Formation Control of Multi-agent Systems with Stochastic Switching Topology and Time-Varying Communication Delays

Dong Xue, Jing Yao, Jun Wang and Yafeng Guo

Abstract

This paper formulates and studies the distributed formation problems of multi-agent systems (MAS) with randomly switching topologies and time-varying delays. The nonlinear dynamic of each agent at different time-interval corresponds to different switching mode which reflects the changing of traveling path in practical systems. The communication topology of the system is switching among finite modes which are governed by a finite-state Markov process. On the basis of artificial potential functions (APFs), a formation controller is designed in a general form. Sufficient conditions for stochastic formation stability of the multi-agent system are obtained in terms of Lyapunov functional approach and linear matrix inequalities (LMIs). Some heuristic rules to design a formation controller for the MAS are then presented. Finally, specific potential functions are discussed and corresponding simulation results are provided to demonstrate the effectiveness of the proposed approach.

Index Terms

Formation control, multi-agent systems, switching topology, time-varying delays, switching paths, stochastic stability.

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I. INTRODUCTION

In In recent years we have witnessed a growing recognition and attention of distributed coordination of multi-agent systems (MASs) across a wide range of disciplines, due to increasing technological advances in communication and computation. Coordination algorithms have applied in cooperative control of unmanned air/underwater vehicles (UAVs) and spacecraft [13], [18], formation control [13], [14], distributed sensor networks [15], and attitude alignment of clusters of satellites [16]. As one of the most important and fundamental issues in the coordination control of multi-agent systems, formation aims to achieve and maintain a desired structure which depends on the specific task. A formation algorithm (or strategy) is an interaction principle that specifies the information exchange between agents. Numerous methods have been applied to deal with these problems, such as leader-follower [17], [18], virtual structure [19], potential functions [11], [12], [20], etc.

Artificial potential functions have been widely developed for robot navigation and coordination control of multi-agent systems including formation, path-planning, collision, obstacle avoidance, etc [6], [20]. Derived from the potential force laws between agents-agents, agents-targets, and agents-obstacles, diverse potential functions are employed in multi-agent systems to achieve complicated behaviors. It is crucial to design artificial potential functions because different potentials, even employed in the same multi-agent system, might result in unpredicted and undesired performances. In particular, the limitation of existing multiple local minima in the potential function leads to a non-reachable problem. Thus, in this paper we show that by choosing an appropriate potential function the multi-agent systems will follow a prescribed trajectory and keep a desirable shape.

In the past, many papers are devoted to the formation problems in the continuous- or discrete-time dynamics. In [2], a typical continuous-time consensus model was described, which introduced a directed graph to model the connection topology and considered directed networks with fixed and switching topology, and undirected networks with communication time-delays and fixed topology. Consensus problems in discrete-time multi-agent systems with fixed topology are explored in [7]. But many multi-agent systems are hybrid in the sense that they exhibit both discrete- and continuous-state dynamics. Note that dynamical behaviors of multi-agent system are subject to not only agent dynamics but also communication topology. As an important class of hybrid systems, switching systems which consist of a family of subsystems and are controlled by some logical rules, are used to describe the communication connections of the multi-agent systems [4], [24]. Based on the graph theory and nonnegative matrix theory, the asynchronous consensus problems of continuous-time multi-agent systems with time-dependent communication topology and time-varying delays are studied in [1]. However, most papers concerning switching topologies are failed to illustrate the specific switching mechanism among the subsystems. In this paper, a finite-state Markov process is introduced to describe the jumping communication topologies.

Furthermore, the interconnection communication delays among agents have to be taken into deliberation in practical problems. It is well known that researching formation problems with switching topologies are more challenging than that with fixed topologies, specifically when time-varying delays are involved. In [2], the consensus problem of continuous-time multi-agent systems with communication delays is discussed. However, it was often assumed that time-delays are constants [2], [8], [21]. Moreover, the discriminating time-dependent delays are taken into account in [1] and asynchronous consensus problems are investigated later. Since the multi-agent systems modeled in this paper are composed of homogenous agents which have the same communication capability, we assume that the time-varying delays are identical for each agent.

In this paper, based on artificial potential function and behavior rules of agents a distributed formation strategy for a multi-agent system is presented. In the real-world multi-agent systems, one may face the following issues:

1). The communication topology of agents is randomly switching, even the dynamic behaviors of each agent are switching.

2). It is inevitable that there exist communication delays, which are commonly time-varying even unknown.

Thus, a formation problem for multiple agents with stochastic switching topology and time-varying communication delays is discussed in this paper. Specifically, a switching nonlinear function is presented to characterize the different changes of navigation-track in the real world, and the switching communication topology is determined by a Markov chain taking values in a finite set. Then the stochastic Lyapunov functional is employed for the theoretical analysis of this time-delay system, which modeled by delayed differential equations. The sufficient conditions are provided in terms of a set of linear matrix inequalities (LMIs) and each LMI corresponds to one possible subsystem.

This paper is organized as follows. In Section II, a model of multi-agent system with switching communication topology and time-varying delays is presented. The stochastic formation-stability analysis is performed based on a stochastic Lyapunov functional in Section III. Section IV contains some numerical examples with specific potential functions. Finally, in Section V, concluding remarks are stated.

II. Problem Formulation

Let $\mathbf{J} = [t_0, +\infty)$, $\mathbf{R}_+ = (0, +\infty)$, $\overline{\mathbf{R}}_- = [0, +\infty)$ and $\mathbf{R}^n$ denote the n-dimensional Euclidean space. For $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$, denote the norm of x as $\|x\| := \Big(\sum\limits_{i=1}^{n} x_i^2 \Big)^{\frac{1}{2}}$. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denotes the maximum and minimum eigenvalue of corresponding matrix, respectively. In the sequel, if not explicitly stated, matrices are assumed to have compatible dimensions, and identity matrix of order n is denoted as I_n (or simply I if no confusion arises). $\mathbf{E}[\cdot]$ stands for the mathematical expectation. The asterisk $*$ in a matrix is used to denote a term induced by symmetry.

Many real-world multi-agent systems have the following properties: every agent has its own dynamic behaviors which may switch among different modes, and each corresponds to one navigating path in this paper; the agents can exchange their information, such as velocity and position in world coordinate system, through wired or wireless communications, but the interconnection structure of system is time-varying; the exchanged information is often with time-delays, and specially, which may be (randomly) time-varying and unknown. Consider a multi-agent system consisting of N identical nodes with communication connections, with each agent being an n-dimensional dynamical system. This dynamical node is described by

$$\dot{x}_i(t) \;=\; f^{\sigma_1}(t, x_i) + \sum_{j=1}^{N} D_{ij}(\sigma_2(t)) x_j(t - \tau(t)) + B u_i(t), \tag{2.1}$$

where $i = 1, 2, \ldots, N$, $x_i = (x_{i_1}, \ldots, x_{i_n})^\top \in \mathbf{R}^n$ are the states variables of agent i, and B is known to be positive matrix. $D(\sigma_2(t)) = (D_{ij}(\sigma_2(t)))_{N \times N}$ are the switching coupling configuration matrix of MAS, describing the communication relationships of agents. $D_{ij}(\sigma_2(t))$ are functions of the random jumping process $\sigma_2(t)$, which is a continuous-time discrete-state Markov jump process, i.e. $\sigma_2(t)$ takes discrete values in a predetermined finite set $\mathcal{M} = \{1, 2, \cdots, m_2\}$

