Effect of Large Disturbances on the Local Behavior of Nonlinear Physically Interconnected Systems

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Motivation – Sensitivity in electric power systems: Subcritical instability & Ill-conditionedness

- [2]: In *highly loaded situations*, inherent tolerance to local small variations reduces and the power systems moves from being elastic to *brittle.*

- [3]: Increasing load leads to *increasing condition number* of power flow Jacobian. *Voltage instability* is related to singularity.

$$\Delta \boldsymbol{y} = \boldsymbol{J}_{PF}^{-1} \Delta \boldsymbol{x} \qquad \boldsymbol{y} = (\boldsymbol{\Theta} \quad \boldsymbol{V})' \text{ Voltage angle / magnitude}$$

$$\boldsymbol{x} = (\boldsymbol{P} \quad \boldsymbol{Q})' \text{ Active / reactive powers}$$

$$\kappa(\boldsymbol{J}_{\mathrm{PF}}) = ||\boldsymbol{J}_{\mathrm{PF}}|| \cdot ||\boldsymbol{J}_{\mathrm{PF}}^{-1}|| = \frac{\sigma_{\max}(\boldsymbol{J}_{\mathrm{PF}})}{\sigma_{\min}(\boldsymbol{J}_{\mathrm{PF}})}, \qquad \sigma_{\min} \searrow 0, \quad \kappa \nearrow \infty$$

$$\kappa \approx 10^3$$

- [4]: *Interacting complex modes* may cause *subcritical* oscillatory instability.

→Q1: *What is the role of ill-conditioning rather than singularity?*

→Q2: *How are static behavior and dynamic behavior interrelated?*

Approach: Gateaux differentials & Constrained optimization

- The first variation of a nonlinear function has an expression as *inner product* via the formalism of *Gateaux* differentials, i.e.

$$\delta\lambda = \lim_{\tau \to 0+} \frac{\lambda^+(\boldsymbol{z}_{\mathrm{opt}} + \tau \delta \boldsymbol{z}_{\mathrm{opt}}(\boldsymbol{d})) - \lambda(\boldsymbol{z}_{\mathrm{opt}})}{\tau} = \tau \langle \boldsymbol{\mathcal{S}}_{\boldsymbol{d}}, \boldsymbol{d} \rangle$$

→ ***Eigenvalue moves along gradient*** *(most sensitive dir.)*

-*Gradient* determined via *constrained (Lagrangian) optimization*

→ *Define the **Lagrangian**: (max spectral deviation s.t. EVP)*

$$\mathcal{L}(\lambda^+, \boldsymbol{v}, \boldsymbol{\mu}, \boldsymbol{z}_{\mathrm{opt}}^+(\boldsymbol{d})) = ||\delta\lambda|| - \langle \boldsymbol{\mu}, \left[\lambda^+ \boldsymbol{B} - \boldsymbol{A}(\boldsymbol{z}_{\mathrm{opt}}^+(\boldsymbol{d}))\right] \boldsymbol{v} \rangle$$

→ ***Stationarity & Gateaux differential***

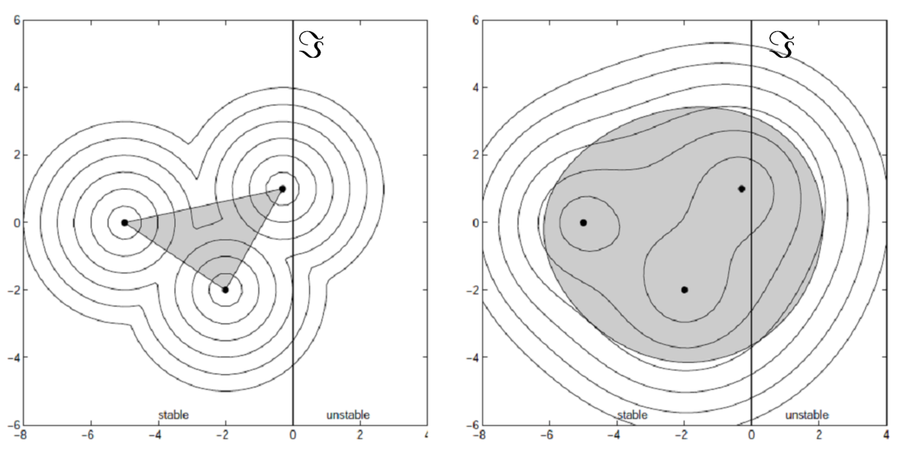
$$\delta \mathcal{L} \stackrel{!}{=} 0 \Rightarrow \boldsymbol{\mathcal{S}}_{\boldsymbol{d}}$$

