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LEHRSTUHL FÜR FINANZMANAGEMENT UND KAPITALMÄRKTE

# Multifactor Capital Asset Pricing Models

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*"The moral justification of capitalism lies in the fact that it is the only system consonant with man's rational nature, that it protects man's survival qua man, and that its ruling principle is: justice."*

Ayn Rand, Capitalism: The Unknown Ideal

# *Abstract*

This thesis consists of three studies on asset pricing. In the first study international bank stock returns from July 1991 to June 2011 are analysed within the Fama-French framework. In the US, Europe and Japan banks appear to have a higher exposure to market risk with increasing market value of equity and even after controlling for the standard risk factors of the Fama-French-Carhart model the risk-adjusted bank stock returns are still highly correlated in all three regions indicating a bank specific industry effect. In the second study the risk factors of the Fama-French model are analysed towards their interdependencies with default and disaster risk. It is shown that the size and value factor can to a large part be explained by default and disaster risk and that these factors can thus be seen as proxies for these two kinds of fundamental risk. In the third study a model incorporating rational inattention into asset pricing is developed. With the help of the model the traditional notion of market efficiency is challenged by a concept of attention driven market efficiency. Further, the model offers a micro-level explanation of Carhart's momentum risk factor within a rational agent framework and is able to portray the dynamics of asset prices during long-term shifts.

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*For my father.*

Chapter 1

# The Theory of Asset Pricing

## 1.1 Introduction

Asset pricing theory deals with the understanding of the prices or values of claims to uncertain future cash flows. This thesis encompasses three different studies on the theory and empirics of asset pricing after a short general introduction to this area of research.

The studies are ordered from only being empirical to being mostly theoretical in nature, starting with a quasi descriptive study of international bank stock returns within the Fama-French framework. In the second study it is shown that the size and the value factor of the Fama-French three-factor model are mostly measuring compensation for default and disaster risk. In the third and final study a new asset pricing model based on the concepts of rational inattention and the overlapping generations framework is developed and its implications towards market efficiency and momentum trading are pointed out.

## 1.2 An Overview of Asset Pricing

To understand asset pricing it is necessary to first understand the concept of market efficiency. The main implication of market efficiency is the absence of arbitrage opportunities due to competitive trading. Every possibility to generate a risk free profit should vanish since all market participants would do the corresponding trade and the prices would converge to a state where no arbitrage opportunities exist any longer. In an efficient market in its equilibrium state thus all information is included in the price of an asset.

Ross (1978) and Harrison and Kreps (1979) show that under the absence of arbitrage opportunities the price of any asset $i$ is the weighted sum of future payoffs weighted by their state probability and a state dependent discount factor:

$$p_{i,t} = \sum_{s} \Pi_{t+1}(s)\, m_{t+1}(s)\, x_{i,t+1}(s) \tag{1.1}$$

$\Pi_{t+1}(\cdot)$ represents the state probability, $m_{t+1}(\cdot)$ the state dependent discount factor and $x_{i,t+1}(\cdot) = p_{i,t+1}(\cdot) + d_{i,t+1}(\cdot)$ the next period payoff consisting of price and dividend.

Generalizing this concept prices of any asset $i$ are given by:[1]

$$p_{i,t} = \mathbb{E}\left[m_{t+1} x_{i,t+1}\right] \tag{1.2}$$

with $m_{t+1} = f\left(data, parameters\right)$ as the stochastic (state dependent) discount factor, which in general can be a function of data and other parameters.

Inserting $x_{i,t+1} = p_{i,t+1} + d_{i,t+1}$ iteratively equation (1.2) shows that the price of an asset is given by the expected discounted value of its future dividends.

[1] Cochrane (2001), p. xv

From these basic concepts three main branches of research can be derived. The first and second deal with the construction and interpretation of the stochastic discount factor, the third is purely empirical.

### 1.2.1 Consumption CAPM

The first branch of research on asset pricing is built on the rational investor assumption. The idea behind this concept is to link the investors preference or utility to the asset price by the means of the stochastic discount factor. Maybe the simplest model to incorporate this link is the CCAPM as developed by Merton (1973), Lucas (1978), and Breeden (1979).

From the Euler equation of macroeconomic consumption theory the investors preferences can be described as:

$$p_{i,t} = \mathbb{E}_t \left[ \beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} x_{i,t+1} \right] \tag{1.3}$$

$u\left(\cdot\right)$ is the investors utility function, $c_t$ the investors period $t$ consumption, $x_{i,t+1}$ the assets next period payoff, and $\beta$ the investors inter-temporal rate of substitution.

This equation is similar to equation (1.2) as it describes the value of an asset as expected discounted future payoffs and setting

$$m_{t+1} = \beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} \tag{1.4}$$

is indeed a valid choice for the stochastic discount factor.

The CCAPM has been quantitatively studied in great detail for example by Grossman and Shiller (1981), Hansen and Singleton (1983), and Hansen and Jagannathan (1997) and the basic model has been improved and modified extensively since its invention to improve its performance when fitting it to data and to loosen some of the underlying assumptions.

The first suggested addition was the separation of the risk aversion and inter temporal substitution preferences, which are assumed to be driven by the same source in the basic model, see Kreps and Porteus (1978) and Epstein and Zin (1989).

The next enhancement of the model was modifying the utility function altogether and to make it not only dependent on the absolute value of consumption but also on changes in consumption, see Sundaresan (1989), Constantinides (1990), Abel (1990), and Campbell and Cochrane (1999).

Going even further in a next step the concept of heterogeneity in investor preferences was added to the model. In particular, if investors have different attitudes toward risk, the stochastic discount factor will be influenced not just by aggregate consumption but also by its distribution, see Malloy et al. (2009).

The last big addition is including extreme tail risk events in the model. This approach introduced by Gabaix (2012) is especially important in the context of this thesis, since the disaster risk factor developed in chapter 3 is based on the model of Gabaix.

### 1.2.2 Behavioural Finance

Although the third essay in this thesis can be attributed to the domain of behavioural models or at least represents a cross-over between a behavioural model and a rational agent model, it does not fit in the second branch of asset pricing research, behavioural finance, since it is, due to its quantitative nature, completely different from the research normally counted as belonging to this field.

Behavioural finance tries to explain the dynamics of the stochastic discount factor by the psychological behaviour of the investors and claims to be able to describe empirical phenomenons, which can not be explained by the standard rational investor assumption. This branch of research dates back to Shiller (1981) and Shiller (1984). In these papers Shiller makes the points that the lack of short-term price predictability does not rule out irrational investors, that investors overreact to plausible reasoning, meaning for example convincing story telling, even in the absence of empirical evidence, and that irrational investors can cause mispricing in the short run due to the funding limitations of the rational investors.

From these beginnings a huge amount of studies about behavioural finance was conducted. An excellent overview of behavioural finance and its application to asset pricing is given in Barberis and Thaler (2003).

One of the key issues addressed by behavioural finance is the limit of arbitrage, meaning that rational investors might not be able to trade against market mispricing because of capital limitations, see Miller (1977), DeLong et al. (1990a), and DeLong et al. (1990b). Empirical evidence in favour of this notion are for example differences in prices of twin shares, see Froot and Dabora (1999), and jumps in share prices after index inclusion due to investments of index funds, see Harris and Gurel (1986) and Shleifer (1986).

To understand more of the underlying issues, which cause irrational traders to deviate from fundamental values, one can turn to cognitive psychology. Barberis and Thaler (2003) list the following key components as reasons for irrational investment behaviour: Overconfidence, meaning overconfidence in the precision of one's personal judgement, optimism and wishful thinking, meaning the display of an unrealistically positive view of one's own ability, representativeness, meaning the use of representative heuristics for statistical inference, conservatism, meaning an overrating of the base rate as compared to sample data, believes perseverance, meaning the clinging to a once formed opinion, anchoring, meaning the forming of an estimate from a more or less arbitrary initial guess, and availability biases, meaning the overweighting of more recent and more salient events.

Another important reason for seemingly irrational behaviour can be found with preferences, which are not resembled in the expected utility framework. Some of the better known extensions are weighted utility theory, see Chew and MacCrimmon (1979), disappointment aversion, see Gul (1991), and regret theory, see Bell (1982).

One of the most recent findings from this branch of research is the concept of rational bubble riding. Brunnermeier and Nagel (2004) show that sophisticated investors like hedge funds rather invest in an overpriced asset than shortening it to profit from an initial increase in mispricing.

### 1.2.3 Empirical Studies

The foundations of the empirical branch of research in asset pricing are Markowitz's concept of systematic risk and the CAPM. Markowitz argues that only systematic risk drives stock returns. Systematic risk is the risk that cannot be eliminated by holding a well diversified portfolio.[2] The CAPM on the other hand, in its most basic form developed by Sharpe (1964), Lintner (1965), and Mossin (1966), connects expected individual asset returns with a measure for systematic risk:

$$\mathbb{E}\left[r_i\right] = r_f + \beta_i \left(\mathbb{E}\left[r_m\right] - r_f\right) \tag{1.5}$$

[2] Markowitz (1959), p. 5-7

$r_i$ is the return of the asset, $r_f$ is the risk free rate, and $r_m$ is the market return. How much exposure to systematic risk an asset carries is determined by the $\beta_i$ coefficient.

In many subsequent studies the problem of the CAPM to deal with cross-sectional differences in returns have been pointed out, most prominently earnings to price ratio by Basu (1977), book-to-market ratio by Stattman (1980) and Rosenberg et al. (1985), and finally market capitalization, meaning smaller firms have higher expected returns than larger firms, by Banz (1981).

These and other CAPM anomalies were summarized by Fama and French (1992) and in Fama and French (1993) an augmented CAPM was presented, specifically the Fama and French three-factor model:

$$r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it} \tag{1.6}$$

The investors expected excess return over the risk free rate of an asset is depending on its exposure to the three risk factors excess market return ($mkt_t$), size ($smb_t$), and value ($hml_t$) given by the three corresponding coefficients. Perhaps the main weakness of this three-factor model is the missing underlying reason for why these two additional risk factors should be priced by the market.

The Fama and French model can be further augmented by a fourth momentum factor ($mom_t$) as suggested by Carhart (1997):

$$r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it} \tag{1.7}$$

This factor is based on the observations of Jegadeesh and Titman (1993) that a portfolio long stocks, which performed well in the last three to twelve months, and short stocks, which performed bad, generates significant excess returns. This observation is consistent with the idea of information entering the market step by step, which it should in accordance with information efficient markets not do.

Since these new empirically based risk factors have been introduced many possible explanations for how they fit in fundamental economic theory have been suggested and a lot of additional factors to augment the model have been invented. This overview will conclude with some of these possible explanations and possible additional factors.

Already in Fama and French (1993) the suggestion was made that the size and value factor proxy more fundamental risk factors, for which investors demand an additional compensation when bearing these risks. In this line of reasoning Vassalou and Xing (2004) showed that the size effect can be seen to a large part as a default risk effect and that the value effect has some degree of dependency on default risk as well.

The alternative to explaining these risk factors by fundamental sources of risk is explaining them by their ability to capture mispricing and irrationality in the sense of the behavioural research branch discussed earlier. These explanations are especially popular with the value and the momentum factor.

Lakonishok et al. (1994) argue that the excess return of the *hml* portfolio is due to undervaluation of the value stocks with high book-to-market ratio and overvaluation of the growth stocks with low book-to-market ratio. The subsequent outperformance of the value stock is thus induced by differences in sentiment of the investors and not by fundamental reasons.

For the momentum effect a greater number of possible explanations have been postulated. The most accepted behavioural explanation is the underreaction to news in the short run, which leads to the momentum effect as information is gradually incorporated into the price, and overreaction in the long run.

Among others Hong and Stein (1999) and Hong et al. (2000) study these phenomena.

Apart from the discussed risk factors size, value, and momentum a large number of additional suggestions for risk factors has been made. Among others these encompass default risk, see Vassalou and Xing (2004), liquidity, see Pastor and Stambaugh (2003), labour income, see Jagannathan and Wang (1996), growth in macroeconomic output and investment, see Cochrane (1996), volatility, see Ang et al. (2006), and the minimum variance portfolio, see Scherer (2010).

## 1.3 Placement of this Thesis

The three studies of this thesis add to different branches of research in asset pricing, which can be seen as the all encompassing fundamental question under which every study originated. In the spirit of Campbell et al. (2010), who explain what makes asset pricing such an interesting field of study—"Theorists develop models with testable predictions; empirical researchers document "puzzles"—stylized facts that fail to fit established theories—and this stimulates the development of new theories."—this thesis tries to contribute in many different ways to the research on asset pricing.

The first study, which is purely empirical in nature, adds to the long standing tradition of studies in asset pricing describing and analysing the asset returns in different regions or industries in order to identify particularities and anomalies, which can be studied later towards their theoretical implications. Especially in the aftermath of the 2008 financial crisis an indepth study of international bank stock returns appears to be a valuable empirical contribution.

The second study tries to develop a further understanding on which fundamental risk factors, proxied by the size and the value factor, are priced by the market. It continues the work of Vassalou and Xing (2004) and adds a second factor, disaster risk, which allows not only to describe the size effect as a default risk effect, but also the value effect as a combined default and disaster risk effect.

The third study makes contribution to many different subbranches of asset pricing. First, it introduces a new kind of quantitative behavioural concept to asset pricing on a brought basis, namely rational inattention. Second, since it can still be viewed as a rational investor model, it makes contributions to the concept of how information travels in the market and gives testable implications towards market efficiency. Third, it gives a rational investor micro level explanation for the momentum effect, which is consistent with the behavioural explanation of Hong and Stein (1999).

The rest of this thesis is organized as follows. Each of the chapters 2 to 4 encompasses one of these studies. All three main chapters are self contained and can be read on their own with the exception of a shared bibliography at the end of this thesis. Chapter 5 summarises the results.

Chapter 2

# International Bank Stock Returns

This essay is based on a joint research project on bank stock returns with Maximilian Overkott at the Technische Universität München and represents mostly my contributions to it.

## 2.1 Introduction

Financial firms and in particular banks are mostly excluded from empirical research in risk factor based asset pricing. To the best of our knowledge there exists no study on bank stock returns outside the US market in this field. With this paper we try to close this gap.

Building on two studies for the US market by Schuermann and Stiroh (2006) and Gandhi and Lustig (2013) we sort banks of all four major economic regions US, Europe, Japan, and Asia ex Japan by their market or their book value of equity into ten, five, or three portfolios depending on the regional sample size.

We regress the value weighted returns of these portfolios on the three risk factors introduced by Fama and French (1993), namely the market excess return, the size factor, and the value factor. Additionally we augment the model by Carhart's momentum factor.

For our whole study we use the risk factors provided by Ken French in his online database.[1] These factors were introduced and studied in great detail towards their explanatory power on an international sample in Fama and French (2012).

Like Schuermann and Stiroh (2006) and later Gandhi and Lustig (2013) we find a common risk factor, more pronounced for large banks, which is not captured by the standard risk factors of the Fama-French model in the US. In the three other regions this observation prevails, which might indicate a general bank industry risk factor.

We also find the bank specific size effect reported by Gandhi and Lustig (2013) for the US market, specifically a rising exposure to market risk with increasing bank size. Internationally this effect can also be seen in Japan and in Europe, but only when sorting by market value of equity, which raises the question whether market or book value is the right size measure for banks.

The rest of this chapter is organized as follows. Section 2.2 describes the dataset used in our study. Section 2.3 briefly repeats the Fama-French methodology. In sections 2.4, 2.5, 2.6, and 2.7 we discuss our empirical results for the US, European, Japanese and Asian ex Japan markets respectively. The conclusions are drawn in section 2.8.

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, viewed on October 18th, 2013

## 2.2 Data

As we want to take a comprehensive look at worldwide bank stock returns, our sample consists of banks belonging to the four main financial regions. These are the US, Europe[2], Japan and Asia ex Japan[3]. We obtain our data from Thomson Reuters Datastream/Worldscope and firms with an ICB code of 8355 are labeled as banks. This means we restrict our sample to commercial banks and exclude other financial service firms like asset managers, brokers or investment banks. A firm is admitted to the data sample for each year, in which market value of equity and monthly return data for the twelve months from July to June are available as well as the book value needed for the portfolio constructions on June 30th.

We apply several screens proposed by Ince and Porter (2006) and Schmidt et al. (2011) to take care of the data problems concerning Datastream's raw return data first brought up by Ince and Porter (2006).

The return data is taken from Datastream, accounting data from Worldscope. All data is expressed in USD. Our sample period has a range of 20 years from July 1991 to June 2011.

[2] The Eurozone, the UK and Switzerland.

[3] Hong Kong, Indonesia, Malaysia, Singapore, South Korea, Taiwan, Thailand, and Vietnam. China is omitted given its quasi state-owned banking system, see Walter and Howie (2011).

### 2.2.1 Descriptive Statistics

**Table 2.1:** Number of Banks per Year

| Year | US | EUR | JAP | AeJ |
|---|---|---|---|---|
| 1991 | 231 | 101 | 83 | 13 |
| 1992 | 300 | 115 | 83 | 16 |
| 1993 | 323 | 125 | 85 | 16 |
| 1994 | 374 | 136 | 92 | 27 |
| 1995 | 460 | 137 | 93 | 30 |
| 1996 | 531 | 145 | 93 | 41 |
| 1997 | 597 | 165 | 92 | 41 |
| 1998 | 680 | 155 | 91 | 36 |
| 1999 | 758 | 153 | 90 | 36 |
| 2000 | 790 | 152 | 85 | 44 |
| 2001 | 838 | 146 | 83 | 47 |
| 2002 | 842 | 139 | 85 | 48 |
| 2003 | 829 | 133 | 90 | 51 |
| 2004 | 786 | 136 | 89 | 51 |
| 2005 | 774 | 134 | 90 | 50 |
| 2006 | 762 | 134 | 88 | 53 |
| 2007 | 742 | 129 | 91 | 52 |
| 2008 | 739 | 120 | 93 | 49 |
| 2009 | 718 | 118 | 92 | 49 |
| 2010 | 649 | 108 | 91 | 48 |

Table 2.1 shows the number of banks in our sample for each of the four regions, specifically the United States of America (US), Europe (EUR), Japan (JAP), Asia ex Japan (AeJ) for July of each year from 1991 to 2010. The underlying database is Thomson Reuters Datastream/Worldscope and the data is filtered as proposed by Ince and Porter (2006) and Schmidt et al. (2011). All companies with return and market value data for the whole year as well as the book value needed for the portfolio construction on the 30th of June are included in our sample.

Table 2.1 shows the number of banks for each region from July 1991 to June 2011. The most interesting finding is the comparatively high turnover in the US. Unlike all other regions the US banking industry seems to have low entry costs for a new bank business on the one hand and on the other hand it is not unusual for a bank to go bankrupt or to be taken over. The minimum number of banks is observed in the first year, 1991, with 231. The highest number is found in 2002 with 842. The year 2002 furthermore represents a turning point in the trend of the number of listed banks in the US. Up to this year every year has seen an increase, which then turns into a steady decline.

This may be explained by the following reasons. First, the burst of the dot-com bubble in 2000/2001 has to be named, where a lot of small and highly leveraged firms struggled or went bankrupt. Second, the short crisis caused by the terrorist attacks of 9/11. In the following years the economy recovered, but the number of listed banks still declined. The last big drop occurs in the next to last year of the sample period. From 2009 to 2010 about ten percent of the banks are delisted. The reason for that can only be the 2008 financial crisis, which was caused by a bubble in the US housing market. Especially smaller banks had problems refinancing the toxic assets they had on their balance sheet. But also the big players of the US financial industry struggled to cope with the loses of their subprime mortgages businesses and asked the US government for financial assistance.

While Fanny Mae and Freddy Mac have been bailed out, Lehman Brothers had to file for bankruptcy under chapter 11. This has caused a worldwide fall in equity prices, immense distrust among banks, liquidity and loan shortfalls and hence a severe recession.

Due to these developments another weak point of the global financial system has been disclosed, namely the discrepancies in solvency among the member countries of the European Monetary Union. While the northern countries like Germany have been able to refinance their sovereign debt at very low interest rates, the southern countries like Greece, Italy, Portugal, and Spain have been sanctioned by the markets for their lack of budgetary discipline and weak labour markets. Many banks active in these countries or big borrowers to these countries' governments and firms have gotten into serious troubles as bond prices severely declined. Still under the impression of the consequences of the Lehman Brothers bankruptcy the European governments decided to bail out the banks, which were most affected by the sovereign debt crisis.[4]

Looking at the number of listed banks in Europe, Table 2.1 shows that the increase in the number of banks at the beginning of our sample period is not comparable with the one in the US. Moreover, the peak was already reached in 1997 with 165 banks. Since then there has been a steady decline. In 2010 the number of listed banks nearly equals the one of 1991 again. This is not the case in the US where in 2010 still about three times more banks exist as compared to 1991. We conclude that the variation in the number of banks is far higher in the US than in Europe.

Turning to Japan we observe a nearly constant amount of 90 banks over the whole sample period.

In Asia ex Japan one can observe a sharp increase in the number of listed banks from 13 banks in 1991 to 41 banks in 1997 most likely due to the high growth rates in the emerging economies of Asia during this time. After a small drop probably caused by the Asia crisis of the late 1990's an almost constant number of around 50 listed banks remains.

[4] An exhaustive study of financial crises can for example be found in Reinhart and Rogoff (2009).

**Table 2.2:** Summary Statistics US

| Descriptive Statistics of the Factor Portfolios in the US | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | Pearson Correlations | | | | |
| Factor | Mean | Std. Dev. | *mkt* | *smb* | *hml* | *mom* | *banks* |
| *mkt* | 0.55 | 4.42 | 1 | | | | |
| *smb* | 0.26 | 3.50 | 0.21 | 1 | | | |
| *hml* | 0.34 | 3.38 | -0.26 | -0.35 | 1 | | |
| *mom* | 0.69 | 5.29 | -0.11 | 0.25 | -0.28 | 1 | |
| *banks* | 0.90 | 6.58 | 0.67 | -0.07 | 0.28 | -0.35 | 1 |

Table 2.2 shows the means (first column) and standard deviations (second column) of the four US risk factors $mkt, smb, hml$, and $mom$ and of the market portfolio of banks weighted by their market value of equity. The rest of the table consists of the correlation matrix of these factors. The return data is taken from Ken French's online database and Datastream. The sample period is July 1991 to June 2011.

After having analysed the number of banks in our regional samples we now turn to the descriptive statistics of our bank portfolio returns and regional Fama-French factors. Table 2.2 shows their means and standard deviations as well as their correlation matrix for the US. The market value weighted bank portfolio outperforms the market by 0.35% per month. This premium goes along with an increased standard deviation by a multiple of about 1.5 as compared to the market factor. All four risk factors have a positive premium in the US for our sample period with the momentum factor being the frontrunner with 0.69% per month. The bank portfolio has a highly positive correlation of 0.67 with the market factor and a smaller correlation of 0.28 with the value factor. There exists almost no correlation with the size factor. The negative correlation of -0.35 with the momentum factor will be discussed in more detail in section 2.4.

**Table 2.3:** Summary Statistics Europe

| Descriptive Statistics of the Factor Portfolios in Europe | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | Pearson Correlations | | | | |
| Factor | Mean | Std. Dev. | *mkt* | *smb* | *hml* | *mom* | *banks* |
| *mkt* | 0.61 | 4.95 | 1 | | | | |
| *smb* | -0.04 | 2.35 | -0.16 | 1 | | | |
| *hml* | 0.52 | 2.42 | 0.12 | -0.08 | 1 | | |
| *mom* | 0.99 | 4.27 | -0.30 | 0.10 | -0.26 | 1 | |
| *banks* | 0.73 | 6.42 | 0.76 | -0.30 | 0.33 | -0.45 | 1 |

Table 2.3 shows the means (first column) and standard deviations (second column) of the four European risk factors $mkt, smb, hml$, and $mom$ and of the market portfolio of banks weighted by their market value of equity. The rest of the table consists of the correlation matrix of these factors. The return data is taken from Ken French's online database and Datastream. The sample period is July 1991 to June 2011.

The corresponding statistics for Europe are depicted in table 2.3. There banks likewise outperform the market factor, but not to the same extent as in the US. The premiums for the value and the momentum factors on the other hand are more pronounced in Europe with the later reaching almost one percent. While there exists a size premium in the US this holds not true for Europe, where the average return of the *smb* portfolio is even negative with -0.04%. In comparison to the US the correlation structure of the bank portfolio has one main difference. Specifically, the correlation of -0.30 with the size factor is highly negative. The correlation with the other factors is almost the same as in the US. European banks also appear to behave more like value than growth stocks and more counter-cyclically, which can be seen from the positive correlation to the value factor and the negative correlation to the momentum factor respectively.

**Table 2.4:** Summary Statistics Japan

| Descriptive Statistics of the Factor Portfolios in Japan | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | Pearson Correlations | | | | |
| Factor | Mean | Std. Dev. | *mkt* | *smb* | *hml* | *mom* | *banks* |
| *mkt* | -0.07 | 5.95 | 1 | | | | |
| *smb* | -0.08 | 3.34 | 0.11 | 1 | | | |
| *hml* | 0.48 | 2.95 | -0.19 | 0.07 | 1 | | |
| *mom* | 0.14 | 4.66 | -0.18 | -0.15 | -0.22 | 1 | |
| *banks* | -0.28 | 6.54 | 0.68 | 0.03 | 0.01 | -0.24 | 1 |

Table 2.4 shows the means (first column) and standard deviations (second column) of the four Japanese risk factors $mkt, smb, hml$, and $mom$ and of the market portfolio of banks weighted by their market value of equity. The rest of the table consists of the correlation matrix of these factors. The return data is taken from Ken French's online database and Datastream. The sample period is July 1991 to June 2011.

In Japan banks in general perform poorly. Table 2.4 shows that while the market stays more or less constant over our sample period, the bank portfolio loses 0.28% per month. This is most likely caused by the consequences of the Japan and Asia crisis in which banks suffered the most, see Fujii and Kawai (2010). Also in Japan, like in the US, no correlation with the size factor exists. The correlation with the momentum factor is substantially lower as compared to the US and especially as compared to Europe.

The particularities of the momentum effect in Japan have already been pointed out by Fama and French (2012) and Asness et al. (2013). Hanauer (2013) argues that one has to condition the momentum premium in Japan on the overall stock market state. If the market state remains unchanged then the Japanese momentum premium is also significantly positive. Additionally, Japanese banks as a whole are not correlated to the value factor.

**Table 2.5:** Summary Statistics Asia ex Japan

| Descriptive Statistics of the Factor Portfolios in Asia ex Japan | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | Pearson Correlations | | | | |
| Factor | Mean | Std. Dev. | *mkt* | *smb* | *hml* | *mom* | *banks* |
| *mkt* | 0.82 | 6.17 | 1 | | | | |
| *smb* | -0.18 | 3.11 | 0.04 | 1 | | | |
| *hml* | 0.62 | 3.31 | 0.14 | -0.06 | 1 | | |
| *mom* | 0.67 | 4.85 | -0.24 | 0.05 | -0.34 | 1 | |
| *banks* | 0.84 | 6.82 | 0.82 | 0.00 | 0.29 | -0.33 | 1 |

Table 2.5 shows the means (first column) and standard deviations (second column) of the four Asian ex Japan risk factors $mkt, smb, hml$, and $mom$ and of the market portfolio of banks weighted by their market value of equity. The rest of the table consists of the correlation matrix of these factors. The return data is taken from Ken French's online database and Datastream. The sample period is July 1991 to June 2011.

As depicted in table 2.5 the Asian ex Japan banks have almost the same average monthly return as the market and they are also highly correlated with it. Moreover, they are independent from the size factor, which has a negative premium as in Europe. Like in the US and Europe bank stock returns in Asia ex Japan are positively correlated with the value factor and negatively correlated with the momentum factor.

## 2.3 Methodology

In our study we use the regional specific risk factors as provided by Ken French in his online data base. In this section we therefore present the factor construction technique used by Fama and French (2012). All returns are denominated in USD.

$Mkt$ is the excess return of the market, meaning the market value weighted return of all stocks in a sample, over the risk-free rate $r_f$. For the risk free rate Fama and French (2012) take the one-month Treasury Bill rate from Ibotson Associates.

At the end of June of each year $y$, all stocks of a region are sorted into two size groups, big $b$ and small $s$, and three book-to-market groups, high $h$, medium $m$, and low $l$. The top 90% of the aggregated market capitalization at the end of June of year $y$ is taken as the size breakpoint. The book-to-market ratio is calculated as the book value at the fiscal year end of year $y$ divided by the market capitalization at the end of December of year $y-1$. The breakpoints for the book-to-market ratio are the 30th and 70th percentiles.

Afterwards six portfolios ($s/h$, $s/m$, $s/l$, $b/h$, $b/m$, and $b/l$) are constructed and monthly value-weighted returns are calculated for the next twelve months starting from July of year $y$ until June of year $y+1$. The portfolios are updated every year.

From these portfolios the size ($smb$) and the value ($hml$) factor are computed as follows:

$$smb = \frac{\left(r_t^{s/l} - r_t^{b/l}\right) + \left(r_t^{s/m} - r_t^{b/m}\right) + \left(r_t^{s/h} - r_t^{b/h}\right)}{3} \tag{2.1}$$

$$hml = \frac{\left(r_t^{s/h} - r_t^{s/l}\right) + \left(r_t^{b/h} - r_t^{b/l}\right)}{2} \tag{2.2}$$

The momentum factor $mom$ is constructed in a similar manner. Each month all stocks are sorted by their cumulative performance beginning from month $t-11$ till month $t-1$. The 30th and 70th percentiles are used as breakpoints and three groups of stocks losers l, neutral n, and winners w are formed. Sorting stocks by their momentum and size attribute leads to six portfolios $s/l$, $s/n$, $s/w$, $b/l$, $b/n$, and $b/w$. The momentum factor is calculated as follows:

$$mom = \frac{\left(r_t^{s/w} - r_t^{s/l}\right) + \left(r_t^{b/w} - r_t^{b/l}\right)}{2} \tag{2.3}$$

To explain the returns of our size sorted bank stock portfolios later on we either use the Fama-French three-factor model or the Fama-French-Carhart four-factor model:

$$r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it} \tag{2.4}$$

$$r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it} \tag{2.5}$$

$r_{it} - r_{ft}$ is the excess return of portfolio i over the risk-free rate $r_f$ for month $t$ and $mkt_t$, $smb_t$, $hml_t$, and $mom_t$ denote the returns of the risk factor mimicking portfolios constructed as described above. $\beta_i$, $s_i$, $h_i$, and $m_i$ represent their respective coefficients.

## 2.4 US Bank Stocks

First, we look at US bank stocks. We distinguish on the one hand between portfolios sorted by book or market value of equity and on the other hand between regressions on the three-factor Fama-French or four-factor Fama-French-Carhart model, as pointed out in the previous section. The separate analyses of book and market value of equity sorted portfolios are performed because market values are also subject to market beliefs on future returns while book values are a pure balance sheet measure.

Gandhi and Lustig (2013) argue in favour of book value as proposed by Berk (1995), but still do both kinds of sorting. We follow this approach in order to take a comprehensive look at the US banking industry and to also portray potential differences between the two measures. The analysis of European banks in section 2.5 will show that the market value should be the size measure of choice in that region.

### 2.4.1 Book Value

Table 2.6 shows the results of a regression on the three-factor Fama-French model for ten bank stock portfolios sorted by book value of equity in ascending order. Portfolio 1 is formed out of the smallest banks and portfolio 10 out of the biggest.

**Table 2.6:** Risk-adjusted Returns of Book Value Sorted Portfolios of US Commercial Banks I

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 0.00 | 1.66 | 3.91 | 2.98 | 1.61 | 1.80 | 1.06 | -1.37 | -0.72 | -3.87 |
| $mkt$ | 0.29*** | 0.34*** | 0.36*** | 0.37*** | 0.41*** | 0.44*** | 0.47*** | 0.70*** | 0.74*** | 1.28*** |
| $smb$ | 0.19*** | 0.20*** | 0.25*** | 0.16*** | 0.23*** | 0.27*** | 0.28*** | 0.43*** | 0.31*** | -0.21*** |
| $hml$ | 0.35*** | 0.38*** | 0.44*** | 0.35*** | 0.37*** | 0.48*** | 0.42*** | 0.81*** | 0.73*** | 0.94*** |
| adj. $R^2$ | 0.20 | 0.34 | 0.36 | 0.30 | 0.37 | 0.41 | 0.37 | 0.56 | 0.51 | 0.66 |
| 1st pc | 0.28 | 0.25 | 0.25 | 0.28 | 0.28 | 0.33 | 0.34 | 0.38 | 0.37 | 0.37 |

Table 2.6 shows the results of time series regressions on the three Fama-French risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it}$. i are ten portfolios of US commercial banks sorted by book value of equity. $mkt, smb$, and $hml$ are the three Fama-French region specific risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

The resulting $\alpha$'s are clearly size dependent. The values range from -3.87% for the tenth to 3.91% for the third portfolio. The fact that these values are all not significantly different from zero shows that the Fama-French model explains large parts of bank stock returns.

