

Intercell Interference Robustness Tradeoff with Loosened Covariance Shaping

Hans H. Brunner[†], Andreas Dotzler[‡], Wolfgang Utschick[‡], Josef A. Nossek[†]

[†]Institute for Circuit Theory and Signal Processing, [‡]Associate Institute for Signal Processing

Technische Universität München, 80290 Munich, Germany

e-mail: {brunner|dotzler|utschick|josef.a.nossek}@tum.de

Abstract—We consider the downlink of a cellular network with multiple antenna base stations (BS) and single antenna mobile devices (MD). Whenever a BS changes its beamforming strategy, it changes the intercell interference (ICI) at every MD in the whole network. These changes are usually not predictable and a BS cannot keep track of the signal to interference plus noise ratio (SINR) of its associated MDs. Consequently, it is not optimal to choose beamforming strategies and link rate adaptations based on measured or assumed SINR values. We propose an optimized shaping constraint on the transmit covariances to make the ICI more predictable. The remaining ICI instationarity is handled with an expected rate optimization, which takes a sampled probability density function of the ICI into account.

I. INTRODUCTION

Uncertainty in the ICI has a variety of negative effects. For example, the link rate adaption can fail because the SINR during the transmission is unknown. This might lead to unexpected outages. Some operations in higher layers, such as scheduling and resource allocation, also depend on the SINR and cannot be performed optimally based on assumed ICI values.

The problem of the ICI instationarity is already addressed in [1]. An upper bound to the possible rates in systems with ICI instationarity is defined, where the actual SINR is assumed to be known in each time slot. In [2], we proposed to optimize the transmit covariances at each BS based on the expected rate of the associated MDs. With this approach, the system for which the transmit covariances are optimized and the system in which the covariances are utilized become the same.

A different method was suggested in [3]. The covariances of the transmit symbols at each BS are forced to scaled identity matrices, which still leaves room for an optimization of the individual beamforming vectors. This constraint completely removes the uncertainty in the ICI variance and the SINR values of the served MDs can be known at the BSs. But, the shaping constraint on the transmitter also reduces the achievable rates.

However, there is still a large gap between the two methods and the upper bound “ICI aware”, which can only be achieved, if the ICI is known at the transmitter. In this paper, a combination of the expected rate optimization with a less restrictive shaping constraint is proposed. The benefits of the stabilization and the expected rate are combined in this new method. With a loosening factor, the strictness of the shaping constraint is

adjusted. This loosening factor can be translated to a limit on the maximum eigenvalue of the sum transmit covariance.

The ICI could be made available with a second pilot at the cost of an additional overhead, if the BSs synchronize the update of their beamforming [4]. In an idealized network MIMO scenario with fully centralized coordination, the problem of ICI awareness does not arise.

The problem of the instationary ICI can also be mitigated with *hybrid automatic repeat request* (HARQ) with *incremental redundancy* (IR). The authors of [5] present an algorithm, which optimizes the scheduling decisions based on the expected rates, where the effects of HARQ are already taken into account. In [5], [6] the authors show that the ergodic upperbound rate with known SINR values can be reached, if infinitely many retransmissions are allowed at the cost of infinite delay. HARQ is not incorporated in the expected rate optimization in this work, but it can be added in a later work.

The used system model based on the Winner channel model is described in Section II. The instationarity of the ICI and the expected rate optimization are discussed in more detail in Section III. Section IV describes the scaled identity optimization and the newly proposed algorithm with the loosened scaled identity constraint on the transmit covariances is presented in Section V. Simulation results are shown in Section VI.

II. SYSTEM MODEL

A MD in the set \mathcal{K} of all MDs is specified by the tuple $(b, k) \in \mathcal{K}$, where $b \in \mathcal{B}$ identifies the BS in the set \mathcal{B} of all BSs and $k \in \mathcal{K}_b$ the MD in the set \mathcal{K}_b of all MDs in the cell of BS b . In this paper, each BS has N antennas and serves $K = |\mathcal{K}_b|$ single antenna MDs, respectively. The vectors $\mathbf{h}_{\hat{b},b,k} \in \mathbb{C}^N$ contain the channel coefficients between the antennas of BS \hat{b} and MD (b, k) .

