On Formation Control of Networked Multi-agent Systems

Dong Xue, Jing Yao, Guanrong Chen and You-Ling Yu

Abstract

In this paper, a systematic framework is developed for the consensus problem, in particular, for formation control of networked dynamic agents. In view of the complexity of the framework with switching coupling topology and nonlinearity, a new decentralized formation strategy based on artificial potential functions (APF) is proposed. Due to the existence of local minima in the APF, the formation controller is designed to introduce some special functions to settle that limitation. A new concept of relative-position-based formation stability is defined, and a Lyapunov approach is used along with an extended LMI algorithm to analyze the condition for formation stability. Finally, an example with simulation is given to demonstrate the effectiveness of the designed formation controller.

Index Terms

Formation stability, artificial potential function, multi-agent system, switching topology, networked system.

I. INTRODUCTION

Consensus problems for multiple agents systems (MAS) have a long history in biological studies [2], [21], [22]. Examples in nature include schools of fish, herds of animals, and colonies of bacteria [3], [21], [22]. Referring to the literature, it is obvious that consensus behavior has

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certain advantages, such as increasing the possibility of finding foods and avoiding predators. But it requires communications among the group members and individual decision making based on the trajectory and velocity of the group.

Advances in communication and computation have enabled the consensus studies to evolve into the field of engineering applications, such as coordination of autonomous unmanned vehicles (AUV), synchronization of coupled oscillators [6], [23], navigation of satellites in space, and flocking of mobile agents [3], [4]. Based on some fundamental principles in biological systems, the motion formation and obstacle avoidance problems of MAS have attracted considerable attention [6], [7]. This paper only focuses on the formation control of MAS. In general, there are two main control strategies for formation: centralized control and decentralized control. The first one includes the leader-following strategy [8], [24] and virtual structures, while the latter is somewhat similar to the leader-following approach except that the leader is virtual and the formation is seen as a rigid body [19]. In the centralized control architecture, agents need to have global information about the whole system. In this framework, the main problem involves computational complexity and susceptibility to communication failures [7]. On the contrary, decentralized formation mostly relies on the behavior of each agent individually, such as collision avoidance, obstacle avoidance, formation keeping and goal seeking, based on which global performance is achieved from a weighted average of the control actions of all agents [10]. The decentralized approach often includes the use of some artificial potential functions (APF), which have been studied extensively for path-planning of multiple agents in the past decades, for its mathematical simplicity, ease of understanding, and good real-time and high-efficiency performance [1], [3], [4], [7].

A number of recent papers on APF-based controllers for formation suffer from a common difficulty in dealing with local minima that often occur in a potential field environment [1], [8], [10], [14]. Consequently, for any initial condition, the convergence of group motion to a specified formation is generally not ensured. This article proposes a new method to deal with the APF, which could solve the local minima problem effectively. Related works on formation control and coordination of multi-agent systems can be found in, e.g., [16]-[20]. In [16], a feedback linearization technique is proposed using only local information for controller design to exponentially stabilize the relative distances between the agents in the formation. Similar problems were studied based on the concept of control Lyapunov functions, together with formation constraints, to develop a formation control strategy with proved stability of the formation [17], [18]. On the other hand, some results on a group of agents that maintain of the formation geometry, which are found on virtual leaders and artificial potentials for robot interactions, are given in [19]. Afterwards some results in [19] were extended to the case in which the group is moving in a sampled gradient field [20]. Other aspects including time-delays in agents' communications, system uncertainties, and asynchronous consensus, have also been studied [13]-[15]. Notice also that the system frameworks in most studies [1], [3]-[4], [6], [8] omit the nonlinear influence from the environments and the switching coupling topology among the agents.

Motivated by studies on complex dynamical networks [11]-[12], a novel framework is presented in this paper to describe the real dynamical characteristics of the MAS. The main contributions of this paper are as follows: a dynamical framework with switching coupling topology and nonlinearity is developed, to describe the dynamical characteristics of the MAS; a controller which takes advantage of the improved APF is designed to ensure the formation control. Based on the Lyapunov function method, conditions and results are addressed in terms of solutions of LMI (Linear Matrix Inequalities).

The rest of the paper is organized as follows: a dynamical model of the MAS is presented in Section II; a generalized artificial potential function is proposed and then a controller based on this function is designed in Section III; a new concept of formation stability is defined and the formation strategy is validated by using the Lyapunov method in Section IV; a numerical example is presented in Section V; finally, conclusions are drawn in Section VI.

II. MATHEMATICAL MODEL

Let $R^+ = [0, +\infty)$, $J = [t_0, +\infty)$ $(t_0 \geq 0)$, and R^n denote the n-dimensional Euclidean space. For $x = (x_1, \ldots, x_n)^\top \in R^n$, the norm of x is $\|x\| := \Big(\sum_{i=1}^{n} x_i^2 \Big)^{\frac{1}{2}}$. The identity matrix of order n is denoted as I_n (or simply I).

Consider a multi-agent system consisting of N individuals in an n-dimensional Euclidean space. Every agent can be represented by a node in a complex network, and every edge represents a communication link between two agents. Suppose that each agent can sense information about the position of other agents, and the agents can exchange their information such as the position in every step. Moreover, all agents know the desired formation shape and trajectory after initiation.