Looking in more detail at the differences between small and large sized banks one observes clearly size dependent loadings on the market index. They range from 0.29 for the smallest to 1.28 for the biggest banks. For portfolios one to nine the increase is somewhat linear, but from portfolio nine to portfolio ten the regression coefficient increases by 0.54. This finding could imply that bigger banks run a bigger leverage as compared to smaller ones and that the exposure to market risk varies due to a different structure of the customer base. Gandhi and Lustig (2013) find a much lower value of 0.91 for their tenth portfolio[5], while our result is in line with the one of Schuermann and Stiroh (2006), who report a value of 1.22.

The loadings on the size factor reveal some interesting patterns too. While the negative estimator of -0.21, significant at the 1% level, for the biggest banks is not surprising, it is noticeable that we find a nearly flat structure of loadings up to the seventh portfolio and a sharp increase to 0.43 for the eighth portfolio. Therefore we can clearly reject a linear structure in the size factor, which might have been expected from the sorting by book value of equity.

This shows on the one hand that it might make a difference to sort on book or market value of equity and on the other hand that the size factor, which is built up of stocks from all industries, might not fully resemble size differences of bank stock returns. Again, our results are in line with those of Schuermann and Stiroh (2006) and somewhat different from those of Gandhi and Lustig (2013). For the biggest banks Schuermann and Stiroh (2006) get a value of -0.33 and Gandhi and Lustig (2013) a value of 0.05.

[5]Recall that Gandhi and Lustig (2013) use CRSP data and another sample period.

Looking at the value factor coefficient estimates are increasing with bank size. The bigger the bank the more it behaves like a value stock. The tenth portfolio has a regression coefficient of nearly one on the value factor while the first portfolio only has an exposure of 0.35.

It is this finding which represents the largest difference to the two related studies of US bank stock returns. Gandhi and Lustig (2013) find a more or less flat structure with loadings ranging from 0.32 to 0.42, while Schuermann and Stiroh (2006) report values only between 0.26 and 0.27 for their two bank stock portfolios. The differences could occur because of the different time periods and the fact that we use Datastream instead of CRSP data.

Overall the Fama-French three-factor model is able to explain a significant amount of the bank stock portfolios' variances. The adjusted $R^2$ measure shows values from 0.20 for the smallest and 0.66 for the biggest banks. This structure is well known with factor models as they are better capable to explain the return patterns of bigger than those of smaller companies, see Fama and French (2012).

The most meaningful insight can be gained from the analysis of the regression residuals. The first principal component of the risk-adjusted bank stock portfolio returns offers a deeper insight towards the behaviour of bank stocks. Since there is a flat structure in the loadings of the first principal component on each size portfolio one might speak of a bank specific industry effect. There exists no size dependent difference in loadings. This result confirms Gandhi and Lustig (2013) and especially Schuermann and Stiroh (2006), who were the first to explore this phenomenon.

In this context one has to keep in mind that the principal component analysis is not able to reveal economic relationships but only statistical ones. Nevertheless risk-adjusted returns of banks of all sizes are still highly correlated, which is a strong indicator towards a fundamental reason behind this observation. Hence, the work of undisclosing the underlying risk factor of the first principal component is yet to be done. The only thing we can conclude is that there exists a connection between all size groups of publicly traded banks, which increases or decreases their stock prices at the same time and might resemble a bank specific risk factor.

### 2.4.2 Market Value

After having analysed the results for our first choice size measure, namely the book value of equity[6], we now turn to our second choice, the market value of equity. The results are shown in table 2.7.

[6]Recall that Gandhi and Lustig (2013) argue in favour of the book value as this is a pure balance sheet measure and is not effected by market expectations.

**Table 2.7:** Risk-adjusted Returns of Market Value Sorted Portfolios of US Commercial Banks I

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | -0.70 | 3.45 | 3.31 | 2.40 | 1.21 | 1.56 | 0.22 | -0.88 | -0.37 | -3.97 |
| $mkt$ | 0.32*** | 0.36*** | 0.40*** | 0.44*** | 0.32** | 0.47*** | 0.61*** | 0.68*** | 0.72*** | 1.28*** |
| $smb$ | 0.23*** | 0.32*** | 0.27*** | 0.21*** | 0.15 | 0.25*** | 0.34*** | 0.48*** | 0.39*** | -0.22*** |
| $hml$ | 0.45*** | 0.40*** | 0.45*** | 0.48*** | 0.32* | 0.49*** | 0.67*** | 0.77*** | 0.77*** | 0.92*** |
| adj. $R^2$ | 0.19 | 0.32 | 0.34 | 0.30 | 0.12 | 0.43 | 0.56 | 0.56 | 0.53 | 0.66 |
| 1st pc | 0.36 | 0.32 | 0.31 | 0.37 | 0.42 | 0.29 | 0.29 | 0.26 | 0.26 | 0.25 |

Table 2.7 shows the results of time series regressions on the three Fama-French risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it}$. i are ten portfolios of US commercial banks sorted by market value of equity. $mkt, smb$, and $hml$ are the three Fama-French region specific risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

Differences as compared to table 2.6 are rather small. One can observe the same trend patterns in the risk factors, in the adjusted $R^2$ measure and also in the first principal component. We can conclude, first, that it makes no big difference sorting bank stocks by book or market value of equity in the US, which means that the ratio of book to market value of banks is almost constant within each size group and, second, that the Fama-French factors can account for a lot of the return variations, but there are still size dependent differences especially in the mean excess returns.

The similarities of the results of both analyses are due to the fact that there is a large overlap of portfolios sorted by book or market value of equity in the US, as table 2.8 expresses.[7]

**Table 2.8:** Market and Book Value Overlapping in the US

| | Book Value | | | | |
|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) |
| Market Value | Number of Year Obs. | | | | |
| 1(low) | 2052 | 451 | 40 | 9 | 0 |
| 2 | 464 | 1507 | 524 | 45 | 1 |
| 3 | 32 | 549 | 1481 | 466 | 12 |
| 4 | 3 | 30 | 491 | 1786 | 231 |
| 5(high) | 1 | 4 | 4 | 235 | 2305 |

Table 2.8 shows the overlap of book and market value of equity sorted portfolios of US bank stocks. The portrayed quantities are bank year observations, aggregated over the whole sample period which ranges from July 1991 to June 2011.

[7]Here only five size categories are used for a better comparison to tables 2.11 and 2.16 later on.

### 2.4.3 Momentum

To gain further insights we now introduce the momentum factor as an additional risk factor. As in section 2.4.1 and 2.4.2 a distinction between a sorting on the book or market value of equity doesn't yield different findings. Thus we will concentrate on the results based on the book value of equity in table 2.9. For the sake of completeness, table 2.10 shows the results when sorting by market value of equity.

**Table 2.9:** Risk-adjusted Returns of Book Value Sorted Portfolios of US Commercial Banks II

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | -0.24 | 1.61 | 4.09 | 3.25 | 1.79 | 2.00 | 0.91 | -0.88 | -0.73 | -1.81 |
| *mkt* | 0.30*** | 0.34*** | 0.35*** | 0.36*** | 0.40*** | 0.44*** | 0.47*** | 0.69*** | 0.74*** | 1.23*** |
| *smb* | 0.19*** | 0.20*** | 0.25*** | 0.17*** | 0.24*** | 0.27*** | 0.28*** | 0.44*** | 0.31*** | -0.15* |
| *hml* | 0.36*** | 0.38*** | 0.43*** | 0.34*** | 0.36*** | 0.47*** | 0.43*** | 0.79*** | 0.73*** | 0.86*** |
| *mom* | 0.02 | 0.00 | -0.02 | -0.03 | -0.02 | -0.02 | 0.01 | -0.05 | 0.00 | -0.19*** |
| adj. $R^2$ | 0.20 | 0.34 | 0.36 | 0.30 | 0.37 | 0.40 | 0.37 | 0.56 | 0.51 | 0.67 |
| 1st pc | 0.28 | 0.25 | 0.26 | 0.28 | 0.28 | 0.33 | 0.35 | 0.37 | 0.37 | 0.36 |

Table 2.9 shows the results of time series regressions on the four Fama-French-Carhart risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it}$. i are ten portfolios of US commercial banks sorted by book value of equity. $mkt, smb, hml$, and $mom$ are the four Carhart risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

**Table 2.10:** Risk-adjusted Returns of Market Value Sorted Portfolios of US Commercial Banks II

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | -0.75 | 3.48 | 3.46 | 2.24 | 1.01 | 1.75 | 0.50 | -0.58 | -0.22 | -1.94 |
| $mkt$ | 0.32*** | 0.36*** | 0.40*** | 0.44*** | 0.33** | 0.46*** | 0.60*** | 0.67*** | 0.72*** | 1.23*** |
| $smb$ | 0.23*** | 0.32*** | 0.28*** | 0.21*** | 0.15 | 0.25*** | 0.35*** | 0.49*** | 0.39*** | -0.16* |
| $hml$ | 0.45*** | 0.40*** | 0.45*** | 0.48*** | 0.33* | 0.48*** | 0.66*** | 0.76*** | 0.76*** | 0.84*** |
| $mom$ | 0.01 | 0.00 | -0.01 | 0.01 | 0.02 | -0.02 | -0.03 | -0.03 | -0.01 | -0.19*** |
| adj. $R^2$ | 0.19 | 0.31 | 0.34 | 0.30 | 0.12 | 0.43 | 0.56 | 0.56 | 0.53 | 0.67 |
| 1st pc | 0.36 | 0.32 | 0.31 | 0.37 | 0.42 | 0.29 | 0.29 | 0.26 | 0.26 | 0.24 |

Table 2.10 shows the results of time series regressions on the four Fama-French-Carhart risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it}$. i are ten portfolios of US commercial banks sorted by market value of equity. $mkt, smb, hml$, and $mom$ are the four Carhart risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% levels respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. Estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component (1st pc) extracted from the residuals of all regressions.

At first glance the momentum factor does not seem to have a significant impact on bank stock performance. The loadings of portfolios one to nine are virtually zero. Only the biggest banks have a significantly negative exposure to this risk factor. This means that they behave more like stocks that had a poor performance in the last few months, which was already suggested in table 2.2. Since momentum gains the highest returns in bull markets and suffers substantial losses during transitions between market states, see Asem and Tian (2010), this negative relationship gives a first hint towards a possible crisis insurance in the returns of the biggest banks in the US.

The estimators for the other risk factors hardly change, except for the loading of the tenth portfolio on the size factor, which becomes more negative and significant. The adjusted $R^2$ measure of the tenth portfolio is only slightly increased to 0.67. Gandhi and Lustig (2013) do not control for the momentum factor. However, in their working paper, Gandhi and Lustig (2011), they show that a portfolio long the biggest and short the smallest banks has a highly significantly negative exposure of -0.27 to this risk factor. Hence, our results are in line with their preliminar findings.

Schuermann and Stiroh (2006) do not look at momentum at all.

The first region we want to compare with the US is Europe. This is done in the following section.

## 2.5 Europe

For European banks we want to distinguish again between a portfolio sorting based on the book and the market value of equity. Europe will be the only region where this distinction leads to absolutely different findings. We will first start with the results based on the book value of equity and then turn to the market value, as we have done already in the previous chapter.

**Table 2.11:** Market and Book Value Overlapping in Europe

| | Book Value | | | | |
|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) |
| Market Value | Number of Year Obs. | | | | |
| 1(low) | 65 | 116 | 185 | 172 | 5 |
| 2 | 129 | 103 | 182 | 111 | 8 |
| 3 | 128 | 131 | 90 | 164 | 20 |
| 4 | 133 | 86 | 42 | 85 | 187 |
| 5(high) | 88 | 97 | 34 | 1 | 319 |

Table 2.11 shows the overlap of book and market value of equity sorted portfolios of European bank stocks. The portrayed quantities are bank year observations, aggregated over the whole sample period which ranges from July 1991 to June 2011.

Looking at table 2.11 it is obvious that there is huge variation in the composition of portfolios regarding the choice of sorting measure. This insight does not only hold true for the whole European cross-section but also for the majority of single countries. Especially Swiss banks stand out to have highly dispersing book-to-market ratios.

### 2.5.1 Book Value

**Table 2.12:** Risk-adjusted Returns of Book Value Sorted Portfolios of European Commercial Banks I

| i | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | -7.05* | -1.38 | -2.18 | -2.24 | -6.67* |
| *mkt* | 0.99*** | 0.89*** | 0.70*** | 0.50*** | 0.97*** |
| *smb* | -0.29* | -0.39*** | -0.52*** | 0.17** | -0.48*** |
| *hml* | 0.58*** | 0.49*** | 0.40** | 0.39*** | 0.77*** |
| adj. $R^2$ | 0.60 | 0.58 | 0.40 | 0.43 | 0.63 |
| 1st pc | 0.48 | 0.45 | 0.52 | 0.24 | 0.48 |

Table 2.12 shows the results of time series regressions on the three Fama-French risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it}$. i are five portfolios of European commercial banks sorted by book value of equity. *mkt*, *smb*, and *hml* are the three Fama-French region specific risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loading of the first principal component extracted from the residuals of all regressions.

From a first look at table 2.12 it is obvious that there are no clear structures as observed in the US, see table 2.6. The banks sorted into the first and fifth portfolio appear to have similar characteristics.[8] These two portfolios are the only ones having a significant $\alpha$ of -7.05% and -6.67% per year respectively. The first row of table 2.12 further reveals a negative excess return for all sizes of banks. None of the portfolios is able to gain a positive $\alpha$, despite the fact that the market weighted portfolio of all banks outperforms the market over the whole sample period as could be seen in table 2.3.

[8]Recall that we do not sort the European banks into ten but five portfolios because of the smaller sample size.

The regression coefficients on the market factor underline the observation of a similar return process for the first and the fifth portfolio. Both have a coefficient of virtually one. Also the loadings of the other banks defile common intuition. While one would expect an increase, the opposite is true: Values drop from 0.89 for the second to 0.50 for the fourth group.

Even more surprising are the regression coefficients on the size factor. The smallest banks, based on their book value of equity, have a negative regression coefficient of -0.29 on it. This highlights once again the uniqueness of the European banking system with its high dispersion between book and market value of equity. Furthermore, all estimators except the one of portfolio four are negative. This shows that a sorting based on the book value of equity for European banks does not reveal the return structures, which are familiar from looking at the US bank market. The book value of European banks seems to be fully independent of the respective market value and it is doubtful that it is the size measure of choice.

On the other hand the loadings on the *hml* portfolio do not reveal such unusual patterns. All portfolios exhibit a significantly positive coefficient whereas the fifth portfolio behaves the most like a portfolio of value stocks. This is not surprising as these banks have the highest book values of equity and the value factor is the return difference between stocks with a high and with a low book-to-market ratio.

As with US banks the loadings of the first principal component, extracted from all residuals, highlight the potential of a common additional bank specific risk factor. All values lie in the range of 0.45 to 0.52 with the fourth portfolio being the exception with a loading of only 0.24.

### 2.5.2 Market Value

**Table 2.13:** Risk-adjusted Returns of Market Value Sorted Portfolios of European Commercial Banks I

| i | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | 2.05 | 0.96 | -0.72 | -4.62 | -5.41 |
| *mkt* | 0.41*** | 0.45*** | 0.53*** | 0.78*** | 0.95*** |
| *smb* | 0.27*** | 0.29*** | 0.20*** | 0.00 | -0.55*** |
| *hml* | 0.39*** | 0.41*** | 0.46*** | 0.61*** | 0.63*** |
| adj. $R^2$ | 0.32 | 0.36 | 0.51 | 0.57 | 0.65 |
| 1st pc | 0.32 | 0.41 | 0.37 | 0.55 | 0.54 |

This table shows the results of time series regressions on the three Fama-French risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it}$. i are five portfolios of European commercial banks sorted by market value of equity. $mkt, smb$, and $hml$ are the three Fama-French region specific risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loading of the first principal component extracted from the residuals of all regressions.

After having analysed the results based on sorting the banks by their book value of equity we now turn to the results based on the respective market value. This is shown in table 2.13. Having seen quite surprising figures in the last paragraph, this analysis will guide us back to more expected ones. The $\alpha$'s show a descending order from 2.05% to -5.41% per year. The first two portfolios offer a positive, the last three a negative risk-adjusted return, which means that there exists a risk premium of more than seven percent for the smallest banks as compared to their bigger counterparts. These results go along with increasing regression coefficients on the market and the value factor. Recall that a sorting with respect to the book value of equity revealed strong similarities of the biggest and smallest banks and a v-formed structure of loadings on the *mkt* and the *hml* portfolio.

The estimators for the size factor follow an inverse relationship, as could be expected. The smallest banks load significantly positive with a coefficient of 0.27 and the biggest significantly negative with a coefficient of -0.55.

As in the US the three-factor Fama-French model is capable to explain bank returns in a reasonable fashion in Europe. The adjusted $R^2$ measures range from 0.32 for the first to 0.65 for the last portfolio.

A very interesting result lies in the first principal component of the regressions' residuals. In the US, see table 2.6, this component loads equivalently on all size portfolios. This finding is not the same for Europe. The first principal component loads still positively on all portfolios, but the magnitude of each loading is clearly linked to the size of the banks in the respective portfolio. While the loadings on the first three portfolios are found to lie between 0.32 and 0.41 a significant jump to values above 0.50 occurs for portfolios four and five. Nevertheless risk-adjusted returns are still correlated over all bank sizes and we can still speak of a bank industry specific factor represented by the first principal component of residuals.

### 2.5.3 Momentum

Like in section 2.4.3 we will now introduce the momentum factor as an additional risk factor. In contrast to the US where this factor hardly plays an economically important role, except for the biggest banks, it does so in Europe. The influence is strongly visible with both of our sorting measures. Table 2.3 has already shown that the bank portfolio has a negative correlation with the European momentum factor. We will first look at the impact of the momentum factor on the sorting based on the book value of equity and then turn to the sorting based on the market value.

**Table 2.14:** Risk-adjusted Returns of Book Value Sorted Portfolios of European Commercial Banks II

| i | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | -2.21 | 1.18 | 2.51 | -1.00 | -0.62 |
| *mkt* | 0.92*** | 0.85*** | 0.63*** | 0.49*** | 0.88*** |
| *smb* | -0.27 | -0.38** | -0.49*** | 0.18** | -0.45*** |
| *hml* | 0.46** | 0.43*** | 0.29* | 0.36*** | 0.62*** |
| *mom* | -0.30*** | -0.16 | -0.29*** | -0.08 | -0.38*** |
| adj. $R^2$ | 0.62 | 0.58 | 0.43 | 0.43 | 0.67 |
| 1st pc | 0.48 | 0.46 | 0.52 | 0.25 | 0.47 |

Table 2.14 shows the results of time series regressions on the four Fama-French-Carhart risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it}$. i are five portfolios of European commercial banks sorted by book value of equity. $mkt, smb, hml$, and $mom$ are the four Carhart risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

The first row of table 2.14 reveals a big increase in the $\alpha$'s, especially for the smallest and biggest banks. Values increase from -7.05% to -2.21% and from -6.67% to -0.62% respectively and become insignificant. This change is almost exclusively caused by the estimated loadings on the momentum factor as all other coefficients hardly change in comparison to table 2.12. All sizes of banks load negatively on the momentum factor, especially portfolios one, three, and five with coefficients of -0.30, -0.29, and -0.38, significant at the 1%

level. Hence, the similarities of large and small banks classified by their book value of equity prevail with the momentum factor.

Adding the momentum factor does not induce any other changes in the loadings and the interpretation of the first principal component. Values stay rather constant compared to table 2.12. Due to the significant regression coefficients of the momentum factor the adjusted $R^2$ measures improve slightly for all portfolios.

Overall table 2.14 reveals an important role of momentum in European bank stock returns by explaining a large fraction of the partly significant abnormal returns especially of portfolios one and five.

**Table 2.15:** Risk-adjusted Returns of Market Value Sorted Portfolios of European Commercial Banks II

| i | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | 2.07 | -0.26 | 0.02 | -1.92 | -0.28 |
| *mkt* | 0.41*** | 0.47*** | 0.52*** | 0.75*** | 0.88*** |
| *smb* | 0.27*** | 0.28*** | 0.20*** | 0.01 | -0.53*** |
| *hml* | 0.39*** | 0.44*** | 0.45*** | 0.55*** | 0.50*** |
| *mom* | 0.00 | 0.08 | -0.05 | -0.17** | -0.32*** |
| adj. $R^2$ | 0.31 | 0.36 | 0.51 | 0.58 | 0.68 |
| 1st pc | 0.33 | 0.44 | 0.37 | 0.54 | 0.52 |

Table 2.15 shows the results of time series regressions on the four Fama-French-Carhart risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it}$. i are five portfolios of European commercial banks sorted by market value of equity. $mkt, smb, hml$, and $mom$ are the four Carhart risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

Table 2.15 shows the results for the same analysis but based on portfolios sorted on the market value of equity. This table underpins the fact of large banks behaving like losers in the Carhart (1997) meaning. The only significant regression coefficients can be found for the fourth and fifth portfolio, which turn out to be negative. These results further strengthen the necessity of differentiating between sorting by book and market value of equity.

Looking at changes of the other estimators one can see that while results for the size factor remain unchanged, values for the biggest banks regarding the market and value factor have declined compared to table 2.14. This shows once more the importance of the momentum factor for the portfolio of the largest banks. Adjusted $R^2$ measures and loadings of the first principal component are virtually unchanged.

At this point, although there exists no theoretical inclination for this implication, it appears as if it is the market value and not the book value of equity, which is the relevant characteristic used by the market for differentiating banks regarding their size. That in the US book value appears to work well, might only be the case because sorting by book value leads to very similar portfolios as the sorting by market value.

After the US and Europe the next region we want to analyse is Japan.

## 2.6 Japan

The analysis of Japanese banks reveals results distinct from those obtained in the US and Europe. While the main findings, like in the US and in difference to Europe, do not change whether one uses the book or alternatively the market value of equity as a size measure, a risk-adjusted premium for small banks in comparison to large banks cannot be observed. This premium is detected both in the US and in Europe when sorting banks by their market value of equity. In both regions smaller banks even show occasionally a positive $\alpha$, see tables 2.6-2.10 and tables 2.13-2.15.

In Japan banks generally perform poorly. In every regression every size sorted portfolio earns a negative risk-adjusted return. Furthermore a comparison between small and large banks shows that the larger ones earn a premium over the smaller ones. These findings could be the result of the severe economic crisis in Japan of the early 1990s and the following stagnation. The consequence has been a period of very low interest rates which has lasted until today and has jeopardized the prime revenue sources of a bank in conjunction with a decreased credit demand. This could have led investors to avoid Japanese bank stocks. A proof towards this assumption has yet to be ascertained and is not part of the scope of this study.[9]

**Table 2.16:** Market and Book Value Overlapping in Japan

| | Book Value | | | | |
|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) |
| Market Value | Number of Year Obs. | | | | |
| 1(low) | 308 | 51 | 4 | 0 | 0 |
| 2 | 41 | 214 | 86 | 5 | 4 |
| 3 | 9 | 72 | 187 | 77 | 11 |
| 4 | 5 | 11 | 65 | 220 | 49 |
| 5(high) | 0 | 2 | 14 | 48 | 296 |

Table 2.16 shows the overlap of book and market value of equity sorted portfolios of Japanese bank stocks. The portrayed quantities are bank year observations, aggregated over the whole sample period which ranges from July 1991 to June 2011.

[9]For further details on the Japanese banking industries see Fujii and Kawai (2010).

### 2.6.1 Book and Market Value

Since the choice of book or market value of equity as size measure leads to even more similar results than in the US we will focus on the results based on book value, which are shown in table 2.17. Table 2.18 presents results based on market value.

**Table 2.17:** Risk-adjusted Returns of Book Value Sorted Portfolios of Japanese Commercial Banks I

| i | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | -11.63*** | -9.50*** | -9.22*** | -6.69*** | -6.63 |
| *mkt* | 0.34*** | 0.31*** | 0.43*** | 0.42*** | 0.95*** |
| *smb* | 0.27*** | 0.20*** | 0.16** | 0.03 | -0.20 |
| *hml* | 0.18*** | 0.26*** | 0.36*** | 0.29*** | 0.33 |
| adj. $R^2$ | 0.30 | 0.22 | 0.34 | 0.32 | 0.47 |
| 1st pc | 0.30 | 0.40 | 0.42 | 0.41 | 0.63 |

This table shows the results of time series regressions on the three Fama-French risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h \cdot hml_t + \epsilon_{it}$. i are five portfolios of Japanese commercial banks sorted by book value of equity. *mkt*, *smb*, and *hml* are the three Fama-French region specific risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loading of the first principal component extracted from the residuals of all regressions.

**Table 2.18:** Risk-adjusted Returns of Market Value Sorted Portfolios of Japanese Commercial Banks I

| i | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | -9.65*** | -8.45*** | -8.25*** | -7.64*** | -6.99 |
| *mkt* | 0.36*** | 0.33*** | 0.34*** | 0.39*** | 0.95*** |
| *smb* | 0.24*** | 0.25*** | 0.17** | 0.04 | -0.19 |
| *hml* | 0.23*** | 0.29*** | 0.28*** | 0.32*** | 0.33* |
| adj. $R^2$ | 0.31 | 0.25 | 0.28 | 0.30 | 0.48 |
| 1st pc | 0.34 | 0.41 | 0.39 | 0.41 | 0.63 |

Table 2.18 shows the results of time series regressions on the three Fama-French risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it}$. $i$ are five portfolios of Japanese commercial banks sorted by market value of equity. $mkt, smb$, and $hml$ are the three Fama-French region specific risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loading of the first principal component extracted from the residuals of all regressions.

As already mentioned, the most important observation for Japan is that all kinds of banks perform poorly compared to all other regions. This conclusion is evident looking at the $\alpha$'s in table 2.17. All risk-adjusted returns are highly negative and are arranged in a decreasing order (in absolute values) from the first to the last portfolio. The smallest banks have a risk-adjusted loss of around 12% per year, the largest banks of around 7%. This structure is very surprising since in the US and in Europe small banks earn a premium as compared to large banks.

The loadings of portfolios one to four on the market factor are more or less flat ranging from 0.31 to 0.43 while the regression coefficient of the biggest banks is about double in magnitude. This is in line with what we find for the biggest banks in the US, see table 2.6. All loadings are highly significant.

When looking at the size factor one discovers a decreasing order of correlations with the last two being insignificantly different from zero. Hence, also in Japan the size factor cannot fully account for size dependent differences in bank stock returns.

Such a structure is not discovered for the value factor. All values lie in between 0.18 to 0.36. This figures are very small compared to those obtained in the US and Europe, see tables 2.6 and 2.13. There, loadings are rather monotonically increasing from portfolio one to ten or five, respectively. Furthermore, the estimators of the size and value factor of the largest banks have a large nominal value but are insignificantly different from zero. This means, that there is a lot uncertainty in the estimators.

Further noteworthy are the rather disappointing adjusted $R^2$ measures of 0.22 to 0.47. Compared to the US and Europe bank stock returns in Japan are worse explained by a regional three-factor model. This might be another reason for the big and often highly significant $\alpha$'s.[10] Explanations for this poor performance are yet to be found.

In contrast, the loadings of the first principal component are in line with our previous results for the other two regions. Also in Japan the structure is rather flat with a small peak with the biggest banks.

This means, that there should again be a common risk factor behind bank stocks' residual returns. Additionally, our results so far have shown that returns of banks located in the three main financial markets worldwide are everything else than integrated. Risk-adjusted returns are not of the same magnitude even if one uses region specific risk factors, which is generally seen

[10] If a model is not able to account reasonably for the return variation, the pricing error, which is resembled by the $\alpha$, might increase.

to be superior to using world wide factors, see Fama and French (2012). Thus there appears to exist a region specific bank risk factor.

### 2.6.2 Momentum

Like in the US and Europe we will now look at the results when additionally regressing on the momentum factor.

**Table 2.19:** Risk-adjusted Returns of Book Value Sorted Portfolios of Japanese Commercial Banks II

| i | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | -11.76*** | -9.68*** | -9.26*** | -6.60*** | -5.94 |
| *mkt* | 0.35*** | 0.32*** | 0.43*** | 0.41*** | 0.92*** |
| *smb* | 0.28*** | 0.21*** | 0.17** | 0.03 | -0.22 |
| *hml* | 0.20*** | 0.28*** | 0.37*** | 0.28*** | 0.26 |
| *mom* | 0.03 | 0.05 | 0.01 | -0.03 | -0.19 |
| adj. $R^2$ | 0.30 | 0.22 | 0.34 | 0.31 | 0.48 |
| 1st pc | 0.31 | 0.41 | 0.42 | 0.41 | 0.62 |

Table 2.19 shows the results of time series regressions on the four Fama-French-Carhart risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it}$. i are five portfolios of Japanese commercial banks sorted by book value of equity. $mkt, smb, hml$, and $mom$ are the four Carhart risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

**Table 2.20:** Risk-adjusted Returns of Market Value Sorted Portfolios of Japanese Commercial Banks II

| i | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | -9.86*** | -8.68*** | -8.33*** | -7.55*** | -6.31 |
| *mkt* | 0.37*** | 0.34*** | 0.35*** | 0.39*** | 0.92*** |
| *smb* | 0.24*** | 0.26*** | 0.17** | 0.03 | -0.22 |
| *hml* | 0.25*** | 0.31*** | 0.29*** | 0.31*** | 0.26 |
| *mom* | 0.06 | 0.06 | 0.02 | -0.02 | -0.18 |
| adj. $R^2$ | 0.32 | 0.25 | 0.27 | 0.30 | 0.48 |
| 1st pc | 0.34 | 0.41 | 0.39 | 0.41 | 0.62 |

This table shows the results of time series regressions on the four Fama-French-Carhart risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it}$. i are five portfolios of Japanese commercial banks sorted by market value of equity. $mkt, smb, hml$, and $mom$ are the four Carhart risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

As tables 2.19 and 2.20 show, adding momentum to the explanatory risk factors does not change the overall results. In both specifications, meaning a sorting based on the book value and one on the market value of equity, no loading on that factor is significant. Hence, all other regression coefficients are not affected and also the adjusted $R^2$ measures and the loadings of the first principal component stay unchanged. Nevertheless a declining trend is observable in the loadings from the smallest to the largest Japanese banks. This fact has already been detected in the US, see tables 2.9 and 2.10, and in Europe, see table 2.15.

As Hanauer (2013) explains momentum in stock returns, especially in Japan, is conditional on market dynamics. This means that a transition from one state of the market to the other shrinks momentum profits dramatically. The negative loadings of the biggest banks therefore provide an insurance against unexpected changes in the state of the economy, similar to what could be observed in the US. These banks will underperform in bullish and outperform in bearish markets.

The last region we will analyze is Asia ex Japan.

## 2.7 Asia ex Japan

In this section we want to look at bank stock returns in Asia ex Japan. In this region we will not find most of the characteristic patterns we have observed so far.

### 2.7.1 Book and Market Value

Recall that because of the small amount of banks in these countries only three portfolios are formed.

**Table 2.21:** Risk-adjusted Returns of Book and Market Value Sorted Portfolios of Asian ex Japan Commercial Banks I

| Book Value | | | | Market Value | | | |
|---|---|---|---|---|---|---|---|
| i | 1 | 2 | 3 | i | 1 | 2 | 3 |
| $\alpha$ | -9.18 | -3.87 | -4.15 | $\alpha$ | -3.97 | -9.82** | -3.08 |
| $mkt$ | 0.90*** | 0.78*** | 0.91*** | $mkt$ | 0.91*** | 0.86*** | 0.89*** |
| $smb$ | 0.12 | 0.19 | -0.10 | $smb$ | 0.25 | 0.21 | -0.08 |
| $hml$ | 0.72*** | 0.19** | 0.37*** | $hml$ | 0.83*** | 0.22** | 0.36**** |
| adj. $R^2$ | 0.45 | 0.44 | 0.70 | adj. $R^2$ | 0.33 | 0.43 | 0.71 |
| 1st pc | 0.82 | 0.51 | 0.27 | 1st pc | 0.91 | 0.40 | 0.14 |

Table 2.21 shows the results of time series regressions on the three Fama-French risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it}$. i are three portfolios of Asian ex Japan commercial banks sorted by book or market value of equity. $mkt, smb$, and $hml$ are the three Fama-French region specific risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loading of the first principal component extracted from the residuals of all regressions.

