How Much Coordination is Needed for Robust Broadcasting over Arbitrarily Varying Bidirectional Broadcast Channels

Rafael F. Schaefer
Department of Electrical Engineering
Princeton University
Princeton, NJ 08544, USA

Holger Boche
Lehrstuhl für Theoretische Informationstechnik
Technische Universität München
80290 München, Germany

***Abstract*—The paradigm shift from an exclusive to a shared use of frequencies comes along with the necessity of new concepts since interference will be ubiquitous. Resulting channels may vary in an arbitrary and unknown manner from channel use to channel use. This is the *arbitrarily varying channel (AVC)*. Here, coordination resources such as *common randomness* or *correlated sources* have been shown to be important for reliable communication; especially for symmetrizable AVCs where deterministic approaches with pre-specified encoder and decoder fail. Here, we study the *arbitrarily varying bidirectional broadcast channel (AVBBC)*, which is motivated by the broadcast phase of bidirectional relaying. Here, a relay establishes a bidirectional communication between two nodes while sharing resources with other coexisting networks. The question is asked how much coordination is needed for reliable communication. The capacity region of the AVBBC is established and it is shown that for a transmission of block length n no more than $\mathcal{O}(\log n)$ outputs of the correlated source suffices to achieve the same as with the much stronger resource of common randomness. Such weak coordination resources of correlated sources can easily be realized by broadcasting a signal (e.g. via satellite) observed by all users only as correlated versions.**

I. Introduction

Recent research developments reveal a paradigm shift from exclusively allocating certain frequency bands to sharing them with others. Most of current systems such as cellular systems usually operate on exclusive frequency bands. In contrast to this, there are future systems such as ad-hoc or sensor networks which will operate on shared resources in an uncoordinated and self-organizing way. The main issue of this development is that interference will be ubiquitous making it to one of the major impairments in future wireless networks. Since such induced interference can no longer be coordinated between coexisting networks, there is the need of new concepts especially for the frequency usage.

In wireless networks with several uncoordinated transmitters, each receiver receives the signal he is interested in but also interfering signals from other transmitters. If there is no a priori knowledge about applied transmit strategies of all other transmitters such as modulation or coding schemes, there is no knowledge about the induced interference. Hence, all users must be prepared for the worst, which is a channel that may vary in an unknown and arbitrary manner from channel use to channel use. The concept of *arbitrarily varying channels (AVC)* [1–3] provides a suitable and robust model for such communication scenarios.

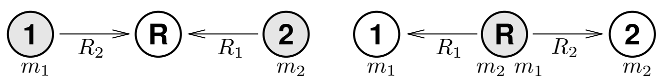
It has been shown that the capacity of an AVC highly depends on how encoder and decoder are coordinated within one transmitter-receiver link. In particular, the traditional deterministic approach with pre-specified encoder and decoder fails in the case of symmetrizable channels resulting in zero capacity [3]. Roughly speaking, such channels can emulate a valid input, which makes it impossible for the decoder to decide on the correct codeword. Unfortunately, many channels of practical importance fall in the category of symmetrizable channels [3]. This makes *coordination resources* such as *common randomness (CR)* [1, 2] or *correlated sources (CS)* [4] even more important and often necessary for reliable communication. They allow transmitter and receiver to coordinate their choice of encoder and decoder. Such more sophisticated strategies outperform deterministic approaches as they yield a positive capacity even in the case of symmetrizable channels.

In this paper, we study *bidirectional relaying*, or two-way relaying, for arbitrarily varying channels. This concept advantageously exploits the bidirectional information flow of communication which allows to reduce the inherent loss in spectral efficiency induced by half-duplex relays [5, 6]. Not surprisingly, it is a promising candidate to improve the overall performance and coverage in wireless networks such as ad-hoc, sensor, and even cellular systems.

The concept of bidirectional relaying can perfectly be applied to three-node networks, where a relay node establishes a bidirectional communication between two other nodes using a two-phase decode-and-forward protocol as shown in Fig. 1. In the first phase, the multiple access phase, both nodes transmit their messages to the relay node which is assumed to decode both messages. Thus, the first phase is the classical and well-known multiple access channel (MAC). In the second phase,

The work of Rafael F. Schaefer was supported by the German Research Foundation (DFG) under Grant WY 151/2-1. The work of Holger Boche was supported in part by the German Research Foundation (DFG) under Grant BO 1734/25-1 and in part by the German Ministry of Education and Research (BMBF) under Grant 01BQ1050.

978-1-4799-2003-7/14/$31.00



(a) MAC phase (b) BBC phase

Fig. 1. Bidirectional relaying in a three-node network, where nodes 1 and 2 exchange their messages m_1 and m_2 with the help of the relay node using a decode-and-forward protocol [27].

the bidirectional broadcast phase, the relay re-encodes and transmits both messages in such a way that both receiving nodes can decode their intended message using their own message from the previous phase as side information. Note that this scenario is different to the classical broadcast channel insofar that the receiving nodes have complementary side information. Therefore this broadcast scenario is known as *bidirectional broadcast channel (BBC)*.

Capacity results for the BBC can be found in [7, 8] for discrete memoryless channels and in [9] for MIMO Gaussian channels. Subsequently, optimal transmit strategies for the multi-antenna case are then analyzed in [10–12]. Besides the decode-and-forward protocol [7–13], there are also approaches based on amplify-and-forward [13, 14], compress-and-forward [15, 16], compute-and-forward [17, 18], or noisy network coding [19].

The initial MAC phase for arbitrarily varying channels is specified by the arbitrarily varying multiple access channel (AVMAC), which is well understood [20–24]. Therefore we concentrate in this paper on the succeeding BBC phase for arbitrarily varying channels. This is specified by the *arbitrarily varying bidirectional broadcast channel (AVBBC)* and first results can be found in [25, 26].