= Necessary optimality conditions
& Conditional equations for the gradient

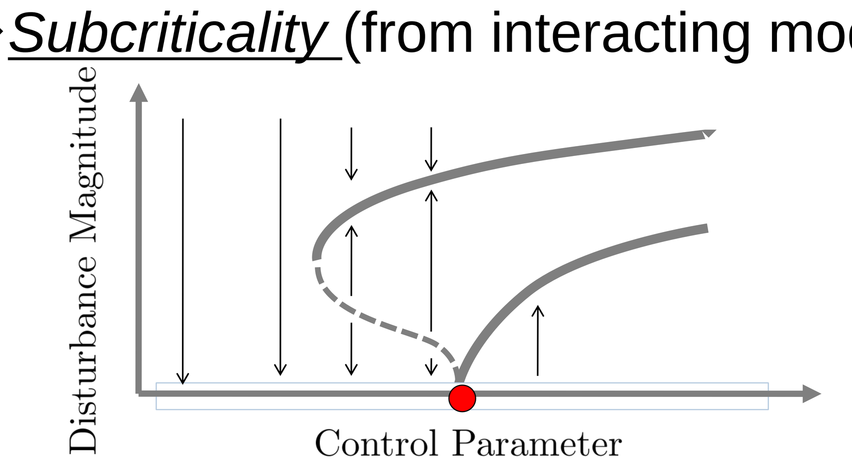
Differences to classical eigenvalue sensitivity
→Indendent of chosen coordinate system! [5]
→Disturbance input vector contains structural information
(Matrix valued perturbation of local dynamics results from (nodal) disturbance input vector)

Effect of transport mechanisms in distributed physical systems

[1]:
→*Ill-conditioned* linearized dynamics
→ Eigenvalue *sensitivity*



→*Subcriticality* (from interacting modes)



Problem Setting: Physically Interconnected systems

System class:

→ System of *nonlinear DAEs*

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \qquad \boldsymbol{J}_{\mathrm{PF}} = \nabla_{\boldsymbol{y}} \boldsymbol{g}|_{\boldsymbol{z}_{\mathrm{opt}}}$$
$$\boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y})$$

→ has *steady state* operating point (with state $\boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y})'$)

$$\boldsymbol{z}_{\mathrm{opt}} \leftarrow \boldsymbol{\mathcal{R}}(\boldsymbol{z}) := \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \\ \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \end{bmatrix} = \boldsymbol{0}$$

→ serving as set-point for *controlled local dynamics*

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \Delta \boldsymbol{x} \\ \boldsymbol{0} \end{pmatrix} = \underbrace{\begin{bmatrix} \nabla_{\boldsymbol{x}} \boldsymbol{f}|_{\boldsymbol{z}_{\mathrm{opt}}} & \nabla_{\boldsymbol{y}} \boldsymbol{f}|_{\boldsymbol{z}_{\mathrm{opt}}} \\ \nabla_{\boldsymbol{x}} \boldsymbol{g}|_{\boldsymbol{z}_{\mathrm{opt}}} & \nabla_{\boldsymbol{y}} \boldsymbol{g}|_{\boldsymbol{z}_{\mathrm{opt}}} \end{bmatrix}}_{\nabla_{\boldsymbol{z}} \boldsymbol{\mathcal{R}}(\boldsymbol{z})|_{\boldsymbol{z}_{\mathrm{opt}}} = \boldsymbol{A}(\boldsymbol{z}_{\mathrm{opt}})} \begin{pmatrix} \Delta \boldsymbol{x} \\ \Delta \boldsymbol{y} \end{pmatrix}$$