with transition probability matrix $\nabla = [\pi_{rl}]$ given by

$$\begin{aligned} & Pr\{\sigma_2(t+\Delta) = l|\sigma_2(t) = r\} \\ = & \begin{cases} \pi_{rl}\Delta + o(\Delta), & r \neq l, \\ 1 + \pi_{rr}\Delta + o(\Delta), & r = l, \end{cases} \end{aligned} \tag{2.2}$$

where $\Delta > 0$, $\pi_{rl} \geq 0$ is the mode transition rate from r to $l (r \neq l)$ and

$$\pi_r := \pi_{rr} = -\sum_{\substack{l=1 \\ l \neq r}}^{m_2} \pi_{rl},$$

for each mode r ($r = 1, 2, \ldots, m_2$), and $o(\Delta)/\Delta \to 0$ as $\Delta \to 0$. To simplify the notation, $D_{ij}(\sigma_2(t))$ will be denoted by $D_{ij}^{\sigma_2}$. If there is a connection between agent i and j ($j \neq i$), then $D_{ij}^r = D_{ji}^r > 0$; otherwise, $D_{ij}^r = D_{ji}^r = 0$, and the diagonal elements of matrix D^r are defined as

$$D_{ii}^r = -\sum_{\substack{j=1 \\ j \neq i}}^{N} D_{ij}^r = -\sum_{\substack{j=1 \\ j \neq i}}^{N} D_{ji}^r.$$

Thus, it is easily proved that the following framework is equivalent to (2.1)

$$\dot{x}_i(t) \;\; = \;\; f^{\sigma_1}(t, x_i) + \sum_{j=1}^{N} D_{ij}^{\sigma_2}\Big[x_j(t-\tau(t)) - x_i(t-\tau(t))\Big] + Bu_i(t). \tag{2.3}$$

Vector-valued functions $f^{\widetilde{r}}(t, x_i) \in \mathbf{R}^n$ are continuously differentiable, representing the dynamic trajectory of MAS (2.1). For multi-agent system (2.1), assume that, for all $i = 1, \ldots, N$ and $\widetilde{r} = 1, \ldots, m_1$, $f^{\widetilde{r}}(t, x_i)$ satisfy Lipschitz condition with respect to x_i, i.e., for any $x_i(t) \in \mathbf{R}^n$ and $x_j(t) \in \mathbf{R}^n$, there exists a positive constant ϕ such that

$$\|f^{\widetilde{r}}(t, x_j) - f^{\widetilde{r}}(t, x_i)\| \leq \phi\|x_j - x_i\|. \tag{2.4}$$

Switching signal $\sigma_1 : \overline{\mathbf{R}}_- \to \{1, 2, \cdots, m_1\}$ is a piecewise constant function. In different time interval, each subsystem of MAS (2.1) corresponds to distinct switching mode. Similar to the complex spatio-temporal switching network [24], $f^{\sigma_1}(t, x_i) \in \{f^1, \ldots, f^{m_1}\}$ and $D^{\sigma_2} \in \{D^1, \ldots, D^{m_2}\}$ take constant mode at every time interval between two consecutive switching times.

Remark 2.1: The switching signal σ_1 is supposed to depend on time t in this paper, but it is available for the switching signal depending on events in practical environment. For a multi-agent system with collective computational abilities, when an emergent event is sensed, e.g. obstacle approaching, the corresponding strategy is trigged, e.g. changing traveling track. However, these switching frameworks lie on practical performance of agents, but for convenience of theoretical analysis, only the time-dependent switching is considered in this paper (the main results could be extended to other switching signals).

Furthermore, the time-varying delay $\tau(t) > 0$ is a continuously differentiable function treated as

$$0 \leq \tau(t) \leq p < \infty, \ \dot{\tau}(t) \leq q < 1, \ \forall t \geq t_0. \tag{2.5}$$

and $\tau(t)$ is written by τ in the following discussion.

Remark 2.2: In most real engineering, time-delay does not change promptly. Specifically when the multi-agent systems get the predetermined formation shape, the agents will keep the fixed relative positions in the formation till another task is trigged. Thus, the communication delays between agents fluctuate smoothly, i.e. $\dot{\tau}(t) \leq q < 1$. Similarly with most discussion concerned homogenous agents [2], [5], the communication delays are assumed to have uniformly function in this paper.

In general, the negative gradient of the potential function is interpreted as an artificial force acting on the agents and instructing their motion, i.e. $F^A(y) = -\nabla_y J^A(\|y\|)$ and $F^R(y) = -\nabla_y J^R(\|y\|)$, where y is a relative position vector between agents, J^A and J^R are artificial potential functions of the attraction and repulsion between individuals, respectively. The formation controller $u_i(t)$ will be explored based on the artificial potential force which includes attractive force F^A and repulsive force F^R. Commonly, the attractive term is used to keep the compactness of system, and the repulsive term is used to ensure collision avoidance.

Then, the formation controller for agent i is given by

$$u_i(t) = -\nabla_{x_i} \sum_{\substack{j=1 \\ j \neq i}}^{N} J(\|x_j - x_i - w_{ij}\|),$$

where $w_{ij} \in \mathbf{R}^n$ is the desired formation vector of agent i and agent j with the properties

$$w_{ij} = -w_{ji} \ \ and \ \ w_{ii} = 0. \tag{2.6}$$

The distance $|w_{ij}|$ is the equilibrium distance at which the attraction and the repulsion get balance. The potential functions can be specified based on the different structure and/or behavior of the MAS. However, different potentials might result in different performance even for the same multi-agent system [20]. Associated with the real-world formation and the characteristics of potential functions, we suppose $J(\|x_j - x_i - w_{ij}\|)(i, j = 1, \ldots, N)$ satisfy the following assumptions:

(A) $J(\|x_j - x_i - w_{ij}\|)$ have a unique minimum at a desired position if and only if $x_j - x_i = w_{ij}$,

$$\nabla_{x_i} \sum_{\substack{j=1 \\ j \neq i}}^{N} J(\|x_j - x_i - w_{ij}\|) = 0.$$

When $\|x_j - x_i\| > \|w_{ij}\|$, $F^A(\|x_j - x_i - w_{ij}\|) > F^R(\|x_j - x_i - w_{ij}\|)$; when $\|x_j - x_i\| < \|w_{ij}\|$, $F^A(\|x_j - x_i - w_{ij}\|) < F^R(\|x_j - x_i - w_{ij}\|)$.

(B) There exist corresponding functions $g(\|x_j - x_i - w_{ij}\|) : \overline{\mathbf{R}} \to \mathbf{R}_+$ such that

$$\begin{aligned} & -\nabla_{x_i} J(\|x_j - x_i - w_{ij}\|) \\ = \; & (x_j - x_i - w_{ij}) g(\|x_j - x_i - w_{ij}\|). \end{aligned} \tag{2.7}$$

For simplicity, $g(\|x_j - x_i - w_{ij}\|)$ is written as g_{ij}, for all $i, j = 1, 2, \ldots, N$. Note that the directions of the potential force and the vector $x_j(t) - x_i(t) - w_{ij}$ should be the same. Assume that $g_{ij} \geq \underline{g} > 0$, where

$$\underline{g} = \min_{i,j=1,\ldots,N} g_{ij}. \tag{2.8}$$

Remark 2.3: It is worth mentioning that existence of multiple local minima in the potential function results in achieving only local convergence to the desired formation. Due to the limitation of local minima, Assumption $(\mathbf{A})$ is necessary condition assuring the achievement of formation.

