On Multicasting Prioritized Messages

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Problem setup
Problem setup

- Ahlswede, Li, Cai and Yeung (2000)
- Avestimehr, Diggavi and Tse (2007)
Problem setup: prioritized messages

\[ W_1, W_2, W_3 \]

Transmitter

Communication Media

Receiver \[ \hat{W}_1 \]
Receiver \[ \hat{W}_2 \]
Receiver \[ \hat{W}_1, \hat{W}_2, \hat{W}_3 \]
Receiver \[ \hat{W}_1, \hat{W}_2, \hat{W}_3 \]
Receiver \[ \hat{W}_1, \hat{W}_2, \hat{W}_3 \]

Video Streaming over Heterogeneous Networks
Scalable Video Coding (SVC standard)

- Korner and Marton (1977); Nair and El-Gamal (2008)
- Ngai and Yeung (2004), Erez and Feder (2003), and Ramamoorthy and Wessel (2009)
Problem setup: objective

- A high priority (common) message of rate $R_1$ and a low priority (private) message of rate $R_2$
- public receivers and private receivers
- What are the ultimate communication rates?
- Optimal or Near optimal communication schemes?
Problem setup: objective

- A high priority (common) message of rate $R_1$ and a low priority (private) message of rate $R_2$
- public receivers and private receivers
- What are the ultimate communication rates?
- Optimal or Near optimal communication schemes?
Outline

1. Combination networks
2. The challenge
3. Linear superposition coding
4. More than two public receivers...
   - A pre-encoding approach
   - A block Markov encoding scheme
5. Optimality results
6. Why are combination networks useful?
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6. Why are combination networks useful?
A combinatorial network model: combination networks
A combinatorial network model: combination networks
A combinatorial network model: combination networks

- A simple combinatorial model to capture the interaction of the signals
- Connections to linear deterministic broadcast channels
A simple combinatorial model to capture the interaction of the signals
Connections to linear deterministic broadcast channels
A simple combinatorial model to capture the interaction of the signals
Connections to linear deterministic broadcast channels
A combinatorial network model: combination networks

\[ W_1 = [w_{1,1}] \]
\[ W_2 = [w_{2,1}, w_{2,2}] \]

A simple combinatorial model to capture the interaction of the signals
Connections to linear deterministic broadcast channels
$m = 2$ public receivers, 2 private receivers
Notation

- \( m = 2 \) public receivers, 2 private receivers
- \( \mathcal{E}_s, s \subseteq \{1, 2\} \): the set of all resources connected to (and only to) every public receiver \( i \in S \)
Notation

- $m = 2$ public receivers, 2 private receivers
- $\mathcal{E}_S$, $S \subseteq \{1, 2\}$: the set of all resources connected to (and only to) every public receiver $i \in S$
- $\mathcal{E}_S^p$, $S \subseteq \{1, 2\}, p \in \{3, 4\}$: in $\mathcal{E}_S$ but also connected to private receiver $p$
Outline

1 Combination networks
2 The challenge
3 Linear superposition coding
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The challenge

\[ W_1 = [w_{1,1}] \]
\[ W_2 = [w_{2,1}, w_{2,2}] \]
The challenge

\[ W_1 = [w_{1,1}] \]

\[ W_2 = [w_{2,1}, w_{2,2}] \]
The challenge

\[ W_1 = [w_{1,1}] \]

\[ W_2 = [w_{2,1}, w_{2,2}] \]
The challenge

\[ W_1 = [w_{1,1}] \]
\[ W_2 = [w_{2,1}, w_{2,2}] \]
**The challenge**

\[ W_1 = [w_{1,1}] \]

\[ W_2 = [w_{2,1}, w_{2,2}] \]
The challenge

Combination networks
Linear superposition coding
More than two public receivers...
Optimality results
Why are combination networks useful?