The rest of the paper is organized as follows. In Section II we introduce the system model for the AVBBC. Coordination resources and different code concepts are then presented in Section III. Section IV discusses the deterministic capacity and the common-randomness-assisted capacity. Then the case of correlated sources is presented in Section V. Finally, the paper ends with a conclusion in Section VI.[1]

II. Channel Model

For the bidirectional broadcast (BBC) phase we assume that the relay has successfully decoded both messages the nodes have sent in the previous multiple access (MAC) phase. It remains for the relay to re-encode and broadcast both messages in such a way that both receiving nodes can decode their intended messages with the help of their own message from the previous phase.

The communication is affected by a channel which may vary in an unknown and arbitrary manner from channel use to channel use throughout the whole transmission of a codeword. To model such a behavior, we introduce a finite state set $\mathcal{S}$. Further, let $\mathcal{X}$ and $\mathcal{Y}_i$, $i = 1,2$, be finite input and output sets. Then for a fixed state sequence $s^n \in \mathcal{S}^n$ of length n and input and output sequences $x^n \in \mathcal{X}^n$ and $y_i^n \in \mathcal{Y}_i^n$, $i = 1,2$, the discrete memoryless broadcast channel is given by $W^n(y_1^n, y_2^n|x^n, s^n) = \prod_{k=1}^n W(y_{1,k}, y_{2,k}|x_k, s_k)$. We denote its marginal channels by $W_i^n(y_i^n|x^n, s^n) = \prod_{k=1}^n W_i(y_{i,k}|x_k, s_k)$, $i = 1,2$.

Definition 1: The discrete memoryless *arbitrarily varying broadcast channel* is the family

$$\mathfrak{W} := \big\{\big(W_1^n(\cdot|\cdot, s^n), W_2^n(\cdot|\cdot, s^n)\big) : s^n \in \mathcal{S}^n\big\}.$$

We further define the "marginal" arbitrarily varying channels (AVCs) to nodes 1 and 2 by $\mathcal{W}_i := \{W_i^n(\cdot|\cdot, s^n) : s^n \in \mathcal{S}^n\}$, $i = 1,2$.

Further, for any probability distribution $\rho \in \mathcal{P}(\mathcal{S})$ we denote the averaged broadcast channel by

$$\overline{W}_\rho(y_1, y_2|x) := \sum_{s\in\mathcal{S}} W(y_1, y_2|x, s)\rho(s) \tag{1}$$

and the corresponding averaged marginal channels by $\overline{W}_{\rho,i}(y_i|x) := \sum_{s\in\mathcal{S}} W_i(y_i|x, s)\rho(s)$, $i = 1,2$, respectively.

III. Coordination Resources and Code Concepts

It has been shown that *coordination resources* are important and often necessary to establish reliable communication over AVCs [1–4] and so it is for the *arbitrarily varying bidirectional broadcast channel (AVBBC)*. Accordingly, we introduce different forms of coordination resources and their corresponding code concepts. For this purpose, let $\mathcal{M}_i := \{1, ..., M_{i,n}\}$ be the set of messages at node i, $i = 1,2$, which is also known at the relay node. We further use the abbreviation $\mathcal{M} := \mathcal{M}_1 \times \mathcal{M}_2$.

A. Deterministic Codes

If no resources for coordination are available, a deterministic code design with pre-specified encoder and decoders has to be used.

Definition 2: A *deterministic* $(n, M_{1,n}, M_{2,n})$*-code* $\mathcal{C}_{\text{det}}$ for the AVBBC $\mathfrak{W}$ is a system

$$\big\{(x_m^n, \mathcal{D}_{m_2|m_1}, \mathcal{D}_{m_1|m_2}) : m \in \mathcal{M}\big\}$$

consisting of codewords

$$x_m^n \in \mathcal{X}^n, \tag{2}$$

one for each message $m = (m_1, m_2) \in \mathcal{M}_1 \times \mathcal{M}_2 = \mathcal{M}$, and decoding sets at nodes 1 and 2

$$\mathcal{D}_{m_2|m_1} \subset \mathcal{Y}_1^n \quad \text{and} \quad \mathcal{D}_{m_1|m_2} \subset \mathcal{Y}_2^n. \tag{3}$$

For given $m_1 \in \mathcal{M}_1$ at node 1 the decoding sets must be disjoint, i.e., $\mathcal{D}_{m_2|m_1} \cap \mathcal{D}_{m_2'|m_1} = \emptyset$ for $m_2' \neq m_2$, and, similarly, for given $m_2 \in \mathcal{M}_2$ at node 2 the decoding sets must satisfy $\mathcal{D}_{m_1|m_2} \cap \mathcal{D}_{m_1'|m_2} = \emptyset$ for $m_1' \neq m_1$.

[1] *Notation:* Discrete random variables are denoted by capital letters and their realizations and ranges by lower case and script letters; $\mathbb{N}$ and $\mathbb{R}_+$ are the sets of positive integers and non-negative real numbers; $I(\cdot;\cdot)$ is the traditional mutual information; $\mathcal{P}(\cdot)$ is the set of all probability distributions; interior$(\cdot)$ denotes the interior of a set; $\mathcal{O}(\cdot)$ is the big-O notation; lhs := rhs means the value of the right hand side (rhs) is assigned to the left hand side (lhs); lhs =: rhs is defined accordingly.

Then for the deterministic code $\mathcal{C}_{\text{det}}$, the average probability of decoding error for state sequence $s^n \in \mathcal{S}^n$ is given by

$$\bar{e}_n(s^n) := \frac{1}{|\mathcal{M}|} \sum_{m \in \mathcal{M}} W^n\big((\mathcal{D}_{m_2|m_1}, \mathcal{D}_{m_1|m_2})^c | x_m^n, s^n\big).$$

Further, we define $\bar{e}_n = \max_{s^n \in \mathcal{S}^n} \bar{e}_n(s^n)$.