Remark 2.4: The term $-\nabla_{x_i} J(\|x_j - x_i - w_{ij}\|)$ in the Assumption $(\mathbf{B})$ represents the potential force between the individuals, which is a vector quantity involving the direction $\frac{x_j - x_i - w_{ij}}{\|x_j - x_i - w_{ij}\|}$. The term g_{ij} determines the attraction-repulsion relationship between the individuals. Regarded the fact that the direction of potential force should be consistent with the vector $x_j - x_i - w_{ij}$ and the processing of proof, the constraint of $g_{ij} \geq \underline{g} > 0$ are given. Compared with the conditions on potential function in [20], [26], the existence of lower bound of g_{ij} is necessary as the coupling term and nonlinear term are considered in the multi-agent system (2.1), which will be explicitly illustrated in the next section.

Similarly to our previous work in [6], for practical application, the MAS has the limited utilization range, i.e. $\max_{i,j=1,\ldots,N} \|x_j - x_i\| < \mathbf{L}$, where $\mathbf{L}$ represents the maximum utilization range. For the predefined formation vectors, there exists $\overline{w} = \max_{i,j=1,\ldots,N} \|w_{ij}\|$, and commonly $\overline{w} < \mathbf{L}$.

Initially, it may seem as if the lower bound $\underline{g}$ is restrictive assumption, since g_{ij} must be known for all $i, j = 1, 2, \ldots, N$. However, note that once the knowledge of $J(\|x_j - x_i - w_{ij}\|)$ is known and associated with the constraints of limited utilization range of MAS, computing $\underline{g}$ is straightforward. For the above assumption, it is satisfied by many potential functions, and certainly by those considered in [11], [20].

Thus, rewrite the formation protocol of multi-agent system (2.1) as

$$u_i(t) = \sum_{\substack{j=1 \\ j \neq i}}^{N} (x_j - x_i - w_{ij})\, g_{ij}. \tag{2.9}$$

In the rest of the paper, we will solve the stochastic formation stability of the multi-agent system (2.1) with controller (2.9). Then according to the conditions derived from the formation analysis, the design criteria of MAS distributed controller for each agent are provided.

III. Analysis of Formation Stability

In this section, stochastic formation stability of the multi-agent system with communication delay and switching topology is presented.

Before proceeding to the theoretical analysis, define a disagreement vector

$$e_{ij}(t) = X_{ij}(t) - w_{ij},$$

where $X_{ij}(t) = x_j(t) - x_i(t)$ is a disagreement of relative position between agent i and j. According to (2.3), the time derivative of $e_{ij}(t)$ is

$$\begin{aligned} \dot{e}_{ij}(t) &= \dot{x}_j(t) - \dot{x}_i(t) \\ &= f^{\sigma_1}(t, x_j) - f^{\sigma_1}(t, x_i) + \sum_{k=1}^{N} \left(D_{jk}^{\sigma_2} X_{jk}(t-\tau) - D_{ik}^{\sigma_2} X_{ik}(t-\tau) \right) \\ &\quad + B \sum_{k=1}^{N} \left(g_{jk} e_{jk}(t) - g_{ik} e_{ik}(t) \right). \end{aligned} \tag{3.1}$$

It is easy to see that $\dot{e}_{ij}(t) = \dot{X}_{ij}(t)$, for $i, j = 1, 2, \ldots, N$. Obviously, if all the disagreement vectors $e_{ij}(t)$ $(i, j = 1, 2, \ldots, N)$ uniformly asymptotically tend to zero, then the dynamical system (2.1) realizes formation stability. Namely, the formation stability of system (2.1) is now equivalent to the problem of stabilizing the system (3.1) using a suitable choice of the control law, such that

$$\lim_{t \to \infty} \sum_{i=1}^{N} \sum_{j=1}^{N} \|e_{ij}(t)\| = 0.$$

In the subsequent discussion, assume that for all $\delta \in [-\tau, 0]$, a scalar $\varepsilon > 0$ exists such that

$$\|e_{ij}(t+\delta)\| \leq \varepsilon \|e_{ij}(t)\|. \tag{3.2}$$

As indicated by [22], this assumption does not bring the conservatism, since ε can be chosen arbitrarily.

The following definitions and Lemma are needed to facilitate the development of the main results of this paper.

Definition 3.1: (Formation Stability)[6] If the formation controller $u(t)$ makes $\|x_j(t) - x_i(t) - \Delta_{ij}\| \to 0$ hold for $(i,j) = \{\, (i,j) \mid i,j = 1,2,\ldots,N\}$ when $t \to \infty$, then the multi-agent system is said to be asymptotically formation stabilizable.

Lemma 3.1: For a positive definite matrix P and any vectors $x, y \in \mathbf{R}^n$, the matrix inequality $2x^\top Py \leq x^\top Px + y^\top Py$ holds.

Definition 3.2: The formation of multi-agent system (2.1) is said to be stochastically stabilizable if, for all initial mode $\sigma_2(0) \in \mathcal{M}$, there exists a formation control law satisfying

$$\lim_{T\to\infty} \mathbf{E}\left\{\int_0^T \sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(t)e_{ij}(t)dt|e(0), \sigma_2(0)\right\} \leq \sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(0)\widetilde{U}e_{ij}(0),\ i,j = 1,\ldots,N \tag{3.3}$$

where $\widetilde{U}$ is a symmetric positive definite matrix and $e(0)$ is initial condition which defined as

$$e(0) = [e_{11}^\top(t), \ldots, e_{1N}^\top(t), \ldots, e_{ij}^\top(t), \ldots, e_{NN}^\top(t)]^\top|_{t=0}.$$

In order to achieve stochastic formation stability, we design a formation controller to guarantee the multi-agent system to form a desired shape asymptotically.

Theorem 3.1: The formation multi-agent system (2.1) is stochastically stabilizable, for all $\widetilde{r} = 1,\ldots,m_1$, $r = 1,\ldots,m_2$, $i,j = 1,\ldots,N$, if positive definite matrices $Q > 0$ and $P^r > 0$, and a constant $\underline{g} > 0$ exist, satisfying the coupling matrix inequalities

$$\begin{bmatrix} \Xi^r & -D_{ij}^r P^r & 0 & -D_{ij}^r P^r \\ * & -\widetilde{q}Q & 0 & 0 \\ * & * & -\underline{g}B^\top P^r + \frac{\phi^2}{N}I & \underline{g}B^\top P^r \\ * & * & * & -\underline{g}B^\top P^r \end{bmatrix} < 0, \tag{3.4}$$

where

$$\Xi^r = Q + \frac{1}{N}\sum_{l}^{m_2}\pi_{rl}P^l - P^r(\underline{g}B - \frac{1}{N}P^r),$$

and $\widetilde{q} = 1 - q > 0$.

Proof: Let the topology mode at time t be D^r, that is $\sigma_2(t) = r \in \mathcal{M}$. Choose the stochastic Lyapunov functional in the form of

$$\begin{aligned} & V(e(t), \sigma_2(t) = r) \equiv V(e, r) \\ = \ & \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(e_{ij}^{\top}(t) \frac{P^r}{N} e_{ij}(t) + \int_{t-\tau}^{t} e_{ij}^{\top}(\theta) Q e_{ij}(\theta) d\theta \right), \end{aligned}$$

where Q is a constant positive definite matrix, and P^r is a constant positive definite matrix for each r.