Mixing of the common and private messages is necessary; but in a controlled manner

One has to reveal (partial) information about the private message to public receivers!
Main Results

An achievable rate-region using a standard linear superposition encoding schemes.

capacity region for two public and any number of private receivers.
Main Results

- An achievable rate-region using a standard **linear superposition encoding** schemes.
  - capacity region for **two public** and **any number of private** receivers.
- The rate-region is enlarged by employing a proper **pre-encoding** at the transmitter.
  - capacity region for **three (or fewer) public** and **any number of private** receivers.
- A **block Markov encoding** scheme may improve both previous schemes.
  - capacity region for **three (or fewer) public** and **any number of private** receivers.
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Rate splitting and linear superposition coding

- let \( W = [w_{1,1} \ldots w_{1,R_1} w_{2,1} \ldots w_{2,R_2}]^T \)
- let \( X = A \cdot W \)
- reveal information about the private messages to public receivers through a **zero-structured encoding matrix**
- a linear superposition coding scheme

\[
A = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\[
R_2 = \alpha_{\{1,2\}} + \alpha_{\{2\}} + \alpha_{\{1\}} + \alpha_{\phi}
\]
Rate splitting and linear superposition coding

- let $W = [w_{1,1} \ldots w_{1,R_1} w_{2,1} \ldots w_{2,R_2}]^T$
- let $X = A \cdot W$
- reveal information about the private messages to public receivers through a zero-structured encoding matrix
- a linear superposition coding scheme

\[
A = \begin{bmatrix}
R_1 & \alpha_{\{1,2\}} & \alpha_{\{1\}} & \alpha_{\{2\}} & \alpha_{\phi} \\
0 & 0 & 0 & 0 & \epsilon_{\{1,2\}} \\
0 & 0 & \epsilon_{\{1\}} & \epsilon_{\{2\}} & \epsilon_{\phi}
\end{bmatrix}
\]

\[
R_2 = \alpha_{\{1,2\}} + \alpha_{\{2\}} + \alpha_{\{1\}} + \alpha_{\phi}
\]

- choose appropriate parameters, and complete the matrix
Rate-region I

A rate pair \((R_1, R_2)\) is achievable if there exist variables 
\(\alpha_{\phi}, \alpha_{\{1\}}, \alpha_{\{2\}}, \alpha_{\{1,2\}}\), s.t.

Structural constraints:
\[
\alpha_S \geq 0 \quad \forall S \subseteq \{1, 2\}
\]
\[
R_2 = \sum \alpha_S
\]

Decoding constraints at public receiver \(i \in \{1, 2\}\):
\[
R_1 + \sum_{S \ni i} \alpha_S \leq \sum_{S \ni i} |\mathcal{E}_S|
\]

Decoding constraints at private receiver \(p\):
\[
R_2 \leq \sum_{S \in \mathcal{T}} \alpha_S + \sum_{S \in \mathcal{T}^c} |\mathcal{E}_S^p| \quad \forall \mathcal{T} \subseteq 2^{\{1,2\}} \text{ superset saturated}
\]
\[
R_1 + R_2 \leq \sum_{S \subseteq \{1,2\}} |\mathcal{E}_S^p|
\]

The converse holds for two public and any number of private receivers, characterizing the capacity region.
**Theorem**

Rate \((R_1, R_2)\) is achievable if and only if

\[
R_1 \leq \min \left( |E\{1\}| + |E\{1,2\}|, |E\{2\}| + |E\{1,2\}| \right)
\]

\[
R_1 + R_2 \leq \min_{p \in I_2} \left\{ |E^p| + |E^p|_1 + |E^p|_2 + |E^p|_{1,2} \right\}
\]

\[
2R_1 + R_2 \leq \min_{p \in I_2} \left\{ |E\{1\}| + 2|E\{1,2\}| + |E\{2\}| + |E^p|_\phi \right\}
\]
Two public and any number of private receivers

Theorem

Rate \((R_1, R_2)\) is achievable if and only if

\[
R_1 \leq \min \left( |E_{\{1\}}| + |E_{\{1,2\}}|, |E_{\{2\}}| + |E_{\{1,2\}}| \right)
\]

\[
R_1 + R_2 \leq \min_{p \in I_2} \left\{ |E_\phi^p| + |E_{\{1\}}^p| + |E_{\{2\}}^p| + |E_{\{1,2\}}^p| \right\}
\]

\[
2R_1 + R_2 \leq \min_{p \in I_2} \left\{ |E_{\{1\}}| + 2|E_{\{1,2\}}| + |E_{\{2\}}| + |E_\phi^p| \right\}
\]

\[
R_1 \leq 2 \\
R_1 + R_2 \leq 3 \\
2R_1 + R_2 \leq 4
\]
Two public and any number of private receivers