Block fading is assumed, where the channel stays constant for T_{block} transmit symbols. The achievable, normalized rate of MD (b, k) , within the capacity region of the MIMO broadcast channel with dirty paper coding, can be expressed as

$$r_{b,k} = \ln \left(\frac{\sigma^2 + \theta_{b,k} + \sum_{\hat{k} \geq k} \mathbf{h}_{b,b,\hat{k}}^H \mathbf{Q}_{b,\hat{k}} \mathbf{h}_{b,b,\hat{k}}}{\sigma^2 + \theta_{b,k} + \sum_{\hat{k} > k} \mathbf{h}_{b,b,\hat{k}}^H \mathbf{Q}_{b,\hat{k}} \mathbf{h}_{b,b,\hat{k}}} \right), \quad (1)$$

$$\theta_{b,k} = \sum_{\hat{b} \in \mathcal{B} \setminus b} \mathbf{h}_{\hat{b},b,k}^H \mathbf{Q}_{\hat{b}} \mathbf{h}_{\hat{b},b,k}, \quad (2)$$

where $\mathbf{Q}_{b,k} \in \mathbb{C}^{N \times N}$ is the transmit covariance matrix for MD (b,k) and $\sum_k \mathbf{Q}_{b,k} = \mathbf{Q}_b \in \mathbb{C}^{N \times N}$ is the sum transmit covariance matrix of BS b . $\sum_{\hat{k} < k} \mathbf{h}_{b,b,\hat{k}}^H \mathbf{Q}_{b,\hat{k}} \mathbf{h}_{b,b,k}$ is the variance of the intracell interference with dirty paper coding, $\theta_{b,k}$ is the variance of the received ICI, and $\sigma^2 = \sigma_\eta^2 + \theta_{bg}$ is the sum variance of the thermal noise σ_η^2 and the background ICI. The Gaussian background ICI θ_{bg} models the BSs further away than the closest 57 BSs for a given signal variance per transmit antenna. All BSs have to satisfy the transmit power constraint $\text{tr}(\mathbf{Q}_b) \leq P$. The CSI measurements are assumed error free and the costs of any signaling overhead are ignored.

III. INSTATIONARITY OF THE INTERCELL INTERFERENCE

In the considered scenario, the BSs do not coordinate their beamforming. The channels between an MD and the interfering BSs are not measured. The interference is regarded as noise. Even in scenarios with cooperation among the BSs this type of interference cannot be completely eliminated. Cooperation is always limited in realistic systems, because the measurement of all interference channels and a coordination of all beamformers in the network cannot be implemented [7], [8]. This interference over the unmeasured channels scales with the common transmit power at the BSs and such systems are always interference limited.

In addition, the variance of the interfering symbols at the receivers cannot be known before the transmission. The BSs are assumed to calculate their transmit covariances in a distributed manner and the update process is not synchronized between the BSs. Even if all BSs would update their beamforming at the same time, the ICI could not be known before the BSs have chosen their transmit covariances. The ICI at each MD will change the moment any BS applies a new transmit covariance. Therefore, the BSs compute their transmit covariances based on assumed ICIs $\tilde{\theta}_{b,k}$. The BSs are blind to the ICI changes and take the risk, that the actual ICI $\theta_{b,k}$ increases and the MD cannot decode the transmitted symbols or that $\theta_{b,k}$ decreases and valuable resources are wasted [1]. This problem can be formulated as

$$\tilde{r}_{b,k} = \begin{cases} \tilde{r}_{b,k} = r_{b,k} |_{\theta_{b,k} = \tilde{\theta}_{b,k}}, & \text{for } \tilde{\theta}_{b,k} \geq \theta_{b,k}, \\ 0, & \text{for } \tilde{\theta}_{b,k} < \theta_{b,k}, \end{cases} \quad (3)$$

where $\tilde{r}_{b,k}$ and $r_{b,k}$ are the assumed and achievable rate with the assumed ICI $\tilde{\theta}_{b,k}$, respectively.

