

# 3D-Finite Element Model of the Human Cochlea Including Fluid-Structure Couplings

Frank Böhnke Wolfgang Arnold

Department of Otolaryngology, Technical University of Munich, Germany

## Key Words

Cochlea · Perilymph · Basilar membrane · Finite element method · Wave propagation

## Abstract

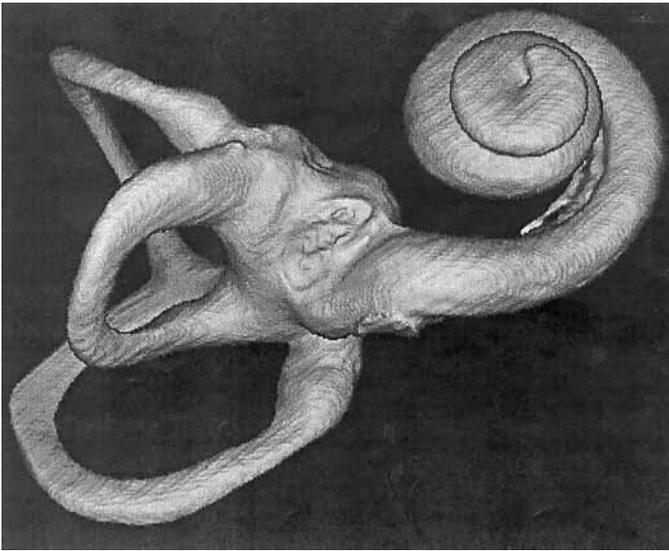
The propagation of acoustic waves in the inner ear in vivo could not be quantified completely yet. This is in particular true in conjunction with the micromechanical structures of the organ of Corti, though these data are important for the explanation and discussion of clinical measurements like otoacoustic emissions and auditory brainstem responses. To access these problems a three-dimensional mechanical model of the cochlea including the fluid-structure couplings is developed and evaluated numerically by finite elements. Although the complex cochlear partition is covered by passive mechanical elements, the results fit early experiments (1928), which studied the wave propagation in the cochlea with fresh human cadavers [G. von Békésy: Experiments in Hearing. New York, McGraw-Hill, 1960]. Additionally it is now easy to calculate the mechanical input impedance of the cochlea. These results agree with recent experiments [S.N. Merchant et al.: Hear Res 1996;97:30–45].

## Introduction

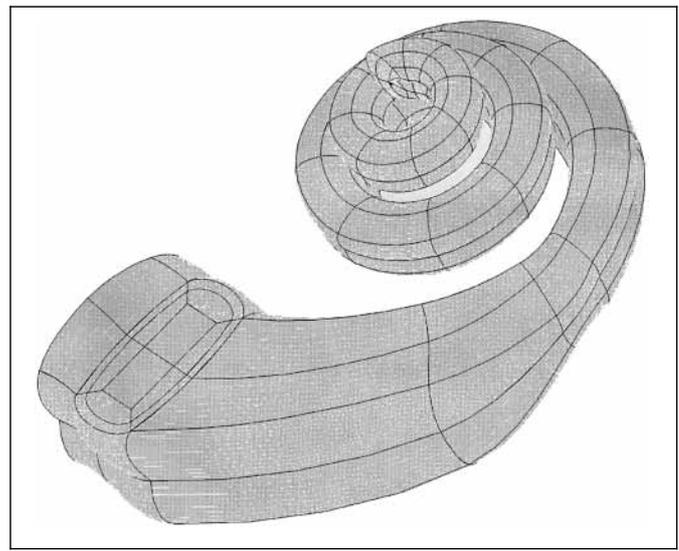
The human ear processes acoustic signals for communication, warning and pleasure. It consists mainly of the outer ear canal, the middle and the inner ear and the central auditory system. The inner ear is separated into the vestibular system, which is necessary for orientation in space, and the cochlea (Latin: *cochlea*, the snail), which is able to detect pressure with a frequency bandwidth of 10 octaves (20 Hz–20 kHz) and a dynamic range of 120 dB ( $F = 10^6$ ) in its healthy state.