Consider the weak infinitesimal operator $\mathcal{A}$ [22] of the stochastic process $\{\sigma_2(t)\}$ $(t \geq 0)$, is give by

$$\begin{aligned} \mathcal{A}V(e(t), \sigma_2(t)) \ = \ & \lim_{\Delta \to \infty} \frac{1}{\Delta} \Big[\mathbf{E}\{ V(e(t+\Delta), \sigma(t+\Delta)) | e(t), \sigma_2(t) = r \} - V(e(t), \sigma_2(t) = r) \Big] \\ = \ & \sum_{i=1}^{N} \sum_{j=1}^{N} e_{ij}^{\top} \left(\frac{P^r}{N} \dot{e}_{ij} + \sum_{l=1}^{m_2} (\frac{\pi_{rl} P^l}{2N}) e_{ij} \right) \\ & + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \Big[e_{ij}^{\top}(t) Q e_{ij}(t) - (1 - \dot{\tau}) e_{ij}^{\top}(t-\tau) Q e_{ij}(t-\tau) \Big] \\ \leq \ & \sum_{i=1}^{N-1} \sum_{j>i}^{N} \Big[e_{ij}^{\top} \frac{2P^r}{N} \Big(f^{\widetilde{r}}(t, x_j) - f^{\widetilde{r}}(t, x_i) \Big) \Big] + V_1(e(t), r) + V_2(e(t), r) \\ & - \sum_{i=1}^{N-1} \sum_{j>i}^{N} \left[(1-q) e_{ij}^{\top}(t-\tau) Q e_{ij}(t-\tau) - e_{ij}^{\top}(t) \Big(\frac{1}{N} \sum_{l=1}^{m_2} \pi_{rl} P^l + Q \Big) e_{ij}(t) \right], \end{aligned} \tag{3.5}$$

where

$$V_1(e(t), r) = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} e_{ij}^{\top}(t) P^r (D_{jk}^r X_{jk}(t-\tau) - D_{ik}^r X_{ik}(t-\tau))$$

and

$$V_2(e(t), r) = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} e_{ij}^{\top}(t) \left(P^r B + B^T P^r \right) \times \left(g_{jk} e_{jk}(t) - g_{ik} e_{ik}(t) \right).$$

From Lemma 3.1 and inequality (2.4), the first term of inequality (3.5) becomes for all $\widetilde{r} =$

$1, \ldots, m_1$

$$
\begin{aligned}
& e_{ij}^{\top}(t) \frac{P^r}{N} \left(\widetilde{f^r}(t, x_j) - \widetilde{f^r}(t, x_i) \right) \\
\leq \;& e_{ij}^{\top}(t) \frac{P^r (P^r)^{\top}}{2N} e_{ij}(t) + \| \widetilde{f^r}(t, x_j) - \widetilde{f^r}(t, x_i) \|^2 \\
\leq \;& e_{ij}^{\top}(t) \frac{P^r (P^r)^{\top}}{2N} e_{ij}(t) + \frac{\phi^2}{2N} X_{ij}^{\top}(t) X_{ij}(t). \qquad (3.6)
\end{aligned}
$$

Since $e_{ij}(t) = -e_{ji}(t)$, $e_{ii}(t) = 0$, the $V_1(e(t), r)$ can be

$$
\begin{aligned}
V_1(e(t), r) &= -\frac{2}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} D_{jk}^r e_{ji}^{\top}(t) P^r X_{jk}(t-\tau) \\
&= -\frac{2}{N} \sum_{i=1}^{N} \sum_{j=1}^{N-1} \sum_{j<k}^{N} D_{jk}^r e_{ji}^{\top}(t) P^r X_{jk}(t-\tau) - \frac{2}{N} \sum_{i=1}^{N} \sum_{j=1}^{N-1} \sum_{k<j}^{N} D_{jk}^r e_{ji}^{\top}(t) P^r X_{jk}(t-\tau) \\
&= -\frac{2}{N} \sum_{i=1}^{N} \sum_{j=1}^{N-1} \sum_{j<k}^{N} D_{jk}^r e_{ji}^{\top}(t) P^r X_{jk}(t-\tau) - \frac{2}{N} \sum_{i=1}^{N} \sum_{k=1}^{N-1} \sum_{j<k}^{N} D_{kj}^r e_{ki}^{\top}(t) P^r X_{kj}(t-\tau).
\end{aligned}
$$

Noticed that $e_{ji}(t) + e_{ik}(t) = e_{jk}(t)$, thus

$$
\begin{aligned}
V_1(e(t), r) &= -\frac{2}{N} \sum_{i=1}^{N} \sum_{j=1}^{N-1} \sum_{j<k}^{N} D_{jk}^r e_{jk}^{\top}(t) P^r X_{jk}(t-\tau) \\
&= -2 \sum_{i=1}^{N-1} \sum_{j>i}^{N} D_{ij}^r e_{ij}^{\top}(t) P^r X_{ij}(t-\tau) \\
&= -2 \sum_{i=1}^{N-1} \sum_{j>i}^{N} D_{ij}^r e_{ij}^{\top}(t) P^r e_{ij}(t-\tau) - 2 \sum_{i=1}^{N-1} \sum_{j>i}^{N} D_{ij}^r e_{ij}^{\top}(t) P^r w_{ij}. \qquad (3.7)
\end{aligned}
$$

Similar to $V_1(e(t), r)$ and by the properties of w_{ij} $(i, j = 1, \ldots, N)$ showed in (2.5), $V_2(e(t), r)$ is treated as:

$$
\begin{aligned}
V_2(e(t), r) &= -\sum_{i=1}^{N-1} \sum_{j>i}^{N} g_{ij} e_{ij}^{\top}(t) \left(P^r B + B^{\top} P^r \right) e_{ij}(t) \\
&\leq -\underline{g} \sum_{i=1}^{N-1} \sum_{j>i}^{N} e_{ij}^{\top}(t) \left(P^r B + B^{\top} P^r \right) e_{ij}(t) \\
&= -\underline{g} \sum_{i=1}^{N-1} \sum_{j>i}^{N} \Big(e_{ij}^{\top}(t) P^r B e_{ij}(t) + w_{ij}^{T} B^{\top} P^r w_{ij} \\
&\quad + X_{ij}^{\top}(t) B^{\top} P^r X_{ij}(t) - 2 X_{ij}^{\top}(t) B^{\top} P^r w_{ij} \Big). \qquad (3.8)
\end{aligned}
$$

