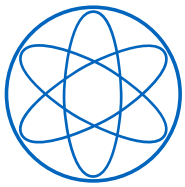
Technische Universität München

Neutrino masses and spontaneously broken flavor symmetries

Dissertation

by

Christian Staudt



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Abstract

We study the phenomenology of supersymmetric flavor models. We show how the predictions of models based on spontaneously broken non–Abelian discrete flavor symmetries are altered when we include so–called Kähler corrections. Furthermore, we discuss anomaly–free discrete R symmetries which are compatible with SU(5) unification. We find a set of symmetries compatible with suppressed Dirac neutrino masses and a unique symmetry consistent with the Weinberg operator. We also study a pseudo–anomalous $\mathrm{U}(1)_R$ symmetry which explains the fermion mass hierarchies and, when amended with additional singlet fields, ameliorates the fine–tuning problem.

Zusammenfassung

Wir untersuchen die Phänomenologie von supersymmetrischen Flavormodellen. Wir zeigen, dass die Vorhersagen von Modellen, die auf spontan gebrochenen nichtabelschen diskreten Flavorsymmetrien basieren, sich ändern, sobald wir sogenannte Kählerkorrekturen berücksichtigen. Wir diskutieren außerdem anomaliefreie diskrete R Symmetrien, die kompatibel mit SU(5)–Vereinheitlichung sind. Wir finden mehrere Symmetrien, die mit unterdrückten Dirac Neutrinomassen kompatibel sind, und eine einzigartige Symmetrie, die mit dem Weinbergoperator kompatibel ist. Wir untersuchen auch eine pseudoanomale $\mathrm{U}(1)_R$ Symmetrie, welche die fermionische Massenhierarchie erklärt und welche, wenn man sie mit zusätzlichen Singlets erweitert, das „fine–tuning“ verbessert.

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Chapter 1

Introduction

The recent discovery of the Higgs boson, with a mass of (125 ± 1) GeV [1,2], marks a triumph for the standard model (SM) of particle physics, which has been an extremely successful theory in the last decades. However, the SM is still challenged by several problems. First, it is clear that the SM cannot be the fundamental theory since gravity is not incorporated. Secondly, there is no reasonable candidate field for dark matter, which accounts for 27% of the energy density of the universe [3], within the SM. Finally, there is the cosmological constant problem.

The smallness of the Higgs mass demonstrates the hierarchy problem of the SM since the Higgs mass, in principle, should receive large quantum corrections due to loop effects [4,5]. The experimental value, therefore, requires a fine–tuning of the different contributions to the Higgs mass in order to cancel each other, which is considered "unnatural" [6].

The hierarchy problem can be solved by the introduction of supersymmetry (SUSY) [7], a spacetime symmetry that introduces for each SM particle a supersymmetric partner, which differs in spin by $1/2$. The fermion loop corrections to the Higgs mass, which cause the hierarchy problem, are then systematically canceled by the bosonic partner of the fermions, which contribute with an opposite sign to the Higgs mass. Therefore, supersymmetry is one of the most common extensions of the standard model, especially since SUSY also provides a viable dark matter candidate and improves gauge coupling unification [8].

However, SUSY also suffers from several problems, like the μ problem. Here, the supersymmetric Higgs mass needs to be sufficiently suppressed in order to avoid the reintroduction of the hierarchy problem. We will later show that this problem can be solved with the help of R symmetries, i.e. symmetries that do not commute with the supersymmetry generators.

Additionally, experiments, measuring a rather heavy Higgs boson [1, 2], put pressure on supersymmetric models since the simplest SUSY models predict a lighter Higgs boson. Solving this issue either requires, again, fine–tuning or an extension of the simplest SUSY models. We will discuss such extensions later in this thesis.

So far we have not discussed another important issue of the standard model, the prediction of massless neutrinos. In the last decades many experiments confirmed the existence of neutrino masses [9–19] and recently experiments have even entered the precision phase of measuring the neutrino mixing parameters, e.g. the mixing angle θ_{13} was shown to be much larger than zero [20–25], contrary to previous expectations. Neutrino masses are, therefore, in obvious contradiction to the SM and require physics beyond the standard model, e.g. the see–saw mechanism [26–29], in order to explain the origin of neutrino masses.

Many models try to explain the neutrino mixing pattern with the help of a spontaneously broken flavor symmetry. However, the recent measurement of a non–vanishing θ_{13} challenged many of these predictions. We show in this thesis that the predictions of such models might be, in general, dramatically modified as soon as one considers effects from the Kähler potential,

so–called Kähler corrections. Even though the Kähler potential is a function specific to supersymmetry, the implications of the Kähler corrections apply to the non–supersymmetric case as well.

The outline of this thesis is as follows: In chapter 2 we introduce the basic concepts of supersymmetry and discuss several supersymmetric example models. Furthermore, we present a method that allows us to compute the superpotential to all orders. Chapter 3 discusses the nature and origin of neutrino masses and we also provide current experimental results for the mixing parameters. In chapter 4 we show how the μ problem can be solved by relating it to supersymmetry breaking while also allowing for viable neutrino masses. We achieve this with the help of R symmetries. In chapter 5 we introduce the flavor problem of the (supersymmetric) SM. As a possible solution thereof we discuss flavor symmetries: first a pseudo–anomalous $\mathrm{U}(1)_R$ symmetry that explains the fermion mass hierarchies and then non–Abelian discrete symmetries for the neutrino mixing. Chapter 6 focuses on corrections to the predictions of such non–Abelian discrete flavor symmetries by considering additional Kähler potential terms and their implications. In chapter 7 we combine many of the previous ideas by building a model based on a pseudo–anomalous $\mathrm{U}(1)_R$ symmetry with a residual $\mathbb{Z}_4^R$ that explains all the fermion mass hierarchies and solves the hierarchy problem without any fine–tuning. The neutrino mixing in such models might be described through a non–Abelian flavor symmetry. We end this thesis in chapter 8 with our conclusions.

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Chapter 2

Supersymmetric models

One of the most persuasive extensions of the standard model is supersymmetry. SUSY solves the hierarchy problem by relating bosons and fermions to each other. This is done by introducing an anti–commuting symmetry generator $\mathcal{Q}$ such that

$$\mathcal{Q}|\text{Boson}\rangle \;=\; |\text{Fermion}\rangle, \qquad\qquad \mathcal{Q}|\text{Fermion}\rangle \;=\; |\text{Boson}\rangle, \tag{2.1}$$

i.e. for each SM particle there exists a superpartner whose spin differs by $1/2$, which cancels the unwanted corrections to the Higgs mass. However, SUSY not only solves the hierarchy problem, it also provides an attractive dark matter candidate and improves gauge coupling unification compared to the SM [8].

In this chapter we briefly review several supersymmetric models, starting with the most famous one, the minimal supersymmetric standard model (MSSM). With the help of the MSSM we introduce the superpotential and the Kähler potential, while also discussing the idea of R symmetries and SUSY breaking.[1] We then introduce extensions of the MSSM, in particular the recently proposed Dirac NMSSM [36]. This chapter ends with a review of how to determine the superpotential to all orders with the help of the Hilbert basis [37–39].

2.1 The minimal supersymmetric standard model

We briefly introduce the basic concepts of supersymmetry and the MSSM in the following sections, for a more detailed review cf. [34, 35] and references therein.

2.1.1 Basic SUSY

In $\mathcal{N} = 1$ supersymmetric theories, ordinary (Minkowski) spacetime with the coordinates x^μ, where $\mu = 0, 1, 2, 3$, is extended by the two Grassmann variables $\theta_\alpha, \theta^\dagger_{\dot\alpha}$, where $\alpha, \dot\alpha = 1, 2$. Therefore, points in "superspace" are given by the coordinates $x^\mu, \theta_\alpha, \theta^\dagger_{\dot\alpha}$. This means that we have to promote our usual fields from functions of the spacetime coordinate x^μ to superfields, which are also functions of the superspace coordinates $\theta_\alpha, \theta^\dagger_{\dot\alpha}$, e.g. $\Psi(x, \theta, \theta^\dagger)$. A so–called chiral superfield is given by

$$\Psi = \varphi(y) + \sqrt{2}\,\theta\,\psi(y) + \theta\,\theta\,F(y)\;, \tag{2.2}$$

with $y^\mu = x^\mu + \mathrm{i}\,\theta\,\sigma^\mu\,\theta^\dagger$. Here, φ is a complex scalar, ψ is a two–component fermion and F is an auxiliary field, i.e. it has no kinetic term in the Lagrangian. Chiral superfields Ψ have the property that

$$D^\dagger_{\dot\alpha}\,\Psi = 0\;, \tag{2.3}$$

[1] In this introductory part we closely follow [34, 35].

and

$$D_\alpha \, \Psi^* = 0 \tag{2.4}$$

for anti–chiral superfields. Here, D_α is the chiral covariant derivative.

In order to build a supersymmetric Lagrangian one has to build a function of such chiral superfields and integrate over the coordinates $\theta_\alpha, \theta^\dagger_{\dot\alpha}$, taking into account that they are Grassmann variables, i.e. we can only integrate with the measures $\int \mathrm{d}^2\theta, \int \mathrm{d}^2\theta^\dagger$ and $\int \mathrm{d}^2\theta \, \mathrm{d}^2\theta^\dagger$. In the following sections we discuss the two most important supersymmetric functions, the superpotential $\mathscr{W}$ and the Kähler potential K; the discussion of the gauge kinetic function can be found in one of the reviews [34,35]. However, let us first determine the necessary superfields for a phenomenologically appealing model, i.e. a model that contains the SM with minimal ingredients.

2.1.2 The MSSM

The minimal supersymmetric standard model is a supersymmetric theory with minimal particle content. As the standard model, the MSSM has the gauge group $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ and its particle matter content is summarized in table 2.1.

Q_i	$\overline{U}_i$	$\overline{D}_i$	L_i	$\overline{E}_i$	H_u	H_d
$\left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right)$	$\left(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right)$	$\left(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}\right)$	$\left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)$	$(\mathbf{1}, \mathbf{1}, 1)$	$\left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right)$	$\left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right)$

Table 2.1: Matter content of the MSSM and their representation under the SM gauge group $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$. The index $i = 1, 2, 3$ denotes the generations.

Compared to the SM there is one additional Higgs field, which is necessary in order to write down a phenomenologically viable theory, e.g. to write down all SM Yukawa couplings,[2] as we will discuss in more detail in the next section.

2.1.3 The superpotential

Arguably, the most important ingredient for any supersymmetric theory is the superpotential $\mathscr{W}$. It is a gauge invariant, holomorphic function of the chiral superfields. Due to its holomorphicity, it is related to the supersymmetric Lagrangian through

$$\mathscr{L} \supset \int \mathrm{d}^2\theta \, \mathscr{W}(\Psi) + \text{c.c.} \, . \tag{2.5}$$

Given the field content of the MSSM from table 2.1 we can write down its superpotential,

$$\mathscr{W} = Y_u \, Q \, \overline{U} \, H_u + Y_d \, Q \, \overline{D} \, H_d + Y_e \, L \, \overline{E} \, H_d + \mu \, H_u \, H_d + \dots \, , \tag{2.6}$$

where we have suppressed family indices. The first three terms lead to the usual Yukawa interaction terms in the Lagrangian and the ellipsis contains terms that violate either baryon or total lepton number, e.g. they would promote proton decay.[3] They are given by

$$\mathscr{W} \supset L \, L \, \overline{E} + L \, H_u + Q \, L \, \overline{D} + \overline{U} \, \overline{D} \, \overline{D} \, . \tag{2.7}$$

[2] The additional Higgs field is also needed for anomaly cancellation.

[3] Note that due to the holomorphicity of $\mathscr{W}$ we had to introduce an additional Higgs field to be able to write down all Yukawa terms.

These operators are usually forbidden by an additionally imposed $\mathbb{Z}_2$ symmetry, called R parity or matter parity. Under this symmetry all matter fields have charge 1 and the Higgs fields have charge 0. Furthermore, the superspace variable θ also has charge 1.[4] On top of forbidding the dangerous operators in equation (2.7), the symmetry also ensures that the lightest supersymmetric particle is stable, since SM fermions have zero charge, whereas, their superpartners have charge 1 and SUSY particles can, therefore, only be created in pairs.

After imposing matter parity the MSSM superpotential is given by

$$\mathscr{W}_{\text{MSSM}} = Y_u\, Q\, \overline{U}\, H_u + Y_d\, Q\, \overline{D}\, H_d + Y_e\, L\, H_d\, \overline{E} + \mu\, H_u\, H_d\ . \tag{2.8}$$

So far we have not addressed the so–called μ term of $\mathscr{W}_{\text{MSSM}}$, which is a parameter with mass dimension 1 in front of the Higgs bilinear $H_u\, H_d$, i.e. it is a supersymmetric mass term for the Higgs bilinear. If the μ term is unsuppressed, meaning that there is no reason for it to be of the size of the electroweak scale, the hierarchy problem is reintroduced.[5] This issue is called the μ problem and it can be solved either by ensuring that the size of the μ term is of the correct order or by forbidding the term by some symmetry argument. As it turns out, only so–called R symmetries, which we introduce briefly in section 2.1.5, can forbid the μ term, assuming anomaly freedom [40]. We discuss this further in chapter 4 and provide a phenomenologically appealing example of a μ term prohibiting symmetry in section 4.2.4.

2.1.4 The Kähler potential

We have just introduced the superpotential, which is a holomorphic function of chiral superfields necessary to build a supersymmetric Lagrangian. Another supersymmetric function is the Kähler potential K, which, unlike the superpotential, is a real function of both chiral and antichiral superfields,

$$\mathscr{L} \supset \int \mathrm{d}^2\theta\, \mathrm{d}^2\theta^\dagger\, K(\Psi, \Psi^*) + \left(\int \mathrm{d}^2\theta\, \mathscr{W}(\Psi) + \text{c.c.} \right)\ . \tag{2.9}$$

At tree level the Kähler potential is canonical, e.g. for the lepton sector we have

$$K_{\text{canonical}} \supset \left(L^f\right)^\dagger\, \delta_{fg}\, L^g + \left(R^f\right)^\dagger\, \delta_{fg}\, R^g\ , \tag{2.10}$$

with f, g being indices in flavor space and L, R denoting the left– or right–handed leptons, respectively. A canonical Kähler potential ensures correct kinetic terms in the Lagrangian since for a chiral superfield, cf. equation (2.2), the D term of $\Psi^\dagger\, \Psi$, i.e. the component proportional to $(\theta\theta)(\theta^\dagger\theta^\dagger)$, is given by

$$\Psi^\dagger\Psi \supset (\theta\theta)(\theta^\dagger\theta^\dagger) \left[\partial^\mu\varphi^*\, \partial_\mu\varphi + \mathrm{i}\, \psi^\dagger\, \overline{\sigma}^\mu\, \partial_\mu\psi + F^* F \right] + \dots\ , \tag{2.11}$$

where the ellipsis stand for total derivative terms we can ignore [34]. Therefore, integrating over all θ and $\theta^\dagger$ components gives the correct kinetic terms.

[4] Therefore, matter parity is sometimes considered an R symmetry; however, it is not a proper R symmetry as we discuss in section 2.1.5.

[5] We discuss the connection between the μ term and the Higgs mass in a bit more detail in section 2.2.1.

2.1.5 R symmetries

Supersymmetry allows for an additional type of symmetry, so–called R symmetries, to be present. The R symmetry generator does not commute with the SUSY generator $\mathcal{Q}$, hence, the coordinates θ and $\theta^\dagger$ also transform under this symmetry with charges q_θ and $-q_\theta$, respectively.[6] This implies that if a given Lagrangian is supposed to be invariant under an R symmetry, allowed Kähler potential terms have to have R charge

$$q_K = 0 \,, \tag{2.12}$$

whereas terms from the superpotential have R charge

$$q_{\mathscr{W}} = 2\, q_\theta \,. \tag{2.13}$$

This follows from equation (2.9) and from the transformation property of $\mathrm{d}\theta$, which has R charge $-q_\theta$.

Given an Abelian R symmetry we can also determine the charges of the superfield components. Assuming a chiral superfield Ψ has R charge r_Ψ, we can use equation (2.2) and determine the charges of its scalar φ and fermionic component ψ, as well as the R charge of its F term, i.e.

$$r_\varphi = r_\Psi \,, \qquad r_\psi = r_\Psi - r_\theta \quad \text{and} \quad r_F = r_\psi - 2\, r_\theta \,. \tag{2.14}$$

This discussion shows that the in section 2.1.3 introduced R parity, a supposed $\mathbb{Z}_2^R$ symmetry, is not a proper R symmetry since the superpotential has the charge

$$\begin{aligned} q_{\mathscr{W}} &= 2\, q_\theta \mod 2 \\ &= 0 \mod 2 \,. \end{aligned} \tag{2.15}$$

Hence, we see, in general, that there is no R symmetry of order 2. Later we discuss two types of R symmetries further, discrete $\mathbb{Z}_M^R$ symmetries in chapter 4 and then a continuous $\mathrm{U}(1)_R$ symmetry in chapter 5.

2.1.6 SUSY breaking

Supersymmetry, besides its many benefits, cannot be an exact theory, otherwise superpartners would have been discovered in experiments already. Therefore, any realistic supersymmetric model must contain SUSY breaking and, analogously, to electroweak symmetry breaking we would expect SUSY to be broken spontaneously. There is a myriad of spontaneously broken SUSY models and given the vast amount of literature on the topic, we refer to the review by Martin [34] for more details.

Here, we briefly introduce general soft supersymmetry–breaking terms, i.e. the breaking terms are of positive mass dimension and no quadratic divergences to scalar masses occur. In a general Lagrangian, we can have the following possible soft breaking terms [34],

$$\begin{aligned} \mathscr{L}_{\text{soft}} = &\left[-\left(\frac{1}{2}\, M_a\, \lambda^a\, \lambda^a + \frac{1}{6}\, A^{ijk}\, \varphi_i\, \varphi_j\, \varphi_k + \frac{1}{2}\, b^{ij}\, \varphi_i\, \varphi_j + t^i\, \varphi_i \right) + \text{h.c.} \right] \\ &- \left(\widetilde{m}^2 \right)^i_{\ j}\, \varphi^{j*}\, \varphi_i \,, \end{aligned} \tag{2.16}$$

[6] We are only considering Abelian R symmetries here. Obviously, similar statements are possible for non–Abelian R symmetries [41].

where the φ_i are the scalar components of the superfields and the λ^a are the superpartners of the SM gauge bosons, the gauginos. In the MSSM we now have three mass terms M_a, one for each gauge group, three so–called A terms in one–to–one correspondence to the Yukawa matrices and the b term for the Higgs bilinear, which in the MSSM is called the $B\mu$ term. Furthermore, there is one slepton mass matrix $\widetilde{m}$ for every MSSM field from table 2.1, which we call the soft masses. The couplings t^i vanish in the MSSM since there are no gauge singlets in the theory.[7]

The A terms and the soft masses in the MSSM are 3×3 matrices in flavor space, and initially there is no reason why these matrices should be diagonal or constrained in any way, thus, introducing a large arbitrariness into the theory. Altogether we have order 100 new parameters introduced through soft SUSY breaking into MSSM [34, 42] and in section 5.1 we discuss the implications and problems following from this fact, focusing, in particular, on the soft masses.

Let us discuss the origin of SUSY breaking. It is generally assumed that supersymmetry is broken in some hidden sector and that its breaking is mediated to the visible sector, e.g. the MSSM, through some particle interaction. The soft breaking terms from equation (2.16) then occur through the F term of the messenger particle. If, for example, SUSY is broken through gravitational interactions in the hidden sector, the breaking scheme is called gravity mediation [43,44], which we assume from here on. In this case one usually considers a spurion field X whose F term breaks SUSY, i.e. $X = \theta\,\theta\,F_X$ with $\langle F_X \rangle \neq 0$. In such a scenario the A terms are generated in the superpotential through

$$\mathscr{W} \supset \frac{X}{\Lambda_{\text{soft}}} \left(Y^X\right)_{fgh} \Psi^f\,\Psi^g\,\Psi^h\,, \tag{2.17}$$

whereas the soft masses are generated in the Kähler potential,

$$K \supset \left(\frac{X}{\Lambda_{\text{soft}}}\, n_{fg} \left(\Psi^f\right)^\dagger \Psi^g + \text{h.c.}\right) + \frac{X^\dagger X}{\Lambda_{\text{soft}}^2}\, k_{fg} \left(\Psi^f\right)^\dagger \Psi^g\,. \tag{2.18}$$

Here, Λ_{soft} is the messenger scale, and in the case of gravity mediation it is given by $\Lambda_{\text{soft}} = M_{\text{P}}$. The couplings Y^X, n and k are matrices in flavor space and, as stated above, at first there are no constraints on their structure. The vacuum expectation value (VEV) of X breaks SUSY, therefore, the soft masses and A terms from equation (2.16) are created. The order parameter of this symmetry breaking is for gravity mediation the gravitino mass $m_{3/2}$ [43, 44], which can be estimated as

$$m_{3/2} \sim \frac{\langle F_X \rangle}{M_{\text{P}}}\,, \tag{2.19}$$

and due to equation (2.18) it should be comparable to the other sparticle masses.

2.2 Beyond the MSSM

In the previous section we introduced the basic concepts of supersymmetry and the MSSM. Now, we want to focus on next–to–minimal supersymmetric models first by showing the

[7] This changes if we introduce right–handed neutrinos.

shortcomings of the MSSM and then how these can be improved or even removed by extending the theory.

For example, the MSSM suffers from a proton decay problem through the presence of dimension four and five operators, e.g. $Q\,Q\,Q\,L$. Even though such a term is suppressed, it still promotes proton decay to a phenomenologically unacceptable level. We present a symmetry that forbids such operators in chapter 4 and now discuss another problem of the MSSM, the μ problem.

2.2.1 The μ problem and the Higgs mass in the MSSM

A main concern of the MSSM is the μ term in equation (2.8), which, if unsuppressed, reintroduces the hierarchy problem into the MSSM. However, a small μ term from the start is considered unnatural unless it is due to a symmetry argument.

Let us examine why an unsuppressed μ term is such a big problem by looking at the connection between the μ term and the measured Higgs mass, which has recently been determined by the LHC to be (125 ± 1) GeV [1, 2]. The issue is highlighted when we look at the Z boson mass in the MSSM, which is given by [34]

$$m_Z^2 = \frac{|m_{H_d}^2 - m_{H_u}^2|}{\sqrt{1 - \sin^2(2\,\beta)}} - m_{H_u}^2 - m_{H_d}^2 - 2\,|\mu|^2 \,. \tag{2.20}$$

Here, m_{H_u,H_d}^2 are the Higgs scalar masses from SUSY breaking and β determines the ratio between the two Higgs VEVs with

$$\tan(\beta) = \frac{v_u}{v_d} \quad \text{with} \quad v = \sqrt{v_u^2 + v_d^2} = 246\ \text{GeV} \,, \tag{2.21}$$

where $v_{u,d}$ are the VEVs of the neutral scalar components of H_u and H_d, respectively. In order for equation (2.20) to be in agreement with experiments all new parameters have to be within the order of the Z boson mass, including, in particular, the μ term.[8] However, such a value for μ is unnatural and, therefore, we need a symmetry argument to forbid the μ term or suppress its size to acceptable values. We present such a symmetry and a related mechanism in chapter 4.

Even if we assume that the μ problem is solved, e.g. through some symmetry, there are still issues matching equation (2.20) with experiments. The MSSM Higgs scalar mass m_h^2 is naturally small [34], therefore, the supersymmetric parter of the top quark, the stop, needs to have a relatively large mass in order to lift the Higgs mass to its measured value. However, the stop masses also contribute to $m_{H_u}^2$, as can be seen in the renormalization group (RG) equation [36]

$$\mu_{\text{Ren}}\,\frac{\mathrm{d}}{\mathrm{d}\mu_{\text{Ren}}}\,m_{H_u}^2 = \frac{3}{8\,\pi^2}\,y_t^2\,\left(m_{\widetilde{Q}_3}^2 + m_{\widetilde{t}_R}^2\right) + \dots \,, \tag{2.22}$$

where μ_{Ren} describes the renormalization scale. Hence, large left– or right–handed stop masses, $m_{\widetilde{Q}_3}$ and $m_{\widetilde{t}_R}$, respectively, increase $m_{H_u}^2$, which means that we need significant cancellations between contributions in equation (2.20) for an experimentally viable Z boson mass. This is considered the fine–tuning problem of the MSSM, since the parameters have to be tuned at a level of at least 1% [45]. For a more detailed discussion of this issue we refer to [34] and references therein.

[8] Alternatively there can be cancellations between the terms which is a fine–tuning problem.

2.2.2 The NMSSM and the GNMSSM

One way to reconcile the Higgs mass and supersymmetry without tuning the stop mass is to extend the MSSM. In the next–to–minimal supersymmetric standard model (NMSSM) [46–48] we add a SM singlet superfield N to the MSSM. This field interacts with Higgs fields trough the superpotential and we get

$$\mathscr{W} \supset \lambda\, N\, H_u\, H_d + \mu\, H_u\, H_d + \mathscr{W}(N)\ , \tag{2.23}$$

where λ is a dimensionless coupling and $\mathscr{W}(N)$ the singlet superpotential. Let us assume that we have $\mathscr{W}(N) = \frac{\mu_N}{2}\, N^2$, where μ_N is the supersymmetric singlet mass, and the scalar potential [47]

$$V_{\text{soft}} \supset m_N^2\, |N|^2\ , \tag{2.24}$$

with the SUSY breaking soft mass m_N. The Higgs mass is then raised through the soft mass and the superpotential interactions [36]

$$\Delta m_h^2 = \lambda^2\, v^2\, \sin^2(2\,\beta)\, \left(\frac{m_N^2}{\mu_N^2 + m_N^2}\right)\ . \tag{2.25}$$

Such a term raises the Higgs mass for a considerably large m_N, but there are some caveats considering the size of this soft mass. It is clear that for very small $m_N \ll \mu_N$, the extra contribution to the Higgs mass in equation (2.25) decouples. However, the singlet soft mass m_N contributes to the Higgs soft masses $m_{H_{u,d}}^2$ at one–loop level, e.g. in the RG equation for $m_{H_u}^2$ [36],

$$\mu_{\text{Ren}}\, \frac{\mathrm{d}}{\mathrm{d}\mu_{\text{Ren}}}\, m_{H_u}^2 = \frac{3}{8\,\pi^2}\, y_t^2\, \left(m_{\widetilde{Q}_3}^2 + m_{\widetilde{t}_R}^2 + m_N^2\right) + \dots\ , \tag{2.26}$$

which depends on m_N^2, hence, requiring it to be of similar size as the Higgs mass.

Raising the Higgs mass to acceptable values with a large m_N^2 in equation (2.25), also increases $m_{H_u}^2$ through equation (2.26), and we need substantially large fine–tuning in order to make equation (2.20) experimentally viable again, thus, spoiling naturalness.

The conventional NMSSM comes with a discrete $\mathbb{Z}_3$ symmetry in order to forbid an unsuppressed linear term N in the superpotential. Under this symmetry all fields have charge 1, which allows $N\, H_u\, H_d$ and a term N^3 in $\mathscr{W}(N)$, which can also be used in order to raise the Higgs mass to an acceptable value [48]. However, this symmetry suffers from a domain wall problem [49, 50] and also does not allow the Weinberg operator. This makes this symmetry, and hence the NMSSM, even more undesirable and we might want to consider a more general extension of the MSSM.

A more general singlet extension of the MSSM with one singlet field N is the so–called generalized NMSSM (GNMSSM) [48, 51] with the superpotential

$$\mathscr{W} \ = \ \xi\, N + \frac{1}{2}\, \mu_N\, N^2 + \frac{1}{3}\, \kappa\, N^3 + \mathscr{W}_{\text{MSSM}}\ , \tag{2.27}$$

where ξ, μ_N and κ are order one parameters with mass dimension $2, 1$ and 0, respectively. This model needs to suppress the first two parameters such that their sizes are $m_{3/2}^2$ and $m_{3/2}$, respectively, in order to be phenomenologically viable. Then, it does not suffer as much

from the fine–tuning problems of the NMSSM [51, 52]. However, we do not discuss this here in further detail since we cannot use the GNMSSM later in chapter 7 for model building reasons, and we switch in the next section to an alternative modification of the original NMSSM proposal, which solves the fine–tuning problem of the (N)MSSM and is compatible with our method in chapter 7.

2.2.3 The Dirac NMSSM

Since the NMSSM does not solve the fine–tuning problem of the MSSM completely, the authors of [36] have modified the NMSSM proposal by adding an additional singlet $\overline{N}$ to the NMSSM setting. Then, they introduce additional U(1) symmetries in such a way that N and $\overline{N}$ only receive a Dirac mass M. The superpotential of the theory is given by

$$\mathscr{W} \;=\; \lambda\, N\, H_u\, H_d + M\, N\, \overline{N} + \mu\, H_u\, H_d + \mathscr{W}_{\text{Yukawas}}\;, \tag{2.28}$$

and the $\mathrm{U}(1)_{\mathrm{PQ}} \times \mathrm{U}(1)_{\overline{N}}$ Peccei–Quinn–like symmetries are summarized in table 2.2.

	H_u	H_d	N	$\overline{N}$	μ	M
$\mathrm{U}(1)_{\mathrm{PQ}}$	1	1	-2	-2	-2	4
$\mathrm{U}(1)_{\overline{N}}$	0	0	0	1	0	-1

Table 2.2: Extra $\mathrm{U}(1)_{\mathrm{PQ}} \times \mathrm{U}(1)_{\overline{N}}$ Peccei–Quinn–like symmetries of the Dirac NMSSM.

We can see that the $\mathrm{U}(1)_{\overline{N}}$ symmetry effectively couples $\overline{N}$ to the dimensionful parameter M, therefore, forbidding a coupling to the Higgs bilinear $H_u\, H_d$. Also, these symmetries forbid other unwanted terms, for example, large tadpoles for the singlets. However, there is no reason for these symmetries to be present at first, especially since the massive parameters μ and M are charged under them and break them explicitly. We address this issue again later in chapter 7.

Compared to the NMSSM there is one new term in the scalar potential after SUSY breaking,

$$V_{\text{soft}} \supset m_N^2\, |N|^2 + m_{\overline{N}}^2\, |\overline{N}|^2\;. \tag{2.29}$$

The soft mass $m_{\overline{N}}^2$ contributes to the new Higgs quartic coupling $|\lambda\, H_u\, H_d|^2$ and the SM–like Higgs mass is increased by [36]

$$\Delta m_h^2 \supset \lambda^2\, v^2\, \sin^2(2\,\beta) \left(\frac{m_{\overline{N}}^2}{M^2 + m_{\overline{N}}^2} \right)\;. \tag{2.30}$$

This term looks very similar to equation (2.25), hence, we might worry about fine–tuning. However, since $\overline{N}$ only couples through a dimensionful parameter, i.e. M, to all of the NMSSM setting, its soft mass does not contribute to the RGEs for m_{H_u,H_d}^2 [36]. In fact the RGE for the up–type Higgs at one–loop level is given by equation (2.26), i.e. only m_N^2 contributes [36]. This allows for a large $m_{\overline{N}}^2$ without any fine–tuning and since equation (2.30) does not vanish for $m_{\overline{N}} \gg M$, we can raise the Higgs mass to phenomenologically viable size.

The authors of [36] also include a study of the fine–tuning for the Dirac NMSSM. There, they fix the Higgs mass to (125 ± 1) GeV and then compute the fine–tuning in dependence

of the Dirac mass M and the soft mass $m_{\overline{N}}$.[9] As a result, they find a large parameter region for the Dirac NMSSM with acceptable fine–tuning: it is given by $700\,\text{GeV} \lesssim M \lesssim 5\,\text{TeV}$ and $m_{\overline{N}} \gtrsim 2\,\text{TeV}$. The least fine–tuned region can be found for $M \sim 2\,\text{TeV}$ and $m_{\overline{N}} \gtrsim 10\,\text{TeV}$, and their benchmark model has $\mu_{\text{eff}} := \mu + \lambda \langle N \rangle = 150\,\text{GeV}$, $M = 1\,\text{TeV}$ and $m_{\overline{N}} = 10\,\text{TeV}$. They also require $m_N \lesssim 1\,\text{TeV}$ in order to avoid the fine–tuning problem of the NMSSM. We show later in chapter 7 how we can realize such a scenario naturally in some flavor models, while incorporating additional nice features [33].

2.3 The superpotential to all orders

Having introduced the basic concept of SUSY in the previous sections, we see that the superpotential plays an important role in any theory. Therefore, it seems natural to try to determine the full superpotential for a given theory. This entails finding all possible holomorphic gauge invariant terms, so–called monomials, and also ensuring that they have the correct R charge, if any field is charged under an R symmetry.

Let us for the moment start without an R symmetry present; we discuss monomials for R symmetries later in section 2.3.3. For a given theory with gauge group G we assume that there is a finite set of holomorphic gauge invariant monomials, which we call basis monomials, and that we can combine these basis monomials in order to construct the full superpotential. In this way a superpotential term, i.e. any gauge invariant monomial $\mathscr{M}$, can be written as

$$\mathscr{M} \;=\; \prod_{i=1}^{H} \mathscr{M}_i^{\eta_i} \qquad \text{with } \eta_i \;\in\; \mathbb{N}_0 \;, \tag{2.31}$$

where $\mathscr{M}_i$ are the basis monomials and H the finite number of basis monomials. Now all we need to do is to find all basis monomials $\mathscr{M}_i$ and we can determine the superpotential to all orders.

Unfortunately, even constructing all basis monomials for the MSSM is rather complicated [53]. However, in [37] it was found that for a particular set of gauge symmetries, U(1) symmetries in [37], we can determine the so–called Hilbert basis in order to determine all holomorphic gauge invariant monomials. In this procedure, the Hilbert basis method, each Hilbert basis vector corresponds to a holomorphic gauge invariant monomial and by determining the full Hilbert basis we also find the full basis of the monomials. In the following sections we review how this procedure works for several gauge groups and then we turn to the case of R symmetries [39].

Note that holomorphic gauge invariant monomials are in one–to–one correspondence with D–flat directions [54], and, therefore, determining the Hilbert basis for a theory does not only allow us to compute the superpotential to all orders, but we also automatically find all D–flat directions in the given theory.

2.3.1 Hilbert basis for continuous non–R symmetries

In [37] it has been shown how to compute the Hilbert basis in the case of U(1) symmetries. Let us start with one U(1) symmetry and F fields ϕ_i, then a monomial

$$\mathscr{M} = \prod_{i=1}^{F} (\phi_i)^{n_i} \qquad \text{with } n_i \;\in\; \mathbb{N}_0 \;, \tag{2.32}$$

[9] For a more detailed discussion of the considered fine–tuning measure, cf. [36].

is gauge invariant if

$$\sum_{i=1}^{F} n_i \, q_i = 0 \; , \tag{2.33}$$

where q_i are the U(1) charges of the respective fields. Since we want to find holomorphic gauge invariant monomials we need the powers n_i to be integers and $n_i \geq 0$. Writing the powers and the charges into vectors this can be understood as the scalar product of two vectors, i.e. $q^T \cdot n = 0$ where $q^T = (q_1, \ldots, q_F)$ and $n = (n_1, \ldots, n_F)^T$.

This can be generalized to L U(1) symmetries and possible solutions are then found by solving

$$\sum_{i=1}^{F} n_i \, q_i^{(j)} = 0 \quad \forall j \in \{1, \ldots, L\} \; , \tag{2.34}$$

which is the same as substituting the charge vector q with a charge matrix Q and solving

$$Q \cdot n = 0 \quad \text{where} \quad Q \in \mathbb{Z}^{L \times F} \quad \text{and} \quad n \in \mathbb{N}_0^F \; . \tag{2.35}$$

It was pointed out in [37] that this problem is well–known in mathematics [55–57] and that these so–called homogeneous linear Diophantine equations can be solved with the help of computer algorithms [58, 59].

The solutions are given by the Hilbert basis $\mathscr{H}$, which a is a minimal set of vectors,

$$\mathscr{H} = \{h^{(1)}, \ldots, h^{(H)}\} \; , \tag{2.36}$$

where $h^{(i)}$ are the Hilbert basis vectors. Note that H is the finite length of the basis which cannot be predicted but has to be determined within the algorithm. With this basis at hand, we can write every solution n of equation (2.35) as

$$n = \sum_{i=1}^{H} \eta_i \, h^{(i)} \quad \text{with} \quad \eta_i \in \mathbb{N}_0 \; . \tag{2.37}$$

Here, every solution n corresponds to a holomorphic gauge invariant monomial and every basis vector $h^{(i)}$ corresponds to a basis monomial $\mathscr{M}_i$ from equation (2.31).

In [38, 39] we pointed out that we can also find holomorphic gauge invariant monomials for SU(N) gauge theories by assigning each field U(1) charges according to the Cartan subalgebras of the gauge group. In the case of a SU(N) gauge group, which has rank $N-1$, we can assign $N-1$ U(1) charges given by the diagonal generators of the SU(N). We can then again construct a charge matrix Q and solve equation (2.35). The caveat of this procedure is that we have to "translate" the solutions and that we also find redundant or vanishing ones which we have to remove from the set. A review of this process can be found in [38, 39] or has been automated in an associated `Mathematica` package [60].

2.3.2 Hilbert basis for discrete non–R symmetries

For discrete Abelian $\mathbb{Z}_M$ symmetries we can use a similar method [39]. The invariance of a monomial under a $\mathbb{Z}_M$ is given when

$$\sum_{i=1}^{F} n_i \, p_i = 0 \mod M \; , \tag{2.38}$$

where p_i are the discrete charges of the fields. This is easily translated in a vector equation if we introduce a dummy field with charge $-M$ and write

$$(-M, p_1, \ldots, p_F) \cdot \begin{pmatrix} m \\ n_1 \\ \vdots \\ n_F \end{pmatrix} = 0 \qquad \text{with } m \in \mathbb{N}_0 \,. \tag{2.39}$$

We can again compute the Hilbert basis for this equation; however, we have to truncate the found basis vectors and discard the first entry because it only represents the dummy field and not any physical field.

