Incentive Mechanisms for Hierarchical Spectrum Markets

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***Abstract*—We study spectrum allocation mechanisms in hierarchical multi-layer markets which are expected to proliferate in the near future according to the evolving spectrum policy reform proposals. We consider the scenario that arises when a governmental agency sells spectrum channels to Primary Operators (POs) who subsequently resell them to Secondary Operators (SOs) through auctions. We show that these hierarchical markets do not result in a socially efficient spectrum allocation which is aimed by the agency, due to lack of coordination among the entities in different layers and the inherently selfish revenue-maximizing strategy of POs. In order to reconcile these opposing objectives, we propose an incentive mechanism which aligns the strategy of the POs with the objective of the agency. This pricing-based scheme constitutes a method for hierarchical market regulation. A basic component of the mechanism is a novel auction scheme which enables POs to allocate their spectrum by balancing their derived revenue and the welfare of the SOs. Our analytical and numerical results indicate that the proposed incentive mechanism leads to significant system performance improvement in terms of social welfare.**

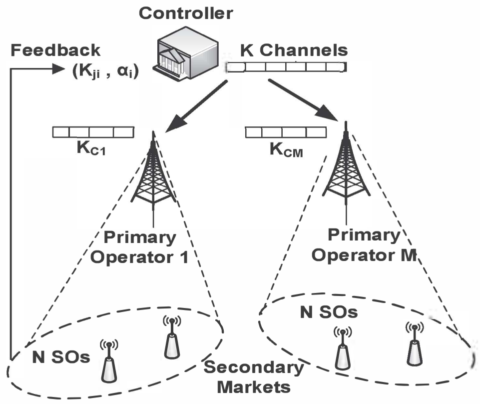


Fig. 1. A three-layer hierarchical spectrum market with one Controller, M primary operators and N secondary operators under each PO.

I. Introduction

A. Motivation

Despite the fact that spectrum is a scarce and expensive to obtain resource, significant amount of it remains idle and unexploited by legitimate owners, [1]. A prominent proposed solution for this problem is the reform of the spectrum allocation policy and the deployment of dynamic spectrum (DS) markets. Spectrum should be allocated in a finer spatio-temporal scale to the interested buyers, the so-called primary operators (POs) [2] and the POs should be able to lease their idle channels to secondary operators (SOs) [3], who serve fewer users in smaller areas. This hierarchical allocation is expected to increase spectrum utilization and related business models have already appeared in the market [4]. Nevertheless, market-based methods for spectrum management are not a panacea and should be carefully employed.

In these hierarchical markets the objective of the agency, which we call hereafter *controller* (CO), is to allocate the spectrum efficiently, i.e. to maximize the aggregate social welfare from its use. However, this objective cannot be achieved because of the following reasons: (i) the **coordination problem**, and the (ii) **conflicting objectives problem**. The first problem emerges when the CO assigns the spectrum to the POs without knowledge on the needs of the SOs (secondary demand). The second problem arises due to the inherently selfish behavior of POs who resell their spectrum in order to maximize their revenue. Clearly, this strategy contradicts the goal of the controller.

In this paper we study spectrum allocation in these hierarchical markets and propose an *incentive mechanism* that improves their efficiency by addressing the above two issues. The mechanism is deployed by the controller who acts as *regulator* and incentivizes the POs to redistribute their spectrum in a socially aware fashion. We consider a basic setting depicted in Figure 1, where each PO is a monopolist and has a certain clientele of SOs. Monopolies are expected to arise often in these markets because the POs obtain the exclusive spectrum use rights for certain areas or because they collude and act effectively as one single seller. The spectrum allocation from the CO to the POs and from the POs to the SOs is accomplished through auction-based mechanisms since there is lack of information about the spectrum demand in each layer. When the CO does not intervene in the market, the POs are expected to employ an optimal auction [5] which maximizes their expected revenue but induces efficiency loss, i.e. the channels are not always allocated to the operators with the highest spectrum demand.