Then associated (3.6), (3.7) and (3.8) with (3.5), the weak infinitesimal $\mathcal{A}V$ becomes

$$\begin{aligned}
&\mathcal{A}V(e(t),\sigma_2(t))\\
\leq\ & \sum_{i=1}^{N-1}\sum_{j>i}^{N}\bigg\{e_{ij}^\top(t)\Big(Q+\frac{1}{N}\sum_{l=1}^{m_2}\pi_{rl}P^l-P^r(\underline{g}B-\frac{1}{N}P^r)\Big)e_{ij}(t)\\
&+2\underline{g}X_{ij}^\top(t)B^\top P^r w_{ij}-\underline{g}w_{ij}^\top B^\top P^r w_{ij}-2D_{ij}^r e_{ij}^\top(t)P^r w_{ij}-X_{ij}(t)\left(\underline{g}B^\top P^r-\frac{\phi^2}{N}I\right)X_{ij}(t)\\
&-\widetilde{q}e_{ij}^\top(t-\tau)Qe_{ij}(t-\tau)-2D_{ij}^r e_{ij}^\top(t)P^r e_{ij}(t-\tau)\bigg\}\\
=\ & \sum_{i=1}^{N-1}\sum_{j>i}^{N}\xi_{ij}^\top(t)\Omega^r\xi_{ij}(t)<0,
\end{aligned}$$

where

$$\xi_{ij}(t)=\begin{bmatrix} e_{ij}^\top(t) & e_{ij}^\top(t-\tau) & X_{ij}^\top(t) & w_{ij}^\top\end{bmatrix}^\top,$$

and

$$\Omega^r=\begin{bmatrix}\Xi^r & -D_{ij}^rP^r & 0 & -D_{ij}^rP^r\\ * & -\widetilde{q}Q & 0 & 0\\ * & * & -\underline{g}B^\top P^r+\frac{\phi^2}{N}I & \underline{g}B^\top P^r\\ * & * & * & -\underline{g}B^\top P^r\end{bmatrix}.$$

Clearly, it is easy to prove that $\|e_{ij}\|<\|\xi_{ij}\|$, and note that $\Omega^r<0$ and $P^r>0$. Thus, for $t>0$

$$\begin{aligned}
&\frac{\mathcal{A}V(e,r)}{V(e,r)}\\
\leq\ & \frac{\sum_{i=1}^{N-1}\sum_{j>i}^{N}\xi_{ij}^\top(t)\Omega^r\xi_{ij}(t)}{\sum_{i=1}^{N-1}\sum_{j>i}^{N}\Big(e_{ij}^\top(t)\frac{P^r}{N}e_{ij}(t)+\int_{t-\tau}^{t}e_{ij}^\top(\theta)Qe_{ij}(\theta)d\theta\Big)}\\
=\ & \frac{-\sum_{i=1}^{N-1}\sum_{j>i}^{N}\xi_{ij}^\top(t)(-\Omega^r)\xi_{ij}(t)}{\sum_{i=1}^{N-1}\sum_{j>i}^{N}\Big(e_{ij}^\top(t)\frac{P^r}{N}e_{ij}(t)+\int_{t-\tau}^{t}e_{ij}^\top(\theta)Qe_{ij}(\theta)d\theta\Big)}\\
\leq\ & -\min_{r\in\mathcal{M}}\left\{\frac{\lambda_{\min}(-\Omega^r)}{\lambda_{\max}(P^r)/N+p\varepsilon^2\lambda_{\max}(Q)}\right\}.
\end{aligned}$$

Define

$$\alpha:=\min_{r\in\mathcal{M}}\left\{\frac{\lambda_{\min}(-\Omega^r)}{\lambda_{\max}(P^r)/N+p\varepsilon^2\lambda_{\max}(Q)}\right\}.$$

Obviously, $\alpha > 0$, so $\mathcal{A}V(e,r) \leq -\alpha V(e,r)$. Then using Dynkin's formula [23], for all $\sigma_2(0) \in \mathcal{M}$, one has

$$\begin{aligned}
& \mathbf{E}\left\{V(e(t),\sigma_2(t))\right\} - V(e(0),\sigma_2(0)) \\
= \;& \mathbf{E}\left\{\int_0^t \mathcal{A}V(e(s),\sigma_2(s))ds\right\} \\
\leq \;& -\alpha \int_0^t \mathbf{E}\left\{V(e(s),\sigma_2(s))\right\} ds.
\end{aligned}$$

The Gronwall-Bellman lemma [23] makes

$$\mathbf{E}\{V(e(t),\sigma_2(t))\} \leq \exp(-\alpha t) V(e(0),\sigma_2(0)).$$

Since $Q > 0$, one gets

$$\mathbf{E}\left\{\int_{t-\tau}^t \left(\sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(s) Q e_{ij}(s)\right) ds\right\} > 0.$$

Thus,

$$\begin{aligned}
& \mathbf{E}\left\{\frac{1}{N}\sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(t) P^r e_{ij}(t) | e(0),\sigma_2(0)\right\} \\
= \;& \mathbf{E}\left\{V(e,r)|e(0),\sigma_2(0)\right\} - \mathbf{E}\left\{\int_{t-\tau}^t \left(\sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(s) Q e_{ij}(s)\right) ds | e(0),\sigma_2(0)\right\} \\
\leq \;& \exp(-\alpha t) V(e(0),r).
\end{aligned}$$

Then, one can obtain

$$\begin{aligned}
& \mathbf{E}\left\{\int_0^T \sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(t) P^r e_{ij}(t) dt | e(0),\sigma_2(0)\right\} \\
\leq \;& N\int_0^T \exp(-\alpha t) dt V(e(0),r) \\
= \;& -\frac{N}{\alpha}[\exp(-\alpha T) - 1] V(e(0),r).
\end{aligned} \tag{3.9}$$

Taking limit as $T \to \infty$, matrix inequality (3.9) yields

$$\begin{aligned}
& \lim_{T\to\infty} \mathbf{E}\left\{\int_0^T \sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(t) P^r e_{ij}(t) dt | e(0),\sigma_2(0)\right\} \\
\leq \;& \lim_{T\to\infty}\left\{-\frac{N}{\alpha}[\exp(-\alpha T) - 1] V(e(0),r)\right\} \\
\leq \;& \frac{N}{\alpha}\sum_{i=1}^{N-1}\sum_{j>i}^{N}\left[e_{ij}^\top(0)\Big(\frac{\lambda_{\max}(P^r)}{N} + \varepsilon^2 p \lambda_{\max}(Q)\Big) I e_{ij}(0)\right].
\end{aligned}$$

Since $P^r > 0$, for each $r \in \mathcal{M}$, one has

$$\begin{aligned}&\lim_{T\to\infty} \mathbf{E}\left\{\int_0^T \sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(t)e_{ij}(t)dt|e(0),\sigma_2(0)\right\}\\ \leq\ & \sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(0)\widetilde{U}e_{ij}(0),\end{aligned}$$

where

$$\widetilde{U} = \max_{r\in\mathcal{M}}\left\{\frac{\lambda_{\max}(P^r)+\varepsilon^2 pN\lambda_{\max}(Q)}{\alpha\lambda_{\min}(P^r)}\right\} I.$$

Based on Definition (3.2), one can prove that the formation of multi-agent system (2.1) under control law (2.9) is stochastically stable. ■

Remark 3.1: Notice the diagonal elements in inequality (3.4), and it is easy to find that the Assumption (B) can guarantees these diagonal terms to be negative. In order to achieve the desired formation, the controller is required to trade off the effect from the coupling term and nonlinearity as modeled in (2.1).

Remark 3.2: It should be noted that the proposed conditions (3.4) are formulated in terms of linear matrix inequalities (LMIs). Therefore, by using MATLAB LMI Toolbox, for a given multi-agent system, the upper bound $\underline{g}$ can be efficiently calculated by optimizing a generalized eigenvalue problem from LMIs (3.4). Moreover, the results in this paper can be easily extended to the multi-agent systems with uncertainty and diverse time-varying delays.