Looking at table 2.21 one cannot observe any patterns in the $\alpha$'s or loadings for Asia ex Japan. Moreover, distinguishing between sorting by book or market value of equity does not make any difference. All portfolios exhibit a negative $\alpha$ in the range of -10% to -3% per year and have a regression coefficient on the market factor of around 0.9.

The loadings on the *smb* portfolio are ordered decreasingly, but all are insignificantly different from zero. Furthermore, the regression coefficients on the *hml* portfolio indicate that in this region the smallest banks behave nearly perfectly as value stocks while the remaining two portfolios have lower regression coefficients.

The adjusted $R^2$ measures are increasing with size while the opposite is true for the loadings of the first principal component of residuals. Still the loadings of the first principal component are all positive, which means that we can still find correlated risk-adjusted returns over all three portfolios.

**Table 2.22:** Market and Book Value Overlapping in Asia ex Japan

| | Book Value | | | | |
|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) |
| Market Value | Number of Year Obs. | | | | |
| 1(low) | 143 | 20 | 6 | 1 | 0 |
| 2 | 18 | 106 | 29 | 1 | 1 |
| 3 | 9 | 24 | 86 | 34 | 4 |
| 4 | 0 | 5 | 34 | 84 | 31 |
| 5(high) | 0 | 0 | 1 | 35 | 127 |

Table 2.22 shows the overlap of book and market value of equity sorted portfolios of Asian ex Japan bank stocks. The portrayed quantities are bank year observations, aggregated over the whole sample period which ranges from July 1991 to June 2011.

Table 2.22 shows that for Asia ex Japan the market and book value sortings are again resulting in similar portfolios, even though not as pronounced as in the US.[11]

### 2.7.2 Momentum

Like for all other regions we will now introduce the momentum factor as an additional risk factor.

[11] Five size categories are used to allow for comparability with the three other regions.

**Table 2.23:** Risk-adjusted Returns of Book and Market Value Sorted Portfolios of Asian ex Japan Commercial Banks II

| Book Value | | | | Market Value | | | |
|---|---|---|---|---|---|---|---|
| i | 1 | 2 | 3 | i | 1 | 2 | 3 |
| $\alpha$ | -5.08 | -2.27 | -2.48 | $\alpha$ | 0.46 | -6.41 | -1.65 |
| *mkt* | 0.86*** | 0.77*** | 0.89*** | *mkt* | 0.86*** | 0.82*** | 0.87*** |
| *smb* | 0.14 | 0.19 | -0.09 | *smb* | 0.27 | 0.22* | -0.08 |
| *hml* | 0.58*** | 0.13 | 0.31*** | *hml* | 0.68*** | 0.10 | 0.31*** |
| *mom* | -0.31** | -0.12 | -0.13** | *mom* | -0.34* | -0.26*** | -0.11* |
| adj. $R^2$ | 0.47 | 0.44 | 0.71 | adj. $R^2$ | 0.34 | 0.45 | 0.71 |
| 1st pc | 0.81 | 0.52 | 0.27 | 1st pc | 0.91 | 0.39 | 0.14 |

Table 2.23 shows the results of time series regressions on the four Fama-French-Carhart risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it}$. i are three portfolios of Asian ex Japan commercial banks sorted by book or market value of equity. $mkt, smb, hml$, and $mom$ are the four Carhart risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

Table 2.23 shows the regressions for Asia ex Japan after adding the momentum factor as an explanatory variable. While the results regarding adjusted $R^2$ measures and the first principal component do not change, the momentum factor plays a non neglectable role for the results of each single regression. All portfolios load negatively on it and the regression coefficients are mostly significant.

Compared to the three-factor model loadings on the market risk factor are only slightly changed, but the negative correlation of the momentum and value factor as reported in table 2.5 might have caused a reduction in the loadings on the later for all portfolios, especially for the smallest banks. The $\alpha$'s of all portfolios increase significantly. However, all but one portfolio still earn negative mean excess returns and all $\alpha$'s are not significantly different from zero.

## 2.8 Conclusion

Although financial firms and in particular banks represent an important industry sector worldwide, there exists no empirical study on bank stock returns in the Fama-French framework outside the US. With this study we close this gap.

Our analysis of bank stock returns from July 1991 till June 2011 of all four major markets suggests the following five primary results.

First, banks in the US, Europe, and Japan have an increasing loading on market risk with market capitalization. This observation also holds true for US and Japanese banks when sorting by book value of equity.

Second, given the particularities of the results in Europe when sorting by book value instead of market value and the fact, that everywhere except in Europe the portfolios resulting from sorting by book and market value are very similar, the question is raised, if it is market value, which is the relevant size measure. This purely empirical finding contradicts current theoretical explanations, see Berk (1995) and Gandhi and Lustig (2013).

Third, in all four regions, even after accounting for all other risk factors, there remains a positive correlation over all market or book value of equity sorted portfolios. This is evident from the uniformly positive loadings of the first principal component extracted from the residual returns of the size or book value sorted portfolio. Thus there appears to exist a bank specific additional risk factor in all regions, which is not captured by the Fama-French-Carhart factors.

Fourth, in the emerging economies of Asia ex Japan the particular structures found in the US, European, and Japanese market are non-existent or far less pronounced. This may be due to the small sample size as compared to the other regions, but it is more likely a result of a missing integrated common market.

Fifth, in the US, Europe and Japan we find a negative loading on the momentum factor with the biggest banks. This indicates to some degree a countercyclical behaviour. The biggest banks act at least partly as the losers in the definition of the momentum factor. The main question posed by this finding is why this is the case. The momentum factor does not represent a fundamental form of risk and therefore cannot be interpreted easily. Especially in light of the recent discussion of “too big to fail” this particularity in the returns of the biggest banks appears to be a promising subject of future research.

## 2.9 Additional Materials

This section contains the results for the US banking market when sorting only into three or five portfolios, should one wish to compare by quantiles and not by similar portfolio size.

**Table 2.24:** Risk-adjusted Returns of Book and Market Value Sorted Portfolios of US Commercial Banks I

| Book Value | | | | Market Value | | | |
|---|---|---|---|---|---|---|---|
| i | 1 | 2 | 3 | i | 1 | 2 | 3 |
| $\alpha$ | 2.36 | 2.03 | -3.67 | $\alpha$ | 2.52 | 1.3 | -3.69 |
| *mkt* | 0.34*** | 0.44*** | 1.22*** | *mkt* | 0.36*** | 0.43*** | 1.22*** |
| *smb* | 0.19*** | 0.27*** | -0.17** | *smb* | 0.28*** | 0.19*** | -0.16** |
| *hml* | 0.38*** | 0.46*** | 0.91*** | *hml* | 0.44*** | 0.42*** | 0.91*** |
| adj. $R^2$ | 0.33 | 0.47 | 0.66 | adj. $R^2$ | 0.34 | 0.32 | 0.66 |
| 1st pc | 0.28 | 0.38 | 0.88 | 1st pc | 0.45 | 0.50 | 0.74 |

Table 2.24 shows the results of time series regressions on the three Fama-French risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it}$. i are three portfolios of US commercial banks sorted by book or market value of equity. $mkt$, $smb$, and $hml$ are the three Fama-French region specific risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

**Table 2.25:** Risk-adjusted Returns of Book and Market Value Sorted Portfolios of US Commercial Banks II

| Book Value | | | | Market Value | | | |
|---|---|---|---|---|---|---|---|
| i | 1 | 2 | 3 | i | 1 | 2 | 3 |
| $\alpha$ | 2.58 | 2.32 | -1.75 | $\alpha$ | 2.65 | 1.06 | -1.72 |
| $mkt$ | 0.34*** | 0.44*** | 1.18*** | $mkt$ | 0.36*** | 0.43*** | 1.18*** |
| $smb$ | 0.20*** | 0.28*** | -0.11 | $smb$ | 0.28*** | 0.19*** | -0.11 |
| $hml$ | 0.38*** | 0.45*** | 0.83*** | $hml$ | 0.44*** | 0.43*** | 0.84*** |
| $mom$ | -0.02 | -0.03 | -0.18*** | $mom$ | -0.01 | 0.02 | -0.18*** |
| adj. $R^2$ | 0.33 | 0.47 | 0.68 | adj. $R^2$ | 0.34 | 0.32 | 0.68 |
| 1st pc | 0.29 | 0.40 | 0.87 | 1st pc | 0.48 | 0.53 | 0.70 |

Table 2.25 shows the results of time series regressions on the four Fama-French-Carhart risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it}$. i are three portfolios of US commercial banks sorted by book or market value of equity. $mkt, smb, hml$, and $mom$ are the four Carhart risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

**Table 2.26:** Risk-adjusted Returns of Book and Market Value Sorted Portfolios of US Commercial Banks III

| Book Value | | | | | | Market Value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| i | 1 | 2 | 3 | 4 | 5 | i | 1 | 2 | 3 | 4 | 5 |
| $\alpha$ | 0.63 | 3.26 | 1.71 | -0.54 | -3.71 | $\alpha$ | 1.39 | 2.56 | 1.15 | -0.56 | -3.77 |
| *mkt* | 0.31*** | 0.36*** | 0.43*** | 0.6*** | 1.25*** | *mkt* | 0.34*** | 0.42*** | 0.37*** | 0.66*** | 1.24*** |
| *smb* | 0.19*** | 0.2*** | 0.25*** | 0.35*** | -0.18** | *smb* | 0.27*** | 0.23*** | 0.17** | 0.43*** | -0.18** |
| *hml* | 0.36*** | 0.39*** | 0.43*** | 0.63*** | 0.92*** | *hml* | 0.4*** | 0.46*** | 0.36*** | 0.73*** | 0.91*** |
| adj. $R^2$ | 0.29 | 0.37 | 0.42 | 0.54 | 0.66 | adj. $R^2$ | 0.26 | 0.34 | 0.23 | 0.59 | 0.66 |
| 1st pc | 0.34 | 0.34 | 0.41 | 0.47 | 0.62 | 1st pc | 0.46 | 0.49 | 0.50 | 0.37 | 0.40 |

Table 2.26 shows the results of time series regressions on the three Fama-French risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it}$. i are five portfolios of US commercial banks sorted by book or market value of equity. $mkt, smb$, and $hml$ are the three Fama-French region specific risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

**Table 2.27:** Risk-adjusted Returns of Book and Market Value Sorted Portfolios of US Commercial Banks IV

| Book Value | | | | | | Market Value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| i | 1 | 2 | 3 | 4 | 5 | i | 1 | 2 | 3 | 4 | 5 |
| $\alpha$ | 0.57 | 3.5 | 1.91 | -0.45 | -1.73 | $\alpha$ | 1.46 | 2.52 | 0.93 | -0.28 | -1.78 |
| *mkt* | 0.31*** | 0.36*** | 0.42*** | 0.59*** | 1.2*** | *mkt* | 0.33*** | 0.42*** | 0.38*** | 0.65*** | 1.19*** |
| *smb* | 0.19*** | 0.2*** | 0.25*** | 0.36*** | -0.12 | *smb* | 0.27*** | 0.22*** | 0.17** | 0.44*** | -0.13 |
| *hml* | 0.36*** | 0.38*** | 0.42*** | 0.62*** | 0.85*** | *hml* | 0.4*** | 0.46*** | 0.37*** | 0.72*** | 0.84*** |
| *mom* | 0.00 | -0.02 | -0.02 | -0.01 | -0.18*** | *mom* | -0.01 | 0.00 | 0.02 | -0.03 | -0.19*** |
| adj. $R^2$ | 0.28 | 0.37 | 0.41 | 0.54 | 0.67 | adj. $R^2$ | 0.26 | 0.33 | 0.22 | 0.59 | 0.67 |
| 1st pc | 0.35 | 0.35 | 0.42 | 0.48 | 0.59 | 1st pc | 0.47 | 0.50 | 0.51 | 0.37 | 0.38 |

This table shows the results of time series regressions on the four Fama-French-Carhart risk factors: $r_{it} - r_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + m_i \cdot mom_t + \epsilon_{it}$. i are five portfolios of US commercial banks sorted by book or market value of equity. $mkt, smb, hml$, and $mom$ are the four Carhart risk factors taken from Ken French's homepage. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The $\alpha$'s have been annualized by multiplying by 12 and are expressed in percentage. The estimation period ranges from July 1991 to June 2011. Newey-West standard errors with three lags are applied to correct for autocorrelation and heteroscedasticity. The last row shows the loadings of the first principal component extracted from the residuals of all regressions.

Chapter 3

# Default and Disaster Risk in Equity Returns

## 3.1 Introduction

The size and the value factor subsume at least part of the exposure of an asset to many different types of risk. Even though the size and the value factor are therefore able to explain a lot of cross-sectional excess returns of stocks, they lack a sound theoretical explanation and the specific risk source which is priced by the market cannot be identified. Many studies have therefore investigated the possibility of explaining parts of the size and value effect by looking at a more fundamental risk concept.

Perhaps the most studied source of fundamental risk in recent research is default risk, see most prominently Vassalou and Xing (2004), but also Griffin and Lemmon (2002), Campbell et al. (2008), and Garlappi and Yan (2011).

Vassalou and Xing (2004) argue that the size effect is more or less a default risk effect and that the value effect has some interdependencies with default risk as well.

In this study we additionally introduce a new source of risk, namely disaster risk, which measures the resilience of an asset against the crisis state of the market. Specifically, a firm which is robust against the crisis state has a low disaster risk exposure.

Afterwards we show that the value effect can be explained as a disaster insurance against the crisis state of the market and that the value effect actually has a negative premium in the non-crisis state after controlling for default risk. This is in line with its interpretation as an insurance against the crisis state. The value effect thus can be explained as an overlapping of a disaster risk effect and a default risk effect. This is a theoretically more sound explanation than other prevailing behavioural ones, see e.g. Lakonishok et al. (1994).

For the size effect we repeat the empirical study of Vassalou and Xing (2004) and add to their observations an analysis of the interdependencies of size and default risk over the crisis and the non-crisis state of the market. We come to the same conclusions, but find a weaker link between size and default risk in the crisis state as compared to the non-crisis state.

We further argue that with default risk and disaster risk we can construct an alternative model within the Fama-French framework. Although the Fama-French model has a better empirical performance, both models are not able to explain cross-sectional returns outside their own framework better than the nested CAPM model. This leads to the question to what extend the performance of Fama-French style models is only driven by some sort of overlapping effects, see Lewellen et al. (2010) and Wallmeier and Tauscher (2013). Furthermore, the question arises if cross-sectional adjusted $R^2$ measures, especially over cross-sectional portfolios constructed with the same attributes as the model's factors, can be seen as a good test of model performance, see Lewellen et al. (2010).

The rest of this paper is organized as follows. In section 3.2 we outline the concepts of disaster and default risk and introduce the Merton model as well as the Gabaix model. In section 3.3 a short review of the Fama-French methodology is given. The data set used for our study is described in section 3.4. Section 3.5 provides the empirical results, specifically looking at first at the descriptive statistics, then performing an analysis of the crisis and the non-crisis parts of the sample, and in the end looking in detail at the value and the size factor as proxies for default and disaster risk. Section 3.6 compares the CAPM, the Fama-French model, and a default and disaster risk model. Conclusions are drawn in Section 3.7.

## 3.2 Default and Disaster Risk

Equity, for example in the model of Jarrow and Turnbull (1992), can be seen as the debt with last seniority and no maturity, which pays dividends instead of coupons. From this point of view equity should obviously be very sensitive to default risk. So far three different approaches have been made to incorporate default risk in asset pricing models: First, looking at credit spreads of corporate bonds, e.g. Feldhütter and Lando (2008) and Friewald et al. (2013). Second, taking credit ratings from accounting based measures as for example in Griffin and Lemmon (2002). Third, using a structural model, i.e. the Merton model, to determine credit risk.

Both the first and the second approach have the severe downside of an obvious selection bias, since not all firms issue corporate bonds. Additionally, these approaches also lead to a too small sample size of companies used for the model construction in general, which makes empirical studies unstable especially when dealing with subsamples.

Furthermore, a liquid and well developed debt market is necessary to obtain reliable default risk data from credit spreads. The same holds true for ratings, which also are not available for all firms and not always provide up to date information. Additionally, a high degree of market integration between bond and equity market, which might not be given in every case and at every point in time, is implicitly assumed, when using bond data for equity pricing models.

Consequently, we will follow the third approach. The first contribution to default risk in asset pricing in this manner was provided by Vassalou and Xing (2004), who use a Merton model to estimate default risk on an individual firm basis and use the resulting default risk factor as an overall market default risk factor. Building on these findings, Kang and Kang (2009) use the same basic idea of measuring default risk with a Merton model to construct a Fama-French style factor for default risk.

At this point it is important to stress that the Merton model has no longer to deliver correct default spreads, but only an ordering of all firms in the sample by default risk. The highly problematic performance of the Merton model when comparing it with real default spreads is therefore not such a serious issue any more.[1] Vassalou and Xing (2004) specifically make the point that a default measure constructed in this manner can only be seen as a proxy compared to other more sophisticated measures, which however, as stated above, are ruled out for different reasons. Therefore we will follow this line of reasoning and go a step further like Garlappi and Yan (2011), who switch from the physical trend to the risk free rate within the underlying model. This step definitively substitutes the concept of obtaining a "real" default probability under the physical probability measure with the concept of obtaining a proxy measure for default risk in a framework better suited to asset pricing.

[1]Schönbucher (2003), p. 284-286

Though default risk is commonly found to be a factor in asset pricing there has been some contradictions towards the sign of the premium for default risk. Where Vassalou and Xing (2004), Kang and Kang (2009) and Friewald et al. (2013) find a positive risk premium, others report a default risk pricing puzzle. For example Griffin and Lemmon (2002) and most recently Garlappi and Yan (2011) report a negative premium.

A different argument for firms close to default altogether was made by Campbell et al. (2008), who state that very distressed firms are simply not suited for empirical reconciliation due to a too high degree of uncertainty in the stock price.

Another effect, which has received a lot of attention in the aftermath of the financial crisis of 2008, is disaster risk. The idea that different resilience to a disaster is priced as a risk factor was first proposed by Rietz (1988). Barro (2006) showed that the disaster frequency is high enough to make it relevant and Gabaix (2012) developed a CCAPM style model with a disaster component. Based on this model Gandhi and Lustig (2013) showed that recovery rates, induced by differences in government guarantees, are highly important for explaining differences in bank equity yields over the last 40 years in the US.

Further noteworthy is the empirical finding of Ruenzi and Weigert (2012), who showed by looking at market to firm coskewness, which can be seen as the empirical counterpart to the resilience concept, that investors receive compensation for holding stocks with a strong sensitivity to extreme market downturns.

Two other studies, Kelly and Jiang (2012) and Kelly and Jiang (2013), look at the phenomenon of extreme tail risk specifically for hedge fund performance. They show that funds or assets, which perform particularly bad during extreme tail risk events, substantially outperform during normal periods. This is the same concept underlying the model of Gabaix, but going exactly in the different direction, meaning from the crisis to the non-crisis state.

We will take a new approach to empirically quantify disaster risk by constructing a Fama-French style disaster risk factor from a proxy measure for individual firm disaster risk based on the Gabaix model.

### 3.2.1 Measuring Credit Risk

For our purposes we define the default of a firm to be equal to the total value of its assets dropping below a default threshold. We assume this threshold to be the nominal value of total debt. To model this relationship we build on the Merton model, see Merton (1974), which is in turn based on the Black Scholes model, see Black and Scholes (1973). In this model the value of the firm's assets, $A_t$, is assumed to follow a geometric Brownian motion. Due to the geometric nature of the assumed model, we perform a small linear translation of the total debt level and the market capitalisation to ensure a properly working model, when total debt levels are close to zero. The dynamics of the value of firm's assets are given by:

$$\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dW_t \tag{3.1}$$

We denote the drift term by $\mu_A$ and the annualized asset volatility by $\sigma_A$.

The value of equity, $E_0$, is given by a call option on the firm's assets with strike price equal to the nominal value of total debt, $D_T$, at maturity T:

$$E_0 = \mathbb{E}\left[\max\left[A_T - D_T, 0\right]\right] \tag{3.2}$$

From the Black Scholes model, which uses a risk neutral probability measure with drift $r_f$, the value of equity can be derived as:

$$E_0 = A_0 \Phi(d_1) - D_T \exp(-r_f)\,\Phi(d_2) \tag{3.3}$$

$\Phi(\cdot)$ denotes the cumulative standard normal distribution function, $r_f$ is the risk free rate of interest and $d_1$ and $d_2$ are given by:

$$d_1 = \frac{\ln\left(A_0 e^{r_f} T / D_T\right) + \frac{1}{2}\sigma_A^2 T}{\sigma_A \sqrt{T}} \tag{3.4}$$

$$d_2 = d_1 - \sigma_A \sqrt{T} \tag{3.5}$$

One possible proxy for the firm's default risk is the default spread, which can be obtained by using the relationship $D_0 = A_0 - E_0 = D_T \exp(-(r_f + s)T)$ and is given by:

$$s = \frac{1}{T} \ln\left(\frac{D_T}{A_0 - E_0}\right) - r_f \tag{3.6}$$

The main problem with using the Merton model is choosing the right initial values or estimating the initial values correctly. The risk free rate, the current value of equity, and the nominal value of debt can all be observed on the market. More problematic are the choices of $\mu_A$ and $\sigma_A$. They cannot be measured in advance and a historic estimation, even though frequently applied in research, is often problematic, since invariance of $\mu_A$ and $\sigma_A$ between the observation and the prognosis period is implicitly assumed.

Using a firm specific trend for the asset value to explain equity returns might in addition lead to a logical circle, since the equity returns, which we would like to explain, are part of this trend. Additionally, estimating the trend from historic data would also incorporate some loading on a momentum factor as a byproduct, which might not be a desired outcome.

In risk management the real world probability measure needs to be taken.

For our purpose, as already stated earlier, it is more reasonable to follow Garlappi and Yan (2011) and switch to the risk neutral probability space by substituting $\mu_A$ with $r_f$. Doing this we lose the direct probability of default argument implied by the model, since we change from the real world to the risk neutral probability measure, but we are able to address the much more severe reasoning and estimation problems. In particular the model's capability of delivering an ordering of firms by credit risk should not be diminished, but more likely even enhanced by avoiding a huge source of estimation uncertainty.

Apart from the problem associated with the estimation of $\sigma_A$ from historic data, the fact that $A_t$ is not observable poses further problems when finding a value for $\sigma_A$. Given the limited availability of implied volatility data on an individual firm level, since it can only be obtained for firms with liquidly traded options written on their individual stock, there is no alternative to using historic volatility data without going down the selection bias and small sample size road of the credit spread models.

The standard approach for this hidden variable problem is to estimate $A_0$ and $\sigma_A$ simultaneously using the relationship

$$\sigma_E = \Phi(d_1)\frac{A_0}{E_0}\sigma_A. \tag{3.7}$$

Solving (3.3) and (3.7) simultaneously delivers $A_0$ and $\sigma_A$ from $E_0$, $D_T$, and $T$, which all can be observed, and $\sigma_E$, which is estimated from 12 months of historic return data.

### 3.2.2 Measuring Disaster Risk

To construct the proxy for measuring disaster risk on an individual firm basis, meaning a proxy for the recovery rate of firms dividends in case of a financial disaster, we use the Gabaix model, see Gabaix (2012), in its modified version introduced by Gandhi and Lustig (2013).

The macroeconomic environment is based on the models of Rietz (1988) and Barro (2006), in which a disaster may happen with a probability $p_t$ and only a part $F_t$ of the firm's dividends remains.

In our model we assume the following stochastic discount factor with a normal and a disaster component:

$$M_{t+1} = M^N_{t+1} \cdot 1 \text{ in state without financial disaster} \tag{3.8}$$

$$M_{t+1} = M^N_{t+1} \cdot M^D_{t+1} \text{ in state with financial disaster} \tag{3.9}$$

During the normal state of the market the stochastic discount factor is completely specified by the normal (Gaussian) risk components, meaning risk, which is unrelated to disaster.

We assume further that the normal component of the stochastic discount factor is linear in the normal risk factors, specifically the market return, *smb*, *hml*, and default risk:

$$M^N_{t+1} = b^t f_{t+1} \tag{3.10}$$

We also assume that the normal risk factors $f_t$ are independent of the disaster realization and that $p_t$ is constant and uncorrelated to $f_t$.

For the dividend process of a firm or a portfolio of firms $i$ we consider the following specifications:

$$\Delta \log D^i_{t+1} = \Delta \log D^{i,N}_{t+1} \text{ in states without disaster} \tag{3.11}$$

$$\Delta \log D^i_{t+1} = \Delta \log D^{i,N}_{t+1} + \log F^i_t \text{ in states with disaster} \tag{3.12}$$

$\Delta \log D^{i,N}_{t+1}$ is the normal component of dividend growth. $1 \geq F^i_t > 0$ can be thought of as the recovery rate. When a disaster occurs a fraction $F^i_t$ of the future dividends doesn't get wiped out. $F_t^i = 0$ would mean that the firm being completely expropriated and $F_t^i = 1$ would mean that there is no dividend loss.

The recovery rate will vary across firms depending on the properties of their business model, financing structure, and asset structure, see Akgun and Gibson (2001) and Gandhi and Lustig (2013).

The resilience of a firm conditional on a disaster happening in $t+1$, $H_t^i$, is defined by:[2]

$$H_t^i = p_t \mathbb{E}\left[M_{t+1}^D F^i - 1\right] \tag{3.13}$$

In the simplest CCAPM setup this translates to:[3]

$$H_t^i = p_t \mathbb{E}\left[\left(F_{t+1}^C\right)^{-\gamma} F^i - 1\right] \tag{3.14}$$

Where $F_{t+1}^C$ is the shock to consumption and $\gamma$ the coefficient of relative risk aversion.

Further it can be derived:[3]

$$\mathbb{E}\left[\hat{R}_{t+1}^i\right] = \exp\left(r - h_t^i\right) \tag{3.15}$$

with

$$\begin{aligned}
\mathbb{E}\left[\hat{R}_{t+1}^i\right] &= \mathbb{E}\left[R_{t+1}^i\right] - \beta^{\mathbf{i}}\lambda && (3.16)\\
r &= \log R_t = \log \mathbb{E}\left[M_{t+1}^N\right]^{-1} && (3.17)\\
h_t^i &= \log\left(1 + H_t^i\right) && (3.18)
\end{aligned}$$

[2] Gabaix (2012), p. 652
[3] Gandhi and Lustig (2013), p. 18

$\mathbb{E}\left[\hat{R}^i_{t+1}\right]$ is the log return conditional on no disaster realization after adjusting for normal risk exposure and $r$ is the rate of return of an asset with zero resilience.

Additionally, when only looking at a sample without disaster realization, the average normal risk-adjusted return will be given by:[4]

$$\mathbb{E}\left[\hat{R}^i_{t+1}\right] \approx \exp(\bar{r} - \bar{h}^i) \tag{3.19}$$

$\bar{h}^i$ denotes the average resilience of firm or portfolio i and $\bar{r}$ denotes the average rate of return of an asset with zero resilience. When looking at two different portfolios, differences in $\alpha$, meaning the excess returns in normal times after correcting for the normal risk factors, can now be directly attributed to differences in average resilience to the disaster state of different firms:[5]

$$\alpha^I - \alpha^{II} = \bar{h}^{II} - \bar{h}^I \tag{3.20}$$

Taking II as the market portfolio, which implies $\alpha = 0$ when regressing among other factors on the market factor, we can define the relative resilience $\hat{\bar{h}}^I$ compared to the market portfolio on an individual firm basis as:

$$\hat{\bar{h}}^I = -\alpha^I \tag{3.21}$$

With $\hat{\bar{h}}^I$ we now have found a firm specific proxy measure for disaster risk, which allows us to construct a Fama-French style mimicking portfolio of disaster risk. There are however some downsides to this measure. First, it is an ex-post measure. Second, $\hat{\bar{h}}^I$ is constant for each firm over the whole duration of our analysis.

[4]Gandhi and Lustig (2013), p. 18
[5]Gandhi and Lustig (2013), p. 19

The observations, from which we will draw most of our inference, belong however to the crisis state part of our sample, which ensures that we are not just looking at an empirical ex-post phenomenon. For our empirical analysis we will count a month as a crisis month, if the market return is negative by more then 1.6 standard deviations. This leads to about 5% of crisis time or 12 crisis observations in our sample which is in accordance with what is suggested in Gabaix (2012).

## 3.3 Fama-French Methodology

The framework used in this study is the Fama-French three-factor model, which we extend by two additional factors of portfolios mimicking default and disaster risk. In general we follow Schmidt et al. (2011) in the standard approach for portfolio factor construction.

The size and value factor portfolios are constructed in the same manner as in Fama and French (1993). A size and a book-to-market attribute based on the firm's properties at the end of the year before is assigned to each company. A firm counts as small (S) with a market value below the median and as big (B) otherwise. Further it has a low (L) book-to-market ratio, if it is below the 30% quantile, high (H) above the 70% quantile, and medium (M) in between. For the six resulting groups of firms the value weighted monthly returns are calculated $R_t^{S/L}, R_t^{S/M}, R_t^{S/H}, R_t^{B/L}, R_t^{B/M}, R_t^{B/H}$ and the returns of the factor mimicking portfolios are given as follows:

$$smb = \frac{\left(R_t^{S/L} - R_t^{B/L}\right) + \left(R_t^{S/M} - R_t^{B/M}\right) + \left(R_t^{S/H} - R_t^{B/H}\right)}{3} \tag{3.22}$$

$$hml = \frac{\left(R_t^{S/H} - R_t^{S/L}\right) + \left(R_t^{B/H} - R_t^{B/L}\right)}{2} \tag{3.23}$$

This approach is taken to minimize correlation between the risk factor portfolios and additionally to enhance the weight of firms with low market capitalisation in the factor portfolios to ensure a better cross-sectional fit. The firm's attributes are calculated anew for every year.

We will forgo the construction of the size factor by cumulative market capitalization as suggested by Fama and French (2012), since this would create a methodological inconsistency when using sequentially sorted portfolios as in Vassalou and Xing (2004) later on and more importantly since this has no methodological equivalent for the new risk factor corresponding to size, default risk.

The default risk factor portfolio and the disaster risk factor portfolio are constructed in the same manner by assigning the attributes low default risk (LD) and high default risk (HD) to firms below and above the median of the proxy for default risk. In the same way a low (LDP) and high (HDP) disaster premium is attributed to a firm, meaning a high recovery rate and a low recovery rate respectively, below the 30% quantile and above the 70% quantile. The two mimicking portfolios are constructed as follows:

$$der = \frac{\left(R_t^{S/HD} - R_t^{S/LD}\right) + \left(R_t^{B/HD} - R_t^{B/LD}\right)}{2} \tag{3.24}$$

$$dir = \frac{\left(R_t^{S/LDP} - R_t^{S/HDP}\right) + \left(R_t^{B/LDP} - R_t^{B/HDP}\right)}{2} \tag{3.25}$$

*Der* denotes the default risk factor and *dir* stands for the disaster risk factor. In difference to the other three risk factors the proxy measure for the disaster risk portfolio is constant for each firm over the whole duration of our analysis as indicated before when constructing the proxy measure for disaster risk. Since we are using value weighted portfolios we control for size in the factor construction to ensure a proper representation of firms with a smaller market capitalisation.

## 3.4 Data Description

The data used for the empirical part consists of monthly return data of US companies from July 1991 till June 2011. All data is obtained from Thomson Reuters Datastream/Worldscope. To obtain a solid data basis the data was filtered as proposed by Ince and Porter (2006) or Schmidt et al. (2011) and for example applied in Hanauer et al. (2013).

Only companies are considered for each year of our sample period for which all necessary data is available in a valid form, specifically, return and market data for the twelve months from July to June and market value, book value, nominal level of debt, and 12 months historic return data for the portfolio construction at the 30th of June, as well as at least one year of non-disaster return data over the whole sample duration. All financial firms are excluded from the sample, since the Merton model doesn't deliver empirically reasonable results for financial firms.