A dynamical framework with N nodes is thus described by

$$\dot{x}_i = f(t) + w \sum_{\substack{j=1 \\ j\neq i}}^{N} D_{ij}^{\sigma} \Gamma(x_j - x_i) + \alpha u_i, \quad i = 1, 2, \ldots, N \tag{2.1}$$

where $t \in J, x_i \in R^n$ denotes the position vector of individual agent i, vector function $f(t) \in R^n$ is continuously differentiable, representing the group motion (i.e. trajectory dynamics of MAS), scalar $w > 0$ denotes the communication coupling strength between agents, $\Gamma \in R^{n\times n}$ is a $0-1$ matrix linking the coupling variable. For example, in a 2-dimensions MAS, when $\Gamma = diag(1,1)$, which means that two couple agent i and agent j are diagonally linked through their x coordinate and y coordinate. $D = \left(D_{ij}^r\right)_{N\times N}$ is the communication coupling configuration matrix of the network, representing the communication coupling relation between the agents: if there is a connection between agent i and agent j $(j \neq i)$, then $D_{ij}^r = D_{ji}^r = 1$ ($r = 1, 2, \ldots, m$); otherwise, $D_{ij}^r = D_{ji}^r = 0$, where m is the number of switch modes. Furthermore, the diagonal elements of matrix D are defined by $D_{ii}^r = -\sum\limits_{j=1, j\neq i}^{N} D_{ij}^r = -\sum\limits_{j=1,j\neq i}^{N} D_{ji}^r$. Switch signal $\sigma : R^+ \to \{1, 2, \cdots, m\}$, represented by $\{\sigma_k\}$ according to $(t_{k-1}, t_k] \to \sigma_k \in \{1, 2, \cdots, m\}$, is a piecewise constant function. The time sequence $\{t_k\}$ satisfies $t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$, and $t_1 > t_0$. When $t \in (t_{k-1}, t_k]$, the σ_kth subsystem is activated [11]. $\alpha \in R^{n\times n}$ is the input matrix of vector $u_i \in R^n$; u_i denotes the external controller, which will be derived from an artificial potential function in the next section.

III. ARTIFICIAL POTENTIAL FUNCTION

In this section, a new Artificial Potential Function (APF) is introduced to guide the multi-agent system to finish their tasks. There are many studies focusing on the APF [1], [3], [4], [13]. The potential function can be divided into two parts: attractive potential, and repulsive potential. Here is a typical sample of APF [1]:

$$\begin{aligned} J(t, y) &= J_a(t, y) + J_r(t, y) \\ &= a(1 - e^{-\frac{\|y\|^2}{c^2}}) + b\, e^{\frac{-\|y\|^2}{d^2}}, \ c > d, \end{aligned} \tag{3.1}$$

where J_a and J_r represent the attraction and repulsion between the agents, respectively, $\|y\| = \left(\sum\limits_{i=1}^{n} y_i^2\right)^{\frac{1}{2}}$ denotes the distance between two agents, and parameters a, b, c, and d are positive constants representing the strengths and effect ranges of the attractive and repulsive forces, respectively. One can easily verify that if $\nabla J(t,y)=0$, the agents reach the desired distance $\|y\|=\frac{c^2d^2}{c^2-d^2}|\ln\frac{ad^2}{bc^2}|$. The corresponding force is given by the negative gradient of (3.1):

$$\begin{aligned} F(t,y) &= F_a(t,y)+F_r(t,y) \\ &= -\nabla J(t,y) \\ &= 2\,y\left(\frac{b}{d^2}e^{-\frac{\|y\|^2}{d^2}}-\frac{a}{c^2}e^{-\frac{\|y\|^2}{c^2}}\right), \end{aligned} \tag{3.2}$$

where F_a represents the attraction and it has a long range, whereas F_r represents the repulsion and it has a short range. However, the potential function (3.1) has local minima. As long as the distances between agents enter into the local minima, the agents will keep that distances and can't arrive at the desired positions. Fig.1 shows an example where $y\in R^1$ $a=3$, $b=1$, $c=2$, and $d=0.7$ are used, from which one can find that there are two local minima.

In order to overcome such a local minima problem, here propose a new configuration where the total potential has a multiplicative and additive structure between the potentials of attraction and repulsion, in the following form:

$$\begin{aligned} J(t,y) &= \frac{1}{a}J_rJ_a+J_a \\ &= b\,e^{-\frac{\|y-\Delta\|^2}{d^2}}\left(1-e^{-\frac{\|y-\Delta\|^2}{c^2}}\right) \\ &\quad +a\left(1-e^{-\frac{\|y-\Delta\|^2}{c^2}}\right),\ c>d, \end{aligned} \tag{3.3}$$

where $\Delta>0$ is the desired distance between agents. So, the corresponding force can be rewritten as

$$\begin{aligned} F(t,y) &= -\nabla J(t,y) \\ &= 2(y-\Delta)\Bigg[b\left(\frac{1}{c^2}+\frac{1}{d^2}\right)\,e^{-(\frac{1}{c^2}+\frac{1}{d^2})(\|y-\Delta\|^2)} \\ &\quad +\frac{a}{c^2}e^{-\frac{\|y-\Delta\|^2}{c^2}}-\frac{b}{d^2}e^{-\frac{\|y-\Delta\|^2}{d^2}}\Bigg]. \end{aligned} \tag{3.4}$$