→ *Local dynamics determined by **spectral properties**:*

$$\boldsymbol{A}(\boldsymbol{z}_{\mathrm{opt}}) \boldsymbol{v} = \lambda \boldsymbol{B} \boldsymbol{v}$$

Consider a steady forcing, so that $\boldsymbol{\mathcal{R}}(\boldsymbol{z}_{\mathrm{opt}}^+) = \boldsymbol{d}$, then,

$$\boldsymbol{d} \xrightarrow{\boldsymbol{\mathcal{R}} + \boldsymbol{d} = \boldsymbol{0}} \delta \boldsymbol{z} = \boldsymbol{z}_{\mathrm{opt}}^+ - \boldsymbol{z}_{\mathrm{opt}} \xrightarrow{\boldsymbol{A}\boldsymbol{v} = \lambda \boldsymbol{B} \boldsymbol{v}} \delta\lambda = \lambda^+ - \lambda$$

→Q 3: *How to quantify the change in eigenvalue ?*

Main result & Implications: Power flow Jacobian & Subcriticality

→A3: *Theorem*: Estimate for deviations

$$\delta\lambda = \tau \left\langle \left(\left[\nabla \boldsymbol{\mathcal{R}}(\boldsymbol{z})|_{\boldsymbol{z}_{\mathrm{opt}}} \right]^{-1} \right)^* \boldsymbol{\mathcal{S}}_{\boldsymbol{z}}, \boldsymbol{d} \right\rangle$$

Spectral sensitivity $\boldsymbol{\mathcal{S}}_{\boldsymbol{z}} = \left[\nabla_{\boldsymbol{z}} [\boldsymbol{A}(\boldsymbol{z}_{\mathrm{opt}}) \boldsymbol{v}]|_{\boldsymbol{z}_{\mathrm{opt}}} \right]^* \boldsymbol{w}$

Adjoint eigenvector $\boldsymbol{w} = \boldsymbol{\mu}$

→ A2: *Inverse of steady state **Jacobian** acts as **gain matrix** on eigenvalue senisitivity!*

→ A1: *Condition number ~ worst case **amplification***

In simple models $\boldsymbol{J}_{\mathrm{PF}} = \nabla_{\boldsymbol{z}} \boldsymbol{\mathcal{R}}$

$$||\boldsymbol{J}_{\mathrm{PF}}^{-1}|| = \frac{1}{\sigma_{\min}(\boldsymbol{J}_{\mathrm{PF}})} \xrightarrow{\text{loading}\uparrow} \infty$$

Else, relations between $\boldsymbol{J}_{\mathrm{PF}}$ & $\nabla_{\boldsymbol{z}} \boldsymbol{\mathcal{R}}$ can be characterized using function maps (and their gradients) as introduced in [6]

Discussion & Outlook

→ *Power flow not considered in linear models* (for RT control) but interaction of the two has caused recent large blackouts!

→ Large amplification possible scenario for *subcritical Hopf-bifurcation*

→ New, *combined static/dynamic analysis tools*, as in [7]

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[4] Ian Dobson, J. Zhang, S. Greene, H. Engdahl, and Peter W. Sauer, "Is strong modal resonance a precursor to power system oscillations?", IEEE Transactions on Circuits and Systems I, 2001
[5]: P. Meliga, D. Sipp, and J.M. Chomaz, "Open-Loop Control of Compressible Afterbody Flows Using Adjoint Methods", Physics of Fluids, 2010
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[7]: H. Mangesius , M. Huber, T. Hamacher, S. Hirche, "A Framework to Quantify Technical Flexibility in Power Systems Based on Reliability Certificates", IEEE ISGT, 2013

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