Definition 3: A rate pair $(R_1, R_2) \in \mathbb{R}_+^2$ is said to be *deterministically achievable* for the AVBBC $\mathfrak{W}$ if for any $\delta > 0$ there exists an $n(\delta) \in \mathbb{N}$ and a sequence of $(n, M_{1,n}, M_{2,n})$-codes $\mathcal{C}_{\text{det}}$ such that for all $n \geq n(\delta)$ we have

$$\frac{1}{n} \log M_{1,n} \geq R_2 - \delta \quad \text{and} \quad \frac{1}{n} \log M_{2,n} \geq R_1 - \delta$$

while

$$\max_{s^n \in \mathcal{S}^n} \bar{e}_n(s^n) \leq \lambda_n$$

with $\lambda_n \to 0$ as $n \to \infty$. The *deterministic capacity region* $\mathcal{R}_{\text{det}}(\mathfrak{W})$ of the AVBBC $\mathfrak{W}$ is the set of all achievable rate pairs.

B. Common-Randomness-Assisted Codes

If *common randomness (CR)* is available at the relay and nodes 1 and 2, then they can use this resource to coordinate their choice of encoder and decoders. This is modeled by a random variable Γ on $\mathcal{G}_n$. Then, codewords (2) and decoding sets (3) depend on the particular realization $\gamma \in \mathcal{G}_n$.

Definition 4: A *CR-assisted* $(n, M_{1,n}, M_{2,n}, \Gamma)$*-code* $\mathcal{C}_{\text{CR}}$ for the AVBBC $\mathfrak{W}$ is a system

$$\big\{\big(x_m^n(\gamma), \mathcal{D}_{m_2|m_1}(\gamma), \mathcal{D}_{m_1|m_2}(\gamma)\big) : m \in \mathcal{M}, \gamma \in \mathcal{G}_n\big\}$$

together with a random variable Γ uniformly distributed on $\mathcal{G}_n$.

Here, $\mathcal{G}_n$ is a finite set of deterministic $(n, M_{1,n}, M_{2,n})$-codes, cf. Definition 2. Thus, the number of deterministic codes contained in the CR-assisted code $\mathcal{C}_{\text{CR}}$ is given by $|\mathcal{G}_n|$. Each realization $\gamma \in \mathcal{G}_n$ corresponds to a particular code and determines which one is selected out of the whole ensemble.

Then for the CR-assisted code $\mathcal{C}_{\text{CR}}$, the average probability of decoding error for $s^n \in \mathcal{S}^n$ becomes

$$\bar{e}_{\text{CR},n}(s^n) := \frac{1}{|\mathcal{M}|} \sum_{m \in \mathcal{M}} \frac{1}{|\mathcal{G}_n|} \sum_{\gamma \in \mathcal{G}_n} \times W^n\big((\mathcal{D}_{m_2|m_1}(\gamma), \mathcal{D}_{m_1|m_2}(\gamma))^c | x_m^n(\gamma), s^n\big)$$

and further $\bar{e}_{\text{CR},n} = \max_{s^n \in \mathcal{S}^n} \bar{e}_{\text{CR},n}(s^n)$.

The definitions of a *CR-assisted achievable* rate pair and the *CR-assisted capacity region* $\mathcal{R}_{\text{CR}}(\mathfrak{W}, \Gamma)$ of the AVBBC $\mathfrak{W}$ follow accordingly.

C. Correlation-Assisted Codes

A weaker form of coordination resources are *correlated sources (CS)* given by $(U^n, V_1^n, V_2^n)_{n=1}^\infty$ with $I(U; V_1) > 0$ and $I(U; V_2) > 0$, where the relay observes U^n and nodes 1 and 2 observe V_1^n and V_2^n. Then, the encoder of the relay depends on $u^n \in \mathcal{U}^n$ and the decoders on $v_1^n \in \mathcal{V}_1^n$ and $v_2^n \in \mathcal{V}_2^n$ respectively. The corresponding communication problem is depicted in Fig. 2.

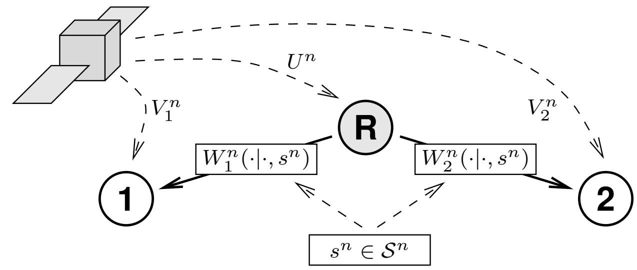


Fig. 2. Transmission over an arbitrarily varying bidirectional broadcast channel (AVBBC). A source signal is broadcasted as coordination resource, which is received by the relay and nodes 1 and 2 as correlated versions (U^n, V_1^n, V_2^n) with $I(U; V_1) > 0$ and $I(U; V_2) > 0$.

Remark 1: Note that correlated sources are in fact a weaker resource than common randomness. It has been shown that in general it is impossible to extract common randomness from correlated sources. More precisely, it is not robust in the sense that the set of all probability distributions, which would allow common randomness extraction, is closed, nowhere dense, and has zero Lebesgue measure, cf. [28, Theorem 1 and Remark 2].

Definition 5: A *CS-assisted* $(n, M_{1,n}, M_{2,n}, (U, V_1, V_2))$*-code* $\mathcal{C}_{\text{CS}}$ for the AVBBC $\mathfrak{W}$ is a system

$$\big\{\big(x_m^n(u^n), \mathcal{D}_{m_2|m_1}(v_1^n), \mathcal{D}_{m_1|m_2}(v_2^n)\big) : m \in \mathcal{M}, u^n \in \mathcal{U}^n, v_1^n \in \mathcal{V}_1^n, v_2^n \in \mathcal{V}_2^n\big\}$$

where $(u^n, v_1^n, v_2^n) \in \mathcal{U}^n \times \mathcal{V}_1^n \times \mathcal{V}_2^n$ are chosen according to $P_{U^n V_1^n V_2^n} = \prod_{i=1}^n P_{UV_1V_2} \in \mathcal{P}(\mathcal{U}^n \times \mathcal{V}_1^n \times \mathcal{V}_2^n)$.