Accordingly, we propose an incentive mechanism based on which the CO charges each PO in proportion to the inefficiency that is caused by its spectrum redistribution decisions. This way, the POs are induced to allocate their spectrum using a new auction scheme which produces less revenue for them but more welfare for the SOs. This is a *novel multi-item auction* where the objective of the auctioneer is a linear combination of its revenue and the valuations of the bidders. The balance between the objective of the POs and the SOs is tuned by a scalar parameter which is determined by the CO and captures its regulation policy. A basic component of the incentive mechanism is a feedback loop through which the SOs provide side information to the controller about the resource allocation decisions of the POs.

B. Related Work and Contribution

Primary operators are considered revenue maximizing entities and hence they are expected to use an optimal auction mechanism. Optimal auctions were introduced by Myerson [5] for single item allocation and extended later for multiple items [6], [7]. They ensure the highest expected revenue for the auctioneer, compared to any other type of auction, but they induce efficiency loss [8], [9]: it is not guaranteed that the auctioned items will be allocated to the bidders with the highest valuations. On the other hand, Vickrey-Clarke-Groves (VCG) auctions constitute the best option for achieving an efficient allocation under a variety of settings and assumptions [9]. However, they often exhibit a high computational and communication complexity that makes their implementation an extremely difficult -if not impossible- task [10].

The interaction of primary and secondary operators is usually modeled as a monopoly market. For example, in [11] the authors consider a setting where each primary license holder sells its idle spectrum channels to a set of secondary users and show that the optimal auction yields higher profit but results in inefficient allocation. A similar monopolistic setting is considered in [12], [13] and [14]. These works analyze exclusively the primary - secondary operators interaction without taking into account the hierarchical structure of the spectrum markets.

This aspect is studied in [15] where the authors consider a multi-layer market and present a mechanism to match demand and supply in the interrelated spectrum markets. Similar models have been considered in [16], [17] and [18]. However, in these studies the demand is considered known and there is no conflict among the objectives of the various entities. Finally, in [19], the authors consider a general hierarchical communication market and explain that due to the different objectives among the 1^{st}-layer auctioneer and the intermediaries, the overall resource allocation is either inefficient or untruthful.

Unlike previous works, in our setting, the entities in the different layers have conflicting interests and there is lack of information about the actual demand in each layer. The intermediaries (2^{nd} layer auctioneers) have (self-) valuations for the spectrum, and are allowed to select the auction scheme that yields for them the maximum possible revenue. Our work is inspired by the sponsored search (keyword) auction mechanisms [20], which assign the search engines advertising slots by taking into account the feedback from the *clickers*.

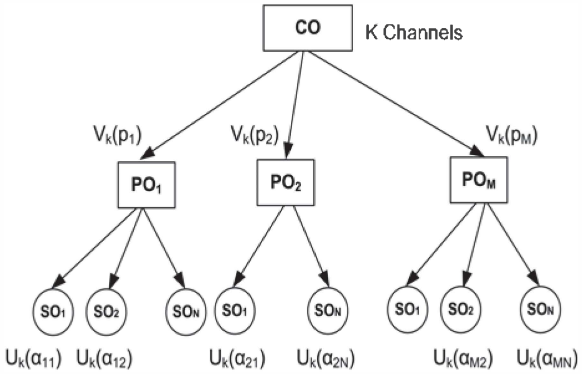


Fig. 2. System Model. The CO has K channels which allocates to M POs. Each PO leases its idle channels to N SOs. $V_k(p_j)$ is the valuation of PO j for the k^{th} channel and $U_k(\alpha_{ji})$ the respective valuation of the SO i under PO j. Variables p_j and α_{ji} represent the operators types.

Similar concepts can be used for the allocation of spectrum as we suggested in [21]. Here, we take a further step towards this direction by giving a detailed methodology.