It is obvious that this can be extended to several, e.g. K, discrete symmetries by introducing K dummy fields. We construct a charge matrix, solve for the Hilbert basis, and truncate the found solutions of the first K elements. The truncated Hilbert basis vectors then again represent basis monomials of equation (2.31).

2.3.3 Hilbert basis for Abelian R symmetries

Let us consider the situation for Abelian R symmetries [39]. At first, the situation seems to be very similar, as a monomial is invariant under a $\mathbb{Z}_N^R$ symmetry if it fulfills a condition like in equation (2.38). The only difference is that the monomial has to have the charge of the superpotential instead of zero, i.e. for a $\mathbb{Z}_N^R$ symmetry where the superpotential has R charge 2 we get

$$\sum_{i=1}^{F} n_i \, r_i = 2 \mod M \,, \tag{2.40}$$

where the r_i are the R charges of the fields.[10] We can translate this into a vector equation by introducing another dummy field, so we get

$$(-2, -N, r_1, \ldots, r_F) \cdot \begin{pmatrix} \ell \\ m \\ n_1 \\ \vdots \\ n_F \end{pmatrix} = 0 \qquad \text{with} \quad \ell \in \{0, 1\} \,. \tag{2.41}$$

Obviously, for $\ell = 1$ this is equivalent to equation (2.40). Solutions with $\ell = 0$ describe monomials with R charge 0 and we can always multiply such a term to a monomial with R charge 2 and still have a valid superpotential term. Therefore, we split the found solutions into two parts: so–called inhomogeneous solutions n_{inhom}, which have $\ell = 1$ and describe possible superpotential terms, as well as homogeneous monomials n_{hom} with $\ell = 0$, which describe monomials with vanishing R charge. In this way we can describe a general solution n to equation (2.41) as

$$n = n_{\text{inhom}}^{(i)} + \sum_{h=1}^{H_0} \eta_h \, n_{\text{hom}}^{(h)} \qquad \text{with} \quad \eta_h \in \mathbb{N}_0 \,, \tag{2.42}$$

[10] In general the superpotential has R charge $2\,q_\theta$ and we simply chose $q_\theta = 1$ as an example.

where $n^{(i)}_{\text{inhom}}$ is any of the inhomogeneous solutions and H_0 is the number of homogeneous solutions. As we can see, every inhomogeneous solution on its own is equivalent to a superpotential term, unlike the homogeneous terms, which always need an inhomogeneous one in order to be permitted, and of course all of these have to be truncated just like in section 2.3.2.

Note that it is also possible to determine the Hilbert basis for $\mathrm{U}(1)_R$ symmetries by simply searching for solutions that fulfill

$$\sum_{i=1}^{F} n_i\, r_i = 2\ , \tag{2.43}$$

i.e. only introducing one dummy field for the superpotential charge.

Similar to all previous cases we can generalize this procedure to several R symmetries, like in section 2.3.2 we need to add a dummy field for every symmetry and one overall dummy field for the superpotential charge.

2.3.4 Usage of the `Mathematica` package

It is possible to compute the Hilbert basis for a gauge group consisting of several $\mathrm{U}(1)$, $\mathrm{SU}(N)$, $\mathbb{Z}_N$, $\mathbb{Z}_N^R$ and $\mathrm{U}(1)_R$ symmetries. We just need to combine all previous procedures and build one large charge matrix with several dummy fields and then truncate the found basis vectors accordingly. For the case of $\mathrm{U}(1)$ and $\mathrm{SU}(N)$ symmetries this has been automatized in a `Mathematica` package [60] accompanying the publication of [38, 39]. As input parameters the package uses the given fields, the gauge groups involved and the representations of the fields. Since the publication of [38, 39] our `Mathematica` package has been updated in order to also be able to compute holomorphic gauge invariant monomials for discrete R and non–R symmetries, as well as $\mathrm{U}(1)_R$ symmetries.

Therefore, the package allows us to find holomorphic gauge invariant monomials, and thus D–flat directions, for a given theory. Since it is capable of distinguishing between monomials with zero R charge and monomials with the superpotential charge, we can also use it in order to determine the Kähler potential for a theory by introducing conjugate fields with opposite charge and then determining all homogeneous monomials, i.e. possible Kähler potential terms due to their zero R charge.

Chapter 3

Neutrino mixing and masses

We review the experimentally well–known fact that neutrinos are massive particles and, thus, they oscillate. We introduce the standard parametrization of neutrino mixing and review the oscillation process. We also present the current experimental results, in particular, of the mixing angles. Finally, we end the chapter by briefly discussing the possible origin and nature of neutrino masses with a slight focus on the see–saw mechanism.

3.1 Neutrinos have mass and oscillate

The disappearance of solar neutrinos was a long–standing problem in particle physics [61–63], which was solved by the introduction of neutrino masses. Today, due to a large number of experiments [9–19], the existence of neutrino masses is well–established. These experiments also revealed that neutrinos have at least three generations and that their masses are all very small, i.e. the heaviest neutrino is lighter than a few eV. Note that for the rest of this thesis we assume that we have three neutrino generations.

Massive neutrinos imply that their flavor eigenstates do not have to be their mass eigenstates; therefore, we cannot diagonalize the neutrino mass matrix and the charged lepton Yukawa matrix simultaneously. For Majorana neutrinos, which we discuss below in section 3.3,

$$V_{\nu,L}^T \, m_\nu \, V_{\nu,L} = \mathrm{diag}(m_1, m_2, m_3) \; , \tag{3.1}$$

$$V_{e,R}^\dagger \, m_e \, V_{e,L} = \mathrm{diag}(m_e, m_\mu, m_\tau) \; , \tag{3.2}$$

the matrices $V_{\nu,L}$ and $V_{e,L}$ are not equal to each other. The Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix U_{PMNS} describes the mismatch between both transformations, which is also present for Dirac neutrinos, and is defined as

$$U_{\mathrm{PMNS}} := V_{e,L}^\dagger \, V_{\nu,L} \; . \tag{3.3}$$

Neutrinos oscillate [64–66] since U_{PMNS} is not equal to the identity matrix, and the relationship between flavor eigenstates $\nu_{e,\mu,\tau}$ and mass eigenstates $\nu_{1,2,3}$ is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\mathrm{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \; . \tag{3.4}$$

3.1.1 Standard parametrization of neutrino oscillations

The mixing matrix U_{PMNS} describes the details of neutrino oscillations and is parametrized by three mixing angles and several phases. We can decompose the PMNS matrix into three

matrices

$$U_{\text{PMNS}} = \text{diag}\left(\text{e}^{\text{i}\delta_e},\, \text{e}^{\text{i}\delta_\mu},\, \text{e}^{\text{i}\delta_\tau}\right) \cdot V(\theta_{12},\, \theta_{13},\, \theta_{23},\, \delta) \cdot \text{diag}\left(\text{e}^{-\text{i}\alpha_1/2},\, \text{e}^{\text{i}\alpha_2/2},\, 1\right) , \tag{3.5}$$

the first matrix carries the unphysical fermionic phases $\delta_{e,\mu,\tau}$ and the third matrix contains the Majorana phases $\alpha_{1,2}$. The Majorana phases are only present if neutrinos are Majorana particles, a possibility we discuss below in section 3.3, and these phases influence the rate of potential neutrinoless double–β decay [67, 68].

The matrix V in equation (3.5) is the lepton sector equivalent of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [69, 70] and it is given by

$$\begin{aligned} V &:= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-\text{i}\delta_{\text{CP}}} \\ 0 & 1 & 0 \\ -s_{13}\, e^{\text{i}\delta_{\text{CP}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, \text{e}^{-\text{i}\delta_{\text{CP}}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, \text{e}^{\text{i}\delta_{\text{CP}}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, \text{e}^{\text{i}\delta_{\text{CP}}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, \text{e}^{\text{i}\delta_{\text{CP}}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, \text{e}^{\text{i}\delta_{\text{CP}}} & c_{23}\, c_{13} \end{pmatrix} , \end{aligned} \tag{3.6}$$

where δ_{CP} is the CP violating phase. Here, s_{ij} stands for $\sin\theta_{ij}$ and c_{ij} denotes $\cos\theta_{ij}$, where the θ_{ij} are the mixing angles between the different generations and a measure of the probability for two generations to mix, i.e. for $\theta_{ij} = 0$ there is no mixing between the two generations. Conventionally, θ_{12} is called the solar, θ_{23} the atmospheric and θ_{13} the reactor neutrino mixing angle. These angles fulfill $0 \leq \theta_{12} \leq \pi/4$ and $0 \leq \theta_{13}\,, \theta_{23} \leq \pi/2$ [71].

3.1.2 Neutrino oscillations

Since neutrinos have non–zero masses there is a non–vanishing probability that if we see a neutrino ν_ℓ of flavor $\ell = e,\, \mu,\, \tau$ at a point $x = 0$, we might find a neutrino of flavor ℓ' at distance $x = L$. Experiments use this in order to detect neutrino oscillations, cf. section 3.2, and by varying the so–called baseline L these experiments become sensitive to a particular oscillation process.

With help of the just introduced standard parametrization we can analytically determine the probability for a neutrino of flavor ℓ to oscillate into flavor ℓ', which we denote with $P\left(\nu_\ell \to \nu_{\ell'}\right)$. We usually assume that a neutrino ν_ℓ is a superposition of the massive neutrino eigenstates ν_j and according to equation (3.4) this can be written as

$$|\nu_\ell\rangle \;=\; \sum_{j=1,2,3} U_{\ell,j}\, |\nu_j\rangle \;, \tag{3.7}$$

where $U = U_{\text{PMNS}}$. Using a plane wave ansatz for the mass eigenstates and the ultrarelativistic limit we can now express the probability $P\left(\nu_\ell \to \nu_{\ell'}\right)$ in dependence on the baseline L and the initial energy of the neutrino E. The general form is given by [71]

$$\begin{aligned} P\left(\nu_\ell \to \nu_{\ell'}\right) = \left|\langle \nu_{\ell'} | \nu_\ell \rangle\right|^2 \;&= \delta_{\ell\ell'} - 4 \sum_{j>k} \text{Re}\left(U^*_{\ell j}\, U_{\ell' j}\, U_{\ell k}\, U^*_{\ell' k}\right) \sin^2\left(\frac{\Delta m^2_{jk}}{4\,E}\, L\right) \\ &\quad + 2 \sum_{j>k} \text{Im}\left(U^*_{\ell j}\, U_{\ell' j}\, U_{\ell k}\, U^*_{\ell' k}\right) \sin\left(\frac{\Delta m^2_{jk}}{2\,E}\, L\right) . \end{aligned} \tag{3.8}$$

We can see that the result depends on the squared mass differences $\Delta m^2_{jk} = m^2_j - m^2_k$ and that the dependency on the angles is given by the different contributions of the mixing matrix U. As we see from equation (3.8), at least two neutrinos ν_j need to have different masses for neutrino oscillations to occur.

Depending on the situation, this formula simplifies and the main contribution comes from a particular mixing angle θ_{ij}, i.e. varying the baseline L and the initial energy E allows us to probe for different neutrino oscillations. This is being done in many experiments and we are going to discuss several results in the following section.

3.2 Current experimental data

In section 3.1.1 we introduced the standard parametrization for neutrino oscillations, most importantly the three mixing angles θ_{ij}. In the last decades many experiments have measured these angles with great accuracy [9–19]. Most recently, the discovery of a non–zero θ_{13} was a great success for experiments [20–25] and a challenge for theoretical models, since many models predict a small or vanishing θ_{13}. In fact, several popular models predict the so–called tri–bi–maximal mixing pattern, which we discuss in section 5.3.1. Table 3.1 summarizes the current best fit values for the mixing angles, based on two global analyses [72, 73]. Note that the authors of [72] find two disconnected best fit values for θ_{23}, one of them in agreement with [73]. How to resolve this issue is an ongoing debate.

	θ_{12}	θ_{13}	θ_{23}
Team I [72] ($\pm 1\sigma$):	$\left(33.36^{+0.81}_{-0.78}\right)^\circ$	$\left(8.66^{+0.46}_{-0.46}\right)^\circ$	$\left(40.0^{+2.1}_{-1.5}\right)^\circ \oplus \left(50.4^{+1.3}_{-1.3}\right)^\circ$
$\sin^2 \times 10^1$ ($\pm 1\sigma$)	$3.02^{+0.13}_{-0.12}$	$0.227^{+0.023}_{-0.024}$	$4.13^{+0.37}_{-0.25} \oplus 5.94^{+0.21}_{-0.22}$
Team II [73] ($\pm 1\sigma$):	$\left(33.6^{+1.1}_{-1.0}\right)^\circ$	$\left(8.93^{+0.46}_{-0.48}\right)^\circ$	$\left(38.4^{+1.4}_{-1.2}\right)^\circ$
$\sin^2 \times 10^1$ ($\pm 1\sigma$)	$3.07^{+0.18}_{-0.16}$	$0.241^{+0.025}_{-0.025}$	$3.89^{+0.24}_{-0.20}$

Table 3.1: Current best fit values from Gonzalez et al. [72] (Team I) and from Fogli et al. [73] (Team II) for the mixing angles.

The absolute scale of neutrino masses is experimentally very hard to determine [74] and the current model–independent upper bound is given for the total mass of all neutrino generations. It is determined through cosmological constraints [75] to be

$$m_{\text{tot}} = \sum m_\nu < 0.23\,\text{eV}\ . \tag{3.9}$$

It is also unknown if the neutrino masses are hierarchical and, if so, what kind of hierarchy they have. The different spectra are

$$\text{degenerate:} \qquad m_1 \sim m_2 \sim m_3\ , \tag{3.10a}$$

$$\text{normal hierarchy:} \qquad m_1 < m_2 \ll m_3\ , \tag{3.10b}$$

$$\text{inverted hierarchy:} \qquad m_3 \ll m_1 < m_2\ . \tag{3.10c}$$

The lower bound for the neutrino masses is dependent on the assumed neutrino mass hierarchy, for normal hierarchy the lower bound is $m_{\text{tot}} > 0.06\,\text{eV}$, and for inverse hierarchy $m_{\text{tot}} > 0.1\,\text{eV}$ [72].

On the other hand, the mass differences between the three generations Δm_{ij} are experimentally accessible [76, 77] and their best fit value can be found together with the best fit value for the CP violating phase δ_{CP} in table 3.2. Note that for these quantities it is also important to distinguish between the cases of normal and inverted hierarchy.

	δ_{CP} (N) $\oplus$ (I)	$\Delta\, m^2_{21}$ [10^{-5} eV^2]	$\Delta\, m^2_{31}$ (N) $\oplus$ $\Delta\, m^2_{32}$ (I) [10^{-3} eV^2]
Team I [72] ($\pm 1\sigma$):	$\left(300^{+66}_{-138}\right)^\circ$	$7.50^{+0.18}_{-0.19}$	$2.473^{+0.070}_{-0.067} \oplus -2.427^{+0.042}_{-0.065}$
Team II [73] ($\pm 1\sigma$):	$\left(194^{+51}_{-55}\right)^\circ \oplus \left(196^{+69}_{-46}\right)^\circ$	$7.54^{+0.26}_{-0.22}$	$2.43^{+0.06}_{-0.10} \oplus -2.42^{+0.11}_{-0.07}$

Table 3.2: Current best fit values from Gonzalez et al. [72] (Team I) and from Fogli et al. [73] (Team II) for δ_{CP} as well as the mass differences for normal (N) and inverted (I) hierarchy.

Comparing the values of table 3.1 and table 3.2 to what we know from mixing in the quark sector [69–71], we can see that neutrino mixing is different, i.e. the mixing angles are large, and we discuss this in more detail in chapter 5 when we come to the flavor problem.

3.3 Origin and nature of neutrino masses

The origin and nature of neutrino masses remains an unsolved problem in the SM, since it predicts massless neutrinos. Therefore, an extension of the SM is necessary in order to explain neutrino masses. This can be done by either including new, higher–dimensional operators or by adding additional particles to the model, and we discuss examples of both ideas in the following.

There is still uncertainty regarding whether neutrinos are Dirac or Majorana mass particles, where the latter means that they are their own anti-particles. Unfortunately, experiments have trouble distinguishing between these two cases. One way to resolve this question is by searching for neutrinoless double–β decays, which are mediated by exchanging light Majorana neutrinos; however, the searches were so far unsuccessful [78, 79].

One proposal to extend the (MS)SM, in order to accommodate massive neutrinos, is the introduction of the so–called Weinberg operator $L\,H_u\,L\,H_u$, a higher–dimensional operator describing neutrinos as Majorana particles. Obviously, we would need to explain the origin of such a dimension–five operator as well as deal with the fact that this operator violates lepton number conservation.

Another extension of the (MS)SM to account for neutrino masses is to include additional gauge singlets, the right–handed neutrinos $\overline{\nu}_i$, in the particle spectrum. The neutrinos would be Dirac particles and their Yukawa coupling, for the MSSM, would be given by

$$\mathscr{W} \supset Y_\nu\, L\, H_u\, \overline{\nu}\ . \tag{3.11}$$

This is a particularly nice and simple proposal, but unfortunately it is also problematic since the Yukawa coupling seems to be quite unnatural. We know that the size of the neutrino

masses has to be in the eV or sub– eV, range but the Higgs VEV v_u is of the order of 100 GeV, and so the couplings in the neutrino Yukawa matrices would have to be very small,

$$Y_\nu^{ij} \sim \frac{m_\nu}{v_u} \sim 10^{-11} \,. \tag{3.12}$$

Therefore, this simple extension cannot be a viable option unless we can explain the size of this unnaturally small number.[1] Also, with this approach the neutrino hierarchies and mixing would be expected to be similar to the quark sector, which as we know is not the case.

However, so far, we have ignored that the Majorana mass term for the right–handed neutrinos is also an allowed operator when we introduce right–handed neutrinos. This leads us to another proposal explaining neutrino masses, the famous see–saw mechanism [26–29].[2] In this scenario, we have the neutrino Yukawa coupling and the Majorana mass term,

$$\mathscr{W} \supset Y_\nu \, L \, H_u \, \overline{\nu} + M_R \, \overline{\nu} \, \overline{\nu} \,, \tag{3.13}$$

where we suppress flavor indices. Here, M_R is the Majorana mass of the right–handed neutrinos and the entries in Y_ν are $\mathcal{O}(1)$, therefore, in contrast to equation (3.12).

In order to explain the see–saw mechanism, let us for a moment assume that we only have one left– and one right–handed neutrino, ν and $\overline{\nu}$, respectively. We can then rewrite the superpotential terms in order to include the full neutrino mass matrix $\mathcal{M}$,

$$\mathscr{W} \supset \Psi_\nu^T \, \mathcal{M} \, \Psi_\nu \quad \text{with} \quad \Psi_\nu = \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} \,, \tag{3.14}$$

with

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \,, \tag{3.15}$$

where m_D is the Dirac neutrino mass proportional to Y_ν. We can compute the eigenvalues of the matrix $\mathcal{M}$, and if we assume the Majorana mass for the right–handed neutrinos to be very heavy, i.e. $M_R \gg m_D$, we get the two eigenvalues $m_{1,2}$ to be

$$m_1 \sim \frac{m_D^2}{M_R} \quad \text{and} \quad m_2 \sim M_R \,. \tag{3.16}$$

We can see that by increasing M_R, pushing the right–handed neutrino out of the observable spectrum, we decrease the other mass eigenvalue.

In reality we know that we have three left–handed neutrinos and if we assume to have three right–handed neutrinos, we get for $\mathcal{M}$ a 6×6 matrix,

$$\mathcal{M} = \left(\begin{array}{c|c} 0 & m_D \\ \hline m_D & M_R \end{array}\right) \,, \tag{3.17}$$

[1] In section 4.2 we provide a set of symmetries justifying such a scenario by relating the Yukawa coupling to SUSY breaking.

[2] We focus here on the type I see–saw mechanism.

and each entry here is now a 3×3 matrix. The situation does not change compared to the one just discussed above, we end up with three light and three heavy eigenvalues with masses similar to equation (3.16). The effective light neutrino mass matrix M_ν can be computed via the see–saw formula

$$M_\nu \sim m_D \, M_R^{-1} \, m_D^T \, . \tag{3.18}$$

Again, we see that we relate the smallness of the observed neutrino masses to very heavy right–handed neutrinos. If we plug in the experimentally known values, we end up with a see–saw scale Λ_ν close to the GUT scale, which hints at a unification scenario.

The see–saw mechanism is a very good approach in order to explain the smallness of the left–handed neutrino masses while simultaneously explaining the non–observance of right–handed neutrinos due to their large mass. Furthermore, it is rather naturally implemented in SUSY SO(10) GUT theories, since a gauge singlet, i.e. a right–handed neutrino, is part of every **16** representation and the theory allows for the necessary Yukawa coupling and for a heavy Majorana mass term for the right–handed neutrinos.

Chapter 4

The μ term, R symmetries and neutrino masses

In this chapter we present a solution the μ problem based on R symmetries and we show how neutrino masses can be related to this. We explore several constraints for R symmetries that allow for Dirac and Majorana neutrino masses and we end the chapter with a classification of these symmetries.

4.1 Solving the μ problem

We saw in section 2.1 that the MSSM suffers from the so–called μ problem, i.e. the question why the size of the μ term is of the order of the electroweak scale. In this section we present a solution to this problem based on the Giudice–Masiero mechanism [80] and we also discuss how we can forbid the μ term with the help of R symmetries.

4.1.1 The Giudice–Masiero mechanism

A solution to the μ problem is given by the Giudice–Masiero mechanism [80]. In this mechanism the holomorphic Higgs bilinear mass term is created effectively in the Kähler potential

$$K \supset k_{H_u H_d} \frac{X^\dagger}{M_\mathrm{P}} H_u H_d + \text{h.c.} \,, \tag{4.1}$$

where $k_{H_u H_d}$ is a coefficient, M_P is the Planck scale and X is the SUSY breaking spurion field with $X = \theta\,\theta\,F_X$, cf. equation (2.19). The above Kähler potential gives rise to an effective superpotential if F_X acquires a VEV,

$$\mathscr{W}_\text{eff} \;\supset\; k_{H_u H_d} \frac{\langle F_X \rangle}{M_\mathrm{P}} H_u H_d \;=:\; \mu_\text{eff}\, H_u H_d \,. \tag{4.2}$$

Therefore, the Giudice–Masiero mechanism successfully explains the existence of an (effective) μ term, and for $\langle F_X \rangle \sim m_{3/2}\, M_\mathrm{P}$ it is of the order of the gravitino mass, i.e. $\mu_\text{eff} \sim m_{3/2}$.

There is a caveat to this idea. The solution proposed by Giudice–Masiero creates the μ term effectively, hence, it has to be absent in the first place for the mechanism to actually work. We want to forbid the μ term by a symmetry; however, the potential symmetries have several constraints. First of all, there are arguments that in any model of quantum gravity there are no global symmetries and that all symmetries are gauged, see e.g. [81] for discussion. Secondly, since the MSSM allows for precision gauge unification, any additional symmetry must not spoil unification and, therefore, should commute with SU(5). Finally, anomaly

freedom is also a necessary constraint for any symmetry since otherwise quantum gravity would introduce inconsistencies [82–85]. As we show in the following section, imposing the last two constraints requires that the symmetry is an R symmetry [86, 87] and since there are no continuous R symmetries in the MSSM [88], we focus our following discussion and classification on discrete, Abelian R symmetries of the order M, i.e. $\mathbb{Z}_M^R$ symmetries (cf. [41] for non–Abelian R symmetries).

4.1.2 Only R symmetries can forbid the μ term

As stated above we focus our discussion on $\mathbb{Z}_M^R$ symmetries and we also impose anomaly freedom as well as compatibility with SU(5) unification. Let us recall the $\mathbb{Z}_M^R$ anomaly coefficients of the MSSM, where we assign SU(5)–universal $\mathbb{Z}_M^R$ charges, $q_{\overline{\mathbf{5}}}$ for the $\overline{\mathbf{5}}$ multiplet $(\overline{D}^f, L^f)$ and $q_{\mathbf{10}}$ for the $\mathbf{10}$ multiplet $(Q^f, \overline{U}^f, \overline{E}^f)$, here $f = 1, 2, 3$ is the flavor index. With these assignments at hand the anomaly coefficients are given by [30, 87]

$$A_3^R := A_{\mathrm{SU}(3)_\mathrm{C}-\mathrm{SU}(3)_\mathrm{C}-\mathbb{Z}_M^R} = \frac{1}{2}\sum_{f=1}^{3}\left(3q_{\mathbf{10}}^f + q_{\overline{\mathbf{5}}}^f\right) - 3q_\theta \ , \tag{4.3a}$$

$$A_2^R := A_{\mathrm{SU}(2)_\mathrm{L}-\mathrm{SU}(2)_\mathrm{L}-\mathbb{Z}_M^R} = \frac{1}{2}\sum_{f=1}^{3}\left(3q_{\mathbf{10}}^f + q_{\overline{\mathbf{5}}}^f\right) + \frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 5q_\theta \ , \tag{4.3b}$$

$$A_1^R := A_{\mathrm{U}(1)_Y-\mathrm{U}(1)_Y-\mathbb{Z}_M^R} = \frac{1}{2}\sum_{f=1}^{3}\left(3q_{\mathbf{10}}^f + q_{\overline{\mathbf{5}}}^f\right) + \frac{3}{5}\left[\frac{1}{2}\left(q_{H_u} + q_{H_d}\right) - 11q_\theta\right] \ . \tag{4.3c}$$

Obviously, q_θ is the superspace charge, which does not necessarily have to be scaled to one [30].

Furthermore, we know that in order for the MSSM to unify at the GUT scale, we require anomalies to be universal and to cancel via the Green–Schwarz (GS) mechanism [89]

$$A_3^R = A_2^R = A_1^R = \rho \mod \eta \quad \text{with} \quad \eta := \begin{cases} M/2 \ , & \text{if } M \text{ even} \ , \\ M \ , & \text{if } M \text{ odd} \ . \end{cases} \tag{4.4}$$

Here, ρ is a constant which is proportional to the discrete shift of the GS axion. Hence, it is an indicator if the GS mechanism is at work since for $\rho = 0$ there is no axion shift and the symmetry is then anomaly–free without the use of the GS mechanism.

Another important constraint for the $\mathbb{Z}_M^R$ symmetries comes from the 't Hooft anomaly matching condition [6, 90], which is particularly interesting since we look at symmetries which are compatible with SU(5) unification. At the GUT level there is only the anomaly coefficient $A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R}$; however, by looking at the SU(5) subgroups $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ we can introduce the two anomaly coefficients $A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)_\mathrm{C}^2-\mathbb{Z}_M^R}$ and $A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)_\mathrm{L}^2-\mathbb{Z}_M^R}$, which at the GUT level fulfill the condition

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)_\mathrm{C}^2-\mathbb{Z}_M^R} = A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)_\mathrm{L}^2-\mathbb{Z}_M^R} \ . \tag{4.5}$$

The SU(5) anomaly coefficient and, therefore, the ones for $\mathrm{SU}(3)_\mathrm{C}$ and $\mathrm{SU}(2)_\mathrm{L}$ [30, 87] are

given by

$$A_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(5)^2-\mathbb{Z}_M^R} + 5q_\theta \ , \tag{4.6a}$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} + 5q_\theta \ , \tag{4.6b}$$

$$A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} + A^{\text{extra}}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} + 5q_\theta \ , \tag{4.6c}$$

where the last terms are the contributions from the gauginos, which for $\mathrm{SU}(3)_{\mathrm{C}}$ and $\mathrm{SU}(2)_{\mathrm{L}}$ can be split into two parts

$$\mathrm{SU}(3)_{\mathrm{C}}: \quad 5q_\theta = q_{\text{adj}}^{\mathrm{SU}(3)_{\mathrm{C}}} + q_{\text{GUT}}^{\mathrm{SU}(3)_{\mathrm{C}}} = 3q_\theta + \frac{1}{2}\cdot 2\cdot 2\cdot q_\theta \ , \tag{4.7a}$$

$$\mathrm{SU}(2)_{\mathrm{L}}: \quad 5q_\theta = q_{\text{adj}}^{\mathrm{SU}(2)_{\mathrm{L}}} + q_{\text{GUT}}^{\mathrm{SU}(2)_{\mathrm{L}}} = 2q_\theta + \frac{1}{2}\cdot 2\cdot 3\cdot q_\theta \ . \tag{4.7b}$$

The first contributions are from the adjoint representations of the respective group, whereas the last terms come from the extra gauginos of the GUT multiplets.

In order to retrieve the SM, we have to break the GUT symmetry while leaving the $\mathbb{Z}_M^R$ symmetry intact. This will remove the GUT contributions $q_{\text{GUT}}^{\mathrm{SU}(N)}$ from equations (4.7). After GUT breaking we have two new anomaly coefficients

$$A^{\text{SU(5) broken}}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} = A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} - q_{\text{GUT}}^{\mathrm{SU}(3)_{\mathrm{C}}} = A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} - 2q_\theta \ , \tag{4.8a}$$

$$A^{\text{SU(5) broken}}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} = A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} - q_{\text{GUT}}^{\mathrm{SU}(2)_{\mathrm{L}}} = A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} - 3q_\theta \ , \tag{4.8b}$$

which should be equal in order for the anomaly coefficients to be universal, due to 't Hooft anomaly matching condition. Since we assume for the matter content to be equal, i.e. $A^{\text{matter}}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} = A^{\text{matter}}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R}$, we get for their differences

$$\begin{aligned} A^{\text{SU(5) broken}}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} - A^{\text{SU(5) broken}}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} &= A^{\mathrm{SU}(5)}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} - A^{\mathrm{SU}(5)}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} + q_\theta \\ &= A^{\text{extra}}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} - A^{\text{extra}}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} + q_\theta \ , \\ &\overset{!}{=} 0 \mod \eta \end{aligned} \tag{4.9}$$

which can only be true if there are contributions of split multiplets in $A^{\text{extra}}_{\mathrm{SU}(N)^2-\mathbb{Z}_M^R}$. Therefore, as a direct consequence of 't Hooft anomaly matching we have split multiplets after GUT breaking, i.e. below the GUT scale.

Naturally, we now wonder where these split multiplets come from. Since SM matter fields come in complete SU(5) representations, the Higgs multiplets are obvious candidates to cancel the difference from equation (4.9), and using this gives us

$$\begin{aligned} A^{\text{extra}}_{\mathrm{SU}(2)^2_{\mathrm{L}}-\mathbb{Z}_M^R} - A^{\text{extra}}_{\mathrm{SU}(3)^2_{\mathrm{C}}-\mathbb{Z}_M^R} &= q_\theta \mod \eta \\ \frac{1}{2}\left(q_{H_u} + q_{H_d} - 2q_\theta\right) &= q_\theta \mod \eta \\ q_{H_u} + q_{H_d} &= 4q_\theta \mod 2\eta \\ q_{H_u} + q_{H_d} &= 2q_{\not W} \mod 2\eta \ . \end{aligned} \tag{4.10}$$

We can now easily see that for an R symmetry $\mu\, H_u\, H_d$ is not an allowed superpotential term, since $2q_{\mathscr{W}}$ does not equal $q_{\mathscr{W}}$ when $q_{\mathscr{W}} \neq 0$.[1] Therefore, non–R symmetries, which have $q_\theta = q_{\mathscr{W}} = 0$, cannot forbid the μ term.

Furthermore, we can constrain the order of our $\mathbb{Z}_M^R$ symmetry. The Giudice–Masiero mechanism creates an effective μ term of the order of the electroweak scale; however, this requires the term $H_u\, H_d$ to be neutral under the R symmetry, i.e. $q_{H_u} + q_{H_d} = 0$. Using this in equation (4.10), we get

$$2q_{\mathscr{W}} = 4q_\theta = 0 \mod M \,, \tag{4.11}$$

therefore, setting

$$q_\theta = \frac{M}{4} \,, \tag{4.12}$$

which means that the order M of our symmetry has to be divisible by 4.

4.2 The relation between R symmetries and neutrino masses

So far we have discussed how to solve the μ problem with the help of the Giudice–Masiero mechanism and anomaly–free, SU(5)–compatible $\mathbb{Z}_M^R$ symmetries. In this section we want to show that there is a set of symmetries with these properties, and the connection between such symmetries and neutrino masses. The only additional constraint so far was that the order M is divisible by 4 and we now discuss what else we have to impose for either Majorana or Dirac neutrinos.

4.2.1 Requirements for neutrino mass models

Before we distinguish between Dirac and Majorana neutrinos, we also demand that the symmetries allow for the usual Yukawa couplings. This puts further restrictions on the charges. First of all, in order for the up– and down–type Yukawa couplings to be allowed we require

$$Y_u: \quad 2q_{\mathbf{10}} + q_{H_u} = q_{\mathscr{W}} \mod M \,, \tag{4.13a}$$

$$Y_d: \quad q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} + q_{H_d} = q_{\mathscr{W}} \mod M \,, \tag{4.13b}$$

where equation (4.13b) is the same condition as for the charged lepton Yukawa Y_e. Putting those conditions together we get

$$3q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} + q_{H_u} + q_{H_d} = 2q_{\mathscr{W}} \mod M \,. \tag{4.14}$$

The Giudice–Masiero mechanism should be allowed, i.e. $q_{H_u} + q_{H_d} = 0$, and hence

$$3q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} = 2q_{\mathscr{W}} \mod M \;\neq\; q_{\mathscr{W}} \mod M \,. \tag{4.15}$$

Since we know from equation (4.12) that $2q_{\mathscr{W}} = 4q_\theta = M$, this sets the relation between $q_{\mathbf{10}}$ and $q_{\overline{\mathbf{5}}}$ to

$$q_{\overline{\mathbf{5}}} = -3q_{\mathbf{10}} \mod M \,. \tag{4.16}$$

[1] Obviously, this is only true for non–trivial R symmetries, i.e. $M \geq 3$.

Furthermore, equation (4.15) automatically forbids dangerous dimension–five operators $\mathbf{10\,10\,10\,\overline{5}}$. Dimension–four operators $\mathbf{10\,\overline{5}\,\overline{5}}$ are also immediately absent from the superpotential since

$$q_{\mathbf{10}} + 2q_{\overline{\mathbf{5}}} = -5q_{\mathbf{10}} \mod M \,, \tag{4.17}$$

describes the R charge of such an operator, which is forbidden if $-5q_{\mathbf{10}} \neq q_{\mathscr{W}}$. Therefore, we see that demanding for the usual Yukawa couplings to be present automatically forbids problematic higher dimensional operators.

Now, if our $\mathbb{Z}_M^R$ symmetries meet all the above constraints, distinguishing between Dirac and Majorana neutrinos mainly comes down to either allowing or forbidding the existence of the Weinberg operator. However, for Dirac neutrinos we also have to think about the Dirac neutrino Yukawa coupling, which we will do in the next section.

4.2.2 Dirac neutrinos

We now take a closer look at Dirac neutrinos. This means that the $\mathbb{Z}_M^R$ symmetries have to forbid the existence of the Weinberg operator $L\,H_u\,L\,H_u$ and we want the operator $L\,H_u\,\overline{\nu}$ to be neutral under the $\mathbb{Z}_M^R$, i.e.

$$q_{\overline{\nu}} = -q_{H_u} - q_L \mod M \,, \tag{4.18}$$

where $\overline{\nu}$ is the right–handed neutrino. We demand the latter in order to create an effective neutrino mass term from the Kähler potential, analogously to the Giudice–Masiero mechanism. This is easily achieved since there are no further constraints on $\overline{\nu}$ and we can adjust its value freely [91–93]. If $L\,H_u\,\overline{\nu}$ is neutral, we can have

$$K \supset k_{LH_u\overline{\nu}}\,\frac{X^\dagger}{M_{\mathrm{P}}^2}\,L\,H_u\,\overline{\nu} + \text{h.c.} \tag{4.19}$$

in the Kähler potential, where $k_{LH_u\overline{\nu}}$ is a dimensionless coefficient. Recall from equation (4.1) that X is the spurion field that breaks SUSY with $\langle F_X\rangle \sim m_{3/2}\,M_{\mathrm{P}}$ and, therefore, we have an effective Dirac neutrino Yukawa coupling

$$\mathscr{W}_{\text{eff}} \;\supset\; k_{LH_u\overline{\nu}}\,\frac{\langle F_X\rangle}{M_{\mathrm{P}}^2}\,L\,H_u\,\overline{\nu} \;\sim\; k_{LH_u\overline{\nu}}\,\frac{m_{3/2}}{M_{\mathrm{P}}}\,L\,H_u\,\overline{\nu}\,, \tag{4.20}$$

which provides a direct connection between the size of the μ term, $\mu \sim m_{3/2}$, and the smallness of neutrino masses through their effective Yukawa coupling

$$Y_\nu \;\sim\; \frac{m_{3/2}}{M_{\mathrm{P}}} \;\sim\; \frac{\mu}{M_{\mathrm{P}}}\,. \tag{4.21}$$

This condition leads to realistic neutrino masses if we assume $m_{3/2}$ to be in the TeV range.

We have collected all conditions on our $\mathbb{Z}_M^R$ symmetries and we have established a link between the size of the μ term and Dirac neutrino masses. We can look for symmetries fulfilling all these conditions and our results, up to order $M = 36$, are summarized in table 4.1. All these 15 symmetries are inequivalent, which has been checked by computing their Hilbert basis from section 2.3, i.e. if two symmetries predict the same Hilbert basis, they are equivalent.

M	$q_{\mathbf{10}}$	$q_{\overline{\mathbf{5}}}$	q_{H_u}	q_{H_d}	q_θ	ρ	$q_{\overline{\nu}}$
4	0	0	2	2	1	1	2
4	2	2	2	2	1	1	0
8	1	5	2	6	2	2	1
12	1	9	4	8	3	3	11
12	2	6	2	10	3	3	4
12	4	0	10	2	3	3	2
16	1	13	6	10	4	4	13
24	1	21	10	14	6	6	17
28	1	25	12	16	7	7	19
28	2	22	10	18	7	7	24
28	4	16	6	22	7	7	6
32	1	29	14	18	8	8	21
36	1	33	16	20	9	9	23
36	2	30	14	22	9	9	28
36	4	24	10	26	9	9	2

Table 4.1: Anomaly–free, SU(5)–compatible and discrete $\mathbb{Z}_M^R$ symmetries that forbid the μ term and the Weinberg Operator while allowing the Giudice–Masiero mechanism and small Dirac neutrino masses. We checked up to order $M = 36$.

