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Consensus Analysis of Networked Multi-agent Systems☆

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Abstract

This paper discusses the problem of consensus for multi-agent systems and convergence analysis. The decentralized consensus control strategy is implemented based on artificial potential functions (APF). Due to the existence of local minima in the APF, some special functions are introduced to settle that limitation. A convergence of consensus protocol is defined to investigate the consensus problem and based on Lyapunov approach sufficient conditions for this convergence principle are established. The main contribution of this paper is to provide a valid decentralized consensus algorithm that overcomes the difficulties caused by nonlinearity and switching coupling topology, and therefore has its obvious practical applications. Finally, simulation results are included. © 2010 Published by Elsevier Ltd

Keywords: Consensus, artificial potential function, convergence analysis, switching topology, multi-agent system.

1. INTRODUCTION

In recent years, decentralized coordination of multi-agent systems (MAS) has become an active area of research and attracted much interest from rather diverse disciplines including animal behavior, system control theory, biophysics, social science, and computer science. Due to the advance in communication and computation consensus study has evolved into the field of engineering applications, such as scheduling of automated highway systems, cooperative control of unmanned aerial vehicles (AUV), formation control of satellite clusters, synchronization of coupled oscillators, flocking of multiple robotic systems, etc [2].

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In the study of cooperative control of multi-agent systems, consensus means to agree upon certain quantity of interest, such as a representation of the center and shape of a formation, the direction of motion for a multi-agent swarm, and the target trajectory, which depends on the states of systems. A consensus algorithm (or strategy) is an interaction principle that specifies the information exchange among multi-agent systems [7]. Such problem is usually called consensus problem, which is one of the foundation of decentralized control field.

Consensus problems for MAS have been studied for a long time [7, 8], and consensus algorithms have applied in formation control [1], flocking [8] and sensor networks [11]. In [14], Vicsek et al. proposed a discrete-time model of multi-autonomous agents, and demonstrated that without any exogenous control all the agents move in the same direction when the noise is small and the density is large. The linearized Vicsek's model was studied by Jadbabaie et al [13] and it was proved that all the agents converge to a common steady state provided that the interconnection graph is jointly connected. Furthermore, in [12] Moreau proposes that more communication do not always necessarily lead to faster convergence and may eventually even lead to a loss of convergence. On the other hand, if the interconnection graph of multi-agent system is not globally interconnected, it needs to introduce an external controller to make system reach consensus. The decentralized approaches like artificial potential functions (APF) have been studied extensively for path-planning of multiple agents in the past decades [1, 2]. In [2] the authors present a swarm model with individuals that move in one-dimensional space according to an attractant/repellent or a nutrient profile. In view of recent papers on APF-based control for consensus, a fundamental problem is the existence of local minima that may occur in a potential field [1]. This article proposes a novel method to deal with the APF which could solve the local minima problem effectively.

Convergence analysis for consensus is equivalent to prove that in an Euclidean space all the state of agents is asymptotically stable [5], [12]. In general, there are two main methods to analyze this problem. The first one is algebraic graph theory [7, 8], and the other one is nonlinear methods which include Lyapunov's direct method [4], set-value Lyapunov theory [12], nonnegative matrix theory and characteristic equation theory. This article selects the second method to analyze the decentralized consensus algorithm.

Furthermore, in view of recent studies [1, 2], most systematic frameworks omit the nonlinear influence from the environments, the couplings and switching topologies among the agents. Motivated by recent results on complex dynamical network [3, 4], a novel framework is presented in this paper to describe the nature characteristics of MAS, such as the nonlinearity and switching coupling topology. Other consensus researches including flocking for multi-agent systems, time-delays in agents' communication and switching topology, system uncertainty, and asynchronous consensus, can be found in [7]-[9], [12].

The paper is organized as follows: a dynamical model of the MAS is introduced and a controller based on a general artificial potential function is designed in Section II; the convergence analysis and technical proof are specified in Section III; in Section IV, specific potential functions are introduced to show how to eliminate the local minima; finally, conclusions are made in Section V.