Now, a decentralized controller for formation of multi-agent system (2.1) is suggested as follows:

$$\begin{aligned} u_i &= -\nabla_{x_i} J(t,x) \\ &= \sum_{\substack{j=1 \\ j\neq i}}^{N} 2\left(x_j - x_i - \Delta_{ij}\right) \\ &\quad \left[b\left(\frac{1}{d^2}+\frac{1}{c^2}\right) e^{-(\frac{1}{d^2}+\frac{1}{c^2})(\|x_j-x_i-\Delta_{ij}\|^2)} \right. \\ &\quad \left. +\frac{a}{c^2} e^{-\frac{\|x_j-x_i-\Delta_{ij}\|^2}{c^2}} - \frac{b}{d^2} e^{-\frac{\|x_j-x_i-\Delta_{ij}\|^2}{d^2}} \right], \\ &\qquad\qquad c > d, \end{aligned} \tag{3.5}$$

where $\|x_j - x_i\|$ is the distance between agents j and i; and Δ_{ij} are formation distance vectors, therefore $\|\Delta_{ij}\|$ are the desired formation distances between them. Note that the Δ_{ij} in [1], [8] are scalars not vectors, so the rigid positions of agents are not ensured. However, the formation here is not only dependent on the desired distance $\|\Delta_{ij}\|$, but also on the agent positions. Furthermore, one has $\Delta_{ij} = -\Delta_{ji}$ and $\Delta_{ii} = 0$. In order to simplify the equation, define

$$\begin{aligned} \varphi_{ij} &= 2\left[b\left(\frac{1}{d^2}+\frac{1}{c^2}\right) e^{-(\frac{1}{d^2}+\frac{1}{c^2})(\|x_j-x_i-\Delta_{ij}\|^2)} \right. \\ &\quad \left. +\frac{a}{c^2} e^{-\frac{\|x_j-x_i-\Delta ij\|^2}{c^2}} - \frac{b}{d^2} e^{-\frac{\|x_j-x_i-\Delta_{ij}\|^2}{d^2}} \right]. \end{aligned} \tag{3.6}$$

Substituting φ_{ij} into (3.5), one obtains

$$u_i = \sum_{\substack{j=1 \\ j\neq i}}^{N} \varphi_{ij}\left(x_j - x_i - \Delta_{ij}\right) \tag{3.7}$$

Remark 3.1: In mechanics, the force between two points is $\overrightarrow{F} = |F|\overrightarrow{e}$, where $\overrightarrow{F}$ means force vector, $|F|$ illustrates force strength and $\overrightarrow{e}$ represents the unit vector along the direction of two points. Similarly in (3.4), direction of the force is consistent with vector $y - \Delta_{ij}$. Based on this analysis, $\varphi_{ij} > 0$ is necessary in controller (3.7).

Furthermore, with respect to communication range limitations of the mobile agents, some constraints should be imposed: $\max_{i,j=1,2,\ldots,N,} \|x_j - x_i\| < 2\Re$, where $2\Re$ represents the maximum range within which agents are able to contact with each other; $\Delta = \max_{i,j=1,2,\ldots,N,} \|\Delta_{ij}\|$. In practice, the maximal formation distance of multi-agent systems should be much shorter than communication ranges of agents, i.e. $\Delta < 2\Re$.

The objective here is to analyze how to choose parameters a, b, c, d. Firstly, for convenience, define $\varepsilon_{ij} = \|x_j - x_i - \Delta_{ij}\|$, and by above analysis get $2\Re - \Delta \leq \varepsilon_{ij} \leq 2\Re + \Delta$.

Then, according to $\varphi_{ij} > 0$ in Remark 3.1, there exist positive constants a, b, c, d satisfying following inequality

$$\begin{aligned} \frac{a}{b} &> \frac{\frac{1}{d^2}e^{-\frac{\varepsilon_{ij}^2}{d^2}} - (\frac{1}{d^2}+\frac{1}{c^2})e^{-(\frac{1}{d^2}+\frac{1}{c^2})\varepsilon_{ij}^2}}{\frac{1}{c^2}e^{-\frac{\varepsilon_{ij}^2}{c^2}}} \\ &= \frac{c^2}{d^2}e^{-(\frac{1}{d^2}-\frac{1}{c^2})\varepsilon_{ij}^2} - \left(1+\frac{c^2}{d^2}\right)e^{-\frac{\varepsilon_{ij}^2}{d^2}} \\ &= \rho(\varepsilon_{ij}). \end{aligned} \tag{3.8}$$

Notice that $\rho(\varepsilon_{ij})$ is a monotonic bounded function. In order to derive the upper bound $\rho(\varepsilon_{ij})$, the parameters c and d are chosen firstly. Considered the actual communication of agents, effect ranges of the attractive and repulsive forces c and d should be within the maximum communication range of agent, i.e. $d < c \leq 2\Re$. Then find the maximum value of $\rho(\varepsilon_{ij})$ by a software such as MATLAB.

For example, with $\Re = 5$, $\Delta = 2$, $c = 2$, and $d = 0.5$, the function $\rho(\varepsilon_{ij})$ is illustrated in Fig.2. One has the maximum value of $\rho(\varepsilon_{ij}) = 9.4069 \times 10^{-104}$, so select $\frac{a}{b} > 9.4069 \times 10^{-104}$ to guarantee $\varphi_{ij} > 0$. Notice that the maximum value of $\rho(\varepsilon_{ij})$ is infinitely closed to zero, so for positive constants a, b, inequality (3.8) is easy to hold, i.e. $\varphi_{ij} > 0$. Set $a = 2$, $b = 3$, the potential (3.3) has only one minimum as shown in Fig.3. Correspondingly, Fig.4 illustrates that when $\|y\| = \Delta = 2$, the potential force $F(t, y) = 0$, namely $F_a = F_r$; when $\|y\| > \Delta$, $F_a > F_r$; conversely, when $\|y\| < \Delta$, one has $F_a < F_r$.