**Theorem**

Rate \((R_1, R_2)\) is achievable if and only if

\[
R_1 \leq \min \left( |\mathcal{E}_{\{1\}}| + |\mathcal{E}_{\{1,2\}}|, |\mathcal{E}_{\{2\}}| + |\mathcal{E}_{\{1,2\}}| \right)
\]

\[
R_1 + R_2 \leq \min_{p \in I_2} \left\{ |\mathcal{E}_{\phi}^p| + |\mathcal{E}_{\{1\}}^p| + |\mathcal{E}_{\{2\}}^p| + |\mathcal{E}_{\{1,2\}}^p| \right\}
\]

\[
2R_1 + R_2 \leq \min_{p \in I_2} \left\{ |\mathcal{E}_{\{1\}}| + 2|\mathcal{E}_{\{1,2\}}| + |\mathcal{E}_{\{2\}}| + |\mathcal{E}_{\phi}^p| \right\}
\]

Diagram:

- **S**
- **D_1**
- **D_2**
- **D_3**

- **R_1 \leq 2**
- **R_1 + R_2 \leq 3**
- **2R_1 + R_2 \leq 4**
Two public and any number of private receivers

Theorem

Rate \((R_1, R_2)\) is achievable if and only if

\[
R_1 \leq \min \left( |E_1| + |E_{1,2}|, |E_2| + |E_{1,2}| \right)
\]

\[
R_1 + R_2 \leq \min_{p \in \mathcal{I}_2} \left\{ |E^p_\phi| + |E^p_1| + |E^p_2| + |E^p_{1,2}| \right\}
\]

\[
2R_1 + R_2 \leq \min_{p \in \mathcal{I}_2} \left\{ |E_1| + 2|E_{1,2}| + |E_2| + |E^p_\phi| \right\}
\]
Combination networks

The challenge

Linear superposition coding

More than two public receivers...

Optimality results

Why are combination networks useful?

Two public and any number of private receivers

Theorem

Rate \((R_1, R_2)\) is achievable if and only if

\[
R_1 \leq \min \left( |E\{1\}| + |E\{1,2\}|, |E\{2\}| + |E\{1,2\}| \right)
\]

\[
R_1 + R_2 \leq \min_{p \in I_2} \left\{ |E^p_\phi| + |E^p\{1\}| + |E^p\{2\}| + |E^p\{1,2\}| \right\}
\]

\[
2R_1 + R_2 \leq \min_{p \in I_2} \left\{ |E\{1\}| + 2|E\{1,2\}| + |E\{2\}| + |E^p_\phi| \right\}
\]
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When there are more than two public receivers...

(0, 2) is not achievable using the previous scheme!

\[ W_1 = [] \]
\[ W_2 = [w_{2,1}, w_{2,2}] \]
When there are more than two public receivers...

- (0, 2) is not achievable using the previous scheme!

\[
W_1 = []\\
W_2 = [w_{2,1}, w_{2,2}]
\]

The private information revealed to different subsets of public receivers need not be independent
Appropriate pre-encoding

\[ W_1 = \left[ \right] \]
\[ W_2 = [w_{2,1}, w_{2,2}] \]

\[ S \]
\[ X_{\{1\}} \]
\[ X_{\{2\}} \]
\[ X_{\{3\}} \]
\[ D_1 \]
\[ D_2 \]
\[ D_3 \]
\[ D_4 \]
\[ D_5 \]
\[ D_6 \]

- pre-encode \( W_2 = [w_{2,1}, w_{2,2}]^T \) into \( W'_2 = [w'_{2,1}, w'_{2,2}, w'_{2,3}] \)
- now use an structured encoding matrix

\[
\begin{bmatrix}
X_{\{1\}} \\
X_{\{2\}} \\
X_{\{3\}}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
w'_{2,1} \\
w'_{2,2} \\
w'_{2,3}
\end{bmatrix}.
\]
Rate-region II

A rate pair \((R_1, R_2)\) is achievable if there exist variables \(\alpha_\phi, \alpha\{1\}, \alpha\{2\}, \alpha\{1,2\}\), s.t.

Structural constraints:
\[
\alpha_S \geq 0 \quad \forall \phi \neq S \subseteq \{1, 2\}
\]
\[
R_2 = \sum \alpha_S
\]

Decoding constraints at public receiver \(i \in \{1, 2\}\):
\[
R_1 + \sum_{S \ni i} \alpha_S \leq \sum_{S \ni i} |\mathcal{E}_S|
\]

Decoding constraints at private receiver \(p \in I_2\):
\[
R_2 \leq \sum_{S \in \mathcal{T}} \alpha_S + \sum_{S \in \mathcal{T}^c} |\mathcal{E}_S^p| \quad \forall \mathcal{T} \subseteq 2^{\{1,2\}} \text{ superset saturated}
\]
\[
R_1 + R_2 \leq \sum_{S \subseteq \{1,2\}} |\mathcal{E}_S^p|
\]