Most optimizations in the literature utilize for the assumed ICI the expectation of the ICI, an ICI realization from a previous step, or ignore this problem at all. This results in a mismatch between the utility of the optimization and the actual performance measure. To counteract this problem, the expectation of the rate with respect to the random ICI variance $\Theta_{b,k}$ is considered, which can take any value $\theta_{b,k} \geq 0$, as proposed in [2]:

$$E_{\Theta_{b,k}}[\tilde{r}_{b,k}] = \tilde{r}_{b,k} F_{\Theta_{b,k}}(\tilde{\theta}_{b,k}), \quad (4)$$

where $F_{\Theta_{b,k}}(\tilde{\theta}_{b,k}) = P(\tilde{\theta}_{b,k} \geq \theta_{b,k})$ is the probability, that the transmission is successful or the *cumulative distribution*

function (CDF) of the random ICI variance evaluated at $\tilde{\theta}_{b,k}$. With this step, the sum utility

$$R = \sum_{(b,k) \in \mathcal{K}} \tilde{r}_{b,k} F_{\Theta_{b,k}}(\tilde{\theta}_{b,k}), \quad (5)$$

is reached, which corresponds to the performance measure. As the interference is regarded as noise, the utility of a MD depends only on the beamforming of the serving BS. The network utility optimization splits into individual cell sum utility optimizations:

$$R_{\text{expected},b} = \max_{\mathbf{Q}_{b,k} \geq \mathbf{0}, \tilde{\theta}_{b,k} \geq 0 \forall (b,k) \in \mathcal{K}_b} \sum_{(b,k) \in \mathcal{K}_b} \tilde{r}_{b,k} F_{\Theta_{b,k}}(\tilde{\theta}_{b,k}), \quad (6)$$

s.t. $\text{tr}(\mathbf{Q}_b) \leq P$.

The transmit covariances $\mathbf{Q}_{b,k}$ and the assumed ICI $\tilde{\theta}_{b,k}$ in problem (6) can be optimized with an alternating optimization as described in [2]. For a fixed assumed ICI, the probability of a successful transmission is fixed and a weighted utility optimization with respect to the transmit covariances remains. For fixed transmit covariances, the assumed ICI can be optimized with a root finding algorithm.

To perform the described procedure, the CDFs of the ICI at each associated MD need to be available at the serving BS. The CDFs can be approximated with long term measurements at the MDs. It could also be possible to estimate a rough CDF directly based on the channel measurements. This would not require any additional measurements and feedback for the CDF. If the update process at the BSs is synchronized, it will be possible to measure $\theta_{b,k}$ with a second pilot, which removes the uncertainty in the ICI afterwards but increases the overhead [4].

IV. SCALED IDENTITY CONSTRAINT

With the interference stabilization method from [3], the sum transmit covariances at the BSs are constraint to scaled identity matrices. Note, that the individual transmit covariances for the MDs can still be optimized. With the stabilization, a BS always knows the ICI at its associated MDs during the data transmission. The ICI is measured before the beamforming is selected. Due to the shaping constraint, the ICI does not change during the optimization of the transmit covariances. There is no need of measuring the interference channels or any statistics about the ICI. The problem of interference awareness disappears at the cost of a restriction on the transmit covariances, which reduces the possible rates. The CDF of the random ICI variance becomes a unit step function at the measured ICI. Again, the utility of an MD only depends on the beamforming of the serving BS and, therefore, the network utility optimization splits into individual utility optimizations per cell. The optimization can be formulated as

$$R_{\text{identity},b} = \max_{\mathbf{Q}_{b,k} \geq \mathbf{0} \forall (b,k) \in \mathcal{K}_b} \sum_{(b,k) \in \mathcal{K}_b} r_{b,k}, \quad (7)$$

s.t. $\mathbf{Q}_b \preceq \frac{P}{N} \mathbf{I}$,

The cell utility optimization in the downlink can be transformed to an uplink optimization [3]

$$\begin{aligned} \min_{\Omega_b \succeq \mathbf{0}} \quad & \max_{q_{b,k} \geq 0 \forall (b,k) \in \mathcal{K}_b} \sum_{(b,k) \in \mathcal{K}_b} r_{b,k}^{\text{MAC}}, \\ \text{s.t.} \quad & \sum_{(b,k) \in \mathcal{K}_b} q_{b,k} \leq P, \quad \text{tr}(\Omega_b) = N, \end{aligned} \quad (8)$$

$$r_{b,k}^{\text{MAC}} = \ln \frac{\left| \Omega_b + \sum_{\hat{k} \leq k} q_{b,\hat{k}} \tilde{\mathbf{h}}_{b,b,\hat{k}} \tilde{\mathbf{h}}_{b,b,\hat{k}}^H \right|}{\left| \Omega_b + \sum_{\hat{k} < k} q_{b,\hat{k}} \tilde{\mathbf{h}}_{b,b,\hat{k}} \tilde{\mathbf{h}}_{b,b,\hat{k}}^H \right|}, \quad (9)$$

where $\Omega_b \in \mathbb{C}^{N \times N}$ is the noise covariance matrix at BS b and $q_{b,k}$ is the transmit power allocated to MD (b,k) in the uplink. $\tilde{\mathbf{h}}_{b,b,k} = \frac{1}{\sqrt{\sigma^2 + \theta_{b,k}}} \mathbf{h}_{b,b,k}$ is the effective channel between the regarded BS and the MD (b,k) . This optimization is a saddle point problem, which can be solved with an alternating algorithm. The cell utility maximizing power allocation and the cell utility minimizing noise realization are found in turns.