2. MATHEMATICAL MODEL

Let $\boldsymbol{J} = [t_0, +\infty)\,(t_0 \geq 0)$, $R_+ = [0, +\infty)$. N and R^n denote, respectively, the set of natural numbers and n-dimensional Euclidean space. For $x = (x_1, \ldots, x_n)^\top \in R^n$, the norm of x is

$\|x\|:=\left(\sum_{i=1}^{n} x_i^2\right)^{\frac{1}{2}}$. Correspondingly, for $A=(a_{ij})_{n\times n}\in R^{n\times n}$, $\|A\|:=\lambda_{\max}^{\frac{1}{2}}\left(A^TA\right)$. The identity matrix of order n is denoted as I_n (or simply I if no confusion arises).

In general, a multi-agent system consisting of N individuals in an n-dimensional Euclidean space can be considered as a complex network. Each agent can be considered as a node in the complex network, and each edge represents a communication link between two agents. Suppose that each agent can sense and update the information of multi-agent system such as the positions of other agents and the preassigned path in every step. Such a dynamical network is described by

$$\dot{x}_i \quad = \quad Ax_i + f(t,x_i) + \tau\sum_{\substack{j=1\\ j\neq i}}^{N} D_{ij}^{\sigma}\Gamma(x_j - x_i) + Bu_i. \tag{1}$$

where $i=1,2,\ldots,N$,$t\in \boldsymbol{J}$, A,B are known to be real constant matrices with appropriate dimensions. $f:\boldsymbol{J}\times R^n\to R^n$ is a continuously differentiable nonlinear function representing the group motion (i.e. trajectory dynamics of MAS), which is assumed to be identical for each agent. $x_i=(x_{i1},x_{i2},\cdots,x_{in})\in R^n$ are the state variables of node i and mean positions of agent i in this paper. Scalar $\tau>0$ denotes the communication coupling strength between agents and $\Gamma\in R^{n\times n}$ is a $0-1$ matrix linking the coupling variables. $D^r=\left(D_{ij}^r\right)_{N\times N}$ is the communication coupling configuration matrix: if there is a connection between agent i and agent $j(j\neq i)$, then $D_{ij}^r=D_{ji}^r=1$, $(r=1,2,\ldots,m)$; otherwise, $D_{ij}^r=D_{ji}^r=0$, where m means the amount of switch modes. Furthermore, the diagonal elements of matrix D^r are defined by$D_{ii}^r=-\sum_{j=1,j\neq i}^{N} D_{ij}^r=-\sum_{j=1,j\neq i}^{N} D_{ji}^r$, where $i=1,2,\ldots,N$.

Switch signal $\sigma: R_+\to\{1,2,\cdots,m\}$ represented by $\{\sigma_k\}$ according to $(t_{k-1},t_k]\to\sigma_k\in\{1,2,\cdots,m\}$, is a piecewise constant function. When $t\in(t_{k-1},t_k]$, the σ_kth subsystem is activated [3]. The time sequence $\{t_k\}$ satisfies $t_1<t_2<\cdots<t_k<\cdots,\lim_{k\to\infty}t_k=\infty$, and $t_1>t_0$. u_i denotes the external controller, and it is derived from artificial potential function as the following discussion.

There are many studies focusing on the artificial potential function (APF) [1]-[2], which can be divided into two types: attractive potential and repulsive potential. Here, a decentralized controller for consensus of multi-agent system (1) is suggested as follows: $u_i=-\nabla_{x_i}J(x)=-\nabla_{x_i}\sum_{\substack{j=1\\ j\neq i}}^{N} J(\|x_j-x_i\|)$, where $x^T=\left[x_1^T,x_2^T,\cdots,x_N^T\right]\in R^{n\times N}$, $J:R_+\to R$ is a potential function, $\|x_j-x_i\|=\left(\sum_{k=1}^{n}(x_{j_k}-x_{i_k})^2\right)^{\frac{1}{2}}$ and $J_{ij}(\|x_j-x_i\|)$ denote the distance and the potential function between two agents, respectively. For the convergence purpose, the potential function $J(x)$ is required to have a unique minimum at a desired distance between the agent based on the convergence conditions.