The converse holds for three (or fewer) public and any number of private receivers, characterizing the capacity region.
Beyond pre-encoding: dependency through time

how to achieve rate pair $(R_1 = 0, R_2 = 2)$?
Beyond pre-encoding: dependency through time

- how to achieve rate pair \((R_1 = 0, R_2 = 2)\)?
- \((R_1 = 0, R'_2 = 3)\) is achievable using the linear superposition encoding scheme, over the extended channel
Beyond pre-encoding: dependency through time

- How to achieve rate pair \((R_1 = 0, R_2 = 2)\)?
- \((R_1 = 0, R'_2 = 3)\) is achievable using the linear superposition encoding scheme, over the extended channel.
- Use it to achieve rate pair \((0, 2)\) over the original network: block Markov encoding and backwards decoding.
Beyond pre-encoding: dependency through time

\[ w_1[t], \ w_1[t + 1] = [] \]

\[ w'_2[t], \ w'_2[t + 1] = [w'_{2,1}[t + 1], w'_{2,2}[t + 1], w'_{2,3}[t + 1]] \]

- how to achieve rate pair \((R_1 = 0, R_2 = 2)\)?
- \((R_1 = 0, R'_2 = 3)\) is achievable using the linear superposition encoding scheme, over the extended channel
- use it to achieve rate pair \((0, 2)\) over the original network: block Markov encoding and backwards decoding
Beyond pre-encoding: dependency through time

\[ w_1[t], \quad w_1[t+1] = [] \]

\[ w'_2[t], \quad w'_2[t+1] = [w'_{2,1}[t+1], w'_{2,2}[t+1], w'_{2,3}[t+1]] \]

- how to achieve rate pair \((R_1 = 0, R_2 = 2)\)?
- \((R_1 = 0, R'_2 = 3)\) is achievable using the linear superposition encoding scheme, over the extended channel
- use it to achieve rate pair \((0, 2)\) over the original network: block Markov encoding and backwards decoding
Beyond pre-encoding: dependency through time

\[ w_1[t], \ w_1[t+1] = [w'] \]

\[ w'_2[t], \ w'_2[t+1] = [w'_{2,1}[t+1], w'_{2,2}[t+1], w'_{2,3}[t+1]] \]

- how to achieve rate pair \((R_1 = 0, R_2 = 2)\)?
- \((R_1 = 0, R'_2 = 3)\) is achievable using the linear superposition encoding scheme, over the extended channel
- use it to achieve rate pair \((0, 2)\) over the original network: block Markov encoding and backwards decoding
A rate pair \((R_1, R_2)\) is achievable if there exist \(\alpha_{\phi}, \alpha_{\{1\}}, \alpha_{\{2\}}, \alpha_{\{1,2\}}\), s.t.

\[
\begin{align*}
\alpha_{\{1,2\}} & \geq 0, \quad \alpha_{\{1\}} + \alpha_{\{1,2\}} \geq 0, \quad \alpha_{\{2\}} + \alpha_{\{1,2\}} \geq 0 \\
\alpha_{\{1\}} + \alpha_{\{2\}} + \alpha_{\{1,2\}} & \geq 0 \\
\alpha_{\phi} + \alpha_{\{1\}} + \alpha_{\{2\}} + \alpha_{\{1,2\}} & \geq 0
\end{align*}
\]

\[
R_2 = \sum_{S} \alpha_S
\]

Decoding constraints at public receiver \(i \in \{1, 2\}:\)

\[
\sum_{S \ni i} \alpha_S \leq \sum_{S \in T} \alpha_S + \sum_{S \in T^c, S \ni i} |E_S| \quad \forall T \subseteq \{\{i\}\} \text{ superset saturated}
\]

\[
R_1 + \sum_{S \ni i} \alpha_S \leq \sum_{S \ni i} |E_S|
\]

Decoding constraints at private receiver \(p:\)

\[
R_2 \leq \sum_{S \in T} \alpha_S + \sum_{S \in T^c} |E_S^p| \quad \forall T \subseteq 2^{\{1,2\}} \text{ superset saturated}
\]

\[
R_1 + R_2 \leq \sum_{S \subseteq \{1,2\}} |E_S^p|
\]

The converse holds for three (or fewer) public and any number of private receivers, characterizing the capacity region.
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Optimality results