IV. FORMATION STABILITY ANALYSIS

In this section, the above-developed framework is applied to study formation stability of multi-agent systems. First, rewrite the dynamical framework (2.1) as

$$\begin{aligned} \dot{x}_i = f(t) + w\sum_{\substack{j=1\\ j\neq i}}^{N} D_{ij}^{\sigma}\Gamma(x_j - x_i) + \alpha\sum_{\substack{j=1\\ j\neq i}}^{N}\varphi_{ij}\left(x_j - x_i - \Delta_{ij}\right), \\ i = 1, 2, \ldots, N. \end{aligned} \tag{4.1}$$

Then, define the following notation:

$$\overline{\varphi} = \min_{i,j=1,2,\ldots,N,}\varphi_{ij}, \tag{4.2}$$

$$X_{ij} = x_j - x_i.$$

Thus, from (4.1) one can obtain

$$\begin{aligned}\dot{X}_{ij} &= w\Gamma \sum_{k=1}^{N} \left(D^{\sigma}_{jk} X_{jk} - D^{\sigma}_{ik} X_{ik} \right) \\ &\quad + \alpha \sum_{k=1}^{N} \left(\varphi_{jk}(X_{jk} - \Delta_{jk}) - \varphi_{ik}(X_{ik} - \Delta_{ik}) \right), \\ &\qquad\qquad i, j = 1, 2, \ldots, N. \end{aligned} \tag{4.3}$$

In the subsequent discussion, the following concepts of formation stability and Schur Complement for multi-agent systems are needed.

Definition 4.1: (Formation Stability) The system (2.1) is said to be asymptotically formation stable if a protocol u_i makes $\|X_{ij} - \Delta_{ij}\| \to 0$ when $t \to \infty$, where $(i,j) = \{\, (i,j) \mid i,j = 1, 2, \ldots, N\}$ and Δ_{ij} is a desired inter-agent relative-position vector.

Lemma 4.1: ([25]) (Schur Complement) Suppose that

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

is a symmetric matrix, where $S_{11} \in R^{n\times n}$. Then, the following three conclusions are equivalent:

(i) $S < 0$;

(ii) $S_{11} < 0$, $S_{22} - S_{12}^{\top} S_{11}^{-1} S_{12} < 0$;

(iii) $S_{22} < 0$, $S_{11} - S_{12}^{\top} S_{22}^{-1} S_{12} < 0$.

Theorem 4.1: Consider the multi-agent system (2.1) with a formation controller given by (3.5). If

$$M_{ij} = \begin{bmatrix} Q_{ij} & 0 & 0 \\ 0 & Q_{ij} & N\overline{\varphi}\alpha \\ 0 & N\overline{\varphi}\alpha & P_{ij} \end{bmatrix} < 0 \tag{4.4}$$

hold for all $i, j = 1, 2, \ldots, N$ and $r = 1, 2, \ldots, m$, where

$$Q_{ij} = -NwD^{r}_{ij}\Gamma - N\overline{\varphi}\alpha,$$

$$P_{ij} = NwD^{r}_{ij}\Gamma - N\overline{\varphi}\alpha,$$

and $\overline{\varphi}$ is given by (4.2), then for any initial states $x_i(0) \in R^n$, the formation of multi-agent system (2.1) is aymptotically stable.

Proof: Define a common Lyapunov function:

$$V(X_{ij}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\|X_{ij}-\Delta_{ij}\|^2,$$

where $X_{ij} = x_j - x_i$. Its time derivative is

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}\sum_{j=1}^{N}\left(X_{ij}-\Delta_{ij}\right)^\top \dot{X}_{ij} \\ &= w\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\Big(D_{jk}^{\sigma}\left(X_{ij}-\Delta_{ij}\right)^\top \Gamma X_{jk} \\ &\quad -D_{ik}^{\sigma}\left(X_{ij}-\Delta_{ij}\right)^\top \Gamma X_{ik}\Big) \\ &\quad +\alpha\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\Big[\varphi_{jk}\left(X_{ij}-\Delta_{ij}\right)^\top\left(X_{jk}-\Delta_{jk}\right) \\ &\quad -\varphi_{ik}\left(X_{ij}-\Delta_{ij}\right)^\top\left(X_{ik}-\Delta_{ik}\right)\Big]. \end{aligned} \tag{4.5}$$

Since the coupling configuration matrix D is symmetric and $X_{ij} = -X_{ji}$, $\Delta_{ij} = -\Delta_{ji}$, the first sum of (4.5) is

$$\begin{aligned} S_1 &= -w\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\Big[D_{jk}^{\sigma}\left(X_{ji}-\Delta_{ji}\right)^\top \Gamma X_{jk} \\ &\quad +D_{ik}^{\sigma}X_{ik}^\top\Gamma\left(X_{ij}-\Delta_{ij}\right)\Big]. \end{aligned} \tag{4.6}$$

In order to reformulate the second term to be similar to the first in (4.6), rename the summation index i by j in the second term so as to obtain

$$S_1 = -2w\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}D_{jk}^{\sigma}\left(X_{ji}-\Delta_{ji}\right)^\top\Gamma X_{jk}\,.$$

Since $X_{jj}-\Delta_{jj}=0$, the equation can be extended as

$$\begin{aligned} S_1 &= -2w\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N}D_{jk}^{\sigma}\left(X_{ji}-\Delta_{ji}\right)^\top\Gamma X_{jk} \\ &\quad -2w\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j<k}^{N}D_{jk}^{\sigma}\left(X_{ji}-\Delta_{ji}\right)^\top\Gamma X_{jk}. \end{aligned}$$