Let us now discuss some explicit example models. We start with the $\mathbb{Z}_8^R$ symmetry since there is only one of this order. The Hilbert superpotential basis for this example is given by its inhomogeneous monomials (with R charge $2q_\theta$)

$$\overline{\nu}^4 \ ; \ \left(L\,L\,\overline{E}\right)\,\overline{\nu} \ ; \ L\,H_d\,\overline{E} \ ; \ \left(L\,L\,\overline{E}\right)^4 \ ; \ \left(L\,L\,\overline{E}\right)^2\,(L\,H_u)^2 \ ; \ (L\,H_u)^4 \ , \tag{4.22}$$

while the homogeneous monomials (with zero R charge) are

$$\begin{aligned}
&\overline{\nu}^8 \ ; \ L\,H_u\,\overline{\nu} \ ; \ (L\,H_u)^8 \ ; \ \left(L\,L\,\overline{E}\right)^5\,\overline{\nu} \ ; \ \left(L\,L\,\overline{E}\right)^4\,\left(L\,H_d\,\overline{E}\right) \ ; \\
&H_u\,H_d \ ; \ \left(L\,L\,\overline{E}\right)\,\overline{\nu}^5 \ ; \ \left(L\,H_d\,\overline{E}\right)\,\overline{\nu}^4 \ ; \ \left(L\,L\,\overline{E}\right)^2\,\left(L\,H_d\,\overline{E}\right)\,(L\,H_u)^2 \ ; \\
&\left(L\,L\,\overline{E}\right)^8 \ ; \ \left(L\,H_d\,\overline{E}\right)^2 \ ; \ \left(L\,L\,\overline{E}\right)\,\left(L\,H_d\,\overline{E}\right)\,\overline{\nu} \ ; \ \left(L\,L\,\overline{E}\right)^2\,\overline{\nu}^2 \ ; \\
&\left(L\,L\,\overline{E}\right)^3\,(L\,H_u) \ ; \ \left(L\,H_d\,\overline{E}\right)\,(L\,H_u)^4 \ ; \ \left(L\,L\,\overline{E}\right)\,(L\,H_u)^3 \ .
\end{aligned} \tag{4.23}$$

Here, we omitted all quark operators for brevity. Recall from section 2.1.5 that any superpotential term has to have R charge $2q_\theta = q_{\not W}$, therefore, they either consist of an inhomogeneous term or of an inhomogeneous term times one or several homogeneous terms. Hence, this model forbids the μ term at the perturbative level while allowing for an effective μ term and an effective Dirac neutrino Yukawa since $L\,H_u\,\overline{\nu}$ and $H_u\,H_d$ are neutral. Furthermore, the model forbids the Weinberg operator but the usual Yukawa couplings are present,

i.e. $L\,H_d\,\overline{E}$ has the correct R charge.[2] This model allows us to write down the following terms in the Kähler potential,

$$K \supset X^\dagger \left(\frac{k_{H_u H_d}}{M_\mathrm{P}} H_u\,H_d + \frac{k_{LH_u\overline{\nu}}}{M_\mathrm{P}^2}\,L\,H_u\,\overline{\nu} + \frac{k_{QQQL}}{M_\mathrm{P}^3}\,Q\,Q\,Q\,L \right) + \text{h.c.}\,. \tag{4.24}$$

Assigning a VEV to the spurion field X, hence, breaking SUSY and the R symmetry, creates an effective superpotential of the form

$$\mathscr{W}_\mathrm{eff} \;\sim\; m_{3/2}\,H_u\,H_d + \frac{m_{3/2}}{M_\mathrm{P}}\,L\,H_u\,\overline{\nu} + \frac{m_{3/2}}{M_\mathrm{P}^2}\,Q\,Q\,Q\,L\;, \tag{4.25}$$

where we find the desired effective μ term and Dirac neutrino Yukawa coupling of the right order, cf. equation (4.21). We also see that an effective dimension–five operator $Q\,Q\,Q\,L$ has been created, which is in agreement with equation (4.16) since $\mathbf{10\,10\,10\,\overline{5}}$ operators have zero R charge. However, this term is heavily suppressed and, therefore, well within experimental bounds for proton decay. Let us further note that assigning a VEV to the spurion breaks the $\mathbb{Z}_8^R$ symmetry down to a non–R $\mathbb{Z}_4$ symmetry, with the charge assignment

$$\begin{pmatrix} q_{\mathbf{10}} & q_{\overline{\mathbf{5}}} & q_{H_u} & q_{H_d} & q_{\overline{\nu}} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 \end{pmatrix}\,. \tag{4.26}$$

This symmetry gets broken down further when the Higgs scalars attain their VEVs and we end up with the familiar matter parity.

Other examples of the Hilbert basis for the $\mathbb{Z}_{12}^R$ symmetries can be found in appendix A. Now, we discuss some problems of the Dirac neutrino models with the help of the $\mathbb{Z}_4^R$ examples.

4.2.3 Problems of models with Dirac neutrinos

Let us investigate two more examples from table 4.1, the two $\mathbb{Z}_4^R$ symmetries. For the first $\mathbb{Z}_4^R$, the inhomogeneous monomials are given by

$$\overline{\nu}\;;\; L\,H_u\;;\; L\,H_d\,\overline{E}\;\;, \tag{4.27}$$

and the homogeneous monomials contain $\overline{\nu}^2$. Here, we again omit terms with quarks. The basis for the second $\mathbb{Z}_4^R$ is similar, its inhomogeneous monomials are

$$L\,H_u\;;\; L\,H_d\,\overline{E}\;\;, \tag{4.28}$$

and this time the homogeneous monomials contain $\overline{\nu}$.

These configurations are somewhat troublesome since they violate matter parity, e.g. $L\,H_u$. Furthermore, both symmetries allow to create a non–perturbative right–handed neutrino mass term, i.e. $\overline{\nu}^2$, which would be of the size of the μ term.[3] However, we can easily solve these problems by imposing the $\mathbb{Z}_4$ from equation (4.26) on both models. After this additional symmetry is imposed, both $\mathbb{Z}_4^R \times \mathbb{Z}_4$ symmetries have the same Hilbert basis, i.e. they are equivalent. For this configuration the inhomogeneous monomials are given by

$$L\,H_d\,\overline{E}\;;\; \left(L\,L\,\overline{E}\right)\,\overline{\nu}\;;\; \left(L\,L\,\overline{E}\right)\,(L\,H_u)^3\;;\; \left(L\,L\,\overline{E}\right)^3\,(L\,H_u)\;, \tag{4.29}$$

[2] As well as $Q\,\overline{U}\,H_u$ and $Q\,\overline{D}\,H_d$ which are not displayed here.

[3] Which can be interesting if we want a see–saw model with masses around the TeV scale.

and the homogeneous ones are

$$\begin{aligned}
&\overline{\nu}^4 \ ; \ H_u\, H_d \ ; \ (L\, H_u)\, \overline{\nu} \ ; \ (L\, H_u)^4 \ ; \ \left(L\, H_d\, \overline{E}\right)\, (L\, H_u)\, \left(L\, L\, \overline{E}\right)^3 \ ; \\
&\left(L\, H_d\, \overline{E}\right)^2 \ ; \ \left(L\, L\, \overline{E}\right)\, \left(L\, H_d\, \overline{E}\right)\, \overline{\nu} \ ; \ \left(L\, L\, \overline{E}\right)^2\, \overline{\nu}^2 \ ; \\
&\left(L\, L\, \overline{E}\right)^2\, (L\, H_u)^2 \ ; \ \left(L\, H_d\, \overline{E}\right)\, (L\, H_u)^3\, \left(L\, L\, \overline{E}\right) \ ; \ \left(L\, L\, \overline{E}\right)^4 \ .
\end{aligned} \tag{4.30}$$

This is a much more satisfactory setting since the matter parity violating terms are again forbidden while maintaining the desirable features.

Unfortunately, there is a more problematic feature of all our $\mathbb{Z}_M^R$ models. So far, we have only considered superpotential terms that do not include quarks. However, if we take a look at the second term in equation (4.22) and in equation (4.29), we see the operator $L\, L\, \overline{E}\, \overline{\nu}$. By itself, this operator is not alarming, but we should notice that in GUT language this is a $\overline{\mathbf{5}}\, \overline{\mathbf{5}}\, \mathbf{10}$ operator contracted with $\overline{\nu}$ and any other such operator is allowed, e.g. $\overline{U}\, \overline{D}\, \overline{D}\, \overline{\nu}$. Furthermore, this operator is always present, since we want $L\, H_u\, \overline{\nu}$ to have zero R charge, cf. equation (4.18),

$$q_{\overline{\nu}} + q_{H_u} + q_{\overline{\mathbf{5}}} = 0 \mod M \ , \tag{4.31}$$

and putting this together with the conditions that we have in order to allow for down–type Yukawa couplings, equation (4.13b),

$$q_{\mathbf{10}} + q_{\overline{\mathbf{5}}} + q_{H_d} = q_{\mathscr{W}} \mod M \ , \tag{4.32}$$

we get

$$q_{\mathbf{10}} + 2q_{\overline{\mathbf{5}}} + q_{H_u} + q_{H_d} + q_{\overline{\nu}} = q_{\mathscr{W}} \mod M \ . \tag{4.33}$$

Since the Giudice–Masiero mechanism requires that the Higgs bilinear is neutral under the R symmetry, i.e. $q_{H_u} + q_{H_d} = 0$, we get

$$q_{\mathbf{10}} + 2q_{\overline{\mathbf{5}}} + q_{\overline{\nu}} = q_{\mathscr{W}} \mod M \ , \tag{4.34}$$

therefore, $\overline{U}\, \overline{D}\, \overline{D}\, \overline{\nu}$ is always allowed. This operator causes problems, in particular, when we look at the first two field generations since the superpotential term

$$\mathscr{W} \supset \frac{k_{\overline{U}\, \overline{D}\, \overline{D}\, \overline{\nu}}}{M_{\mathrm{P}}}\, \overline{U}_1\, \overline{D}_1\, \overline{D}_2\, \overline{\nu} \ , \tag{4.35}$$

mediates proton decay via gluino exchange. This means that the process $p \to K^+ + \overline{\nu}$ would be observed; which has not been the case so far.

One way to potentially circumvent this problem is to assume that SUSY breaking follows mirage mediation [94–96], where heavy squarks can naturally occur and suppress this process sufficiently.

Another option is to change the setting in such a way that the discrete $\mathbb{Z}_M^R$ symmetries originate from a pseudo–anomalous $\mathrm{U}(1)_R$ symmetry. In this scenario we can still have sufficiently small Dirac neutrino masses, but the potentially dangerous proton decay operator $\overline{U}\, \overline{D}\, \overline{D}\, \overline{\nu}$ can be forbidden by the holomorphicity of the superpotential. We present such symmetries later in section 5.2 and an explicit example in section 7.3.2.

Let us also note that there are other symmetries that forbid proton decay operators in the context of Dirac neutrinos that do not suffer from this problem [97].

4.2.4 Majorana neutrinos and a unique $\mathbb{Z}_4^R$ symmetry

After addressing constraints for Dirac neutrinos in the previous section we now take a look at $\mathbb{Z}_M^R$ symmetries allowing Majorana neutrinos. We require the conditions from section 4.2.1 to be fulfilled as well as the Weinberg operator to be present in the superpotential,

$$2q_{\overline{\mathbf{5}}} + 2q_{H_u} = 2q_\theta \mod M \,. \tag{4.36}$$

Recall that we have a condition from equation (4.13a) for the up–type Yukawa,

$$\begin{aligned} 2q_{\mathbf{10}} &= 2q_\theta - q_{H_u} \mod M \\ &= 2q_{\overline{\mathbf{5}}} + q_{H_u} \mod M \,, \end{aligned} \tag{4.37}$$

where the last line is due to equation (4.36). We also want the down–type Yukawa, equation (4.13b), to be allowed,

$$\begin{aligned} q_{\mathbf{10}} &= 2q_\theta - q_{\overline{\mathbf{5}}} - q_{H_d} \mod M \\ &= q_{\overline{\mathbf{5}}} + 2q_{H_u} - q_{H_d} \mod M \,, \end{aligned} \tag{4.38}$$

where again the last line is due to equation (4.36). Putting these two conditions together we get

$$3q_{H_u} - 2q_{H_d} = 0 \mod M \,. \tag{4.39}$$

We have $q_{H_u} = -q_{H_d}$ since we require the Giudice–Masiero mechanism to be allowed, and we end up with $5q_{H_u} = 0 \mod M$, implying that $q_{H_u} = 0$. Therefore, we get the charges to be

$$q_{H_u} = q_{H_d} = 0 \mod M \,, \tag{4.40a}$$

$$q_{\mathbf{10}} = q_{\overline{\mathbf{5}}} = q_\theta \mod M \,. \tag{4.40b}$$

This follows directly from equation (4.37) and equation (4.38).

Given the charge relations from equation (4.40b) we see that this symmetry commutes with SO(10) and we could define the charge of the **16** multiplet to be $q_{\mathbf{16}} = q_{\mathbf{10}} = q_{\overline{\mathbf{5}}}$. As we know from the end of section 4.1.2, the order M of our $\mathbb{Z}_M^R$ symmetry has to be divisible by four, so the first possible symmetry admitting Majorana neutrinos is a $\mathbb{Z}_4^R$ symmetry, previously discussed in [40, 98], and the matter charges can be found in table 4.2.

	H_u	H_d	$\mathbf{10}$	$\overline{\mathbf{5}}$	θ
$\mathbb{Z}_4^R$	0	0	1	1	1

Table 4.2: $\mathbb{Z}_4^R$ charges for the MSSM fields in SU(5) notation. The superpotential has R charge 2 since the superspace coordinate θ has charge 1.

It has been shown [40] that this symmetry is the only one fulfilling the conditions from section 4.2.1, i.e. being anomaly–free, μ term and proton decay forbidding, while being consistent with SO(10) unification, and that any other symmetry with $M \neq 4$ is just a trivial extension of this $\mathbb{Z}_4^R$ symmetry [87]. Note that this claim is independent of the choice of q_θ [30].

In the previous two sections we showed that there is a class of anomaly–free, SU(5)–compatible, Abelian $\mathbb{Z}_M^R$ symmetries, which connect the size of the μ term directly to the smallness of Dirac neutrino masses. However, in this section we showed that for the case of Majorana neutrinos, there is a unique symmetry with all the desirable features, and which even commutes with SO(10). In the following chapter we continue by taking a closer look at models which describe Dirac and Majorana neutrinos and how we might make phenomenological predictions, especially for neutrino mixing, by introducing a so–called flavor symmetry.

Chapter 5

Flavor model building

In this chapter we discuss the general idea of flavor model building. We first remind the reader of problems regarding flavor in the SM and the MSSM, e.g. the occurrence of flavor changing neutral currents (FCNCs), and we introduce flavor symmetries, which help us deal with this issue. We then examine models with spontaneously broken flavor symmetries. First, we discuss models based on a $\mathrm{U}(1)_R$ symmetry, which explains the fermion hierarchies in the quark and charged lepton sector while also predicting a viable CKM matrix. Second, we discuss non–Abelian flavor symmetries, which in our examples deal with the important issue of neutrino mixing; in particular, we discuss models that predict a tri–bi–maximal (TBM) neutrino mixing pattern.

5.1 The flavor problem and flavor symmetries

The standard model, as well as its supersymmetric extension, suffers from several problems regarding flavor [42, 99]. Foremost, the nature and origin of flavor is not understood, which introduces a large amount of free parameters in our theory, e.g. a softly broken MSSM has order 100 undetermined parameters [34, 42] and these are at first unrelated, cf. section 2.1.6. Furthermore, several differences between the lepton and the quark sector seem to be rather arbitrary. Taking a look at the absolute mass scales of quarks and charged leptons, e.g. of the electron and the top quark, we see that the electron mass $m_e = 511\,\mathrm{keV}$ is much smaller than the top mass $m_t = 173\,\mathrm{GeV}$. The difference gets even more prominent when we consider neutrino masses since we saw in section 3.1 that the sum of the neutrino masses is between $0.02\,\mathrm{eV} \lesssim m_{\mathrm{tot}} \lesssim 0.6\,\mathrm{eV} \ll m_t$. The smallness of neutrino masses can be either explained via the see–saw mechanism, cf. section 3.3, or, for Dirac neutrinos, can be related to R symmetry breaking effects and the size of the μ term, as we showed previously in section 4.2. The question of why there is such a huge difference between lepton and quark masses is, however, so far unresolved.

We know from experimental observation that neutrinos have mass and, therefore, they mix. Currently, we even enter the precision measurement phase of the mixing angles, which turn out to be rather large, cf. section 3.2,

$$9^\circ \lesssim \theta_{ij}^\nu \lesssim 38^\circ \,, \tag{5.1}$$

where θ_{ij}^ν is a neutrino mixing angle. Comparing these to the quark mixing angles θ_{ij}^q, which we get from the CKM matrix [69–71],

$$0.2^\circ \lesssim \theta_{ij}^q \lesssim 13.0^\circ \,, \tag{5.2}$$

we see that there is almost no mixing in the quark sector, whereas there is large mixing for neutrinos. We can compare the structure of the mixing matrices, V_{CKM} and U_{PMNS}

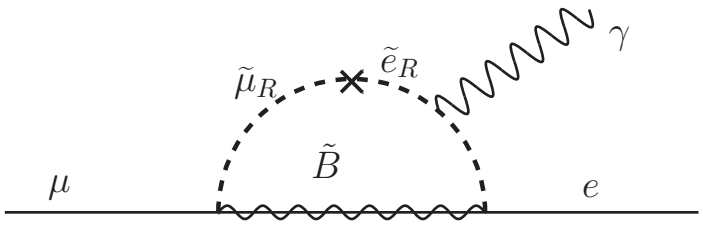


Figure 5.1: Contribution of $(\widetilde{m}^2_{RR})_{21}$ via bino exchange to $\mu \to e\,\gamma$, where the mass insertion is indicated by $\times$.

respectively, by displaying the approximate absolute values of their entries,

$$V_{\text{CKM}} \sim \begin{pmatrix} \approx 1 & 0.2 & 0.001 \\ 0.2 & \approx 1 & 0.01 \\ 0.001 & 0.01 & \approx 1 \end{pmatrix}, \qquad U_{\text{PMNS}} \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}, \tag{5.3}$$

where we can see that the CKM matrix is basically the unit matrix plus small off–diagonal corrections, whereas the PMNS matrix is far from this situation. The presented values of the mixing angles and matrix entries are, at first glance, completely arbitrary and it is not understood why either situation should by realized in Nature. In particular, there is no apparent reason why the mixing in both sectors should be so fundamentally different.

In supersymmetry yet another issue arises due to our lack of understanding for flavor: the possibility of FCNCs. General soft terms, as we have seen in section 2.1.6, are, in principle, new sources of flavor changing neutral currents. Take, for example, a soft mass term for the right–handed sleptons $\widetilde{r}$,

$$\mathcal{L}_{\text{soft}} \supset -\widetilde{r}^\dagger\, \widetilde{m}^2_{RR}\, \widetilde{r}\,, \tag{5.4}$$

where there is no reason that $\widetilde{m}^2_{RR}$ should be diagonal. However, an off–diagonal mass term that mixes the different sleptons, e.g. $(\widetilde{m}^2_{RR})_{21}\, \widetilde{\mu}^*_R\, \widetilde{e}_R$, contributes to flavor–violating processes. Here, the contribution would be to the decay $\mu \to e\,\gamma$, see figure 5.1. Furthermore, there are more terms in the soft breaking Lagrangian which promote flavor–violation, e.g.

$$\mathcal{L}_{\text{soft}} \supset -\widetilde{\ell}^\dagger\, \widetilde{m}^2_{LL}\, \widetilde{\ell} - \widetilde{\ell}\, A_{LR}\, \widetilde{r}\,, \tag{5.5}$$

which contribute to $\mu \to e\,\gamma$ via wino or bino loops if $\widetilde{m}^2_{LL}$ or A_{LR} are non–diagonal. The process $\mu \to e\,\gamma$, however, is unobserved and the experimental bounds are quite stringent, i.e. $\text{Br}(\mu \to e\,\gamma)_{\text{exp}} < 2.4 \times 10^{-12}$ from [71] or the more recent, and even more stringent bound by the MEG collaboration $\text{Br}(\mu \to e\,\gamma)_{\text{exp}} < 5.7 \times 10^{-13}$ [100]. This means that the off–diagonal elements of the soft mass matrices have to be sufficiently small or even absent, but unfortunately, there is no intrinsic reason why this should be the case.

So far, we have addressed many problems regarding flavor. Several of these can be (partially) solved by introducing so–called flavor symmetries, which we discuss in the next section.

5.1.1 Flavor symmetries

Introducing flavor symmetries in the SM or MSSM may help us deal with several of the above mentioned problems and also constrain the large amount of undetermined parameters.

These flavor symmetries may be thought of as the source of the observed patterns of fermion mixing and masses. The canonical approach is to introduce a flavor symmetry G_{F} at a high scale Λ and then break the symmetry spontaneously by assigning VEVs to so–called 'flavon' fields, which are SM singlets in non–trivial representations of the flavor symmetry. In this way we create inter–family relations by connecting the Yukawa couplings to each other, therefore, significantly reducing the amount of free parameters in the theory. Note that an additional GUT symmetry would further reduce the amount of free parameters by relating quarks to leptons.

It is instructive to demonstrate the procedure with the help of a modified version of the familiar Froggatt–Nielsen (FN) mechanism [101]. Let us assume we extend the MSSM with a flavor symmetry $\mathrm{U}(1)_{\mathrm{F}}$; therefore, our gauge group is given by $[\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)]_{\mathrm{SM}} \times \mathrm{U}(1)_{\mathrm{F}}$. We also add some flavon Φ, which is a SM singlet but carries charge under $\mathrm{U}(1)_{\mathrm{F}}$. Furthermore, we assign $\mathrm{U}(1)_F$ charges to the MSSM fields in such a way that the Yukawa couplings are forbidden, i.e. the operators $\Psi_f \, \Psi_g \, \Psi_h$ have a non–zero $\mathrm{U}(1)_{\mathrm{F}}$ charge, where $\Psi_{f,g,h}$ describe matter superfields with the flavor indices f, g, h. However, by assigning appropriate charges to the different fields and picking the right power n_{fgh} of the flavon field, the term

$$\mathscr{W} \supset y^{fgh} \left(\frac{\Phi}{\Lambda}\right)^{n_{fgh}} \Psi_f \, \Psi_g \, \Psi_h \, , \tag{5.6}$$

where y_{fgh} is a dimensionless coupling and Λ the flavor scale, can be made to have vanishing charge under $\mathrm{U}(1)_{\mathrm{F}}$, therefore being a valid superpotential term. The flavor symmetry is unbroken above the scale Λ, which we assume to be around the Planck scale M_{P}. If we now break the flavor symmetry by assigning a VEV to the flavon Φ, we create an effective superpotential of the form

$$\mathscr{W}_{\mathrm{eff}} \supset Y^{fgh} \, \Psi_f \, \Psi_g \, \Psi_h \tag{5.7}$$

with the Yukawa couplings given by

$$Y^{fgh} := y^{fgh} \left(\frac{\langle\Phi\rangle}{\Lambda}\right)^{n_{fgh}} \, . \tag{5.8}$$

This rather simple example illuminates several nice features of flavor symmetries. Usually,

$$\varepsilon := \frac{\langle\Phi\rangle}{\Lambda} \, , \tag{5.9}$$

is called the expansion parameter and it is now related to symmetry breaking effects. Assuming that $\varepsilon < 1$, usually one picks $0.1 \lesssim \varepsilon \lesssim 0.2$, it becomes clear that we can create a mass hierarchy between the different members of a family by choosing appropriate powers n_{fgh}, i.e.

$$n_{fgh} \; = \; -\frac{(R_{\Psi_f} + R_{\Psi_g} + R_{\Psi_h})}{R_\Phi} \, , \tag{5.10}$$

and the appropriate charges of the matter superfields.[1] This mechanism and variants of it have been known for a long time, e.g. if there is only one flavon field, as shown so far, we have the famous Froggatt–Nielsen mechanism [101].

[1] In a somewhat obvious notation R_{Ψ_f} is the $\mathrm{U}(1)_F$ charge of the field Ψ_f and similar for other fields.

Furthermore, we see that by choosing a more sophisticated setting of flavons we can try to explain the flavor mixing pattern by the dynamics of the underlying symmetry, and, for example, we can choose more than one flavon in order to split the flavor group after symmetry breaking into different sectors. If we assume in addition a more sophisticated, e.g. a non–Abelian discrete, flavor symmetry, these sectors might experience residual symmetries after the breaking of the flavor symmetry and the predictions of such models can be inferred from these sector symmetries,[2] therefore being experimentally testable. The only requirement is that we pick a large enough flavor group at the high scale, which we then break down to subgroups in the different sectors via flavon VEV assignment. This can be especially rich in structure if the flavons themselves are in non–trivial irreducible representations of the flavor group because their VEVs might be non–trivially aligned. We demonstrate this mechanism with the help of an A_4 example in section 5.3 where we need three additional flavons in order to create two sectors with different residual symmetries and a rich phenomenology.

In addition to the just discussed $U(1)_F$ symmetry, there is a variety of other possible flavor groups and several models have been constructed. Some of these models are based on other continuous symmetries like SU(2) or SU(3) and, more recently, many models with discrete symmetries have emerged, most of them utilizing discrete non–Abelian symmetries like S_3, S_4, A_4, T′, $\Delta(27)$ and many more. For a review consult [102] and references therein. These models lead to several different kinds of mixing patterns. Models with bi–maximal mixing pattern [103, 104] and tri–bi–maximal mixing pattern [105, 106] have been favored in the past when their results were within experimental limits. Also, models with non–Abelian R symmetries have been recently discussed for the possibility of flavor model building [41].

In the following section we discuss FN models based on a $U(1)_R$ flavor symmetry and afterwards we focus on models with an A_4 or T′ flavor symmetry. Specifically, we examine A_4 which exhibits a tri–bi–maximal mixing pattern in the neutrino sector, i.e. predicting a vanishing θ_{13}. Given the measurement of a rather large θ_{13}, TBM models are currently becoming less popular; however, with the help of an A_4 example, we want to show later in chapter 6 that it might be too early to give up on certain flavor models.

Before we finish our discussion of flavor symmetries and continue with explicit flavor models, we show why flavor symmetries help with dangerous FCNC inducing terms, which we mentioned earlier in section 5.1. Consider the soft SUSY breaking operator

$$\mathcal{L}_{\text{soft}} \supset -\widetilde{\ell}^\dagger \, \widetilde{m}^2_{LL} \, \widetilde{\ell} \, , \tag{5.11}$$

which originates from the Kähler potential term

$$K \supset -\frac{1}{\Lambda^2_{\text{soft}}} k^{fg} \, X^\dagger \, X \, L^\dagger_f \, L_g \, , \tag{5.12}$$

where Λ_{soft} is the soft breaking scale, X is the SUSY breaking spurion and k^{fg} is a matrix in flavor space, which does not have to be diagonal. However, if we assume that L is in an irreducible representation $\boldsymbol{r}$ of some flavor group G_F we get constraints on k^{fg}. Obviously, $k^{fg} \, L^\dagger_f \, L_g$ has to be invariant under a transformation of the flavor symmetry. Written with matrices this means

$$L^\dagger \, k \, L = (\boldsymbol{D}_r \, L)^\dagger \; k \; (\boldsymbol{D}_r \, L) = L^\dagger \left(\boldsymbol{D}_r^\dagger \, k \, \boldsymbol{D}_r \right) L \, , \tag{5.13}$$

[2] We call them sector symmetries since they do not have to be symmetries of the Lagrangian.

where $\boldsymbol{D}_r$ is the representation matrix of $\boldsymbol{r}$. Therefore, we require

$$\boldsymbol{D}_r^\dagger \, k \, \boldsymbol{D}_r \stackrel{!}{=} k \, , \tag{5.14}$$

and by Schur's lemma this can only be fulfilled if $k \propto \mathbb{1}$, therefore, $\widetilde{m}^2_{LL}$ is diagonal. We get the same result for similar terms, e.g. $\widetilde{m}^2_{RR}$ in

$$\mathcal{L}_{\text{soft}} \supset -\widetilde{r}^\dagger \, \widetilde{m}^2_{RR} \, \widetilde{r} \, , \tag{5.15}$$

can only have $\widetilde{m}^2_{RR} \propto \mathbb{1}$ if the $\widetilde{r}_i$ are in an irreducible representation of the flavor group. Diagonal soft breaking terms imply non–existing mixing angles in family space which means that different generation sleptons do not mix, therefore, removing FCNCs at leading order.

However, there is still mixing induced by the so–called A terms, e.g. A_{LR} from

$$\mathcal{L}_{\text{soft}} \supset -\widetilde{\ell} \, A_{LR} \, \widetilde{r} \, . \tag{5.16}$$

We usually assume that the couplings A_{LR} are of similar size as the corresponding Yukawa matrices, i.e.

$$A_{LR} = a_{LR} \, Y_e \, , \tag{5.17}$$

where a_{LR} is some $\mathcal{O}(1)$ coefficient. This only allows for the third families to have large couplings, which is due to their very large mass within experimental limits. This is obviously an assumption; however, it seems reasonable that there should not be a large deviation between the Yukawa matrices and the corresponding A terms, hence we do not consider this assumption to be unnatural. Flavor symmetries can, therefore, forbid dangerous FCNCs at leading order.

5.1.2 Alternatives to flavor symmetries

Before we discuss explicit flavor models in the next section we review briefly some alternatives to flavor symmetries. Consider again the problem of FCNCs in supersymmetry. General soft breaking terms as we have seen them in section 2.1.6 allow for FCNCs since the soft mass matrices, e.g. $\widetilde{m}^2_{RR}$ or $\widetilde{m}^2_{LL}$ from

$$\mathcal{L}_{\text{soft}} \supset -\widetilde{r}^\dagger \, \widetilde{m}^2_{RR} \, \widetilde{r} - \widetilde{\ell}^\dagger \, \widetilde{m}^2_{LL} \, \widetilde{\ell} \, , \tag{5.18}$$

do not have to be diagonal in family space. However, a popular scheme is just to assume SUSY breaking is "flavor universal" and that due to this universality such matrices are diagonal. As above, this renders all mixing effects in the soft terms irrelevant besides the ones coming from the so–called A terms, e.g. A_{LR}, which again we assume to be of the same order as the Yukawa couplings, i.e. the mixing is within experimental limits. These two assumptions together are usually referred to as "soft supersymmetry–breaking universality" and it is quite popular in gravity–mediated models such as minimal supergravity (MSUGRA) and the constrained minimal supersymmetric models (CMSSM) [34]. Obviously, a sensible UV complete theory would be required to explain these assumptions somehow.

Another possibility to circumvent the danger of FCNCs is to assume, or to explain, that squark masses are so large that they suppress FCNCs sufficiently. This seems to be contradictory to the original purpose of supersymmetry, i.e. explaining the hierarchy problem;

however, since the LHC excludes more and more parameter space for small squark masses, these models are becoming more attractive [107].

When we only consider the neutrino mixing sector, there is an interesting idea to explain the mixing pattern called anarchy [108]. In this scheme the neutrino mixing pattern does not follow a certain pattern or symmetry, but all the entries of the mixing matrix are of similar size and randomly distributed. This idea works particularly well for large mixing angles, as we have partially seen in the neutrino case. It has been recently argued [109] that the idea of anarchy is still compatible with current experimental limits and that measuring θ_{23} has, at this point, the most power to distinguish between anarchy or theoretical models. However, anarchy models lack any predictivity and, therefore, we return to flavor models in the next sections.

5.2 $\mathrm{U}(1)_R$ Froggatt–Nielsen models with residual symmetries

So far the only explicit flavor model we have discussed were FN models [101] based on a U(1) symmetry. However, in a recent paper [110] the possibility of having FN models with a $\mathrm{U}(1)_R$ symmetry was also discussed. There, the general idea is that these models have a pseudo–anomalous (or "omalous" [111]) $\mathrm{U}(1)_R$ symmetry, which explains the fermion mass hierarchies while also exhibiting a residual symmetry, the $\mathbb{Z}_4^R$ symmetry from section 4.2.4, which forbids certain unwanted operators.

In this section we first discuss the originally proposed models, then we argue that the found models, in particular the charge assignment of the fields, can be improved and present more appealing examples. Before summarizing the features of such models we also briefly discuss the neutrino mixing in the different sets of models.

5.2.1 $\mathrm{U}(1)_R$ FN models

The authors of [110] proposed FN models with a $\mathrm{U}(1)_R$ instead of a U(1) symmetry. This situation is similar to the one in equation (5.6), i.e. an operator $Y^{fgh}\,\Psi_f\,\Psi_g\,\Psi_h$ is created in the superpotential through assigning the flavon field Φ a VEV,

$$\mathscr{W} \supset \left(\frac{\Phi}{\Lambda}\right)^{n_{fgh}} \Psi_f\,\Psi_g\,\Psi_h\,, \tag{5.19}$$

where we ignore this time undetermined $\mathcal{O}(1)$ coefficients. The key difference to the case around equation (5.6) is that here the powers n_{fgh} are given by

$$n_{fgh} \;=\; \frac{2\,R_\theta - (R_{\Psi_f} + R_{\Psi_g} + R_{\Psi_h})}{R_\Phi} \tag{5.20}$$

since we are considering an R symmetry. We see that the operators without the flavon fields have R charge $2\,R_\theta - n\,R_\Phi$, with $n \in \mathbb{N}$ since the superpotential is holomorphic. When the flavon now acquires a VEV we, like in the original FN case, generate the hierarchies due to the different family charges and the symmetry breaking parameter is again given by $\varepsilon := \frac{\langle\Phi\rangle}{\Lambda}$, the expansion parameter, and we usually set $\varepsilon \sim 0.2$.

There is a huge amount of possible charge assignments and, subsequently, FN models. The authors of [110], therefore, impose several constraints on the symmetries. First, the symmetries should explain the fermion mass hierarchies and mixing patterns, as in the

original FN model [101]. Secondly, they should be anomaly–free with respect to the Green–Schwarz mechanism [89]. Thirdly, the U(1)$_R$ symmetry is spontaneously broken when the flavon acquires a VEV and the charge assignment should allow for a residual $\mathbb{Z}_4^R$ symmetry as in section 4.2.4. This, together with other considerations [110], already fixes the charge of the superspace coordinate θ to $R_\theta = \pm\frac{1}{4}\,R_\Phi$.

These constraints influence the charge assignment of the fields and the field charges are summarized in equation (B.1) and equation (B.2) in appendix B.1. Note that the constraints are parametrized by additional integers. The most important ones are: $x \in \{0,1,2,3\}$, which determines the top to bottom mass ratio, $y \in \{-1,0,1\}$, which parametrizes the CKM mixing matrix and $z \in \{0,1\}$, which gives the ratio between the lepton masses. Using these parameters the Yukawa matrices are schematically given by [112]

$$\begin{aligned} Y_u \;&\sim\; \begin{pmatrix} \varepsilon^8 & \varepsilon^{5+y} & \varepsilon^{3+y} \\ \varepsilon^{7-y} & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^{5-y} & \varepsilon^2 & 1 \end{pmatrix}, \qquad Y_d \;\sim\; \varepsilon^x \begin{pmatrix} \varepsilon^4 & \varepsilon^{3+y} & \varepsilon^{3+y} \\ \varepsilon^{3-y} & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^{1-y} & 1 & 1 \end{pmatrix}, \\ Y_e \;&\sim\; \varepsilon^x \begin{pmatrix} \varepsilon^{4+z} & \varepsilon^{2-\Delta^L_{21}} & \varepsilon^{-\Delta^L_{31}} \\ \varepsilon^{4+z+\Delta^L_{21}} & \varepsilon^2 & \varepsilon^{-\Delta^L_{32}} \\ \varepsilon^{4+z+\Delta^L_{31}} & \varepsilon^{2+\Delta^L_{32}} & 1 \end{pmatrix}, \end{aligned} \tag{5.21}$$

with $\Delta^L_{ij} = R_{L_i} - R_{L_j}$ and where we ignore the undetermined $\mathcal{O}(1)$ coefficients. In our convention the superpotential reads

$$\mathscr{W} \;\supset\; (Y_u)_{ij}\, Q_i\, \overline{U}_j\, H_u + (Y_d)_{ij}\, Q_i\, \overline{D}_j\, H_d + (Y_e)_{ij}\, L_i\, \overline{E}_j\, H_d\,. \tag{5.22}$$

After imposing those constraints one finds 3×34 different models and their field charges are normalized with respect to the flavon charge, which in [110] is given by $R_\Phi = -1$ and, therefore, $R_\theta = \pm\frac{1}{4}$. The authors categorize the models according to the value of the y parameter, i.e. $y = -1, 0, 1$, and then number them from #1–12, where for each number there is a sub–structure according to the value of x, e.g. model #1a has $x = 0$, cf. [110] for more details.

As mentioned above, the models found are supposed to be anomaly–free through the Green–Schwarz mechanism, and, therefore, the authors of [110] demand anomaly universality for the SM gauge group, cf. equation (4.4) for the discrete case,

$$A_{\mathrm{SU(3)^2_C\text{–}U(1)}_R} = A_{\mathrm{SU(2)^2_L\text{–}U(1)}_R} = A_{\mathrm{U(1)^2_Y\text{–}U(1)}_R} = \rho\,, \tag{5.23}$$

where ρ is a constant. They also impose that $A_{\mathrm{U(1)}^2_R\text{–}\mathrm{U(1)_Y}} = 0$, which is solved by setting

$$\begin{aligned} R_{H_d} \;=\; &\frac{1}{3(14\,R_\theta - 18 - 3x - 2z)}\,\big[3\,R_{L_1}(12 - 16\,R_\theta + 2x + 3z) + 18 \\ &+ 2\,\Delta^L_{21}(6 - 8\,R_\theta + x + z) + 2\,\Delta^L_{31}(3 - 8\,R_\theta + x + z) - 156\,R_\theta \\ &+ x(14\,R_\theta - 36 - 6x) + z(-2z - 5x - 12\,R_\theta) - 18y + 104\,R_\theta^2\big]\,. \end{aligned} \tag{5.24}$$

Due to this anomaly constraint the 3×34 models almost always have highly fractional charges, or very large integer values after normalization, cf. table 5.1.