From the Schur complement, one can find the matrix inequalities (3.4) are equivalent to

$$\begin{aligned}&\begin{bmatrix}\varpi^r & 0 & -D_{ij}^r P^r\\ * & -\underline{g}B^\top P^r + \frac{\phi^2}{N}I & \underline{g}B^\top P^r\\ * & * & -\underline{g}B^\top P^r\end{bmatrix}\\ =&\begin{bmatrix}Q+\sum\limits_{l=1}^{m_2}\frac{\pi_{rl}}{N}P^l-\underline{g}P^rB & 0 & -D_{ij}^r P^r\\ * & -\underline{g}B^\top P^r & \underline{g}B^\top P^r\\ * & * & -\underline{g}B^\top P^r\end{bmatrix}\\ &+\underbrace{\begin{bmatrix}P^r\left(\frac{(D_{ij}^r)^2Q^{-1}}{\widetilde{q}}+\frac{1}{N}I\right)P^r & 0 & 0\\ * & \frac{\phi^2}{N}I & 0\\ * & 0 & 0\end{bmatrix}}_{\tilde{A}} < 0,\end{aligned}$$

where $r \in \mathcal{M}$ and

$$\varpi^r \;=\; Q + \frac{1}{N}\sum_{l=1}^{m_2} \pi_{rl} P^l - \underline{g} P^r B + P^r \left(\frac{(D_{ij}^r)^2 Q^{-1}}{\widetilde{q}} + \frac{1}{N} I \right) P^r.$$

With respect to the nonnegative of $\tilde{A}$, one can get

$$\begin{bmatrix} Q + \sum_{l}^{m_2} \frac{\pi_{rl}}{N} P^l - \underline{g} P^r B & 0 & -D_{ij}^r P^r \\ * & -\underline{g} B^\top P^r & \underline{g} B^\top P^r \\ * & * & -\underline{g} B^\top P^r \end{bmatrix} \;<\; 0. \tag{3.10}$$

Let $Z^r = (P^r)^{-1}$, $Y^r = \underline{g} Z^r$, $R = Q^{-1}$ and define $K^r = diag(Z^r, Z^r, Z^r)$. Pre- and post-multiplying (3.10) by K^r, one can see that the coupled matrix inequalities (3.10) are equivalent to the following matrix inequalities:

$$\begin{bmatrix} \Pi^r & 0 & -D_{ij}^r Z^r \\ * & -Y^r B^\top & Y^r B^\top \\ * & * & -Y^r B^\top \end{bmatrix} < 0, \tag{3.11}$$

where

$$\Pi^r \;=\; Z^r R^{-1} Z^r + \sum_{l}^{m_2} \frac{\pi_{rl}}{N} Z^r (Z^l)^{-1} Z^r - B Y^r.$$

Then the inequalities (3.11) are in turn equivalent to the following LMIs for $r = 1, \ldots, m_2$

$$\begin{bmatrix} -BY^r + \frac{\pi_r}{N} Z^r & 0 & -D_{ij}^r Z^r & \chi^r \\ * & -Y^r B^\top & Y^r B^\top & 0 \\ * & * & -Y^r B^\top & 0 \\ * & * & * & -\Upsilon^r \end{bmatrix} < 0, \tag{3.12}$$

where

$$\begin{aligned} \chi^r \;=\; & \Big[\sqrt{\pi_{r1}/N} Z^r, \; \cdots \; \sqrt{\pi_{r,r-1}/N} Z^r, \\ & \sqrt{\pi_{r,r+1}/N} Z^r, \; \cdots \; \sqrt{\pi_{rm_2}/N} Z^r \; Z^r \Big] \end{aligned} \tag{3.13}$$

and

$$\Upsilon^r = diag\left(Z^1 \; \cdots \; Z^{r-1} \; Z^{r+1} \; \cdots \; Z^{m_2} \; R \right). \tag{3.14}$$

According to above derivation, the formation stability results are summarized in the following theorem based on LMIs.

Theorem 3.2: The formation of multi-agent system (2.1) is stochastically stabilizable, for $r = 1, \ldots, m_2$, if there exist positive definite matrices R, Z^r and Y^r, such that the coupled LMIs (3.12) hold, where χ^r and Υ^r are given by (3.13) and (3.14), respectively. And the minimum of functionals g_{ij} can be calculated from $\underline{g} = \inf_{r\in\mathcal{M}} \frac{\|Y^r\|_2}{\|(Z^r)\|_2}$.

Note that if the communication topology has only one fixed form, the multi-agent system (2.1) reduces to a deterministic one. In the subsequent theorem, we present the formation stability property for the deterministic multi-agent system (2.1).

Theorem 3.3: For the multi-agent system (2.1) with fixed topology and time-varying delay, if there exist matrices $P > 0$, $Q > 0$ and a constant $\underline{g}$ such that the following LMIs

$$\begin{bmatrix} \tilde{\tilde{\Xi}} & -D_{ij}P & 0 & -D_{ij}P \\ * & -\tilde{q}Q & 0 & 0 \\ * & * & -\underline{g}B^\top P + \frac{\phi^2}{N}I & \underline{g}B^\top P \\ * & * & * & -\underline{g}B^\top P \end{bmatrix} < 0 \tag{3.15}$$

hold, for all $i, j = 1, \ldots, N$, where

$$\tilde{\tilde{\Xi}} = Q - P(\underline{g}B - \frac{1}{N}P),$$

then the formation of multi-agent system (2.1) is asymptotically stabilizable.

From above theorems, we can conclude an algorithm of formation controller design for multi-agent system with stochastic switching topology and time-varying communication delays.

(1) According to the Assumption (**A**) and (**B**), choose a potential function which is differentiable and has a unique minimum.

(2) For all $i, j = 1, \ldots, N$, validate the lower bound of g_{ij} is existed and satisfied the constraint $\underline{g} > 0$. If the lower bound $\underline{g} > 0$ does not exist, then adjust the parameters of potential function and go to (2).

(3) Solve (3.4) and, verify the positive definiteness of Q and P^r ($r = 1, \ldots, m_2$). If (3.4) does not have positive definite solutions P^r and Q, regulate the parameters of potential functions and go to (2).

IV. Numerical examples

In this section, some examples are conducted to show the effectiveness of the proposed theoretical results. As discussed above the formation of multi-agent systems may have distinct performance based on different potential function J.

Firstly, according to assumption of potential function, the formation controller $u_i(t)$ is chosen as in [6]

$$\begin{aligned} u_i(t) &= 2\sum_{\substack{j=1\\ j\neq i}}^{N} e_{ij}(t)\Bigg(\frac{C_a}{L_a^2}\exp\left(-\frac{\|e_{ij}(t)\|^2}{L_a^2}\right) \\ &\quad -\frac{C_r}{L_r^2}\exp\left(-\frac{\|e_{ij}(t)\|^2}{L_r^2}\right)+C_r\left(\frac{1}{L_r^2}+\frac{1}{L_a^2}\right) \\ &\quad \times\exp\left(-(\frac{1}{L_r^2}+\frac{1}{L_a^2})(\|e_{ij}(t)\|^2)\right)\Bigg), \end{aligned} \tag{4.1}$$

with constraints $L_a > L_r$ and

$$\frac{C_a}{C_r} > \frac{L_a^2}{L_r^2}\exp\left(-(\frac{1}{L_r^2}-\frac{1}{L_a^2})\|e_{ij}(t)\|^2\right)-\left(1+\frac{L_a^2}{L_r^2}\right)\exp\left(-\frac{\|e_{ij}(t)\|^2}{L_r^2}\right),$$