Renaming j in the second term as k, and noticing the symmetry of D^σ, one can rewrite S_1 as

$$\begin{aligned} S_1 &= -2w \sum_{i=1}^{N} \sum_{k=1}^{N-1} \sum_{j>k}^{N} D^\sigma_{jk} \left(X_{ji} - \Delta_{ji}\right)^\top \Gamma X_{jk} \\ &\quad -2w \sum_{i=1}^{N} \sum_{j=1}^{N-1} \sum_{k<j}^{N} D^\sigma_{jk} \left(X_{ki} - \Delta_{ki}\right)^\top \Gamma X_{kj} \\ &= -2w \sum_{i=1}^{N} \sum_{k=1}^{N-1} \sum_{j>k}^{N} D^\sigma_{jk} \Big[\left(X_{ji} - \Delta_{ji}\right)^\top \\ &\quad + \left(X_{ik} - \Delta_{ik}\right)^\top \Big] \Gamma X_{jk} \, . \end{aligned}$$

One can then verify that $X^\top_{ji} + X^\top_{ik} = x^\top_i - x^\top_j + x^\top_k - x^\top_i = X^\top_{jk}$ and $\Delta^\top_{ji} + \Delta^\top_{ik} = \Delta^\top_{jk}$, thus

$$\begin{aligned} S_1 &= -2w \sum_{i=1}^{N} \sum_{k=1}^{N-1} \sum_{j>k}^{N} D^\sigma_{jk} \left(X_{jk} - \Delta_{jk}\right)^\top \Gamma X_{jk} \\ &= -2Nw \sum_{i=1}^{N-1} \sum_{j>i}^{N} D^\sigma_{ij} X^\top_{ij} \Gamma \left(X_{ij} - \Delta_{ij}\right) . \end{aligned} \tag{4.7}$$

In order to transform it to a quadratic form, an term $Nw \sum_{i=1}^{N-1} \sum_{j>i}^{N} D^\sigma_{ij} \Delta^\top_{ij} \Gamma \left(X_{ij} - \Delta_{ij}\right)$ is added to and then subtracted from (4.7), to obtain

$$\begin{aligned} S_1 &= -2Nw \sum_{i=1}^{N-1} \sum_{j>i}^{N} D^\sigma_{ij} X^\top_{ij} \Gamma \left(X_{ij} - \Delta_{ij}\right) \\ &\quad +Nw \sum_{i=1}^{N-1} \sum_{j>i}^{N} D^\sigma_{ij} \Delta^\top_{ij} \Gamma \left(X_{ij} - \Delta_{ij}\right) \\ &\quad -Nw \sum_{i=1}^{N-1} \sum_{j>i}^{N} D^\sigma_{ij} \Delta^\top_{ij} \Gamma \left(X_{ij} - \Delta_{ij}\right) \\ &= -Nw \sum_{i=1}^{N-1} \sum_{j>i}^{N} D^\sigma_{ij} \left(X_{ij} - \Delta_{ij}\right)^\top \Gamma \left(X_{ij} - \Delta_{ij}\right) \\ &\quad -Nw \sum_{i=1}^{N-1} \sum_{j>i}^{N} D^\sigma_{ij} X^\top_{ij} \Gamma X_{ij} \\ &\quad +Nw \sum_{i=1}^{N-1} \sum_{j>i}^{N} D^\sigma_{ij} \Delta^\top_{ij} \Gamma \Delta_{ij}. \end{aligned} \tag{4.8}$$

Then, the second sum of (4.5) is similarly calculated, as follows:

$$\begin{aligned} S_2 &= -2\alpha \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \varphi_{jk} \left(X_{ji} - \Delta_{ji}\right)^\top \left(X_{jk} - \Delta_{jk}\right) \\ &= -2\alpha \sum_{i=1}^{N} \sum_{k=1}^{N-1} \sum_{j>k}^{N} \varphi_{jk} \left(X_{ji} - \Delta_{ji}\right)^\top \left(X_{jk} - \Delta_{jk}\right) \end{aligned}$$

$$
\begin{aligned}
& -2\alpha\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j<k}^{N}\varphi_{jk}\left(X_{ji}-\Delta_{ji}\right)^{\top}\left(X_{jk}-\Delta_{jk}\right) \\
= & -2\alpha\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N}\varphi_{jk}\left(X_{ji}-\Delta_{ji}\right)^{\top}\left(X_{jk}-\Delta_{jk}\right) \\
& -2\alpha\sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{k<j}^{N}\varphi_{jk}\left(X_{ki}-\Delta_{ki}\right)^{\top}\left(X_{kj}-\Delta_{kj}\right) \\
= & -2\alpha\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j>k}^{N}\varphi_{jk}\left(X_{jk}-\Delta_{jk}\right)^{\top}\left(X_{jk}-\Delta_{jk}\right) \\
= & -2N\alpha\sum_{i=1}^{N-1}\sum_{j>i}^{N}\varphi_{ij}\left(X_{ij}-\Delta_{ij}\right)^{\top}\left(X_{ij}-\Delta_{ij}\right)\ .
\end{aligned}
$$

In terms of (4.2), one has

$$
S_2 \leq -2N\overline{\varphi}\alpha\sum_{i=1}^{N-1}\sum_{j>i}^{N}\left(X_{ij}-\Delta_{ij}\right)^{\top}\left(X_{ij}-\Delta_{ij}\right)
$$

and so

$$
\begin{aligned}
S_2 \leq & -N\overline{\varphi}\alpha\sum_{i=1}^{N-1}\sum_{j>i}^{N}\left(X_{ij}-\Delta_{ij}\right)^{\top}\left(X_{ij}-\Delta_{ij}\right) \\
& -N\overline{\varphi}\alpha\sum_{i=1}^{N-1}\sum_{j>i}^{N}X_{ij}^{\top}X_{ij}-N\overline{\varphi}\alpha\sum_{i=1}^{N-1}\sum_{j>i}^{N}\Delta_{ij}^{\top}\Delta_{ij} \\
& +N\overline{\varphi}\alpha\sum_{i=1}^{N-1}\sum_{j>i}^{N}X_{ij}^{\top}\Delta_{ij}+N\overline{\varphi}\alpha\sum_{i=1}^{N-1}\sum_{j>i}^{N}\Delta_{ij}^{\top}X_{ij}.
\end{aligned}
\tag{4.9}
$$