However, in the next section we argue that this does not need to be the case since we can solve the $A_{\mathrm{U(1)}^2_R\text{–}\mathrm{U(1)_Y}} = 0$ constraint without adjusting R_{H_d}.

#	R_{H_d}	R_{H_u}	R_{Q_1}	R_{Q_2}	R_{Q_3}	$R_{\overline{U}_1}$	$R_{\overline{U}_2}$	$R_{\overline{U}_3}$	$R_{\overline{D}_1}$	$R_{\overline{D}_2}$	$R_{\overline{D}_3}$	R_{L_1}	R_{L_2}	R_{L_3}	$R_{\overline{E}_1}$	$R_{\overline{E}_2}$	$R_{\overline{E}_3}$
6b	$\frac{253}{130}$	$-\frac{123}{130}$	$\frac{3199}{780}$	$\frac{2419}{780}$	$\frac{859}{780}$	$\frac{4169}{780}$	$\frac{1829}{780}$	$\frac{269}{780}$	$-\frac{427}{780}$	$-\frac{1207}{780}$	$-\frac{1207}{780}$	$-\frac{339}{260}$	$-\frac{339}{260}$	$-\frac{339}{260}$	$\frac{1523}{260}$	$\frac{743}{260}$	$\frac{223}{260}$
6c	$\frac{353}{150}$	$-\frac{203}{150}$	$\frac{3869}{900}$	$\frac{2969}{900}$	$\frac{1169}{900}$	$\frac{4999}{900}$	$\frac{2299}{900}$	$\frac{499}{900}$	$-\frac{137}{900}$	$-\frac{1037}{900}$	$-\frac{1037}{900}$	$-\frac{269}{300}$	$-\frac{269}{300}$	$-\frac{269}{300}$	$\frac{1813}{300}$	$\frac{913}{300}$	$\frac{313}{300}$

Table 5.1: Examples with highly fractional $\mathrm{U}(1)_R$ charges from [110] for $z = 0$, $x = 1, 2$ and the CKM parameter $y = 0$.

5.2.2 Removing the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_Y}$ anomaly coefficient

The authors of [110] need to impose $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_Y} = 0$ and, hence, they set R_{H_d} as in equation (5.24). However, we want to follow the discussion by Banks and Dine [85] and allow for extra states, which are chiral w.r.t. $\mathrm{U}(1)_R$, to be present.[3] We allow this since we do not assume to posses a UV complete model. We choose the R charges of the extra states in such a way that they decouple at a high scale, i.e. at $\mathrm{U}(1)_R$ breaking, but still cancel the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_Y}$ anomaly while shifting the $A_{\mathrm{SM}^2-\mathrm{U}(1)_R}$ anomaly coefficients universally.

Let us introduce, for example, an extra pair of SU(2) doublets, $\overline{Y} = (\mathbf{1}, \mathbf{2})_{1/2}$ and $Y = (\mathbf{1}, \mathbf{2})_{-1/2}$ as well as an extra pair of $\mathrm{SU}(3)_C$ triplets $Z = (\mathbf{3}, \mathbf{1})_{1/3}$ and $\overline{Z} = (\overline{\mathbf{3}}, \mathbf{1})_{-1/3}$ with R charges $R_{\overline{Z}} = R_Y = r$ and $R_Z = R_{\overline{Y}} = -r - n\,R_\phi + 2\,R_\theta$ with $r \in \mathbb{Q}$ and $n \in \mathbb{N}$. Assigning the charges in such a way allows for superpotential terms

$$\mathscr{W} \supset \frac{\Phi^n}{\Lambda^{n-1}}\, Z\, \overline{Z} + \frac{\Phi^n}{\Lambda^{n-1}}\, Y\, \overline{Y}\,. \tag{5.25}$$

This automatically sets $n \geq 0$ due to the holomorphicity of the superpotential. In fact, we need $n \neq 0$, otherwise the field pairs would not be chiral w.r.t. $\mathrm{U}(1)_R$ and already be massive before $\mathrm{U}(1)_R$ breaking. After we break the $\mathrm{U}(1)_R$ to its discrete subgroup $\mathbb{Z}_M^R$ we get effective mass terms for both pairs.

Introducing such field pairs shifts the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_Y}$ anomaly in a non–trivial way by

$$\begin{aligned}\Delta A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_Y} &= 2\sqrt{\frac{3}{5}}\left[-(r - R_\theta)^2 + (-r - n\,R_\Phi + R_\theta)^2\right] \\ &= 2\sqrt{\frac{3}{5}}\left[(n\,R_\Phi)^2 - 2\,n\,R_\Phi\,(R_\theta - r)\right]\,.\end{aligned} \tag{5.26}$$

We can now, for given R_Φ and R_θ, always choose this shift in such a way that it cancels the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_Y}$ anomaly coefficient. The SM anomaly coefficients all shift universally due to presence of the extra field pairs, and in SU(5) normalization this is given by

$$\begin{aligned}\Delta A_{\mathrm{SU}(3)_C^2-\mathrm{U}(1)_R} &= \Delta A_{\mathrm{SU}(2)_L^2-\mathrm{U}(1)_R} = \Delta A_{\mathrm{U}(1)_Y^2-\mathrm{U}(1)_R} \\ &= \frac{1}{2}\left[(r - R_\theta) + (-r - n\,R_\Phi + R_\theta)\right] = -\frac{n\,R_\Phi}{2}\,.\end{aligned} \tag{5.27}$$

The shift in $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_Y}$ does not spoil the universality here.

In section 5.2.3 we show an example of the just presented procedure; however, we can also just drop the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_Y} = 0$ constraint since it does not lead to new solutions, as

[3] The authors of [110] exclude this possibility since they want to write down a UV complete model from the beginning.

we can see when we perform a hypercharge shift. This means that it is always allowed to simply shift the $\mathrm{U}(1)_R$ charges of the fields by $\alpha\, q_\mathrm{Y}$, where α is a field–invariant constant and q_Y the hypercharge of the field. If we consider the example #6b from table 5.1 we can set $R_{H_u} = 0$ by choosing $\alpha = \frac{123}{65}$. Then using this α for all fields in table 5.1, i.e.

$$R_\text{new} \;=\; 12 \left(R_{\text{model \#6b}} + \frac{123}{65}\, q_\mathrm{Y} \right) , \tag{5.28}$$

we get the new field charges in table 5.2, where the factor 12 normalizes the charges to integer value. Note that the charges are the same as if we simply drop the anomaly coefficient $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}}$ all together and set $R_{H_u} = 0$ by hand.

5.2.3 Models with modified charges

As we have just seen, we can always fulfill the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}}$ anomaly condition; therefore, we can adjust the R charge of the Higgs fields,

$$R_{H_d} + R_{H_u} = -z + 8\, R_\theta \,, \tag{5.29}$$

through the hypercharge shift. We set $R_{H_u} = 0$, thus $R_{H_d} = -z + 8\, R_\theta$, whereas the charge relations of the other fields, cf. equation (B.1), remain unchanged. Putting all of this together we can now again compute the charges for the models in [110] and we find more appealing charges [33]. We also start with $R_\Phi = -1$ and $R_\theta = \pm\frac{1}{4}$ and then normalize the results to integer value. The results can be found in appendix B.2. This leads to two different possible flavon charges, either -4 or -12, i.e. after symmetry breaking we end up with a $\mathbb{Z}_4^R$ or $\mathbb{Z}_{12}^R$, respectively, residual symmetry. The $\mathbb{Z}_4^R$ symmetry is hereby the desired one which we described in section 4.2.4. Furthermore, we can decompose the $\mathbb{Z}_{12}^R$ symmetry into a $\mathbb{Z}_4^R \times \mathbb{Z}_3$ symmetry, where the $\mathbb{Z}_4^R$ again is the desired one and the $\mathbb{Z}_3$ is the non–trivial center of $\mathrm{SU}(3)_\mathrm{C}$, $\mathbb{Z}_3^{\mathrm{SU}(3)_\mathrm{C}}$. Since this does not constrain any SM couplings and the symmetry is non–anomalous we can ignore the $\mathbb{Z}_3^{\mathrm{SU}(3)_\mathrm{C}}$ factors, hence, the $\mathbb{Z}_3$ symmetry from here on.

All of this can be illustrated with the help of example #6b from table 5.1. After dropping the unnecessary anomaly constraint we recomputed the field charges in table 5.2 and the charges simplify significantly. Furthermore, we see that we end up with a $\mathbb{Z}_{12}^R$ symmetry

	ϕ	θ	H_d	H_u	Q_1	Q_2	Q_3	$\overline{U}_1$	$\overline{U}_2$	$\overline{U}_3$	$\overline{D}_1$	$\overline{D}_2$	$\overline{D}_3$	L_1	L_2	L_3	$\overline{E}_1$	$\overline{E}_2$	$\overline{E}_3$
$\mathrm{U}(1)_R$	-12	3	12	0	53	41	17	49	13	-11	1	-11	-11	-27	-27	-27	93	57	33
$\mathbb{Z}_{12}^R$	0	3	0	0	7	7	7	11	11	11	11	11	11	3	3	3	3	3	3
$\mathbb{Z}_4^R$	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\mathbb{Z}_3^{\mathrm{SU}(3)_\mathrm{C}}$	0	0	0	0	1	1	1	2	2	2	2	2	2	0	0	0	0	0	0

Table 5.2: Example #6b for $x = 2$ and $y = z = 0$ where $\mathbb{Z}_{12}^R = \mathbb{Z}_4^R \times \mathbb{Z}_3^{\mathrm{SU}(3)_\mathrm{C}}$. We rescaled the $\mathbb{Z}_4^R$ charges..

after the flavon acquires its VEV, which is the desired $\mathbb{Z}_4^R$ symmetry times the non–trivial

center of $\mathrm{SU}(3)_\mathrm{C}$. Let us also note that the same thing happens in all other examples with $R_\Phi = -12$ in appendix B.2.

We use another example in order to show how to cancel the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}}$ anomaly by introducing additional fields: example #12c with $y = -1$ from appendix B.2. Setting $R_{H_u} = 0$ and normalizing the charges to integer values gives us a more appealing charge assignment than in [110], and the charges are summarized in table 5.3. Again, we can see that after $\mathrm{U}(1)_R$ breaking we obtain the familiar $\mathbb{Z}_4^R$ symmetry.

	ϕ	θ	H_d	H_u	Q_1	Q_2	Q_3	$\overline{U}_1$	$\overline{U}_2$	$\overline{U}_3$	$\overline{D}_1$	$\overline{D}_2$	$\overline{D}_3$	L_1	L_2	L_3	$\overline{E}_1$	$\overline{E}_2$	$\overline{E}_3$
$\mathrm{U}(1)_R$	-4	1	8	0	15	15	7	19	3	-5	7	-1	-1	-5	-5	-9	27	19	15
$\mathbb{Z}_4^R$	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 5.3: Example #12c for $x = 1, y = -1$ and $z = 0$.

In section 5.2.2 we said that we can introduce additional fields in order to cancel the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}}$ anomaly, which allows us to set $R_{H_u} = 0$, and this can be achieved by introducing field pairs $Y - \overline{Y}$ and $Z - \overline{Z}$, which, since they have a non–vanishing R charge, shift the anomaly

$$\begin{aligned}\Delta A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}} &= 2\sqrt{\frac{3}{5}}\left[(n\,R_\Phi)^2 - 2\,n\,R_\Phi\,(R_\theta - r)\right] \\ &= 2\sqrt{\frac{3}{5}}\left[16\,n^2 + 8\,n\,(1-r)\right]\,. \end{aligned} \tag{5.30}$$

Obviously, we can now find n and r in order to cancel the anomaly, which for the charges in table 5.3 is given by

$$A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}} = \sqrt{\frac{3}{5}}\,750\,. \tag{5.31}$$

The first integer solution for n and r, fulfilling

$$A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}} + \Delta A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}} \;=\; 0\,, \tag{5.32}$$

is given by $n = 1$ and $r = -42$. Hence, we get the R charges

$$R_Z \;=\; R_{\overline{Y}} \;=\; 48 \qquad \text{and} \qquad R_{\overline{Z}} \;=\; R_Y \;=\; -42\,. \tag{5.33}$$

The superpotential terms of the field pairs then occur with one flavon field multiplied, i.e.

$$\mathscr{W} \supset \Phi\,Z\,\overline{Z} + \Phi\,Y\,\overline{Y}\,, \tag{5.34}$$

which means that they decouple after the flavon acquires its VEV.

As mentioned in section 5.2.2 the $A_{\mathrm{SM}^2-\mathrm{U}(1)_R}$ anomaly coefficients are shifted universally,

$$\Delta A_{\mathrm{SU}(3)_\mathrm{C}^2-\mathrm{U}(1)_R} \;=\; \Delta A_{\mathrm{SU}(2)_\mathrm{L}^2-\mathrm{U}(1)_R} \;=\; \Delta A_{\mathrm{U}(1)_\mathrm{Y}^2-\mathrm{U}(1)_R} \;=\; -\frac{n\,R_\Phi}{2} \;=\; 2\,. \tag{5.35}$$

We see on the one hand that we can cancel the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}}$ anomaly by introducing extra field pairs $Y - \overline{Y}$ and $Z - \overline{Z}$, which acquire a large mass and decouple after $\mathrm{U}(1)_R$ breaking. On the other hand, the $A_{\mathrm{SM}^2-\mathrm{U}(1)_R}$ coefficients were shifted universally.

We can also use example #12c to show how the type I see–saw mechanism works in the models [110]. We have to introduce right–handed neutrinos $\overline{\nu}_i$ and allow for a Dirac and Majorana mass term, i.e. schematically we get

$$\mathscr{W} \supset \left(\frac{\Phi}{\Lambda}\right)^p L\, H_u\, \overline{\nu} + \left(\frac{\Phi}{\Lambda}\right)^q \Lambda\, \overline{\nu}\, \overline{\nu}\,, \tag{5.36}$$

where $p, q \geq 0$ are integers. After the flavon acquires a VEV we get an effective Dirac Yukawa matrix Y_ν and a Majorana mass term for the right–handed neutrinos M_R.

For our example #12c in table 5.3 we can pick

$$R_{\overline{\nu}_1} \;=\; 23, \quad R_{\overline{\nu}_2} \;=\; 19 \quad \text{and} \quad R_{\overline{\nu}_3} \;=\; 15\,, \tag{5.37}$$

which allows for the Dirac and Majorana mass term to be present,

$$Y_\nu \;\sim\; \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^1 \end{pmatrix}\,, \qquad M_R \;\sim\; \Lambda \begin{pmatrix} \varepsilon^{11} & \varepsilon^{10} & \varepsilon^9 \\ \varepsilon^{10} & \varepsilon^9 & \varepsilon^8 \\ \varepsilon^9 & \varepsilon^8 & \varepsilon^7 \end{pmatrix}\,, \tag{5.38}$$

i.e. the type I see–saw mechanism works. For this example we end up with a normal mass hierarchy and an absolute neutrino mass scale of $m^\nu_{\text{abs}} \sim \mathcal{O}(0.1)\text{eV}$, where we assume $\Lambda \sim M_\text{P}$ and $\varepsilon \sim 0.2$. However, this does not predict fully accurate neutrino mixing angles and we discuss this further in the following section.

5.2.4 Neutrino mixing in U(1)$_R$ FN models

As we mentioned before, the models in [110] use the type I see–saw mechanism to create neutrino masses, which are of the right order of magnitude. The charges of the right–handed neutrinos have to be chosen such that they allow for an effective Dirac Yukawa matrix and a Majorana mass term for the right–handed neutrinos, see equation (5.36). Also, the charges should not allow for too large or too small neutrino masses after we apply the see–saw formula.

As we showed in the previous section with the help of an example, this can be easily done and we end up with viable neutrino masses. However, the predicted neutrino mixing is not necessarily realistic, i.e. the mixing angles might be of the wrong size. We can see this when we look at the neutrino mass matrices M_ν predicted by the different models. As it turns out these are independent of the charges of the right–handed neutrinos and using the see–saw formula, cf. equation (3.18), we get

$$M_\nu \sim \begin{pmatrix} \varepsilon^{-2\,\Delta^L_{31}} & \varepsilon^{-2\,\Delta^L_{31}+\Delta^L_{21}} & \varepsilon^{-\Delta^L_{31}} \\ \varepsilon^{-2\,\Delta^L_{31}+\Delta^L_{21}} & \varepsilon^{2\,(\Delta^L_{21}-\Delta^L_{31})} & \varepsilon^{\Delta^L_{21}-\Delta^L_{31}} \\ \varepsilon^{-\Delta^L_{31}} & \varepsilon^{\Delta^L_{21}-\Delta^L_{31}} & 1 \end{pmatrix} \varepsilon^{\mathcal{Z}}\,, \tag{5.39}$$

where $\mathcal{Z}$ is negative integer with $-9 \leq \mathcal{Z} \leq -5$; its precise value is chosen in order to predict accurate neutrino masses and can be found in [110]. Since the mass matrix solely depends on the Δ^L_{ij}, up to the $\text{e}^{\mathcal{Z}}$ factor, only three different patterns occur, regardless of x, y, z. We have ignored $\mathcal{O}(1)$ coefficients so far.

Therefore, for all models #1–2 as well as #11–12, which have $\Delta^L_{21} = 0$ and $\Delta^L_{31} = -1$, the neutrino mass matrix is given by

$$M_\nu^{(\mathrm{A})} \sim \begin{pmatrix} \varepsilon^2 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon^2 & \varepsilon \\ \varepsilon & \varepsilon & 1 \end{pmatrix} \varepsilon^{\mathcal{Z}} \,. \tag{5.40}$$

Such a mass matrix only predicts semi–realistic neutrino mixing, in particular, the atmospheric mixing angle θ_{23} turns out to be suppressed by a factor of ε. Therefore, we need some fine–tuning between the $\mathcal{O}(1)$ coefficients to overcome this. On the other hand, the angles θ_{12} and θ_{13} are roughly of the correct size.

All models #3–4, where $\Delta^L_{21} = \Delta^L_{31} = -1$, have the neutrino mass matrix

$$M_\nu^{(\mathrm{B})} \sim \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} \varepsilon^{\mathcal{Z}} \,, \tag{5.41}$$

and in general predict mixing angles of correct order of magnitude. From this point of view, these models are somewhat preferred.

A different situation is given for models #5–10, which have $\Delta^L_{21} = \Delta^L_{31} = 0$, and, therefore, the mass matrix

$$M_\nu^{(\mathrm{C})} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \varepsilon^{\mathcal{Z}} \,. \tag{5.42}$$

Such a mass matrix usually gives too large of a reactor mixing angle θ_{13} since none of its entries is suppressed. This problem can be overcome in two ways, either by fine–tuning the $\mathcal{O}(1)$ coefficients or, looking at the structure of $M_\nu^{(\mathrm{C})}$, by introducing a non–Abelian flavor symmetry. For these models #5–10 we might be tempted to dismiss the problem altogether and refer to the anarchy scheme, introduced in section 5.1.2, which fits well with this pattern. However, as we mentioned above, this scheme is rather undesirable due to its lack of predictivity, thus it only provides an interesting alternative.

In summary, we showed that in order for the type I see–saw mechanism to work, only models #3–4 do not require fine–tuning or additional ingredients, e.g. a non–Abelian flavor symmetry. Nonetheless, all models are somehow capable of providing (semi–)realistic neutrino mixing. A caveat is that models #9–10 often predict too large neutrino masses and are, therefore, not experimentally viable.

An alternative solution for all models is to consider the case of Dirac neutrinos only [33]. As we have seen in section 4.2.2 it is possible to have strongly suppressed Dirac Yukawa matrices Y_ν for the right–handed neutrinos from R symmetries. In section 4.2.2 this was done by creating the Dirac Yukawa matrices effectively from the Kähler potential, cf. equation (4.19),

$$K \supset \frac{X^\dagger}{M_{\mathrm{P}}^2} L\, H_u\, \overline{\nu} \quad \curvearrowright \quad \mathscr{W}_{\mathrm{eff}} \supset Y_\nu\, L\, H_u\, \overline{\nu} \quad \text{with} \quad Y_\nu \sim \frac{m_{3/2}}{M_{\mathrm{P}}} \,, \tag{5.43}$$

which means that the Yukawa matrices are due to the supersymmetry breaking spurion X. Such a scenario can be realized by choosing large negative R charges for the right–handed

neutrinos and hence for the spurion X, i.e. they have the same sign as the flavon Φ. Then, the Majorana mass term for the $\overline{\nu}_i$ and the Dirac Yukawa matrix are forbidden by holomorphicity and we end up with Dirac neutrinos as in equation (5.43).

Besides being an interesting alternative to the type I see–saw case, this also does not suffer from the potential proton decay problem as in section 4.2.3 since an operator $\overline{U}\,\overline{D}\,\overline{D}\,\overline{\nu}$, like in equation (4.35), is forbidden by the holomorphicity of the superpotential, cf. section 7.3.2 for an explicit example.

5.2.5 Summary of $\mathrm{U}(1)_R$ FN models

In the previous sections we showed that Froggatt–Nielsen models based on a $\mathrm{U}(1)_R$ symmetry allow us to address several issues simultaneously. A major caveat of the original models [110] was that they had highly fractional charges; however, we showed in section 5.2.2 that we can remove the unnecessary anomaly constraint $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}} = 0$ that was responsible for these charges and in section 5.2.3 we presented more appealing charge assignments for some examples. A summary of all improved charges can be found in appendix B.2 and now we briefly summarize the main features of the models again.

The models have the familiar $\mathbb{Z}_4^R$ symmetry, cf. section 4.2.4, as a residual symmetry, they are anomaly–free due to the GS mechanism and forbid the μ term as well as dangerous proton decay operators. Furthermore, the models predict phenomenologically viable fermion masses and hierarchies, in particular for the quark and charged lepton masses as well as for the CKM matrix. The neutrino mixing is more complicated and was either explained by the type I see–saw mechanism or through Dirac neutrinos alone. We often need some fine–tuning to make the models phenomenologically acceptable or a non–Abelian flavor symmetry. Therefore, we focus in the rest of this chapter on such symmetries and the predictions they make for neutrino mixing.

5.3 Models with discrete, non–Abelian flavor symmetries

In the previous section we discussed models that mainly focused on the fermion mass hierarchies and the CKM mixing matrix. However, we saw in section 5.1 that the mixing in the lepton sector, i.e. the neutrino mixing, is especially interesting since it is so different from the CKM case. For the rest of this chapter we, therefore, focus our discussion on the lepton sector and on how to address the issues there with non–Abelian flavor symmetries since several interesting models with discrete, non–Abelian flavor symmetries exist. In particular, we focus on supersymmetric extensions of the SM with an additional flavor symmetry at the high scale, which gets spontaneously broken by assigning VEVs to some or many flavons. We start by reviewing in detail a popular model [113,114] based on the alternating group A_4 [106], also called tetrahedral group, in order to demonstrate how flavor model building works for discrete non–Abelian flavor symmetries. We then continue with another well–known model based on the double cover of the alternating group, T' [115].

5.3.1 A model based on an A_4 flavor symmetry

The discrete, non–Abelian group A_4 is the alternating group on four elements, i.e. the even permutations of a set with four elements, which is the symmetry group of a tetrahedron. It

is necessary to recall the basic group properties of A_4 before we continue. We choose a basis where the group is generated by

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} , \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} , \qquad \text{with} \qquad \omega = \mathrm{e}^{\frac{2\pi\mathrm{i}}{3}} . \quad (5.44)$$

The symmetry has four inequivalent irreducible representations, three of them one–dimensional and one three–dimensional. The singlets are denoted by $\mathbf{1}$, $\mathbf{1'}$ and $\mathbf{1''}$, whereas the triplet is denoted by $\mathbf{3}$. There are several multiplication laws and the non–trivial ones are

$$\mathbf{1'} \otimes \mathbf{1'} = \mathbf{1''} , \qquad \mathbf{1''} \otimes \mathbf{1''} = \mathbf{1'} , \qquad \mathbf{1'} \otimes \mathbf{1''} = \mathbf{1} , \quad (5.45)$$

with the most important one being

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{1''} \oplus \mathbf{3}_\mathrm{s} \oplus \mathbf{3}_\mathrm{a} . \quad (5.46)$$

Here, we call $\mathbf{3}_\mathrm{s}$ the symmetric and $\mathbf{3}_\mathrm{a}$ antisymmetric triplet contraction. Given two triplets $\boldsymbol{a}$ and $\boldsymbol{b}$, in components their contractions look like

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1}} = a_1 b_1 + a_2 b_3 + a_3 b_2 , \quad (5.47a)$$

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1'}} = a_3 b_3 + a_1 b_2 + a_2 b_1 , \quad (5.47b)$$

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1''}} = a_2 b_2 + a_1 b_3 + a_3 b_1 , \quad (5.47c)$$

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{3}_\mathrm{s}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{pmatrix} , \quad (5.47d)$$

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{3}_\mathrm{a}} = \mathrm{i}\sqrt{\frac{3}{2}}\begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{pmatrix} , \quad (5.47e)$$

where $(\boldsymbol{a} \otimes \boldsymbol{b})_{\boldsymbol{R}}$ means that $\boldsymbol{a}$ and $\boldsymbol{b}$ are contracted to the representation $\boldsymbol{R}$. Please note the important (complex) coefficients in front of the triplets, which are often absent in the literature. In appendix C we review another basis for A_4 and its connection to the just presented one.

Having the basic group properties at hand, we can now start reviewing the previously mentioned model by Altarelli et al. [113, 114] which will give us tri–bi–maximal mixing for neutrinos. In this model all SM matter fields are in irreducible representations of A_4. All the charged leptons transform as singlets: e_R as $\mathbf{1}$, μ_R as $\mathbf{1''}$ and τ_R as $\mathbf{1'}$. The Higgs fields H_u and H_d transform as trivial singlets $\mathbf{1}$, whereas the left–handed lepton doublets transform as a triplet $\mathbf{3}$. There are additional fields in this model: three flavons, which will break the flavor symmetry. There are two triplets, Φ_ν and Φ_e, as well as another trivial A_4 singlet ξ. In order to distinguish between the two flavon triplets and to achieve tri–bi–maximal mixing, the model is also amended by a $\mathbb{Z}_4$ symmetry. Φ_e and the Higgs fields stay invariant under this symmetry, whereas Φ_ν and ξ change their signs. The leptons transform with opposite complex phases, i.e. $L \to \mathrm{i}\,L$ and $R \to -\mathrm{i}\,R$. The field content is summarized in table 5.4.

	e_R	μ_R	τ_R	L	H_u	H_d	Φ_ν	Φ_e	ξ
A_4	$\mathbf{1}$	$\mathbf{1''}$	$\mathbf{1'}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
$\mathbb{Z}_4$	3	3	3	1	0	0	2	0	2

Table 5.4: Summary of the A_4 model by Altarelli et al. [113, 114].

Assigning the just listed charges allows us to write down the following superpotential, split into two sectors,

$$\mathscr{W}_\nu = \frac{\lambda_1}{\Lambda\,\Lambda_\nu}\left\{[(L\,H_u)\otimes(L\,H_u)]_{\mathbf{3}_\mathrm{s}}\otimes\Phi_\nu\right\}_{\mathbf{1}} + \frac{\lambda_2}{\Lambda\,\Lambda_\nu}\,[(L\,H_u)\otimes(L\,H_u)]_{\mathbf{1}}\,\xi\,, \tag{5.48}$$

$$\mathscr{W}_e = \frac{h_e}{\Lambda}\,(\Phi_e\otimes L)_{\mathbf{1}}\,H_d\,e_R + \frac{h_\mu}{\Lambda}\,(\Phi_e\otimes L)_{\mathbf{1'}}\,H_d\,\mu_R + \frac{h_\tau}{\Lambda}\,(\Phi_e\otimes L)_{\mathbf{1''}}\,H_d\,\tau_R\,, \tag{5.49}$$

where $\lambda_{1,2}$ and $h_{e,\mu,\tau}$ are dimensionless couplings. Λ and Λ_ν denote the flavor scale and the see–saw scale, respectively. After electroweak symmetry breaking we can substitute the Higgs fields for their VEVs, i.e.

$$\langle H_u\rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix} \quad \text{and} \quad \langle H_d\rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}\,. \tag{5.50}$$

Then, using the multiplication laws from equation (5.47), we want to compute mass matrices for the neutrinos and for the charged leptons from the different sectors

$$\mathscr{W}_\nu = \frac{1}{2}L^T\,m_\nu\,L \qquad \text{and} \qquad \mathscr{W}_e = R^T\,m_e\,L\,. \tag{5.51}$$

m_ν gets contributions from the Φ_ν term

$$\begin{aligned}\left\{[(L\,H_u)\otimes(L\,H_u)]_{\mathbf{3}_\mathrm{s}}\otimes\Phi_\nu\right\}_{\mathbf{1}} &\to \frac{v_u^2}{\sqrt{2}}\left\{\begin{pmatrix} 2L_1^2 - 2L_2\,L_3 \\ 2L_3^2 - 2L_1\,L_2 \\ 2L_2^2 - 2L_1\,L_3 \end{pmatrix}\otimes\Phi_\nu\right\}_{\mathbf{1}} \\ &= \sqrt{2}\,v_u^2\left[\left(L_1^2 - L_2\,L_3\right)\Phi_{\nu 1} + \left(L_3^2 - L_1\,L_2\right)\Phi_{\nu 3} + \right. \\ &\quad \left.\left(L_2^2 - L_1\,L_3\right)\Phi_{\nu 2}\right]\,,\end{aligned} \tag{5.52a}$$

and from the ξ term

$$[(L\,H_u)\otimes(L\,H_u)]_{\mathbf{1}}\,\xi \to v_u^2\left[L_1^2 + 2L_2\,L_3\right]\xi\,. \tag{5.52b}$$

This gives us for m_ν

$$m_\nu = \frac{v_u^2}{\Lambda\,\Lambda_\nu}\begin{pmatrix} 2\lambda_2\,\xi + 2\sqrt{2}\,\lambda_1\,\Phi_{\nu 1} & -\sqrt{2}\,\lambda_1\,\Phi_{\nu 3} & -\sqrt{2}\,\lambda_1\,\Phi_{\nu 2} \\ -\sqrt{2}\,\lambda_1\,\Phi_{\nu 3} & 2\sqrt{2}\,\lambda_1\,\Phi_{\nu 2} & 2\lambda_2\,\xi - \sqrt{2}\,\lambda_1\,\Phi_{\nu 1} \\ -\sqrt{2}\,\lambda_1\,\Phi_{\nu 2} & 2\lambda_2\,\xi - \sqrt{2}\,\lambda_1\,\Phi_{\nu 1} & 2\sqrt{2}\,\lambda_1\,\Phi_{\nu 3} \end{pmatrix}\,. \tag{5.53}$$

The charged lepton Yukawa matrix is computed similarly from the terms

$$(\Phi_e \otimes L)_{\mathbf{1}}\ H_d\, e_R \to v_d\,(L_1\,\Phi_{e1} + L_2\,\Phi_{e3} + L_3\,\Phi_{e2})\, e_R\ , \tag{5.54a}$$
$$(\Phi_e \otimes L)_{\mathbf{1}'}\ H_d\, \mu_R \to v_d\,(L_3\,\Phi_{e3} + L_1\,\Phi_{e2} + L_2\,\Phi_{e1})\, \mu_R\ , \tag{5.54b}$$
$$(\Phi_e \otimes L)_{\mathbf{1}''}\ H_d\, \tau_R \to v_d\,(L_2\,\Phi_{e2} + L_1\,\Phi_{e3} + L_3\,\Phi_{e1})\, \tau_\mathrm{R}\ , \tag{5.54c}$$

which results in m_e

$$m_e = \frac{v_d}{\Lambda} \begin{pmatrix} h_e\,\Phi_{e1} & h_e\,\Phi_{e3} & h_e\,\Phi_{e2} \\ h_\mu\,\Phi_{e2} & h_\mu\,\Phi_{e1} & h_\mu\,\Phi_{e3} \\ h_\tau\,\Phi_{e3} & h_\tau\,\Phi_{e2} & h_\tau\,\Phi_{e1} \end{pmatrix}\ . \tag{5.55}$$

The A_4 symmetry gets now broken by assigning VEVs to the flavons, and they are given by

$$\langle \Phi_\nu \rangle = (v, v, v)^T\ , \tag{5.56a}$$
$$\langle \Phi_e \rangle = (v', 0, 0)^T\ , \tag{5.56b}$$
$$\langle \xi \rangle = w\ . \tag{5.56c}$$

A key ingredient of this model is the structure of these VEVs, the so–called VEV alignment. After breaking the symmetry in such a way the charged lepton Yukawa matrix becomes diagonal,

$$m_e = v_d\ \mathrm{diag}\,(y_e,\, y_\mu,\, y_\tau)\ , \tag{5.57}$$

where $y_{e,\mu,\tau} = h_{e,\mu,\tau}\,\frac{v'}{\Lambda}$. However, the neutrino mass matrix reads as

$$m_\nu = \begin{pmatrix} a+2d & -d & -d \\ -d & 2d & a-d \\ -d & a-d & 2d \end{pmatrix}\ , \tag{5.58}$$

where we define $a = 2\lambda_2\,\frac{v_u^2}{\Lambda_\nu}\,\frac{w}{\Lambda}$ and $d = \sqrt{2}\lambda_1\,\frac{v_u^2}{\Lambda_\nu}\,\frac{v}{\Lambda}$.

After determining the mass matrices of both sectors we can compute the neutrino mixing matrix. As we have shown at the beginning of section 3.1, the neutrino mixing matrix is the mismatch between the diagonalization matrices for the charged lepton and the neutrino mass matrices

$$U_\mathrm{PMNS} = V_{e,L}^\dagger\, V_{\nu,L}\ . \tag{5.59}$$

In the A_4 example this is rather easy since m_e is already diagonal, hence $V_{e,L}^\dagger = \mathbb{1}$ and therefore $U_\mathrm{PMNS} = V_{\nu,L}$. The neutrino mass matrix is diagonalized independently of a, d by the tri–bi–maximal mixing matrix

$$U_\mathrm{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}\ , \tag{5.60}$$

which means that the overall neutrino mixing matrix U_PMNS is given by the tri–bi–maximal mixing matrix U_TBM, originally proposed by Harrison et al. [105].

5.3.2 Deviations from tri–bi–maximal mixing

The main prediction of the just discussed A_4 model is the tri–bi–maximal neutrino mixing pattern where the mixing angles are given by

$$\theta_{12} = \arctan\frac{1}{\sqrt{2}} \approx 35.3^\circ\,, \qquad \theta_{13} = 0^\circ\,, \qquad \theta_{23} = 45^\circ\,, \tag{5.61}$$

which used to be a good description of experimental data, e.g. ten years ago all values were well within limits [71]. However, as we already mentioned in section 5.1.1 current measurements strongly disfavor TBM since θ_{13} and θ_{23} in TBM do not agree with the global fit of experimental data from table 3.1. In particular, the measurement of a large rather than zero θ_{13} is in great tension with TBM predictions,

$$\theta_{13}^{\text{exp}} - \theta_{13}^{\text{TBM}} \approx 9^\circ\,. \tag{5.62}$$

There are a lot of models not based on A_4 suffering from this or similar problems. They either predict TBM as well [105,106] or make very strong predictions for other mixing patterns, e.g. bi–maximal mixing [103,104], which are also ruled out by experiments. However, often not all holomorphic contributions to the mixing parameters were accounted for and higher–order superpotential corrections have to be taken into account, e.g.

$$\mathscr{W}_\nu \supset \frac{1}{\Lambda_\nu\,\Lambda^n}\mathcal{O}_n\,(L\,H_u)\,(L\,H_u) \quad \text{and} \quad \mathscr{W}_e \supset \frac{1}{\Lambda^n}\widetilde{\mathcal{O}}_n\,R\,L\,H_d\,, \tag{5.63}$$

where $\mathcal{O}_n$ and $\widetilde{\mathcal{O}}_n$ are flavon polynomials of the order n. We can then use higher–order corrections, i.e. $n > 1$, to achieve agreement with experiments. The structure and order n of these terms are also dictated by the chosen symmetry through clever assignment of the representations or charges.

The benefit of including higher–order superpotential terms is that models remain calculable and with the help of Hilbert basis computations from section 2.3 it is possible to compute a basis for the superpotential, therefore, determining the superpotential to all orders. However, one has to take into account many more terms and their different contributions, which makes viable experimental predictions quite hard, especially while ensuring that very high orders become negligible. Furthermore, with the help of the Hilbert basis method it might be also possible to construct models where higher–order terms are forbidden to all orders, therefore, fixing the predictions to small order terms.

Another issue for the predictions of most models is that very often they are not invariant under the renormalization group (RG); therefore, corrections from the RG equations can create changes in the predicted mixing angles [116, 117], even though these corrections are loop–suppressed. A somewhat related issue is that most models base their predictions solely on the holomorphic superpotential, which, as we demonstrate in chapter 6, might not be sufficient due to potentially large corrections from the Kähler potential. Before we continue discussing such corrections we briefly review another model which does not predict the TBM mixing pattern.

5.3.3 A model based on a T′ flavor symmetry

So far we discussed a model based on A_4 which gives us TBM mixing, but there is a plethora of other possible groups and mixing patterns. One very well–known and interesting example

is to pick T′ as the flavor symmetry [115]. T′ is the double–covering group of A_4 and it also contains three irreducible singlet representations **1**, **1′**, **1″** and one triplet **3**. In contrast to A_4, there are also three doublets contained in the group, **2**, **2′**, **2″**.