The human cochlea has a spiral form with 2.5 windings. Its largest dimension is only about 4 mm and its straight average length is 34 mm. The cochlea consists of three channels which are filled with lymph. Two of these channels are separated by the cochlear partition which consists of the basilar membrane, the organ of Corti, the stereocilia and the tectorial membrane. The organ of Corti contains about 3,000 inner hair cells, which transform mechanical displacements into electrical currents, about 12,000 outer hair cells and a large number of mainly supporting cells. All these cells work together highly nonlinearly. However, their precise interaction is not completely understood.

For understanding the complex processes in the cochlea, the development and evaluation of a structural mechanical finite element model is presented. The peri-



**Fig. 1.** Microtomographic image of the fluid-filled caves of the human inner ear.



**Fig. 2.** Geometric model of the scala tympani and scala vestibuli of the human cochlea.

lymph is considered as inviscous and compressible. The fluid-structure couplings between the perilymph and the stapes footplate, the round window membrane and the elastic cochlear partition are covered in three dimensions for the first time.

### Morphology and Methods

Figure 1 is a microtomographic image of the fluid-filled caves of the human inner ear. Further information on the imaging of microtomographic pictures are given in this issue [1].

To simulate the propagation of mechanical waves in the human cochlea we developed a three-dimensional finite element model which neglects micromechanical elements of the cochlear partition for the moment, but takes the three-dimensional fluid-structure system with its curved geometry into consideration [2]. The three-dimensional mechanical model of the cochlea is shown in figure 2. The areas which cover the kidney-shaped cross-sections of the scala vestibuli and scala tympani are located to form the curved geometry of the cochlea. The points on the boundary of these cross-sections are connected by spline functions. Because the model is built by a low number of volumes (240), inadmissibly high curvatures may result. Therefore the maximum physically permissible warping factors, which are given by the finite element package, must not be exceeded.

#### Cochlear Partition

The cochlear partition is idealized by two orthotropic elastic shells with variable width (80–500  $\mu\text{m}$ ) and thickness (7.5–2.5  $\mu\text{m}$ ) from the base (stapes) to the apex (helicotrema) of the cochlea [3].

The cochlear partition is divided into two identical shells because the fluid can be coupled to the upper or lower area of the shell solely with the used FE package. Their Young's moduli are  $E_x = 100 \text{ MPa}$  in the transverse and  $E_y = 10 \text{ kPa}$  in the longitudinal direction according to the stiff fibers embedded into the softer tissue of the basilar membrane. Of course, the areas of the shells which are on the opposite side of the adjoining fluid are coupled among themselves. Figure 3 shows one of these shells which represents the cochlear partition. The continuous decrease of thickness and the increasing width from base to apex can be seen.

All Poisson's ratios (transverse expansions) of the shell are set to zero for the moment. This may result in a reduced accuracy of the shell dynamics when the displacement based finite elements are used because the elements may lock [4]. The damping is included specifically as material damping of the shell and chosen as  $\beta = 10^{-5}$ . It is worth noting that this is the only damping considered and therefore the cause for complex displacements and the phase shifts, which are found in the results.

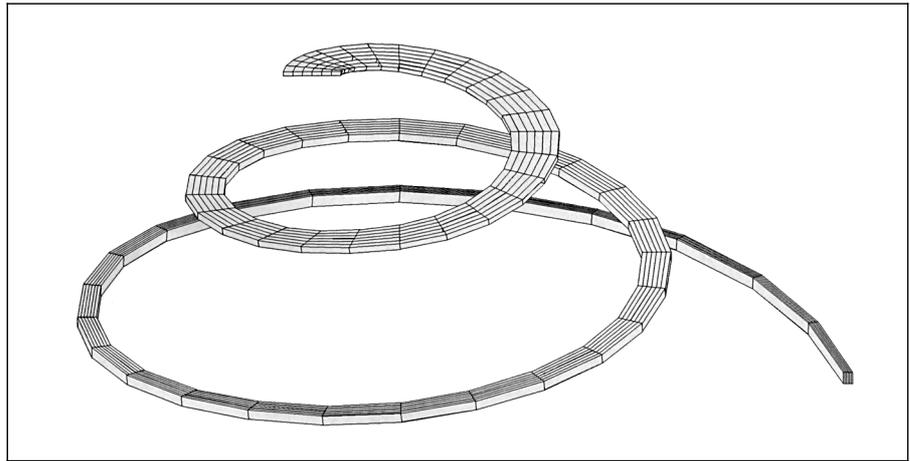
#### Stapes Footplate of the Oval Window and the Round Window Membrane

The Young's modulus of the shell which represents the stapes footplate is  $E_{St} = 12 \text{ GPa}$ . The Young's modulus of the annular ligament is  $E_{AL} = 700 \text{ kPa}$ , which was determined experimentally for elastin [5]. The round window membrane is idealized by an elastic shell with the lower Young's modulus  $E_{RW} = 9.8 \text{ MPa}$  and an average thickness  $t_{RW} = 100 \mu\text{m}$ . The lowest thickness was determined to 56  $\mu\text{m}$  in the center of the round window membrane [6].