In the subsequent discussion, assume $J_{ij}(\|x_j-x_i\|)$ satisfy:

(i) The potentials $J_{ij}(\|x_j-x_i\|)$ are symmetric and satisfy $\nabla_{x_i}J_{ij}(\|x_j-x_i\|)=-\nabla_{x_j}J_{ij}(\|x_j-x_i\|)$.

(ii) There exist corresponding functions $\eta_{ij}:R_+\to R$ such that

$$-\nabla_{x_i}J_{ij}(\|x_j-x_i\|)=(x_j-x_i)\eta_{ij}(\|x_j-x_i\|). \tag{2}$$

(iii) For all $i,j=1,2,\ldots,N$, $\eta_{ij}(\|x_j-x_i\|)>0$ always hold.

In order to simplify following calculation, $\eta_{ij}(\|x_j - x_i\|)$ are written as η_{ij} in short. The necessity of this assumption will be analyzed in Remark 3.1 in Section 3. So, the controller of the multi-agent system 1 can be written as

$$u_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} -\nabla_{x_i} J(\|x_j - x_i\|) = \sum_{\substack{j=1 \\ j\neq i}}^{N} \left(x_j - x_i\right) \eta_{ij}, \tag{3}$$

3. CONVERGENCE ANALYSIS

With the above preparation, the main results of this paper are presented in this section. Firstly, rewrite the dynamical framework (1) as

$$\dot{x}_i = Ax_i + f(t, x_i) + \tau \sum_{\substack{j=1 \\ j\neq i}}^{N} D_{ij}^{\sigma} \Gamma (x_j - x_i) + B \sum_{\substack{j=1 \\ j\neq i}}^{N} \eta_{ij} \left(x_j - x_i\right), i, j = 1, 2, \ldots, N. \tag{4}$$

In the subsequent discussion, the following notations are defined

$$\overline{\eta} = \min_{i,j=1,2,\ldots,N} \eta_{ij}, \tag{5}$$

$$\lambda = \max_{i,j=1,2,\ldots,N} \left(\lambda_{max}(A) + \phi_{ij}(t)\right), \tag{6}$$

$X_{ij} = x_j - x_i$, where $i, j = 1, 2, \ldots, N$, and $t \in \boldsymbol{J}$. $\phi_{ij}(t)$ are continuous bounded functions on $\boldsymbol{J}$, which satisfy $\left(f(t, x_j) - f(t, x_i)\right)^{\top} X_{ij} \leq \phi_{ij}(t) X_{ij}^{\top} X_{ij}$, where $x_i, x_j \in R^n$, $i, j = 1, 2, \ldots, N$.

According to (4) the time derivative of X_{ij} is

$$\dot{X}_{ij} = AX_{ij} + f(t, x_j) - f(t, x_i) + \tau \sum_{k=1}^{N} \left(D_{jk}^{\sigma} \Gamma X_{jk} - D_{ik}^{\sigma} \Gamma X_{ik}\right) + B \sum_{k=1}^{N} \left(\eta_{jk} X_{jk} - \eta_{ik} X_{ik}\right),$$

where $i, j = 1, 2, \ldots, N$.

The following definition is needed to facilitate the development of the main results.

Definition 3.1. *The system (4) is said to reach consensus, if the state of every agent asymptotically converges to an n-dimensional agreement space characterized by the following equation:* $x_1 = x_2 = \ldots = x_N$.

Theorem 3.1. *For the multi-agent system (4), if the following inequality*

$$\lambda I_n - N\overline{\eta}B - \tau N D_{ij}^{r} \Gamma < 0 \tag{7}$$

holds for all $i, j = 1, 2, \ldots, N$ *and* $r = 1, 2, \ldots, m$, *where* $\overline{\eta}$ *and* λ *are given by (5) and (6). Then consensus is asymptotically reached for arbitrary initial states* $x_i(0) \in R^n$.