The multiplication law for the contraction of two triplets is the same in both groups, cf. equation (5.47). The contraction between doublets and triplets is given by

$$\mathbf{2}, \mathbf{2'}, \mathbf{2''} \otimes \mathbf{3} \;=\; \mathbf{2} \oplus \mathbf{2'} \oplus \mathbf{2''} \,, \tag{5.64}$$

which in components for the doublet $\boldsymbol{a}$ and the triplet $\boldsymbol{b}$ is given by,

$$\begin{aligned}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\mathbf{2},\mathbf{2'},\mathbf{2''}} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} &= \begin{pmatrix} \sqrt{2}\, a_2\, b_2 + a_1\, b_1 \\ \sqrt{2}\, a_1\, b_3 - a_2\, b_1 \end{pmatrix}_{\mathbf{2},\mathbf{2'},\mathbf{2''}} \oplus \begin{pmatrix} \sqrt{2}\, a_2\, b_3 + a_1\, b_2 \\ \sqrt{2}\, a_1\, b_1 - a_2\, b_2 \end{pmatrix}_{\mathbf{2'},\mathbf{2''},\mathbf{2}} \\ &\quad \oplus \begin{pmatrix} \sqrt{2}\, a_2\, b_1 + a_1\, b_3 \\ \sqrt{2}\, a_1\, b_2 - a_2\, b_3 \end{pmatrix}_{\mathbf{2''},\mathbf{2},\mathbf{2'}} \,. \end{aligned} \tag{5.65}$$

Furthermore, we have multiplications of the doublets among each other, e.g.

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3} \,, \quad \mathbf{2'} \otimes \mathbf{2'} = \mathbf{1''} \oplus \mathbf{3} \,, \quad \mathbf{2''} \otimes \mathbf{2''} = \mathbf{1'} \oplus \mathbf{3} \,. \tag{5.66}$$

All of these and other relations can be found, for instance, in [118] and they are especially important when we discuss non–holomorphic corrections in the next chapter.

However, first, let us discuss the well–known model by Chen et al. [115], which is based on an $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{T'}$ gauge group. Since it is a GUT model it contains **10** and $\overline{\mathbf{5}}$ representations which are put in different representations of T′: the three generations of $\overline{\mathbf{5}}$ in a triplet, and the first two generations of the **10** in a doublet. The third **10** generation, as well as the Higgs multiplets, are singlets under T′. The GUT symmetry is broken by a higher–dimensional multiplet. In addition to the GUT field content, the model has an extended flavon sector, which is summarized in table 5.5. As we can see, the model is also amended by two Abelian $\mathbb{Z}_{12}$ symmetries in order to limit the allowed operators of the Lagrangian. As above in the

	ϕ	ϕ'	ψ	ψ'	ζ	N	ξ	η
T′	**3**	**3**	**2′**	**2**	**1″**	**1′**	**3**	**1**
$\mathbb{Z}_{12}$	3	2	6	9	9	3	10	10
$\mathbb{Z}_{12}$	3	6	7	8	2	11	0	0

Table 5.5: Flavon content of the T′ model.

A_4 example, we have to break the flavor symmetry by assigning VEVs to the flavons along certain directions, which are given by

$$\begin{aligned} \langle \phi \rangle = \phi_0 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \,, \quad \langle \phi' \rangle \;=\; \phi'_0 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \,, \quad \langle \xi \rangle \;=\; \xi_0 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \,, \\ \langle \psi \rangle = \psi_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \,, \quad \langle \psi' \rangle \;=\; \psi'_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \,, \end{aligned} \tag{5.67}$$

while the one–dimensional fields ζ, N and η, acquire non–trivial values.

In the quark sector this VEV assignment leads to a realistic CKM matrix [115], which we do not discuss further since we focus on the lepton sector. There, a lepton mixing pattern is realized which is very close to TBM,

$$\theta_{12} \approx 33^\circ , \quad \theta_{23} = 45^\circ \quad \text{and} \quad \theta_{13} \approx 3^\circ , \tag{5.68}$$

and the deviations are related to the Cabibbo angle θ_c through SU(5) GUT relations [115],

$$\theta_{13} = \frac{\theta_\text{c}}{3\sqrt{2}} . \tag{5.69}$$

Furthermore, this model started an ongoing discussion on group theoretical origins of CP violation[4] and predicts an experimentally valid absolute neutrino mass scale.

Unfortunately, the predictions of this model have been ruled out by the measurement of a large θ_{13}, since

$$\theta_{13}^\text{exp} - \theta_{13}^{\text{T}'} \approx 6^\circ , \tag{5.70}$$

as we know from table 3.1. This and other experimental inaccuracies can be accommodated by including yet another singlet into the flavon sector [121].

It is interesting to see if the predictions of the original model can be saved by including corrections from the non–holomorphic sector, which is what we discuss in the following chapter, first for general flavor models, then for A_4 and T'.

[4] See [119, 120] for the current discussion of CP violation from flavor symmetries.

Chapter 6

Corrections due to Kähler potential terms

In the last chapter we looked at flavor models that, leaving the case of the $U(1)_R$ FN models aside for the moment, describe lepton mixing solely based on the superpotential of the given model. In this chapter we show that this is not enough since there exist corrections coming from the non–holomorphic Kähler potential of the models. As it turns out, these corrections are sizable and (almost) impossible to forbid, contrary to some previous statements in the literature [122, 123]. We derive these corrections for a generic, an A_4 and a T' model, and we end this chapter by discussing the implications for mixing angle predictions and flavor changing neutral currents.

In this chapter we only focus on the lepton sector of the given models. Let us note that similar statements and corrections are possible for the quark sector as well, but this strongly depends on the flavor structure of your model. Due to simplicity, we mainly use the A_4 example from section 5.3.1, i.e. the lepton sector, in order to explain the Kähler corrections.

6.1 Kähler corrections

In section 2.1.4 we introduced the Kähler potential as a real function of chiral and antichiral superfields which is canonical, i.e. diagonal, at tree level. For the lepton sector the Kähler potential, in general, can be written as

$$K \supset L^\dagger \mathscr{K}_L L + R^\dagger \mathscr{K}_R R \tag{6.1}$$

with the left–handed lepton doublets $L = (L_1, L_2, L_3)$ and the right–handed lepton singlets $R = (R_1, R_2, R_3)$. Here, $\mathscr{K}_{L/R}$ is a Hermitian matrix, the so–called Kähler metric. As we have seen in section 2.1.4 we attain the canonical Kähler potential for $\mathscr{K}_{L/R} = \mathbb{1}$.

In section 5.1.1 we also introduced flavor symmetries and we briefly mentioned higher–order correction terms from the superpotential. It is straightforward to argue that the Kähler potential should also contain all possible terms which are allowed by the flavor symmetry, meaning that we have to include higher–order terms,

$$K = K_{\text{canonical}} + \Delta K \,, \tag{6.2}$$

where ΔK contains contractions of L^f and R^f and their Hermitian conjugates with the flavons

$$\Delta K = \left(L^f\right)^\dagger \left(\Delta\mathscr{K}_L\right)_{fg} L^g + \left(R^f\right)^\dagger \left(\Delta\mathscr{K}_R\right)_{fg} R^g \,. \tag{6.3}$$

Here, $\Delta\mathscr{K}_L$ and $\Delta\mathscr{K}_R$ are Hermitian matrices in flavor space which do not have to be diagonal; their specific structure is determined by the chosen flavor symmetry, the flavon

content of the model and the VEV structure. Especially, when the flavons acquire their VEVs, as described in section 5.3, i.e. breaking the flavor symmetry, these matrices modify the Kähler metric.

In order to return to a canonical Kähler potential, we have to rotate the involved fields to account for such higher–order Kähler terms. These field redefinitions then, in turn, affect all operators in the superpotential where redefined fields occur, in particular the Majorana mass matrix and the charged lepton Yukawa matrix. Therefore, this affects the neutrino mixing parameters, which were originally fixed through the flavor symmetry. This can easily be seen when we take a look at Kähler potential part for the lepton doublets L,

$$K_L = (L_f)^\dagger \, (\mathbb{1} + \Delta\mathscr{K}_L)^{fg} \, (L_g) \qquad \rightarrow \qquad K_L = (L')^\dagger_f \mathbb{1}^{fg} (L')_g \; , \tag{6.4}$$

where we redefined the lepton doublet $L \to L'$ in order to attain a canonical Kähler potential. We have to put this redefinition into the superpotential as well, therefore, changing the Majorana mass matrix of the neutrinos,

$$\mathscr{W}_\nu = \frac{1}{2} (L')^T \, m_\nu \, L' \qquad \rightarrow \qquad \mathscr{W}_\nu = \frac{1}{2} L^T \, m'_\nu \, L \; , \tag{6.5}$$

where $m_\nu \neq m'_\nu$. Similar things will happen for the charged lepton Yukawa matrix and, in general, the neutrino mixing matrix will be affected by $\Delta\mathscr{K}_L$, its size being determined by the flavon VEV over the flavor scale.

6.1.1 Effects of canonical normalization

Before we discuss in detail how such field redefinitions can be parametrized and what effects they have, let us clarify our assumptions. Our starting point is a model where the leptonic mixing parameters, the masses, angles and complex phases, are determined through a flavor symmetry, e.g. as we did in chapter 5. However, we do not assume any particular model and our discussion does not depend on the underlying model. Therefore, the only input we consider is the charged lepton Yukawa matrix Y_e and the Majorana neutrino mass matrix m_ν, and for computational reasons we choose a basis where the former is diagonal. This means that the set of input parameters is given by six masses, three charged lepton and three neutrino ones, and the nine mixing parameters we know from section 3.1.1 including the so–called "unphysical" phases $\delta_{e,\mu,\tau}$. Furthermore, we assume that so far the Kähler potential is canonical and higher–order terms have not yet been taken into account.

Now, let us assume we are given a model with a spontaneously broken flavor symmetry explaining the neutrino mixing pattern. The question naturally arises how we can parametrize higher–order Kähler corrections and what effect they have. The contractions of MSSM fields with the flavons can lead to corrections to the canonical Kähler potential, i.e. a change in the Kähler metric in equation (6.1), which is then given by the following Hermitian matrices

$$\mathscr{K}_{L/R} \; = \; \mathbb{1} + \Delta\mathscr{K}_{L/R} \; , \tag{6.6}$$

and due to the presence of $\Delta\mathscr{K}_{L/R}$ the leptons L and R are not canonically normalized anymore. Therefore, we have to rotate the fields in order to remove off–diagonal terms,

$$L' = H_L \, L \; , \tag{6.7a}$$
$$R' = H_R \, R \; , \tag{6.7b}$$

where L' and R' are now again canonically normalized. The matrices $H_{L/R}$ are Hermitian and we have the relation $\Delta\mathscr{K}_{L/R} = H^2_{L/R}$.

The presence of the Kähler corrections $\Delta\mathscr{K}_{L/R}$ forces us to rotate the fields. We assume that these corrections are due to higher order flavor corrections, i.e. they are suppressed by powers of the flavon VEV over the fundamental scale, hence it is reasonable to argue that they can be parametrized by

$$\Delta\mathscr{K}_L = -2\,x_1\,P_L\ , \tag{6.8a}$$
$$\Delta\mathscr{K}_R = -2\,x_2\,P_R\ , \tag{6.8b}$$

where $x_{1,2}$ are infinitesimal parameters and $P_{L/R}$ Hermitian but not necessarily diagonal. We choose a factor -2 for later convenience. Hence, we have $H_{L/R} = \mathbb{1} - x_{1,2}\,P_{L/R}$, to first order in $x_{1,2}$, and for the fields in equation (6.7)

$$L' := (\mathbb{1} - x_1\,P_L)\ L\ , \tag{6.9a}$$
$$R' := (\mathbb{1} - x_2\,P_R)\ R\ . \tag{6.9b}$$

These field redefinitions lead to changes in the neutrino mixing matrix, i.e. in the superpotential, as we already hinted at in equation (6.5). The change up to linear order in x_1 is given by

$$\begin{aligned}\mathscr{W}_\nu &= \frac{1}{2}L^T\,m_\nu^0\,L \\ &\simeq \frac{1}{2}\left[(\mathbb{1}+x_1\,P_L)\ L'\right]^T\ m_\nu^0\ \left[(\mathbb{1}+x_1\,P_L)\ L'\right] \\ &\simeq \frac{1}{2}L'^T\,m_\nu^0\,L' + \frac{1}{2}x_1\,L'^T\ \left(P_L^T\,m_\nu^0 + m_\nu^0\,P_L\right)\ L'\ ,\end{aligned} \tag{6.10}$$

where $m_\nu^0 = m_\nu(x_1 = 0)$. We can now define the neutrino mass operator in dependence of x_1

$$m_\nu(x_1) = m_\nu^0 + x_1\ \left(P_L^T\,m_\nu^0 + m_\nu^0\,P_L\right)\ , \tag{6.11}$$

and, more importantly, the change of m_ν with respect to x_1. This differential equation is given by

$$m_\nu'(x_1) \equiv \frac{\mathrm{d}}{\mathrm{d}x_1}m_\nu(x_1)\ =\ P^T\,m_\nu^0 + m_\nu^0\,P\ , \tag{6.12}$$

and from now on we use the prime notation for the derivative with respect to x_1.[1]

We want to find an analytic expression for the change in mixing parameters with respect to x_1 and for an unspecified Hermitian P_L. Since equation (6.12) has the same form as the RG equation for the neutrino mass operator (cf. equation (B.5) of [116]), we can use the same methods as in [116] in order to find such a solution.

Consider the x_1–dependent mixing matrix $m_\nu(x_1)$. We diagonalize it using an unitary diagonalization matrix $U_\nu(x_1)$, which will also be x_1–dependent, i.e.

$$U_\nu^T(x_1)\,m_\nu(x_1)\,U_\nu(x_1)\ =\ D_\nu(x_1)\ =\ \mathrm{diag}\,(m_1(x_1), m_2(x_1), m_3(x_1))\ , \tag{6.13}$$

[1] The derivative with respect to x_2 can also occur but the difference will be clear within context.

and therefore,

$$m_\nu(x_1) = U_\nu^*(x_1)\, D_\nu(x_1)\, U_\nu^\dagger(x_1)\ . \tag{6.14}$$

If we now plug this expression into equation (6.12), evaluating the result at $x_1 = 0$, we get

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x_1}\left(U_\nu^*(x_1)\, D_\nu(x_1)\, U_\nu^\dagger(x_1)\right)\Big|_{x_1=0} &= (U_\nu')^*\, D_\nu\, U_\nu^\dagger + U_\nu^*\, D_\nu\, (U_\nu')^\dagger + U_\nu^*\, D_\nu'\, U_\nu^\dagger \\ &= P_L^T\, U_\nu^*\, D_\nu\, U_\nu^\dagger + U_\nu^*\, D_\nu\, U_\nu^\dagger\, P_L\ ,\end{aligned} \tag{6.15}$$

where the first line equals the left–hand side and the second line corresponds to the right–hand side of equation (6.12), the latter being evaluated at $x_1 = 0$. In order to further simplify this result we multiply the equation by U_ν^T from the left and by U_ν from the right, giving us

$$U_\nu^T\, (U_\nu')^*\, D_\nu + D_\nu\, (U_\nu')^\dagger\, U_\nu + D_\nu' \ = \ \widetilde{P}_L^T\, D_\nu + D_\nu\, \widetilde{P}_L\ , \tag{6.16}$$

where we define

$$\widetilde{P}_L \ = \ U_\nu^\dagger\, P_L\, U_\nu \ = \ U_{\mathrm{PMNS}}^\dagger\, P_L\, U_{\mathrm{PMNS}}\ . \tag{6.17}$$

Here, we use the fact that $U_e(0,0) = \mathbb{1}$ since we start in a basis where the charged lepton Yukawa matrix is diagonal, hence $U_{\mathrm{PMNS}} = U_\nu$.

6.1.2 Analytic formulae

For any unitary matrix U we can define a matrix T

$$T \ := \ U^\dagger\, U'\ , \tag{6.18}$$

which is anti–Hermitian.[2] Since T is anti–Hermitian it has nine independent parameters

$$u \ := \ \{\mathrm{Re}\, T_{12}, \mathrm{Re}\, T_{13}, \mathrm{Re}\, T_{23}, \mathrm{Im}\, T_{11}, \mathrm{Im}\, T_{12}, \mathrm{Im}\, T_{13}, \mathrm{Im}\, T_{22}, \mathrm{Im}\, T_{23}, \mathrm{Im}\, T_{33}\}\ . \tag{6.19}$$

Using the standard parametrization U_{PMNS} of neutrino oscillations from section 3.1.1 for U in equation (6.18) we can define a $T_{\mathrm{PMNS}} = U_{\mathrm{PMNS}}^\dagger\, U'_{\mathrm{PMNS}}$, and, hence, we can completely determine all entries for a vector u_{PMNS}. Since T_{PMNS} is linear in the derivatives of the mixing parameters we can find a matrix A which maps

$$\xi \ := \ \{\theta_{12}', \theta_{13}', \theta_{23}', \delta', \delta_e', \delta_\mu', \delta_\tau', \varphi_1', \varphi_2'\}\ , \tag{6.20}$$

onto u_{PMNS}, i.e.

$$A\, \xi = u_{\mathrm{PMNS}}\ . \tag{6.21}$$

Using this and T_{PMNS} we can then determine A. Applying the inverted matrix A^{-1} to u_{PMNS} will give us analytic equations for the change in mixing parameters, e.g. θ_{12}' is given by

$$\theta_{12}' = (\xi)_1 = \left(A^{-1}\, u_{\mathrm{PMNS}}\right)_1\ , \tag{6.22}$$

and similarly for the other mixing parameters.

[2] This follows directly from $\frac{\mathrm{d}}{\mathrm{d}x}\mathbb{1} = \frac{\mathrm{d}}{\mathrm{d}x}\left(U^\dagger\, U\right) = (U')^\dagger\ U + U^\dagger\, U' = 0$.

Therefore, we found a general mapping A for any u in order to determine the change in mixing angles. We can can apply this universal A, or rather A^{-1}, to any u and get analytic formulae for the change in mixing parameters. Hence, we can now also define T_ν with the x_1–dependent diagonalization matrix $U_\nu(x_1)$ and put this into equation (6.16) and completely determine T_ν, thus u. If we apply A^{-1} then to this vector we compute the analytic change in mixing angles in dependence on x_1 and P_L.

Now we just need to determine T_ν and we do this by putting its definition into equation (6.16), which we can rewrite as

$$D'_\nu \;=\; \widetilde{P}_L^T\, D_\nu + D_\nu\, \widetilde{P}_L + D_\nu\, T_\nu - T_\nu^*\, D_\nu \;. \tag{6.23}$$

We can see from this equation that the left–hand side is diagonal and real, which in turn has to apply to the right–hand side as well. Therefore, we get

$$m'_i \;=\; 2\,(\widetilde{P}_L)_{ii}\, m_i + \left((T_\nu)_{ii} - (T_\nu^*)_{ii}\right) m_i \;, \tag{6.24}$$

which again has to be real. However, since the first term on the right–hand side is real, because P_L and hence $\widetilde{P}_L$ are Hermitian, and the second term is imaginary, the latter needs to vanish. This leaves us with the condition

$$\mathrm{Im}(T_\nu)_{ii} \;=\; 0 \;. \tag{6.25}$$

Furthermore, we need to consider the off–diagonal terms of equation (6.23), which need to vanish. This gives us the conditions

$$m_i\,(T_\nu)_{ij} - (T_\nu)^*_{ij}\, m_j \;=\; -(\widetilde{P}_L^T)_{ij}\, m_j - m_i\,(\widetilde{P}_L)_{ij} \;, \tag{6.26}$$

such that

$$\mathrm{Re}(T_\nu)_{ij} = -\frac{m_j\ \mathrm{Re}(\widetilde{P}_L)_{ji} + m_i\ \mathrm{Re}(\widetilde{P}_L)_{ij}}{m_i - m_j} \;=\; -\frac{m_i + m_j}{m_i - m_j}\ \mathrm{Re}(\widetilde{P}_L)_{ij} \;, \tag{6.27a}$$

$$\mathrm{Im}(T_\nu)_{ij} = -\frac{m_j\ \mathrm{Im}(\widetilde{P}_L)_{ji} + m_i\ \mathrm{Im}(\widetilde{P}_L)_{ij}}{m_i + m_j} \;=\; -\frac{m_i - m_j}{m_i + m_j}\ \mathrm{Im}(\widetilde{P}_L)_{ij} \;. \tag{6.27b}$$

These equations, together with equation (6.25), now allow us to fully determine T_ν.

Then, if we apply the inverse of the matrix A from equation (6.21) to the vector u, using the just computed components from T_ν as its entries, we get analytic formulae for the change in mixing parameters for general x_1 and P_L. Furthermore, we see that including higher–order terms in the Kähler potential modifies the mixing parameters, in particular the mixing angles θ_{ij}, and we call these changes Kähler corrections.

So far we only considered changes in the neutrino mass operator, but also the charged lepton sector changes due to the field redefinitions in equation (6.9),

$$\begin{aligned}
\mathscr{W}_e &= -R^T\, Y_e^0\, L + \mathrm{h.c.} \\
&= -R'^T \left((H_R)^{-1}\right)^T\, Y_e^0\, (H_L)^{-1}\, L' + \mathrm{h.c.} \\
&\simeq -R'^T\, (\mathbb{1} + x_2\, (P_R)^T)\, Y_e^0\, (\mathbb{1} + x_1\, P_L)\, L' + \mathrm{h.c.} \\
&\simeq -R'^T\, (Y_e^0 + x_2\, (P_R)^T\, Y_e^0 + x_1\, Y_e^0\, P_L)\, L' + \mathrm{h.c.} \;.
\end{aligned} \tag{6.28}$$

Here, the charged lepton Yukawa matrix can be diagonalized by a bi–unitary transformation and we get the unitary matrix $U_e(x_1, x_2)$ acting on the left side. As we can see this matrix depends on x_1 and x_2. Next, we follow a similar procedure as before, e.g. introducing the matrix T_e, but now it is convenient to split T_e into two parts, one matrix $T_e^{x_1}$ that corresponds to the changes due to x_1 and another part $T_e^{x_2}$ that corresponds to the changes from x_2.

We can again determine all terms in T_e and, therefore, the full T matrices in order to get the change in mixing angles. Putting everything together we get [32]

$$T^{x_1} = -U_{\text{PMNS}}^{\dagger}\, T_e^{x_1}\, U_{\text{PMNS}} + T_\nu \,, \tag{6.29a}$$

$$T^{x_2} = -U_{\text{PMNS}}^{\dagger}\, T_e^{x_2}\, U_{\text{PMNS}} \,. \tag{6.29b}$$

Using this result and the matrix A from equation (6.21) we can now derive the change in mixing parameters to first order in x_1 and x_2, i.e. due to P_L and P_R, respectively. The resulting analytic formulae for the Kähler corrections can be found as a `Mathematica` package [124]. Note that without specifying any initial conditions the formulae are very lengthy but they might simplify significantly as we see in section 6.3.2.

6.2 Kähler corrections to example models

In the previous section we showed how Kähler corrections give rise to changes in mixing parameters and we derived analytic formulae describing these changes. The question remains what terms give rise to these corrections. We already mentioned that they are given by contractions of matter fields with flavons. In detail, we are going to discuss two forms of such corrections: linear and quadratic ones.

If we consider, for instance, the lepton doublets L of a given model, assuming they are in an irreducible representation $\boldsymbol{r}$ of some flavor group G_{F}, then linear, i.e. first order corrections will be given by

$$\Delta K_{\text{linear}} = \sum_i \frac{\kappa_{\Phi_i}}{\Lambda} L_{\boldsymbol{r}}^{\dagger}\, (L \otimes \Phi_i)_{\boldsymbol{r}} + \text{h.c.} \,, \tag{6.30}$$

where Φ_i are some of the flavons of the theory and κ_{Φ_i} is a dimensionless coupling. As always, Λ describes the flavor scale. This is an allowed Kähler potential term if $(L \otimes \Phi_i)$ is contracted to the same representation $\boldsymbol{r}$ as L is in, therefore, we can only consider flavons Φ_i whose representation allows such a contraction. When the flavons Φ_i receive their VEV we change the Kähler metric $\mathscr{K}_{fg}$ from its canonical form

$$\mathscr{K}_{fg} = \delta_{fg} + \Delta\mathscr{K}_{fg} = \delta_{fg} + (\alpha P_{fg} + \text{h.c.}) \,, \tag{6.31}$$

with α being a constant times an infinitesimal parameter. Then, $\Delta\mathscr{K}_{fg}$, or rather P_{fg}, leads through canonical normalization to the changes discussed in the previous section.

The linear corrections are only suppressed by one power of the expansion parameter $\langle\Phi_i\rangle/\Lambda$, making them rather large. It is trivial to find such corrections if the flavon is in the singlet representation of the flavor group G_{F}; however, in such cases the flavon VEV does not change the mixing parameters since it will only change the overall normalization and, therefore, we will not consider singlet contributions from here on, neither in linear, nor in quadratic corrections.

There is a general caveat to the discussion of first–order terms. Obviously, the linear corrections are only possible if $L_{\boldsymbol{r}}$ and $(L \otimes \Phi_i)_{\boldsymbol{r}}$ are in the same irreducible representation of G_F and of the overall gauge group. However, if the flavons are charged under any additional symmetry, e.g. an Abelian $\mathbb{Z}_N$, such terms will be forbidden. Furthermore, even if the model does not have any additional symmetry in the first place, it is easy to find one that charges the Φ_i.

The situation is different when we consider second–order terms. These quadratic corrections are of the form, again for the lepton doublets,

$$\Delta K_\mathrm{quadratic} = \sum_{i,j} \sum_{\boldsymbol{r},\boldsymbol{r'}} \frac{\kappa_{\Phi_{i,j}}}{\Lambda^2} \left((L \otimes \Phi_j)^\dagger_{\boldsymbol{r}} \otimes (L \otimes \Phi_i)_{\boldsymbol{r'}} \right)_{\mathbf{1}} \,, \tag{6.32}$$

where $\kappa_{\Phi_{i,j}}$ is a dimensionless coupling and the sum over $\boldsymbol{r}, \boldsymbol{r'}$ represents all possible representations $(L \otimes \Phi_{i,j})$ can be contracted to. Obviously, both factors have to be in the same representation, thus $\boldsymbol{r} = \boldsymbol{r'}$. A similar problem as for the linear contractions arises since a general term as we have it in equation (6.32) can always be forbidden if the various flavons $\Phi_{i,j}$ are charged differently under some additional symmetry. Thus, we only consider terms where $i = j$, simplifying our equation to

$$\Delta K_\mathrm{quadratic} = \sum_{i} \sum_{\boldsymbol{r}} \frac{\kappa_{\Phi_i}}{\Lambda^2} \, (L \otimes \Phi_i)^\dagger_{\boldsymbol{r}} \, (L \otimes \Phi_i)_{\boldsymbol{r}} \,. \tag{6.33}$$

where the contraction to the singlet representation was omitted. In the following, we always imply that we contract Kähler potential terms to the singlet $\mathbf{1}$.

Such terms cannot be forbidden by any conventional symmetry and induce corrections to the mixing parameters, which are suppressed by two powers of the expansion parameter $\langle \Phi_i \rangle / \Lambda$. Furthermore, all flavor models based on spontaneously broken flavor symmetries have them and we can write similar terms for the right–handed leptons as well.

6.2.1 An A_4 example

After introducing the general structure of Kähler corrections we now consider explicit models and we start with the A_4 example by Altarelli et al. [113, 114] from section 5.3.1. The model has two flavon triplets Φ_ν and Φ_e,[3] and since L is also in the triplet representation, we can write the following linear terms for the lepton doublet

$$\Delta K_\mathrm{linear} = \sum_{i \in \{\mathrm{a,s}\}} \left(\frac{\kappa^{(i)}_{\Phi_\nu}}{\Lambda} \, L^\dagger \, (L \otimes \Phi_\nu)_{\mathbf{3}_i} + \frac{\kappa^{(i)}_{\Phi_e}}{\Lambda} \, L^\dagger \, (L \otimes \Phi_e)_{\mathbf{3}_i} \right) + \mathrm{h.c.} \,, \tag{6.34}$$

where the index i differentiates between contractions to the symmetric or antisymmetric triplets $\mathbf{3}_{a,s}$, cf. equation (5.47). We get four different corrections after the flavons acquire

[3] Recall that we do not consider singlets for the Kähler corrections.

their VEVs. The contractions for a generic triplet Φ in components is given by

$$L^\dagger (L\otimes\Phi)_{\mathbf{3}_\mathrm{s}} = \frac{1}{\sqrt{2}}\Big[(L_1^\dagger)(2\,L_1\Phi_1 - L_2\Phi_3 - L_3\Phi_2) + (L_2^\dagger)(2\,L_3\Phi_3 - L_1\Phi_2 - L_2\Phi_1) + (L_3^\dagger)(2\,L_2\Phi_2 - L_1\Phi_3 - L_3\Phi_1)\Big]\,, \tag{6.35a}$$

$$L^\dagger (L\otimes\Phi)_{\mathbf{3}_\mathrm{a}} = \mathrm{i}\sqrt{\frac{3}{2}}\Big[(L_1^\dagger)(L_2\Phi_3 - L_3\Phi_2) + (L_2^\dagger)(L_1\Phi_2 - L_2\Phi_1) + (L_3^\dagger)(L_3\Phi_1 - L_1\Phi_3)\Big]\,. \tag{6.35b}$$

The Kähler metric $\mathscr{K}_{fg}$ changes when the flavons acquire their VEVs, see equation (5.56). For the flavon Φ_ν with VEV $\langle\Phi_\nu\rangle = (v,v,v)^T$ the deviation from the canonical form $\Delta\mathscr{K}_{fg}$ is given by

$$(\Delta\mathscr{K})^{(\mathrm{s})}_{\Phi_\nu} = \kappa^{(s)}_{\Phi_\nu}\frac{v}{\Lambda}\frac{1}{\sqrt{2}}P^{(\mathrm{s})}_{\langle\Phi_\nu\rangle} + \mathrm{h.c.}\,, \tag{6.36a}$$

$$(\Delta\mathscr{K})^{(\mathrm{a})}_{\Phi_\nu} = \mathrm{i}\,\kappa^{(a)}_{\Phi_\nu}\frac{v}{\Lambda}\sqrt{\frac{3}{2}}P^{(\mathrm{a})}_{\langle\Phi_\nu\rangle} + \mathrm{h.c.}\,, \tag{6.36b}$$

with

$$P^{(\mathrm{s})}_{\langle\Phi_\nu\rangle} = \begin{pmatrix} 2 & -1 & -1 \\ -1 & -1 & 2 \\ -1 & 2 & -1 \end{pmatrix}\,, \tag{6.37a}$$

$$P^{(\mathrm{a})}_{\langle\Phi_\nu\rangle} = \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}\,. \tag{6.37b}$$

However, for Φ_e with $\langle\Phi_e\rangle = (v',0,0)^T$ we get the Kähler corrections

$$(\Delta\mathscr{K})^{(\mathrm{s})}_{\Phi_e} = \kappa^{(s)}_{\Phi_e}\frac{v'}{\Lambda}\frac{1}{\sqrt{2}}P^{(\mathrm{s})}_{\langle\Phi_e\rangle} + \mathrm{h.c.}\,, \tag{6.38a}$$

$$(\Delta\mathscr{K})^{(\mathrm{a})}_{\Phi_e} = \mathrm{i}\,\kappa^{(a)}_{\Phi_e}\frac{v'}{\Lambda}\sqrt{\frac{3}{2}}P^{(\mathrm{a})}_{\langle\Phi_e\rangle} + \mathrm{h.c.}\,, \tag{6.38b}$$

with the P matrices

$$P^{(\mathrm{s})}_{\langle\Phi_e\rangle} = \mathrm{diag}(2,-1,-1)\,, \tag{6.39a}$$

$$P^{(\mathrm{a})}_{\langle\Phi_e\rangle} = \mathrm{diag}(0,-1,1)\,. \tag{6.39b}$$

In the model from section 5.3.1 only terms with Φ_e, i.e. equation (6.39a) and equation (6.39b), are inducing changes since Φ_ν is charged under an additional $\mathbb{Z}_4$ shaping symmetry. However, as we discussed above we could introduce additional symmetries to forbid the Φ_e contributions as well.

This is not possible for quadratic terms of the form $(L\otimes\Phi)^\dagger(L\otimes\Phi)$, with $\Phi = \Phi_{\nu,e}$. Checking equation (5.47) we can see that there are six possible correction terms from each flavon,

$$\begin{aligned} &(L\otimes\Phi)^\dagger_{\mathbf{1}}(L\otimes\Phi)_{\mathbf{1}}\,, \quad (L\otimes\Phi)^\dagger_{\mathbf{1}'}(L\otimes\Phi)_{\mathbf{1}'}\,, \quad (L\otimes\Phi)^\dagger_{\mathbf{1}''}(L\otimes\Phi)_{\mathbf{1}''}\,, \\ &(L\otimes\Phi)^\dagger_{\mathbf{3}_a}(L\otimes\Phi)_{\mathbf{3}_a}\,, \quad (L\otimes\Phi)^\dagger_{\mathbf{3}_s}(L\otimes\Phi)_{\mathbf{3}_s}\,, \quad (L\otimes\Phi)^\dagger_{\mathbf{3}_a}(L\otimes\Phi)_{\mathbf{3}_s}\,. \end{aligned} \tag{6.40}$$

As we will see later, there are only five structurally different contributions $P_{\mathrm{I},\ldots,\mathrm{V}}$, cf. equation (6.52). Given a generic triplet Φ, for example $(L\otimes\Phi)^\dagger_{\mathbf{1}}\,(L\otimes\Phi)_{\mathbf{1}}$ is then given by

$$(L\otimes\Phi)^\dagger_{\mathbf{1}}\,(L\otimes\Phi)_{\mathbf{1}} = (L_1\,\Phi_1 + L_2\,\Phi_3 + L_3\,\Phi_2)^\dagger\,(L_1\,\Phi_1 + L_2\,\Phi_3 + L_3\,\Phi_2)\ , \tag{6.41}$$

which gives the Kähler correction

$$\Delta\mathscr{K} = \kappa\,\frac{1}{\Lambda^2}\,P + \text{h.c.}\ , \tag{6.42}$$

with

$$P_\Phi = \begin{pmatrix} \Phi_1^\dagger\,\Phi_1 & \Phi_1^\dagger\,\Phi_3 & \Phi_1^\dagger\,\Phi_2 \\ \Phi_3^\dagger\,\Phi_1 & \Phi_3^\dagger\,\Phi_3 & \Phi_3^\dagger\,\Phi_2 \\ \Phi_2^\dagger\,\Phi_1 & \Phi_2^\dagger\,\Phi_3 & \Phi_2^\dagger\,\Phi_2 \end{pmatrix}\ . \tag{6.43}$$

We now substitute the Φ_ν for the generic flavon Φ and after acquiring its VEV, $\langle\Phi_\nu\rangle = (v,v,v)^T$, we get

$$v^2\,P_{\langle\Phi_\nu\rangle} = v^2\,P_{\mathrm{IV}} = v^2\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\ , \tag{6.44}$$

whereas, when we use Φ_e instead, with $\langle\Phi_e\rangle = (v',0,0)^T$, we get the result

$$(v')^2\,P_{\langle\Phi_e\rangle} = (v')^2\,P_{\mathrm{I}} = (v')^2\ \mathrm{diag}(1,0,0)\ . \tag{6.45}$$

Another possible second–order contraction from equation (6.40) is $(L\otimes\Phi)^\dagger_{\mathbf{3}_s}(L\otimes\Phi)_{\mathbf{3}_a}$, which for a generic triplet Φ gives us

$$\begin{aligned}(L\otimes\Phi)^\dagger_{\mathbf{3}_s}(L\otimes\Phi)_{\mathbf{3}_a} = \mathrm{i}\,\frac{\sqrt{3}}{2}\,\Big[&\left(2\,L_1^\dagger\Phi_1^\dagger - L_2^\dagger\Phi_3^\dagger - L_3^\dagger\Phi_2^\dagger\right)(L_2\Phi_3 - L_3\Phi_2) \\ &+ \left(2L_3^\dagger\Phi_3^\dagger - L_2^\dagger\Phi_1^\dagger - L_1^\dagger\Phi_2^\dagger\right)(L_1\Phi_2 - L_2\Phi_1) \\ &+ \left(2L_2^\dagger\Phi_2^\dagger - L_1^\dagger\Phi_3^\dagger - L_3^\dagger\Phi_1^\dagger\right)(L_3\Phi_1 - L_1\Phi_3)\Big]\ ,\end{aligned} \tag{6.46}$$

and, therefore, the P matrix

$$P = \mathrm{i}\,\frac{\sqrt{3}}{2}\begin{pmatrix} -\Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3 & 2\Phi_1^\dagger\Phi_3 + \Phi_2^\dagger\Phi_1 & -2\Phi_1^\dagger\Phi_2 - \Phi_3^\dagger\Phi_1 \\ -2\Phi_2^\dagger\Phi_3 - \Phi_1^\dagger\Phi_2 & -\Phi_3^\dagger\Phi_3 + \Phi_1^\dagger\Phi_1 & 2\Phi_2^\dagger\Phi_1 + \Phi_3^\dagger\Phi_2 \\ 2\Phi_3^\dagger\Phi_2 + \Phi_1^\dagger\Phi_3 & -2\Phi_3^\dagger\Phi_1 - \Phi_2^\dagger\Phi_3 & \Phi_2^\dagger\Phi_2 - \Phi_1^\dagger\Phi_1 \end{pmatrix}. \tag{6.47}$$

For $\langle\Phi\rangle = \langle\Phi_\nu\rangle = (v,v,v)^T$ this gives the Kähler correction

$$\Delta\mathscr{K} = 3\,\frac{\sqrt{3}}{2}\,\kappa_{\mathrm{V}}\,\frac{v^2}{\Lambda}\,P_{\mathrm{V}} + \text{h.c.}\ , \tag{6.48}$$

with

$$P_{\mathrm{V}} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}\ . \tag{6.49}$$

However, using $\langle\Phi\rangle = \langle\Phi_e\rangle = (v',0,0)^T$ we get

$$\Delta\mathscr{K} = \mathrm{i}\,\frac{\sqrt{3}}{2}\,\kappa\,\frac{(v')^2}{\Lambda}\,P + \text{h.c.}\,, \tag{6.50}$$

with

$$P = \mathrm{diag}(0,1,-1) = P_{\mathrm{II}} - P_{\mathrm{III}}\,. \tag{6.51}$$

These are only four of the in total possible twelve P matrices and the others can be found in appendix D.1.