which guarantees $g_{ij} > 0$. Obviously, the potential function developed in (4.1) has a unique minimum at a desired value, when $X_{ij}(t) = w_{ij}$ for all $i, j = 1, 2, \dots, N$. L_a L_r C_a, and C_r are positive parameters representing ranges and strengths of attraction and repulsion, respectively. Let $L_a = 0.5$ $L_r = 0.32$ $C_a = 21$, $C_r = 0.8$, $N = 4$, $n = 2$, $B = I$ and the time-varying delay $\tau(t) = 0.5\sin t$ in the multi-agent system (2.1). The coupling configuration matrix are designed to be stochastically switching with equal probability between two modes:

$$D^1 = \begin{bmatrix} -0.64 & 0.32 & 0 & 0.32 \\ * & -0.64 & 0.32 & 0 \\ * & * & -0.64 & 0.32 \\ * & * & * & -0.64 \end{bmatrix},$$

and

$$D^2 = \begin{bmatrix} -0.64 & 0 & 0.32 & 0.32 \\ * & -0.32 & 0 & 0.32 \\ * & * & -0.64 & 0.32 \\ * & * & * & -0.96 \end{bmatrix}.$$

Moreover, set

$$f(t, x_i) = 0.3 * [\cos(0.5 * x_{i_1}(t)), \sin(0.25 * x_{i_2}(t))]^\top,$$

and one can get $\phi = 0.13$ and $\underline{g} = 0.4021$ in this example. The goal of this task is to drive this MAS to keep a formation of square. From theorem 3.1, the formation of multi-agent system with switching topology and time delay (2.1) is stochastically stable. Fig.1 shows four agents achieve the predefined formation from the initial positions $(1.1, 3.0)^T$, $(3.0, 4.0)^T$, $(3.7, 3.2)^T$, and $(2.6, 2.0)^T$. An error function

$$d(t) = \sum_{i=1}^{N-1} \sum_{j>i}^{N} (\|x_j(t) - x_i(t)\| - \|w_{ij}\|)$$

is given to visualize the effectiveness of formation which is showed in Fig.2. From Fig.1 and Fig.2, one can find the multi-agent system with time-varying delays obtains the desired formation shape in a short period of time.

Furthermore, the multi-agent system will perform more complex tasks following the switching track scheme showed in Table I with a square formation. The initial position of agents are given as $(1.0, 1.2)^T$, $(2.0, 4.0)^T$, $(6.7, 3.0)^T$, and $(4.2, 2.0)^T$. It should be mentioned that the predesigned tracks in this example could be derived from embedded microprocessor in each agent which are responsible for collecting information from environment and providing the accessible path. The trajectories of the system (2.1 exploring in a cave-like scenario and corresponding formation error are described in Fig.3 and Fig.4, which demonstrates the effectiveness of the proposed formation protocol (2.9) in this paper.

V. Conclusions

The formation protocol of a multi-agent system with stochastic switching topology and time-varying delays is addressed in this paper. The formation controller based on artificial potential functions has been designed in a general form. By introducing a disagreement function, the formation problem of multi-agent system is translated into the stochastic stability of an error system. Then by employing stochastic Lyapunov functional approach and linear matrix inequalities, the sufficient conditions for formation keeping of the MAS are obtained. The main contribution of this paper is to provide a valid distributed formation algorithm that overcomes the difficulties caused by unreliable communication channels, such as stochastic information transmission, switching communication topology, and time-varying communication delays. Therefore this approach possesses great potential in practical applications. Finally, examples have been provided to verify the effectiveness of the proposed approach.

REFERENCES

[1] F. Xiao and L. Wang, Asynchronous Consensus in Continuous-Time Multi-Agent Systems with Switching Topology and Time-Varying Delays, *IEEE Transactions on Automatic Control*, vol. 53, 2008, pp.1804-1816.

[2] R. Olfati-Saber and R.M. Murray, Consensus Problems in Networks of Agents with Switching Topology and Time-Delays, *IEEE Transactions on Automatic Control*, vol. 49, 2004, pp. 1520-1532.

[3] P. Lin and Y. Jia, Distributed rotating formation control of multi-agent systems, *Systems and Control Letters*, vol. 59, 2010, pp. 587-595.

[4] J. Hu and Y. Hong, Leader-following coordination of multi-agent systems with coupling time delays, *Physica A*, vol. 374, 2007, pp. 853-863.

[5] P. Lin and Y. Jia, Consensus of a Class of Second-Order Multi-Agent Systems with Time-Delay and Jointly-Connected Topologies, *IEEE Transactions on Automatic Control*, vol. 55, 2010, pp. 778-784.

[6] D. Xue, J. Yao, G. Chen and Y. Yu, Formation Control of Networked Multi-Agent Systems, *IET Control Theory and Application*, vol. 4, 2010, pp. 2168-2176.

[7] F. Xiao, L. Wang and A. Wang, Consensus Problems in Discrete-Time Multi-Agent Systems with Fixed Topology, *Journal Mathematical Analysis and Applicatioin*, vol. 322, 2006, pp. 587-598.

[8] D. Lee and M.W. Spong, Agreement with Non-Uniform Information Delays, *in American Control Conference*, Minneapolis, MN, 2006, pp. 756-761.

[9] Y. Sun and L. Wang, Consensus of Multi-Agent Systems in Directed Networks with Nonuniform Time-Varying Delays, *IEEE Tansactions on Automatic Control*, vol. 54, 2009, pp. 1607-1613.

[10] X. Zhang and Q. Han, Robust H_∞ Filtering for A Class of Uncertain Linear Systems with Time-Varying Delay, *Automatica*, vol. 44, 2008, pp. 157-166.

[11] V. Gazi and K.M. Passino, Stability Analysis of Social Foraging Swarms, *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, vol. 34, 2004, pp. 539-557.

[12] J. Ren, K.A. McIsaac, R.V. Patel and T.M. Peters, A Potential Fild Model Using Generalized Sigmoid Functions, *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, vol. 37, 2007, pp. 477-484.

[13] X. Wang, V. Yadav and S.N. Balakrishnan, Cooperative UAV Formation Flying with Obstacle/Collision Avoidance, *IEEE Transaction on Control Systems Technology*, vol. 15, 2007, pp. 672-679.

[14] F. Zhang, Geometric Cooperative Control of Particle Formations, *IEEE Transactions on Automatic Control*, vol. 5, 2010, pp. 800-803.

[15] A. Tahbaz-Salehi and A. Jadbabaie, Distributed Coverage Verification in Sensor Networks without Location Information *IEEE Transation on Automatic Control*, vol. 55, 2010, pp. 1837-1849.

[16] I. Chang, S. Park and K. Choi, Nonlinear Attitude Control of A Tether-Connected Multi-Satellite in Three-Dimensional Space, *IEEE Transactions on Aerospace and Electronic Systems*, vol. 46, 2010, pp. 1950-1968.

[17] G.L. Mariottini, F. Morbidi, D. Prattichizzo, N. Michael, G. Pappas and K. Daniilidis, Vision-Based Location for Leader-Follower Formation Control, *IEEE Transactions on Robotics*, vol. 25, 2009, pp. 1431-1438.

[18] E. Yang and D. Gu, Nonlinear Formation-Keeping and Mooring Control of Multiple Autonomous Underwater Vehicles of Multiple Autonomous Underwater Vehicles, *IEEE/ASME Transactions on Mechatronics*, vol. 12, 2007, pp. 164-178.