Proof. Without loss of generality, define a common Lyapunov function: $V(X_{ij}) = \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N}\|X_{ij}\|^2$, where $X_{ij} = x_j - x_i$. Its corresponding time derivative is

$$\begin{aligned}
\dot{V} &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(X_{ij}\right)^\top \dot{X}_{ij} \\
&= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij}^\top A X_{ij} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij}^\top \left(f(t,x_j) - f(t,x_i)\right) + \frac{1}{2}\tau\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\Big(D_{jk}^\sigma X_{ij}^\top \Gamma X_{jk} \\
&\quad - D_{ik}^\sigma X_{ij}^\top \Gamma X_{ik}\Big) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(\eta_{jk} X_{ij}^\top B X_{jk} - \eta_{ik} X_{ij}^\top B X_{ik}\right) \\
&\le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} X_{ij}^\top A X_{ij} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \phi_{ij}(t) X_{ij}^\top X_{ij} + \frac{1}{2}\tau\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\Big(D_{jk}^\sigma X_{ij}^\top \Gamma X_{jk} \\
&\quad - D_{ik}^\sigma X_{ij}^\top \Gamma X_{ik}\Big) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(\eta_{jk} X_{ij}^\top B X_{jk} - \eta_{ik} X_{ij}^\top B X_{ik}\right) \qquad (8)
\end{aligned}$$

Noticed that the coupling configuration matrix D^σ is symmetric and $X_{ij} = -X_{ji}$, thus the third sum of (8) is $\Psi_3 = -\tau\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\left(D_{jk}^\sigma X_{ji}^\top \Gamma X_{jk} + D_{ik}^\sigma X_{ik}^\top \Gamma X_{ij}\right)$.

In order to make the second term become identical to the first, rename the summation index i by j in the second term, then $\Psi_3 = -\tau\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} D_{jk}^\sigma X_{ji}^\top \Gamma X_{jk}$.

One can verify that $X_{jj} = 0$, so as to the equation can be extended as

$$\Psi_3 = -\tau\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N} D_{jk}^\sigma X_{ji}^\top \Gamma X_{jk} - \tau\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j<k}^{N} D_{jk}^\sigma X_{ji}^\top \Gamma X_{jk}\,.$$

Change over j and k in the second term, so Ψ_3 turns to

$$\Psi_3 = -\tau\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N} D_{jk}^\sigma X_{ji}^\top \Gamma X_{jk} - \tau\sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{k<j}^{N} D_{jk}^\sigma X_{ki}^\top \Gamma X_{kj} = -\tau\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N} D_{jk}^\sigma\left(X_{ji}^\top + X_{ik}^\top\right)\Gamma X_{jk}\,.$$

Since $X_{ji}^\top + X_{ik}^\top = \left[x_i^\top - x_j^\top + x_k^\top - x_i^\top\right] = X_{jk}^\top$, thus

$$\Psi_3 = -\tau\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N} D_{jk}^\sigma X_{jk}^\top \Gamma X_{jk} = -N\tau\sum_{i=1}^{N-1}\sum_{j>i}^{N} D_{ij}^\sigma X_{ij}^\top \Gamma X_{ij}. \qquad (9)$$

Then the fourth sum of (8) is similarly analyzed, as follows:

$$\begin{aligned}
\Psi_4 &= -\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\eta_{jk} X_{ji}^\top B X_{jk} = -\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N}\eta_{jk} X_{ji}^\top B X_{jk} - \sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j<k}^{N}\eta_{jk} X_{ji}^\top B X_{jk} \\
&= -\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N}\eta_{jk} X_{ji}^\top B X_{jk} - \sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{k<j}^{N}\eta_{jk} X_{ki}^\top B X_{kj}
\end{aligned}$$

$$= \quad -\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N}\eta_{jk}X_{jk}^{\top}BX_{jk} = -N\sum_{i=1}^{N-1}\sum_{j>i}^{N}\eta_{ij}X_{ij}^{\top}BX_{ij}.$$

In terms of (5), one has

$$\Psi_4 \leq -N\overline{\eta}\sum_{i=1}^{N-1}\sum_{j>i}^{N}X_{ij}^{\top}BX_{ij}, \tag{10}$$

where $\overline{\eta}$ is defined in (5).