Discussions delegated to the end of the presentation, if of your interest!
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Connections with linear deterministic broadcast channels

\[ Y_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \end{bmatrix} \]

\[ Y_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} \]

\[ Y_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} \]

\[ Y_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} \]
Connections with linear deterministic broadcast channels

\[
Y_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \end{bmatrix}
\]

\[
Y_2 = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ x_3 \end{bmatrix}
\]

\[
Y_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 + x_3 \\ x_4 \end{bmatrix}
\]

\[
Y_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 + x_2 \\ x_2 + 3x_3 + 2x_4 \end{bmatrix}
\]
Connections with linear deterministic broadcast channels

\[ Y_1 = H_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \end{bmatrix} \]

\[ Y_2 = H_2 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ x_3 \end{bmatrix} \]

\[ Y_3 = H_3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 + x_3 \\ x_4 \end{bmatrix} \]

\[ Y_4 = H_4 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 + x_2 \\ x_2 + 3x_3 + 2x_4 \end{bmatrix} \]
Connections with linear deterministic broadcast channels

\[
Y_1 = H_1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \end{bmatrix}
\]

\[
Y_2 = H_2 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ x_3 \end{bmatrix}
\]

\[
Y_3 = H_3 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 + x_3 \\ x_4 \end{bmatrix}
\]

\[
Y_4 = H_4 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 + x_2 \\ x_2 + 3x_3 + 2x_4 \end{bmatrix}
\]
The capacity region of a linear deterministic broadcast channel with two public receivers and any number of private receivers is given by

\[
\begin{align*}
R_1 & \leq \min_{i \in I} r\{i\} \\
R_1 + R_2 & \leq \min_{i \in I_2} r\{i\} \\
2R_1 + R_2 & \leq \min_{i \in I_2} \{r\{1\} + r\{2\} + r\{1,2,i\} - r\{1,2\}\},
\end{align*}
\]

where the size of $\mathbb{F}$ is larger than $K$. The rates given above are expressed in $\log |\mathbb{F}|(\cdot)$.

- $r\{i\} \triangleq \text{rank}(H_i)$
- $r\{i_1, \ldots, i_{|\mathcal{S}|}\} \triangleq \text{rank} \begin{bmatrix} H_{i_1} \\ \vdots \\ H_{i_{|\mathcal{S}|}} \end{bmatrix}$
### Example

<table>
<thead>
<tr>
<th></th>
<th>H1</th>
<th>H2</th>
<th>H3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 0 0 0</td>
<td>1 0 0 0</td>
<td>1 0 1 1</td>
</tr>
<tr>
<td>2</td>
<td>0 0 1 1</td>
<td>0 1 0 0</td>
<td>1 0 0 0</td>
</tr>
<tr>
<td>3</td>
<td>0 1 1 1</td>
<td>0 1 1 1</td>
<td>0 1 1 1</td>
</tr>
</tbody>
</table>
Example

\[ \mathbf{H}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \]

\[ \mathbf{H}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \]

\[ \mathbf{H}_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \]

\[ r_1 = r_2 = 2 \]

\[ r_3 = 3 \]

\[ r_{12} = 3 \]

\[ r_{123} = 3 \]
Example

\[ \mathbf{H}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \]

\[ \mathbf{H}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \]

\[ \mathbf{H}_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \]

\[
\begin{align*}
  r_1 &= r_2 = 2 \\
  r_3 &= 3 \\
  r_{12} &= 3 \\
  r_{123} &= 3
\end{align*}
\]
Example

\[
\mathbf{H}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \\
\mathbf{H}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \\
\mathbf{H}_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}
\]

\[r_1 = r_2 = 2\]
\[r_3 = 3\]
\[r_{12} = 3\]
\[r_{123} = 3\]

\[R_1 \leq 2\]
\[R_1 + R_2 \leq 3\]
\[2R_1 + R_2 \leq 4\]
Example

$$H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$r_1 = r_2 = 2$$

$$r_3 = 3$$

$$r_{12} = 3$$

$$r_{123} = 3$$

$$R_1 \leq 2$$

$$R_1 + R_2 \leq 3$$

$$2R_1 + R_2 \leq 4$$
Summary

- Studied the problem of multicasting prioritized messages over combination networks
Summary

- Studied the problem of multicasting prioritized messages over combination networks

- Combination networks turn out to be a rich class of networks and a rich class of linear deterministic broadcast channels
Summary

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- Generalizing these schemes to linear deterministic broadcast channels seems very promising