[19] Y. Gu, B. Seanor, G. Campa, M.R. Napolitano, L. Rowe, S. Gururajan and S. Wan, Design and Flight Testing Evaluation of Formation Control Laws *IEEE Transactions on Control Systems Technology*, vol. 14, 2006, pp. 1105-1112.

[20] J. Yao, R. Ordonez and V. Gazi, Swarm Tracking Using Artificial Potentials and Sliding Mode Control, *in 45th IEEE Conference on Decision and Control*, San Diego, CA, USA, 2006, pp. 4670-4675.

[21] Z. Wang, F. Yang, D.W.C. Ho and X. Liu, Robust H_∞ Filtering for Stochastic Time-Delay Systems with Missing Measurements, *IEEE Transactions on Signal Processing*, vol. 54, 2006, pp. 2579-2587.

[22] Y. Cao and J. Lam, Robust H_∞ Control of Uncertain Markovian Jump Systems with Time-Delay, *IEEE Transactions on Automatic Control*, vol. 45, 2000, pp. 77-83.

[23] H.J. Kushner, Stochastic Stabiility and Control, *New York: Academic Press*, 1967.

[24] J. Yao, H.O. Wang, Z. Guan and W. Xu, Passive Stability and Synchronization of Complex Spatio-Temporal Switching Networks with Time Delays *Automatica*, vol. 45, 2009, pp. 1721-1728.

[25] H. Liu, J. Lu, J. Lu and D.J. Hill, Structure Identification of Uncertain general complex dynamical networks with time delay, *Automatica*, vol. 45, 2009, pp. 1799-1807.

[26] V. Gazi, Swarm Aggregations Using Artificial Potenails and Sliding-Mode Control, *IEEE Transactions on Robotics*, vol. 21, 2005, pp. 1208-1214.

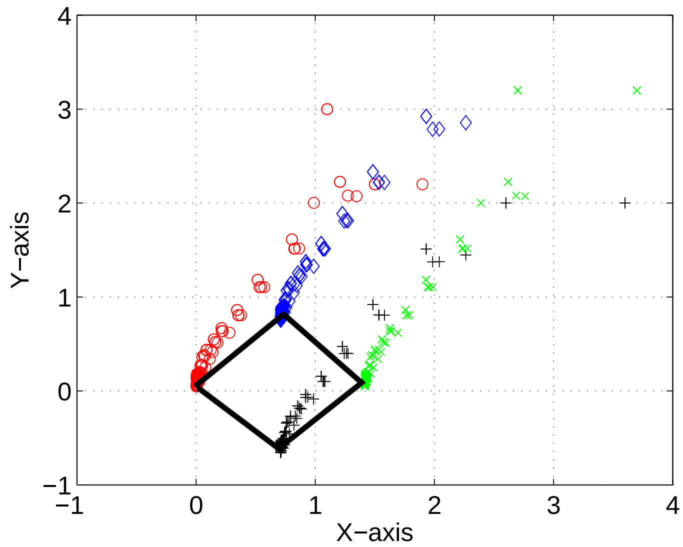

Fig. 1. Formation of multi-agent system with stochastic switching topology and time-varying delay

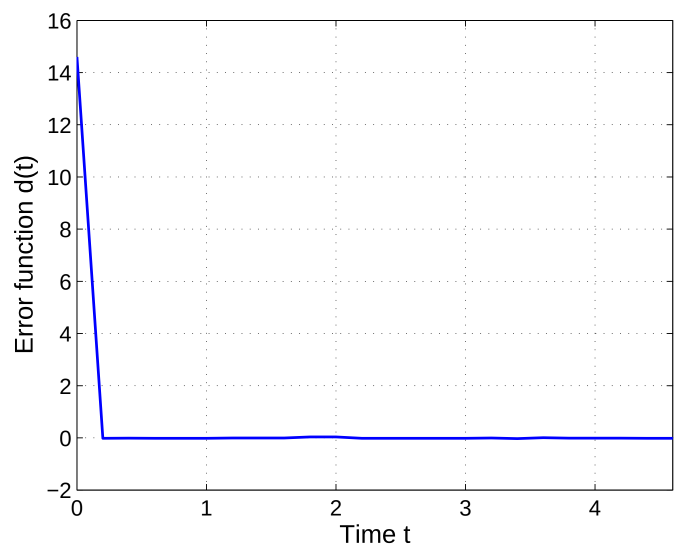

Fig. 2. Formation disagreement of multi-agent system with stochastic switching topology and time-varying delay

TABLE I

TRACK TABLE

Period of time t/(s)	$f^{\sigma_1}(t, x_i)$
$0 - 10$	$[3, 0]^\top$
$10 - 21$	$[0, 0.5]^\top$
$21 - 29$	$[1, 0]^\top$
$29 - 31$	$[4, 0.8]^\top$
$31 - 36$	$[3, 0]^\top$
$36 - 38$	$[4, -0.8]^\top$
$38 - 40$	$[3, 0]^\top$

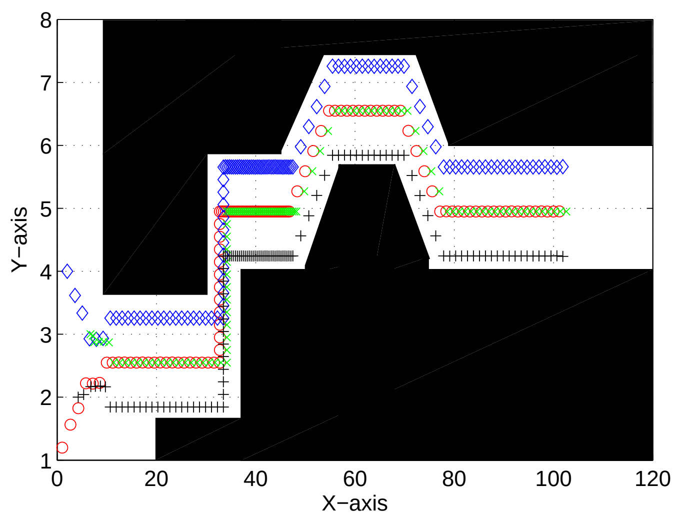

Fig. 3. Formation of multi-agent system with switching trajectories

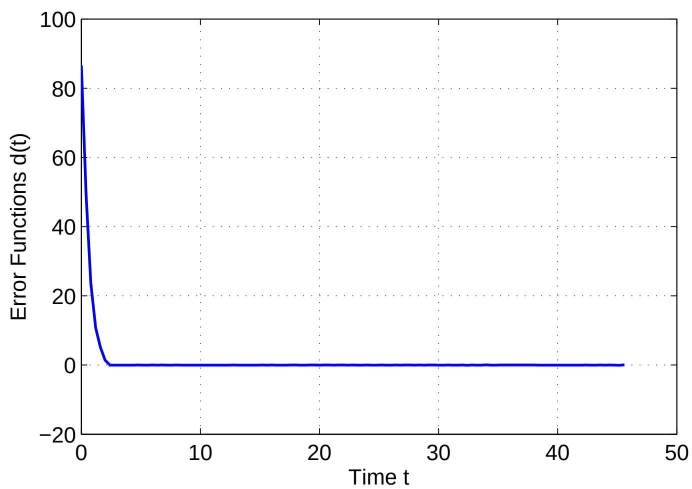

Fig. 4.   Formation disagreement of multi-agent system with switching trajectories