Then for the CS-assisted code $\mathcal{C}_{\text{CS}}$, the average probability of decoding error for $s^n \in \mathcal{S}^n$ becomes

$$\bar{e}_{\text{CS},n}(s^n) := \frac{1}{|\mathcal{M}|} \sum_{m \in \mathcal{M}} \sum_{u^n, v_1^n, v_2^n} P_{U^n V_1^n V_2^n}(u^n, v_1^n, v_2^n) \times W^n\big((\mathcal{D}_{m_2|m_1}(v_1^n), \mathcal{D}_{m_1|m_2}(v_2^n))^c | x_m^n(u^n), s^n\big)$$

and further $\bar{e}_{\text{CS},n} = \max_{s^n \in \mathcal{S}^n} \bar{e}_{\text{CS},n}(s^n)$.

The definitions of a *CS-assisted achievable* rate pair and the *CS-assisted capacity region* $\mathcal{R}_{\text{CS}}(\mathfrak{W}, (U, V_1, V_2))$ of the AVBBC $\mathfrak{W}$ follow accordingly.

In Definition 5 the relay is required to observe the whole sequence $u^n \in \mathcal{U}^n$ prior to encoding as each symbol of the codeword depends on the whole observation, i.e., $x_m^n(u^n) = (x_{m,1}(u^n), x_{m,2}(u^n), ..., x_{m,n}(u^n))$. The coordination resource can further be weakened by considering causal encoding, where the relay uses only its current observation u_i, $i = 1, ..., n$ for encoding, i.e.,

$$x_m^n(u^n) = \big(x_{m,1}(u_1), x_{m,2}(u_2), ..., x_{m,n}(u_n)\big). \tag{4}$$

Note that the decoding remains the same, since both receiving nodes use block decoding which starts after having received the whole sequences $y_1^n \in \mathcal{Y}_1^n$ and $y_2^n \in \mathcal{Y}_2^n$ respectively. Then the whole coordination resources $v_1^n \in \mathcal{V}_1^n$ and $v_2^n \in \mathcal{V}_2^n$ are also available at that time.

Then, the definitions of a *causally CS-assisted achievable* rate pair and the *causally CS-assisted capacity region* $\mathcal{R}_{\text{CS}}(\mathfrak{W}, (U, V_1, V_2)_{\text{causal}})$ of the AVBBC $\mathfrak{W}$ follow accordingly.

IV. Deterministic and CR-Assisted Capacities

Based on the capabilities of the coordination resources, it is easy to see that the capacity regions can be ordered in the following way

$$\begin{aligned}\mathcal{R}_{\text{det}}(\mathfrak{W}) &\subseteq \mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)_{\text{causal}})\\ &\subseteq \mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)) \subseteq \mathcal{R}_{\text{CR}}(\mathfrak{W},\mathit{\Gamma}).\end{aligned} \tag{5}$$

Next we briefly discuss the two extreme cases in (5). These are the cases where either no coordination resources or the strongest form, i.e., common randomness, are available. For this purpose we define the region

$$\overline{\mathcal{R}}(\mathfrak{W}) := \bigcup_{P_X\in\mathcal{P}(\mathcal{X})}\Big\{(R_1,R_2)\in\mathbb{R}_+^2\colon R_1 \leq \inf_{\rho\in\mathcal{P}(\mathcal{S})} I(X;\overline{Y}_{1,\rho})$$
$$R_2 \leq \inf_{\rho\in\mathcal{P}(\mathcal{S})} I(X;\overline{Y}_{2,\rho})\Big\}$$

where $\overline{Y}_{i,\rho}$ is the random variable associated with the output of the averaged channel $\overline{W}_{i,\rho}$, $\rho\in\mathcal{P}(\mathcal{S})$, cf. (1).

To state the capacity region $\mathcal{R}_{\text{det}}(\mathfrak{W})$ for the deterministic approach with no coordination resources, we further need the concept of symmetrizability.

Definition 6: An AVBBC $\mathfrak{W}$ is *$\mathcal{Y}_i$-symmetrizable* if there exists a channel $\sigma:\mathcal{X}\to\mathcal{P}(\mathcal{S})$ such that

$$\sum_{s\in\mathcal{S}} W_i(y_i|x,s)\sigma(s|x') = \sum_{s\in\mathcal{S}} W_i(y_i|x',s)\sigma(s|x)$$

holds for all $x,x'\in\mathcal{X}$ and $y_i\in\mathcal{Y}_i$. This means, the channel $\widetilde{W}_i(y_i|x,x') = \sum_{s\in\mathcal{S}} W_i(y_i|x,s)\sigma(s|x')$ is symmetric in x,x' for all $x,x'\in\mathcal{X}$ and $y_i\in\mathcal{Y}_i$, $i=1,2$.

With this concept, we are able to completely characterize the deterministic capacity region $\mathcal{R}_{\text{det}}(\mathfrak{W})$ of the AVBBC $\mathfrak{W}$.

Theorem 1 ([25, 26]): If and only if the AVBBC $\mathfrak{W}$ is non-$\mathcal{Y}_1$-symmetrizable and non-$\mathcal{Y}_2$-symmetrizable, then the deterministic capacity region $\mathcal{R}_{\text{det}}(\mathfrak{W})$ is

$$\mathcal{R}_{\text{det}}(\mathfrak{W}) = \overline{\mathcal{R}}(\mathfrak{W}).$$

If the AVBBC $\mathfrak{W}$ is $\mathcal{Y}_1$-symmetrizable or $\mathcal{Y}_2$-symmetrizable, then $\text{interior}(\mathcal{R}_{\text{det}}(\mathfrak{W}))=\emptyset$.