V. LOOSENED COVARIANCE SHAPING

By loosening the strict shaping constraint, a controlled instationarity of the ICI can be introduced. This instationarity can be handled by optimizing the expected rate. With this approach, the two different techniques dealing with the ICI awareness problem can be combined. A tradeoff between the covariance shaping with the scaled identity and the expected rate method can be found. The combined local optimization reads as

$$\begin{aligned} R_{\text{loose},\alpha,b} = \quad & \max_{\mathbf{Q}_b \succeq \mathbf{0}, \tilde{\theta}_{b,k} \geq 0 \forall (b,k) \in \mathcal{K}_b} \sum_{(b,k) \in \mathcal{K}_b} \mathbb{E}_{\Theta_{\alpha,b,k}} [\tilde{r}_{b,k}], \\ \text{s.t.} \quad & \mathbf{Q}_b \preceq \alpha \frac{P}{N} \mathbf{I}, \quad \text{tr}(\mathbf{Q}_b) \leq P, \end{aligned} \quad (10)$$

where $\alpha \geq 1$ loosens the shaping constraint. Note, that the statistics of the random ICI variance $\Theta_{\alpha,b,k}$ depend on the shaping constraint. For $\alpha = 1$, the shaping constraint is strict and there is no uncertainty in the ICI. For $\alpha \geq N$ the constraint $\mathbf{Q}_b \preceq \alpha \frac{P}{N} \mathbf{I}$ is not binding and the statistics of the random ICI are the same as for the expected rate optimization without any shaping constraint.

With the eigenvalues $\lambda_{b,n}$ ($n = 1, \dots, N$) of \mathbf{Q}_b and $\text{tr}(\mathbf{Q}_b) = \sum_n \lambda_{b,n}$, problem (10) can be written as

$$\begin{aligned} R_{\text{loose},\alpha,b} = \quad & \max_{\mathbf{Q}_b \succeq \mathbf{0}, \tilde{\theta}_{b,k} \geq 0 \forall (b,k) \in \mathcal{K}_b} \sum_{(b,k) \in \mathcal{K}_b} \tilde{r}_{b,k} F_{\Theta_{\alpha,b,k}}(\tilde{\theta}_{b,k}), \\ \text{s.t.} \quad & \max(\lambda_{b,n}) \leq \alpha \frac{P}{N}, \quad \sum_n \lambda_{b,n} \leq P, \end{aligned} \quad (11)$$

where $F_{\Theta_{\alpha,b,k}}(\tilde{\theta}_{b,k})$ is the probability that the transmission is successful, i.e., the CDF of the ICI evaluated at $\tilde{\theta}_{b,k}$, where all BSs use the loosening factor α .

The transmit covariances $\mathbf{Q}_{b,k}$ and the assumed ICI $\tilde{\theta}_{b,k}$ in problem (11) can be optimized with an alternating optimization. For fixed transmit covariances, the problem simplifies to

$$\max_{\tilde{\theta}_{b,k} \geq 0 \forall (b,k) \in \mathcal{K}_b} \sum_{(b,k) \in \mathcal{K}_b} \tilde{r}_{b,k} F_{\Theta_{\alpha,b,k}}(\tilde{\theta}_{b,k}). \quad (12)$$

Setting the derivation of problem (12) with respect to the assumed ICI $\tilde{\theta}_{b,k}$ to zero yields

$$\frac{\partial \tilde{r}_{b,k}}{\partial \tilde{\theta}_{b,k}} F_{\Theta_{\alpha,b,k}}(\tilde{\theta}_{b,k}) + \tilde{r}_{b,k} f_{\Theta_{\alpha,b,k}}(\tilde{\theta}_{b,k}) = 0, \quad (13)$$

where $f_{\Theta_{\alpha,b,k}}$ is the probability density function of $\Theta_{\alpha,b,k}$. The assumed ICI $\tilde{\theta}_{b,k}$, which fulfills equation (13), can be found with a root finding algorithm.