As we mentioned above, and can see in appendix D.1, most of the contractions lead to identical P matrices, which is the reason for our up to this point random naming, e.g. P_{II} or P_{III}. It turns out, we can summarize all possible corrections for this model in terms of five matrices, $P_{\mathrm{I}}\,\ldots\,P_{\mathrm{V}}$, which we have all encountered so far. We can summarize them by

$$P_{\mathrm{I}} = \mathrm{diag}(1,0,0)\,, \quad P_{\mathrm{II}} = \mathrm{diag}(0,1,0) \quad \text{and} \quad P_{\mathrm{III}} = \mathrm{diag}(0,0,1)\,, \tag{6.52a}$$

which come from contractions of L with Φ_e, and by

$$P_{\mathrm{IV}} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad \text{and} \quad P_{\mathrm{V}} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}\,, \tag{6.52b}$$

which are contributions due to Φ_ν. Since $\langle\Phi_e\rangle = (v',0,0)^T$, the contribution in the Kähler potential of the first three matrices is proportional to $(v')^2$, and since $\langle\Phi_\nu\rangle = (v,v,v)^T$, the contributions of the last two P matrices is proportional to v^2. In section 6.3 we discuss the implications of some of the just presented corrections for the explicit model and we see that they induce significant changes in mixing parameters, despite their quadratic suppression.

We mentioned above that there are also corrections from the right–handed leptons and we want to briefly comment on this. In the A_4 model the right–handed leptons are in the singlet representations $\mathbf{1}, \mathbf{1'}$ and $\mathbf{1''}$. Therefore, the higher–order Kähler potential terms can only look like

$$K_{\text{right}} = \sum_{i=1}^{3} \sum_{n=e,\nu} \frac{\kappa_i^n}{\Lambda^2}\,(R_i\,\Phi_n)^\dagger\,(R_i\,\Phi_n) = \sum_{i=1}^{3} \sum_{n=e,\nu} \kappa_i^n \left(R_i^\dagger\,R_i\right) \left(\frac{\Phi_n^\dagger\,\Phi_n}{\Lambda^2}\right)\,, \tag{6.53}$$

where κ_i^n are dimensionless couplings. Due to this structure, we only get diagonal corrections from the right–handed leptons, i.e. the associated P matrix is given by $P_{\mathrm{R}} = \mathrm{diag}(\alpha_1,\alpha_2,\alpha_3)$, where the coefficients are unrelated and depend on the flavon VEVs and the couplings κ_i^n. However, such corrections can only affect the mass eigenvalues of the charged lepton Yukawa matrix, since we assumed that we are working in a basis where the latter is diagonal.

A variation of the discussed A_4 model could also contain flavons in the $\mathbf{1'}$ and $\mathbf{1''}$ representation, which could lead to non–diagonal corrections in the right setup; however, just as in the case of linear corrections this can easily be forbidden by an additional symmetry since this requires either just one flavon, therefore being a first–order term, or two different flavons.

6.2.2 A T′ example

In the previous section we discussed Kähler corrections for an A_4 example, but of course the T′ example [115] from section 5.3.3 also gives rise to Kähler corrections, and, since we have an extended flavon sector in this model, we expect an even greater variety in possible correction terms.

Taking a look at the flavon content in table 5.5 shows that there are no allowed first–order corrections since all flavons are charged under one of the $\mathbb{Z}_{12}$ symmetries. Furthermore, we have the same corrections as in the A_4 case, i.e. the P matrices from equation (6.52a) and equation (6.52b) are also present. Beyond these terms there are also quadratic corrections coming from the flavon doublets ψ and ψ'. To be specific,

$$(L\otimes\psi)^\dagger_{\mathbf{2}}\,(L\otimes\psi)_{\mathbf{2}}\ , \qquad (L\otimes\psi)^\dagger_{\mathbf{2'}}\,(L\otimes\psi)_{\mathbf{2'}} \qquad \text{and} \qquad (L\otimes\psi)^\dagger_{\mathbf{2''}}\,(L\otimes\psi)_{\mathbf{2''}}\ , \tag{6.54}$$

and there are the same contractions with ψ', but no mixed terms due to their different $\mathbb{Z}_{12}$ charges.

Taking the multiplication law from equation (5.65) we get e.g. the contribution

$$\begin{aligned}(L\otimes\psi')^\dagger_{\mathbf{2'}}\,(L\otimes\psi')_{\mathbf{2'}} = &\left(\sqrt{2}\,\psi'_2\,L_3+\psi'_1\,L_2\right)^\dagger\left(\sqrt{2}\,\psi'_2\,L_3+\psi'_1\,L_2\right)+\\ &+\left(\sqrt{2}\,\psi'_1\,L_1-\psi'_2\,L_2\right)^\dagger\left(\sqrt{2}\,\psi'_1\,L_1-\psi'_2\,L_2\right)\ ,\end{aligned} \tag{6.55}$$

which gives us the Kähler correction

$$\Delta\mathscr{K} = \kappa_{\mathbf{2'2'}}\begin{pmatrix} 2\,(\psi'_1)^2 & -\sqrt{2}\,(\psi'_1)^\dagger\,\psi'_2 & 0\\ -\sqrt{2}\,\psi'_1\,(\psi'_2)^\dagger & (\psi'_1)^2+(\psi'_2)^2 & \sqrt{2}\,(\psi'_1)^\dagger\,\psi'_2\\ 0 & \sqrt{2}\,\psi'_1\,(\psi'_2)^\dagger & 2\,(\psi'_2)^2\end{pmatrix}+\text{h.c.}\ . \tag{6.56}$$

ψ' has the VEV $\langle\psi'\rangle=(\psi'_0,\,\psi'_0)^T$, which gives us

$$\Delta\mathscr{K} \;=\; \kappa_{\mathbf{2'2'}}\,(\psi'_0/\Lambda)^2\,P+\text{h.c.} \qquad \text{with} \qquad P=\begin{pmatrix}2 & -\sqrt{2} & 0\\ -\sqrt{2} & 2 & \sqrt{2}\\ 0 & \sqrt{2} & 2\end{pmatrix}\ . \tag{6.57}$$

Substituting ψ' for ψ, the same term gives us for $\langle\psi\rangle=(\psi_0,0)^T$

$$\Delta\mathscr{K} \;=\; \kappa_{\mathbf{2'2'}}\,(\psi_0/\Lambda)^2\,P+\text{h.c.} \qquad \text{with} \qquad P=\text{diag}(2,1,0)\ . \tag{6.58}$$

In appendix D.2 we compute all other terms from equation (6.54) with the flavons ψ, ψ' and plug in their VEVs. For the corrections coming from the contraction of L and ψ we get the P matrices

$$P_{\text{i}} \;=\; \text{diag}(0,2,1)\ , \quad P_{\text{ii}} \;=\; \text{diag}(1,0,2) \quad \text{and} \quad P_{\text{iii}} \;=\; \text{diag}(2,1,0)\,, \tag{6.59a}$$

which are proportional to $(\psi_0)^2$. The contractions with ψ' lead to

$$\begin{aligned}P_{\text{iv}} &= \begin{pmatrix}2 & \sqrt{2} & -\sqrt{2}\\ \sqrt{2} & 2 & 0\\ -\sqrt{2} & 0 & 2\end{pmatrix}\ , \quad P_{\text{v}} = \begin{pmatrix}2 & 0 & \sqrt{2}\\ 0 & 2 & -\sqrt{2}\\ \sqrt{2} & -\sqrt{2} & 2\end{pmatrix} \quad \text{and}\\ P_{\text{vi}} &= \begin{pmatrix}2 & -\sqrt{2} & 0\\ -\sqrt{2} & 2 & \sqrt{2}\\ 0 & \sqrt{2} & 2\end{pmatrix}\ ,\end{aligned} \tag{6.59b}$$

which are all proportional to $(\psi_0')^2$.

As we already mentioned, these are additional corrections in the T′ model since the P matrices $P_{\mathrm{I}}, \ldots, P_{\mathrm{V}}$ of the A_4 model from equation (6.52a) and equation (6.52b) are also allowed by the group and flavon structure.

6.2.3 General P matrices

So far we discussed specific examples for Kähler corrections; however, it is also possible to make general statements about the correction terms. The higher–order contributions have to be Hermitian and, therefore, the P matrices can be expressed in form of nine Hermitian basis matrices

$$P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad P_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{6.60a}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} , \quad P_5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} , \quad P_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{6.60b}$$

$$P_7 = \begin{pmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad P_8 = \begin{pmatrix} 0 & 0 & -\mathrm{i} \\ 0 & 0 & 0 \\ \mathrm{i} & 0 & 0 \end{pmatrix} , \quad P_9 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\mathrm{i} \\ 0 & \mathrm{i} & 0 \end{pmatrix} . \tag{6.60c}$$

In this language, we can also express our previous results, e.g. the P matrices for T′ from equation (6.59a) and equation (6.59b),

$$\begin{aligned} &P_{\mathrm{i}} = 2\,P_3 + P_6 , \qquad P_{\mathrm{ii}} = P_1 + 2\,P_6 , \qquad P_{\mathrm{iii}} = 2\,P_1 + P_3 , \\ &P_{\mathrm{iv}} = 2\,(P_1 + P_3 + P_6) + \sqrt{2}\,(P_2 - P_4) , \\ &P_{\mathrm{v}} = 2\,(P_1 + P_3 + P_6) + \sqrt{2}\,(P_4 - P_5) , \\ &P_{\mathrm{vi}} = 2\,(P_1 + P_3 + P_6) + \sqrt{2}\,(P_5 - P_2) , \end{aligned} \tag{6.61}$$

which makes the application of our formulae from section 6.1.2 much easier.

6.3 Implications

In the previous sections we showed that corrections due to Kähler potential terms occur in models based on (non–)Abelian flavor symmetries and in section 6.1.2 we derived general formulae, which describe how the mixing angles change. Now, we want to apply this knowledge in practice, i.e. we want to see how the correction terms, which we derived for explicit examples in section 6.2, affect the mixing angles for these well–known models. Afterwards, we discuss the effects of these Kähler corrections on VEV alignment and FCNCs.

6.3.1 Corrections for general P matrices

Before we start discussing explicit examples, let us remind ourselves that all correction terms from section 6.2 can be described with the help of nine Hermitian basis matrices P, cf. equation (6.60), and we can summarize the Kähler corrections for the P matrices by

assuming certain initial conditions, e.g. bi–maximal or tri–bi–maximal mixing. We take the Kähler potential to be

$$K \;=\; L^\dagger \,(1 + x_L \, P_L)\; L + R^\dagger \, R \,, \tag{6.62}$$

where in this scenario only the left–handed sector is modified and we take P_L to be one of the nine P matrices from equation (6.60). For all matrices we take $x_L = 0.01$ and we use for the mass differences the PDG [71] values, cf. section 3.2. Furthermore, we fix the absolute mass scale of the neutrinos by setting $m_1 = 0.01\,\text{eV}$. Then, using bi–maximal mixing as our initial conditions, i.e.

$$\begin{aligned} &\theta_{12} = \frac{\pi}{4} \,, \quad \theta_{13} \;=\; 0 \,, \quad \theta_{23} \;=\; \frac{\pi}{4} \,, \quad \delta \;=\; \text{undefined} \,, \\ &\delta_e = \pi \,, \quad \delta_\mu \;=\; \pi \,, \quad \delta_\tau \;=\; 0 \,, \quad \varphi_1 \;=\; \varphi_2 \;=\; 2\pi \,, \end{aligned} \tag{6.63}$$

we get the results in table 6.1. On the other hand, table 6.2 contains the results when we

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
$\Delta\theta_{12}$ [°]:	-1.0	0.20	0.51	0.20	1.0	0.51	0	0	0
$\Delta\theta_{13}$ [°]:	0	-0.12	-0.016	0.12	0	0.016	-0.076	0.076	0.012
$\Delta\theta_{23}$ [°]:	0	-0.023	-0.23	0.023	-0.29	0.23	0	0	0

Table 6.1: Changes of the mixing angles under Kähler corrections of the form $\Delta K = x\, L^\dagger \, P_i \, L$ for $x = 0.01$ (cf. equation (6.60)) starting from bi–maximal mixing.

use tri–bi–maximal mixing as our initial condition, i.e.

$$\begin{aligned} &\theta_{12} = \arcsin \frac{1}{\sqrt{3}} \,, \quad \theta_{13} \;=\; 0, \quad \theta_{23} \;=\; \frac{\pi}{4} \,, \quad \delta \;=\; \text{undefined} \,, \\ &\delta_e = \pi \,, \quad \delta_\mu \;=\; \pi \,, \quad \delta_\tau \;=\; 0 \,, \quad \varphi_1 \;=\; \varphi_2 \;=\; 2\pi \,. \end{aligned} \tag{6.64}$$

We should note that in both initial conditions the phase δ is not defined since θ_{13} is zero.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
$\Delta\theta_{12}$ [°]:	-0.96	-0.28	0.48	-0.28	0.96	0.48	0	0	0
$\Delta\theta_{13}$ [°]:	0	-0.12	-0.015	0.12	0	0.015	-0.073	0.073	0.012
$\Delta\theta_{23}$ [°]:	0	-0.021	-0.24	0.021	-0.29	0.24	0	0	0

Table 6.2: Changes of the mixing angles under Kähler corrections of the form $\Delta K = x\, L^\dagger \, P_i \, L$ for $x = 0.01$ (cf. equation (6.60)) starting from tri–bi–maximal mixing.

Therefore, we ensured for all P matrices that the transition of δ at $\theta_{13} = 0$ is analytical [32].

6.3.2 Mixing angles in the A_4 example

In section 5.3.1 we discussed a flavor model based on A_4 which predicted tri–bi–maximal mixing. However, we showed in section 6.2.1 that Kähler corrections occur for such a

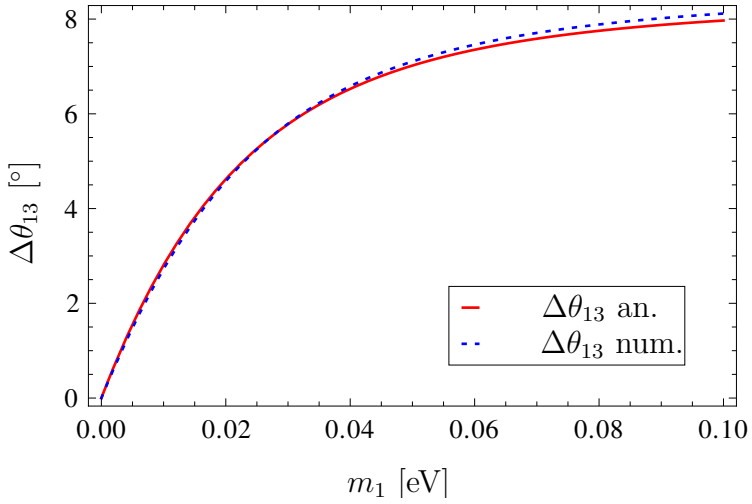


Figure 6.1: The effect of the Kähler correction ΔK on θ_{13}, shown in equation (6.65) for $\kappa_{\mathrm{V}}\, v^2/\Lambda^2 = (0.2)^2$. The continuous line plots equation (6.66) while the dashed line represents a numerical computation.

setting. In fact, there are five possible correction terms described by the matrices $P_{\mathrm{I},\ldots,\mathrm{V}}$ in equation (6.52). Take for example the correction due to P_{V}, which originates from $(L \otimes \Phi_\nu)^\dagger_{\mathbf{3}_s} (L \otimes \Phi_\nu)_{\mathbf{3}_a}$. Following its derivation in equation (6.47) we can determine the additional Kähler potential term

$$\Delta K \;=\; \kappa_{\mathrm{V}} \cdot \frac{v^2}{\Lambda^2} \cdot 3\sqrt{3} \cdot (L^f)^\dagger \, (P_{\mathrm{V}})_{fg} \, (L^g) \;, \tag{6.65}$$

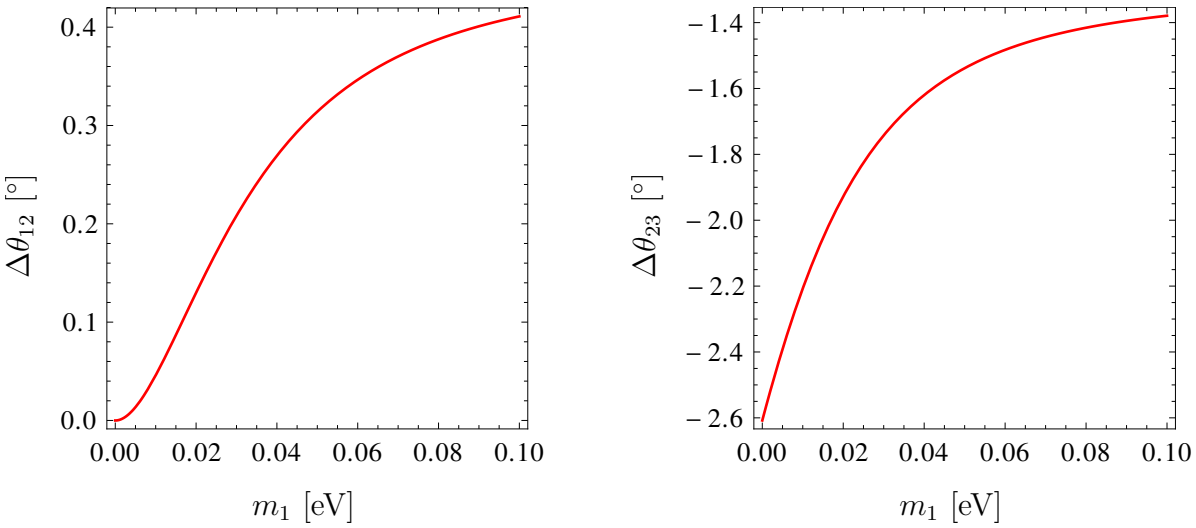
where κ_{V} denotes an undetermined Kähler coefficient. Using our analytic formulae from section 6.1.2 here and setting the initial conditions to tri–bi–maximal mixing we get a rather simple formula for the change in θ_{13},

$$\begin{aligned} \Delta\theta_{13} &= \kappa_{\mathrm{V}} \cdot \frac{v^2}{\Lambda^2} \cdot 3\sqrt{3} \cdot \frac{1}{\sqrt{2}} \left(\frac{2m_1}{m_1 + m_3} + \frac{m_e^2}{m_\mu^2 - m_e^2} + \frac{m_e^2}{m_\tau^2 - m_e^2} \right) \\ &\simeq \kappa_{\mathrm{V}} \cdot \frac{v^2}{\Lambda^2} \cdot 3\sqrt{6} \; \frac{m_1}{m_1 + m_3} \;, \end{aligned} \tag{6.66}$$

where the m_i are the neutrino masses and we simplified the formula by ignoring the small contributions of the charged leptons. This even simplifies further since for very large m_1, compared to the mass differences Δm_{ij}, we get ${}^{m_1}\!/_{(m_1+m_3)} \to {}^1\!/_2$.

Using the PDG [71] values for the mass differences we can plot $\Delta\theta_{13}$ only depending on m_1. The result is shown in figure 6.1. Note that we set the ratio of VEV v over scale Λ to 0.2 and choose the Kähler coefficient $\kappa_{\mathrm{V}} = 1$. As we can see in this figure the correction to θ_{13} is rather large; recall that the angle is zero in the uncorrected model. In particular $\Delta\theta_{13} \approx 8.4°$ for large m_1 as can be seen from our analytic formula in equation (6.66) for $m_1 \gg \Delta m_{ij}$.

If we now consider the change in the other two mixing angles, θ_{12} and θ_{23}, we would assume from table 6.2 that there is no change due to P_{V} since there is no change in mixing angles due to P_7, P_8 or P_9, which make up P_{V}. However, the values in table 6.2, and also in table 6.1, are computed for a fixed mass scale and only using the linear approximation



(a) $\Delta\theta_{12}$. (b) $\Delta\theta_{23}$.

Figure 6.2: Changes of (a) θ_{12} and (b) θ_{23} due to the Kähler correction ΔK shown in equation (6.65) for $\kappa_{\mathrm{V}}\, v^2/\Lambda^2 = (0.2)^2$, which were computed numerically.

for the change in mixing angles. But we just showed that e.g. θ_{13} experiences large Kähler corrections, which in turn also influences the other mixing angles. We, therefore, computed numerically the change in θ_{12} and θ_{23} [117] depending on the size of m_1. The results can be found in figure 6.2, again using $\kappa_{\mathrm{V}}\, v^2/\Lambda^2 = (0.2)^2$ and the PDG [71] values for the mass differences. We can see in these plots that, in contrast to θ_{13}, the change in mixing angles is much smaller and, therefore, θ_{12} and θ_{23} remain almost unaffected [31, 32].

If we assume that the Kähler corrections are only created by P_{V}, we could make the model phenomenologically competitive since the current experimental value for θ_{13} is between 8° and 9°, cf. table 3.1, which is compatible with figure 6.1. Also, the other angles do not change much, as we see in figure 6.2, and the tri–bi–maximal predictions for them are within the acceptable reach. Kähler corrections due to the matrix P_{V} are also interesting because they might create CP violation since the entries of P_{V} are complex. Especially, this could be of interest in regard with the ongoing discussion of CP violation from group theory and in flavor model building [119, 120].

However, so far we only discussed corrections due to P_{V} and not due to any of the other matrices we computed in section 6.2.1. These matrices also contribute to the correction of the mixing angles and their effects might be quite different. If we consider for example the Kähler corrections due to P_{IV}, which generated by the singlet contractions, e.g. $(L \otimes \Phi_\nu)^\dagger_{\mathbf{1}}\, (L \otimes \Phi_\nu)_{\mathbf{1}} + \mathrm{h.c}$, we get the additional Kähler potential term

$$\Delta K \;=\; \kappa_{\mathrm{IV}} \cdot \frac{v^2}{\Lambda^2} \cdot 2 \cdot (L^f)^\dagger\, (P_{\mathrm{IV}})_{fg}\, (L^g)\;, \tag{6.67}$$

where κ_{IV} is an undetermined Kähler coefficient. Using our analytic formulae we can now

determine the change e.g. in θ_{12}, and this is given by

$$\Delta\theta_{12} \;=\; \kappa_{\mathrm{IV}} \cdot \frac{v^2}{\Lambda^2} \cdot \sqrt{2} \cdot \frac{\left(m_\mu^2\, m_\tau^2 - m_e^4\right)}{\left(m_e^2 - m_\mu^2\right)\left(m_e^2 - m_\tau^2\right)} \;, \tag{6.68}$$

which only depends on the charged lepton masses and not on the neutrino masses. We can plug in the masses and get a change $\Delta\theta_{12} \approx 3.2^\circ$ for $\kappa_{\mathrm{IV}} = 1$ and $v/\Lambda = 0.2$. For θ_{23} we get a similar result, also independent of the neutrino masses, and we get a mixing angle change $\Delta\theta_{23} \approx -2.3^\circ$. θ_{13} is not changed by the Kähler correction from P_{IV}. Note that there is an ambiguity concerning κ_{IV} since its sign is undetermined and therefore also the signs of the $\Delta\theta_{ij}$; either way, one of the angles, i.e. θ_{12} or θ_{23}, is driven away from its best fit value while $\Delta\theta_{13} = 0$.

We further discuss the general implications of these results for flavor model building in section 6.3.4.

6.3.3 Mixing angles in the T′ example

In section 5.3.3 we described a flavor model based on the non–Abelian group T′ which predicts almost tri–bi–maximal mixing but with $\theta_{13} \approx 3^\circ$. However, Kähler corrections occur in this model, as we have seen in section 6.2.2. In particular, the just for A_4 described matrices $P_{\mathrm{I},\ldots,\mathrm{V}}$ occur, due to the T′ triplets, as well as additional corrections from the matrices $P_{\mathrm{i},\ldots,\mathrm{vi}}$, cf. equation (6.59). The latter are generated from the T′ doublets. Naturally, we can ask ourselves if the same mixing angle changes as for A_4 arise in the T′ model.

Even though the same correction terms as in A_4 occur, the results are different in T′. The reason for this is given by the different initial conditions in both models. The A_4 model starts from tri–bi–maximal mixing, contrary to the T′ model which predicts $\theta_{12} \approx 33^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} \approx 3^\circ$. Furthermore, our formulae are only applicable in a situation where the charged lepton Yukawa matrix is diagonal, which is also not the case for the T′ model. Hence, we first have to perform a basis transformation in order to diagonalize the charged lepton Yukawa matrix. This will not affect the mixing matrix and, thus, also not the mixing angles, since it is a basis transformation. However, the structure of the correction matrices P changes.

So far we determined the P matrices in section 6.2.2 from a Kähler potential term $(L \otimes \Phi)^\dagger_{\boldsymbol{r}}\, (L \otimes \Phi)_{\boldsymbol{r}}$ which leads, after VEV insertion, to the Kähler correction

$$\Delta K \;\supset\; L^\dagger\, P\, L + \mathrm{h.c.}\;, \tag{6.69}$$

in a basis where the charged lepton Yukawa matrix is non–diagonal. In order to use our formulae we have to diagonalize the Yukawa matrix, which redefines the charged leptons L through some matrix V. Therefore, we also need to redefine the P matrix

$$L^\dagger\, P\, L \;\to\; (V\, L)^\dagger\, P\, V\, L \;=\; L^\dagger\, \widetilde{P}\, L\;, \tag{6.70}$$

where we have $\widetilde{P} := V^\dagger\, P\, V$.

Using this method, and the correct initial conditions, we can now analyze the effects of the Kähler corrections for the T′ model. We start by looking at the matrix P_{V}, or rather $\widetilde{P}_{\mathrm{V}}$, which had a rather large effect in the A_4 model. We have to consider that the T′ model predicts absolute neutrino masses, e.g. $m_1 = 0.0156\,\mathrm{eV}$, hence the mass scale is fixed and

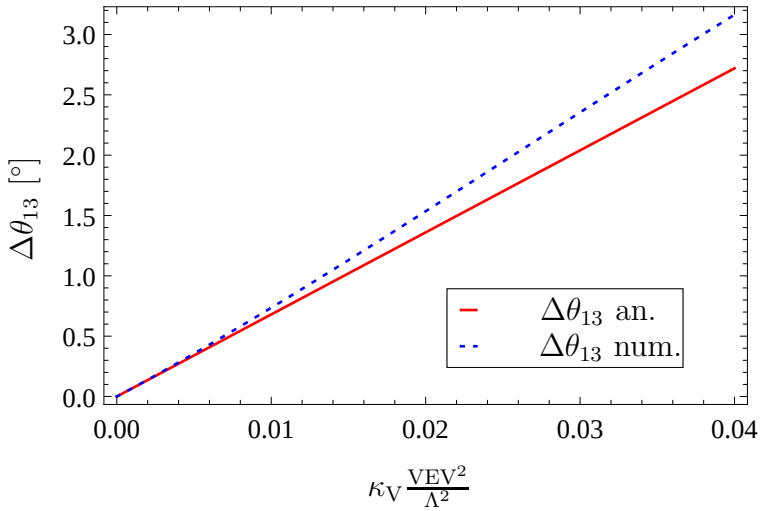


Figure 6.3: Change of θ_{13} in the T′ model due to the Kähler correction from the matrix P_V. We distinguish between the results of the analytical formulae (continuous line) and a numerical computation (dashed line).

we can only vary the expansion parameter $x = \mathrm{VEV}^2/\Lambda^2$ and some Kähler coefficient κ_V, which we set $\kappa_V = 1$. The change in θ_{13} can be found in figure 6.3.

From this plot we see that the maximal $\Delta\theta_{13}$ we can get is around 3°, which would give us $\theta_{13} \approx 6°$. There is the possibility that several flavons contribute to this angle, but this requires for their effects to align and to be of similar strength in order to get $\theta_{13} \approx 9°$, which is rather unrealistic. Furthermore, the original T′ model [115] needs to set rather small VEVs in which case the Kähler corrections become irrelevant.

Let us now consider corrections that are not present in A_4 models, i.e. corrections due to the matrices $P_{i,\ldots,vi}$ from equation (6.59). For example, we can pick P_{vi}, which is due to the contraction $(L \otimes \psi')^\dagger_{\mathbf{2'}}(L \otimes \psi')_{\mathbf{2'}}$ in equation (6.55), where ψ' is a T′ flavon doublet. Before we compute the change in mixing angles, we have to again modify the matrix and, hence, get $\widetilde{P}_{vi}$ in a basis where the charged lepton Yukawa matrix is diagonal. Then using the correct initial conditions and neutrino masses we can plot the mixing angle change, e.g. $\Delta\theta_{23}$, over the expansion parameter VEV^2/Λ^2 in figure 6.4.

This plot shows us that $\Delta\theta_{23}$ does not get particularly large even for $\kappa_{vi}\,\mathrm{VEV}^2/\Lambda^2 = (0.2)^2$. Also, one should consider the smallness of the VEVs in the original T′ model, besides for one flavon, which would again render the effect to be insignificant.

6.3.4 General comments on flavor symmetries

In the previous sections we discussed two interesting models and the effects that Kähler corrections have in these cases. For both models we solely focus on quadratic corrections of the left–handed leptons since linear corrections, although they would be less suppressed, can easily be forbidden by an additional symmetry. The corrections for the right–handed fields are not considered since they only change the overall normalization or can be forbidden as well.

For the first model, based on A_4, we generate phenomenologically viable $\theta_{13} \approx 8°$ through

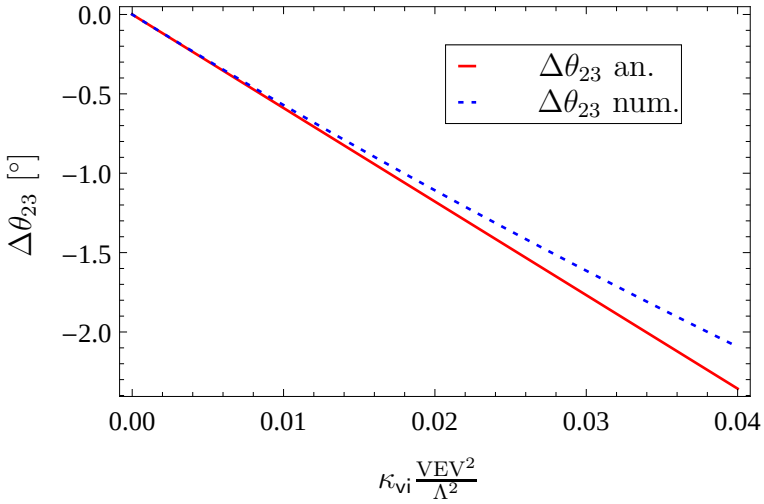


Figure 6.4: Change of θ_{23} in the T′ model due to the Kähler correction from the matrix P_{vi}. We distinguish between the results of the analytical formulae (continuous line) and a numerical computation (dashed line).

the consideration of the Kähler corrections. However, this was based only on the analysis of one correction matrix P_{V} while ignoring four other such matrices. We showed that for the case of P_{IV} the effects are very different and actually do not improve the situation since either θ_{12} or θ_{23} are driven away from their best fit value, and similar situations occur for the other P matrices, i.e. their effects are not compatible with experimental measurements [31, 32].

The situation is different for the second model, based on T′. Due to the changed initial conditions, e.g. non–zero θ_{13} and non–diagonal charged lepton Yukawa matrix, the effects of Kähler corrections on the mixing angles are smaller. Even if we assume a much larger VEV over scale ratio than is given in the model, recall that in the original model [115] the ratio is usually quite small, we can lift e.g. θ_{13} only up to $\approx 6^\circ$ [32].

We derived analytic formulae in section 6.1.1 and we also provide a `Mathematica` package [124], which allows us to apply our discussion to other concrete models with some flavor group G_F. As we mentioned previously, quadratic corrections with one flavon Φ, i.e. $(L \otimes \Phi)^\dagger_{\boldsymbol{r}} \, (L \otimes \Phi)_{\boldsymbol{r}}$ where $\boldsymbol{r}$ is a representation of G_F, cannot be forbidden by a conventional symmetry and will always be present; thus one always has to consider Kähler corrections when building a flavor model. The only way to avoid such corrections is to decrease the size of the flavon VEV, which is usually not allowed for all flavons since the expansion parameter of the symmetry breaking is given by VEV over scale and, therefore, determines the size of the couplings.

The effects of the Kähler corrections show on the one hand that it might be premature to rule out certain flavor models solely on their predictions from the superpotential. It seems possible that previously dismissed models can become phenomenologically interesting if we consider corrections from the Kähler potential, which bares hope for some simple models. On the other hand, we showed that the inclusion of Kähler corrections can destroy the nice properties of rather sophisticated constructions since they are difficult to control and (almost) impossible to forbid. Therefore, very fine–tuned models might be spoiled by the inclusion of the effects.

This generates a difficult to control problem. One way out is to try to build viable models, which include all effects from the superpotential and the Kähler potential. This can be rather cumbersome since it is unclear how to control the Kähler corrections in a particular way. Another option is to address precisely this part of the problem and try to gain a better understanding of the Kähler potential and its terms. Especially the undetermined Kähler coefficients play an important role since their size controls the effect of the Kähler corrections. We might be able to compute them analytically, e.g. through higher–dimensional calculations in string theory [125–127] or wave function overlaps [128, 129]. However, so far, for our purposes no significant progress has been made, thus we need to be careful when making predictions in flavor model building, in particular, when they are solely based on the superpotential.

6.3.5 VEV alignment

We have seen that the Kähler corrections might have severe effects on the mixing angles and it is, therefore, somewhat natural to ask if they have further implications. One important aspect of flavor model building is the issue of VEV alignment, i.e. the particular form the flavon VEVs take in a given theory, and one might wonder if the Kähler corrections affect this alignment, e.g. by changing the F–flatness conditions of the given theory. As it turns out [32] this is not the case since the conditions are the same for a canonical and for the corrected Kähler potential.

Furthermore, the Kähler corrections also do not change the D–flatness conditions since for the usual models there is only the SM gauge group present and the flavon is a singlet with respect to this group, e.g. in the A_4 model. If there is a larger gauge group and the flavon is also charged under this group the situation is more complicated and one has to check this by hand. An exception to this is if there is only one VEV acquiring field that is also charged under the larger gauge group; then, the D–flatness conditions automatically remain unchanged [32], e.g. for our T′ example.

Therefore, we see that for most models the Kähler corrections do not affect the VEV alignment of a given model.

6.3.6 FCNCs

At the end of section 5.1.1 we discussed the importance of flavor symmetries in order to forbid dangerous flavor changing neutral currents. However, the discussion in section 5.1.1 does not take into account the possibility of Kähler corrections, which might induce FCNCs. Let us recall the relevant Kähler potential soft–breaking terms from section 2.1.6

$$K \supset \left(\frac{X}{\Lambda_{\text{soft}}} \, n_{fg} \left(L^f\right)^\dagger L^g + \text{h.c.} \right) + \frac{X^\dagger X}{\Lambda_{\text{soft}}^2} \, k_{fg} \left(L^f\right)^\dagger L^g + L \to R \,, \tag{6.71}$$

and the soft–breaking superpotential terms, i.e. the A terms

$$\mathscr{W} \supset \frac{X}{\Lambda_{\text{soft}}} \left(Y^X\right)_{fg} L^f \, R^g \, H_d \,, \tag{6.72}$$

where we replaced the superfield Ψ from equation (2.18) and from equation (2.17) with the appropriate lepton or Higgs fields. Since we assume a flavor symmetry to be present, we

know from our discussion in section 5.1.1 that the matrices n and k are diagonal in flavor space and we can define

$$(n_{L/R})_{fg} = \kappa_{L/R}\,\delta_{fg}\ , \tag{6.73a}$$
$$(k_{L/R})_{fg} = \kappa'_{L/R}\,\delta_{fg}\ , \tag{6.73b}$$

with $\kappa_{L/R}$ and $\kappa'_{L/R}$ being order one coefficients. We also assume that we start from a canonical Kähler potential before we take the flavon corrections into account, i.e. the Kähler metric $\mathscr{K}$ is diagonal as well, $\left(\mathscr{K}_{L/R}\right)_{fg} = \delta_{fg}$.

These Kähler potential and superpotential terms lead to the soft–breaking Lagrangian

$$\mathscr{L}_{\text{soft}} \supset -(\widetilde{\ell}^f)^\dagger\,(\widetilde{m}^2_{\text{LL}})_{fg}\,\widetilde{\ell}^g - (\widetilde{r}^f)^\dagger\,(\widetilde{m}^2_{\text{RR}})_{fg}\,\widetilde{r}^g - \left(\widetilde{\ell}^f\,(A_{\text{LR}})_{fg}\,\widetilde{r}^g + \text{h.c.}\right)\ , \tag{6.74}$$

where $\widetilde{\ell}$ and $\widetilde{r}$ denote the left– and right–handed slepton fields, respectively. As we described in equation (2.19), the supersymmetry breaking parameter is defined by the spurion VEV over the breaking scale and for our purposes here we define $\widetilde{M}^2 = \frac{|\langle F_X\rangle|^2}{\Lambda^2_{\text{soft}}}$. Using this we can express the terms in equation (6.74) by

$$\left(\widetilde{m}^2_{\text{LL/RR}}\right)_{fg} = \widetilde{M}^2\,\left[\kappa'_{L/R} + |\kappa_{L/R}|^2\right]\,\delta_{fg}\ , \tag{6.75a}$$
$$(A_{\text{LR}})_{fg} = \sqrt{\widetilde{M}^2}\,\left[(Y_e^X)_{fg} + (n_L)_{fi}\,(Y_e)_{ig} + (Y_e)_{fi}\,(n_R)_{ig}\right]\ , \tag{6.75b}$$

where we used equation (6.73) in the first line and where Y_e is the usual electron Yukawa coupling.