The case $\text{interior}(\mathcal{R}_{\text{det}}(\mathfrak{W})) = \emptyset$ for a symmetrizable AVBBC becomes intuitively reasonable, if one realizes the following. Roughly speaking, a symmetrizable channel can emulate the input of a valid codeword so that the interfering sequence may look like another valid codeword. Then, the receiver receives a superimposed version of two valid codewords making it impossible for him to decide on the correct one. Thus, reliable communication can no longer be guaranteed which results in $\text{interior}(\mathcal{R}_{\text{det}}(\mathfrak{W}))=\emptyset$.

Since such a deterministic approach with predetermined encoder and decoders fails in this case, one is interested in more sophisticated strategies that work well also for symmetrizable channels. This is where the coordination resources enter the picture.

Theorem 2 ([25]): The CR-assisted capacity region $\mathcal{R}_{\text{CR}}(\mathfrak{W},\mathit{\Gamma})$ of the AVBBC $\mathfrak{W}$ is

$$\mathcal{R}_{\text{CR}}(\mathfrak{W},\mathit{\Gamma}) = \overline{\mathcal{R}}(\mathfrak{W}).$$

Thus, CR-assisted strategies achieve the same rates as deterministic strategies but also in the case of symmetrizable channels. Theorems 1 and 2 then yields the following characterization.

Remark 2: The deterministic capacity region $\mathcal{R}_{\text{det}}(\mathfrak{W})$ displays the following dichotomy behavior:

$$\mathcal{R}_{\text{det}}(\mathfrak{W}) = \mathcal{R}_{\text{CR}}(\mathfrak{W},\mathit{\Gamma}) \qquad \text{if interior}(\mathcal{R}_{\text{det}}(\mathfrak{W}))\neq\emptyset.$$

Thus, the deterministic approach with no coordination resources either equals the CR-assisted approach with the "strongest" type of coordination resources or else yields an empty capacity region.

A crucial observation for the analysis of the CS-assisted strategies is the following.

Remark 3: From [25, Section V-A and Lemma 1] we know that the amount of common randomness needed to achieve the CR-assisted capacity region $\mathcal{R}_{\text{CR}}(\mathfrak{W},\mathit{\Gamma})$ of the AVBBC $\mathfrak{W}$ is quadratic in block length. In more detail, for a transmission of block length n, it suffices to use a CR-assisted code which consists of n^2 deterministic codes, i.e., $|\mathcal{G}_n| = n^2$, cf. also Section III-B and Definition 4.

V. CS-Assisted Capacity Region

The previous discussion showed that deterministic strategies have the drawback that they do not work for symmetrizable channels. On the other hand, coordination based on common randomness allowed to overcome such channel conditions to transmit reliably also for symmetrizable channels. However, common randomness is a very "strong" type of coordination resource as all users, i.e., the relay and both receiving nodes, have to observe the same realization of a common random experiment. Such a coordination resource might be hard to realize in practice and the question is whether CR-assisted strategies are necessary to achieve $\overline{\mathcal{R}}(\mathfrak{W})$, cf. Theorem 2, or whether "less" coordination resources are sufficient and if so how far they can be reduced.

In the following we study the "weaker" form of coordination resources given by correlated sources as introduced in III-C. Here, all users observe correlated outputs of a common random source. The relay observes the output $u^n\in\mathcal{U}^n$ and the receiving nodes 1 and 2 the outputs $v_1^n\in\mathcal{V}_1^n$ and $v_2^n\in\mathcal{V}_2^n$ respectively which are drawn according to $P_{UV_1V_2}^n$. To ensure that these outputs are correlated, we require $I(U;V_1)>0$ and $I(U;V_2)>0$. Such a coordination resource is immediately given if all users observe, for example, a synchronization signal from a satellite, cf. Fig. 2.

The next result shows that this "weaker" form of coordination resources already suffices to achieve the same performance as the "stronger" resource of common randomness.

Theorem 3: If $I(U;V_1)>0$ and $I(U;V_2)>0$, then the (causally) CS-assisted capacity regions $\mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2))$ and $\mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)_{\text{causal}})$ of the AVBBC $\mathfrak{W}$ are

$$\mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)) = \mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)_{\text{causal}}) = \overline{\mathcal{R}}(\mathfrak{W}).$$

Proof: We will restrict ourselves to prove the desired result only for causally CS-assisted strategies, i.e.,

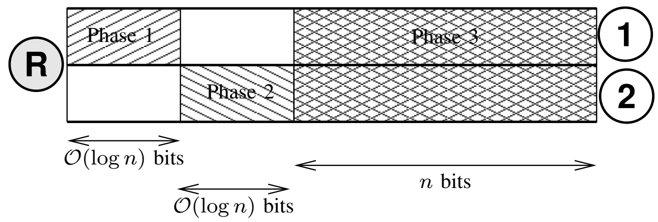


Fig. 3. Three-phase protocol that achieves the causally CS-assisted capacity region $\mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)_{\text{causal}})$ of the AVBBC $\mathfrak{W}$. In the first two phases, the relay uses a causally CS-assisted code for the classical point-to-point AVC to inform both receiving nodes which CR-assisted code is used in the following third phase.

$\mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)_{\text{causal}}) = \overline{\mathcal{R}}(\mathfrak{W})$. Then the corresponding result $\mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)) = \overline{\mathcal{R}}(\mathfrak{W})$ for (non-causally) CS-assisted strategies follows immediately from the ordering of the corresponding regions (5).

In the following we present a causally CS-assisted strategy that achieves all rate pairs $(R_1,R_2) \in \overline{\mathcal{R}}(\mathfrak{W})$. The main idea is to prove this result by concatenating a causally CS-assisted code with a CR-assisted code. The optimality or converse follows then from the fact that $\mathcal{R}_{\text{CR}}(\mathfrak{W},\mathit{\Gamma}) = \overline{\mathcal{R}}(\mathfrak{W})$, cf. Theorem 2, and the ordering $\mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)_{\text{causal}}) \subseteq \mathcal{R}_{\text{CS}}(\mathfrak{W},(U,V_1,V_2)) \subseteq \mathcal{R}_{\text{CR}}(\mathfrak{W},\mathit{\Gamma})$, cf. (5).