**Table 3.1:** Number of Companies per Year

| Year | Firms | Year | Firms |
|---|---|---|---|
| 1991 | 1577 | 2001 | 3898 |
| 1992 | 1692 | 2002 | 3537 |
| 1993 | 1834 | 2003 | 3533 |
| 1994 | 1936 | 2004 | 3362 |
| 1995 | 2865 | 2005 | 3348 |
| 1996 | 3178 | 2006 | 3305 |
| 1997 | 3509 | 2007 | 3269 |
| 1998 | 3769 | 2008 | 3356 |
| 1999 | 4364 | 2009 | 2948 |
| 2000 | 4196 | 2010 | 2770 |
| | | Total | 62246 |

Table 3.1 shows the total number of firms of our sample as of July of each year. The underlying database is Thomson Reuters Datastream/Worldscope and the data is filtered as proposed by Ince and Porter (2006). All companies with return and market value data for the twelve months from July to June as well as market value, book value, nominal value of debt, and 12 months of historic return data for the portfolio construction at the 30th of June are taken for our sample. All financial firms are excluded and every firm must have at least twelve month of non-disaster return data over the whole sample period.

Table 3.1 shows how many companies meet these criteria each year for the US market. Compared to spread based datasets, which would be the other market based approach to measure default risk, see for example Friewald et al. (2013) with a data basis of 675 companies for the US over the duration of January 2001 to April 2010, the data basis can be considered far more brought and in general can be seen to encompass the whole US market.

All values are denominated in USD and for the risk free rate one-month Treasury bill rates from Ibbotson Associates, obtained from Ken French's website[6], are taken.

## 3.5 Empirical Analysis

Table 3.2 shows the mean, the standard deviation, and the Pearson correlations of the three Fama-French factors as well as of the credit risk and the new distress risk factor. It also provides the level of significance for the correlation coefficients and the mean values.

[6]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, viewed January 15th, 2012

**Table 3.2:** Descriptive Statistics of the Five Risk Factors

| Descriptive Statistics of the Factor Portfolios | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | Pearson Correlations | | | | |
| Factor | Mean | Std.Dev. | *mkt* | *smb* | *hml* | *der* | *dir* |
| *mkt* | 0.62* | 4.59 | 1 | | | | |
| *smb* | 0.28 | 3.80 | 0.27*** | 1 | | | |
| *hml* | 0.69** | 5.17 | -0.36*** | -0.44*** | 1 | | |
| *der* | 0.65 | 5.01 | 0.62*** | 0.72*** | -0.45*** | 1 | |
| *dir* | -1.61*** | 3.32 | -0.02 | -0.21*** | 0.70*** | -0.16** | 1 |

Table 3.2 shows the descriptive statistics of the five risk factors used in our study. *Mkt* denotes the excess return of the value weighted market portfolio over the risk free rate, *smb* the excess return of small firms over large firms, *hml* the excess return of firms with high book-to-market ratio over firms with a low book-to-market ratio, *der* the excess return of firms with high default risk over firms with low default risk, and *dir* the excess return of firms with a low disaster premium over firms with a high disaster premium. All estimations are performed using the monthly returns from July 1991 till June 2011. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The standard errors used for the significance tests were adjusted for heteroscedasticity and autocorrelation using Newey-West with 3 lags when applicable.

The mean monthly excess return is found to be 0.62%. Fama and French (1993) report 0.42% for the duration of July 1963 to December 1991. Size delivers a positive premium of 0.28% compared to Fama and French (1993) with 0.27%. The value premium is found to be 0.69% compared to 0.40% in Fama and French (1993). Fama and French (2012) find 0.66% for the market return, 0.24% for the size factor, and 0.33% for the value factor over the duration from November 1991 to March 2011, using a different method for the breakpoint construction of the size attribute, though.

Further we find a premium of 0.65% and -1.61% for the default risk and the disaster risk factor respectively. Kang and Kang (2009) find 1.36% for the Korean market after controlling for the three Fama-French risk factors with a similar factor over the period form July 1995 and June 2007. Unfortunately Vassalou and Xing (2004) and Garlappi and Yan (2011) did not construct a Fama-French style factor, to which we could compare our findings.

This premium is also what could be expected from the underlying theory, since by construction the default risk factor mimicking portfolio is short in companies with low default risk and long in companies with high default risk.

Because the riskier long position should yield higher returns, the portfolio as a whole should have a positive return. Even though our results make sense intuitively, there has been an issue with the sign of the premium for default risk in general. Using different models than the Merton model to quantify default risk Griffin and Lemmon (2002) and Garlappi and Yan (2011) have reported a default risk pricing puzzle. Specifically, they find negative returns on default risk with equities. We will also address this problematic in the light of our findings in the next section.

The negative mean monthly returns of the distress risk factor are a direct result of the factor construction and the ex-post character of the proxy measure used in the factor construction, combined with the large proportion of non-crisis months in the sample. Consequently, there can be no interpretation of the value of the mean monthly return of the disaster risk factor at this point. The negative returns can however be seen as an insurance premium, since the portfolio is long firms with a low distress risk and short firms with a high distress risk.

This should be treated as an assumption of the factor construction process induced by the Gabaix model and not as an empirical result. This also applies to all regressions using the *dir* factor with non-crisis observations.

The mean monthly return of the market excess return is found to be significant at the 10% level and the mean monthly return of the value factor at the 5% level. The significance level of 1% for the distress risk factor as well as it has been with its actual value should again be judged under the light of the ex-post proxy measure.

One aspect of this study is to analyse the interdependencies of the different risk factors. A first insight can be gained by looking at the correlations of all five factors over the whole sample. We find all except two of the correlation coefficients significant at the 1% level. Specifically only the market excess return is not correlated to the disaster risk factor and the correlation coefficient of the default and the desaster risk factor is only significant at the 5% level.

For our primary aim of explaining the size and the value effect by default and disaster risk the very high correlations of the default risk factor with the size factor with 0.72 and of the value factor with the disaster risk factor with 0.70 and with the default risk factor with -0.45 are most notable.

For a more detailed analysis and especially to draw meaningful inferences from our empirical analysis we have to split the sample into crisis state and non-crisis state observations. The main motivation to do this lies of course in the construction principle of the disaster risk factor. By definition the crisis state is the more interesting one since we can expect a different behaviour compared to the non-crisis state, which should at least to some degree be in line with the theory of the Gabaix model, on which the factor construction is based. We will however show that actually all factors have a distinct crisis non-crisis character.

This separation of states is achieved by assuming that the market is in a crisis state when the market return is more than 1.6 standard deviations negative. This is the same definition as used in the proxy construction for the distress risk factor and leads to a total of 12 crisis observations in our sample.

### 3.5.1 Splitting into Crisis and Non-Crisis State

At first we take a look at the descriptive statistics of all five factors in the crisis state of the market as reported in table 3.3.

**Table 3.3:** Descriptive Statistics of the Five Risk Factors in the Crisis State

| Descriptive Statistics of the Factor Portfolios | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | Pearson Correlations | | | | |
| Factor | Mean | Std.Dev. | *mkt* | *smb* | *hml* | *der* | *dir* |
| *mkt* | -10.70*** | 2.80 | 1 | | | | |
| *smb* | -3.16** | 3.87 | 0.62** | 1 | | | |
| *hml* | 4.05* | 7.83 | -0.12 | 0.43 | 1 | | |
| *der* | -8.62*** | 4.75 | 0.78*** | 0.60** | -0.54* | 1 | |
| *dir* | 0.62 | 3.79 | 0.17 | 0.31 | 0.81*** | -0.17 | 1 |

Table 3.3 shows the descriptive statistics of the five risk factors used in our study. *Mkt* denotes the excess return of the value weighted market portfolio over the risk free rate, *smb* the excess return of small firms over large firms, *hml* the excess return of firms with high book-to-market ratio over firms with a low book-to-market ratio, *der* the excess return of firms with high default risk over firms with low default risk, and *dir* the excess return of firms with a low disaster premium over firms with a high disaster premium. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. All estimations are performed using the monthly returns of all months with a more than 1.6 standard deviations negative return.

The mean of the market excess returns is found to be -10.71%, significant at the 1% level. This is what could be expected by only using observations with a market return at least 1.6 standard deviations negative from the original full sample for the crisis sample. More surprising is the change in sign and significance of the size factor with a crisis mean value of -3.16% and a 5% level of significance. The value factor is now no longer significant at the 5% level, but at the 10% level, with a value of 4.05%. The default risk factor, like the size factor, changes its sign and is found to be -8.62%, significant at the 1% level. At last, the disaster factor is no longer significant and is found to be 0.62%, which is in line with what could be expected from the model qualitatively, but not nearly as pronounced as one would hope for.

The reasons for this are probably found with the measure for the individual firm, which is dependent on how well the other risk factors explain the individual stock returns and of course and more profoundly on how much noise remains in the excess returns. Specifically, the problem is that we are not able to differentiate between how much of the excess returns can be attributed to other effects not incorporated in our model—for example idiosyncratic firm performance or macro specific trends can highly effect individual $\alpha$'s—and how much really is caused by the insurance effect as proposed within the Gabaix model.

Therein also lies the reason why the value of the mean of the corresponding distress factor is not suited for interpretation in the non-crisis state since it has an additional negative bias. The same holds true to a smaller extend for the realisations in the crisis state, which will be suffering from the same negative bias as well.

Nevertheless the distress risk factor behaves as predicted and a mean -1.62% monthly return is turned to a positive return precisely in the crisis months. Given the high degree of possible delusion and the qualitatively well fitting outcomes these results can also be judged as an empirical verification of the Gabaix model for the equity market on a brought data basis.

When looking at the dependencies a very high correlation of 0.81, significant at the 1% level, is found between the disaster risk factor and the value factor. The correlation between the default risk and the size factor is reported with 0.60 and the one between the default risk factor and the value factor with -0.54, significant at the 5% and 10% level respectively. It is noteworthy that this is very close to what is found over the full sample. Given the small sample size, which affects estimation accuracy and the significance levels, the only other relevant correlations are between the market excess return and the size factor with 0.62 and between the market excess return and the default risk factor with 0.78.

The main inferences from the presented data are that the disaster risk factor can be seen as a driving factor behind the value effect in the crisis state and that a measure of disaster risk as defined by Gabaix (2012) and Gandhi and Lustig (2013) is feasible for equities.

Before addressing other interesting implications from this data we present the same descriptive statistics for the non-crisis state, which allows a more precise presentation of our findings and their implications afterwards.

**Table 3.4:** Descriptive Statistics of the Five Risk Factors in the Non-Crisis State

| Descriptive Statistics of the Factor Portfolios | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | Pearson Correlations | | | | |
| Factor | Mean | Std.Dev. | *mkt* | *smb* | *hml* | *der* | *dir* |
| *mkt* | 1.27*** | 3.75 | 1 | | | | |
| *smb* | 0.47 | 5.18 | 0.21*** | 1 | | | |
| *hml* | 0.49* | 3.36 | -0.33*** | -0.45*** | 1 | | |
| *der* | 1.18*** | 4.58 | 0.48*** | 0.73*** | -0.42*** | 1 | |
| *dir* | -1.74*** | 3.25 | 0.10 | -0.19*** | 0.69*** | -0.20 | 1 |

Table 3.4 shows the descriptive statistics of the five risk factors used in our study. $Mkt$ denotes the excess return of the value weighted market portfolio over the risk free rate, *smb* the excess return of small firms over large firms, *hml* the excess return of firms with high book-to-market ratio over firms with a low book-to-market ratio, *der* the excess return of firms with high default risk over firms with low default risk, and *dir* the excess return of firms with a low disaster premium over firms with a high disaster premium. All estimations are performed using the monthly returns of all months with a market return higher than 1.6 standart deviations negative. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The standard errors used for the significance tests were adjusted for heteroscedasticity and autocorrelation using Newey-West with 3 lags when applicable.

Table 3.4 reports the findings for all five factors in the non-crisis state. The market excess return has a monthly mean of 1.27% and is significant at the 1% level. The size factor is found to be 0.47%, the value factor 0.49%, significant at the 10% level, the default risk factor 1.18% and the disaster risk factor -1.74%, both significant at the 1% level. The correlation structure closely resembles the one of the full sample.

Having now the descriptive statistics of both the crisis and non-crisis state at hand we can see a clear switch in sign of the premium in the size and the default risk factor from 0.47% and 1.18% in the non-crisis state to -3.16% and -8.62% in the crisis state. It is further interesting that the correlation changes only slightly from 0.73 to 0.60 from the non-crisis to the crisis state, which can be considered high for both states.

The differences in sign of the market excess return and the disaster factor are caused, as already stated earlier, by the sample construction and by the proxy measure.

Apart from this only that there has been no change in sign from the crisis state to the non-crisis state in the value factor and that its mean value is eight times higher in the crisis state appear to be essential. The high returns in the crisis state, even though not significant in the crisis sample due to the small sample size and the high standard deviation, appear to be a specific characteristic of the value effect and the driver behind its significance when looking at the full sample. This means understanding these crisis returns seems to be the key to understand the value effect altogether.

Before going into the deeper analysis of the dependencies of the value and the size effects on default and disaster risk the differences in sign of the default risk factor shall be addressed. As stated earlier, there have been some recent studies, see Griffin and Lemmon (2002) and Garlappi and Yan (2011), suggesting a default risk pricing puzzle. Since we observe both negative and positive returns in our default risk factor we might offer an explanation towards this pricing puzzle.

By definition an investor expects a higher return from an investment in a firm which has a higher probability to run into financial distress in the future and therefore a firm with this characteristic should exhibit higher returns. If the potential risk actually becomes reality, meaning the risk realizes, and the firm goes down the road of financial distress and in the end even default the consequent reevaluation dynamics will result in negative returns. This is the reason for the observed patterns: The positive premium for holding firms with the potential of financial distress during the non-crisis state, and the dynamics of collective reevaluation when this risk realizes during the crisis state of the market.

To understand why the same effect can be observed in the study of Garlappi and Yan (2011) one has to look closely into their way of constructing their portfolios and proxy measures. Garlappi and Yan (2011) take rating grades from Moody's KMV model as the proxy measure, update the portfolios on a monthly basis, and use ten categories of default risk. Using a relatively precise measure for actually identifying companies close to bankruptcy and updating the portfolios every month, the portfolios constructed in such a way will capture the dynamics of the realization process of default risk in the same way we do in the crisis state. Thus there seems actually no default risk pricing puzzle at all, it's just a question of method, specifically, if one is measuring the exposure to risk or the actual realization of risk.

### 3.5.2 Default and Disaster Risk in the Value Effect

We have seen in the analysis performed in the last section that there is a strong dependence of the value factor on the disaster risk factor in the crisis state. This leads to the hypothesis that the value factor can be interpreted as an insurance against the crisis state of the market.

To get a deeper insight into the dependencies of the value factor on the default risk and the disaster risk factor we perform a regression of the value factor on the market excess return, the default risk factor and the disaster risk factor in different combinations over the full, the crisis and the non-crisis sample. The results of this regressions are reported in table 3.5.

**Table 3.5:** *Hml* Regression Analysis

| *hml* (full) | Intercept | *mkt* | *der* | *dir* | adj. $R^2$ |
|---|---|---|---|---|---|
| $\sim mkt + der + dir$ | 2.16*** | -0.17*** | -0.15*** | 0.75*** | 0.63 |
| $\sim der$ | 0.95*** | | -0.29** | | 0.12 |
| $\sim dir$ | 0.19*** | | | 0.80*** | 0.48 |
| ***hml* (crisis)** | **Intercept** | ***mkt*** | ***der*** | ***dir*** | **adj. $R^2$** |
| $\sim mkt + der + dir$ | -1.20 | -0.04 | -0.44 | 1.58*** | 0.63 |
| $\sim der$ | 2.22 | | 0.17 | | -0.08 |
| $\sim dir$ | 3.02* | | | 1.66*** | 0.61 |
| ***hml* (non-crisis)** | **Intercept** | ***mkt*** | ***der*** | ***dir*** | **adj. $R^2$** |
| $\sim mkt + der + dir$ | 2.34*** | -0.26*** | -0.15*** | 0.72*** | 0.66 |
| $\sim der$ | 0.95*** | | -0.30*** | | 0.10 |
| $\sim dir$ | 1.74*** | | | 0.71*** | 0.48 |

Table 3.5 shows the coefficients and the adjusted $R^2$ measures from the regressions of the *hml* factor on the market excess return, the default risk factor, and the disaster risk factor. The first panel uses all monthly return observations from July 1991 till June 2011, the second panel only the months with a more than 1.6 standard deviations negative market return (crisis state), and the third panel only the months with a not more than 1.6 standard deviations negative market return (non-crisis state). Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The standard errors used for the significance tests were adjusted for heteroscedasticity and autocorrelation using Newey-West with 3 lags when applicable.

Looking at the regressions in the crisis state the disaster risk factor is shown to be the driving force behind the value factor as expected from the very high correlation of 0.81 seen in the correlation analysis, see table 3.3. Around 62% of the variation of the value factor can be explained by the disaster risk factor. The disaster risk factor is significant at the 1% level and one quarter of the value premium can be explained, even though the disaster risk factor's premium might be negatively biased as explained above. Moreover, when regressing on the default risk and the disaster risk factor together the default risk factor is not found to be significant in contrast to the non-crisis and full sample regressions. Therefore the value effect appears to be really an insurance effect for the crisis state of the market in the sense of the Gabaix model. Specifically, this effect dominates the influence of the default risk factor on the value factor in the crisis state.

In the non-crisis state the explanatory importance of the disaster risk factor prevails even though at a reduced level of around 48% of variation. The default risk factor is now significant as compared to the crisis sample and stays significant when also regressing on the disaster risk factor. The explanatory power of the default risk factor remains low at only around 11%.

The interpretation of the value effect as an insurance effect against the crisis state of the market, as indicated by the correlation and the regressions analyses and also plausible under the obvious assumption that value stocks are more resilient to the crisis state than growth stocks, bears however the following problem: An insurance against the crisis state can not deliver a positive return during the non-crisis state as the value factor does. Thus when claiming to explain the value effect as an disaster risk effect we arrive at what seems to be a pricing puzzle. Our line of reasoning would only be solid, when there exists a negative return on the value effect in the non-crisis state.

The reason that this seems not to be the case are the interdependencies of the value effect with default risk. As we have already seen the value factor is also correlated with the default risk factor and that only in the crisis state this dependencies are dominated by the ones with the disaster risk factor. We thus follow the approach by Vassalou and Xing (2004) and sequentially sort for five categories of default risk and then for five categories of book-to-market value.

**Table 3.6:** Value Controlled by Default Risk (non-crisis state)

| | Book-to-Market Value | | | | | |
|---|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) | |
| Default Risk | Mean Return Estimates | | | | | (IV+V-I-II)/2 |
| 1(low) | 1.65 | 1.96 | 1.53 | 1.51 | 1.63 | -0.24 |
| 2 | 1.44 | 1.31 | 1.29 | 1.69 | 1.18 | 0.06 |
| 3 | 1.76 | 1.71 | 1.62 | 1.38 | 1.43 | -0.33 |
| 4 | 2.17 | 2.13 | 2.36 | 2.42 | 2.09 | 0.11 |
| 5(high) | 2.48 | 2.63 | 3.02 | 2.49 | 2.64 | 0.01 |

Table 3.6 shows the mean returns of 25 portfolios sequentially sorted first into five default risk categories and then into five book-to-market categories. Additionally the differences of the mean of the portfolios with lower book-to-market ratios (I/II) and the portfolios with higher book-to-market ratios (IV/V) are reported for each default risk category. Only months with a market return not less than 1.6 standard deviations negative in the duration from July 1991 till June 2011 were taken.

Looking at the mean returns of the sequentially sorted portfolios in table 3.6 and the differences between the I/II, meaning the growth, and the IV/V, meaning the value, portfolios, we can see a negative or slightly positive value premium in the stocks not suffering from default risk, specifically the portfolios with a default risk of 1, 2, and 3. Therefore when taking the "disaster" on the individual firm basis, meaning default risk, out of the picture the *hml* factor will behave very similar to the *dir* factor. It costs during normal times but delivers during the crisis state.

This effect is much less pronounced when looking at the whole sample and only stocks with the smallest default risk attribute have a negative premium as can be seen in table 3.7.[7] That is also the reason why the insurance characteristic of the value factor is normally hidden, when not splitting in a crisis and a non-crisis sample first.

Garlappi and Yan (2011) report qualitatively the same effect when doing a similar sorting, but use a different methodology.

[7]See table 3.12 in section 3.8 for the crisis sample.

**Table 3.7:** Value Controlled by Default Risk (full sample)

| | Book-to-Market Value | | | | | |
|---|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) | |
| Default Risk | Mean Return Estimates | | | | | (IV+V-I-II)/2 |
| 1(low) | 1.20 | 1.48 | 1.13 | 1.06 | 1.22 | -0.20 |
| 2 | 0.81 | 0.72 | 0.75 | 1.12 | 0.69 | 0.14 |
| 3 | 0.82 | 0.77 | 0.91 | 0.82 | 0.95 | 0.09 |
| 4 | 0.94 | 0.84 | 1.35 | 1.69 | 1.42 | 0.67 |
| 5(high) | 0.97 | 1.40 | 2.02 | 1.75 | 1.93 | 0.66 |

Table 3.7 shows the mean returns of 25 portfolios sequentially sorted first into five default risk categories and then into five book-to-market categories. Additionally the differences of the mean of the portfolios with lower book-to-market ratios (I/II) and the portfolios with higher book-to-market ratios (IV/V) are reported for each default risk category. All monthly return observations from July 1991 till June 2011 were used.

The fundamental finding for the interpretation of the value effect is that it is a combination of two overlapping risk exposures approximated by the book-to-market ratio. On the one hand the value effect is an insurance effect against the crisis state of the market, which is the dominant effect. On the other hand it is closely related to default risk in so far as the value effect can only be observed with firms which suffer from a higher probability to encounter default in the next year.

The consequent interpretation of the value effect as an insurance against crisis on the individual as well as on the market level makes also sense from a more heuristic point of view since value stock should be more resilient to the crisis state, since its value does not depend so much on investor's sentiment, which will be severely negatively adjusted in the crisis state.

### 3.5.3 Default and Disaster Risk in the Size Effect

Going back to the correlation analysis as a first step of identifying interdependencies between risk factors, see tables 3.2, 3.3, and 3.4, it is clear that disaster risk is not a specifically relevant factor for the size effect.

Default risk on the other hand appears to be highly relevant. Our findings support the results of Vassalou and Xing (2004), who state that the size effect is basically a default risk effect, while we add the observation that the interdependencies stay the same over the crisis and non-crisis state of the market, even though the characteristics of both factors change significantly.

Keeping in mind the high correlation of the size factor and the default risk factor, which also stays nearly identical over the crisis and non-crisis state of the market, we next perform a regression analysis of the size factor on the market excess return, the default risk factor and the disaster risk factor in various combinations and again over the full, the crisis and the non-crisis sample. The results can be seen in table 3.8.

**Table 3.8:** *Smb* Regression Analysis

| *smb* (full) | Intercept | *mkt* | *der* | *dir* | adj. $R^2$ |
|---|---|---|---|---|---|
| $\sim mkt + der + dir$ | -0.22 | -0.29*** | 0.88*** | -0.12 | 0.56 |
| $\sim der$ | -0.19 | | 0.72*** | | 0.51 |
| $\sim dir$ | -0.25 | | | -0.33 | 0.07 |
| *smb* (crisis) | Intercept | *mkt* | *der* | *dir* | adj. $R^2$ |
| $\sim mkt + der + dir$ | 6.82 | 0.91 | 0.02 | -0.40 | 0.35 |
| $\sim der$ | 1.03 | | 0.48** | | 0.29 |
| $\sim dir$ | -2.98** | | | -0.27 | -0.01 |
| *smb* (non-crisis) | Intercept | *mkt* | *der* | *dir* | adj. $R^2$ |
| $\sim mkt + der + dir$ | -0.50 | -0.23** | 0.91*** | -0.15* | 0.56 |
| $\sim der$ | -0.50 | | 0.83*** | | 0.53 |
| $\sim dir$ | -0.04 | | | -0.30 | 0.03 |

Table 3.8 shows the coefficients and the adjusted $R^2$ measures from the regressions of the size factor on the market excess return, the default risk factor, and the disaster risk factor. The first panel uses all monthly return observations from July 1991 till June 2011, the second panel only the months with a more than 1.6 standard deviations negative market return (crisis state), and the third panel only the months with a not more than 1.6 standard deviations negative market return (non-crisis state). Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The standard errors used for the significance tests were adjusted for heteroscedasticity and autocorrelation using Newey-West with 3 lags when applicable.

Table 3.8 shows that the default factor explains with 54% most of the variance of the size factor in the non-crisis state. In the crisis state it performs not so well and can only explain around 29% of the variance of the size factor. This is also reflected by the level of significance found for the credit risk factor with 1% in the non-crisis state and 5% in the crisis state. All other factors only offer marginal improvements in all additionally performed regressions or even have a negative effect on the explanatory performance.

To further show the close relation between default risk and size we again perform a sequential sorting for five categories of default risk and then for five categories of market value. The results for the non-crisis state are reported in table 3.9 and for the crisis state in table 3.10.

**Table 3.9:** Size Controlled by Default Risk (non-crisis state)

| | Market Value | | | | | |
|---|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) | |
| Default Risk | Mean Return Estimates | | | | | (I+II-IV-V)/2 |
| 1(low) | 0.37 | 0.72 | 0.69 | 1.06 | 1.31 | -0.64 |
| 2 | 0.09 | 0.24 | 0.69 | 0.80 | 0.81 | -0.64 |
| 3 | 0.13 | 0.47 | 0.58 | 0.78 | 0.85 | -0.52 |
| 4 | 1.14 | 1.45 | 1.24 | 1.20 | 1.09 | 0.15 |
| 5(high) | 2.43 | 3.15 | 2.76 | 2.31 | 1.41 | 0.93 |

Table 3.9 shows the mean returns of 25 portfolios sequentially sorted first into five default risk categories and then into five market capitalisation categories. Additionally the differences of the mean of the portfolios with lower market capitalisation (I/II) and the portfolios with higher market capitalisation (IV/V) are reported for each default risk category. Only months with a market return not less than 1.6 standard deviations negative in the duration from July 1991 till June 2011 were taken.

**Table 3.10:** Size Controlled by Default Risk (crisis state)

| | Market Value | | | | | |
|---|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) | |
| Default Risk | Mean Return Estimates | | | | | (I+II-IV-V)/2 |
| 1(small) | -5.88 | -7.11 | -7.27 | -6.47 | -6.62 | 0.05 |
| 2 | -8.34 | -8.92 | -8.45 | -9.72 | -9.74 | 1.10 |
| 3 | -11.49 | -11.56 | -13.11 | -12.61 | -14.32 | 1.94 |
| 4 | -13.68 | -13.56 | -16.37 | -18.20 | -19.55 | 5.26 |
| 5(big) | -13.55 | -15.70 | -15.56 | -17.95 | -19.09 | 3.90 |

Table 3.10 shows the mean returns of 25 portfolios sequentially sorted first into five default risk categories and then into five market capitalisation categories. Additionally the differences of the mean of the portfolios with lower market capitalisation (I/II) and the portfolios with higher market capitalisation (IV/V) are reported for each default risk category. Only months with a market return more than 1.6 standard deviations negative in the duration from July 1991 till June 2011 were taken.

If the size effect is not a default risk effect, it should prevail in the same way in each default risk category as it has been in the case without sorting for default risk, see tables 3.3 and 3.4. Looking at the differences in returns between small and big firms in the non-crisis sample we see that actually the opposite is true. We would expect a positive difference in returns but we observe a negative one in three out of five cases. In the crisis sample the picture stays the same. We would expect a negative return, but observe a positive return in all default risk categories. We can take this as an empirical proof that the size effect is at least to a large degree a default risk effect and that this relationship is robust over different market regimes.[8]

Summing up, the size factor can be seen to a large degree as a default risk factor. This interpretation is exactly in line with the findings of Vassalou and Xing (2004), but we also show that this relationship stays the same over the crisis and non-crisis state of the market. Both default risk factor and size factor, have a distinct crisis/non-crisis behaviour and the interdependencies stay the same over both states. This provides further clear evidence towards the interpretation of the size effect as a default risk effect.

## 3.6 Model Performances

After the analysis of the value and size effect as proxy measures for default and disaster risk we next turn our attention to the overall performance of a possible default and disaster risk model in explaining asset returns. The benchmark models will of course be the CAPM and the Fama-French model as the standard factor models based on portfolio construction. We compare and test the cross-sectional performance of the models on the 25 Fama-French portfolios and 25 default and disaster risk sorted portfolios.

[8]See table 3.13 in section 3.8 for the full sample.

The CAPM and the Fama-French model are taken in their usual form:

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot mkt_t + \epsilon_{it} \tag{3.26}$$

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot mkt_t + s_i \cdot smb_t + h_i \cdot hml_t + \epsilon_{it} \tag{3.27}$$

The new default and disaster risk model is constructed by augmenting the CAPM by the default risk and the disaster risk factor:

$$R_{it} - R_{ft} = \alpha_i + \beta_i \cdot mkt_t + de_i \cdot der_t + di_i \cdot dir_t + \epsilon_{it} \tag{3.28}$$

To evaluate the empirical properties of the model we perform a time series regression for all the three models on the 25 Fama-French portfolios and on the 25 default and disaster risk sorted portfolios. The summary statistics for each regression, meaning the GRS statistics, the sum of absolute $\alpha$'s, and the mean adjusted $R^2$ measure, are reported in table 3.11.

**Table 3.11:** Summary Statistics of Model Performances

| Fama-French Port. | GRS | **a** | **R** |
|---|---|---|---|
| CAPM | 2.74 | 0.38 | 0.55 |
| FF | 2.42 | 0.28 | 0.85 |
| DERDIR | 7.97 | 0.79 | 0.71 |
| Default/Disaster Port. | GRS | **a** | **R** |
| CAPM | 5.19 | 0.63 | 0.64 |
| FF | 9.61 | 0.90 | 0.70 |
| DERDIR | 4.35 | 0.40 | 0.75 |

Table 3.11 shows the GRS statistics, the sum of absolute $\alpha$'s, and the mean adjusted $R^2$ measure for the CAPM, the Fama-French model (FF) and the default and disaster risk model (DERDIR) for the 25 Fama-French portfolios in the first panel and for the 25 default and disaster risk sorted portfolios in the second panel. With 25 portfolios and 240 months of return data, the critical values for all three models are: 90%: 1.41; 95%: 1.56; and 99%: 1.86.

The GRS statistics show that non of the models is able to explain the cross-sectional returns of either the Fama-French or the default and disaster risk portfolios. Fama and French (2012) however show with a similar GRS statistics for the Fama-French model on the 25 Fama-French portfolios that, when eliminating micro-caps from the test, meaning the five portfolios with the smallest size attribute, the Fama-French model is able to explain the cross-sectional returns over the remaining 20 Fama-French portfolios.

The Fama-French model achieves an 0.30 increase in the mean adjusted $R^2$ measure and a reduction of the sum of absolute $\alpha$'s by 0.10 as compared to the CAPM over the Fama-French portfolios. It also performs better than the CAPM in the GRS statistics. For the default and disaster risk portfolios the situation is different. The Fama-French model is still able to increase the mean adjusted $R^2$ measure by about 0.05, but the sum of absolute $\alpha$'s increases by

0.26 as compared to the CAPM and the CAPM performs better with the GRS statistics.

The default and disaster risk model on the other hand is not able to perform better than the CAPM in the Fama-French portfolio case, it only increases the mean adjusted $R^2$ measure by about 0.15. The CAPM however has a much better GRS statistics and the sum of absolute $\alpha$'s reported for the CAPM is also 0.41 smaller. With the default and disaster risk portfolios the mean adjusted $R^2$ measure can be increased by about 0.11 by the default and disaster risk model as compared to the CAPM and the sum of absolute $\alpha$'s can be reduced by 0.24. The default and disaster risk model beats the CAPM with the GRS statistics as well.

When comparing the Fama-French model and the default and disaster risk model, each model is able to perform better than the other on the portfolio sorting that is based on the same measures as their mimicking portfolio constructions.

The more surprising revelation is however that the CAPM performs better than the three-factor models if they are not used on portfolios to favour them. The interesting part of this observation is that this is not only reflected in the GRS statistics, where one could make the argument that the higher mean adjusted $R^2$ measure results in the test being more powerful and therefore more likely to reject the hypotheses of all $\alpha$'s being equal to zero, but also in the sum of absolute $\alpha$'s. Specifically, this means that while being better in explaining the variation of returns the three-factor models are actually performing worse than the CAPM when explaining the $\alpha$'s of portfolios which are not constructed to their advantage.

The problem of the default and disaster risk model with the sum of absolute $\alpha$'s measure when comparing it with the CAPM over the 25 Fama-French portfolios might be attributed to the particularities of the disaster risk factor in the non-crisis state. This however is no longer an argument when comparing the CAPM and the Fama-French model over the 25 default and disaster risk sorted portfolios, since both models would have to deal with it.