Now according to (8), (9) and (10), the derivative $\dot{V}$ becomes

$$\begin{aligned}\dot{V} \quad &\leq \quad \frac{1}{2}\lambda\sum_{i=1}^{N}\sum_{j=1}^{N}X_{ij}^{\top}X_{ij} - N\tau\sum_{i=1}^{N-1}\sum_{j>i}^{N}D_{ij}^{\sigma}X_{ij}^{\top}\Gamma X_{ij} - N\overline{\eta}\sum_{i=1}^{N-1}\sum_{j>i}^{N}X_{ij}^{\top}BX_{ij} \\ &= \quad \sum_{i=1}^{N-1}\sum_{j>i}^{N}X_{ij}^{\top}\Big(\lambda I_n - N\overline{\eta}B - \tau ND_{ij}^{\sigma}\Gamma\Big)X_{ij}\end{aligned}$$

Finally $\dot{V}$ turns into a quadratic form, and according to (7) it is easily to have $\dot{V} < 0$. Based on Definition 3.1, consensus of MAS (4) is asymptotically reached. Here is the complete proof of Theorem 3.1.

Remark 3.1. *Notice conditions (7), if $\eta_{ij} > 0$, it is easy to find that the controller can accelerate the convergence, which is identical with the assumption (III). On the other hand, if the $\eta_{ij} < 0$, the controller hinders the convergence.*

Remark 3.2. *As many papers mentioned [3, 4, 15], there is a close relationship between the number of nodes and convergence effect in a multi-agent system consensus. Especially, as the number of nodes increasing, the speed of convergence increases obviously. Similarly, the number of connections in the multi-agent system has the same effect on convergence as the node number, that is, less the elements $D_{ij}^{r} = 0$ in D^{r} in (1), it is easier to get convergence. These results will be illustrated in the following section.*

4. POTENTIAL FUNCTIONS DISCUSSION

As mentioned above, the multi-agent system (1) may have different performance based on the potential function $J_{ij}(\|x_j - x_i\|)$. In this section, we illustrate how the potential function influences the consensus by several examples.

Here, we divide the potential function $J(x)$ into two classes: attractive potential J_a, and repulsive potential J_r. And the controller (3) are the negative gradient of these two terms.

$$J_{ij}(\|x_j - x_i\|) = J_a(\|x_j - x_i\|) + J_r(\|x_j - x_i\|) = a(1 - e^{-\frac{\|x_j - x_i\|^2}{c^2}}) + b\, e^{\frac{-\|x_j - x_i\|^2}{d^2}}, \qquad c > d, \tag{11}$$

where $\|x_j - x_i\| = \Big(\sum_{k=1}^{n}(x_{j_k} - x_{i_k})^2\Big)^{\frac{1}{2}}$ denotes the distance between two agents. The parameters a, b, c, and d are positive constants representing the strengths and effect ranges of the attractive and repulsive force, respectively. It should be emphasized that $-\nabla J_a$ represents the attraction and has a long range, whereas $-\nabla J_r$ represents the repulsion and has a short range, i.e. $c > d$.

So one can obtain,

$$\eta_{ij} = 2\Big(\frac{b}{d^2}e^{-\frac{\|x_j-x_i\|^2}{d^2}} - \frac{a}{c^2}e^{-\frac{\|x_j-x_i\|^2}{c^2}}\Big). \tag{12}$$

Notice that the practical communication ability of agents is limit, so R is defined as the radius of the maximum relative range within which agents are operated, i.e. $\max\limits_{i,j=1,2,\ldots,N,} \|x_j - x_i\| \leq 2R$. Based on Remark 3.1, if the following inequality $\frac{a}{b} < \frac{c^2}{d^2}e^{\frac{1}{c^2}-(\frac{1}{d^2})\|x_j-x_i\|^2}$ are hold, then $\eta_{ij} > 0$. So set $a = 10$, $b = 20$, $c = 4$, $d = 0.5$ and $R = 0.5$, and one can easily find the potential function (12) has two minimal as shown in Fig.1. Therefore, agents will be trapped at the minimum where the total force becomes zero, rather than at a goal position. In other words, the multi-agent systems will have more than one convergence center.