As we can see in equation (6.75a) the soft mass matrix starts out to be diagonal; however, this is before we consider the Kähler corrections. It is also possible to have contractions of a flavon Φ with the soft–breaking terms, e.g.

$$K \supset \kappa'\,\frac{X^\dagger X}{\Lambda^2_{\text{soft}}}\,\frac{1}{\Lambda^2}\,(L\otimes\Phi)^\dagger_{\boldsymbol{r}}\,(L\otimes\Phi)_{\boldsymbol{r}}\ , \tag{6.76}$$

where $\boldsymbol{r}$ is some representation of the flavor group. After the flavon acquires its VEV we get effective coupling matrices, which do not necessarily have to be diagonal

$$(\mathscr{K}_{L/R})_{fg} = \delta_{fg} - 2\,x\,(P_{\text{kin},L/R})_{fg}\ , \tag{6.77a}$$
$$(n_{L/R})_{fg} = \kappa_{L/R}\,\left[\delta_{fg} - 2\,x\,(N_{L/R})_{fg}\right]\ , \tag{6.77b}$$
$$(k_{L/R})_{fg} = \kappa'_{L/R}\,\left[\delta_{fg} - 2\,x\,(P_{\text{soft},L/R})_{fg}\right]\ , \tag{6.77c}$$

where x is the expansion parameter of flavor symmetry breaking. P_{kin} describes the Kähler corrections which we have discussed so far, e.g. in section 6.2, whereas N and P_{soft} come from terms like equation (6.76), i.e. the contractions with the soft–breaking terms, and all of these matrices are unrelated. The effective couplings are suppressed by the expansion parameter x, which is usually given by $(\text{VEV}/\Lambda)^2$ since the terms are normally created through quadratic corrections.[4]

[4] Recall from section 6.2 that linear corrections can always be forbidden by some additional symmetry which is not the case for quadratic ones.

The Kähler metric now contains off–diagonal terms and, as in section 6.1.1, we have to rotate the lepton fields in order to canonically normalize them,

$$L^f \to L'^f = \left[\delta_{fg} + x\,(P_{\text{kin},L})_{fg}\right]\, L^f \ , \tag{6.78a}$$
$$R^f \to R'^f = \left[\delta_{fg} + x\,(P_{\text{kin},R})_{fg}\right]\, R^f \ , \tag{6.78b}$$

and, therefore, we get transformed coupling matrices

$$(n'_{L/R})_{fg} = \kappa_{L/R}\,\left[\delta_{fg} + 2\,x\,\Big((P_{\text{kin},L/R})_{fg} - (N_{L/R})_{fg}\Big)\right] \ , \tag{6.79a}$$
$$(k'_{L/R})_{fg} = \kappa'_{L/R}\,\left[\delta_{fg} + 2\,x\,\Big((P_{\text{kin},L/R})_{fg} - (P_{\text{soft},L/R})_{fg}\Big)\right] \ . \tag{6.79b}$$

Note that we only consider terms which are first order in x, e.g. there are no terms $\propto x^2\,P_{\text{kin}}\,P_{\text{soft}}$. Furthermore, we might have to rotate into a basis where the charged lepton Yukawa matrix is diagonal in order to compare our results to experiments. However, we start with a diagonal Yukawa matrix by assumption, and the off–diagonal terms are induced by corrections of the order x. Therefore, this will not affect n or k at the linear order since we ignore higher order terms.

Inserting these results into equation (6.75a) we get the soft mass matrix for the sleptons, up to linear order in x,

$$\begin{aligned}(\widetilde{m}^2_{\text{LL/RR}})_{fg} \ = \widetilde{M}^2\ \Big\{ & \left(\kappa'_{L/R} + \left|\kappa_{L/R}\right|^2\right)\,\delta_{fg} \\ & +\ 2\,x\,\kappa'_{L/R}\,\Big[(P_{\text{kin},L/R})_{fg} - (P_{\text{soft},L/R})_{fg}\Big] \\ & +\ 2\,x\,\left|\kappa_{L/R}\right|^2\,\Big[(P_{\text{kin},L/R})_{fg} - (N_{L/R})_{fg} + \text{h.c.}\Big]\Big\} \ .\end{aligned} \tag{6.80}$$

We see in this equation that all off–diagonal terms are suppressed by a factor x compared to diagonal terms and one might even think of cases where there are cancellations between P_{kin}, N and P_{soft}, which remove the off–diagonal elements. However, for the following discussion we assume that the off–diagonal terms are of order x.

For the A terms the situation is similar. Before we take the Kähler corrections into account the A terms in equation (6.75b) are, in first order, diagonal since we assume that we are in a basis where the charged lepton Yukawa matrix Y_e is diagonal. However, due to the transformations in equation (6.78) we modify the second and third term in equation (6.75b) and from equation (6.79a) we see that the thus induced off–diagonal terms are suppressed by a factor of x. Also, the smallness of the lepton masses suppresses these entries even further. Furthermore, at the end of section 5.1.1 we argued that there should not be a large deviation in size between the Yukawa couplings and the A terms, so dangerously large entries in Y_e^X should not occur. This assumption remains unchanged since the rotations in equation (6.78) should only affect off–diagonal terms in Y_e^X up to a factor of x times the diagonal entries which are small to begin with. Therefore, this does not pose a threat.

We see in equation (6.80) that the Kähler corrections create off–diagonal terms for the soft masses and we just discussed the case for the A terms. In both cases the off–diagonal entries are at most of order x or even smaller. Nonetheless, FCNCs are induced by such entries, in general through slepton, chargino, higgsino and neutralino loops. This poses experimental constraints on the size of the off–diagonal terms and the strongest constraint comes from

the process $\mu \to e\,\gamma$, see e.g. figure 5.1 in section 5.1. In SUSY, photino and slepton loops contribute to this decay and this is given by [130]

$$\mathrm{Br}(\mu \to e\,\gamma) = \frac{12\,\pi\,\alpha^3}{G_\mathrm{F}^2\,m_\mathrm{SUSY}^4}\left|(\delta_{12})_\mathrm{LL}\,M_3(y) + \frac{\sqrt{y}\,m_\mathrm{SUSY}}{m_\mu}(\delta_{12})_\mathrm{LR}\,M_1(y)\right|^2 + \;(\mathrm{L}\leftrightarrow\mathrm{R})\;. \tag{6.81}$$

Here, m_SUSY is the soft SUSY breaking mass, G_F is the Fermi constant and α is the fine–structure constant. The loop factors $M_1(y)$ and $M_3(y)$ depend on the mass–squared ratio between photino and slepton, $y = m_{\widetilde{\gamma}}^2/m_{\widetilde{\ell}}^2$, where we set $m_{\widetilde{\ell}} \sim m_\mathrm{SUSY}$ and their precise expression can be found in [130]. For the following analysis it is sufficient to state the upper bounds for $M_1(y)$ and $M_3(y)$, which are given by $M_1(y) < 0.5$ and $M_3(y) < 0.083$. The parameters $(\delta_{12})_{LL}$ and $(\delta_{12})_{LR}$ are the so–called mass insertion parameters and they describe the ratio between the off–diagonal and the diagonal entries of either $\widetilde{m}^2$, in the case of $(\delta_{12})_{LL}$, or the A terms, in the case of $(\delta_{12})_{LR}$. We know from equation (6.80) that $(\delta_{12})_{LL} \sim x$ and we know from our discussion of the A terms that the off–diagonal ones are proportional to x times the according Yukawa matrix entry. In this case we can, therefore, estimate its size to be $\widetilde{M}\,x\,y_\mu$ which gives us $(\delta_{12})_{LR} \simeq\; (x\,m_\mu)/m_\mathrm{SUSY}$.

So far we have assumed that the expansion parameter is given maximally by the Cabibbo angle squared, i.e. $x \lesssim 0.04$ and we use this now for our analysis of equation (6.81) in order to determine a bound for the supersymmetric scale m_SUSY. The current experimental limit for the decay rate $\mu \to e\,\gamma$ is given by the MEG collaboration [100], i.e. $\mathrm{Br}(\mu \to e\,\gamma)_\mathrm{exp} < 5.7 \times 10^{-13}$, and for a photino to slepton mass–squared ratio of $y = 5$ we have a lower bound for the soft SUSY breaking mass of $m_\mathrm{SUSY} \geq 1\,\mathrm{TeV}$, whereas for $y = 2$, we get $m_\mathrm{SUSY} \geq 1.4\,\mathrm{TeV}$, and for $y = 0.5$, we have $m_\mathrm{SUSY} \geq 1.8\,\mathrm{TeV}$. As soon as we go to $m_\mathrm{SUSY} \geq 2\,\mathrm{TeV}$, the experimental limits are always satisfied and the result is independent of the photino to slepton mass–squared ratio. [5]

Therefore, it is always possible to fulfill the experimental constraints by adjusting the soft SUSY breaking scale accordingly, thus the Kähler corrections can be sizable without inducing too large FCNCs.

[5] Note that in [32] the experimental limit was given by $\mathrm{Br}(\mu \to e\,\gamma)_\mathrm{exp} < 2.4 \times 10^{-12}$ [71], and the limits on the soft SUSY breaking mass m_SUSY were less strict, e.g. the experimental constraints were always satisfied for $m_\mathrm{SUSY} \geq 1.4\,\mathrm{TeV}$.

Chapter 7

Holomorphic zeros and a $\mathbb{Z}_4^R$ Dirac NMSSM

In chapter 2 we introduced supersymmetry and some supersymmetric models, in particular, the MSSM. We noted that the MSSM, as well as its singlet extension, the NMSSM, suffers from a fine–tuning problem. We also introduced the Dirac NMSSM, which might ameliorate this issue; however, the extra ingredients for this model were not very well motivated and introduced rather ad hoc.

In chapter 5 we discussed flavor symmetries, and we saw in section 5.2 that models based on a $\mathrm{U}(1)_R$ symmetry allow us to explain the fermion mass hierarchies and most mixing angles, especially, if we combine the models with a non–Abelian flavor symmetry for the neutrino mixing, cf. section 5.3.

We want to bring these two discussions together and show how we can obtain a more natural Dirac NMSSM from a $\mathrm{U}(1)_R$ symmetry, a model we call "$\mathbb{Z}_4^R$ Dirac NMSSM" [33]. One of the main reasons that allows us to construct such a model in section 7.2 are the so–called holomorphic zeros, which we will discuss in the following section.

7.1 Holomorphic zeros

In section 5.2 we discussed a pseudo–anomalous $\mathrm{U}(1)_R$ symmetry with a residual $\mathbb{Z}_4^R$. Given such a $\mathrm{U}(1)_R$ symmetry, where the anomaly coefficients $A_{G-G-\mathrm{U}(1)_R}$ are greater than zero, we need a negatively charged field to acquire a VEV in order to cancel the Fayet–Iliopoulos term. In our case, this is the flavon Φ, which has either charge -4 or -12, depending on the normalization of the fields. In such a scenario, any operator $\mathcal{O}$ with negative $\mathrm{U}(1)_R$ charge is forbidden by the holomorphicity of the superpotential since we cannot find a $n > 0$ such that $\Phi^n \mathcal{O}$ is an allowed superpotential term. These are holomorphic zeros [131], a feature of pseudo–anomalous $\mathrm{U}(1)_R$ symmetries.

Take, for example, our models in section 5.2, which break the $\mathrm{U}(1)_R$ symmetry to the well–known $\mathbb{Z}_4^R$ symmetry from section 4.2.4; precisely this $\mathbb{Z}_4^R$ symmetry, where $q_{H_u} = q_{H_d} = 0$ and $q_\theta = 1$, cf. table 4.2, allows for a potentially dangerous superpotential term

$$\mathscr{W} \supset X\, H_u\, H_d\,, \tag{7.1}$$

if the spurion X, which breaks SUSY when its F term acquires a VEV, has R charge 2. Then, this coupling results in a dangerously large $B\mu$ term of the order $m_{3/2}\, M_\mathrm{P}$. If we assume that the $\mathbb{Z}_4^R$ is the residual symmetry of a spontaneously broken pseudo–anomalous $\mathrm{U}(1)_R$ symmetry, we can forbid such a term with holomorphic zeros. This is the case if the spurion X carries $\mathrm{U}(1)_R$ charge

$$R_X \;=\; 2\,R_\theta - R_{H_u} - R_{H_d} + n\,R_\phi \quad \text{with} \quad n > 0\,. \tag{7.2}$$

The coupling $X\,\phi^m\,H_u\,H_d$ is then forbidden by the holomorphicity of the superpotential, even though, from the perspective of the $\mathbb{Z}_4^R$ symmetry, the coupling would be allowed. Therefore, the dangerous $B\mu$ term is forbidden by holomorphic zeros [33], and we use similar charge assignments in the next section in order to forbid other unwanted terms in the superpotential.

Before we use holomorphic zeros to build a more natural Dirac NMSSM, let us note that the holomorphic zeros get in principle lifted by non–perturbative effects [132]. Consider a negatively charged operator $\mathcal{O}$. It is possible that a term

$$\mathscr{W}_{\text{np}} \supset \mathrm{e}^{-b\,S}\,\mathcal{O}\;, \tag{7.3}$$

can be generated and is an allowed superpotential term. Here, S is the dilaton that contains the Green–Schwarz axion a via $S|_{\theta=0} \;=\; s + \mathrm{i}\,a$, and the coefficient b depends on the hidden sector superpotential. Unfortunately, the size of these quantities can only be determined as soon as we know the "full" theory, i.e. a UV completion of the model [33]. Therefore, we can only estimate the size of the non–perturbative effects and it is reasonable to assume that their size does not exceed $m_{3/2}$ [33]. In the following sections we assume that the effects are rather suppressed compared to $m_{3/2}$; nonetheless, we should keep in mind that in principle holomorphic zeros are lifted by non–perturbative effects and that their precise size depends on the hidden sector of the UV complete model.

7.2 How to build a $\mathbb{Z}_4^R$ Dirac NMSSM

In section 5.2 we saw that flavor models with a $\mathrm{U}(1)_R$–derived $\mathbb{Z}_4^R$ symmetry have many appealing features. So far these models predicted a MSSM particle content and, therefore, have the same fine–tuning and Higgs issues. As we reviewed in section 2.2.3, the Dirac NMSSM does not suffer from this problem, and we now want to embed this model within a pseudo–anomalous $\mathrm{U}(1)_R$ symmetry.

Let us remind ourselves what extra ingredients we need in order to obtain the Dirac NMSSM from the usual MSSM. As we saw in section 2.2.3, the Dirac NMSSM superpotential is given by

$$\mathscr{W} \;=\; \lambda\,N\,H_u\,H_d + M\,N\,\overline{N} + \mathscr{W}_{\text{MSSM}}\;, \tag{7.4}$$

where N and $\overline{N}$ are SM singlets, λ is a dimensionless coupling, and M the Dirac mass term. Furthermore, the Higgs mass gets lifted through the soft mass of $\overline{N}$, i.e. we also have the soft term

$$V_{\text{soft}} \supset m_{\overline{N}}\,|\overline{N}|^2\;, \tag{7.5}$$

which we assume to be of order $m_{3/2}$. We achieve this by introducing a SUSY breaking spurion X, which gives us the soft term in equation (7.5) through

$$K \supset \frac{X\,X^\dagger}{M_{\text{P}}^2}\,\overline{N}\,\overline{N}^\dagger\;. \tag{7.6}$$

Hence, the assumption $m_{\overline{N}} = m_{3/2}$ is equivalent to $\langle F_X\rangle = m_{3/2}\,M_{\text{P}}$. In summary, we need to introduce the fields N, $\overline{N}$ and the spurion X.

We need to assign appropriate $\mathrm{U}(1)_R$ charges to the additional fields. The charge of the singlet N is fixed directly due to the charge of the Higgs bilinear,

$$R_N = 2\,R_\theta - R_{H_u} - R_{H_d}\ . \tag{7.7}$$

Given this charge assignment and the fact that $R_{H_u} + R_{H_d} = 0 \mod R_\Phi$, which is given for all models from section 5.2, we can distinguish two cases. In the first case, the charge of the Higgs bilinear is negative and, therefore, we can have an effective linear term for the singlet N. In fact, some terms like $\Phi^n\,N^m$ are allowed in the superpotential, and especially $m = 1$ would be possible. In the second case, the Higgs bilinear has positive $\mathrm{U}(1)_R$ charge and an effective linear term in N is forbidden by holomorphic zeros. The first case is realized in models #1–4 in appendix B.2 and the second one in models #5–12. Thus, from now on we only consider models #5–12 in order to build a $\mathbb{Z}_4^R$ Dirac NMSSM.

Let us return to the charge assignment for the other fields. The Dirac mass M should be generated effectively from the Kähler potential,

$$K \supset \frac{X^\dagger}{M_\mathrm{P}} \left(\frac{\Phi}{\Lambda}\right)^n N\,\overline{N}\ , \tag{7.8}$$

which results in a mass of order $M \sim \varepsilon^n\,m_{3/2}$. Recall that $\varepsilon = \langle\Phi\rangle/\Lambda \sim 0.2$. As we mentioned in section 2.2.3, we need $m_{\overline{N}} \sim m_{3/2}$ to be of order 10 TeV and $700\,\mathrm{GeV} \lesssim M \lesssim 5\,\mathrm{TeV}$ [36] for the Dirac mass M. Therefore, we choose $n = 1$ in equation (7.8) which fixes the spurion charge to

$$R_X \;=\; R_\phi + R_N + R_{\overline{N}}\ . \tag{7.9}$$

The next step is now to fix the charge of the singlet $\overline{N}$, which is connected to the spurion charge R_X. We should remind ourselves that we want to have matter parity as a residual symmetry after SUSY breaking. This in turn means that the spurion X should have even $\mathbb{Z}_4^R$ charge. We can see in equation (7.9) that the first two contributions have even charge, hence $R_{\overline{N}}$ should also have even charge. Therefore, we set

$$R_{\overline{N}} \;=\; \frac{m}{2}\,R_\phi \;=\; -2\,m\,R_\theta \quad \text{with} \quad m \in \mathbb{N}\ , \tag{7.10}$$

and we get for the spurion

$$R_X \;=\; R_N + \frac{m+2}{2}\,R_\phi \;=\; R_N - 2\,(m+2)\,R_\theta\ . \tag{7.11}$$

Here, we use $R_\phi = -4\,R_\theta$ as is the case for models #5–12. Now all parts in equation (7.9) are even and matter parity remains intact after SUSY breaking. Also note that we choose $m > 0$ which forbids unwanted terms for the singlet $\overline{N}$, i.e. an (effective) linear term and a supersymmetric mass for $\overline{N}$ are forbidden by the holomorphicity of the superpotential, just as it is the case for N.

Nonetheless, we also have to ensure that we cannot create such dangerous operators effectively from the Kähler potential. Arguably, the most dangerous operators are the linear terms for the singlets,

$$K \;\supset X^\dagger \left(\frac{\phi}{\Lambda}\right)^{a_1} \left(\frac{\phi^\dagger}{\Lambda}\right)^{a_2} N \quad \curvearrowright \quad \mathscr{W}_\mathrm{eff} \;\supset\; \varepsilon^{a_1+a_2}\,m_{3/2}\,M_\mathrm{P}\,N\ , \tag{7.12a}$$

$$K \;\supset X^\dagger \left(\frac{\phi}{\Lambda}\right)^{\widetilde{a}_1} \left(\frac{\phi^\dagger}{\Lambda}\right)^{\widetilde{a}_2} \overline{N} \quad \curvearrowright \quad \mathscr{W}_\mathrm{eff} \;\supset\; \varepsilon^{\widetilde{a}_1+\widetilde{a}_2}\,m_{3/2}\,M_\mathrm{P}\,\overline{N}\ , \tag{7.12b}$$

where a_i, $\widetilde{a}_i \in \mathbb{N}$. We only consider the largest, i.e. most troublesome, operator, and, hence, we either set $a_1 = 0$ or $a_2 = 0$, and the same for the $\widetilde{a}_i$. These operators turn out to be allowed if we choose m even in equation (7.10), and we would need to choose a very large a_i, or $\widetilde{a}_i$ respectively, in order to suppress their contribution significantly. Therefore, we need to choose m odd.

However, this choice does not forbid effective supersymmetric masses for the singlets N, $\overline{N}$ from the Kähler potential,

$$K \supset \frac{X^\dagger}{M_\mathrm{P}} \left(\frac{\phi}{\Lambda}\right)^{b_1} \left(\frac{\phi^\dagger}{\Lambda}\right)^{b_2} N^2 \quad \curvearrowright \quad \mathscr{W}_\mathrm{eff} \supset \varepsilon^{b_1+b_2}\, m_{3/2}\, N^2 \,, \tag{7.13a}$$

$$K \supset \frac{X^\dagger}{M_\mathrm{P}} \left(\frac{\phi}{\Lambda}\right)^{\widetilde{b}_1} \left(\frac{\phi^\dagger}{\Lambda}\right)^{\widetilde{b}_2} \overline{N}^2 \quad \curvearrowright \quad \mathscr{W}_\mathrm{eff} \supset \varepsilon^{\widetilde{b}_1+\widetilde{b}_2}\, m_{3/2}\, \overline{N}^2 \,, \tag{7.13b}$$

where b_i, $\widetilde{b}_i \in \mathbb{N}$. Such terms occur for odd m and appropriate b_i, $\widetilde{b}_i$. Note that we again always take only one of the b_i, or $\widetilde{b}_i$ respectively, equal to zero in order to consider the largest contribution.

We have potentially dangerous operators for all choices of m; however, we can choose m to be odd, forbidding linear terms as in equation (7.12), and large in such a way that the supersymmetric masses in equation (7.13) are sufficiently suppressed. As it turns out, the supersymmetric mass for $\overline{N}$ is less suppressed than the one for N,[1] and its suppression is related to m through

$$m = \begin{cases} 3 - 2\left(\widetilde{b}_1 - \widetilde{b}_2\right) > 0 & \text{for models \#5–10 ,} \\ 5 - 2\left(\widetilde{b}_1 - \widetilde{b}_2\right) > 0 & \text{for models \#11–12 .} \end{cases} \tag{7.14}$$

Recall that we want $m > 0$ in order for holomorphic zeros to forbid the linear terms, therefore, we have to increase $\widetilde{b}_2$ to achieve a large suppression in equation (7.13). A suppression of the order ε^8, for example, can be achieved by choosing $\widetilde{b}_1 = 0$ and $\widetilde{b}_2 = 8$, and this gives us supersymmetric masses of the order MeV, if we assume $m_{3/2} \sim 10\,\mathrm{TeV}$. According to equation (7.14) we would then need to choose $m = 19$ and hence $R_{\overline{N}} = -38\,R_\theta$ for models #5–10, whereas for models #11–12 we would need $m = 21$ and $R_{\overline{N}} = -42\,R_\theta$.

In summary, we find now that we can constrain the charges of the additional fields to be

$$R_N = 2\,R_\theta - R_{H_u} - R_{H_d} \,, \tag{7.15a}$$

$$R_{\overline{N}} = -2\,m\,R_\theta \,, \tag{7.15b}$$

$$R_X = R_N - 2\,(m+2)\,R_\theta \,, \tag{7.15c}$$

where $m > 0$ is odd and chosen according to Equation (7.14) for an appropriate suppression of the effective supersymmetric masses in Equation (7.13). This is only valid for models #5–12.

Before we discuss an explicit realization of these models, let us note that we can avoid supersymmetric masses for N and $\overline{N}$ altogether if we introduce a $\mathbb{Z}_2$ symmetry under which only the spurion X and the singlet $\overline{N}$ are charged. Such a symmetry forbids all potentially dangerous terms and allows any value $m > 0$. Furthermore, we should mention that the MSSM μ term is also forbidden by holomorphicity but an effective μ term is generated by the singlet VEV $\langle N \rangle$.

[1] We get $b_1 + b_2 = \widetilde{b}_1 + \widetilde{b}_2 + 2$, where either $b_1 = \widetilde{b}_1 = 0$ or $b_2 = \widetilde{b}_2 = 0$ for the largest contribution.

7.3 An example $\mathbb{Z}_4^R$ Dirac NMSSM

In the previous section we showed that we can add the SM singlets N, $\overline{N}$ and the spurion X to a certain subset of the models which we discussed in section 5.2, models #5–12 from appendix B.2, in order to obtain a $\mathbb{Z}_4^R$ Dirac NMSSM. Now we want to present an explicit example of this procedure; for this we use model #6b from section 5.2.3. We find the charges of the MSSM fields in table 5.2, and the $\mathrm{U}(1)_R$ charges of the additional singlets are chosen according to equation (7.15). Thus we get

$$R_N \;=\; -6\,, \qquad R_{\overline{N}} \;=\; -114 \qquad \text{and} \qquad R_X = -132\,, \tag{7.16}$$

which gives us a sufficient suppression for the supersymmetric masses of N and $\overline{N}$, i.e. $\widetilde{b}_2 = 8$ in equation (7.14). The relevant charges for the Dirac NMSSM are summarized in table 7.1. Note that after $\mathrm{U}(1)_R$ breaking we first get a $\mathbb{Z}_{12}^R$ symmetry, which is the product of our

	ϕ	θ	H_d	H_u	N	$\overline{N}$	X	matter
$\mathrm{U}(1)_R$	-12	3	12	0	-6	-114	-132	see Table 5.2
$\mathbb{Z}_4^R$	0	1	0	0	2	2	0	1

Table 7.1: Model #6b from Section 5.2.3 with the fields $N, \overline{N}$ and the spurion X. After the breaking of $\mathrm{U}(1)_R$ we get a $\mathbb{Z}_{12}^R = \mathbb{Z}_4^R \times \mathbb{Z}_3^{\mathrm{SU}(3)_\mathrm{C}}$ symmetry where the $\mathbb{Z}_4^R$ is our desired symmetry.

desired $\mathbb{Z}_4^R$ symmetry times the non–trivial center of $\mathrm{SU}(3)_\mathrm{C}$, and we only display the $\mathbb{Z}_4^R$ charges in table 7.1.

These charges allow us to write down the superpotential

$$\mathscr{W} \;\supset\; \lambda\, N\, H_u\, H_d + \text{Yukawas}\,, \tag{7.17}$$

where λ is a dimensionless constant and the Yukawa couplings give the desired fermion mass hierarchies and mixings as described in section 5.2. The Kähler potential contains the terms

$$K \supset \left(k_1\, \frac{X^\dagger}{M_\mathrm{P}}\, \frac{\phi}{\Lambda}\, N\, \overline{N} + \text{h.c.} \right) + k_2\, \frac{X\, X^\dagger}{M_\mathrm{P}^2}\, \overline{N}\, \overline{N}^\dagger\,, \tag{7.18}$$

where the k_i are dimensionless constants. The first bracket will lead after SUSY breaking to a Dirac mass term of the order $\varepsilon\, m_{3/2}$, i.e. the effective superpotential is given by

$$\mathscr{W}_\mathrm{eff} \;\supset\; \lambda\, N\, H_u\, H_d + M\, N\, \overline{N} + \text{Yukawas}\,, \tag{7.19}$$

with $M \sim \varepsilon\, m_{3/2}$. The second term in equation (7.18) gives us the usual soft breaking mass of the order $m_{3/2}$ for $\overline{N}$

$$V_\mathrm{soft} \supset m_{\overline{N}}^2\, |\overline{N}|^2\,, \tag{7.20}$$

with $m_{\overline{N}} = \sqrt{k_2}\, m_{3/2}$. The benchmark model from section 2.2.3 can now easily be reproduced, e.g. by setting $m_{3/2} \sim \mathcal{O}(10)\,\mathrm{TeV}$ and $\sqrt{k_2} \sim 1.5$, assuming as usual $\varepsilon \sim 0.2$. Such a parameter setting allows us to raise the Higgs mass to a viable value [36].

This procedure can now be reproduced for any model #5–12 from appendix B.2, and we obtain a $\mathbb{Z}_4^R$ Dirac NMSSM. This relies mainly on holomorphic zeros, which forbid unwanted operators, and we use them in the next sections again in order to explain the neutrino mixing in these models. We will use our just discussed example model, but the procedure can be applied to all other models as well [33]; however, we first need to distinguish between see–saw induced and Dirac neutrinos.

7.3.1 See–saw neutrinos

So far our example model #6b did not include any right–handed neutrinos, which we need in order to explain the neutrino mixing. The original models [110] use the type I see–saw mechanism to explain the neutrino mixing. Therefore, we add three right–handed neutrinos $\overline{\nu}_i$ to the model and assign them appropriate R charges, choosing

$$R_{\overline{\nu}_i} = 45 \;, \quad \forall i \;, \tag{7.21}$$

allowing for an appropriate neutrino mass scale [110]. Now, all right–handed neutrinos have $\mathbb{Z}_4^R$ charge 1 after $\mathrm{U}(1)_R$ breaking and we can write down all operators necessary for the see–saw mechanism. The neutrino mass scale of this model is then $m^\nu_{\text{abs}} \sim \mathcal{O}(0.1)\,\text{eV}$,[2] and, as described in section 5.2.4, the neutrino mixing matrix is given by

$$M_\nu \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \tag{7.22}$$

The structure of this matrix invites us to either consider an anarchical scenario for the neutrino mixing or to introduce a non–Abelian flavor symmetry in order to control the mixing angles, cf. section 5.3.

7.3.2 Dirac neutrinos

An alternative to the see–saw induced neutrino masses are Dirac neutrinos, as we briefly mentioned in section 5.2.4. As in section 4.2.2, we want to have sufficiently suppressed Dirac neutrino masses from the Kähler potential. Therefore, we need $X^\dagger\, L\, H_u\, \overline{\nu}$ to be an allowed Kähler potential term. This automatically fixes the charge of the right–handed neutrinos

$$R_{\overline{\nu}_i} \;=\; R_X - R_{L_i} - R_{H_u} \;, \tag{7.23}$$

and for our model we have $R_{\overline{\nu}_i} = -105$ for all i, cf. table 7.1, and hence the right–handed neutrinos have $\mathbb{Z}_4^R$ charge 3 after $\mathrm{U}(1)_R$ breaking. We get the effective superpotential

$$K \;\supset\; \frac{X^\dagger}{M_{\text{P}}^2}\, L\, H_u\, \overline{\nu} \quad \curvearrowright \quad \mathscr{W}_{\text{eff}} \;\supset\; Y_\nu\, L\, H_u\, \overline{\nu} \;, \tag{7.24}$$

with sufficiently suppressed Yukawa couplings, i.e.

$$Y_\nu \;\sim\; \frac{m_{3/2}}{M_{\text{P}}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \tag{7.25}$$

[2] This assumes $\varepsilon \sim 0.2$ and that M_{P} is the fundamental scale of our theory.

again allowing for an interesting neutrino mixing structure.

The fact that the right–handed neutrinos have such large negative charges implies that the Majorana mass term, as well as the superpotential Dirac Yukawa term $L\,H_u\,\overline{\nu}$, is forbidden by holomorphicity. Furthermore, dangerous proton decay operators, such as $\overline{U}\,\overline{D}\,\overline{D}\,\overline{\nu}$ from section 4.2.3, are forbidden by holomorphic zeros as well.

7.4 Features of $\mathbb{Z}_4^R$ Dirac NMSSM models

In this chapter we showed that we can construct a $\mathbb{Z}_4^R$ Dirac NMSSM by adding three additional SM singlets, N, $\overline{N}$ and the spurion X, to the spontaneously broken pseudo–anomalous $\mathrm{U}(1)_R$ flavor models from section 5.2, which all have the $\mathbb{Z}_4^R$ from section 4.2.4 as a residual symmetry after $\mathrm{U}(1)_R$ breaking. The resulting models explain the fermion mass hierarchies and the CKM matrix due to the underlying $\mathrm{U}(1)_R$ symmetry and the flavor–dependent $\mathrm{U}(1)_R$ charges. The models are anomaly–free via the GS mechanism and forbid dangerous proton decay operators because of the residual $\mathbb{Z}_4^R$ symmetry. Implementing the Dirac NMSSM from section 2.2.3 into the $\mathrm{U}(1)_R$ setting allows us to lift the Higgs mass to a phenomenologically viable value with low fine–tuning.

A caveat of the original $\mathrm{U}(1)_R$ models [110] is that they only predict semi–realistic neutrino mixing, cf. section 5.2.4. However, we might overcome this issue by introducing a non–Abelian flavor symmetry, as in section 5.3, which can improve the neutrino mixing pattern significantly. The models support both possible types of neutrino masses, see–saw induced and Dirac neutrinos, where in the latter case the Dirac neutrino Yukawa couplings are sufficiently suppressed due to the same method as in chapter 4.

In summary, we find that by combining several ideas of the previous chapters we can construct $\mathbb{Z}_4^R$ Dirac NMSSM models that explain all fermion mass hierarchies and mixings, including for neutrinos, while having a Higgs mass of suitable size with low fine–tuning [33].

Chapter 8

Conclusions

We started this thesis by briefly reviewing supersymmetry and the Hilbert basis method, with focus on discrete (R) symmetries, which enabled us to determine superpotentials to all orders. We noted that the MSSM suffers from the μ problem and an unnaturally small Higgs mass, and we showed how singlet extensions of the MSSM, in particular the Dirac NMSSM, can lift the Higgs mass without, or with little, fine–tuning. In such a setting we introduced two SM singlets N and $\overline{N}$, which couple to each other via a dimensionful parameter, the Dirac mass M. The singlet N also couples to the Higgs bilinear in the superpotential, thereby naturally raising the Higgs mass.

Furthermore, we examined the well–known fact that neutrinos are massive and, therefore, oscillate. We introduced the standard parametrization of neutrino oscillations and gave a brief overview of the experimental results, including the recent measurement of $\theta_{13} \approx 9^\circ$. The existence of neutrino masses can be explained in several ways and we reviewed the see–saw mechanism for Majorana neutrinos and the possibility of Dirac neutrinos.

In connection to this we analyzed how the different types of neutrino masses can be connected to R symmetries. We found that there are anomaly–free $\mathbb{Z}_M^R$ symmetries, where M is a multiple of 4, which forbid proton decay operators and the μ term. These symmetries are also compatible with the Giudice–Masiero mechanism that allows us to generate a naturally suppressed μ term, therefore, solving the μ problem. We then showed that there is a unique $\mathbb{Z}_4^R$ symmetry compatible with the Weinberg operator and that there is a set of $\mathbb{Z}_M^R$ symmetries where the smallness of the Dirac neutrino Yukawa couplings is related to the smallness of the μ term via $Y_\nu \sim \frac{\mu}{M_{\rm P}}$.

The flavor problem of the (MS)SM was examined and we provided a solution through the introduction of spontaneously broken flavor symmetries. First, we discussed a pseudo–anomalous $\mathrm{U}(1)_R$ symmetry that explains the quark and charged lepton masses and hierarchies as well as the CKM matrix through flavor–dependent $\mathrm{U}(1)_R$ charges. The models based on this $\mathrm{U}(1)_R$ exhibit a residual $\mathbb{Z}_4^R$ symmetry after $\mathrm{U}(1)_R$ breaking, which forbids dangerous proton decay operators. We showed that the original models [110] imposed an unnecessary anomaly constraint and by removing this constraint we obtained much more appealing charge assignments. However, these models do not always provide realistic neutrino mixing patterns and, hence, rely on discrete non–Abelian flavor symmetries or the anarchy scheme. We discussed spontaneously broken discrete non–Abelian flavor symmetry models, giving two examples based on A_4 [113, 114] and T' [115].

These models, and many similar ones, rely solely on the superpotential for their predictions. However, we showed in this thesis that corrections from the Kähler potential are sizable and cannot be forbidden by a conventional symmetry. We illustrated this by considering the A_4 and T' examples, e.g. we showed that for the A_4 model, which originally predicts a tri–bi–maximal mixing pattern, with $\theta_{13} = 0$, a correction to θ_{13} of roughly 8° can be

generated by Kähler potential effects. We argue that, because these corrections are always present, we need to gain a better understanding of the Kähler potential in order to make sensible predictions when using spontaneously broken non–Abelian flavor symmetries.

Subsequently, we built models combining the good features of the Dirac NMSSM and of the pseudo–anomalous $\mathrm{U}(1)_R$ symmetries with a residual $\mathbb{Z}_4^R$. We showed how we can add additional SM singlets to the $\mathrm{U}(1)_R$ scenario and how to assign them charges in order to achieve a Dirac NMSSM setting. We demonstrated that we can employ holomorphic zeros in order to forbid unwanted terms like the linear terms of the singlet fields. Using these techniques, we found a set of models, which we call $\mathbb{Z}_4^R$ Dirac NMSSMs, that explain fermion masses and hierarchies while simultaneously raising the Higgs mass without fine–tuning. We argued that by combining these models with a non–Abelian discrete flavor symmetry we might be able to also explain the neutrino mixing pattern, therefore, closing a loophole of the original models [110]. Our $\mathbb{Z}_4^R$ Dirac NMSSM models are compatible with both Majorana and Dirac neutrinos, and the open question in particle physics which of the two alternatives is realized by Nature, will hopefully be answered by experiments, e.g. [133], in the near future.

The $\mathbb{Z}_4^R$ Dirac NMSSM models have a very appealing phenomenology; however, we built them bottom–up by imposing the $\mathrm{U}(1)_R$ symmetry and the Dirac NMSSM setting by hand, therefore, lacking control over non–perturbative effects and the hidden superpotential. It would be interesting to see if we can obtain such models from the top–down perspective, e.g. from string constructions. Furthermore, string models might also help us gain a better understanding of the Kähler potential, which is necessary in order to make sensible predictions in flavor model building. It seems, therefore, promising to further investigate string constructions, which make non–trivial, top–down predictions for the Higgs and the flavor sector. These predictions might be testable by the upcoming 14 TeV run of the LHC [134,135], by a possible linear collider [136] or by precision neutrino experiments [137, 138].

Appendix A

The Hilbert superpotential bases for models with $\mathbb{Z}_{12}^R$ symmetries

In chapter 4 we discussed symmetries that solve the μ problem with the help of R symmetries. We showed that this can also be related to neutrino masses. In particular for Dirac neutrinos we showed in section 4.2.2 how the smallness of the Dirac Yukawa coupling can be related to the μ term through SUSY breaking.