This raises the question to what extend the increases in the adjusted $R^2$ measures are due to some kind of overlapping effect and caused by the portfolio construction character of this class of asset pricing models and if and to what extend the augmentation of the CAPM by the two Fama-French factors might actually lead to some kind of model induced mispricing, when trying to explain returns not resulting from sorting by size and book-to-market.

That overlapping effects in general have a non-negligible effect was also shown by Wallmeier and Tauscher (2013) with a split sample approach for a broad sample of European stocks, which makes our reasoning at this point feasible. A more general critique on the $R^2$ measure as the main indicator to evaluate asset pricing models and on the Fama-French portfolios as the portfolios used for the testing of asset pricing models is provided in great detail by Lewellen et al. (2010).

We have shown that the default and disaster risk model is competitive against the other asset pricing models in its own domain, but can not compete with the Fama-French model overall, which is the only model, which comes close to explaining cross-sectional returns in a satisfactory manner for one of the 25 test portfolio choices. Moreover we have provided evidence that the CAPM might be better than its augmentations when dealing with cross-sectional returns outside the domain of the specific augmented model.

## 3.7 Conclusion

The main finding of our study is the explanation of the value effect as an overlapping of a disaster risk effect and a default risk effect. Specifically, that value stocks outperform growth stocks in the crisis state of the market and in the non-crisis state when the individual firm has a high risk of running into individual financial distress. In the non-crisis state and with low or medium individual distress risk growth stocks however outperform value stocks. The value effect can thus be seen as an insurance effect against the crisis state on a market wide and on an individual level. The dominant factor however is not the general default risk as measured by a structural model, as investigated by Vassalou and Xing (2004) and Garlappi and Yan (2011), but the tail risk of extreme disaster events. This observation also grounds the value effect on a more fundamental theoretical basis as compared to other prevailing explanations like for example by investor overconfidence, see Lakonishok et al. (1994).

Further, we have verified again that the size effect is mostly a default risk effect and that this relationship is stable over the crisis and non-crisis state of the market with completely different characteristics of the effects under the different market regimes.

In the context of differentiating between the crisis and the non-crisis state of the market we also provided some evidence for a possible explanation of the default risk pricing puzzle as a result of measuring possible risk as opposed to observing the dynamics of occurring risk.

To the best of our knowledge we are also the first who tried to construct a new disaster risk factor based on the Gabaix model and to introduce a new three-factor Fama-French style model based on default and disaster risk as an alternative to the standard Fama-French three-factor model.

This alternative framework allowed us to raise the question to what extend the augmentation of the CAPM leads to real improvements when trying to explain cross-sectional returns of portfolios not sorted by the same proxy measure as the one used for the factor construction of the augmented model.

In light of these observations it would be most interesting to see which other augmentations like momentum, see Carhart (1997), liquidity, see Pastor and Stambaugh (2003), variance, the minimum variance portfolio etc. keep their explanatory powers outside their framework.

## 3.8 Additional Materials

This section contains the tables 3.12 and 3.13 referred to in the footnotes of this chapter.

**Table 3.12:** Value Controlled by Default Risk (crisis state)

| | Book-to-Market Value | | | | | |
|---|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) | |
| Default Risk | Mean Return Estimates | | | | | (IV+V-I-II)/2 |
| 1(low) | -6.68 | -6.67 | -5.83 | -6.79 | -5.95 | 0.31 |
| 2 | -10.09 | -9.61 | -8.65 | -8.96 | -7.99 | 1.38 |
| 3 | -15.59 | -15.58 | -11.49 | -9.06 | -7.35 | 7.38 |
| 4 | -20.55 | -21.57 | -16.14 | -11.03 | -10.34 | 10.38 |
| 5(high) | -25.40 | -20.04 | -15.33 | -11.17 | -10.36 | 11.96 |

Table 3.12 shows the mean returns of 25 portfolios sequentially sorted first into five default risk categories and then into five book-to-market categories. Additionally the differences of the mean of the portfolios with lower book-to-market ratios (I/II) and the portfolios with higher book-to-market ratios (IV/V) are reported for each default risk category. Only months with a market return more than 1.6 standard deviations negative in the duration from July 1991 till June 2011 were taken.

**Table 3.13:** Size Controlled by Default Risk (full sample)

| | Market Value | | | | | |
|---|---|---|---|---|---|---|
| | I(low) | II | III | IV | V(high) | |
| Default Risk | Mean Return Estimates | | | | | (I+II-IV-V)/2 |
| 1(low) | 0.73 | 1.17 | 1.14 | 1.49 | 1.76 | -0.68 |
| 2 | 0.57 | 0.77 | 1.22 | 1.40 | 1.41 | -0.74 |
| 3 | 0.79 | 1.16 | 1.37 | 1.54 | 1.72 | -0.66 |
| 4 | 1.99 | 2.31 | 2.25 | 2.31 | 2.27 | -0.14 |
| 5(high) | 3.35 | 4.23 | 3.81 | 3.47 | 2.59 | 0.76 |

Table 3.13 shows the mean returns of 25 portfolios sequentially sorted first into five default risk categories and then into five market capitalisation categories. Additionally the differences of the mean of the portfolios with lower market capitalisation (I/II) and the portfolios with higher market capitalisation (IV/V) are reported for each default risk category. All monthly return observations from July 1991 till June 2011 were used.

Chapter 4

# Asset Pricing under Rational Inattention

This essay is based on a joint research project on asset pricing and rational inattention with Steve Heinke at the University of Zurich and it represents mostly my contributions to it.

## 4.1 Introduction

Asset prices should fully reflect all available information on the future cash flows associated with holding this asset, see Fama (1970). Thus the riskiness of the asset ought to be the only factor a rational agent trades on. Nowadays information is available in massive amounts, but still the existence of profitable trading strategies beside risk challenges the notions of rational expectation asset pricing and market efficiency, see Jegadeesh and Titman (1993), Lakonishok et al. (1994), and Hong et al. (2000).

Moreover, the existence of funds spending vast amount of money on information processing and data mining just to trade profitable on this informational advantage, underlines that information alone is not the bottleneck when it comes to expectation building. Rather how to deal with this overwhelming amount of information becomes the key question. In other words, in an information rich environment, attention to each piece of information becomes a scarce resource, see Falkinger (2008) and Hefti (2013).

By combining the concept of scarcity of attention with the well developed overlapping generation framework of asset pricing we try to gain new insights into the question why arbitrage opportunities exist and offer an attention driven explanation on why the efficient market hypothesis not always holds.

The theoretical research in this field so far can be divided into two main strands of the literature. The first deals with the question what hinders rational arbitrageurs to take advantage of mispricing in the market and equalize them. An extensive review on these limits to arbitrage arguments can be found in Brunnermeier (2009). The second investigates the specific nature of why agents fail to act fully rational. Common approaches are to assume special preferences, see Barberis et al. (2001), or biased beliefs, see among others Barberis et al. (1998) and Daniel et al. (1998).

While these approaches explain some of the empirically found anomalies, they have the drawback that their set-up is somewhat ad-hoc and that in the long run most of the claimed market anomalies diminish, which is not implied by most of these models, see Fama (1998).

In this paper we develop a model which addresses the problem of modelling the information flow in an asset pricing context and which can be extended by exogenous shocks to allow for momentum trading in the short run and asset prices fully reflecting all fundamentals in the long run.

The concept that information takes time till it is reflected in the price is modelled by constraining the ability of the agent to process all available information. Basically we add some costs and a budget constraint on information processing, leading to a two stage optimization problem. In the first stage the agent chooses the information pieces he values most, in the second stage he decides on the trading quantity of the asset based on the information he has.

Modelling information choice is a prosperous strand of economic research, especially in the field of finance. To the best of our knowledge we contribute to this discussion in the following ways. From the theoretical point of view we add a model of a disaggregated infinite horizon economy allowing for heterogeneity among agents with respect to signals and information processing capacity constraints.

In a next step this structure allows us to distinguish between long-term and short-term effects and how they seep into the economy. Moreover, we can explain the existence of momentum strategies as an outcome of rational actions and show that shifts in the long-term fundamentals need time till they are fully reflected in the price.

Finally, by allowing for heterogeneity in the information processing constraints, we discuss how algorithmic and high-frequency traders turn their capability of processing enormous amounts of data into an informational advantage, which generates excess returns. A similar argument can be used to explain the existence of financial services in general, since pooling resources allows for specialization in information gathering and thus allows for a more efficient use of the scarce resource attention. This efficiency gain can be expressed in terms of excess returns and thus justifies paying other people to let them do the investment decision.

From the empirical perspective we show that our model in its basic form is supported by the data and how this finding effects the concept of market efficiency and its testing. Specifically, we challenge the efficient market hypothesis by a concept of attention driven efficiency. Further, we give an empirical case study example of the more advanced form of our model with the burst of the US subprime bubble.

The rest of this chapter is organised as follows. Section 4.2 gives an overview of the attention literature with a special focus on finance topics. Afterwards, in section 4.3, we introduce one of the simplest versions of our model and theoretical framework highlighting the basic mechanisms behind it. Subsequently, in section 4.4, we extend the basic version of the model to provide testable implications with respect to market efficiency and to incorporate shifts in our framework. Section 4.5 discusses the model with heterogeneous agents. Section 4.6 draws the conclusions.

## 4.2 Literature Review

This study builds on two strands of theoretical literature. On the one hand we build on the literature on rational inattention in order to model information acquisition and attention allocation. On the other hand we use an overlapping generation framework which allows us to model information aggregation processes within a competitive asset market. This short literature review will give an overview on what rational inattention is about and what has been done in this relatively new field of research so far. Afterwards we will shortly portray the overlapping generation models framework.

Models of rational inattention focus on goal driven attention allocation processes. Thus in contrast to stimulus driven attention models, the agent has an active role when deciding which signal he receives, see Hefti (2013).

As an illustrative example consider a fund manager who is in charge of the investment decision. Every morning he gets a newspaper and can decide on how much time he is going to spend on reading the newspaper itself as well as on how much of this time he is going to devote on the economics, finance, and politics section of this newspaper.

Suppose that the more of the newspaper he reads, the better will be his idea of what is going on in the world and therefore he is more likely to make good investment decision, but less spare time remains for doing other relevant tasks. Given there is less time available than required to read the entire newspaper, he faces the problem of how to allocate the given time over the different subsections of the newspaper.

For an investment decision reading the finance section might be most relevant. Nevertheless, reading the politics or general economics section might also be important since certain topics such as general economic policies, decision on warfare, strikes and so on will be discussed there, which could potentially matter for the investment decision as well.

Under the assumption that the investor knows the structure of his newspaper, he can judge the “average” information potential of each section in advance, and thus decides rationally on the allocation of a given time over the subsections as well as on the total reading time by weighting the expected benefits of the optimal reading strategy against the costs of doing something else.

It is important to understand that the investor allocates his given reading time, meaning his mental resources, only according to the ex-ante expected information content of the subsections. Hence the investor’s attention allocation is invariant to fancy headlines, pictures or report framing, which would be the assumption of stimulus-driven attention models.

Sims, see Sims (2003) and Sims (2005), was the first to formalize such an allocation problem, using the concept of entropy from Shannon’s information theory as the measure of informativeness of a channel. In most parts of the literature a channel is simply a signal that is correlated with the future state of the world.

The approach of DeOliveira et al. (2013) generalizes Sims idea with a decision theoretic foundation. In their framework an agent follows a channel $\pi$ providing him with information on the state of the world $\omega \in \Omega$. The channel formalizes the likelihood of receiving a posterior $p$ by updating the prior $\bar{p}$ after an information update. Based on the received information the agent chooses an act out of a set of feasible acts, $f \in F$, where $f$ is a mapping associated with the consequence $f(\omega)$ for each state of the world. In terms of the newspaper example, a channel $\pi$ represents a time allocation over the subsections, while $f$ is the investment decision, for example buying or selling.

The general attention allocation problem can be written as an information acquisition problem:

$$\max_{\pi \in \Pi(\bar{p})} \left[ \int_{\Delta(\Omega)} \underbrace{\max_{f \in F} \left( \int_{\Omega} u\left(f(\omega)\right) p(d\omega) \right)}_{\text{Stage II}} \pi(dp) - c(\pi) \right] \tag{4.1}$$

The utility function on consequences is represented by $u(\cdot)$ and $\Pi(\bar{p})$ is the set of all channels for a given prior $\bar{p}$. The subjective information cost function $c(\cdot)$ captures the costs (in utilities) associated with each amount of information. In our example the costs depend both on the overall reading time and the reading difficulties of the selected subsections in the newspaper example.

The whole problem itself is a two-stage decision. In stage II the agent solves a standard utility maximizing problem for any posterior distribution $p$, meaning for any belief about the states of the world. The objective of stage I is to choose the optimal channel $\pi$ that maximize the information value with respect to the subjective costs.

In our example the investor decides on his optimal trading strategy given some allocation of time over the subsections in stage II. At stage I he chooses overall reading time and attention allocation such that his expected utility of choosing an investment plan is maximized, by weighting up the utility costs of additional reading time against the marginal benefit of obtaining better information, meaning a better posterior, on the state of the world and thus on the prospects of his investment decision.

Therefore the agent is totally rational in the sense that he optimizes both over information acquisition and investment actions. Due to the fact that information is subjectively costly, he will be inattentive to information that is not ex-ante promising to be useful in expectations relative to information costs. Most of the modelling done in rational inattention focuses on the variation of the subjective cost function $c(\cdot)$. See for example Hellwig et al. (2012) for a discussion on information choice technologies.

The rational inattention approach has been applied to diverse macroeconomic sub-fields such as sticky prices, see Sims (2003) and Woodford (2009), differences in the price reactions due to different shocks, see Mackowiak and Wiederholt (2009) and Matejka and McKay (2012), understanding the forward discount puzzle of the uncovered interest rate parity condition, see Bacchetta and VanWincoop (2005), business cycles, see Mackowiak and Wiederholt (2009), and consumption choice with asymmetric responses by wealth shocks, see Tutino (2013), or in finance for studying portfolio allocation decision, see Peng (2005), understanding home bias, see Mondria et al. (2010), sectoral instead of firm specific learning, see Peng and Xiong (2006), and under diversification, see VanNieuwerburgh and Veldkamp (2008). Other fields in economics where the rational inattention approach has been used are coordination games, see Hellwig and Veldkamp (2009), and business studies by investigating a team production problem with task specialization resulting in an emergency of organizational leadership, see Dessein et al. (2013). Veldkamp (2011) is a comprehensive source for further applications of the rational inattention concept.

Most of the literature mentioned above as well as our model build the cost function on the information theoretic concept of mutual information. The agent wants to know more about the normally distributed random variable $X$ with variance $\sigma_X^2$, but can only observe the signal $s$, where $X$ and $s$ have a multivariate normal distribution with conditional variance $\sigma_{X|s}^2$ of $X$. The unconditional entropy $H(\cdot)$ of $X$ is given by $H(X) = \frac{1}{2} log_2(2\pi e \sigma_X^2)$. This can be interpreted as a measure of uncertainty. The conditional entropy of $X$ after observing the signal $s$ is $H(X|s) = \frac{1}{2} log_2(2\pi e \sigma_{X|s}^2)$.

With these measures at hand one can calculate the mutual information the signal $s$ contains about the random variable $X$ and vice versa, by deducting the conditional entropy from the unconditional one $I(X;s) = H(X) - H(X|s)$. Equipped with the quantification of mutual information, limited attention capacities are modelled by a bound $\kappa$ on its per period average:

$$\mathbb{I}(X;s) \leq \kappa \tag{4.2}$$

Having discussed rational inattention we now turn to a short overview of overlapping generation models.

The overlapping generation framework is used within the field of asset pricing mostly when one wants to discuss the information aggregation process as Hellwig (1980) did in his seminal work, where he studied the implications for the information contained in the price when each agent has a different piece of information. He concluded that in large markets only the common element of information that is known to many agents is reflected in the equilibrium price.

Apart from this the overlapping generations framework is also the only framework explicitly modelling agents' interaction in a market and specifically for the financial market this model therefore represents a realistic approach. Biais et al. (2010) rely on these insights and study the equilibrium prices and portfolio selection when there are agents with asymmetric information sets in the market. In this setting the less informed agents face a winner´s curse problem and has to take this into considerations when deciding on the portfolio selection. Indexing fails and there are possibilities to outperform the market and momentum strategies.

This paper builds on this framework, since we are interested in the information choices, how these choices affect asset prices, and how agents interact with each other. Moreover a model based on this framework can be extended to incorporate heterogeneity of agents in two ways, at first in the signal itself and secondly in the information capacity constraint of each agent. We take advantage of both possibilities and are able to derive interesting results showing that attention is a relevant factor when one wants to understand asset price dynamics within a competitive market.

## 4.3 Model

This section starts with introducing the environment of the framework, followed by an analytical derivation of the solution of the simplest case of our model in detail.

### 4.3.1 Framework

There are $N$ assets in the economy and the dividend flow of each asset $n$ follows a stochastic process with a deterministic mean $\mu_n$ and variance $\sigma_n^2$.

The random component is represented by $\epsilon_{nt}$, which is a normally distributed random variable with mean 0 and variance 1. In each period $t$ the old generation $t-1$ is already in the economy owning the assets and a new generation $t$ is born consisting of a continuum of identical agents distributed uniformly on the unit interval with a constant population mass of one. The agents of generation t can only receive noisy signals about the dividends before they can trade. Limited by their information processing capability constraint they have to decide on how noisy the signal of each asset should be.

This implies the following order of events. Each agent $i \in [0,1]$ of the young generation $t$ first decides how to allocate his attention, then he receives his information on the dividends in period $t$ and decides on his trading strategy $q^i_{t+1}$. After this trading takes place, while the old generation $t-1$ will sell all its assets to finance their consumption in period $t$, the young generation $t$ will buy the assets in order so save for consumption in its second period. The prices clear markets. Finally, the dividend realizes and the residual of dividends minus payments for the bought assets will be consumed.

In period $t+1$ generation $t-1$ leaves the economy, generation $t$ will be the old generation and a new generation $t+1$ is born, repeating the procedure above. Figure 4.1 summarizes the order of events.

**Figure 4.1:** Timing of Events

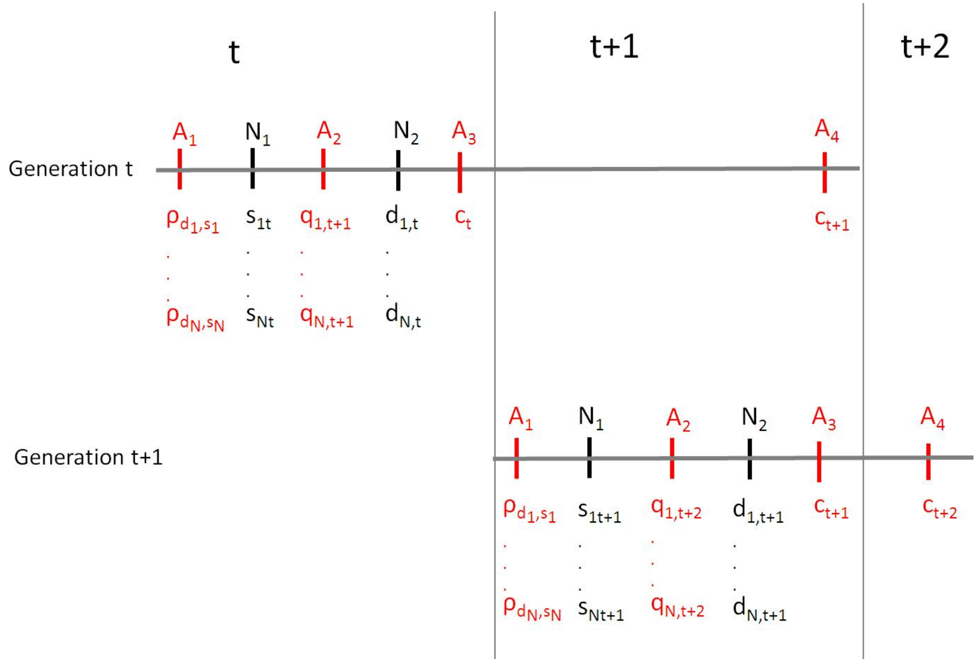


Figure 4.1 shows graphically how the overlapping generation model works. The life of generation t and t+1 are represented by the two grey time lines. $A_{1,2,3,4}$ mark events where the generation has to take an action or make a decision. $N_{1,2}$ stand for an occurrence by nature. $\rho_{d_n,s_n}$ is the correlation between signal n and dividend of asset n. $s_{nt}$ stands for the signal belonging to asset n in time period t. $q_{nt+1}$ is the number of asset n hold from period t to period t+1. $c_t$ and $c_{t+1}$ are the consumptions associated with the investment in period t and t+1 for generation t.

The signal of asset $n$ in period $t$ agent $i$ chooses to observe is taken from the set of all possible signal structures $\Gamma$ and consists of the future dividend plus noise $\tilde{\sigma}_n^i \psi_{nt}^i$, where $\tilde{\sigma}_n^i$ is the scaling parameter of the noise and $\psi_{nt}^i$ is normally distributed with mean 0 and variance 1. The information precision of each signal is measured by the amount of average mutual information $\mathbb{I}(\cdot)$ the signal contains about the dividend. This is also where the assumption of limited attention enters the model, since each agent $i$ has an upper bound $\kappa^i$ on the amount of information he can process $\mathbb{I}(\cdot)$. Thus $\kappa^i$ can be thought of as the maximum information processing capacity. We will refer to (4.5) as information processing constraint.

$$\begin{aligned} d_{n,t} &= \mu_n + \sigma_n \epsilon_{nt} &(4.3)\\ s_{n,t}^i &= \mu_n + \sigma_n \epsilon_{nt} + \tilde{\sigma}_n^i \psi_{nt}^i &(4.4)\\ \mathbb{I}\left(\{d_t\};\{s_t^i\}\right) &\leq \kappa^i &(4.5) \end{aligned}$$

Where $s_t^i$ is the vector of all signals chosen by agent $i$ and $d_t$ is the vector of the stochastic processes of all assets' dividends. The agent's inter-temporal rate of substitution is given by $\beta$.

With this notation at hand one can describe the two-stage decision problem of the agent:

$$\max_{s_t^i \in \Gamma} \mathbb{E}\left[u(c_t^i; s_t^i) + \beta u(c_{t+1}^i; s_t^i) \middle| s_t^i\right] \tag{4.6}$$

subject to the following constraints

$$\begin{aligned} \mathbb{I}\left(\{d_t\};\{s_t^i\}\right) &\leq \kappa^i &(4.7)\\ q_{t+1}^{i*} &= \arg\max_{q_{t+1}^i} \mathbb{E}\left[u(c_t^i; s_t^i) + \beta u(c_{t+1}^i; s_t^i) \middle| s_t^i\right] &(4.8)\\ c_t^i &= q_{t+1}^{i*}\left(d_t - p_t\right) &(4.9)\\ c_{t+1}^i &= q_{t+1}^{i*} p_{t+1} &(4.10) \end{aligned}$$

In the first stage (4.6) the agent decides on the signal structure he wants to receive taking his information processing constraint (4.7) into account. In the second stage he decides on his trading strategy (4.8) given the received signal of the chosen structure and the budget constraints (4.9) and (4.10).

Note that in this framework agent $i$ decides on the level of precision of the signal $s_t^i \in \Gamma$ and not a specific signal itself. Furthermore, following Sims (2003) and using Shannon entropy as an information measure implies that the precision level of a signal can be translated into the correlation between dividend and signal.

**Proposition 4.1** *Information Capacity Constraint*
*Given independent dividends and signals the information capacity constraint (4.5) can be written as:*

$$\underbrace{\frac{1}{2}\log_2\left(\frac{1}{1-\rho_1^2}\right)}_{\kappa_1} + \underbrace{\frac{1}{2}\log_2\left(\frac{1}{1-\rho_2^2}\right)}_{\kappa_2} .... + \underbrace{\frac{1}{2}\log_2\left(\frac{1}{1-\rho_N^2}\right)}_{\kappa_N} \leq \kappa \tag{4.11}$$

*Where $\rho_n$ is the chosen correlation parameter between asset n's dividend and the corresponding signal.*

**Proof:** See section 4.7.1. □

Solving the agent's decision problem one has to start at the second stage, finding the market price dependent on the signals received. Having a solution for the market price depended on the structure of the signals, the first stage of the problem can be addressed by weighting the usefulness of a more precise signal for one asset against less precise signals for all other assets and thus solving the attention allocation problem. The next section will demonstrate this with a particularly simple model.

### 4.3.2 Two Assets and One Agent Group

This section introduces the simplest case of our model in order to get some intuition towards the approach and how the equilibrium will be derived in augmented models afterwards. The results are similar to the ones when using a Lucas tree model such as Peng (2005) or Luo and Young (2010). Thus, this part does not add new insights. However, there are some interesting implications gained on the way of getting there.

We assume the case of an economy with two risky assets and one group of homogeneous agents with mean-variance utility, see Biais et al. (2010):

$$u(c) = \mathbb{E}\left[c\right] - \frac{\gamma}{2} Var\left[c\right] \tag{4.12}$$

$$u(c;s) = \mathbb{E}\left[c|s\right] - \frac{\gamma}{2} Var\left[c|s\right] \tag{4.13}$$

One can think of the two assets as shares in a company, bonds or a portfolio of financial products. Assuming further that one asset has a lower variance than the other one proposition 4.2 says that the agent will allocate more attention on tracking the more risky, meaning the more volatile, asset, since the gain in utility by eliminating uncertainty is higher with this asset. The larger the differences in the variance and the lower the information processing capacity $\kappa$ the more available attention is allocated to the asset with the higher variance.

Moreover, the part of the asset price the agent group cannot acquire any information about, as can be seen in equation (4.16), is unaffected by the attention allocation choice.

The part of the asset price, which the agent group can inform itself about, on the other hand is dependent on the attention allocation. Specifically, the higher the signal precision on one asset, the more weight the agent gives to the signal as compared to the fundamental mean value.

**Proposition 4.2** *Allocation in the Two Asset Case*

*If there are only two assets and one representative agent group with mean-variance utility, the demand vector for the assets $q^*$ is given by:*

$$q^* = \frac{(1-\beta)}{\gamma} A^{-1} \left( \Xi^2 \cdot (s_t - \mu) + \frac{\mu}{1-\beta} - p_t \right) \tag{4.14}$$

*$\Xi$ is the diagonal matrix of optimal correlations $\rho_n^*$ and $A$ the variance of future dividends and prices:*

$$A = \begin{pmatrix} \sigma_1^2(1-\rho_1^{*2}) + \beta\sigma_1^2\rho_1^{*2} & \\ & \sigma_2^2(1-\rho_2^{*2}) + \beta\sigma_2^2\rho_2^{*2} \end{pmatrix} \tag{4.15}$$

*Normalizing the number of shares to one, the price vector $p_t$ is given by:*

$$p_t = \Xi^2 \cdot (s_t - \mu) + \frac{\mu}{1-\beta} - \frac{\gamma}{(1-\beta)} A1 \tag{4.16}$$

*This in turn implies the optimal attention allocation structure, meaning the desired signal precision, which depends on the variance ratio of both dividend processes $\varkappa = \frac{\sigma_1}{\sigma_2}$, as:*

$$\rho_1^* = \begin{cases} \sqrt{1-\left(\frac{1}{4}\right)^\kappa} & if \quad \varkappa^2 \quad > \quad 4^\kappa \\ \sqrt{1-\frac{1}{\varkappa}\left(\frac{1}{2}\right)^\kappa} & if \quad \varkappa^2 \quad \in \quad \left[\frac{1}{4^\kappa}; 4^\kappa\right] \\ 0 & if \quad \varkappa^2 \quad < \quad \frac{1}{4^\kappa} \end{cases} \tag{4.17}$$

$$\rho_2^* = \begin{cases} 0 & if \quad \varkappa^2 \quad > \quad 4^\kappa \\ \sqrt{1-\varkappa\left(\frac{1}{2}\right)^\kappa} & if \quad \varkappa^2 \quad \in \quad \left[\frac{1}{4^\kappa}; 4^\kappa\right] \\ \sqrt{1-\left(\frac{1}{4}\right)^\kappa} & if \quad \varkappa^2 \quad < \quad \frac{1}{4^\kappa} \end{cases} \tag{4.18}$$

**Proof:** The solution to this problem is derived in four steps. The first step derives the asset demand, the second step solves for the market clearing price, the third step simplifies the attention allocation problem, and the fourth step finally solves the attention allocation problem.

**Step 1:** The market clearing price can be derived from the solution of (4.8) given an optimal attention allocation and the received signals $s_t$. The corresponding FOC (first order condition) is given by:

$$0 = \mathbb{E}\left[d_t|s_t\right] + \beta\mathbb{E}\left[p_{t+1}|s_t\right] - p_t - \gamma A q^* \tag{4.19}$$

$$0 = \Xi^2 \cdot (s_t - \mu) + \mu + \beta\mathbb{E}\left[p_{t+1}|s_t\right] - p_t - \gamma A q^* \tag{4.20}$$

This is a stationary problem and all the future periods will be the same in expectations given today's signal:

$$\mathbb{E}\left[p_{t+1}|s_t\right] = \mathbb{E}\left[d_{t+1}|s_t\right] + \beta\mathbb{E}\left[p_{t+2}|s_t\right] - \gamma A q^* \tag{4.21}$$

Iteratively substituting equation (4.21) in equation (4.19) leads to the following representation of the problem:

$$\begin{aligned} 0 = {} & \Xi^2 \cdot (s_t - \mu) + \mu + \sum_{t=1}^{T} \beta^t \left(\mu - \gamma A q^*\right) \\ & + \beta^{T+1}\left(\mathbb{E}\left[p_{t+T+2}\right]\gamma A q^*\right) - p_t - \gamma A q^* \end{aligned} \tag{4.22}$$

This can be simplified by taking the limit of $T \to \infty$:

$$\lim_{T\to\infty} \sum_{t=0}^{T} \beta^t \left(\mu - \gamma A q^*\right) = \frac{\mu}{1-\beta} + \frac{\gamma}{(1-\beta)} A q^* \tag{4.23}$$

$$\lim_{T\to\infty} \beta^{T+1}\left(\mathbb{E}\left[p_{t+T+2}\right]\gamma A q^*\right) = 0 \tag{4.24}$$

Rearranging terms one gets:

$$q^* \quad = \quad \frac{(1-\beta)}{\gamma} A^{-1} \left( \Xi^2 \cdot (s_t - \mu) + \frac{\mu}{1-\beta} - p_t \right) \tag{4.25}$$

Thus (4.25) is the optimal trading strategy of the agent group given the attention allocation $\Xi$ and the received signal $s_t$.

**Step 2:** In equilibrium the agent group has to hold all assets. Normalizing them to one, $q^* = 1$, yields the equilibrium market prices:

$$p_t \quad = \quad \Xi^2 \cdot (s_t - \mu) + \frac{\mu}{1-\beta} - \frac{\gamma}{(1-\beta)} A1 \tag{4.26}$$

**Step 3:** With the market clearing price, the agent can solve the attention allocation problem of the first stage, where he maximizes his utility $(I)$ compared to the case where he receives no signals $(II)$.