Now, according to (4.8) and (4.9), the derivative $\dot{V}$ becomes

$$
\begin{aligned}
\dot{V} = & S_1+S_2 \\
\leq & \sum_{i=1}^{N-1}\sum_{j>i}^{N}\left(X_{ij}-\Delta_{ij}\right)^{\top}Q_{ij}\left(X_{ij}-\Delta_{ij}\right) \\
& +\sum_{i=1}^{N-1}\sum_{j>i}^{N}X_{ij}^{\top}Q_{ij}X_{ij}+\sum_{i=1}^{N-1}\sum_{j>i}^{N}\Delta_{ij}^{\top}P_{ij}\Delta_{ij} \\
& +N\overline{\varphi}\alpha\sum_{i=1}^{N-1}\sum_{j>i}^{N}X_{ij}^{\top}\Delta_{ij}+N\overline{\varphi}\alpha\sum_{i=1}^{N-1}\sum_{j>i}^{N}\Delta_{ij}^{\top}X_{ij} \\
\leq & \sum_{i=1}^{N-1}\sum_{j>i}^{N}\left[\begin{array}{ccc}\left(X_{ij}-\Delta_{ij}\right)^{\top} & X_{ij}^{\top} & \Delta_{ij}^{\top}\end{array}\right]
\end{aligned}
$$

$$\begin{bmatrix} Q_{ij} & 0 & 0 \\ 0 & Q_{ij} & N\overline{\varphi}\alpha \\ 0 & N\overline{\varphi}\alpha & P_{ij} \end{bmatrix} \begin{bmatrix} (X_{ij} - \Delta_{ij}) \\ X_{ij} \\ \Delta_{ij} \end{bmatrix}.$$

Thus, $\dot{V}$ is in a quadratic form, and by the LMI (4.3) it is easily proved that $\dot{V} < 0$. According to Definition 4.1, the predefined formation is reached, completing the proof of Theorem 4.1. ■

Remark 4.1: In (2.1), when $u_i = 0$, $D_{ij}^r = 0, (j = 1, 2, \ldots, N, j \neq i, r = 1, 2, \ldots, m)$, agents in the system are isolated nodes, and dynamics of agents only depend on $f(t)$. On the other hand, when the multi-agent system achieves formation, i.e. $X_{ij} - \Delta_{ij} = 0$, the controller $u_i = 0$ in (2.1). The dynamics trajectory of system (2.1) tends to $f(t) + w \sum_{\substack{j=1 \\ j \neq i}}^{N} D_{ij}^{\sigma} \Gamma(\Delta_{ij})$, so all formatted multi-agents are along with the same trajectory.

Corollary 4.1: Consider the conditions described in Theorem 4.1. The formation stability of multi-agent system (2.1) is equivalent to one of the following inequalities:

(i) $Q_{ij} < 0$, and $P_{ij} - (N\overline{\varphi})^2 \alpha^\top Q_{ij} \alpha < 0$;

(ii) $Q_{ij} < 0$, $P_{ij} < 0$, and $Q_{ij} - (N\overline{\varphi})^2 \alpha^\top P_{ij} \alpha < 0$.

Proof: Let the matrix M_{ij} in (4.4) be decomposed into

$$M_{ij} = \begin{bmatrix} Q_{ij} & M_{ij}^{12} \\ M_{ij}^{21} & M_{ij}^{22} \end{bmatrix},$$

where $M_{ij}^{12} = \left(M_{ij}^{21}\right)^\top = [0 \quad 0]$ and

$$M_{ij}^{22} = \begin{bmatrix} Q_{ij} & N\overline{\varphi}\alpha \\ N\overline{\varphi}\alpha & P_{ij} \end{bmatrix}.$$

By Lemma 4.1, one can find that if $Q_{ij} < 0$ and $M_{ij}^{22} < 0$, then $M_{ij} < 0$. Furthermore, if either of the following conclusions holds then $M_{ij}^{22} < 0$:

(i) $Q_{ij} < 0$, and $P_{ij} - (N\overline{\varphi})^2 \alpha^\top Q_{ij} \alpha < 0$;

(ii) $P_{ij} < 0$, and $Q_{ij} - (N\overline{\varphi})^2 \alpha^\top P_{ij} \alpha < 0$.

Thus, if the conditions in Corollary 4.1 hold, then $M_{ij} < 0$. According to Theorem 4.1, Corollary 4.1 is proved. ■

V. NUMERICAL EXAMPLE

In this section, an example of formation for a multi-agent system is given to demonstrate the effectiveness of the proposed control method.