For a fixed assumed ICI, the probability of a successful transmission is fixed and problem (11) is a weighted sum rate maximization with a limit on the maximum eigenvalue. As described in [9], this problem can be converted to a convex-concave uplink optimization problem

$$\begin{aligned} \min_{\beta_b \geq 0, \Omega_b \succeq \beta_b \mathbf{I}} \quad & \max_{q_{b,k} \geq 0 \forall (b,k) \in \mathcal{K}_b} \sum_{(b,k) \in \mathcal{K}_b} \tilde{r}_{b,k}^{\text{MAC}} F_{\Theta_{\alpha,b,k}}(\tilde{\theta}_{b,k}), \\ \text{tr}(\Omega_b) = \frac{N}{\alpha} (1 + \beta_b (\alpha - 1)) \quad & \sum_{(b,k) \in \mathcal{K}_b} q_{b,k} = P \end{aligned} \quad (14)$$

$$\tilde{r}_{b,k}^{\text{MAC}} = r_{b,k}^{\text{MAC}} \Big|_{\theta_{b,k} = \tilde{\theta}_{b,k}} \quad (15)$$

where β_b controls the weighting between the transmit power and the shaping constraint. This saddle point problem can be solved efficiently with a joint scaled gradient descent, which updates the transmit powers $q_{b,k}$ and the uplink noise Ω_b in parallel. In each step, the updates have to be projected orthogonally onto the constraint set [9].

The complete algorithm, which optimizes the transmit covariances and the assumed ICI powers, is listed in Algorithm 1. The algorithm converges, because the cost function improves in every step and the cost function is limited. Convergence is typically reached after three iterations. ‘‘uplink2downlink’’ is a conversion of the transmit powers and the noise covariance matrix in the uplink to the transmit covariance matrices in the downlink as described, e.g., in [10].

Algorithm 1 Maximization of the Expected Cell Sum Rate with Loosened Covariance Shaping

Require: transmit power P , loosening factor α

- 1: $\tilde{\theta}_{b,k} \leftarrow \mathbb{E}_{\Theta_{\alpha,b,k}}[\theta_{b,k}] \forall k$ \triangleright initialize assumed ICIs
- 2: **while** not converged **do**
- 3: $\tilde{\mathbf{h}}_{b,b,k} \leftarrow \frac{1}{\sqrt{\sigma^2 + \tilde{\theta}_{b,k,i}}} \mathbf{h}_{b,b,k} \forall k$ \triangleright effective channels
- 4: $[q_{b,k} \forall k, \Omega_b] \leftarrow$ solution to problem (14)
- 5: $\mathbf{Q}_{b,k} \leftarrow$ uplink2downlink($q_{b,k}, \Omega_b$)
- 6: $\tilde{\theta}_{b,k} \leftarrow$ solution to problem (13)
- 7: **end while**

VI. SIMULATIONS

For the simulation results, a cellular network with 19 three faced sites and, therefore, 57 BSs is considered. Each BS serves the MDs within the hexagonal shaped cell it covers. The wrap-around method is used to treat all cells equally and the channels are generated according to the 3GPP MIMO urban macro cell model [11]. Every BS has $N = 4$ transmit antennas. In every cell, $K = 4$ MDs are placed uniformly distributed and suffer from a thermal noise variance of $\sigma_n^2 =$

$8.3 \cdot 10^{-14}$ W, respectively. The background interference is set to $\theta_{bg} = 9.53 \cdot 10^{-13} \cdot P$, where P is the transmit power.

The simulations operate on an histogram of ICI realizations instead of the probability distribution. The first round of ICI realizations is generated with scaled identities as transmit covariances for the interfering BSs. New transmit covariances are found with these ICI realizations and these new transmit covariances are used for the calculation of new ICI realizations. Only with the second set of ICI realizations the calculated expectation of the rates and the simulated expectation become equal. The scaled identity matrices are always of full rank, while the second set of covariances is not. A further iteration with the ICI realizations and covariances does not change the results.