We assume $\mathcal{R}_{\text{CR}}(\mathfrak{W},\mathit{\Gamma}) \neq \{(0,0)\}$, since otherwise there is nothing to prove, cf. (5). From Theorem 2 we know that there exists a CR-assisted $(n,M_{1,n},M_{2,n},\mathit{\Gamma})$-code $\mathcal{C}_{\text{CR}}$ that achieves the desired rate pairs $(R_1,R_2) \in \overline{\mathcal{R}}(\mathfrak{W})$. Moreover, we know that such a code requires only polynomial amount of common randomness, i.e., $|\mathcal{G}_n| = n^2$, cf. Remark 3 and [25].

Unfortunately, such a code requires common randomness at all nodes which is not the case in our scenario. The main idea is now based on Ahlswede's *elimination technique* [2]. This means, we aim for converting this CR-assisted code to a causally CS-assisted code which has effectively the same rates and probabilities of error. This is done by concatenating the original codewords of the CR-assisted code by prefixes of negligible length which inform the receiving nodes which code out of the n^2 deterministic codes is actually used.

We propose a three-phase protocol as depicted in Fig. 3. Prior to any transmission, the relay node chooses an element $\gamma \in \mathcal{G}_n$ uniformly at random for the CR-assisted code. In the first two phases of the protocol, the relay uses causally CS-assisted codes to inform both receiving nodes which $\gamma \in \mathcal{G}_n$ was chosen. This indicates which out of the n^2 deterministic codes will be used. After the two phases, the relay and both receiving nodes have established common randomness. Thus, in the third phase, the users can use a CR-assisted code $\mathcal{C}_{\text{CR}}$ where encoder and decoders are chosen according to $\gamma \in \mathcal{G}_n$. We have to ensure that the resources spent in the first two phases to inform the receiving nodes are negligible so that there is no loss in overall rates.

Let us analyze the three phases in more detail. For the first phase, we know from [4] that for the point-to-point AVC the causally CS-assisted capacity equals the CR-assisted capacity, i.e., $R(\mathcal{W}_1,(U,V_1)_{\text{causal}}) = R(\mathcal{W}_1,\mathit{\Gamma})$, since $I(U;V_1) > 0$. Thus, there exists a causally CS-assisted code for the AVC $\mathcal{W}_1$ given by the system

$$\big\{\big(x_\gamma^{k_n}(u^{k_n}),\mathcal{D}_\gamma(v_1^{k_n})\big):\gamma\in\mathcal{G}_n,u^{k_n}\in\mathcal{U}^{k_n},v_1^{k_n}\in\mathcal{V}_1^{k_n}\big\}$$

with codewords $x_\gamma^{k_n}(u^{k_n}) \in \mathcal{X}^{k_n}$ and decoding sets $\mathcal{D}_\gamma(v_1^{k_n}) \in \mathcal{Y}_1^{k_n}$ at node 1. Note that in [4] it is not explicitly stated that the outputs of the correlated sources are available only causally, but a careful inspection of the corresponding proof, cf. [4, Equation (2.2)] reveals that the encoding is restricted to be causal meeting our definition in (4). The average probability of error is

$$\bar{e}^{(1)}_{\text{CS},k_n}(s^{k_n}) = \frac{1}{|\mathcal{G}_n|}\sum_{\gamma\in\mathcal{G}_n}\sum_{u^{k_n},v_1^{k_n}} P_{U^{k_n}V_1^{k_n}}(u^{k_n},v_1^{k_n}) \times W_1^n\big((\mathcal{D}_\gamma(v_1^{k_n}))^c|x_\gamma^{k_n}(u^{k_n}),s^{k_n}\big) \leq \lambda^{(1)}_{\text{CS},k_n}$$

for all $s^{k_n} \in \mathcal{S}^{k_n}$ with $\lambda^{(1)}_{\text{CS},k_n} \to 0$ as $k_n \to \infty$. Since $\mathcal{R}_{\text{CR}}(\mathfrak{W},\mathit{\Gamma}) \neq \{(0,0)\}$ by assumption, we have $R(\mathcal{W}_1,(U,V_1)_{\text{causal}}) > 0$ so that this code achieves positive rates and is therefore suitable for informing the receiving node which $\gamma \in \mathcal{G}_n$ was chosen by the relay.

The same argumentation holds for the second phase as well, where we also find a suitable causally CS-assisted code for the AVC $\mathcal{W}_2$

$$\big\{\big(x_\gamma^{l_n}(u^{l_n}),\mathcal{D}_\gamma(v_2^{l_n})\big):\gamma\in\mathcal{G}_n,u^{l_n}\in\mathcal{U}^{l_n},v_2^{l_n}\in\mathcal{V}_2^{l_n}\big\}$$

with average probability of error

$$\bar{e}^{(2)}_{\text{CS},l_n}(s^{l_n}) = \frac{1}{|\mathcal{G}_n|}\sum_{\gamma\in\mathcal{G}_n}\sum_{u^{l_n},v_2^{l_n}} P_{U^{l_n}V_2^{l_n}}(u^{l_n},v_2^{l_n}) \times W_2^n\big((\mathcal{D}_\gamma(v_2^{l_n}))^c|x_\gamma^{l_n}(u^{l_n}),s^{l_n}\big) \leq \lambda^{(2)}_{\text{CS},l_n}$$

for all $s^{l_n} \in \mathcal{S}^{l_n}$ with $\lambda^{(2)}_{\text{CS},l_n} \to 0$ as $l_n \to \infty$.

Thus, after the first two phases, the relay has successfully informed nodes 1 and 2 about the chosen $\gamma \in \mathcal{G}_n$ so that they can use a CR-assisted code $\mathcal{C}_{\text{CR}}$ given by

$$\big\{\big(x_m^n(\gamma),\mathcal{D}_{m_2|m_1}(\gamma),\mathcal{D}_{m_1|m_2}(\gamma)\big):m\in\mathcal{M},\gamma\in\mathcal{G}_n\big\}$$

with probability of error $\bar{e}^{(3)}_{\text{CR},n} \leq \lambda^{(3)}_{\text{CR},n}$ cf. Definition 4.

