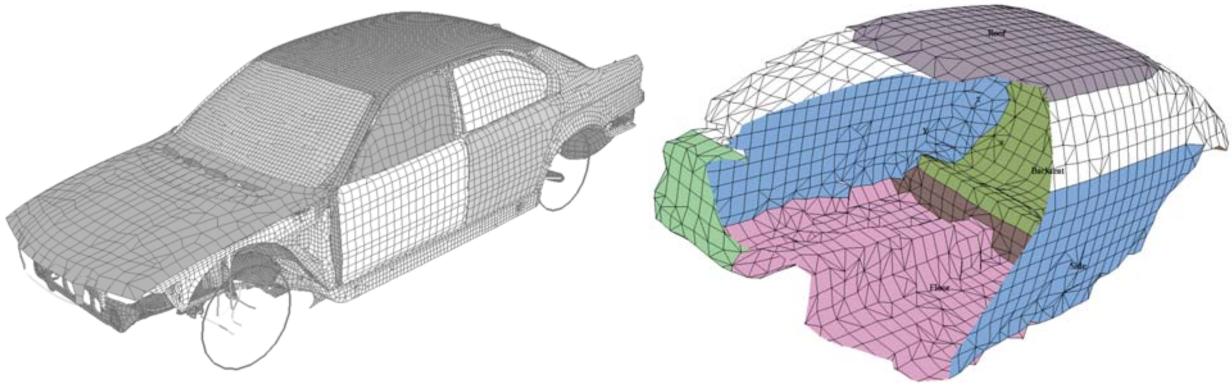


Fig. 4: *Simulated sound pressure on the middle axis of the tube at 600 Hz.*

APPLICATION

A practical application of the derived finite element is shown in figure 5. The fluid surface element has been used to model the sound absorption treatment in the passenger compartment of a BMW luxury sedan. The finite element model of the car consists of a structural part (fig. 5a) with approximately 110.000 shell-, bar-, mass- and spring- elements and a fluid part with 8800 fluid volume elements. Figure 5b) shows a section of the surface of the fluid volume, where various absorption panels of different thickness' (roof, floor, back seats, fire wall etc.) are marked by shaded areas. At the white areas of the fluid surface (mainly where the car windows are located) the rigid wall boundary condition ($B=0$, $Z \to \infty$) was used.

The car body was exited by a constant force of 1 N in the frequency range from 20 Hz - 200 Hz at one of the engine mounts, and the sound pressure at the drivers ear was calculated (fig. 6). For comparison the same calculation was done without absorption panels. As expected, the peaks of the sound pressure are higher, more narrow and shifted to higher frequencies, as compared with the absorption case.



(a) (b)

***Fig. 5**: (a) Finite element model of a BMW sedan, (b) Surface of the fluid volume with several absorption panels.*

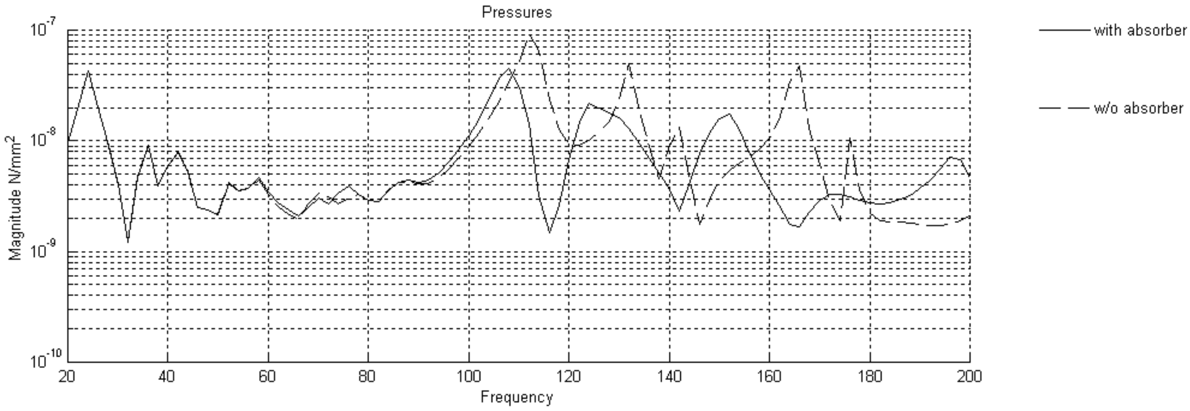


***Fig. 6**: Sound pressure at the drivers ear with and without application of sound absorbing material on the fluid surface.*

CONCLUSION

The developed finite fluid surface element based on the bulk reaction theory of Bliss is an extension to the commonly used normal incidence impedance boundary condition. It improves the calculation of sound absorption effects on the fluid caused by bulk reacting porous sound absorbers. The improvement can be significant, especially for thin layers and lateral incidence, when the bulk reaction is the dominant damping effect.

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