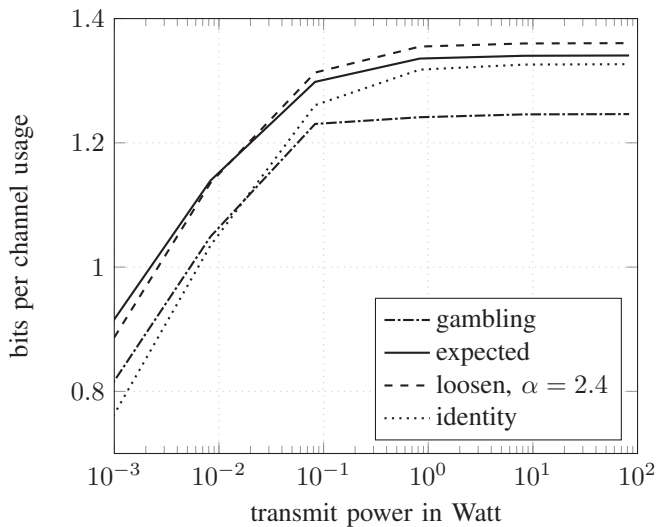


Figure 1. Average user rate over transmit power

Numerical simulations to evaluate the performance of the presented methods can be seen in Figure 1. The normalized average user rate is plotted over the transmit power. The result labeled with “expected” has the rates optimized according to the expected rate algorithm from [2] without any shaping constraint, while “identity” is the interference robustness method with the strict shaping constraint from [3]. Both methods yield substantial improvement compared to the conservative link rate adaption algorithm (“gambling”) from [1] with completely different approaches. The newly proposed algorithm with the loosened shaping constraint is labeled with “loosen”. The loosening factor was selected to $\alpha = 2.4$. This selection gave the best results at high SNR values. All curves saturate for high power because of the ICI. The saturation starts around $P = 1$ W, as it is assumed that all BSs transmit in the same frequency band (reuse 1).

The influence of the loosening on the performance at high SNR values can be seen in Figure 3. $\alpha = 1$ and $\alpha = 4$ are the extreme values, where the loosening converges to the scaled identity and the expected rate algorithm, respectively.

In low SNR scenarios (Figure 3) the effect of the ICI

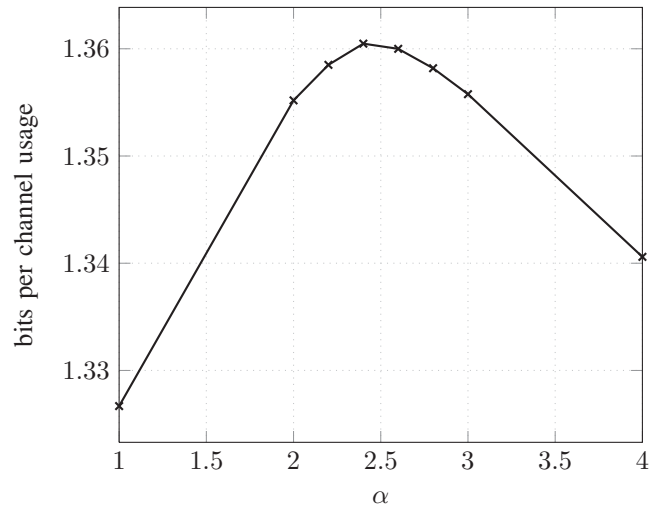


Figure 2. Influence of the loosening at a transmit power of 83 W

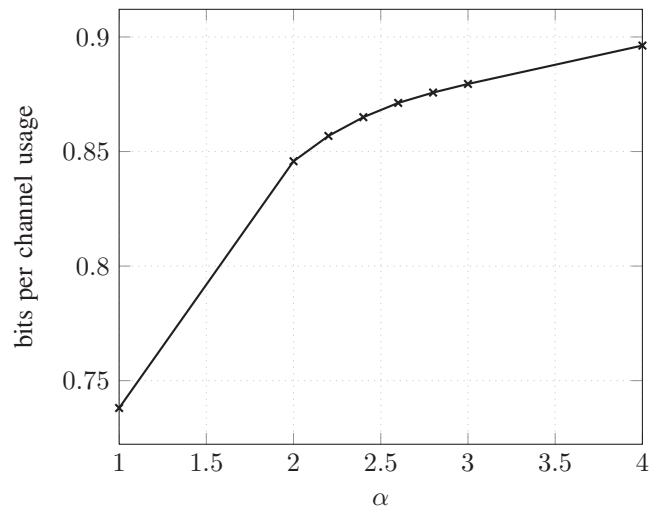


Figure 3. Influence of the loosening at a transmit power of 0.83 mW

vanishes. Therefore, the shaping constraint has no benefit and an unrestricted optimization yields the best results.