Concatenating these codes, we obtain a causally CS-assisted code of block length $\tilde{n} = k_n + l_n + n$ with codewords $(x_\gamma^{k_n}(u^{k_n}),x_\gamma^{l_n}(u^{l_n}),x_m^n(\gamma))$, decoding sets $\mathcal{D}_\gamma(v_1^{k_n}) \times \mathcal{D}_{m_2|m_1}(\gamma)$ and $\mathcal{D}_\gamma(v_2^{l_n}) \times \mathcal{D}_{m_1|m_2}(\gamma)$ at nodes 1 and 2 for all $m \in \mathcal{M}$, $\gamma \in \mathcal{G}_n$, $(u^{k_n},u^{l_n}) \in \mathcal{U}^{k_n+l_n}$, $v_1^{k_n} \in \mathcal{V}_1^{k_n}$, and $v_2^{l_n} \in \mathcal{V}_2^{l_n}$. Its average probability of error is

$$\begin{aligned}\bar{e}_{\text{CS},\tilde{n}} &\leq \max_{s^{\tilde{n}}\in\mathcal{S}^{\tilde{n}}}\big(\bar{e}^{(1)}_{\text{CS},k_n}(s^{k_n}) + \bar{e}^{(2)}_{\text{CS},l_n}(s^{l_n}) + \bar{e}^{(3)}_{\text{CR},n}(s^n)\big)\\ &\leq \lambda^{(1)}_{\text{CS},k_n} + \lambda^{(2)}_{\text{CS},l_n} + \lambda^{(3)}_{\text{CR},n} =: \lambda_{\tilde{n}}\end{aligned}$$

with $\lambda_{\tilde{n}} \to 0$ as $\tilde{n} \to \infty$. Thus, the new concatenated code satisfies the average error criterion.

It remains to check that there is no loss in rates due to the prefixes in the two first phases. As the CR-assisted code used in the third phase of the protocol requires only polynomial amount of common randomness, i.e., $|\mathcal{G}_n| = n^2$, the resources spent for the first two phases for informing the receivers is of the order $k_n = \mathcal{O}(\log n)$ and $l_n = \mathcal{O}(\log n)$ so that $k_n/n \to 0$ and $l_n/n \to 0$. Thus, the rate of the final code from the relay to node 1 is

$$\begin{aligned}\frac{1}{k_n + l_n + n}\log(n^2 M_{2,n}) \\ &= \frac{1}{k_n + l_n + n}\big(\log(n^2) + \log M_{2,n}\big) \\ &= \frac{1}{\frac{k_n}{n} + \frac{l_n}{n} + 1}\Big(\frac{1}{n}\log(n^2) + \frac{1}{n}\log M_{2,n}\Big) \underset{n\to\infty}{\longrightarrow} R_1\end{aligned}$$

since $\frac{1}{\frac{k_n}{n}+\frac{l_n}{n}+1} \to 1$ and $\frac{1}{n}\log(n^2) \to 0$ as $n \to \infty$ and further $\frac{1}{n}\log M_{2,n} = R_1$. Similarly, we get $\frac{1}{k_n+l_n+n}\log(n^2 M_{1,n}) \to R_2$ as $n \to \infty$ for the rate from the relay to node 2. Thus, the overall rates of the final causally CS-assisted code are only negligible affected by the addition of the prefixes. ■

Remark 4: Due to the concatenated structure, the code constructed above is a very special causally CS-assisted code; especially as it uses only negligible small amount of the coordination resources. Anyhow, as this code achieves all rate pairs in $\overline{\mathcal{R}}(\mathfrak{W})$, it is sufficient to achieve the causally CS-assisted capacity region $\mathcal{R}_{\text{CS}}(\mathfrak{W}, (U, V_1, V_2)_{\text{causal}})$.

VI. Conclusion

In this paper we studied the arbitrarily varying bidirectional broadcast channel (AVBBC), which is motivated by the broadcast phase of decode-and-forward bidirectional relaying in the presence of other coexisting wireless networks which use the same (wireless) resources. The corresponding capacity regions for different forms of coordination were characterized. In particular, we derived the capacity region for the case where all nodes observe correlated outputs of a random source. It turned out that the CS-assisted capacity region equals the CR-assisted capacity region. Thus, such "weak" coordination resources as correlated sources suffice to achieve the same performance as for much "stronger" approaches based on common randomness. Moreover, sophisticated strategies based on coordination resources achieve in general higher rates than deterministic approaches with pre-specified encoder and decoders; especially for symmetrizable channels.

References

[1] D. Blackwell, L. Breiman, and A. J. Thomasian, "The Capacities of Certain Channel Classes under Random Coding," *Ann. Math. Stat.*, vol. 31, no. 3, pp. 558–567, 1960.

[2] R. Ahlswede, "Elimination of Correlation in Random Codes for Arbitrarily Varying Channels," *Z. Wahrscheinlichkeitstheorie verw. Gebiete*, vol. 44, pp. 159–175, 1978.

[3] I. Csiszár and P. Narayan, "The Capacity of the Arbitrarily Varying Channel Revisited: Positivity, Constraints," *IEEE Trans. Inf. Theory*, vol. 34, no. 2, pp. 181–193, Mar. 1988.

[4] R. Ahlswede and N. Cai, "Correlated Sources Help Transmission Over an Arbitrarily Varying Channel," *IEEE Trans. Inf. Theory*, vol. 43, no. 4, pp. 1254–1255, Jul. 1997.

[5] B. Rankov and A. Wittneben, "Spectral Efficient Protocols for Half-Duplex Fading Relay Channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 379–389, Feb. 2007.