In summary, the contributions of this work are the following: (i) we analyze the unregulated hierarchical spectrum allocation and show that it is inefficient, (ii) we present an incentive mechanism that motivates the POs to increase the efficiency of their spectrum redistribution, (iii) we introduce the β-optimal auction which achieves a balance between the revenue of the seller (optimality) and the welfare of the buyers (efficiency). This is a new mechanism that can be used also for the allocation of similar resources such as bandwidth, transmission power, etc.

The rest of this paper is organized as follows. In Section II we introduce the system model and in Section III we analyze the hierarchical spectrum allocation without the intervention of the controller. This analysis helps us to describe the incentive mechanism and assess its efficacy in Section IV. Finally in Section V we present our numerical study and conclude in Section VI.

II. System Model

We consider a three-layer hierarchical spectrum market with one *controller* (CO) on top of the hierarchy, a set $\mathcal{M} = \{1, 2, \ldots, M\}$ of *primary operators* (POs) in the second layer and a set $\mathcal{N}_j = \{1, 2, \ldots, N_j\}$, $j \in \mathcal{M}$ of *secondary operators* (SOs) that lie in the third layer under each PO, as it is shown in Figure 2. There exists a set $\mathcal{K} = \{1, 2, \ldots, K\}$ of identical spectrum channels which the CO allocates to the M primary operators. Accordingly, each PO redistributes the channels it acquired among itself and the N_j SOs that lie in its secondary market. The objective of the POs is to incur maximum revenue from reselling the spectrum while satisfying their own needs.

The perceived utility of each operator (PO or SO) for acquiring a channel is represented by a scalar value. Following the law of diminishing marginal returns we consider that each additional channel has smaller additional value increase for the operator. Different operators may have different spectrum needs and hence different channel valuations. For example, an operator with many clients will have very high channel valuations since it can accrue significant revenue. We summarize the different needs of the operators with a real-valued parameter which we call the *type* of the operator [9], [12].

Secondary Operators: In detail, consider a SO $i \in \mathcal{N}_j$, in the secondary market of PO $j \in \mathcal{M}$, with $k-1$ channels at its disposal. Its valuation for acquiring one more channel, i.e. the k^{th} channel, is $U_k(\alpha_{ji}) \in \mathcal{R}^+$ which is assumed to be positive, monotonically increasing and differentiable function of parameter α_{ji}. This is the *type* of the SO and represents its spectrum needs. For example, it can be related to the number of users the operator serves. The SOs types are independent random variables (i.r.v.), $\alpha_{ji} \in \mathcal{A} = (0, A_{max})$, $A_{max} \in R^+$, drawn from the same distribution function $F(\cdot)$ with finite density $f(\cdot)$ on $\mathcal{A}$. We define $\boldsymbol{\alpha}_j = (\alpha_{j1}, \alpha_{j2}, \ldots, \alpha_{jN_j})$. We assume that it is: $U_1(\alpha_{ji}) \geq U_2(\alpha_{ji}) \geq \ldots \geq U_K(\alpha_{ji}) \geq 0$, for each $\alpha_{ji} \in \mathcal{A}$, $i \in \mathcal{N}_j$, $j \in \mathcal{M}$. The SO i pays for the channels an amount of money that is determined by the respective PO j.

Primary Operators: Each PO $j \in \mathcal{M}$ receives K_{cj} channels from the CO at a cost of $Q(K_{cj})$ monetary units and decides how many it will reserve for its own needs, K_{j0}, and how many it will allocate to each one of the N_j SOs at its secondary market, $\mathbf{K}_j = (K_{ji} : i \in \mathcal{N}_j)$. We assume that the valuation of the PO for using the k^{th} additional channel is $V_k(p_j) \in \mathcal{R}^+$ which belongs to a known family of functions $V_k(\cdot)$ and is parameterized by the private i.r.v. $p_j \in \mathcal{P} = (0, P_{max})$, $P_{max} \in R^+$. In analogy with α_{ji}, p_j is the type of the PO and models its spectrum needs, e.g. the number of the PO's clients. The valuation functions are considered positive, monotonically increasing and continuously differentiable w.r.t. the type p_j: $V_1(p_j) \geq \ldots \geq V_K(p_j) \geq 0$.