The problem is to drive four agents to form a square, and to follow a chosen trajectory in the xy-plane. In this example, $N = 4$, $n = 2$. Set $\Delta_{12} = [\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}]'$, $\Delta_{23} = [\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}]'$, $\Delta_{34} = [-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}]'$,$\Delta_{41} = [-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}]'$, $\Delta_{13} = [\sqrt{2}, 0]'$, $\Delta_{24} = [0, -\sqrt{2}]'$, $w = 0.3$, $\Gamma = I$, $\Re = 5$, and $\alpha = I$. For simplicity, the networked multi-agent system (2.1) periodically switchs between two modes with different topologies in every time step: (I) $D^1_{13} = D^1_{24} = 0$ and $D^1_{ij} = 1$ ($i, j = 1, 2, 3, 4$); (II) $D^2_{ij} = 1$ ($i, j = 1, 2, 3, 4$). In (3.4), $a = 8$, $b = 0.3$, $c = 0.7$, $d = 0.2$ and the initialization positions of four agents in plane are $(1.0, 1.2)^\top$, $(2.0, 4.0)^\top$, $(6.7, 3.0)^\top$, and $(4.2, 2.0)^\top$. Moreover, in order to visualize the formation effectiveness, define the formation error as follows:

$$e(t) = \sum_{i=1}^{N-1} \sum_{j>i}^{N} (\|x_j - x_i\| - \|\Delta_{ij}\|) \,.$$

The target trajectory is given by the following parametric vector: $f(t) = \begin{bmatrix} \cos(0.5t), & \sin(0.25t) \end{bmatrix}^\top$, satisfying the conditions in (4.4) and Corollary 4.1.

As one can see from Fig.5 and Fig.6, after a short period of time, the four agents achieve the desired formation and the formation error approaches to 0. Fig. 7 shows paths of the multi-agent system without the controller, and the results clearly illuminate the validity of the formation controller from a different point of view. Finally, Fig. 8 illustrates formation error of MAS (2.1) with the controller u_i based on potential (3.1), which means owing to the existence of local minima, the formation is non-reachable.

VI. CONCLUSIONS

The problem of decentralized formation control for multi-agents systems, with switching coupling topology and nonlinearity, has been studied. A formation controller based on an improved artificial potential function has been designed, and a numerical example has been provided to verify the effectiveness of the proposed approach. The formation control strategy presented in this paper has several advantages: firstly, it successfully eliminates local minima of the artificial potential functions and effectively drives the agents to achieve the desired formation quickly; moreover, the dynamic system framework takes nonlinearity and switching coupling topology into account; last but not least, it is easy to accomplish various shapes of formation, and to follow different preassigned trajectories precisely. This new approach has a good potential to find more applications in the future.

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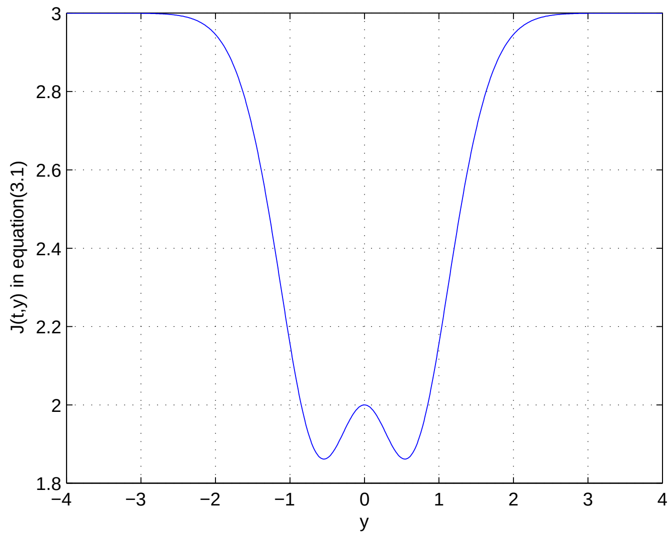