#### Perilymphatic Fluid

For simplification, only two (scala tympani, scala vestibuli) of the three fluid-filled channels (scala tympani, scala vestibuli and scala media) are taken into account. The fluid is considered as linear,

**Fig. 3.** Curved orthotropic shell representing the basilar membrane. The thickness at the base is  $7.5 \mu\text{m}$  and continuously decreasing to  $2.5 \mu\text{m}$  at the helicotrema, which are half the values of the human basilar membrane. The width is continuously increasing from  $80 \mu\text{m}$  at the base of the cochlea to  $500 \mu\text{m}$  near the helicotrema.



inviscous and compressible and therefore we are able to cover fast acoustic waves in the fluid. The bulk modulus  $K$  of the perilymphatic fluid is equal to that of water  $K = 2,250 \text{ MPa}$ . With the fluid density  $\rho_{FI} = 10^3 \text{ [kg/m}^3\text{]}$  the speed of sound in the fluid is:

$$c_{FI} = \sqrt{\frac{K}{\rho_{FI}}} = 1,500 \frac{m}{s} \quad (1)$$

#### Fluid-Structure Coupling

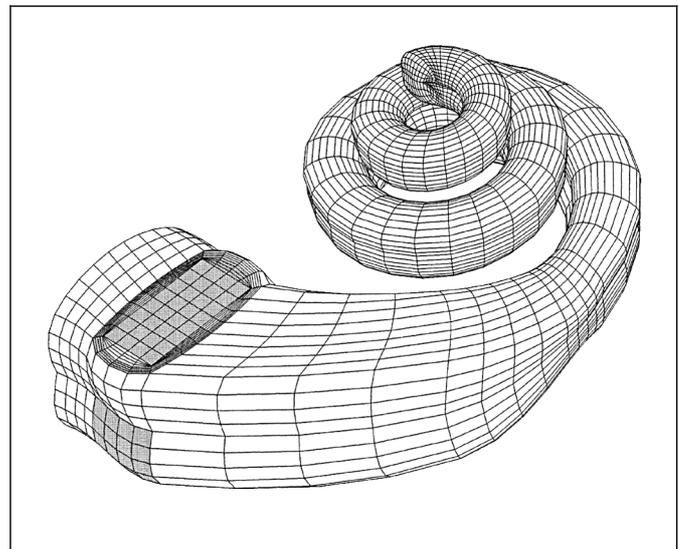
It is necessary to couple the areas of the stapes footplate, the area of the round window and both areas of the cochlear partition (upper and lower) to the lymph. Because the location of the oval and the round window are not orthogonal to the global Cartesian coordinate system, two additional local Cartesian coordinate systems must be introduced. Because shell elements may be coupled to the fluid with only one of their areas, the cochlear partition is represented by two double-curved shells. In each case the outer shell areas are coupled to the fluid. On the opposite sides the shells are coupled to each other and their distance is adjustable.

Figure 4 shows the complete cochlear model discretized with finite elements. The chosen discretization leads to approximately 32,000 finite elements and therefore about 115,000 variables (active degrees of freedom) must be evaluated.

#### Boundary Conditions

To solve the problem numerically, appropriate boundary conditions must be introduced. The perilymph is enclosed by rigid bone. Therefore, the normal components of the fluid displacements must vanish at the bony surrounding areas. Because the fluid is assumed to be inviscous the tangential components do not vanish. If viscosity is covered it is essential to use special boundary conditions, which cover the vanishing of all displacement components at rigid walls.