The attention allocation problem needs to be viewed from an individual agents point of view, who is competing against all other agents of his generation and the following one.

$$\max_{s_t^i \in \Gamma} \mathbb{E} \left[ \underbrace{u(c_t^*; s_t^i) + \beta u(c_{t+1}^*; s_t^i)}_{I} - \underbrace{u\left(c_t^{\Delta}\right) - \beta u(c_{t+1}^{\Delta})}_{II} \middle| s_t^i \right] \tag{4.27}$$

Since the agent can not influence the second period outcomes by his actions, since the single agent within a continuum is a price taker and in equilibrium every agent will hold the same amount of assets, the problem reduces to:

$$\max_{s_t \in \Gamma} \mathbb{E} \left[ u(c_t^*; s_t^i) - u(c_t^{\Delta}) \middle| s_t^i \right] \tag{4.28}$$

The relevant parameter for the signal choice is only the correlation $\rho_n$ between $d_{n,t}$ and $s_{n,t}$. In the context of our model $\rho_n$ is dependent on $\tilde{\sigma}_n$, the only free parameter, since all noise terms are assumed to be independent. The correlation $\rho_n$ therefore only depends on the variance of the additional noise term of the signal:

$$\begin{aligned}
\rho\left(d_{nt}, s_{nt}\right) &= Cor\left(\mu_n + \sigma_n \epsilon_{nt}, \mu_n + \sigma_n \epsilon_n \tilde{\sigma}_n \psi_n\right) && (4.29)\\
&= Cor\left(\sigma_n \epsilon_{nt}, \sigma_n \epsilon_n + \tilde{\sigma}_n \psi_n\right) && (4.30)\\
&= \frac{Cov\left(\sigma_n \epsilon_{nt}, \sigma_n \epsilon_n + \tilde{\sigma}_n \psi_n\right)}{\sqrt{Var\left(\sigma_n \epsilon_{nt}\right) Var\left(\sigma_n \epsilon_n + \tilde{\sigma}_n \psi_n\right)}} && (4.31)\\
&= \frac{\sigma_n^2}{\sqrt{\sigma_n^2\left(\sigma_n^2 + \tilde{\sigma}_n^2\right)}} && (4.32)\\
&= \frac{\sigma_n}{\sqrt{\left(\sigma_n^2 + \tilde{\sigma}_n^2\right)}} && (4.33)\\
&= \rho_n \in ]0, 1] && (4.34)
\end{aligned}$$

Consequently, the signal structure can be seen as only dependent on the correlation parameters $\rho_1$ and $\rho_2$.[1] Combining this with the fact that in equilibrium the expected utility without any informative signal $E\left[u(c_t^{\Delta})\right]$ is constant for all choices of correlation one can reduce the optimization problem regarding the signals to:

$$\max_{\{\rho_1,\rho_2\}\in[0,1]} \mathbb{E}\left[u(d_t; s_t)|s_t\left(\rho_1, \rho_2\right)\right] \tag{4.35}$$

Assuming mean-variance utility it follows that:

$$\max_{\{\rho_1,\rho_2\}\in[0,1]} \mathbb{E}\left[\mathbb{E}\left[d_t|s_t\left(\rho_1,\rho_2\right)\right] - \frac{\gamma}{2} Var\left[d_t|s_t\right]\Big| s_t\left(\rho_1,\rho_2\right)\right] \tag{4.36}$$

Since we have assumed two assets the aggregated dividend is $d_t = d_{1,t} + d_{2,t}$. This leads to the following representation of our optimization problem:

$$\max_{\{\rho_1,\rho_2\}\in[0,1]} \mathbb{E}\left[\mathbb{E}\left[d_{1t} + d_{2t}|s_t\right] - \frac{\gamma}{2} Var\left[d_{1t} + d_{2t}|s_t\right]\Big| s_t\left(\rho_1,\rho_2\right)\right] \tag{4.37}$$

[1] Since for a corner solution, meaning $\rho_n = 0$, the signal is irrelevant, we can simply set $\tilde{\sigma}_n^2 = \eta$ for any $\eta \in \mathbb{N}$ to close the set.

This can be decomposed to:

$$\max_{\{\rho_1,\rho_2\}\in[0,1]} \mathbb{E}\left[d_{1t}|s_t\left(\rho_1,\rho_2\right)\right] + \mathbb{E}\left[d_{2t}|s_t\left(\rho_1,\rho_2\right)\right] - \frac{\gamma}{2}Var\left[d_{1t}+d_{2t}|s_t\left(\rho_1,\rho_2\right)\right] \tag{4.38}$$

As we are looking for the optimal correlation of the signals and the fundamentals, all constant parameters or level variables can be neglected for the optimization problem and the only uncertainty arises in the second moments. Furthermore, both dividend processes are uncorrelated and thus an equivalent optimizing problem is given by:

$$\max_{\{\rho_1,\rho_2\}\in[0,1]} -Var\left[d_{1t}|s_t\left(\rho_1,\rho_2\right)\right] - Var\left[d_{2t}|s_t\left(\rho_1,\rho_2\right)\right] \tag{4.39}$$

Applying the rules for dependent mean and variance of correlated normally distributed variables:[2]

$$\max_{\{\rho_1\rho_2\}\in[0,1]} -\sigma_1^2\left(1-\rho_1^2\right) - \sigma_2^2\left(1-\rho_2^2\right) \tag{4.40}$$

Again ignoring all constant parameters for the optimization, the reduced form is:

$$\max_{\{\rho_1,\rho_2\}\in[0,1]} \sigma_1^2\rho_1^2 + \sigma_2^2\rho_2^2 \tag{4.41}$$

**Step 4:** For tractability we replace $\{\rho_1^2,\rho_2^2\}$ by $\{\xi_1,\xi_2\}$ in the reduced optimization problem:

$$\max_{\{\xi_1,\xi_2\}\in[0,1]} \sigma_1^2\xi_1 + \sigma_2^2\xi_2 \tag{4.42}$$

subject to the information processing constraint

$$\frac{1}{2}\log_2\left(\frac{1}{1-\xi_1}\right) + \frac{1}{2}\log_2\left(\frac{1}{1-\xi_2}\right) \le \kappa \tag{4.43}$$

[2] $(x_1|x_2=a) \sim N\left(\hat{\mu},\hat{\Sigma}\right)$, with $\hat{\mu} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}\left(a-\mu_2\right)$ and $\hat{\Sigma} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$.

Since the objective function is increasing in both choice variables the information processing constraint will be binding at any maximum. Therefore one can rewrite this constraint imposing strict equality. Reformulating the $\log_2$'s to ln's we arrive at:

$$\frac{\ln(\frac{1}{1-\xi_1})}{\ln(2)} + \frac{\ln(\frac{1}{1-\xi_2})}{\ln(2)} = 2\kappa \tag{4.44}$$

$$\exp\left(2\kappa\ln(2)\right) = \frac{1}{1-\xi_1}\frac{1}{1-\xi_2} \tag{4.45}$$

$$\left(1-\xi_1\right)\left(1-\xi_2\right) = \frac{1}{\exp\left(2\kappa\ln(2)\right)} \tag{4.46}$$

$$\xi_1+\xi_2-\xi_1\xi_2 = \underbrace{1-\frac{1}{\exp\left(2\kappa\ln(2)\right)}}_{\alpha} \tag{4.47}$$

$$\xi_1+\xi_2-\xi_1\xi_2-\alpha = 0 \tag{4.48}$$

The first order condition of the corresponding Lagrange auxiliary function

$$\mathcal{L} = \sigma_1^2\xi_1+\sigma_2^2\xi_2+\lambda\left(\alpha-\xi_1-\xi_2+\xi_1\xi_2\right) \tag{4.49}$$

are:

[$\xi_1$ :]

$$\sigma_1^2+\lambda\left(\xi_2-1\right)=0 \tag{4.50}$$

[$\xi_2$ :]

$$\sigma_2^2+\lambda\left(\xi_1-1\right)=0 \tag{4.51}$$

[$\lambda$ :]

$$\alpha-\xi_1-\xi_2+\xi_1\xi_2=0 \tag{4.52}$$

Dividing (4.50) by (4.51)

$$\underbrace{\frac{\sigma_1^2}{\sigma_2^2}}_{\varkappa^2} = \frac{1-\xi_2}{1-\xi_1} \tag{4.53}$$

The inner solution follows from a reformulation of (4.52) leading to the trajectory:

$$\xi_2 = \frac{\alpha - \xi_1}{1 - \xi_1} \tag{4.54}$$

Plugging (4.54) into (4.53) yields:

$$\begin{aligned} \varkappa^2 &= \frac{1 - \frac{\alpha - \xi_1}{1 - \xi_1}}{1 - \xi_1} && (4.55) \\ &= \frac{1 - \alpha}{(1 - \xi_1)^2} && (4.56) \end{aligned}$$

Solving for $(1 - \xi_1)$:

$$\begin{aligned} (1 - \xi_1)^2 &= \frac{1 - \alpha}{\varkappa^2} && (4.57) \\ 1 - \xi_1 &= \pm\sqrt{1 - \alpha}\frac{1}{\varkappa} && (4.58) \end{aligned}$$

Since $\{\xi_1, \xi_2\} \in [0, 1]$, the only plausible solution is given by:

$$\xi_1^* = 1 - \sqrt{1 - \alpha}\frac{1}{\varkappa} \tag{4.59}$$

From (4.53) one can derive the condition for a corner solution, meaning of a state, in which the second asset will be neglected, specifically $\xi_1^* = \alpha = 1 - \left(\frac{1}{4}\right)^\kappa$ and $\xi_2^* = 0$, as:

$$4^\kappa < \varkappa^2 \tag{4.60}$$

Using $\kappa_1 = \frac{1}{2}\log_2\left(\frac{1}{1-\xi_1}\right)$ it follows that:

$$\kappa_1 = \begin{cases} \kappa & if \quad \varkappa^2 > 4^\kappa \\ \frac{1}{2}\kappa + \frac{1}{2}\log_2(\varkappa) & if \quad \varkappa^2 \in \left[\frac{1}{4^\kappa}; 4^\kappa\right] \\ 0 & if \quad \varkappa^2 < \frac{1}{4^\kappa} \end{cases} \tag{4.61}$$

Thus after reconverting $\xi_1$ into $\rho_1$:

$$\rho_1^* = \begin{cases} \sqrt{1-\left(\frac{1}{4}\right)^{\kappa}} & if \quad \varkappa^2 > 4^{\kappa} \\ \sqrt{1-\frac{1}{\varkappa}\left(\frac{1}{2}\right)^{\kappa}} & if \quad \varkappa^2 \in \left[\frac{1}{4^{\kappa}}; 4^{\kappa}\right] \\ 0 & if \quad \varkappa^2 < \frac{1}{4^{\kappa}} \end{cases} \tag{4.62}$$

and due to symmetry:

$$\rho_2^* = \begin{cases} 0 & if \quad \varkappa^2 > 4^{\kappa} \\ \sqrt{1-\varkappa\left(\frac{1}{2}\right)^{\kappa}} & if \quad \varkappa^2 \in \left[\frac{1}{4^{\kappa}}; 4^{\kappa}\right] \\ \sqrt{1-\left(\frac{1}{4}\right)^{\kappa}} & if \quad \varkappa^2 < \frac{1}{4^{\kappa}} \end{cases} \tag{4.63}$$

□

How to derive an numeric solution for the N asset case by an approximation of the resulting boundary conditions is presented in section 4.7.5.

Under the information processing constraint information becomes valuable. The value of the information processing capacity can be measured by the expected excess price the agent group attributes to the asset under the information processing constraint $\kappa$ as compared to the uninformed state.

In the one asset case $\rho^{*2}$ is directly given by

$$\rho^{*2} = \xi^* = \alpha = 1 - \frac{1}{4^{\kappa}} \tag{4.64}$$

Taking expectations of the price in the case without any signal:

$$\mathbb{E}[p] \quad = \quad \frac{\mu - \gamma\sigma^2}{1-\beta} \tag{4.65}$$

Doing the same in the constrained case with a binding information processing constraint (4.5):

$$\mathbb{E}[p] = \frac{\mu - \gamma\sigma^2}{1-\beta} + \gamma\alpha\sigma^2 \tag{4.66}$$

The excess value is bounded by $\gamma\sigma^2$ and we can define the relative excess value, meaning which part of the maximal excess value is achieved by $1 - \frac{1}{4^\kappa}$. Defining the excess value as a function of $\kappa$ leads to:

$$X(\kappa) = \gamma\sigma^2\left(1 - \frac{1}{4^\kappa}\right) \tag{4.67}$$

In the two asset case one can focus on the inner solutions, since corner solutions collapse into the one asset case:

$$\begin{aligned} X(\kappa) &= 2\gamma\left(\left(1 - \frac{\sigma_2}{\sigma_1}\sqrt{\frac{1}{4^\kappa}}\right)\sigma_1^2 + \left(1 - \frac{\sigma_1}{\sigma_2}\sqrt{\frac{1}{4^\kappa}}\right)\sigma_2^2\right) &(4.68)\\ &= 2\gamma\left(\sigma_1^2 + \sigma_2^2\right) - 2\gamma\sqrt{\frac{1}{4^\kappa}}\left(\sigma_1\sigma_2 + \sigma_1\sigma_2\right) &(4.69)\end{aligned}$$

The upper bound is given by $2\gamma\left(\sigma_1^2 + \sigma_2^2\right)$. Obviously this excess value is structurally different as compared to the one asset case.

When looking at the symmetric case $\sigma_1 = \sigma_2$ the excess value changes to:

$$X(\kappa) = 4\gamma\sigma^2\left(1 - \frac{1}{4^\kappa}\right) \tag{4.70}$$

In this case we can again define the relative excess value again as $1 - \frac{1}{4^\kappa}$.

We will study the value of information processing capacity in more detail later on, when having the possibility to interact with other agent groups can lead to actual gains and not only increases in utility reflected by the asset price. For now it is enough to point out the definitive value of information processing capacity within our model.

# 4.4 Model Extensions

Having developed and discussed the simplest form of our model we now turn to extensions of interest. At first we will modify our baseline model to derive testable implications from it. The extension incorporates different groups of agents, who receive different signals, but are otherwise identical. This allows for heterogeneity in information. In a second step, we will add exogenous shifts in the long-term mean of the underlying dividend flow and discuss the resulting implications.

## 4.4.1 Testing Model Implications

In this section we extend the baseline model by incorporating heterogeneity in signals. From the resulting model one can derive first empirical testable hypotheses.

**Proposition 4.3** *Price Variance*
*Assume that there are G agent groups with independent signals and one asset in the economy, then the price formula is given by:*

$$p_t = \rho^{*2}\left(\frac{1}{G}\sum_{g=1}^{G} s_{gt} - \mu\right) + \frac{\mu}{1-\beta} - \frac{\gamma}{(1-\beta)}A1 \tag{4.71}$$

*with*

$$A = \sigma^2(1-\rho^{*2}) + \beta\left(\sigma^2 + \frac{1}{G}\tilde{\sigma}^2\right)\rho^{*4} \tag{4.72}$$

*Consequently the variance of the price of the first asset can be written as:*

$$Var\left(p_t\right) = \left(\sigma^2 + \frac{1}{G}\tilde{\sigma}^2\right)\rho^{*4} \tag{4.73}$$

**Proof:** See section 4.7.2. □

Proposition 4.3 directly links the price variance of an asset to the variance of the underlying dividend process, the attention allocated on the asset, and the number of independent signals, meaning the number of different agent groups G.

For heterogeneously informed market participants, meaning a very high G, the volatility of an asset price depends only on the variance of the dividend and the attention allocation, given a not too small attention allocation and thus not a too big signal variance. On the other hand, if the number of heterogeneously informed groups G decreases the variance of the price increases, as long as the information processing capacity stays constant.

In a next step we will rewrite our model in a time series regression form in order to fit it to data later on. Assuming a large G, which should be the case when looking at an highly liquid asset or basket of assets, the $\frac{1}{G}\tilde{\sigma}^2$ term becomes insignificantly small:

$$Var\left(p_t\right) = \sigma^2\rho^{*4} \tag{4.74}$$

To bring the model to the data one has further to assume an exogenously given attention allocation, since there is no possibility to solve the attention allocation problem without knowing $\kappa$, the number of possible sources of risk, and their fundamental variance. Taking the root and replacing $\rho^{*2}$ by $\alpha\left(\kappa\left(t\right)\right)$, see (4.64), as well as introducing time dependency leads to the following form of the model:

$$\sigma\left(p_t\right)(t) \quad = \quad \sigma\alpha\left(\kappa\left(t\right)\right) \tag{4.75}$$

As our asset we chose the S&P 500 index. A reasonable choice for an exogenously given attention allocation measure on the S&P 500 would be the Google Investing Index ($GII$), which captures all finance related Google searches in the US. Since we are not interested in the S&P 500 itself but in its instantaneous volatility, we take the VIX S&P 500 implied volatility index as a proxy. Furthermore we assume $GII(t) \sim \alpha(\kappa(t))$ with a coefficient of proportionality of $\chi$ and normalize all values.[3] For our regression we take daily data from September 23rd, 2008 until September 23rd, 2013. This is equal to 1282 observations. The VIX S&P 500 is obtained from Datastream and the Google Investing Index from the Google website.[4]

[3] The normalization is performed by subtracting 0.38, which makes the intercept in the later on performed regression approximately zero. Since we don't know the absolute level of attention at any time and only assume relative changes, setting the intercept to zero is a valid normalization choice.

[4] http://www.google.com/finance?cid=2001035, viewed September 23rd, 2013

**Figure 4.2:** S&P 500 Volatility and Google Investing Index

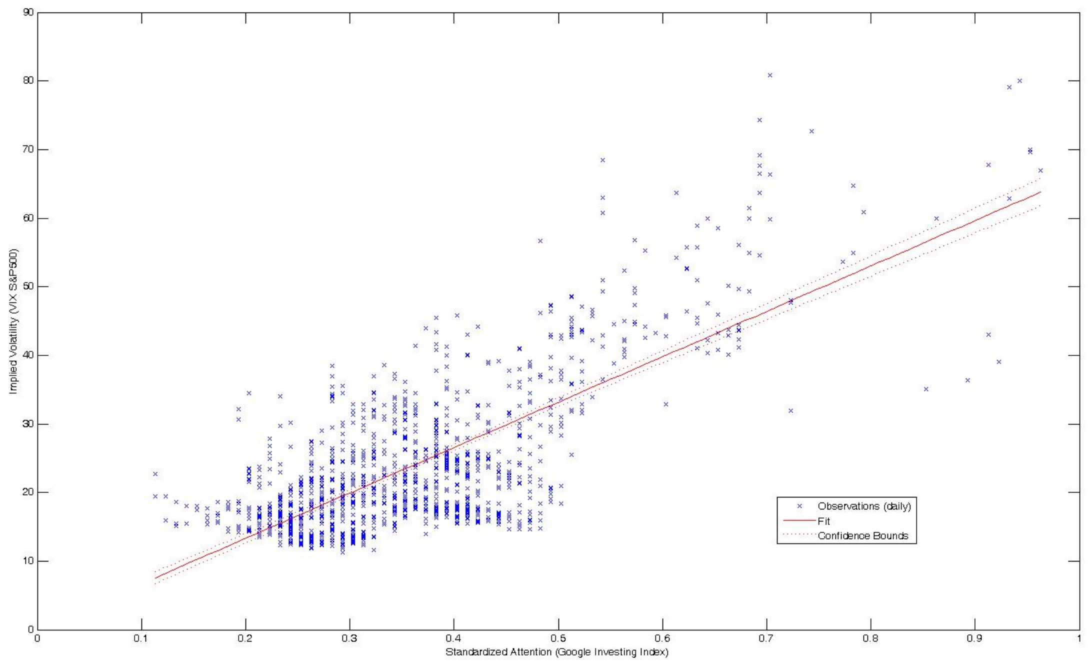


Figure 4.2 shows the VIX S&P500 implied volatility index (Datastream) and the Google Investing Index (Google Finance), normalized by subtracting 0.38, during the time from September 23rd, 2008 until September 23rd, 2013. The number of observations is 1282. Further the resulting fit of the corresponding linear regression is shown, as well as it's 95% confidence bounds. The results of the regression are portrayed in table 4.1.

**Table 4.1:** S&P 500 Volatility and Google Investing Index

| $\sigma\,(S\&P500)\,(t)$ | Intercept | $\chi$ | adj. $R^2$ |
|---|---|---|---|
| $\sim GII\,(t)$ | 0.09 | 66.17*** | 0.55 |

This table shows the results of the time series regression of the VIX S&P 500 implied volatility index on the normalised Google Investing Index. The observation period is consisting of 1282 observations from September 23rd, 2008 until September 23rd, 2013. Statistical significance is indicated by *, **, and *** at the 10%, 5%, and 1% level respectively. The intercept value is a result of the normalisation process and should not be interpreted in this regression. The corresponding F-statistic of the model against the constant model is 159.

If the attention measured by the Google Investing Index would be irrelevant one would see a symmetric cloud and an insignificant regression coefficient $\chi$. As figure 4.2 and our regression analysis show this is not the case. We find an adjusted $R^2$ value of 0.55 and a regression coefficient for the Google Investing Index significant at the 1% level as well as a very high F-statistic of 159 of our model.[5]

Looking at these results the first question, which comes to mind, is of course causality. Since the Google Investing Index is only available as a seven day's average, it consists mostly of data prior to the VIX S&P 500 volatility index.

Normally the next step towards determining causality would be a test of Granger causality, which unfortunately is not feasible in this particular case, since the underlying time series are non-stationary and the Google Investing Index's first difference process is quasi discrete. However, Da et al. (2011) show using Google search data on Bloomberg stock ticker numbers that Google search volume leads other attention measures, such as extreme returns or

[5]The results of this regression should be interpreted with care, since the underlying time seriousness are non-stationary, which might lead to a spurious regression analysis. Given the long-term mean reversion character of volatility and the Google Investing Index it should however not be a critical issue in this case.

news. Thus it is quiet save to assume causality to go the way from attention to volatility and not the other way around.

These findings are also in line with other recent empirical studies. For example Preis et al. (2013) provide evidence for the predictive power of changes in Google search volume and Moat et al. (2013) show the numbers of readers of Wikipedia articles related to financial topics are "early warning signs" for stock markets moves.

These findings have three major implications towards how information travels in financial markets and how efficiently it is incorporated into the price.

First, the information flow, which is represented by the signals in our model, appears to be limited by the capacity to process information and by how much attention or information processing capacity is spent on a particular source of uncertainty.

Second, this clearly contradicts the efficient market hypothesis, at least in the semi-strong and strong form, see Fama (1970). According to the efficient market hypothesis all publicly available information should always be rapidly incorporated into the price and thus there should be no relationship between Google search volume and volatility. It appears however that the attention to the information is relevant, not just its availability.

Third, information is incorporated efficiently into the price if the allocated information processing capacity is sufficiently high. Consequently, the market is supposed to be efficient during repeated events, which make information available like for example earning announcements. All this implies that testing for market efficiency in an event study context as pioneered by Fama et al. (1969) is problematic, since one is testing only market efficiency of processing information conditional on high attention allocation. Thus, rephrasing it, such an event study picks situations, during which information is better processed than in normal times, in order to proof that information is incorporated into the asset prices by financial markets efficiently.

### 4.4.2 One Asset with Shift

The empirical study of the last subsection that has shown how much available information is actually incorporated into the price depends highly on the information processing capacity allocated to this task. Having a solution method for our model at hand, we focus now on the topic of shifts in fundamentals of the dividend process and when they are seen in asset prices. Thus we take advantage of the intertemporal framework of our model.

Specifically, we look at the behaviour of an asset from the end of one equilibrium state given by an exogenous shock in the long-term mean of the dividend process till the next shock. To this end we augment our model by a shift $\tilde{\mu}$ in the fundamental dividend process, which is distributed normally with mean $\mu_0$ and variance $\sigma_2^2$. The agent receives two signals, one about the dividend itself $s_{1t}$ and one about the shift in the mean $s_{2t}$. Thus the dividend and signal processes look as follows:

$$\begin{aligned} d_{1t} &= \mu + \sigma_1 \epsilon_{1t} + \tilde{\mu} && (4.76) \\ \tilde{\mu} &= N(\mu_0, \sigma_2^2) && (4.77) \\ s_{1t} &= \mu_1 + \sigma_1 \epsilon_{1t} + \tilde{\sigma}_1 \psi_{1t} && (4.78) \\ s_{2t} &= \tilde{\mu} + \tilde{\sigma}_2 \psi_{2t} && (4.79) \end{aligned}$$

All noise terms are independent of each other and the correlations, $\rho_1$ and $\rho_2$, of $d_1$ with $s_{1t}$ and $\tilde{\mu}$ with $s_{2t}$ depend only on the variance of the additional noise terms. The best guess of the agent about $\tilde{\mu}$ is denoted with $\mu_t$, which is the weighted mean between the agent's signal on the mean $s_{2t}$ and his previous guess $\mu_{t-1}$, thus $\mu_t = \rho_2^2 s_{2t} + (1-\rho_2^2)\mu_{t-1}$. To ensure a stationary problem and since the agent does not know if a shift has occurred every generation assumes its information about $\tilde{\mu}$ to have variance $\sigma_2^2$, meaning the same quality.

**Proposition 4.4** *Attention Allocation with Shift*
*If there is only one asset with a shift in the mean of the dividend process and one agent group with mean-variance utility, the price for the asset is given by:*

$$p_t = \rho_1^{*2}(s_{1t} - \mu) + \frac{\rho_2^{*2}(s_{2t} - \mu_t) + \mu + \mu_t}{1-\beta} - \frac{\gamma}{(1-\beta)} A1 \tag{4.80}$$

*The variance is given by:*

$$A = \sigma_1^2(1-\rho_1^{*2}) + \frac{\sigma_2^2}{(1-\beta)^2}(1-\rho_2^{*2}) + \beta\sigma_1^2\rho_1^{*2} + \frac{\beta}{(1-\beta)^2}\rho_2^{*2}\sigma_2^2 \tag{4.81}$$

*This implies the attention allocation:*

$$\rho_1^* = \begin{cases} \sqrt{1-\left(\frac{1}{4}\right)^{\kappa}} & if \quad \varkappa^2 > 4^{\kappa} \\ \sqrt{1-\frac{1}{\varkappa}\left(\frac{1}{2}\right)^{\kappa}} & if \quad \varkappa^2 \in \left[\frac{1}{4^{\kappa}}; 4^{\kappa}\right] \\ 0 & if \quad \varkappa^2 < \frac{1}{4^{\kappa}} \end{cases} \tag{4.82}$$

$$\rho_2^* = \begin{cases} 0 & if \quad \varkappa^2 > 4^{\kappa} \\ \sqrt{1-\varkappa\left(\frac{1}{2}\right)^{\kappa}} & if \quad \varkappa^2 \in \left[\frac{1}{4^{\kappa}}; 4^{\kappa}\right] \\ \sqrt{1-\left(\frac{1}{4}\right)^{\kappa}} & if \quad \varkappa^2 < \frac{1}{4^{\kappa}} \end{cases} \tag{4.83}$$

*with* $\varkappa = \frac{\sigma_1}{\sigma_2}(1-\beta)$.

**Proof:** See section 4.7.3. □

The main difference to the model without a shift is that the attention allocation does not only depend on the signal to noise ratio any more. The time preference also influences the attention allocation on the long-term dividend mean or on the dividend today. The less the agent is concerned about tomorrow, meaning the lower the $\beta$, the less he focuses on the mean shift. At this point it needs to be stressed that a constant $\beta$ over all generations is assumed.

**Proposition 4.5** *Intertemporal Attention Allocation*

*The attention allocation varies with the time preferences $\beta$. The higher $\beta$ the lower will be $\rho_1^*$ and the higher will be $\rho_2^*$.*

**Proof:** The proposition follows immediately from $\varkappa = \frac{\sigma_1}{\sigma_2}(1-\beta)$ and its influence on the allocation scheme. □

### 4.4.3 Simulation and Empirical Evidence of the Shift Model

The partial neglect of the fundamentals leads to a lacked adjustment towards the new equilibrium. Figure 4.3 shows one simulated sample price path, where the shift, $\tilde{\mu} = -2$, in period $T$ is considerably large. The simulation was performed in Matlab using $\beta = 0.9$, $\gamma = 0.2$, $\kappa = 0.09$, $\mu = 25$, $\mu_0 = 0$, $\sigma_1^2 = 10$, and $\sigma_2^2 = 0.1$ as the underlying market parameters. Since $\frac{\sigma_1}{\sigma_2}(1-\beta) = 1$ the attention is equally distributed in this case. For the sake of simplicity we will choose the model specification always in such a manner, that attention is allocated equally between the long- and short-term sources of uncertainty when performing simulations.

**Figure 4.3:** Sample Path of a Shift

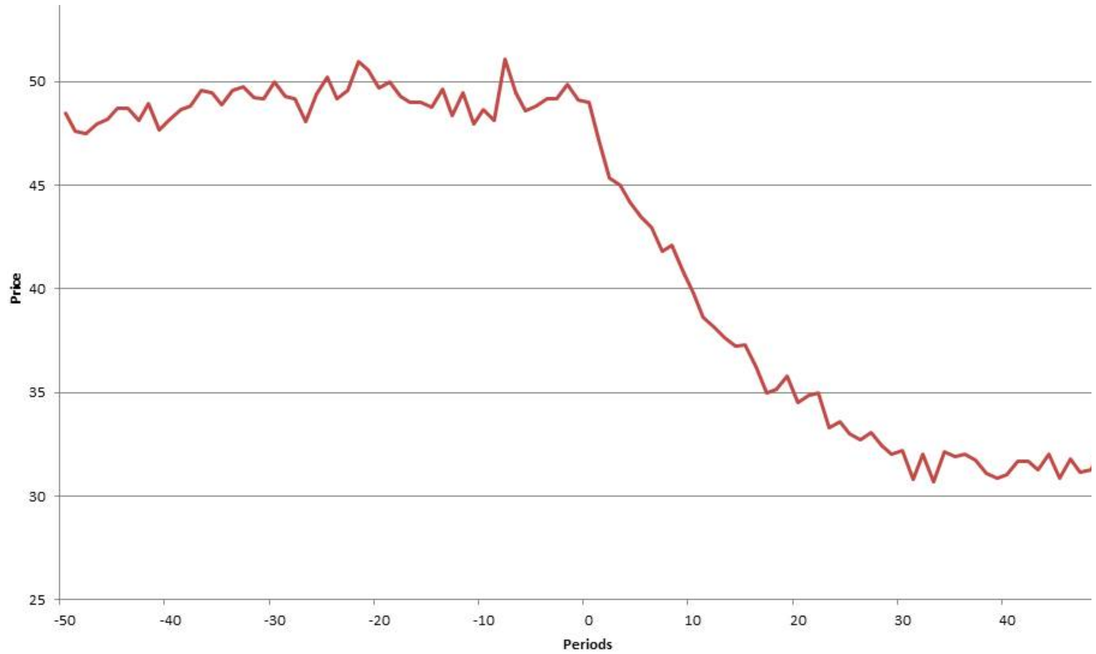


Figure 4.3 shows one sample path of a simulated asset price during a shift. The period of the shift is $t = 0$. The whole graph was simulated with $T = 0$, $\beta = 0.9$, $\gamma = 0.2$, $\kappa = 0.09$, $\mu = 25$, $\mu_0 = 0$, $\sigma_1^2 = 10$, $\sigma_2^2 = 0.1$, and $\tilde{\mu} = -2$ as the underlying market parameters.

One can see that the adjustment time of the price shown in figure 4.3 is considerably long. Obviously the two factors most important for determining the adjustment time are the size of the shift and the information processing capacity $\kappa$.

To give an overview of the interdependencies of these three variables we perform a sensitivity analysis of the adjustment time as a function of the information processing capacity and the shock in the long-term mean of the underlying dividend process. Figure 4.4 shows this analysis for $\kappa \in \{0.10, 0.11, ..., 0.20\}$ and $\tilde{\mu} \in \{1.0, 1.5, ..., 6.0\}$, measured in standard deviations $\sigma_2$. The adjustment time is given by the mean adjustment time over 10000 Monte Carlo simulations at each node and defined as the time it takes the price to reach its new theoretical long-term mean for the first time after the shift occurred.

**Figure 4.4:** Adjustment Time Depended on $\kappa$ and the Shift Size

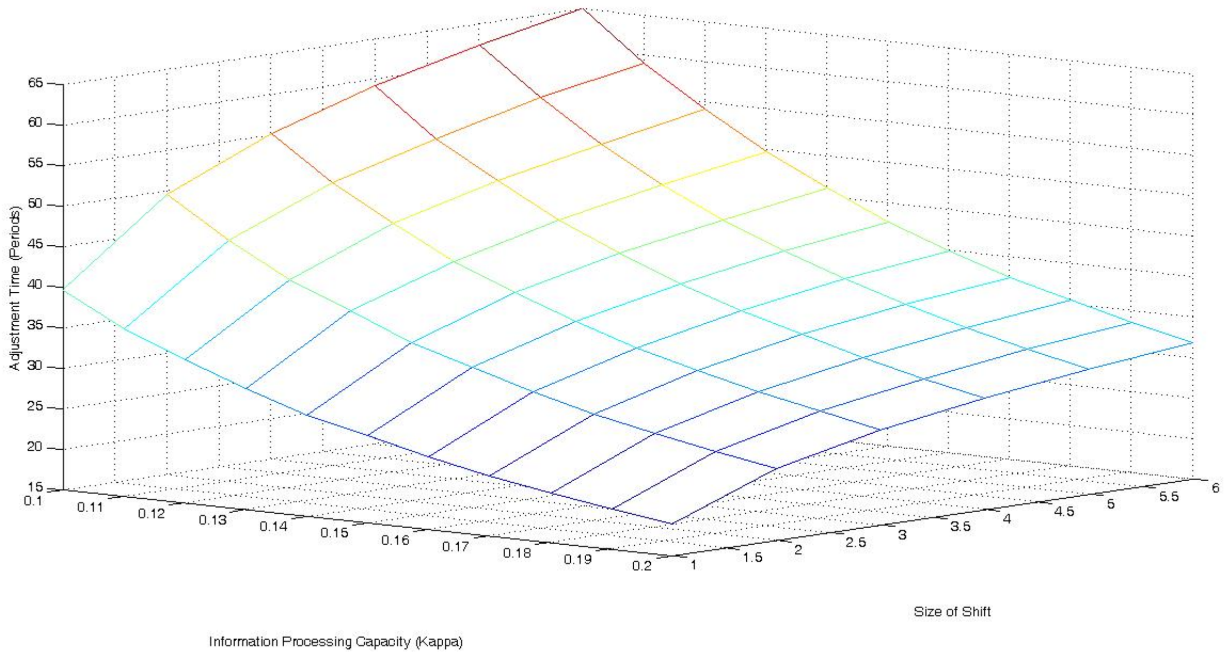


Figure 4.4 shows the mean adjustment time, meaning the time starting from when the shock happened until the price reaches the new theoretically implied long-term mean level for the first time. The graph presents the mean adjustment time of 10000 Monte Carlo simulations at each node using $T = 0$, $\beta = 0.9$, $\gamma = 0.5$, $\mu = 20$, $\mu_0 = 0$, $\sigma_1^2 = 10$, and $\sigma_2^2 = 0.1$ as the underlying market parameters.