We presented a set of $\mathbb{Z}_M^R$ symmetries in table 4.1 that forbid neutrino masses perturbatively and for models with two $\mathbb{Z}_4^R$ symmetries and a $\mathbb{Z}_8^R$ symmetry, we presented the Hilbert superpotential basis. Here, we provide further examples based on the $\mathbb{Z}_{12}^R$ symmetries in table 4.1. Let us recall from section 2.3 that a possible superpotential term $\mathscr{M}$ contains only one inhomogeneous monomial and an arbitrary combination of homogeneous monomials.

There are three examples which have a $\mathbb{Z}_{12}^R$ symmetry, with differing Hilbert basis, i.e. the three $\mathbb{Z}_{12}^R$ symmetries are inequivalent. The first symmetry has the charges

$$\left(\begin{array}{cccccccc} q_{\mathbf{10}} & q_{\overline{\mathbf{5}}} & q_{H_u} & q_{H_d} & q_\theta & \rho & q_{\bar\nu} \end{array} \right) = \left(\begin{array}{cccccccc} 1 & 9 & 4 & 8 & 3 & 3 & 11 \end{array} \right) , \tag{A.1}$$

which leads to the inhomogeneous monomials

$$\begin{aligned} &\left(L\,H_d\,\overline{E}\right)\ ;\ (L\,H_u)^6\ ;\ \bar\nu^6\ ;\ \left(L\,L\,\overline{E}\right)\bar\nu\ ;\\ &\left(L\,L\,\overline{E}\right)^6\ ;\ \left(L\,L\,\overline{E}\right)^4(L\,H_u)^2\ ;\ \left(L\,L\,\overline{E}\right)^2(L\,H_u)^4\ , \end{aligned} \tag{A.2}$$

whereas the homogeneous ones are given by

$$\begin{aligned} &\left(L\,L\,\overline{E}\right)^{12}\ ;\ (L\,H_u)^{12}\ ;\ H_u\,H_d\ ;\ (L\,H_u)\,\bar\nu\ ;\ \left(L\,L\,\overline{E}\right)\left(L\,H_d\,\overline{E}\right)\bar\nu\ ;\\ &\bar\nu^{12}\ ;\ \left(L\,H_d\,\overline{E}\right)(L\,H_u)^6\ ;\ \left(L\,H_d\,\overline{E}\right)^2\ ;\ \left(L\,L\,\overline{E}\right)^7\bar\nu\ ;\\ &\left(L\,L\,\overline{E}\right)(L\,H_u)^5\ ;\ \left(L\,L\,\overline{E}\right)^6\left(L\,H_d\,\overline{E}\right)\ ;\ \left(L\,H_d\,\overline{E}\right)\bar\nu^6\ ;\\ &\left(L\,L\,\overline{E}\right)^2\bar\nu^2\ ;\ \left(L\,H_d\,\overline{E}\right)\left(L\,L\,\overline{E}\right)^4(L\,H_u)^2\ ;\ \left(L\,L\,\overline{E}\right)^5(L\,H_u)\ ;\\ &\left(L\,H_d\,\overline{E}\right)\left(L\,L\,\overline{E}\right)^2(L\,H_u)^4\ ;\ \left(L\,L\,\overline{E}\right)\bar\nu^7\ ;\ \left(L\,L\,\overline{E}\right)^3(L\,H_u)^3\ . \end{aligned} \tag{A.3}$$

The second $\mathbb{Z}_{12}^R$ symmetry has the charge assignment

$$\left(\begin{array}{cccccccc} q_{\mathbf{10}} & q_{\overline{\mathbf{5}}} & q_{H_u} & q_{H_d} & q_\theta & \rho & q_{\bar\nu} \end{array} \right) = \left(\begin{array}{cccccccc} 2 & 6 & 2 & 10 & 3 & 3 & 4 \end{array} \right) , \tag{A.4}$$

which gives us for the inhomogeneous monomials

$$\left(L\,H_d\,\overline{E}\right)\ ;\ \left(L\,L\,\overline{E}\right)^3\ ;\ \left(L\,L\,\overline{E}\right)\bar\nu\ ;\ \left(L\,L\,\overline{E}\right)(L\,H_u)^2\ , \tag{A.5}$$

and for the homogeneous monomials

$$\begin{aligned}
&\left(L\,L\,\overline{E}\right)^6\ ;\ \left(L\,L\,\overline{E}\right)^4\,\bar{\nu}\ ;\ \left(L\,H_d\,\overline{E}\right)\,\left(L\,L\,\overline{E}\right)^3\ ;\ H_u\,H_d\ ;\\
&(L\,H_u)\,\bar{\nu}\ ;\ \bar{\nu}^3\ ;\ (L\,H_u)^3\ ;\ \left(L\,H_d\,\overline{E}\right)\,(L\,H_u)^2\,\left(L\,L\,\overline{E}\right)\ ;\\
&\left(L\,L\,\overline{E}\right)^2\,(L\,H_u)\ ;\ \left(L\,H_d\,\overline{E}\right)^2\ ;\ \left(L\,L\,\overline{E}\right)^2\,\bar{\nu}^2\ ;\ \left(L\,H_d\,\overline{E}\right)\,\left(L\,L\,\overline{E}\right)\,\bar{\nu}\ .
\end{aligned} \tag{A.6}$$

The final $\mathbb{Z}_{12}^R$ symmetry has

$$\left(\begin{array}{cccccccc} q_{\mathbf{10}} & q_{\overline{\mathbf{5}}} & q_{H_u} & q_{H_d} & q_\theta & \rho & q_{\bar{\nu}} \end{array}\right) = \left(\begin{array}{cccccccc} 4 & 0 & 10 & 2 & 3 & 3 & 2 \end{array}\right)\ , \tag{A.7}$$

as its charges, and with these we get the inhomogeneous monomials

$$\bar{\nu}^3\ ;\ \left(L\,H_d\,\overline{E}\right)\ ;\ \left(L\,L\,\overline{E}\right)\,\bar{\nu}\ ;\ (L\,H_u)^3\ ;\ \left(L\,L\,\overline{E}\right)^2\,(L\,H_u)\ , \tag{A.8}$$

and the following homogeneous ones

$$\begin{aligned}
&(L\,H_u)^6\ ;\ \left(L\,H_d\,\overline{E}\right)\,(L\,H_u)^3\ ;\ H_u\,H_d\ ;\ \left(L\,H_d\,\overline{E}\right)\,\left(L\,L\,\overline{E}\right)^2\,(L\,H_u)\ ;\\
&(L\,H_u)\,\bar{\nu}\ ;\ \left(L\,L\,\overline{E}\right)^3\ ;\ \left(L\,L\,\overline{E}\right)\,(L\,H_u)^2\ ;\ \left(L\,H_d\,\overline{E}\right)^2\ ;\\
&\left(L\,H_d\,\overline{E}\right)\,\bar{\nu}^3\ ;\ \left(L\,H_d\,\overline{E}\right)\,\left(L\,L\,\overline{E}\right)\,\bar{\nu}\ ;\ \left(L\,L\,\overline{E}\right)\,\bar{\nu}^4\ ;\ \bar{\nu}^6\ ;\\
&\left(L\,L\,\overline{E}\right)^2\,\bar{\nu}^2\ .
\end{aligned} \tag{A.9}$$

Appendix B

Charge assignments for $\mathrm{U}(1)_R$ FN models

B.1 Charge equations for all fields

In the original computation the authors [110] set $R_\phi = -1$, however, we should note that all integers in this section, i.e. x, y, z, ζ, Δ^H, Δ^L_{ij} and any integer number, are given in units of $-R_\phi$. Therefore, we can return to the general setting by multiplying all additional integers with $-R_\phi$. We chose another possibility to get our results in appendix B.2, we used $R_\phi = -1$ to solve the charge constraints and rescale the resulting charges afterwards [33].

B.1.1 All fields besides the Higgs

The authors of [110] summarized the constraints for the $\mathrm{U}(1)_R$ charges of all fields, besides the Higgs fields, in the following way, with $\Delta^L_{ij} = R_{L_i} - R_{L_j}$,

$$
\begin{aligned}
R_{Q_1} &= \frac{1}{3}\left[\frac{39}{4} - R_{H_d} + x + 2y + z - \zeta - \frac{\Delta^H}{2} - 4R_\theta\right] , && \text{(B.1a)}\\
R_{Q_2} &= R_{Q_1} - 1 - y , && \text{(B.1b)}\\
R_{Q_3} &= R_{Q_1} - 3 - y , && \text{(B.1c)}\\
R_{\overline{U}_1} &= -R_{Q_1} - R_{H_u} + 8 + 2R_\theta , && \text{(B.1d)}\\
R_{\overline{U}_2} &= R_{\overline{U}_1} - 3 + y , && \text{(B.1e)}\\
R_{\overline{U}_3} &= R_{\overline{U}_1} - 5 + y , && \text{(B.1f)}\\
R_{\overline{D}_1} &= -R_{Q_1} - R_{H_d} + 4 + x + 2R_\theta , && \text{(B.1g)}\\
R_{\overline{D}_2} &= R_{\overline{D}_1} - 1 + y , && \text{(B.1h)}\\
R_{\overline{D}_3} &= R_{\overline{D}_1} - 1 + y , && \text{(B.1i)}\\
R_{L_1} &= R_{H_d} + \frac{1}{4}\left(2\Delta^H + 1\right) , && \text{(B.1j)}\\
R_{L_2} &= R_{L_1} - \Delta^L_{31} + z + 3\zeta - 4R_\theta , && \text{(B.1k)}\\
R_{L_3} &= R_{L_1} + \Delta^L_{31} , && \text{(B.1l)}\\
R_{\overline{E}_1} &= -R_{L_1} - R_{H_d} + 4 + x + z + 2R_\theta , && \text{(B.1m)}\\
R_{\overline{E}_2} &= R_{\overline{E}_1} + \Delta^L_{31} - 2 - 2z - 3\zeta + 4R_\theta , && \text{(B.1n)}\\
R_{\overline{E}_3} &= R_{\overline{E}_1} - \Delta^L_{31} - 4 - z , && \text{(B.1o)}
\end{aligned}
$$

where $\zeta \in \{-1, 0\}$. We get Δ^H from demanding the $\mathbb{Z}_4^R$ symmetry after $\mathrm{U}(1)_R$ breaking and its value depends on other model parameters, $\Delta^H = \mathcal{Z} + 2z - \frac{1}{2} - 2\,\Delta^L_{31} - 14\,R_\theta$, where $\mathcal{Z}$ is

a negative integer, which for example determines the neutrino mass scale, cf. section 5.2.4. Its precise value depends on the given model [110].

B.1.2 Higgs charges

In [110], the authors also constrain the Higgs charges

$$\begin{aligned} R_{H_d} &= \frac{1}{3(14\,R_\theta - 18 - 3x - 2z)}\,[3\,R_{L_1}(12 - 16\,R_\theta + 2x + 3z) \\ &\quad + 2\,\Delta^L_{21}(6 - 8\,R_\theta + x + z) + 2\,\Delta^L_{31}(3 - 8\,R_\theta + x + z) - 156\,R_\theta + 18 \\ &\quad + x(14\,R_\theta - 36 - 6x) + z(-2z - 5x - 12\,R_\theta) - 18y + 104\,R_\theta^2\big]\ , \qquad \text{(B.2a)} \\ R_{H_u} &= -R_{H_d} - z + 8\,R_\theta\ , \qquad \text{(B.2b)} \end{aligned}$$

where the one for R_{H_d} comes from demanding that the $A_{\mathrm{U}(1)_R^2-\mathrm{U}(1)_\mathrm{Y}}$ anomaly cancels. We showed in section 5.2.2 though that this constraint can be dropped, therefore, we demand for the Higgs charges

$$\begin{aligned} R_{H_d} &= -R_{H_u} - z + 8\,R_\theta\ , \qquad \text{(B.3a)} \\ R_{H_u} &= 0\ . \qquad \text{(B.3b)} \end{aligned}$$

Here, the charge of H_u is arbitrary and we can always assign a different R_{H_u}.

B.2 Tables of more appealing charge assignments

For the original and for our models we have to solve equation (B.1). However, if we solve also equation (B.3) instead of equation (B.2), we get much more appealing charge assignments, which we summarize here. We also provide the charges for the additional fields N, $\overline{N}$ and X. We set $R_{H_u} = 0$ for all examples and the charges $R_N = 2\,R_\theta - R_{H_d}$, $R_{\overline{N}} = -2\,m\,R_\theta$ and $R_X = R_N - 2\,(m+2)\,R_\theta$ as described in section 7.2. In order to get sufficiently small supersymmetric masses for N and $\overline{N}$, cf. equation (7.14), we choose $m = 19$ for models #5–10 and $m = 21$ for the remaining models #11–12.

We do not provide charges for the additional singlets for models #1–4 since we exclude them as potential Dirac NMSSM candidates [33].

B.2.1 Charges for $y = -1$

#	R_ϕ	R_θ	R_{H_d}	R_{Q_1}	R_{Q_2}	R_{Q_3}	$R_{\overline{U}_1}$	$R_{\overline{U}_2}$	$R_{\overline{U}_3}$	$R_{\overline{D}_1}$	$R_{\overline{D}_2}$	$R_{\overline{D}_3}$	R_{L_1}	R_{L_2}	R_{L_3}	$R_{\overline{E}_1}$	$R_{\overline{E}_2}$	$R_{\overline{E}_3}$	R_N	$R_{\overline{N}}$	R_X
1 a	−4	−1	−12	17	17	9	13	−3	−11	9	1	1	−7	−7	−11	37	25	21			
1 b	−12	−3	−36	55	55	31	35	−13	−37	35	11	11	−21	−21	−33	123	87	75			
1 c	−12	−3	−36	59	59	35	31	−17	−41	43	19	19	−21	−21	−33	135	99	87			
1 d	−4	−1	−12	21	21	13	9	−7	−15	17	9	9	−7	−7	−11	49	37	33			
2 a	−4	−1	−12	19	19	11	11	−5	−13	11	3	3	−9	−9	−13	43	31	27			
2 b	−12	−3	−36	61	61	37	29	−19	−43	41	17	17	−27	−27	−39	141	105	93			
2 c	−12	−3	−36	65	65	41	25	−23	−47	49	25	25	−27	−27	−39	153	117	105			
3 a	−12	−3	−24	47	47	23	43	−5	−29	19	−5	−5	−21	−33	−33	87	75	51			
3 b	−4	−1	−8	17	17	9	13	−3	−11	9	1	1	−7	−11	−11	33	29	21			
3 c	−12	−3	−24	55	55	31	35	−13	−37	35	11	11	−21	−33	−33	111	99	75			
3 d	−12	−3	−24	59	59	35	31	−17	−41	43	19	19	−21	−33	−33	123	111	87			
4 a	−4	−1	−8	19	19	11	11	−5	−13	11	3	3	−9	−13	−13	39	35	27			
4 b	−12	−3	−24	61	61	37	29	−19	−43	41	17	17	−27	−39	−39	129	117	93			
5	−4	1	4	13	13	5	21	5	−3	1	−7	−7	−7	−7	−7	25	13	5	−2	−38	−44
6 a	−12	3	12	41	41	17	61	13	−11	1	−23	−23	−27	−27	−27	81	45	21	−6	−114	−132
6 b	−4	1	4	15	15	7	19	3	−5	3	−5	−5	−9	−9	−9	31	19	11	−2	−38	−44
6 c	−12	3	12	49	49	25	53	5	−19	17	−7	−7	−27	−27	−27	105	69	45	−6	−114	−132
6 d	−12	3	12	53	53	29	49	1	−23	25	1	1	−27	−27	−27	117	81	57	−6	−114	−132
7 a	−12	3	12	41	41	17	61	13	−11	1	−23	−23	−27	−27	−27	81	45	21	−6	−114	−132
7 b	−4	1	4	15	15	7	19	3	−5	3	−5	−5	−9	−9	−9	31	19	11	−2	−38	−44
8 a	−12	3	12	43	43	19	59	11	−13	−1	−25	−25	−33	−33	−33	87	51	27	−6	−114	−132
8 b	−12	3	12	47	47	23	55	7	−17	7	−17	−17	−33	−33	−33	99	63	39	−6	−114	−132
8 c	−4	1	4	17	17	9	17	1	−7	5	−3	−3	−11	−11	−11	37	25	17	−2	−38	−44
8 d	−12	3	12	55	55	31	47	−1	−25	23	−1	−1	−33	−33	−33	123	87	63	−6	−114	−132
9 a	−4	1	4	15	15	7	19	3	−5	−1	−9	−9	−13	−13	−13	31	19	11	−2	−38	−44
9 b	−12	3	12	49	49	25	53	5	−19	5	−19	−19	−39	−39	−39	105	69	45	−6	−114	−132
9 c	−12	3	12	53	53	29	49	1	−23	13	−11	−11	−39	−39	−39	117	81	57	−6	−114	−132
9 d	−4	1	4	19	19	11	15	−1	−9	7	−1	−1	−13	−13	−13	43	31	23	−2	−38	−44
10 a	−12	3	12	55	55	31	47	−1	−25	11	−13	−13	−45	−45	−45	123	87	63	−6	−114	−132
10 b	−12	3	12	59	59	35	43	−5	−29	19	−5	−5	−45	−45	−45	135	99	75	−6	−114	−132
11	−12	3	24	35	35	11	67	19	−5	7	−17	−17	−9	−9	−21	51	27	15	−18	−126	−156
12 a	−12	3	24	37	37	13	65	17	−7	5	−19	−19	−15	−15	−27	57	33	21	−18	−126	−156
12 b	−12	3	24	41	41	17	61	13	−11	13	−11	−11	−15	−15	−27	69	45	33	−18	−126	−156
12 c	−4	1	8	15	15	7	19	3	−5	7	−1	−1	−5	−5	−9	27	19	15	−6	−42	−52

B.2.2 Charges for $y = 0$

#	R_ϕ	R_θ	R_{H_d}	R_{Q_1}	R_{Q_2}	R_{Q_3}	$R_{\overline{U}_1}$	$R_{\overline{U}_2}$	$R_{\overline{U}_3}$	$R_{\overline{D}_1}$	$R_{\overline{D}_2}$	$R_{\overline{D}_3}$	R_{L_1}	R_{L_2}	R_{L_3}	$R_{\overline{E}_1}$	$R_{\overline{E}_2}$	$R_{\overline{E}_3}$	R_N	$R_{\overline{N}}$	R_X
1 a	−12	−3	−36	59	47	23	31	−5	−29	19	7	7	−21	−21	−33	111	75	63			
1 b	−4	−1	−12	21	17	9	9	−3	−11	9	5	5	−7	−7	−11	41	29	25			
1 c	−12	−3	−36	67	55	31	23	−13	−37	35	23	23	−21	−21	−33	135	99	87			
1 d	−12	−3	−36	71	59	35	19	−17	−41	43	31	31	−21	−21	−33	147	111	99			
2 a	−12	−3	−36	65	53	29	25	−11	−35	25	13	13	−27	−27	−39	129	93	81			
2 b	−4	−1	−12	23	19	11	7	−5	−13	11	7	7	−9	−9	−13	47	35	31			
2 c	−12	−3	−36	73	61	37	17	−19	−43	41	29	29	−27	−27	−39	153	117	105			
3 a	−12	−3	−24	55	43	19	35	−1	−25	11	−1	−1	−21	−33	−33	87	75	51			
3 b	−12	−3	−24	59	47	23	31	−5	−29	19	7	7	−21	−33	−33	99	87	63			
3 c	−4	−1	−8	21	17	9	9	−3	−11	9	5	5	−7	−11	−11	37	33	25			
3 d	−12	−3	−24	67	55	31	23	−13	−37	35	23	23	−21	−33	−33	123	111	87			
4 a	−12	−3	−24	65	53	29	25	−11	−35	25	13	13	−27	−39	−39	117	105	81			
4 b	−4	−1	−8	23	19	11	7	−5	−13	11	7	7	−9	−13	−13	43	39	31			
5	−12	3	12	47	35	11	55	19	−5	−5	−17	−17	−21	−21	−21	75	39	15	−6	−114	−132
6 a	−12	3	12	49	37	13	53	17	−7	−7	−19	−19	−27	−27	−27	81	45	21	−6	−114	−132
6 b	−12	3	12	53	41	17	49	13	−11	1	−11	−11	−27	−27	−27	93	57	33	−6	−114	−132
6 c	−4	1	4	19	15	7	15	3	−5	3	−1	−1	−9	−9	−9	35	23	15	−2	−38	−44
6 d	−12	3	12	61	49	25	41	5	−19	17	5	5	−27	−27	−27	117	81	57	−6	−114	−132
7 a	−12	3	12	49	37	13	53	17	−7	−7	−19	−19	−27	−27	−27	81	45	21	−6	−114	−132
7 b	−12	3	12	53	41	17	49	13	−11	1	−11	−11	−27	−27	−27	93	57	33	−6	−114	−132
8 a	−4	1	4	17	13	5	17	5	−3	−3	−7	−7	−11	−11	−11	29	17	9	−2	−38	−44
8 b	−12	3	12	55	43	19	47	11	−13	−1	−13	−13	−33	−33	−33	99	63	39	−6	−114	−132
8 c	−12	3	12	59	47	23	43	7	−17	7	−5	−5	−33	−33	−33	111	75	51	−6	−114	−132
8 d	−4	1	4	21	17	9	13	1	−7	5	1	1	−11	−11	−11	41	29	21	−2	−38	−44
9 a	−12	3	12	53	41	17	49	13	−11	−11	−23	−23	−39	−39	−39	93	57	33	−6	−114	−132
9 b	−4	1	4	19	15	7	15	3	−5	−1	−5	−5	−13	−13	−13	35	23	15	−2	−38	−44
9 c	−12	3	12	61	49	25	41	5	−19	5	−7	−7	−39	−39	−39	117	81	57	−6	−114	−132
9 d	−12	3	12	65	53	29	37	1	−23	13	1	1	−39	−39	−39	129	93	69	−6	−114	−132
10 a	−4	1	4	21	17	9	13	1	−7	1	−3	−3	−15	−15	−15	41	29	21	−2	−38	−44
10 b	−12	3	12	67	55	31	35	−1	−25	11	−1	−1	−45	−45	−45	135	99	75	−6	−114	−132
11	−12	3	24	43	31	7	59	23	−1	−1	−13	−13	−9	−9	−21	51	27	15	−18	−126	−156
12 a	−4	1	8	15	11	3	19	7	−1	−1	−5	−5	−5	−5	−9	19	11	7	−6	−42	−52
12 b	−12	3	24	49	37	13	53	17	−7	5	−7	−7	−15	−15	−27	69	45	33	−18	−126	−156
12 c	−12	3	24	53	41	17	49	13	−11	13	1	1	−15	−15	−27	81	57	45	−18	−126	−156

B.2.3 Charges for $y = +1$

#	R_ϕ	R_θ	R_{H_d}	R_{Q_1}	R_{Q_2}	R_{Q_3}	$R_{\overline{U}_1}$	$R_{\overline{U}_2}$	$R_{\overline{U}_3}$	$R_{\overline{D}_1}$	$R_{\overline{D}_2}$	$R_{\overline{D}_3}$	R_{L_1}	R_{L_2}	R_{L_3}	$R_{\overline{E}_1}$	$R_{\overline{E}_2}$	$R_{\overline{E}_3}$	R_N	$R_{\overline{N}}$	R_X
1 a	−12	−3	−36	67	43	19	23	−1	−25	11	11	11	−21	−21	−33	111	75	63			
1 b	−12	−3	−36	71	47	23	19	−5	−29	19	19	19	−21	−21	−33	123	87	75			
1 c	−4	−1	−12	25	17	9	5	−3	−11	9	9	9	−7	−7	−11	45	33	29			
1 d	−12	−3	−36	79	55	31	11	−13	−37	35	35	35	−21	−21	−33	147	111	99			
2 a	−12	−3	−36	73	49	25	17	−7	−31	17	17	17	−27	−27	−39	129	93	81			
2 b	−12	−3	−36	77	53	29	13	−11	−35	25	25	25	−27	−27	−39	141	105	93			
2 c	−4	−1	−12	27	19	11	3	−5	−13	11	11	11	−9	−9	−13	51	39	35			
3 a	−4	−1	−8	21	13	5	9	1	−7	1	1	1	−7	−11	−11	29	25	17			
3 b	−12	−3	−24	67	43	19	23	−1	−25	11	11	11	−21	−33	−33	99	87	63			
3 c	−12	−3	−24	71	47	23	19	−5	−29	19	19	19	−21	−33	−33	111	99	75			
3 d	−4	−1	−8	25	17	9	5	−3	−11	9	9	9	−7	−11	−11	41	37	29			
4 a	−12	−3	−24	73	49	25	17	−7	−31	17	17	17	−27	−39	−39	117	105	81			
4 b	−12	−3	−24	77	53	29	13	−11	−35	25	25	25	−27	−39	−39	129	117	93			
5	−12	3	12	55	31	7	47	23	−1	−13	−13	−13	−21	−21	−21	75	39	15	−6	−114	−132
6 a	−4	1	4	19	11	3	15	7	−1	−5	−5	−5	−9	−9	−9	27	15	7	−2	−38	−44
6 b	−12	3	12	61	37	13	41	17	−7	−7	−7	−7	−27	−27	−27	93	57	33	−6	−114	−132
6 c	−12	3	12	65	41	17	37	13	−11	1	1	1	−27	−27	−27	105	69	45	−6	−114	−132
6 d	−4	1	4	23	15	7	11	3	−5	3	3	3	−9	−9	−9	39	27	19	−2	−6	−12
7 a	−4	1	4	19	11	3	15	7	−1	−5	−5	−5	−9	−9	−9	27	15	7	−2	−38	−44
7 b	−12	3	12	61	37	13	41	17	−7	−7	−7	−7	−27	−27	−27	93	57	33	−6	−114	−132
8 a	−12	3	12	59	35	11	43	19	−5	−17	−17	−17	−33	−33	−33	87	51	27	−6	−114	−132
8 b	−4	1	4	21	13	5	13	5	−3	−3	−3	−3	−11	−11	−11	33	21	13	−2	−38	−44
8 c	−12	3	12	67	43	19	35	11	−13	−1	−1	−1	−33	−33	−33	111	75	51	−6	−114	−132
8 d	−12	3	12	71	47	23	31	7	−17	7	7	7	−33	−33	−33	123	87	63	−6	−114	−132
9 a	−12	3	12	61	37	13	41	17	−7	−19	−19	−19	−39	−39	−39	93	57	33	−6	−114	−132
9 b	−12	3	12	65	41	17	37	13	−11	−11	−11	−11	−39	−39	−39	105	69	45	−6	−114	−132
9 c	−4	1	4	23	15	7	11	3	−5	−1	−1	−1	−13	−13	−13	39	27	19	−2	−38	−44
9 d	−12	3	12	73	49	25	29	5	−19	5	5	5	−39	−39	−39	129	93	69	−6	−114	−132
10 a	−12	3	12	71	47	23	31	7	−17	−5	−5	−5	−45	−45	−45	123	87	63	−6	−114	−132
10 b	−4	1	4	25	17	9	9	1	−7	1	1	1	−15	−15	−15	45	33	25	−2	−38	−44
11	−4	1	8	17	9	1	17	9	1	−3	−3	−3	−3	−3	−7	17	9	5	−6	−42	−52
12 a	−12	3	24	53	29	5	49	25	1	−11	−11	−11	−15	−15	−27	57	33	21	−18	−126	−156
12 b	−4	1	8	19	11	3	15	7	−1	−1	−1	−1	−5	−5	−9	23	15	11	−6	−42	−52
12 c	−12	3	24	61	37	13	41	17	−7	5	5	5	−15	−15	−27	81	57	45	−18	−126	−156

Appendix C

Alternative A_4 basis

In chapter 5 we described flavor models based on non–Abelian discrete symmetries and in section 5.3.1 we used A_4 as an example symmetry group. So far we used A_4 in basis generated by the matrices

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} , \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} , \qquad \text{with} \qquad \omega = \mathrm{e}^{\frac{2\pi\mathrm{i}}{3}} , \tag{C.1}$$

which gives us the multiplication law in equation (5.47). However, it also possible, and widely used, to generate A_4 with a different set of basis generators

$$\widetilde{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} , \qquad \widetilde{T} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} . \tag{C.2}$$

In this basis the multiplication laws are the same, the trivial ones

$$\mathbf{1'} \otimes \mathbf{1'} = \mathbf{1''} , \qquad \mathbf{1''} \otimes \mathbf{1''} = \mathbf{1'} , \qquad \mathbf{1'} \otimes \mathbf{1''} = \mathbf{1} , \tag{C.3}$$

and the most important one

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{1''} \oplus \mathbf{3}_\mathrm{s} \oplus \mathbf{3}_\mathrm{a} . \tag{C.4}$$

The only difference is how these transformations look like in components, i.e. assuming we have two triplets $\boldsymbol{a}$ and $\boldsymbol{b}$ we get for $\mathbf{3} \otimes \mathbf{3}$

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1}} = a_1\, b_1 + a_2\, b_2 + a_3\, b_3 , \tag{C.5a}$$

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1'}} = a_1\, b_1 + \omega\, a_2\, b_2 + \omega^2\, a_3\, b_3 , \tag{C.5b}$$

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{1''}} = a_1\, b_1 + \omega^2\, a_2\, b_2 + \omega\, a_3\, b_3 , \tag{C.5c}$$

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{3}_\mathrm{s}} \sim \begin{pmatrix} a_2\, b_3 + b_3\, a_2 \\ a_3\, b_1 + b_3\, a_1 \\ a_1\, b_2 + b_1\, a_2 \end{pmatrix} , \tag{C.5d}$$

$$(\boldsymbol{a} \otimes \boldsymbol{b})_{\mathbf{3}_\mathrm{a}} \sim \begin{pmatrix} a_2\, b_3 - b_3\, a_2 \\ a_3\, b_1 - b_3\, a_1 \\ a_1\, b_2 - b_1\, a_2 \end{pmatrix} . \tag{C.5e}$$

This new basis is related to our previously used one through the unitary transformation matrix

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} . \tag{C.6}$$

We can relate the two bases through the transformation $\widetilde{S} = U_\omega \, S \, U_\omega^\dagger$ and $\widetilde{T} = U_\omega \, T \, U_\omega^\dagger$. It is important to note that this transformation also relates the different flavon VEVs, i.e. the VEV $(v, v, v)^T$ in one basis is equivalent to the VEV $(v', 0, 0)^T$ in the other basis, and vice versa.

Appendix D

A_4 and T' contractions

D.1 A_4 contractions

In section 6.2.1 we presented a few of the possible Kähler corrections for the A_4 example. As we stated in equation (6.40), six contractions are possible for a given flavon triplet Φ

$$\begin{aligned}&(L\otimes\Phi)^\dagger_{\mathbf{1}}\,(L\otimes\Phi)_{\mathbf{1}}\ , \qquad (L\otimes\Phi)^\dagger_{\mathbf{1}'}\,(L\otimes\Phi)_{\mathbf{1}'}\ , \qquad (L\otimes\Phi)^\dagger_{\mathbf{1}''}\,(L\otimes\Phi)_{\mathbf{1}''}\ ,\\ &(L\otimes\Phi)^\dagger_{\mathbf{3}_a}\,(L\otimes\Phi)_{\mathbf{3}_a}\ , \qquad (L\otimes\Phi)^\dagger_{\mathbf{3}_s}\,(L\otimes\Phi)_{\mathbf{3}_s}\ , \qquad (L\otimes\Phi)^\dagger_{\mathbf{3}_a}\,(L\otimes\Phi)_{\mathbf{3}_s}\ .\end{aligned} \tag{D.1}$$

Since we have two different flavon triplets, Φ_ν and Φ_e, there will be 12 terms in total which we now present in the following.

We start with Φ_ν, recalling that $\langle\Phi_\nu\rangle = (v,v,v)^T$. As it turns out,

$$(L\otimes\Phi_\nu)^\dagger_{\mathbf{1}}\,(L\otimes\Phi_\nu)_{\mathbf{1}}\ , \qquad (L\otimes\Phi_\nu)^\dagger_{\mathbf{1}'}\,(L\otimes\Phi_\nu)_{\mathbf{1}'}\ , \qquad (L\otimes\Phi_\nu)^\dagger_{\mathbf{1}''}\,(L\otimes\Phi_\nu)_{\mathbf{1}''}\ , \tag{D.2}$$

all give the same Kähler corrections after Φ_ν acquires its VEV, which are represented by the P matrix

$$P = \begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}\ . \tag{D.3}$$

Also $(L\otimes\Phi_\nu)^\dagger_{\mathbf{3}_a}\,(L\otimes\Phi_\nu)_{\mathbf{3}_a}$ and $(L\otimes\Phi_\nu)^\dagger_{\mathbf{3}_s}\,(L\otimes\Phi_\nu)_{\mathbf{3}_s}$ give the same correction,

$$P = \frac{3}{2}\begin{pmatrix}2 & -1 & -1\\ -1 & 2 & -1\\ -1 & -1 & 2\end{pmatrix}\ . \tag{D.4}$$

The last one due to Φ_ν acquiring its VEV is coming from $(L\otimes\Phi_\nu)^\dagger_{\mathbf{3}_a}\,(L\otimes\Phi_\nu)_{\mathbf{3}_s}$ and given by

$$P = \begin{pmatrix}0 & \mathrm{i} & -\mathrm{i}\\ -\mathrm{i} & 0 & \mathrm{i}\\ \mathrm{i} & -\mathrm{i} & 0\end{pmatrix}\ . \tag{D.5}$$

Obviously, all these corrections are proportional to v^2.

Let us now consider Φ_e with $\langle \Phi_e \rangle = (v', 0, 0)^T$. Here, all contractions result in diagonal matrices, i.e.

$$(L \otimes \Phi_e)^\dagger_{\mathbf{1}} (L \otimes \Phi_e)_{\mathbf{1}} \longrightarrow P = \mathrm{diag}(1,0,0) , \tag{D.6a}$$
$$(L \otimes \Phi_e)^\dagger_{\mathbf{1'}} (L \otimes \Phi_e)_{\mathbf{1'}} \longrightarrow P = \mathrm{diag}(0,1,0) , \tag{D.6b}$$
$$(L \otimes \Phi_e)^\dagger_{\mathbf{1''}} (L \otimes \Phi_e)_{\mathbf{1''}} \longrightarrow P = \mathrm{diag}(0,0,1) , \tag{D.6c}$$
$$(L \otimes \Phi_e)^\dagger_{\mathbf{3}_s} (L \otimes \Phi_e)_{\mathbf{3}_s} \longrightarrow P = \mathrm{diag}(4,1,1) , \tag{D.6d}$$
$$(L \otimes \Phi_e)^\dagger_{\mathbf{3}_a} (L \otimes \Phi_e)_{\mathbf{3}_a} \longrightarrow P = \mathrm{diag}(0,1,1) , \tag{D.6e}$$
$$(L \otimes \Phi_e)^\dagger_{\mathbf{3}_a} (L \otimes \Phi_e)_{\mathbf{3}_s} \longrightarrow P = \mathrm{diag}(0,1,-1) . \tag{D.6f}$$

which are all proportional to $(v')^2$.

D.2 T' contractions

In section 6.2.2 we presented a few of the contractions for the T' example and here we want to summarize all of the ones due to the doublet contractions, i.e. the additional ones to the already known triplet contractions. In this model we have two doublets, ψ and ψ', and according to equation (6.54) three possible contractions per flavon with the left–handed lepton triplet L, e.g. for ψ we have

$$(L \otimes \psi)^\dagger_{\mathbf{2}} (L \otimes \psi)_{\mathbf{2}} , \qquad (L \otimes \psi)^\dagger_{\mathbf{2'}} (L \otimes \psi)_{\mathbf{2'}} \qquad \text{and} \qquad (L \otimes \psi)^\dagger_{\mathbf{2''}} (L \otimes \psi)_{\mathbf{2''}} , \tag{D.7}$$

and the same terms for ψ', so six new terms in total.

Let us start with the field ψ and recall that ψ is in the $\mathbf{2'}$ of T' and acquires the VEV $\langle \psi \rangle = (\psi_0, 0)^T$. Using this we get the three diagonal Kähler corrections

$$(L \otimes \psi)^\dagger_{\mathbf{2}} (L \otimes \psi)_{\mathbf{2}} \longrightarrow P = \mathrm{diag}(0,1,2) , \tag{D.8a}$$
$$(L \otimes \psi)^\dagger_{\mathbf{2'}} (L \otimes \psi)_{\mathbf{2'}} \longrightarrow P = \mathrm{diag}(2,1,0) , \tag{D.8b}$$
$$(L \otimes \psi)^\dagger_{\mathbf{2''}} (L \otimes \psi)_{\mathbf{2''}} \longrightarrow P = \mathrm{diag}(1,0,2) , \tag{D.8c}$$

which are all proportional to ψ_0^2.

Consider the field ψ', a $\mathbf{2}$ under T' with the VEV $\langle \psi' \rangle = (\psi'_0, \psi'_0)^T$, which gives us three different P matrices,

$$(L \otimes \psi')^\dagger_{\mathbf{2}} (L \otimes \psi')_{\mathbf{2}} \longrightarrow P = \begin{pmatrix} 2 & \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & 2 & 0 \\ -\sqrt{2} & 0 & 2 \end{pmatrix} , \tag{D.9a}$$
$$(L \otimes \psi')^\dagger_{\mathbf{2'}} (L \otimes \psi')_{\mathbf{2'}} \longrightarrow P = \begin{pmatrix} 2 & -\sqrt{2} & 0 \\ -\sqrt{2} & 2 & \sqrt{2} \\ 0 & \sqrt{2} & 2 \end{pmatrix} , \tag{D.9b}$$
$$(L \otimes \psi')^\dagger_{\mathbf{2''}} (L \otimes \psi')_{\mathbf{2''}} \longrightarrow P = \begin{pmatrix} 2 & 0 & \sqrt{2} \\ 0 & 2 & -\sqrt{2} \\ \sqrt{2} & -\sqrt{2} & 2 \end{pmatrix} , \tag{D.9c}$$

which are all proportional to $(\psi'_0)^2$.

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