The shells which represent the cochlear partition must be supported correctly. The inner boundary at the lamina spiralis ossea is clamped and therefore all displacements and rotations vanish there. On the opposite side at the ligamentum spirale, the shells are simply supported. Therefore, rotations relating to the longitudinal (cochlea) direction may be different from zero. Rotations relating to the two axes orthogonal to the longitudinal axis and all displacements vanish at the outer border of the shells at the ligamentum spirale.



**Fig. 4.** Cochlear model discretized with finite elements.

The boundaries of the shell, which represents the round window, are clamped. Therefore, all displacements and rotations vanish at the rim of the round window.

#### Acoustic Input Impedance of the Cochlea

The acoustic input impedance  $Z_C$  of the cochlea is defined as the ratio between the pressure  $P_S$  at the stapes footplate and the velocity  $v_S$  of the displaced volume. Under the assumption of harmonic excitation, this ratio becomes:

$$Z_C = \frac{P_S}{j\omega X_S A_S} \quad (2)$$

$X_S$  is the displacement of the stapes in z-direction,  $A_S$  is the stapes area,  $\omega$  is the circular frequency and  $j$  is the imaginary unit indicating a complex value.

## Results

### Wave Propagation in the Cochlea

The external load is applied as a pressure on the stapes footplate area. Its amplitude is  $P_S = 1$  Pa, which is equal to the sound pressure level  $L_e = 94$  dB(SPL) in all cases.

Our results show calculations using external harmonic loads of three different frequencies:  $f_1 = 100$  Hz,  $f_2 = 2,000$  Hz and  $f_3 = 10,000$  Hz. The frequency range is not limited, and therefore a static analysis and a frequency, e.g.  $f = 20,000$  Hz, could additionally be applied.

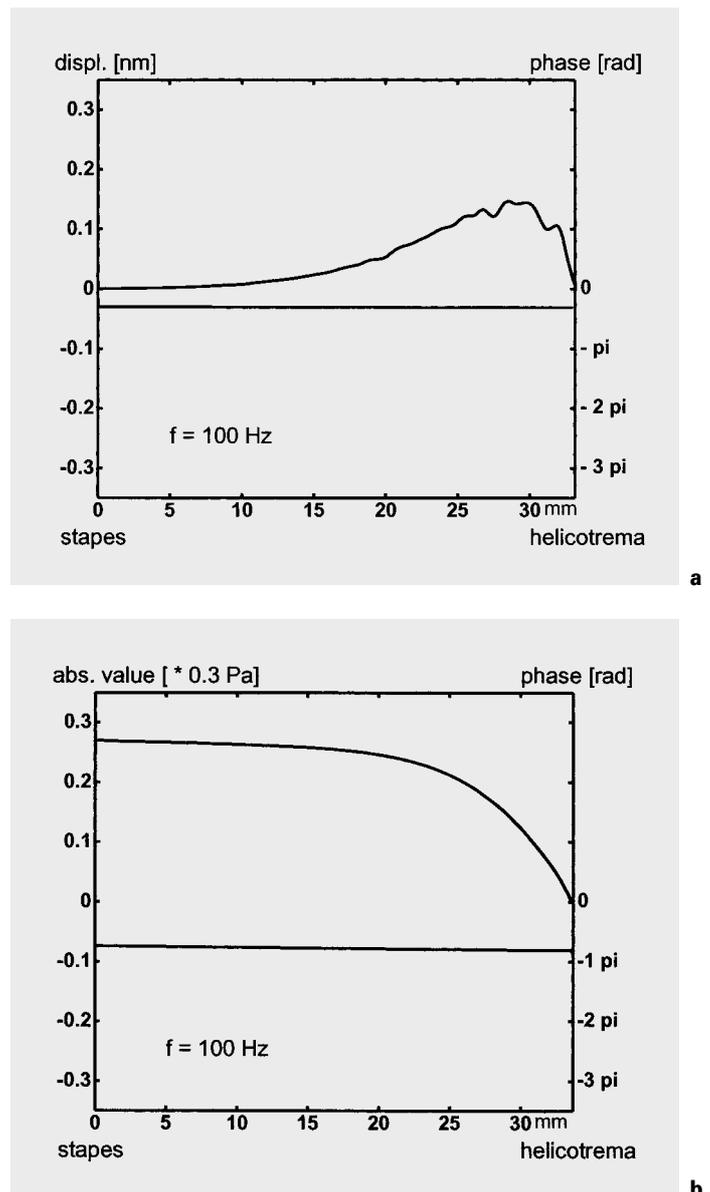
For the external pressure with the lowest frequency  $f_1 = 100$  Hz, the maximum absolute value of the displacement (0.14 nm) of the cochlear partition is at a distance of 30 mm from the stapes footplate near the helicotrema (fig. 5a). The horizontal line marks the constant phase shift. The whole cochlear partition is moving in phase. Figure 5b shows the pressure distribution at the cochlear partition for the same frequency. The absolute value of the pressure decreases continuously from 0.08 Pa to a value near 0 Pa with a constant phase shift.