Looking at figure 4.4 one can see an overproportional increase in the adjustment time with lower information capacity $\kappa$ and an underproportional increase with shift size.

Having illustrated the properties of our model by two simulation studies we turn now to an empirical example. Perhaps the best example data-wise of our model is the burst of the US subprime bubble, since it represents a major shift in an asset price during a period for which Google search data is available.

Figure 4.5 shows the Case-Shiller Home Price 20 City Composite index and the cumulative Google Trend search results of "subprime" in the period of January 2006 to December 2011. The Case-Shiller Home Price 20 City Composite is an index of the home prices of the 20 major metropolitan areas in the US. The index is published monthly by Standard & Poor's. It uses the Karl Case and Robert Shiller method of a house price index, which is a modified version of the weighted repeat sales methodology.[6] The cumulative Google Trend search results of "subprime" were directly obtained from Google Trend.[7] Since there exists a base rate of non-financial related searches for "subprime" only each monthly value in excess of the long-term sample mean of 12 in the original measure of Google Trends were used.

[6] http://www.spindices.com/index-family/real-estate/sp-case-shiller, viewed September 25th, 2013

[7] http://www.google.de/trends/, viewed September 23rd, 2013

**Figure 4.5:** Case-Shiller Home Price Index and Google Trend Search "Subprime"

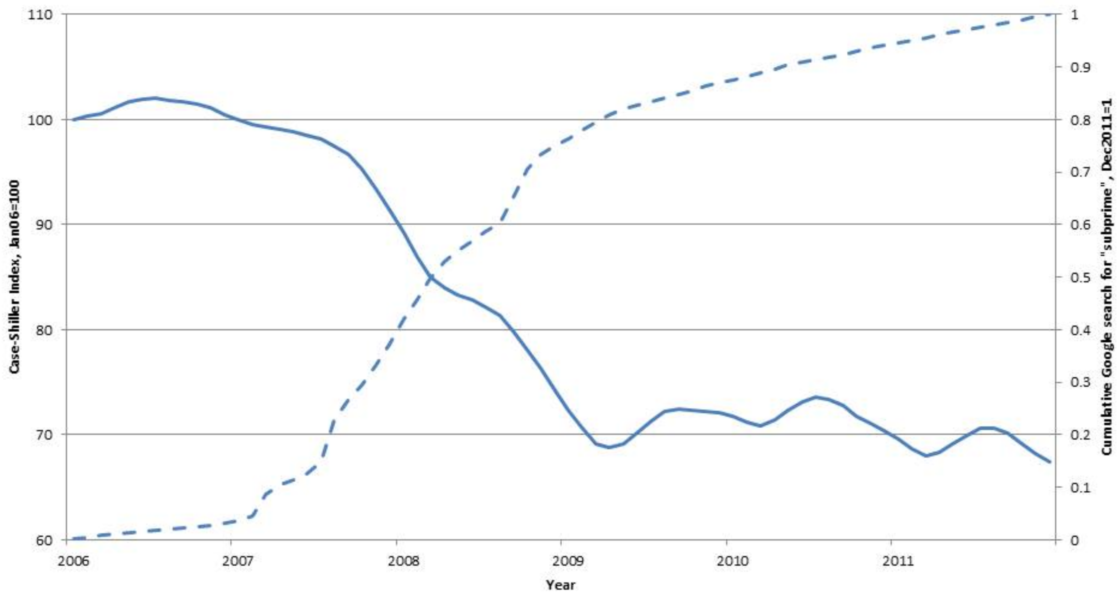


Figure 4.5 shows the Case-Shiller Home Price 20 City Composite index (left axis) with January 2006 as basis 100 and the cumulative Google Trend search results for "subprime" (right axis) with its December 2011 set to 1. The data of the Case-Shiller index is obtained from Datastream. The cumulative Google Trend search results of "subprime" were directly obtained from Google Trend. Since there exists a base rate of non-financial related searches for "subprime" each monthly value in excess of the long-term sample mean of 12 in the original measure of Google Trend was used.

As one can see, the increase in Google search volume for "subprime" presides the major downturn in the second half of 2007 in the Case-Shiller Home Price Index.

During the price correction more and more available information about the housing market is absorbed by market participants informing themselves, which is measured by the number of cumulative Google searches for this topic. This also implies that traders with an informational advantage could gain a lot out of their obtained information. We will return to this question in section 4.5.2, where we explicitly model a market of heterogeneous agents with respect to their information processing capacity constraint during a shift.

Going back to the question of how efficient information is processed by the market, the burst of the subprime bubble is a good example for this not being the case and how it takes a long time for the information to spread as more and more market participants inform themselves about the underlying situation. It is important to point out again that only singular events can provide evidence toward market inefficiencies as discussed in section 4.4.1.

### 4.4.4 Implications for the Momentum Effect

The model also allows us to discuss momentum trading. The fundamental requirement for a momentum effect are phases of uniformly positive or negative expected excess returns, which are empirically resembled by a phase of autocorrelation in excess returns, see Biais et al. (2010). In the context of our model excess returns are returns in excess of dividends.

Before giving a formal proof of universally positive or negative expected excess returns during a shift, we give a simulated example of our model showing correlated excess returns during a shift. The autocorrelation of the full sample is close to zero. But during the transition phase from period 100 till period 125 the autocorrelation is substantially higher with 0.24.

**Figure 4.6:** Momentum Trading

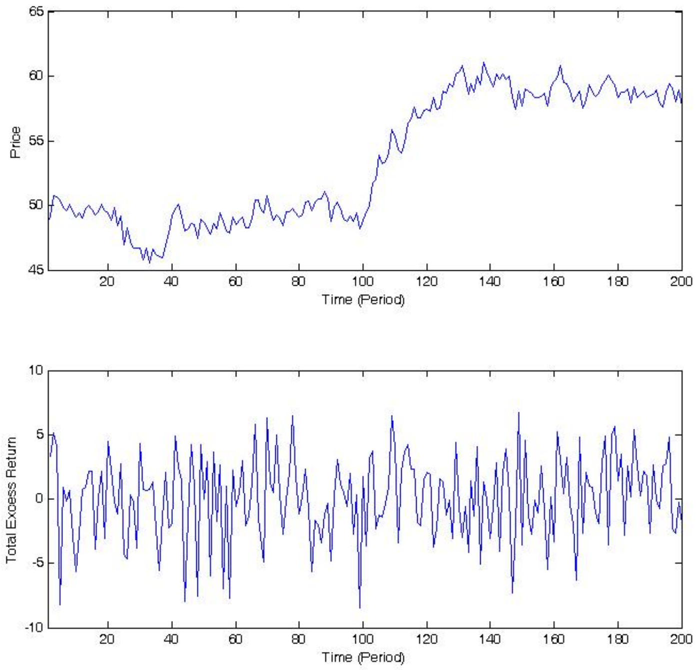


Figure 4.6 shows the asset sample path, with a positive shift in the long-term mean of the dividend process in period 100. The graph below shows the excess returns, meaning the returns in excess of dividends. $\beta = 0.9$, $\gamma = 0.2$, $\kappa_1 = 0.20$, $\kappa_2 = 0.25$, $\mu = 10$, $\sigma_1^2 = 5$, $\sigma_2^2 = 0.05$, and $\tilde{\mu} = 2$ are the parameters of the underlying market. The autocorrelation of the excess returns over the full sample is -0.08 and 0.24 during the shift from period 100 till period 125.

**Proposition 4.6** *Momentum Effect*

*During a shift, meaning a phase of length $\tau \in \mathbb{N}$ with $\tilde{\mu} < \mu_{t+i}$ or $\tilde{\mu} > \mu_{t+i}$ $\forall i \leq \tau$, the expected excess return under an extended outsider information set*[8] *is either uniformly positive or negative.*

[8] The extended outside information set includes the information that a shift is happening, which of course is no information the agent could obtain from only taking into account the signals he receives.

**Proof:** See section 4.7.4. □

The intuition is that during a positive (negative) shift, the expected excess return conditional on the shift, meaning if the agent knows that there has been a shift, is also positive (negative). This means that there is a statistical autocorrelation if one looks ex-post at the data, but ex-ante, in the moment the agent has to decide how to invest, the agent cannot be sure about the shift and its size since its only source of information is the signal he receives.

Consequently, the agent is not able to take advantage of this momentum trading opportunity on a single asset. Leaving the scope of our model and assuming an economy with many assets, which are partly shifting at any given time, one would be able to exploit these autocorrelations with a momentum trading strategy, even though one does not know if any particular asset is really shifting or not.

## 4.5 Investors with Different Information Processing Capabilities

In modern trading, computer models and algorithms are taking over more and more human decision making. For example high-frequency traders are involved in almost 70% of all dollar volume trades, see Brogaard (2010). The reasons for this development are simple, their algorithms can process more information in a shorter time.

The next extension of the baseline framework models such advantages in information processing capacity and shows how these advantages effect the optimal asset allocation and the returns for different agent groups. Our framework thus allows to discuss informational advantages by allowing for different information processing capacity constraints.

From a more general perspective differences in information processing capacity can also reflect the infrastructure and expertise on interpreting available data of a financial services provider as compared to an amateur investor. In this case one would interpret the financial service industry as a seller of information processing capacity.

### 4.5.1 Two Assets and Two Types of Agents

We add to the simplest case of a two assets economy as described in section 4.3.2 two possible types of agents differing from each other by their information processing capabilities. Let the fraction of group one with information processing constraint $\kappa^1$ be $\lambda$. Its information allocation will be denoted by $\rho_{11}$ and $\rho_{12}$. The same applies for group two with an information processing constraint $\kappa^2$, representing a fraction of the population of $1-\lambda$ and an information allocation of $\rho_{21}$ and $\rho_{22}$.

Given an optimal attention allocation the optimal asset allocation of both investor groups is given by:[9]

$$\begin{aligned} q_1^* &= \frac{(1-\beta)}{\gamma}\left(A_1+\tilde{A}\right)^{-1}\left(\Xi_1^2\cdot(s_{1t}-\mu)+\frac{\mu}{1-\beta}-p_t\right) \quad (4.84)\\ q_2^* &= \frac{(1-\beta)}{\gamma}\left(A_2+\tilde{A}\right)^{-1}\left(\Xi_2^2\cdot(s_{2t}-\mu)+\frac{\mu}{1-\beta}-p_t\right) \quad (4.85)\end{aligned}$$

$A_1$ and $A_2$ are the variances associated with the dividends for each of the investor groups and $\tilde{A}$ the variances of future prices. In order to shorten notation we define $\tau_i$ as:

$$\begin{aligned} \tau_1 &= \frac{(1-\beta)}{\gamma}\left(A_1+\tilde{A}\right)^{-1} \quad (4.86)\\ \tau_2 &= \frac{(1-\beta)}{\gamma}\left(A_2+\tilde{A}\right)^{-1} \quad (4.87)\end{aligned}$$

[9]For the attention allocation process a fixed trading strategy $\bar{q}=1$ needs to be assumed as a technical assumption.

In equilibrium the sum of all assets has to equal the total supply of one:

$$1 = \lambda\tau_1\left(\Xi_1^2\cdot(s_{1t}-\mu)+\frac{\mu}{1-\beta}-p_t\right) \tag{4.88}$$

$$+(1-\lambda)\,\tau_2\left(\Xi_2^2\cdot(s_{2t}-\mu)+\frac{\mu}{1-\beta}-p_t\right) \tag{4.89}$$

Solving for the price this leads to:

$$p_t=\frac{\mu}{1-\beta}+\omega\left(\Xi_1^2\cdot(s_{1t}-\mu)\right)+(1-\omega)\left(\Xi_2^2\cdot(s_{2t}-\mu)\right)-\Omega 1 \tag{4.90}$$

with

$$\omega = (\lambda\tau_1+(1-\lambda)\,\tau_2)^{-1}\,\lambda\tau_1 \tag{4.91}$$

$$\Omega = (\lambda\tau_1+(1-\lambda)\,\tau_2)^{-1} \tag{4.92}$$

Thus the variance matrices for the two investor groups are given by:

$$\tilde{A} = \beta[\left(\omega\Xi_1^2+(1-\omega)\,\Xi_2^2\right)^2 \tag{4.93}$$

$$+\left(\omega^2\left(\Xi_1^2-\Xi_1^4\right)+(1-\omega)^2\left(\Xi_2^2-\Xi_2^4\right)\right)]\begin{pmatrix}\sigma_1^2 & \\ & \sigma_2^2\end{pmatrix} \tag{4.94}$$

$$A_i = \begin{pmatrix}\sigma_1^2 & \\ & \sigma_2^2\end{pmatrix}\left(1-\begin{pmatrix}\rho_{i1}^{*2} & \\ & \rho_{i2}^{*2}\end{pmatrix}\right) \tag{4.95}$$

The values for $\omega$ are implicitly given as the solution to the following equation:

$$\omega = \left(\lambda\left(A_1+\tilde{A}\right)^{-1}+(1-\lambda)\left(A_2+\tilde{A}\right)^{-1}\right)^{-1}\lambda\left(A_1+\tilde{A}\right)^{-1} \tag{4.96}$$

Unfortunately there exists no closed form solution to equation (4.96) and thus the value of $\omega$ has to be obtained numerically. Since however $\omega$ can be precomputed the loss as compared to a closed form solution is rather small. Economically $\omega$ represents the fact that $\lambda$ needs to be adjusted for the average informedness of the agent group, since a better informed group will trade more in the market as compared to a less informed group of the same size.

Figures 4.7 and 4.8 show the asset allocation and the cumulative excess returns, meaning the cumulative returns in excess of dividends normalised by group size, for each of the two agent groups with different information processing capacities.

**Figure 4.7:** Value of Information Processing Capacity I

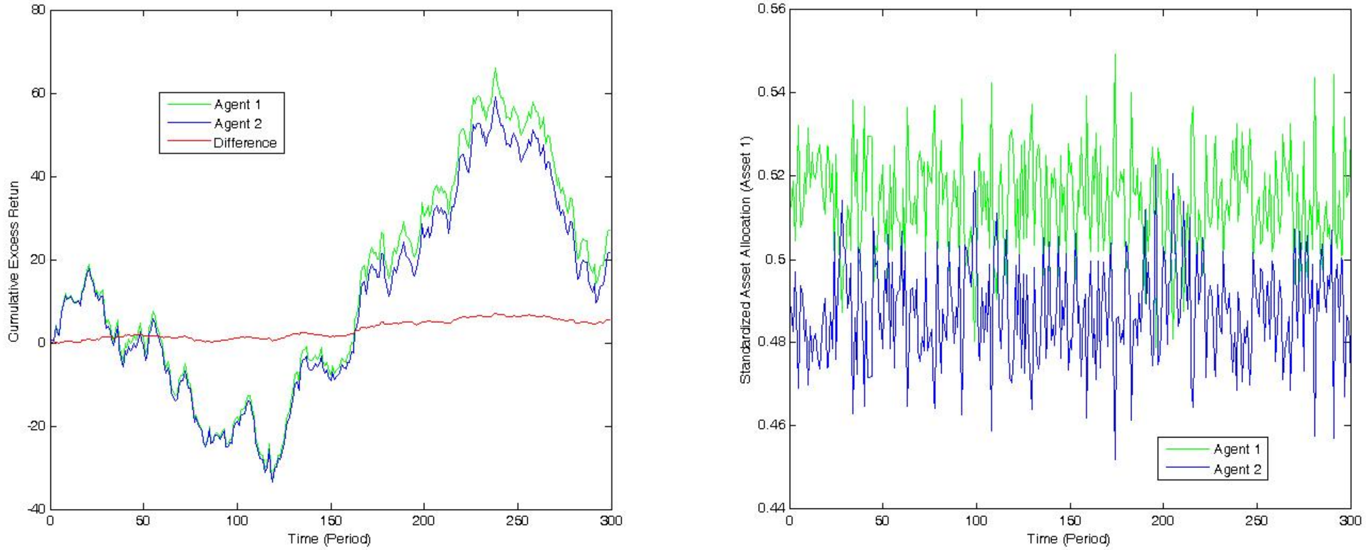


Figure 4.7 shows the cumulative excess return standardized by group size and the asset allocation of the first asset of two equally big agent groups with different information processing capacities trading on two assets. The parameters of the underlying market are taken as $\beta = 0.9$, $\gamma = 0.5$, $\lambda = 0.5$, $\kappa^1 = 0.45$, $\kappa^2 = 0.25$, $\mu_{1,2} = 10$, $\sigma^2_{1,2} = 4$. The length of the simulation is 300 time periods.

**Figure 4.8:** Value of Information Processing Capacity II

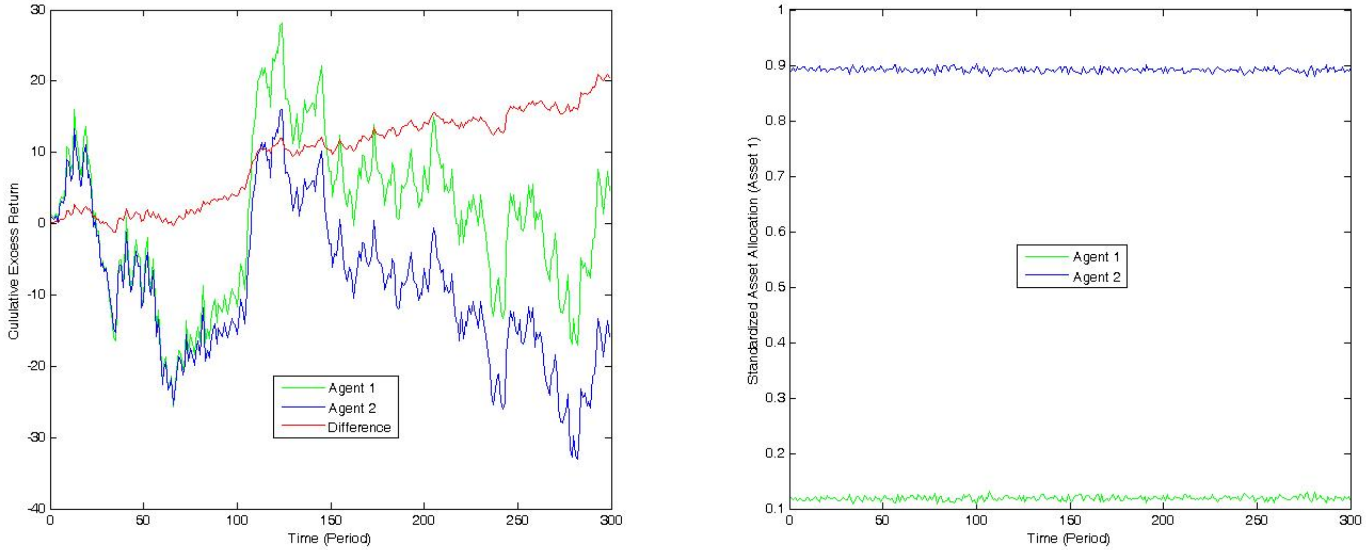


Figure 4.8 shows the cumulative excess return standardized by group size and the asset allocation of the first asset of two differently big agent groups with different information processing capacities trading on two assets. The size of agent group one is only $\lambda = 0.1$. The parameters of the underlying market are taken as $\beta = 0.9$, $\gamma = 0.5$, $\lambda = 0.1$, $\kappa^1 = 0.45$, $\kappa^2 = 0.25$, $\mu_{1,2} = 10$, $\sigma^2_{1,2} = 4$. The length of the simulation is 300 time periods.

The simulation parameters of figure 4.7 and 4.8 differ only in the relative size $\lambda$ of agent group one, which has a higher information processing capacity. This shows that for the group with a higher information processing capacity constraint the value of $\kappa$ depends on their own fraction on the whole population of agents.

Thus the smaller the group of fast learners is the higher is their excess return (standardized by the group size). This simply reflects the fact, that the group with better information processing capacity constraints faces less competition on information the smaller they are, thus the higher will be the margin for each individual trader.

### 4.5.2 One Asset with Shift

As mentioned before the intertemporal framework of our model also allows us to discuss such coups like the successful bet of John Paulson on the burst of the subprime bubble of the US housing market from an informational processing and attention allocation point of view. Abstracting from the individual case one can interpret it as an informational advantage from a higher information processing constraint within the environment of a shock in the fundamentals.

Model-wise we add to our model of section 4.4.2 two groups of agents with different information processing constraints. In addition to the last section, each group of agents has now current beliefs $\mu_{1t}$, $\mu_{2t}$ of the long-term mean, which they update every period. This allows for heterogeneity in the dynamics in the long-term mean expectations. The optimal asset allocation of both groups is given by:

$$\begin{aligned} q_1^* &= \frac{(1-\beta)}{\gamma}\left(A_1+\tilde{A}\right)^{-1} && (4.97)\\ &\left(\rho_1^{*2}\left(s_{11t}-\mu\right)+\frac{\rho_2^{*2}\left(s_{12t}-\mu_t\right)+\mu+\mu_{1t}}{1-\beta}-p_t\right) \\ q_2^* &= \frac{(1-\beta)}{\gamma}\left(A_2+\tilde{A}\right)^{-1} && (4.98)\\ &\left(\rho_1^{*2}\left(s_{21t}-\mu\right)+\frac{\rho_2^{*2}\left(s_{22t}-\mu_t\right)+\mu+\mu_{2t}}{1-\beta}-p_t\right) \end{aligned}$$

Using the same notational short cuts as before the price will be:

$$\begin{aligned} p_t &= \omega\left(\rho_1^{*2}\left(s_{11t}-\mu\right)+\frac{\rho_2^{*2}\left(s_{12t}-\mu_t\right)+\mu+\mu_{1t}}{1-\beta}\right) && (4.99)\\ &+(1-\omega)\left(\rho_1^{*2}\left(s_{21t}-\mu\right)+\frac{\rho_2^{*2}\left(s_{22t}-\mu_t\right)+\mu+\mu_{2t}}{1-\beta}\right) \\ &-\Omega_1 \end{aligned}$$

with

$$\begin{aligned} \omega &= \left(\lambda\tau_1+(1-\lambda)\tau_2\right)^{-1}\lambda\tau_1 && (4.100)\\ \Omega &= \left(\lambda\tau_1+(1-\lambda)\tau_2\right)^{-1} && (4.101) \end{aligned}$$

Thus the variances for the two investor groups are given by:

$$\begin{aligned} \tilde{A} &= \left(\omega\rho_{11}^{*2}+(1-\omega)\rho_{21}^{*2}\right)^2\beta\sigma_1^2 && (4.102)\\ &+\left(\omega^2\frac{\rho_{12}^{*4}}{(1-\beta)^2}+(1-\omega)^2\frac{\rho_{22}^{*4}}{(1-\beta)^2}\right)\beta\sigma_2^2 \\ &+\omega^2\beta\left(\rho_{11}^{*4}\tilde{\sigma}_{11}^2+\frac{\rho_{12}^{*4}}{(1-\beta)^2}\tilde{\sigma}_{12}^2\right) \\ &+\left(1-\omega^2\right)\beta\left(\rho_{21}^{*4}\tilde{\sigma}_{21}^2+\frac{\rho_{22}^{*4}}{(1-\beta)^2}\tilde{\sigma}_{22}^2\right) \\ A_i &= \sigma_1^2\left(1-\rho_{i1}^{*2}\right)+\frac{\sigma_2^2}{1-\beta}\left(1-\rho_{i2}^{*2}\right) && (4.103) \end{aligned}$$

As with the model without shifts the values for $\omega$ are implicitly given as the solution to the following equation:

$$\omega = \left(\lambda\left(A_1+\tilde{A}\right)^{-1}+(1-\lambda)\left(A_2+\tilde{A}\right)^{-1}\right)^{-1}\lambda\left(A_1+\tilde{A}\right)^{-1} \tag{4.104}$$

Having derived the model we now want to compare its implications, which we portray with the help of a simulation case study, with the bet of John Paulson on the burst of the US housing market bubble.

**Figure 4.9:** Heterogeneous Information Processing Capacities and Shift

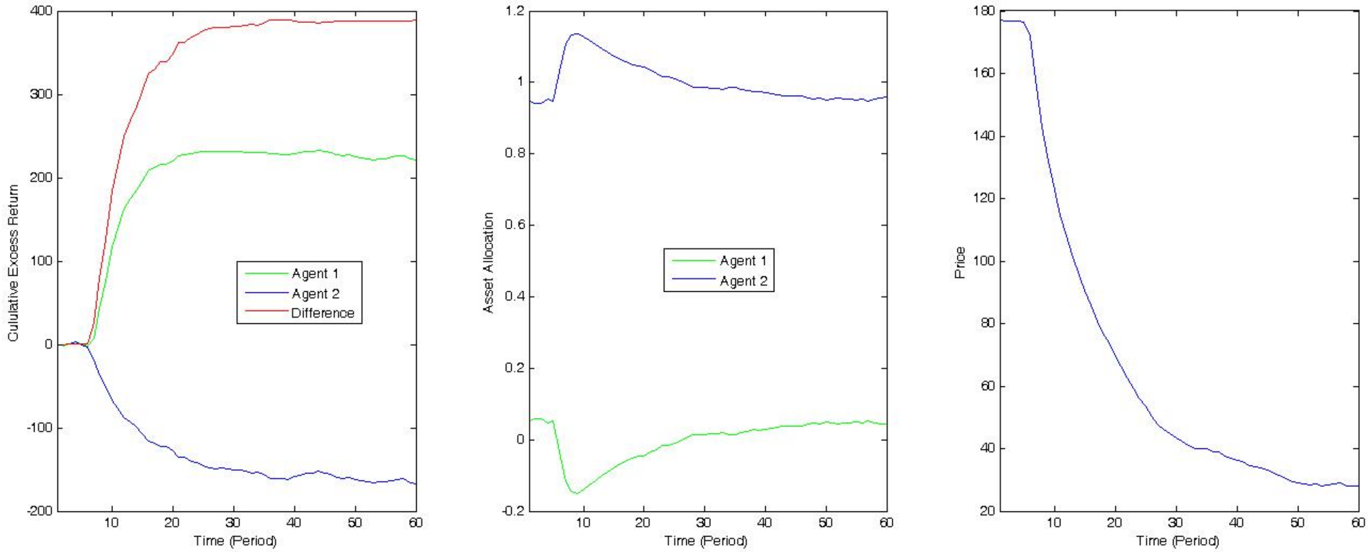


Figure 4.9 shows the cumulative excess return and the asset allocation, standardized by group size, of each agent group and the dynamics of the price of the shifted asset. The simulation time is 60 periods with a shock in period 5, which wipes out most of the assets fundamental value. The model parameters used for the simulation are $\beta = 0.9$, $\gamma = 0.2$, $\lambda = 0.05$, $\kappa^1 = 0.5$, $\kappa^2 = 0.01$, $\mu = 20$, $\mu_{0,1} = 0$, $\mu_{0,2} = 0$, $\sigma_1^2 = 4$, $\sigma_2^2 = 0.04$, $\tilde{\mu} = -15$.

Figure 4.9 shows a simulation of the most extreme case, meaning a small group with a high information processing capacity and a large group with a very small information processing capacity constraint.

As one can see, both groups trade in opposite directions as most of the future value of the asset is wiped out by a shock in period 5. While the group with a high information processing capacity constraint shortens the asset, the other is buying it, since it is not yet aware of the sharp drop.

As mentioned before a real world example to this might be the bet of John Paulson on the burst of US housing market bubble. By shorting his exposure to BBB tranches of subprime mortgage backed securities using credit default swaps he was able to reap a huge profit when the US housing market declined sharply in 2007/2008 and many of the BBB tranches lost all their value. That this was not just a lucky guess, but actually a model example of an agent having a huge advantage in information processing capacity can be seen by the fact that "John Paulson [...] purchased the best database on house-price statistics, commissioned a technology company to help him warehouse it, and hired extra analysts to interpret the numbers".[10]

Wanting to trade on what he perceived as the greatest weakness of the US financial system he allocated a lot of information processing capacity on the US housing market, as one can see from his investments, and was therefore able to exploit the informational advantage during the burst of the bubble. Even though he earned more than $15 billions with this strategy in the end, he lost millions of dollars on the way, since it took a while till the prices dropped finally in 2007/2008.[11]

[10] Mallaby (2010), p. 386

[11] For an in-depth discussion of John Paulson's trade see Mallaby (2010), p. 307-391.

## 4.6 Conclusion

In this paper we introduced a model fusing rational inattention with an overlapping generation model for the financial market. To the best of our knowledge we are the first to combine rational inattention with a real market model and to derive implications towards competitive attention allocation and the choice between short-term and long-term relevant information from it. Going on from these mostly theoretical results we derived the following four main implications from our model.

First, we have shown and empirically tested that the capability to process information and the willingness to allocate this capability towards a specific source of uncertainty is highly relevant in the context of how information travels in the financial market. Thus we challenged the efficient market hypothesis by an alternative framework of attention driven efficiency. Given the idea of attention driven efficiency, we further pointed out that it might be problematic to test market efficiency in general during times of high allocated attention, since the market should be efficient during these times.

Second, extending our basic model by a shift component, we showed how shifts can be seen as a result of limited information processing capacity while still staying in a rational agent framework. Furthermore, we portrayed the plausibility of this concept with an empirical case study of the burst of the US subprime bubble and John Paulson's successful bet on it.

Third, within this shift framework, we were able to give a micro-level explanation of the momentum effect in a rational agent framework without direct arbitrage opportunities.

Fourth, we have shown that within our framework financial services providers can be seen as providers of information processing capacity.

Since this is the first rational inattention model developed to explicitly model information aggregation on the financial market, we believe that there is still a huge potential for other possible applications. Further, there should be ample opportunity to extend and improve on the suggested model. Perhaps most prominently the question of making information available to other parties is not addressed within our model context and would be the most interesting extension of our framework.