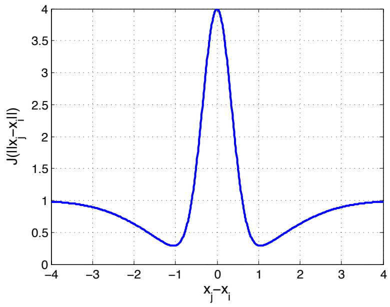

Figure 1: The potential function $J(x_j - x_i)$ in equation (12)

For the purpose of overcoming this local minima problem, a multiplicative and additive structure between the potentials of attraction and repulsion is introduced as following:

$$J_{ij}(\|x_j - x_i\|) = \frac{1}{a}J_rJ_a + J_a = b\,e^{-\frac{\|x_j-x_i\|^2}{d^2}}(1 - e^{-\frac{\|x_j-x_i\|^2}{c^2}}) + a(1 - e^{-\frac{\|x_j-x_i\|^2}{c^2}}), \qquad c > d. \tag{13}$$

After that one has,

$$\eta_{ij} = 2\Big(b(\frac{1}{d^2} + \frac{1}{c^2})e^{-(\frac{1}{d^2}+\frac{1}{c^2})(\|x_j-x_i\|^2)} + \frac{a}{c^2}e^{-\frac{\|x_j-x_i\|^2}{c^2}} - \frac{b}{d^2}e^{-\frac{\|x_j-x_i\|^2}{d^2}}\Big) \tag{14}$$

In order to simplify following calculation, let $\xi_{ij} = \|x_j - x_i\|$, here $0 \leq \xi_{ij} \leq 2R$. Thus, rewrite the equation (14) as $\eta_{ij} = 2\Big(b(\frac{1}{d^2} + \frac{1}{c^2})e^{-(\frac{1}{d^2}+\frac{1}{c^2})\xi_{ij}^2} + \frac{a}{c^2}e^{-\frac{\xi_{ij}^2}{c^2}} - \frac{b}{d^2}e^{-\frac{\xi_{ij}^2}{d^2}}\Big)$. Now, according to Remark 3.1, there exist positive constants a, b, c, and d satisfying the following inequality

$$\frac{a}{b} > \beta(\xi_{ij}). \tag{15}$$

where

$$\beta(\xi_{ij}) = \frac{c^2}{d^2}e^{-(\frac{1}{d^2}-\frac{1}{c^2})\xi_{ij}^2} - \Big(1 + \frac{c^2}{d^2}\Big)e^{-\frac{\xi_{ij}^2}{d^2}}. \tag{16}$$

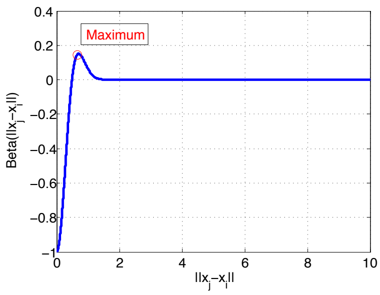

Figure 2: The function $\beta(\xi)$

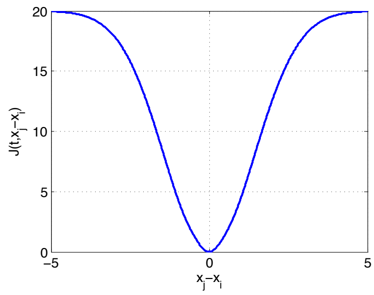

Figure 3: The configured attraction and repulsion potential function $J(\|x_j - x_i\|)$