The benefit of the PO from reselling its spectrum to the respective secondary market is given by the revenue component $H(\mathbf{K}_j, \boldsymbol{\alpha}_j)$. POs may accrue different revenue either because they sell different number of channels or because they have different demand (types of SOs) in the respective secondary market. We define the combined valuation - revenue objective of each PO $j \in \mathcal{M}$ as follows:

$$J(p_j, \boldsymbol{\alpha}_j, K_{j0}, \mathbf{K}_j) = \sum_{k=1}^{K_{j\bullet}} V_k(p_j) + H(\mathbf{K}_j, \boldsymbol{\alpha}_j) \quad (1)$$

Controller: The goal of the controller is to increase the efficiency of the hierarchical spectrum market, i.e. to ensure that the channels are allocated to the operators with the highest possible needs and deter POs from exploiting the SOs. Therefore it acts as regulator and deploys an incentive mechanism to induce a channel allocation that maximizes a balanced sum of the POs' combined objectives and the valuations of the SOs:

$$C(\beta) = \sum_{j=1}^{M} [J(p_j, \boldsymbol{\alpha}_j, K_{j0}, \mathbf{K}_j) + \beta \sum_{i=1}^{N_j} \sum_{k=1}^{K_{ji}} U_k(\alpha_{ji})] \quad (2)$$

where $\beta \in R^+$ is defined by the CO and determines this balance. Notice that the objective of the CO incorporates both the channel valuation of the POs and their revenue components, since the latter are the their motivation for reallocating the spectrum.

III. Unregulated Hierarchical Spectrum Allocation

We begin our study with the unregulated hierarchical spectrum allocation and show that the channels are not allocated efficiently, i.e. to the primary and secondary operators with the highest channel valuations. In order to simplify our analysis we assume that each secondary market has the same number of SOs, i.e. $N_j = N$, $\forall j \in \mathcal{M}$. The presented model and analysis is used in Section IV in order to explain the proposed incentive mechanism.

A. Second Stage: SOs - PO Interaction

We assume that each PO has only partial information about the underneath secondary market. It knows the family of the valuation functions of the SOs, $U_k(\alpha)$, $k \in \mathcal{K}$, and their types distribution function $F(\cdot)$ but not their actual types. To elicit this missing information the PO runs an *optimal auction* where each one of the N SOs submits a bid, $b_i \in \mathcal{A}$ in order to declare its type α_{ji}, i.e. its spectrum needs. The PO collects the bids, $\mathbf{b} = (b_i : i \in \mathcal{N}_j)$, and determines the allocated spectrum and the respective payment for each bidder. Here, the seller (PO) is also interested in the auctioned items and hence it compares its expected revenue from selling a channel with the valuation for using it, $V_k(\cdot)$, before it decides if it will re-allocate it or reserve it.

In this auction, each PO $j \in \mathcal{M}$ finds the optimal allocation, $(\mathbf{K}_j^*, K_{j0}^*)$, of its K_{cj} channels which maximizes its combined objective given by eq. (1) by solving the following **PO Spectrum Allocation Problem**, $(\mathbf{P_{po}})$:

$$\max_{\mathbf{K}_j, K_{j0}} J(p_j, \boldsymbol{\alpha}_j, K_{j0}, \mathbf{K}_j) \quad (3)$$

s.t.

$$K_{j0} + \sum_{i=1}^{N} K_{ji} \leq K_{cj},\ K_{ji},\ K_{j0} \in \{0, 1, 2, \ldots, K_{cj}\} \quad (4)$$

Notice that, initially the POs do not know how many channels they will receive from the CO. Therefore, they solve problem $\mathbf{P_{po}}$ for $K_{cj} = K$.