A harmonic pressure of frequency  $f_2 = 2,000$  Hz at the stapes footplate leads to a maximum displacement (0.2 nm) of the cochlear partition at approximately half the cochlea length (fig. 6a). In this case the maximum phase shift is at least  $-2,160^\circ$  or  $-12\pi$ . The pressure is mainly decreasing from the stapes (0.016 Pa) to the helicotrema to a pressure near 0 Pa (fig. 6b, upper line). It is interesting to note that the phase of the pressure (fig. 6b, lower line) has both negative and positive slopes and does not exceed  $-1.5\pi$ . At  $f_3 = 10$  kHz the maximum absolute value of the displacement (0.23 nm) is at a distance of only 5 mm from the stapes footplate and the maximum phase shift is about  $-360^\circ$  or  $-2\pi$  (fig. 7a). The pressure decreases from 0.33 Pa at the stapes to a value near 0 Pa at the helicotrema. There are only low variations of the pressure phase from the constant shift ( $-0.6\pi$ ) in the basal part of the cochlea (fig. 7b, lower line).

### Acoustic Input Impedance of the Cochlea

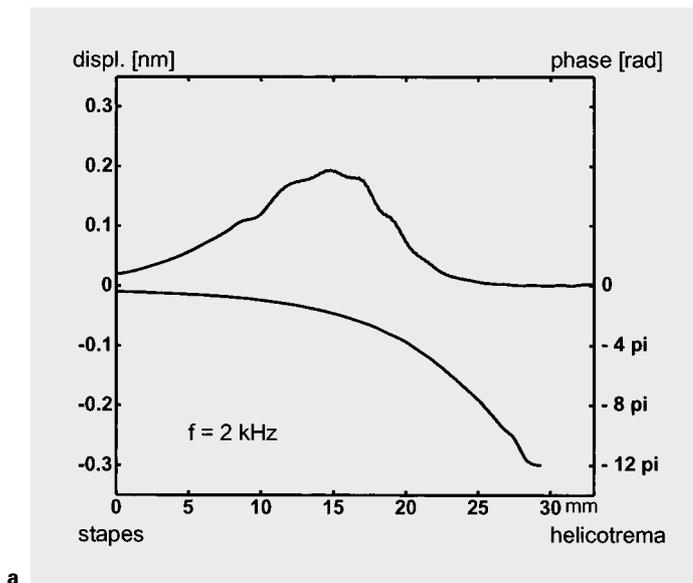
For an external pressure of frequency  $f = 1,000$  Hz and amplitude 1 Pa at the stapes footplate, the displacement of the stapes footplate is evaluated to 0.2 nm, which is also determined from temporal bones of human cadavers [7]. It is interesting to note that this is only twice the diameter of an atom. If the area of the stapes footplate is chosen as  $A_s = 3.6$  mm<sup>2</sup>, the acoustic input impedance of the cochlea model is:

$$Z_C = -j 138.89 \cdot 10^9 \text{ Pa s/m}^3 = -j 138.89 \text{ G}\Omega$$

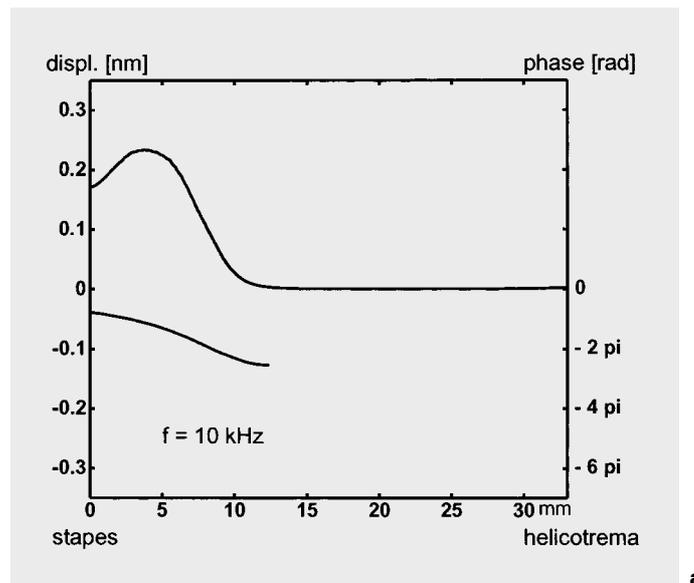


**Fig. 5. a** Absolute displacement and phase of displacement along the cochlear partition with external pressure amplitude 1 Pa and frequency  $f_1 = 100$  Hz. **b** Absolute pressure and phase of pressure at the cochlear partition with external pressure amplitude 1 Pa and frequency  $f_1 = 100$  Hz.

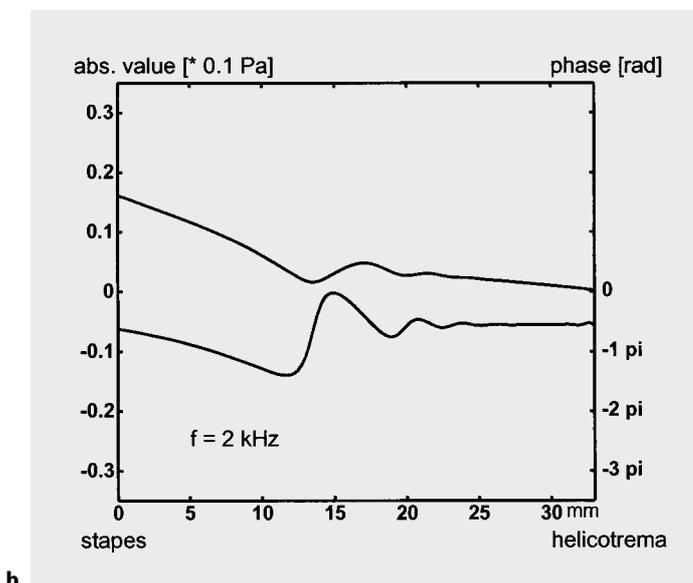
which is the stiffness component. This value is a factor 1.38 above the experimental value [7] and therefore in close proximity. The derived unit ohm ( $\Omega$ ) indicates the acoustic impedance in the meter-kilogram-second (mks) system.



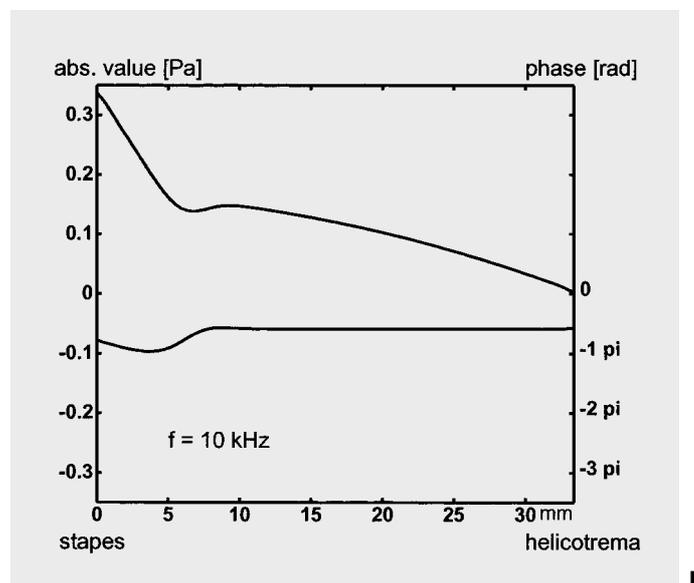
a



a



b



b

**Fig. 6. a** Absolute displacement and phase of displacement along the cochlear partition with external pressure amplitude 1 Pa and frequency  $f_2 = 2,000$  Hz. **b** Absolute pressure and phase of pressure at the cochlear partition with external pressure amplitude 1 Pa and frequency  $f_2 = 2,000$  Hz.