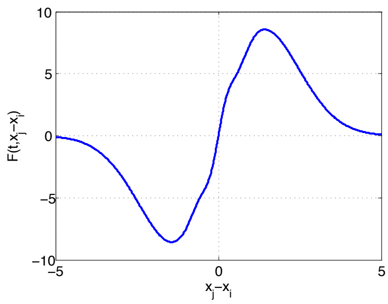

Figure 4: The configured attraction and repulsion force function $F(x_j - x_i)$

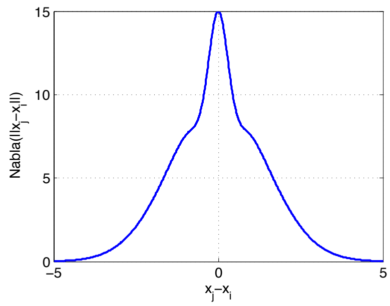

Figure 5: The function η_{ij}

Since $\eta_{ij} > 0$, thus the following inequality can be obtained:

$$\frac{\frac{1}{d^2}e^{-\frac{\xi_{ij}^2}{d^2}} - (\frac{1}{d^2}+\frac{1}{c^2})e^{-(\frac{1}{d^2}+\frac{1}{c^2})\xi_{ij}^2}}{c^2 e^{-\frac{\xi_{ij}^2}{c^2}}} = \frac{c^2}{d^2}e^{-(\frac{1}{d^2}-\frac{1}{c^2})\xi_{ij}^2} - \left(1+\frac{c^2}{d^2}\right)e^{-\frac{\xi_{ij}^2}{d^2}} = \beta(\xi_{ij}) < \frac{a}{b}. \tag{17}$$

One can easily find $\beta(\xi)$ is a monotonic bounded function. In order to derive the upper bound a/b, the parameters c and d are chosen firstly. Then find the maximum value of $\beta(\xi_{ij})$ by a software such as MATLAB.

For example, with $c = 2$, $d = 0.5$ and $R = 5$, the graphs of $\beta(\xi_{ij})$ is illustrated in Fig.2. For the maximum value $\beta(\xi_{ij}) = 0.153$, select $\frac{a}{b} > 0.153$ to guarantee $\eta_{ij} > 0$. When set $a = 20$, $b = 10$, $c = 2$, $d = 0.5$ and $R = 5$, the artificial potential function (13) has only one minimal as shown in Fig.3. Fig.4 illustrates the corresponding force, i.e. $\nabla_{x_i} J_{ij}(\|x_j - x_i\|)$ and η_{ij} is described in Fig.5.

Notice that the form of controller (3) is similar with the coupling term in (1), but it is accessible by setting parameters a, b, c, d to reach an agreement regarding a certain quantity of interest.

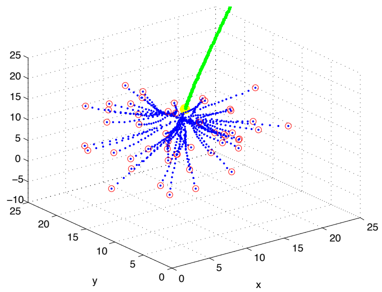

Figure 6: State trajectories of 50 agents in 3-dimension

Substitute the specific controller into (4) and consider a multi-agent system with 50 coupled nodes in which the dynamics of each node is periodically switched between two modes, certainly the results should work for all arbitrary switching systems as well. They are: (I). $D^1_{ij} = 1(i, j = 1, 2, \ldots, 50$, and $j \neq i)$. (II). $D^2_{ij} = 0(j = i + 1$, and $j = i + 2, i = 1, 2, \ldots, 50)$, and others are $D^2_{ij} = 1(i, j = 1, 2, \ldots, 50$, and $j \neq i)$. Select $a = 10$, $b = 3$, $c = 2$, $d = 0.5$, and $n = 3$ here, and the initialization positions of agents are randomly selected in a 3-dimensional space of radius 10, i.e. $R = 10$. Set $\tau = 0.2$, $\Gamma = I_3$, $B = I_3$, and $A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$.

Select the target trajectory as following parametric vector $f(t) = \begin{bmatrix} \cos(0.5t) & \sin(0.25t) & t \end{bmatrix}^\top$. Further, if there is no special explanation, the parameters of other examples are same as this one. With a software such as LMI toolbox in MATLAB, (7) are achieved in this example.