[6] P. Larsson, N. Johansson, and K.-E. Sunell, "Coded Bi-directional Relaying," in *Proc. 5th Scandinavian Workshop on Ad Hoc Networks*, Stockholm, Sweden, May 2005, pp. 851–855.

[7] T. J. Oechtering, C. Schnurr, I. Bjelaković, and H. Boche, "Broadcast Capacity Region of Two-Phase Bidirectional Relaying," *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 454–458, Jan. 2008.

[8] S. J. Kim, P. Mitran, and V. Tarokh, "Performance Bounds for Bidirectional Coded Cooperation Protocols," *IEEE Trans. Inf. Theory*, vol. 54, no. 11, pp. 5235–5241, Nov. 2008.

[9] R. F. Wyrembelski, T. J. Oechtering, I. Bjelaković, C. Schnurr, and H. Boche, "Capacity of Gaussian MIMO Bidirectional Broadcast Channels," in *Proc. IEEE Int. Symp. Inf. Theory*, Toronto, Canada, Jul. 2008, pp. 584–588.

[10] T. J. Oechtering, R. F. Wyrembelski, and H. Boche, "Multiantenna Bidirectional Broadcast Channels - Optimal Transmit Strategies," *IEEE Trans. Signal Process.*, vol. 57, no. 5, pp. 1948–1958, May 2009.

[11] T. J. Oechtering, E. A. Jorswieck, R. F. Wyrembelski, and H. Boche, "On the Optimal Transmit Strategy for the MIMO Bidirectional Broadcast Channel," *IEEE Trans. Commun.*, vol. 57, no. 12, pp. 3817–3826, Dec. 2009.

[12] R. F. Wyrembelski, T. J. Oechtering, H. Boche, and M. Skoglund, "Robust Transmit Strategies for Multiantenna Bidirectional Broadcast Channels," in *Proc. ITG Workshop Smart Antennas*, Dresden, Germany, Mar. 2012, pp. 46–53.

[13] P. Popovski and T. Koike-Akino, *Coded Bidirectional Relaying in Wireless Networks*, ser. New Directions in Wireless Communications Research. Springer US, 2009, ch. 11, pp. 291–316.

[14] H. Q. Ngo, T. Q. S. Quek, and H. Shin, "Amplify-and-Forward Two-Way Relay Networks: Error Exponents and Resource Allocation," *IEEE Trans. Commun.*, vol. 58, no. 9, pp. 2653–2666, Sep. 2009.

[15] C. Schnurr, T. J. Oechtering, and S. Stańczak, "Achievable Rates for the Restricted Half-Duplex Two-Way Relay Channel," in *Proc. Asilomar Conf. Signals, Systems, Computers*, Pacific Grove, CA, USA, Nov. 2007, pp. 1468–1472.

[16] D. Gündüz, E. Tuncel, and J. Nayak, "Rate Regions for the Separated Two-Way Relay Channel," in *Proc. Allerton Conf. Commun., Control, Computing*, Urbana-Champaign, IL, USA, Sep. 2008, pp. 1333–1340.

[17] M. P. Wilson, K. Narayanan, H. D. Pfister, and A. Sprintson, "Joint Physical Layer Coding and Network Coding for Bidirectional Relaying," *IEEE Trans. Inf. Theory*, vol. 56, no. 11, pp. 5641–5654, Nov. 2010.

[18] W. Nam, S.-Y. Chung, and Y. H. Lee, "Capacity of the Gaussian Two-Way Relay Channel to Within $\frac{1}{2}$ Bit," *IEEE Trans. Inf. Theory*, vol. 56, no. 11, pp. 5488–5494, Nov. 2010.

[19] S. H. Lim, Y.-H. Kim, A. El Gamal, and S.-Y. Chung, "Noisy Network Coding," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 3132–3152, May 2011.

[20] J.-H. Jahn, "Coding of Arbitrarily Varying Multiuser Channels," *IEEE Trans. Inf. Theory*, vol. 27, no. 2, pp. 212–226, Mar. 1981.

[21] J. A. Gubner, "On the Deterministic-Code Capacity of the Multiple-Access Arbitrarily Varying Channel," *IEEE Trans. Inf. Theory*, vol. 36, no. 2, pp. 262–275, Mar. 1990.

[22] R. Ahlswede and N. Cai, "Arbitrarily Varying Multiple-Access Channels Part I – Ericson's Symmetrizability Is Adequate, Gubner's Conjecture Is True," *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 742–749, Mar. 1999.

[23] ——, "Arbitrarily Varying Multiple-Access Channels Part II - Correlated Senders' Side Information, Correlated Messages, and Ambiguous Transmission," *IEEE Trans. Inf. Theory*, vol. 45, no. 2, pp. 749–756, Mar. 1999.

[24] M. Wiese and H. Boche, "On the Weakest Resource for Coordination in AV-MACs with Conferencing Encoders," in *Proc. IEEE Inf. Theory Workshop*, Sevilla, Spain, Sep. 2013, pp. 1–5.

[25] R. F. Wyrembelski, I. Bjelaković, and H. Boche, "Coding Strategies for Bidirectional Relaying for Arbitrarily Varying Channels," in *Proc. IEEE Global Commun. Conf.*, Honolulu, HI, USA, Dec. 2009, pp. 1–6.

[26] ——, "On the Capacity of Bidirectional Relaying with Unknown Varying Channels," in *Proc. IEEE Workshop Comp. Adv. Multi-Sensor Adaptive Processing*, Aruba, Dutch Antilles, Dec. 2009, pp. 269–272.

[27] R. F. Wyrembelski, "Robust Coding Strategies and Physical Layer Service Integration for Bidirectional Relaying," Ph.D. dissertation, Technische Universität München, Munich, Germany, Apr. 2012.

[28] H. Boche and J. Nötzel, "Arbitrarily Small Amounts of Correlation for Arbitrarily Varying Quantum Channels," in *Proc. IEEE Int. Symp. Inf. Theory*, Instanbul, Turkey, Jul. 2013, pp. 734–738.