The influence of α on the CDF of the ICI is plotted in Figure 4 for a single MD with a single channel realization. For $\alpha = 1$, the CDF is an unit step function at the mean ICI. The CDFs become flatter for increasing α . The possible ICI values become less predictable.

VII. CONCLUSION

To counteract the problem of instationary ICI, we adapted the scaled identity constraint method. This method stabilizes the ICI powers, while it reduces the possible rates. By loosening the constraint, we could limit the drawback, but the ICI is not completely predictable any more. We handled this unpredictability with the expected rate algorithm. We have shown that the combined algorithm improves the achievable rates.

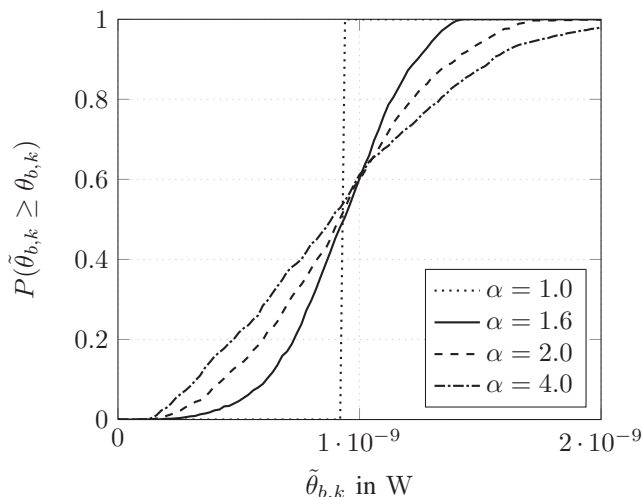


Figure 4. CDF of the ICI for different loosening

REFERENCES

- [1] M.T. Ivrlač and J.A. Nossek, "Intercell-Interference in the Gaussian MISO Broadcast Channel," in *GLOBECOM '07, IEEE Global Telecommunications Conference*, Nov. 2007.
- [2] H.H. Brunner and J.A. Nossek, "Handling Unknown Interference in Cellular Networks with Interference Coordination," in *WSA 2012, International ITG Workshop on Smart Antennas*, Mar. 2012.
- [3] A. Dotzler, M. Riemensberger, W. Utschick, and G. Dietl, "Interference Robustness for Cellular MIMO Networks," in *SPAWC 2012, IEEE International Workshop on Signal Processing Advances in Wireless Communications*, 2012.
- [4] R. Bendlin, H.H. Brunner, M.T. Ivrlač, J.A. Nossek, and Y. Huang, "Two-Phase Scheduling and Leakage-Based Precoding in Wireless Cellular Networks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 3148–3157, 2013.
- [5] H. Shirani-Mehr, H. Papadopoulos, S.A. Ramprasad, and G. Caire, "Joint Scheduling and ARQ for MU-MIMO Downlink in the Presence of Inter-Cell Interference," *IEEE Transactions on Communications*, vol. 59, no. 2, pp. 578–589, 2011.
- [6] S. Mukherjee, S.A. Ramprasad, and Yueqian Li, "Performance Bounds and Improvements to HARQ in LTE under Unknown and Highly Variable Interference Due to MIMO," in *VTC 2011, IEEE Vehicular Technology Conference*, 2011.
- [7] A. Lozano, R. Jr. Heath, and J. Andrews, "Fundamental Limits of Cooperation," *IEEE Transactions on Information Theory*, 2013.
- [8] H.H. Brunner, M.T. Ivrlač, and J.A. Nossek, "Upper Bound to Interference Coordination with Channel State Information Outdating," in *EW 2011, European Wireless Conference*, Apr. 2011.
- [9] Hans H. Brunner, Andreas Dotzler, Wolfgang Utschick, and Josef A. Nossek, "Weighted Sum Rate Optimization with Limited Eigenvalues of the Sum Transmit Covariance," in *preparation*.
- [10] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, Achievable Rates, and Sum-Rate Capacity of Gaussian MIMO Broadcast Channels," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2658–2668, 2003.
- [11] "Spatial Channel Model for Multiple Input Multiple Output (MIMO) Simulations," Tech. Rep. 25.996 V9.0.0, 3rd Generation Partnership Project, Technical Specification Group Radio Access Network, Dec. 2009.