Fig. 1. The attraction/repulsion potential function $J(t, y)$

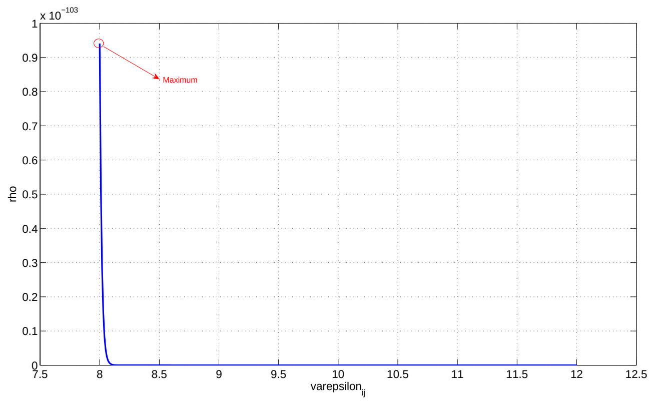

Fig. 2. The function $\rho(\varepsilon_{ij})$

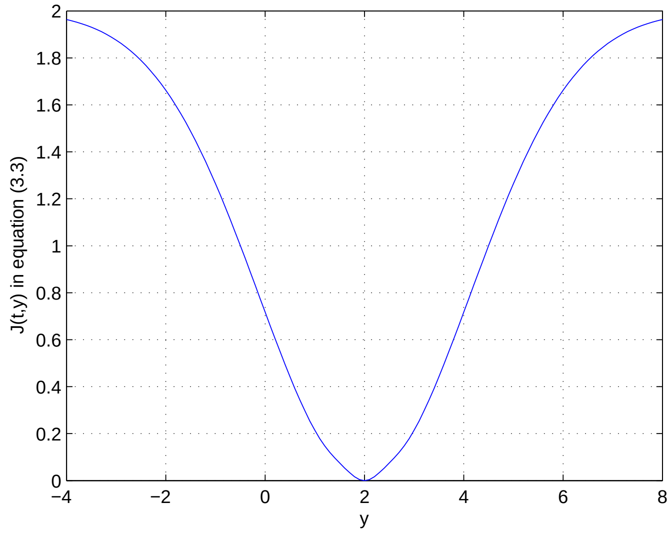

Fig. 3. The configured attraction/repulsion potential function $J(t, y)$

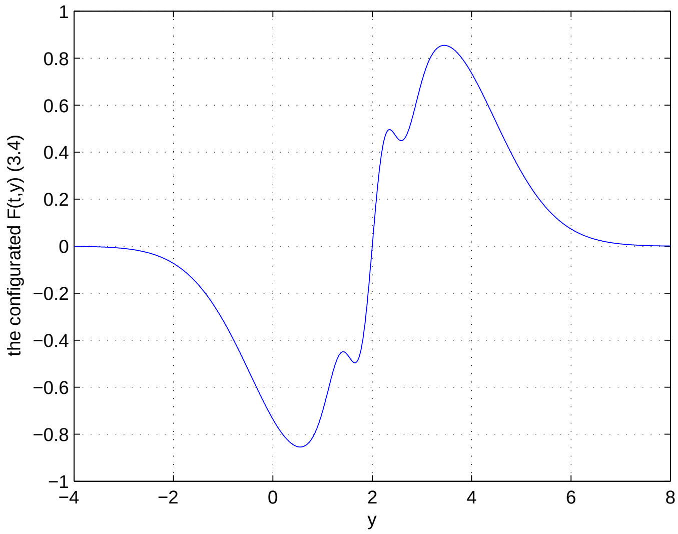

Fig. 4. The configured attraction/repulsion force function $F(t, y)$

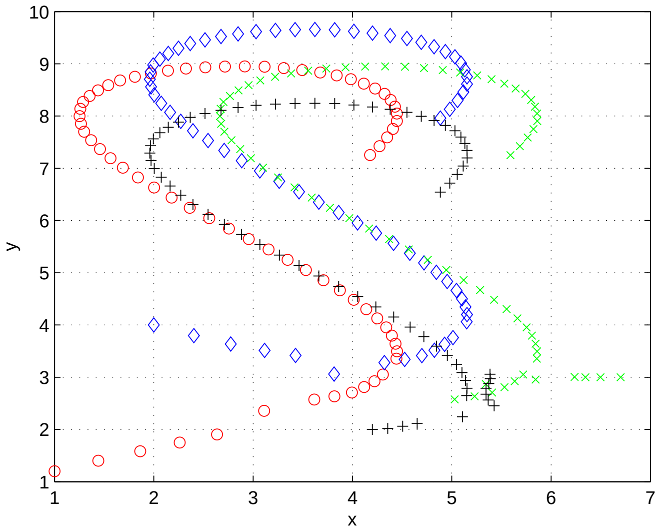

Fig. 5. The paths and formation of four agents under control

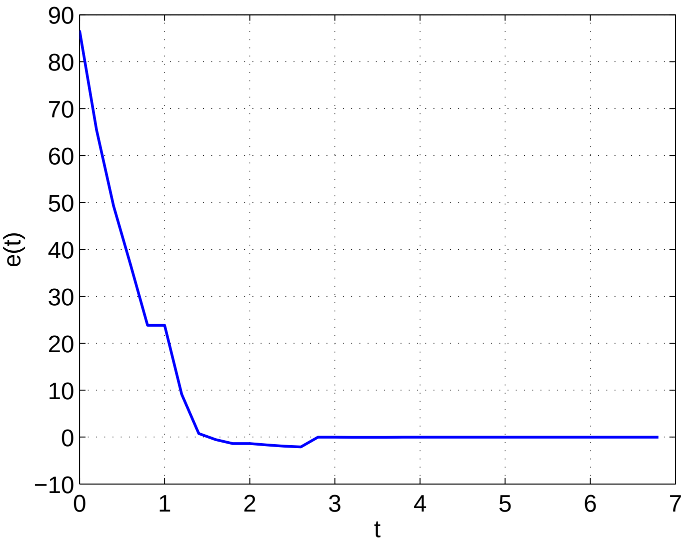

Fig. 6. The formation error of four agents under control

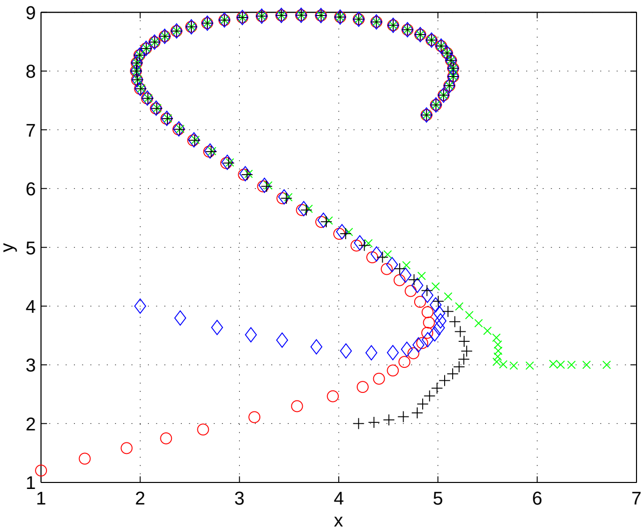

Fig. 7. The paths and formation of four agents without the controller

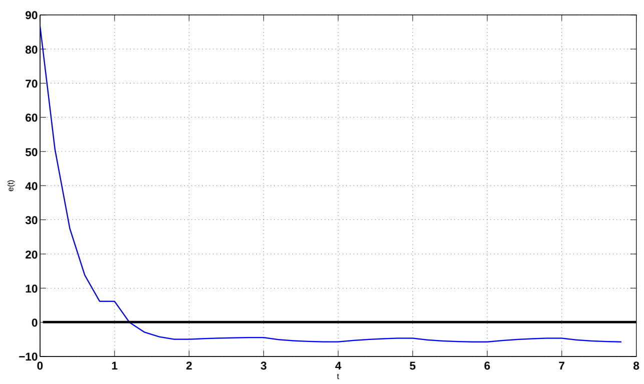

Fig. 8. The formation error of four agents under controller using (3.2)