Moreover, a consensus error is defined to estimate the convergence effectiveness: $e(t) = \sum_{i=1}^{N-1} \sum_{j>i}^{N} (\|x_j - x_i\|)$. From Figs.6 − 8, one can find that after a short period, all the agents already converge to a same state and the consensus error quickly becomes zero. In Fig.6 and Fig.8, the red points represent the initial positions of agents. Notice that when all agents converge to the yellow points, i.e. the multi-agent system gets the convergence, and then they travel together along the green trajectories.

Furthermore, as presented in Remark 3.2, in order to illustrate the number of agents and the nonzero connection D^r_{ij} have relation with the consensus results, respectively, consider two multi-agent systems. One has 25 coupled nodes, and other paraments are completely same with the above example; the other one has the same parameters with the above example except switching modes: (I). $D^1_{ij} = 0(j = |i + 2k - 50|,\ j \neq i,\ i = 1, 2, \ldots, 50, k = 1, 2, \ldots, 25)$, and others are $D^2_{ij} = 1(i, j = 1, 2, \ldots, 50$, and $j \neq i)$. (II). $D^2_{ij} = 0(j = |i + 2k - 51|,\ j \neq i,\ i = 1, 2, \ldots, 50, k = 1, 2, \ldots, 25)$, and others are $D^2_{ij} = 1(i, j = 1, 2, \ldots, 50$, and $j \neq i)$. Compared Figs.9 − 10 with Fig. 7, one can find the convergency time of a multi-agent system with more nodes and connections between agents is shorter.

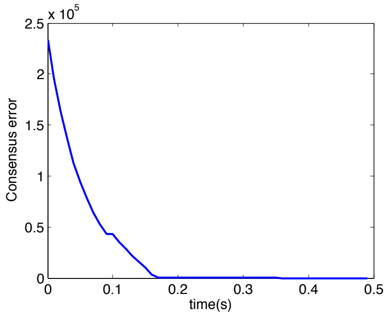

Figure 7: The consensus error of multi-agent systems: $e(t)$

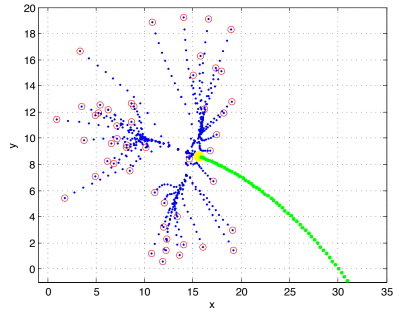

Figure 8: State trajectories of 50 agents in 2-dimension

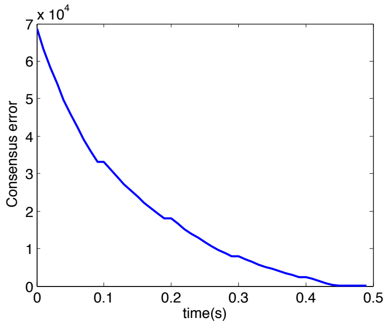

Figure 9: The consensus error of multi-agent systems with 25 nodes

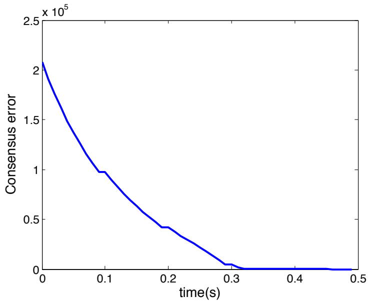

Figure 10: The consensus error of multi-agent systems with other coupling switching topology

5. Conclusions

This paper presents a consensus control strategy for continuous-time multi-agent systems with switching coupling topology and nonlinearity, and it is implemented based on artificial potential function. A consensus controller based on a general potential function is developed into multi-agent system, then conditions of convergence for this system are obtained. Two specific forms of the potential functions are provided to verify the effectiveness of the proposed analysis. It is important to note that one can set the parameters of this controller to accomplish convergence of multi-agent systems without unknown internal coupling. This new approach has a good potential to find more applications in the future.

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