**Fig. 7. a** Absolute displacement and phase of displacement along the cochlear partition with external pressure amplitude 1 Pa and frequency  $f_3 = 10,000$  Hz. **b** Absolute pressure and phase of pressure at the cochlear partition with external pressure amplitude 1 Pa and frequency  $f_3 = 10,000$  Hz.

## Discussion

The calculated displacements and pressures are those we expect in comparison with former experiments [7, 8]. The question for the existence of traveling or standing waves can now easily be answered. The damping, which is

material damping here solely, is the cause for phase shifts of the displacement of the cochlear partition and the pressure in the lymph. If the damping is neglected, real displacements and pressures result and in this case we find standing waves in the cochlea.

It is interesting to note that the maximum displacement of the basilar membrane is of the same order of magnitude (0.2 nm) for the three frequencies, which span a frequency bandwidth of more than 6 octaves. This is in contrast to the values for the pressure. We assume this is a consequence of the dimensions of the basilar membrane, which increases in width and decreases in thickness from the base to the apex of the cochlea respectively.

The acoustic input impedance of the cochlea can be determined numerically now. This is relevant for the realistic termination of middle ear models. A good agreement with recent experimental values is found. The evaluated displacement of the stapes footplate is only 0.2 nm with an external applied pressure  $L_e = 94$  dB(SPL) at the stapes footplate ( $f = 1,000$  Hz). If an increase of the pressure caused by the middle ear by 20 dB is assumed, this displacement (0.2 nm) corresponds to 74 dB(SPL) in the ear canal. The linear extrapolation of the stapes displacement leads to an extremely small value for the hearing threshold at  $L_{HS} = 0$  dB(SPL), namely 0.04 pm (picometer) or 40 fm (femtometer, Fermi).

In a future investigation, the acoustic input impedance of the cochlea will be evaluated in a wider frequency range (20–20,000 Hz).

Because the viscosity of the fluid cannot be included easily in a curved geometry by the finite element package used, we are unable to calculate shearing forces in the fluid. As a consequence, an incompatibility in the formulation of the fluid-structure system results because the

equations describing the mechanical behavior of the shell contain rotations. These cannot be coupled to a fluid which is described by displacements and pressure solely.

## Conclusions

(1) The 3D-finite element model allows the evaluation of the passive mechanical behavior of the human cochlea with arbitrary input pressure at the stapes footplate including all kinds of slow and fast waves in the lymph and the cochlear partition.

(2) The curved geometry of the cochlea and the fluid-structure coupling is covered in three dimensions. Therefore, the micromechanical structures of the organ of Corti and those of the cochlear partition can be easily included in further investigations.

(3) The linear solutions are in close agreement with early experimental results taken from human temporal bones [8]. To cover the nonlinear physical behavior of the cochlea, material and geometrical nonlinearities should be included in future calculations.

## Acknowledgements

We gratefully acknowledge the support of Dr. R. Schmidt (TU-Dresden) and of CADFEM in Grafing (Germany) in helping us using the ANSYS 5.4 finite element software.

## References

- 1 Vogel U: New approach for 3D-imaging and geometry modeling of the human inner ear. *ORL* 1999;61:259–267.
- 2 Böhnke F, Schmidt R: Aspekte der FE-Modellierung der Hörschnecke des Menschen. Proc 15th CAD-FEM Users' Meeting, Fulda, Germany, Oct. 1997, pp 1–11.
- 3 Wada H, Sugawara M, Kobayashi T, Hozawa K, Takasaka T: Measurement of guinea pig basilar membrane using computer-aided three-dimensional reconstruction system. *Hear Res* 1998;120:1–6.
- 4 Bathe KJ: *Finite Element Procedures*. Englewood Cliffs, Prentice-Hall, 1996.
- 5 Carton RW, Dainauskas J, Clark JW: Elastic properties of single elastic fibres. *J Appl Physiol* 1962;17:547–551.
- 6 Nomura Y: Otological Significance of the Round Window. *Adv Otorhinolaryngol*. Basel, Karger, 1984, vol 33, pp 1–162.
- 7 Merchant SN, Ravicz ME, Rosowski JJ: Acoustic input impedance of the stapes and cochlea in human temporal bones. *Hear Res* 1996;97:30–45.
- 8 von Békésy G: *Experiments in Hearing*. New York, McGraw-Hill, 1960.