## 4.7 Mathematical Proofs

### 4.7.1 Proof of Proposition 4.1

If two normally distributed random variables $X$ and $Y$ are correlated with each other with correlation parameter $\rho$ the mutual information, meaning the information one variable contains about the other, can be expressed as the reduction in the entropy of $X$ by observing the other random variable $Y$.

$$I(X,Y) = H(X) - H(X|Y) \tag{4.105}$$

$H(X)$ is the unconditional entropy of $X$ and $H(X|Y)$ is the conditional entropy of the $X$ given $Y$. Both can be calculated using the entropy formula:

$$\begin{aligned} H(X) &= \frac{1}{2}\log_2\left[(2\pi e)^T \det \Omega_X\right] & (4.106) \\ H\left(X|Y\right) &= \frac{1}{2}\log_2\left[(2\pi e)^T \det \Omega_{X|Y}\right] & (4.107) \end{aligned}$$

If $X,Y$ are jointly multivariate normal distributed with $Cov(X_i,X_j) = 0\;\forall i \neq j$, $Cov(Y_i,Y_j) = 0\;\forall i \neq j$, $Cov(X_i,Y_j) = \rho\sigma_X\sigma_Y\;\forall i = j$, $Cov(X_i,Y_j) = 0\;\forall i \neq j$, $Var(X_i) = \sigma_X^2$, and $Var(Y_i) = \sigma_Y^2$ the mutual information (4.105) can also be written in the following way:

$$
\begin{aligned}
I(X,Y) &= H(X) - H(X|Y) && (4.108)\\
&= \frac{1}{2}\log_2\left[(2\pi e)^T \sigma_X^{2T}\right] - \frac{1}{2}\log_2\left[(2\pi e)^T\left(\sigma_X^2 - \sigma_X^2\rho^2\right)^T\right] && (4.109)\\
&= \frac{1}{2}T\log_2\left(\frac{1}{1-\rho^2}\right) && (4.110)
\end{aligned}
$$

Since we are interested in the average information per period, we divide (4.110) by $T$:

$$
\begin{aligned}
\mathbb{I}(X_t,Y_t) &= \frac{1}{T}\frac{1}{2}T\log_2\left(\frac{1}{1-\rho^2}\right) && (4.111)\\
&= \frac{1}{2}\log_2\left(\frac{1}{1-\rho^2}\right) && (4.112)
\end{aligned}
$$

In our case the dividends are independent, as well as the signals. Therefore one can think of independent information processes for each asset and simply take the sum of the amount of information in the given period:

$$
\underbrace{\frac{1}{2}\log_2\left(\frac{1}{1-\rho_1^2}\right)}_{\kappa_1} + \underbrace{\frac{1}{2}\log_2\left(\frac{1}{1-\rho_2^2}\right)}_{\kappa_2} .... + \underbrace{\frac{1}{2}\log_2\left(\frac{1}{1-\rho_N^2}\right)}_{\kappa_N} \leq \kappa \qquad (4.113)
$$

$\square$

### 4.7.2 Proof of Proposition 4.3

We add two groups of investors with the same information processing capacity to the model of section 4.3.2. Group one with relative magnitude $\lambda$ and optimal information allocation $\rho_1^*$ and group two with relative magnitude $1-\lambda$ and optimal information allocation $\rho_2^*$. The resulting optimal asset allocations for each agent group are:

$$q_1^* = \frac{(1-\beta)}{\gamma} A^{-1} \left( \rho_1^{*2} \cdot (s_{1t} - \mu) + \frac{\mu}{1-\beta} - p_t \right) \tag{4.114}$$

$$q_2^* = \frac{(1-\beta)}{\gamma} A^{-1} \left( \rho_2^{*2} \cdot (s_{2t} - \mu) + \frac{\mu}{1-\beta} - p_t \right) \tag{4.115}$$

In equilibrium the following equation has to hold, since all assets, normalised to one, have to be held by the agents:

$$\begin{aligned} 1 &= \frac{(1-\beta)}{\gamma} A^{-1} \\ &\left( \lambda \rho_1^{*2} \cdot (s_{1t} - \mu) + (1-\lambda)\, \rho_2^{*2} \cdot (s_{2t} - \mu) + \frac{\mu}{1-\beta} - p_t \right) \end{aligned} \tag{4.116}$$

Rearranging terms leads to the following price formula:

$$\begin{aligned} p_t &= \lambda \rho_1^{*2} \cdot (s_{1t} - \mu) + (1-\lambda)\, \rho_2^{*2} \cdot (s_{2t} - \mu) \\ &\quad + \frac{\mu}{1-\beta} - \frac{\gamma}{(1-\beta)} A 1 \end{aligned} \tag{4.117}$$

$$= \rho^{*2} \left( (\lambda s_{1t} + (1-\lambda)\, s_{2t}) - \mu \right) + \frac{\mu}{1-\beta} - \frac{\gamma}{(1-\beta)} A 1 \tag{4.118}$$

The variance A is given by:

$$A = \sigma^2 (1 - \rho^{*2}) + \beta \left( \sigma^2 + \left( \lambda^2 + (1-\lambda)^2 \right) \tilde{\sigma}^2 \right) \rho^{*4} \tag{4.119}$$

This collapses to the matrix of the non heterogeneous case for $\lambda = 0$ and $\lambda = 1$. Note that $\rho_1^* = \rho_2^* = \rho^*$.

Generalizing to G equally large groups with independent signals one obtains the price as:

$$p_t = \rho^{*2} \left( \sum_{n=1}^{G} \frac{1}{G} s_{nt} - \mu \right) + \frac{\mu}{1-\beta} - \frac{\gamma}{2(1-\beta)} A 1 \tag{4.120}$$

The variance A with G groups is given by:

$$A = \sigma^2(1-\rho^{*2}) + \beta\left(\sigma^2 + \frac{1}{G}\tilde{\sigma}^2\right)\rho^{*4} \tag{4.121}$$

Thus the price variance for asset one is given by:

$$Var\left(p_t\right) \;=\; \left(\sigma^2 + \frac{1}{G}\tilde{\sigma}^2\right)\rho^{*4} \tag{4.122}$$

□

The variance of the price is therefore not only a question of attention allocation but also a question of how many independent opinions are present. Higher volatility in distress situation may not only result from more attention allocation but also because of more homogeneity, meaning less independent groups.

### 4.7.3 Proof of Proposition 4.4

Within our framework the household problem for the individual agent is given by:

$$\max_{s_t^i\in\Gamma}\mathbb{E}\left[u(c_t^i;s_t^i) + \beta u(c_{t+1}^i;s_t^i)\middle|s_t^i\right] \tag{4.123}$$

subject to the following constraints

$$\begin{aligned}
\mathbb{I}\left(\{d_t\};\left\{s_t^i\right\}\right) &\leq \kappa^i && (4.124)\\
q_{t+1}^{i*} &= \arg\max_{q_{t+1}^i}\mathbb{E}\left[u(c_t^i;s_t^i) + \beta u(c_{t+1}^i;s_t^i)\middle|s_t^i\right] && (4.125)\\
c_t^i &= q_{t+1}^{i*}\left(d_t - p_t\right) && (4.126)\\
c_{t+1}^i &= q_{t+1}^{i*}p_{t+1} && (4.127)
\end{aligned}$$

The agent has mean-variance utility:

$$u(c) = \mathbb{E}\left[c\right] - \frac{\gamma}{2} Var\left[c\right] \tag{4.128}$$

Solving this household problem analogous to Proposition 4.2 yields the following FOC for the quantity of assets the agent wants to hold:

$$\begin{aligned}
0 &= \mathbb{E}\left[d_t|s_t\right] + \beta\mathbb{E}\left[p_{t+1}|s_t\right] - p_t - \gamma A q^* && (4.129)\\
0 &= \rho_1^{*2}\left(s_{1t}-\mu\right) + \rho_2^{*2}\left(s_{2t}-\mu_t\right) + \mu + \mu_t && (4.130)\\
&\quad +\beta\mathbb{E}\left[p_{t+1}|\rho\right] - p_t - \gamma A q^* \\
0 &= \rho_1^{2}\left(s_{1t}-\mu\right) + \rho_2^{*2}\left(s_{2t}-\mu_t\right) + \mu + \mu_t && (4.131)\\
&\quad +\sum_{i=1}^{N}\beta^i\left(\rho_2^{*2}\left(s_{2t}-\mu_t\right) + \mu - \gamma A q^*\right) \\
&\quad +\beta^{N+1}\left(\mathbb{E}\left[p_{t+N+2}\right]\gamma A q^*\right) - p_t - \gamma A q^* \\
0 &= \rho_1^{*2}\left(s_{1t}-\mu\right) + \sum_{i=0}^{N}\beta^i\left(\rho_2^{*2}\left(s_{2t}-\mu_t\right) + \mu + \mu_t - \gamma A q^*\right) && (4.132)\\
&\quad -p_t
\end{aligned}$$

We assume updating of the agent's best guess $\mu_t$ about $\tilde{\mu}$. The agent forms his new opinion on $\tilde{\mu}$ by weighting his signal on the mean $s_{2t}$ and his previous belief $\mu_{t-1}$ by the quality of the signal he receives, thus $\mu_t = \rho_2^2 s_{2t} + (1-\rho_2^2)\mu_{t-1}$. The variance of $\tilde{\mu}$ is assumed to be $\sigma_2^2$ for each generation to ensure a stationary problem. This results in the optimal asset allocation:

$$\begin{aligned}
q^* &= \frac{(1-\beta)}{\gamma}A^{-1} && (4.133)\\
&\quad \left(\rho_1^{*2}\left(s_{1t}-\mu\right) + \frac{\rho_2^{*2}\left(s_{2t}-\mu_t\right)+\mu+\mu_t}{1-\beta} - p_t\right)
\end{aligned}$$

In equilibrium the agent group has to hold all assets, normalized to one:

$$\begin{aligned} 1 &= \frac{(1-\beta)}{\gamma} A^{-1} \\ & \left( \rho_1^{*2} \left( s_{1t} - \mu \right) + \frac{\rho_2^{*2} \left( s_{2t} - \mu_t \right) + \mu + \mu_t}{1-\beta} - p_t \right) \end{aligned} \tag{4.134}$$

Solving for the price leads to:

$$p_t = \rho_1^{*2} \left( s_1 - \mu \right) + \frac{\rho_2^{*2} \left( s_2 - \mu_t \right) + \mu + \mu_t}{1-\beta} - \frac{\gamma}{(1-\beta)} A1 \tag{4.135}$$

The variance A is given by:

$$\begin{aligned} A &= \sigma_1^2 (1-\rho_1^{*2}) + \frac{\sigma_2^2}{(1-\beta)^2} (1-\rho_2^{*2}) + \beta \sigma_1^2 \rho_1^{*2} \\ & + \frac{\beta}{(1-\beta)^2} \rho_2^{*2} \sigma_2^2 \end{aligned} \tag{4.136}$$

With a closed form solution for the price at hand we can turn to the attention allocation problem. Analogue to the proof of proposition 4.2 the simplified optimization problem ignoring all constant parameters is given by:

$$\max_{\{\rho_1, \rho_2\} \in [0,1]} \sigma_1^2 \rho_1^2 + \frac{\sigma_2^2}{(1-\beta)^2} \rho_2^2 \tag{4.137}$$

This is equivalent to the step 4 in the proof of proposition 4.2 when simply replacing $\sigma_2$ by $\frac{\sigma_2}{(1-\beta)}$. □

### 4.7.4 Proof of Proposition 4.6

In equilibrium expected prices for all future periods are identical:

$$\tilde{\mu} = \mu_{t+i} \forall i \in \mathbb{N} \rightarrow \mathbb{E} \left[ R_t \right| \tilde{\mu} = \mu_t] - (\tilde{\mu} + \mu_t) = 0 \tag{4.138}$$

During a (positive) shift, which implies non perfect information processing meaning $\rho_2^2 < 1$ and $\tilde{\mu} > \mu_t, \mu_{t+1}$, the expected excess return is given by:

$$\begin{aligned}
\mathbb{E}\left[R_t|\, shift\right] - (\tilde{\mu} + \mu_t) &= \mathbb{E}\left[\frac{\rho_2^2\left(s_{2t+1} - \mu_{t+1}\right) + \mu + \mu_{t+1}}{1-\beta}\right. && (4.139)\\
&\quad -\frac{\gamma}{(1-\beta)}A1\\
&\quad -\frac{\rho_2^2\left(s_{2t} - \mu_t\right) + \mu + \mu_t}{1-\beta}\\
&\quad \left.+\frac{\gamma}{(1-\beta)}A1\right]\\
&= \frac{1}{1-\beta}\rho_2^2\left(\tilde{\mu} - \mu_{t+1}\right) && (4.140)
\end{aligned}$$

Since $\tilde{\mu} > \mu_{t+1}$ holds because of the ongoing shift equation (4.140) will be positive during a (positive) shift. □

During a negative shift the same arguments hold but it implies a negative expected excess return. The model does not imply a possible excess return for the agent on an individual asset, since he does not have the shift information (which is actually forward looking) in his signal, which encompasses all the information he can acquire.

### 4.7.5 N Asset Allocation

$$\max_{\xi_n \in [0,1]} \sum_{n=1}^{N} \sigma_n^2 \xi_n \tag{4.141}$$

$$\sum_{n=1}^{N} \frac{1}{2}\log_2\left(\frac{1}{1-\xi_n}\right) \leq \kappa \tag{4.142}$$

We first look at the general case of a non-corner solution. Simplifying the boundary condition by only taking into account first order and first order interaction effects we obtain:

$$\sum_{n=1}^{N} \xi_n - \sum_{n=1}^{N} \sum_{j=n+1}^{N} \xi_n \xi_j \quad = \quad \alpha \tag{4.143}$$

As before $\alpha$ is given by:

$$\alpha \quad = \quad 1 - \frac{1}{\exp\left(2\kappa \ln(2)\right)} \tag{4.144}$$

In case of two assets this is equal to:

$$\xi_1 + \xi_2 - \xi_1 \xi_2 - \alpha = 0 \tag{4.145}$$

The FOC for a maximum, meaning setting the normal of the differentiable manifold equal to a multiple of the gradient of the target function, is given by:

$$\begin{pmatrix} 0 & 1 & ... & 1 & \sigma_1^2 \\ 1 & 0 & ... & 1 & \sigma_2^2 \\ & & ... & & \\ 1 & ... & 1 & 0 & \sigma_{N^2} \end{pmatrix} \begin{pmatrix} \xi_1 \\ ... \\ \xi_N \\ \lambda \end{pmatrix} = \begin{pmatrix} 1 \\ ... \\ ... \\ 1 \end{pmatrix} \tag{4.146}$$

We define:

$$\Psi = \begin{pmatrix} 0 & 1 & ... & 1 \\ 1 & 0 & ... & 1 \\ ... & ... & ... & ... \\ 1 & ... & 0 & 1 \\ 1 & ... & 1 & 0 \end{pmatrix} \tag{4.147}$$

Thus we obtain:

$$\xi(\lambda) = \Psi^{-1} \begin{pmatrix} 1 \\ ... \\ ... \\ 1 \end{pmatrix} - \lambda \begin{pmatrix} \sigma_1^2 \\ ... \\ ... \\ \sigma_N^2 \end{pmatrix} \tag{4.148}$$

$\lambda^*$ is given by the following solution to the quadratic equation:

$$\sum_{i=1}^{N} \xi_i(\lambda) - \sum_{i=1}^{N} \sum_{j=i+1}^{N} \xi_i(\lambda)\xi_j(\lambda) = \alpha \tag{4.149}$$

We obtain the optimal allocation as:

$$\xi^*(\lambda^*) = \Psi^{-1} \begin{pmatrix} 1 \\ \dots \\ \dots \\ 1 \end{pmatrix} - \lambda^* \begin{pmatrix} \sigma_1^2 \\ \dots \\ \dots \\ \sigma_N^2 \end{pmatrix} \tag{4.150}$$

□

For $N = 1$ this trivially leads to $\xi^* = \alpha$ and for $N = 2$ in case of an inner solution to:

$$\lambda^* = \frac{\sqrt{1-\alpha}}{\sigma_1 \sigma_2} \tag{4.151}$$

$$\xi^* = \begin{pmatrix} 1 - \sqrt{1-\alpha}\frac{\sigma_2}{\sigma_1} \\ 1 - \sqrt{1-\alpha}\frac{\sigma_1}{\sigma_2} \end{pmatrix} \tag{4.152}$$

This is identical to the model without approximation for the one and the two asset case, since the approximation is exact for up to two assets because only for three and more assets higher order interaction terms exist, which are lost by the approximation.

Chapter 5

# Conclusion

The main findings of the three studies encompassing this thesis can be summarized as, first, finding an industry specific risk factor for bank stock returns worldwide, which cannot be explained by the standard Fama-French or Fama-French-Carhart model, second, the reaffirmation of the interpretation of the Fama-French factors as proxies for more fundamental risk sources, namely default and disaster risk, and third, developing a quantitative model to incorporate the concept of rational inattention into asset pricing and deriving testable implications towards market efficiency and a micro-level explanation for the momentum effect from it.

Apart from finding a bank specific risk factor we have shown that banks in the US, Europe, and Japan have an increasing loading on the market risk factor with increasing market capitalisation. It also appears as if not the book value of equity, as suggested in recent studies, but the market value of equity is the right measure of a bank's size. For all regions except Europe both size measures lead however to similar results. Another intriguing finding was the negative loading on the momentum factor of the biggest banks, while otherwise the momentum factor appears to be rather unimportant when explaining bank stock returns. This finding might especially be interesting in the context of "too big to fail", since it suggests a somewhat countercyclical behaviour.

Turning to the interpretation of the Fama-French factors as proxies for default and disaster risk, we have shown that the size effect is mostly a default risk effect and that this relationship is stable over the crisis and non-crisis state of the market with completely different characteristics of the effects under the different market regimes. The value effect can be seen as an overlapping of an insurance against the crisis state of the market and default risk. Meaning, it has a disaster risk component and a default risk component. Having constructed two alternative risk factors the question was raised to what extend augmentations of the CAPM lead to real improvements, since the augmented models appear to only be capable to outperform the CAPM in explaining cross-sectional returns, if the sorting is done by the same proxy measure as the one used in the factor construction.

Looking at the implications of rational inattention for asset pricing, we have presented a new model of how information travels within financial markets and have presented empirical evidence that the concept of attention driven information processing is more conjugate with market data as compared to the prevailing concept of efficient markets. Augmenting our model by a shift component made it possible to explain shifts in asset prices by a lack of attention on small permanent changes in the fundamentals. This can also be seen as a micro-level explanation of the momentum effect. By a further augmentation of the model through the introduction of heterogeneous information processing capacities we were finally able to give a fundamental interpretation of the financial services industry as providers of information processing capacity. Moreover, the burst of the housing bubble in the US and the successful bet of John Paulson against it were shown to be prime examples of our framework.

Generally this thesis contributed to all traits, which make asset pricing a unique discipline in finance. New anomalies were discovered and described empirically in the international bank sector. Possible explanations for known anomalies in equity returns were developed, both in a traditional rational investor and in a new cross-over quantitative behavioural model environment. New theories towards attention driven anomalies were postulated and empirical implications of them tested.

Especially the analysis and explanation of described anomalies in international bank stock returns and a further improvement on rational inattention models for asset pricing would be most interesting topics for future research in the parts of asset pricing covered by this thesis.

More generally asset pricing still offers infinitely many possibilities for future research both empirical and theoretical. This holds especially true since there still exists no fully coherent theory on how asset prices are fundamentally established by risk, attitudes toward it, and behavioural factors as well as on when the efficient market hypothesis holds and how important the exceptions from it are.

# References


Abel, A. (1990). Asset prices under habit formation and catching up with the Joneses. *American Economic Review 80*, 38–42.

Akgun, A. and R. Gibson (2001). Recovery risk in stock returns. *Journal of Portfolio Management 27*, 22–31.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The cross-section of volatility and expected returns. *Journal of Finance 61*, 259–299.

Asem, E. and G. Y. Tian (2010). Market dynamics and momentum profits. *Journal of Financial and Quantitative Analysis 45*, 1549–1562.

Asness, C. S., T. J. Moskowitz, and L. H. Pedersen (2013). Value and momentum everywhere. *Journal of Finance 68*, 929–985.

Bacchetta, P. and E. VanWincoop (2005). A theory of the currency denomination of international trade. *Journal of International Economics 67*, 295–319.

Banz, R. (1981). The relationship between return and market value of common stocks. *Journal of Financial Economics 9*, 3–18.

Barberis, N., M. Huang, and T. Santos (2001). Prospect theory and asset prices. *The Quarterly Journal of Economics 116*, 1–53.

Barberis, N., A. Shleifer, and R. Vishny (1998). A model of investor sentiment. *Journal of Financial Economics 49*, 307–343.

Barberis, N. and R. Thaler (2003). A survey of behavioral finance. *Handbook of the Economics of Finance 1*, 1053–1128.

Barro, R. (2006). Rare disasters and asset markets in the twentieth century. *The Quarterly Journal of Economics 121*, 823–866.

Basu, S. (1977). Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis. *Journal of Finance 32*, 663–682.

Bell, D. (1982). Regret in decision making under uncertainty. *Operations Research 30*, 961–981.

Berk, J. (1995). A critique of size-related anomalies. *Review of Financial Studies 8*, 275–286.

Biais, B., P. Bossaerts, and C. Spatt (2010). Equilibrium asset pricing and portfolio choice under asymmetric information. *Review of Financial Studies 23*, 1503–1543.

Black, F. and M. Scholes (1973). The pricing of options and corporate liabilities. *The Journal of Political Economy 81*, 637–654.

Breeden, D. (1979). An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics 7*, 265–296.

Brogaard, J. A. (2010). High frequency trading and its impact on market quality. *Working paper*.

Brunnermeier, M. K. (2009). Bubbles: Entry in new palgrave dictionary of economics. *New Palgrave Dictionary of Economics*.

Brunnermeier, M. K. and S. Nagel (2004). Hedge funds and the technology bubble. *Journal of Finance 59*, 2013–2040.

Campbell, J. Y. and J. H. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy 107*, 205–251.

Campbell, J. Y., J. Hilscher, and J. Szilagyi (2008). In search of distress risk. *Journal of Finance 63*, 2899–2939.

Campbell, J. Y., C. Polk, and T. Vuolteenaho (2010). Growth or glamour? Fundamentals and systematic risk in stock returns. *Review of Financial Studies 23*, 305–344.

Carhart, M. M. (1997). On persistence in mutual fund performance. *Journal of Finance 52*, 57–82.

Chew, S. and K. MacCrimmon (1979). Alpha-nu choice theory: an axiomatization of expected utility. *Working paper*.

Cochrane, J. H. (1996). A cross-sectional test of an investment-based asset pricing model. *Journal of Political Economy 104*, 572–621.

Cochrane, J. H. (2001). *Asset Pricing.* Princeton, New Jersey: Princeton University Press.

Constantinides, G. M. (1990). Habit formation: A resolution of the equity premium puzzle. *Journal of Political Economy 98*, 519–543.

Da, Z., J. Engelberg, and P. Gao (2011). In search of attention. *Journal of Finance 66*, 1461–1499.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam (1998). Investor psychology and security market under- and overreactions. *Journal of Finance 53*, 1839–1885.

DeLong, J. B., A. Shleifer, L. H. Summers, and R. Waldmann (1990a). Noise trader risk in financial markets. *Journal of Political Economy 98*, 703–738.

DeLong, J. B., A. Shleifer, L. H. Summers, and R. J. Waldmann (1990b). Positive feedback investment strategies and destabilizing rational speculation. *Journal of Finance 45*, 379–395.

DeOliveira, H., T. Denti, M. Mihm, and M. K. Ozbek (2013). Rationally inattentive preferences. *Working paper*.

Dessein, W., A. Galeotti, and T. Santos (2013). Rational inattention and organizational focus. *CEPR Discussion Paper DP9395*.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*, 937–969.

Falkinger, J. (2008). Limited attention as a scarce resource in information-rich economies. *The Economic Journal 118*, 1596–1620.

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *Journal of Finance 25*, 383–417.

Fama, E. F. (1998). Market efficiency, long-term returns, and behavioral finance. *Journal of Financial Economics 49*, 283–306.

Fama, E. F., L. Fisher, M. C. Jensen, and R. Roll (1969). The adjustment of stock prices to new information. *International Economic Review 10*, 1–21.

Fama, E. F. and K. R. French (1992). The cross-section of expected stock returns. *Journal of Finance 47*, 427–465.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*, 3–56.

Fama, E. F. and K. R. French (2012). Size, value, and momentum in international stock returns. *Journal of Financial Economics 105*, 457–472.

Feldhütter, P. and D. Lando (2008). Decomposing swap spreads. *Journal of Financial Economics 88*, 375–405.

Friewald, N., C. Wagner, and J. Zechner (2013). The cross-section of credit risk premia and equity returns. *Journal of Finance (forthcoming).*

Froot, K. A. and E. M. Dabora (1999). How are stock prices affected by the location of trade? *Journal of Financial Economics 53*, 189–216.

Fujii, M. and M. Kawai (2010). Lessons from Japan's banking crisis, 1991-2005. *ADBI Working Paper Series 222.*

Gabaix, X. (2012). Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. *The Quarterly Journal of Economics 127*, 645–700.

Gandhi, P. and H. N. Lustig (2011). Size anomalies in U.S. bank stock returns: A fiscal explanation. *Working paper*.

Gandhi, P. and H. N. Lustig (2013). Size anomalies in U.S. bank stock returns. *Journal of Finance (forthcoming).* doi:10.1111/ectj.1211, used for page references.

Garlappi, L. and H. Yan (2011). Financial distress and the cross-section of equity returns. *Journal of Finance 66*, 789–822.

Griffin, J. M. and M. L. Lemmon (2002). Book-to-market equity, distress risk, and stock returns. *Journal of Finance 57*, 2317–2336.

Grossman, S. J. and R. J. Shiller (1981). The determinants of the variability of stock market prices. *American Economic Review 71*, 222–227.

Gul, F. (1991). A theory of disappointment aversion. *Econometrica 59*, 667–686.

Hanauer, M. (2013). Is Japan different? Evidence on momentum and market dynamics. *Working paper*.

Hanauer, M., C. Kaserer, and M. S. Rapp (2013). Risikofaktoren und Multifaktormodelle für den deutschen Aktienmarkt (Risk factors and multi-factor models for the German stock market). *Betriebswirtschaftliche Forschung und Praxis 65*, 469–492.

Hansen, L. and R. Jagannathan (1997). Assessing specification errors in stochastic discount factor models. *Journal of Finance 52*, 557–590.

Hansen, L. P. and K. J. Singleton (1983). Stochastic consumption, risk aversion, and the temporal behavior of asset prices. *Journal of Political Economy 91*, 249–265.

Harris, L. and E. Gurel (1986). Price and volume effects associated with changes in the S&P 500 list: New evidence for the existence of price pressures. *Journal of Finance 41*, 851–860.

Harrison, J. M. and D. M. Kreps (1979). Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory 20*, 381–408.

Hefti, A. M. (2013). Attention and competition. *Working paper*.

Hellwig, C., S. Kohls, and L. Veldkamp (2012). Information choice technologies. *American Economic Review 102*, 35–40.

Hellwig, C. and L. Veldkamp (2009). Knowing what others know: Coordination motives in information acquisition. *Review of Economic Studies 76*, 223–251.

Hellwig, M. F. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory 22*, 477–498.

Hong, H., T. Lim, and J. C. Stein (2000). Bad news travels slowly : Size, analyst coverage, and the profitability of momentum strategies. *Journal of Finance 55*, 265–295.

Hong, H. and J. C. Stein (1999). A unified theory of underreaction, momentum trading, and overreaction in asset markets. *Journal of Finance 54*, 2143–2184.

Ince, O. S. and R. B. Porter (2006). Individual equity return data from Thomson Datastream: Handle with care. *Journal of Financial Research 29*, 463–479.

Jagannathan, R. and Z. Wang (1996). The conditional CAPM and the cross-section of expected returns. *Journal of Finance 51*, 3–53.

Jarrow, R. A. and S. M. Turnbull (1992). Credit risk: Drawing the analogy. *Risk Magazine 5*, 63–70.

Jegadeesh, N. and S. Titman (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance 48*, 65–91.

Kang, C. O. and H. G. Kang (2009). The effect of credit risk on stock returns. *Journal of Economic Research 14*, 49–67.

Kelly, B. and H. Jiang (2012). Tail risk and hedge fund returns. *Working paper*.

Kelly, B. and H. Jiang (2013). Tail risk and asset prices. *Working paper*.

Kreps, D. M. and E. L. Porteus (1978). Temporal resolution of uncertainty and dynamic choice theory. *Econometrica 46*, 185–200.

Lakonishok, J., A. Shleifer, and R. W. Vishny (1994). Contrarian investment, extrapolation, and risk. *Journal of Finance 49*, 1541–1578.

Lewellen, J., S. Nagel, and J. Shanken (2010). A skeptical appraisal of asset pricing tests. *Journal of Financial Economics 96*, 175–194.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics 47*, 13–37.

Lucas, R. E. (1978). Asset prices in an exchange economy. *Econometrica 46*, 1429–1446.

Luo, Y. and E. R. Young (2010). Asset pricing under information-processing constraints. *Economics Letters 107*, 26–29.

Mackowiak, B. and M. Wiederholt (2009). Optimal sticky prices under rational inattention. *American Economic Review 99*, 769–803.

Mallaby, S. (2010). *More Money Than God.* New York, New York: The Penguin Press.

Malloy, C. J., T. J. Moskowitz, and A. Vissing-Jorgensen (2009). Long-run stockholder consumption risk and asset returns. *Journal of Finance 64*, 2427–2479.

Markowitz, H. M. (1959). *Portfolio Selection: Efficient Diversification of Investments.* New York, New York: John Wiley & Sons Inc.

Matejka, F. and A. McKay (2012). Simple market equilibria with rationally inattentive consumers. *American Economic Review 102*, 24–29.

Merton, R. C. (1973). An intertemporal asset pricing model. *Econometrica 41*, 867–887.

Merton, R. C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance 29*, 449–470.

Miller, E. M. (1977). Risk, uncertainty and divergence of opinion. *Journal of Finance 32*, 1151–1168.

Moat, H. S., C. Curme, A. Avakian, D. Y. Kenett, H. E. Stanley, and T. Preis (2013). Quantifying Wikipedia usage patterns before stock market moves. *Scientific Reports 3:1801*.

Mondria, J., T. Wu, and Y. Zhang (2010). The determinants of international investment and attention allocation: Using internet search query data. *Journal of International Economics 82*, 85–95.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica 34*, 768–783.

Pastor, L. and R. F. Stambaugh (2003). Liquidity risk and expected stock returns. *The Journal of Political Economy 111*, 642–685.

Peng, L. (2005). Learning with information capacity constraints. *Journal of Financial and Quantitative Analysis 40*, 307–329.

Peng, L. and W. Xiong (2006). Investor attention, overconfidence and category learning. *Journal of Financial Economics 80*, 563–602.

Preis, T., H. S. Moat, and H. E. Stanley (2013). Quantifying trading behavior in financial markets using google trends. *Scientific Reports 3:1684*.

Reinhart, C. M. and K. S. Rogoff (2009). *This Time Is Different: Eight Centuries of Financial Folly*. Princeton, New Jersey: John Wiley & Sons Ltd.

Rietz, T. A. (1988). The equity risk premium: A solution. *Journal of Monetary Economics 22*, 117–131.

Rosenberg, B., K. Reid, and R. Lanstein (1985). Persuasive evidence of market inefficiency. *Journal of Portfolio Management 11*, 9–17.

Ross, S. A. (1978). A simple approach to the valuation of risky streams. *Journal of Business 51*, 453–475.

Ruenzi, S. and F. Weigert (2012). Extreme dependence structures and the cross-section of expected stock returns. *Working paper*.

Scherer, B. (2010). A new look at minimum variance investing. *Working paper*.

Schmidt, P. S., U. von Arx, A. Schrimpf, A. F. Wagner, and A. Ziegler (2011). On the construction of common size, value and momentum factors in international stock markets: A guide with applications. *Swiss Finance Institute Research Paper Series 58*.

Schönbucher, P. J. (2003). *Credit Derivatives Pricing Models: Models, Pricing and Implementation*. Chichester, England: John Wiley & Sons Ltd.

Schuermann, T. and K. J. Stiroh (2006). Visible and hidden risk factors for banks. *Federal Reserve Bank of New York Staff Reports 252*.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance 19*, 425–442.

Shiller, R. J. (1981). The use of volatility measures in assessing market efficiency. *Journal of Finance 36*, 291–304.

Shiller, R. J. (1984). Stock prices and social dynamics. *Brookings Papers on Economic Activity 2*, 457–510.

Shleifer, A. (1986). Do demand curves for stocks slope down? *Journal of Finance 41*, 579–590.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics 50*, 665–690.

Sims, C. A. (2005). Rational inattention : A research agenda. *Discussion Paper Series 1: Economic Studies 34*.

Stattman, D. (1980). Book values and stock returns. *The Chicago MBA: A Journal of Selected Papers 4*, 25–45.

Sundaresan, S. (1989). Intertemporally dependent preferences and the volatility of consumption and wealth. *Review of Financial Studies 2*, 73–89.

Tutino, A. (2013). Rationally inattentive consumption choices. *Review of Economic Dynamics 16*, 421–439.

VanNieuwerburgh, S. and L. Veldkamp (2008). Information acquisition and under-diversification. *NBER Working Paper 13904*.

Vassalou, M. and Y. Xing (2004). Default risk in equity returns. *Journal of Finance 59*, 831–868.

Veldkamp, L. (2011). *Information Choice in Macroeconomics and Finance*. Princeton, New Jersey: Princeton University Press.

Wallmeier, M. and K. Tauscher (2013). A note on the impact of portfolio overlapping in tests of the Fama and French three-factor model. *Working paper*.

Walter, C. E. and F. J. Howie (2011). *Red Capitalism*. Singapore: John Wiley & Sons (Asia) Pte. Ltd.

Woodford, M. (2009). Information-constrained state-dependent pricing. *Journal of Monetary Economics 56*, 100–124.