The maximization of the expected revenue, $(\mathbf{P_{po}})$, can be transformed to a deterministic channel allocation problem. Let us first define the additional expected revenue the PO incurs for assigning the k^{th} channel to SO $i \in \mathcal{N}_j$. In auction theory [6] this is known as the *contribution* of the bidder (here the SOs) and is defined as:

$$\pi_k(b_i) = U_k(b_i) - \frac{dU_k(\alpha)}{d\alpha}\Big|_{\alpha=b_i} \frac{1 - F(b_i)}{f(b_i)} \quad (5)$$

Notice that the *contribution*, i.e. the expected revenue, is directly related to the value the auctioned item has for the buyer. Due to the assumptions we made about our model, the *contributions* are monotonically strictly increasing in the types of the SOs and decreasing in the number of channels. Therefore, the so-called *regularity conditions* [6] are satisfied and the auction problem $\mathbf{P_{po}}$ is *regular*. In this case the channel allocation that maximizes the combined objective of the PO j can be easily derived using the following deterministic allocation and payment rules.

1) PO Optimal Auction Allocation Rule: The auctioneer (PO j) calculates the contributions $\pi_k(b_i)$ of each SO $i \in \mathcal{N}_j$ for all the auctioned channels, $k = 1, \ldots, K_{cj}$, and selects the K_{cj} highest of them. In the sequel, it compares these K_{cj} contributions with its own valuations for the channels and constructs the contribution-valuation vector $\mathbf{X}_j$ which has K_{cj} elements in decreasing order:

$$\mathbf{X}_j = (x_{(l)} : x_{(l)} > x_{(l+1)}, l = 1, \ldots, K_{cj}) \quad (6)$$

Then, the PO simply assigns each channel $l = 1, \ldots, K_{cj}$ to the respective i^{th} SO if $x_{(l)} = \pi_k(b_i)$ or it reserves it for itself if $x_{(l)} = V_k(p_j)$. For example, for a PO with 4 channels and two SOs bidders, a possible instance of X_j is $X_j = (V_1(p_j), \pi_1(b_1), \pi_1(b_2), V_2(p_j))$ which means that the PO will reserve 2 channels for itself and assign one to each SO.

2) PO Optimal Auction Payment Rule: The price that each SO i pays for receiving the k^{th} spectrum channel depends on the bids submitted by all the other SOs, $\mathbf{b}_{-i} = (b_n : n \in \mathcal{N} \setminus \{i\})$. Namely, let us denote with $z_k(b_{-i})$ the minimum bid that the i^{th} SO has to submit in order to acquire the k^{th} channel, [6]:

$$z_k(\mathbf{b}_{-i}) = \inf\{\hat{\alpha}_{ji} \in \mathcal{A} : \pi_k(\hat{\alpha}_{ji}) \geq \max\{0, x_{(K_{cj}+1)}\}\} \quad (7)$$

This means that in order to get the k^{th} item the i^{th} SO has simply to submit a bid high enough to draft its contribution within the first K_{cj} elements of $\mathbf{X}_j$. The actual charged price for each channel is equal to its valuation had it a type equal to this minimum bid [5], [6], [12]. Hence the aggregate payment for the SO is:

$$h(b_i, \mathbf{b}_{-i}) = \sum_{k=1}^{K_{ji}(b_i, \mathbf{b}_{-i})} U_k(z_k(\mathbf{b}_{-i})) \quad (8)$$

Due to the payment and the respective monotonic allocation rule, the auction mechanism is incentive compatible and individual rational [11], [20]. Hence $b_i^* = \alpha_{ji}$, $\forall i \in \mathcal{N}_j$, i.e. the SOs reveal to the POs their actual private information, e.g. the actual number of their users.

B. First Stage: POs - CO Interaction

After learning the demand in their secondary spectrum markets, the POs ask the CO for spectrum. The controller determines the channel distribution, $\mathbf{K}_c = (\{\mathbf{K}_j, K_{j0}\} : j = 1, 2, \ldots, M)$ and the payment $Q(\cdot)$ for each PO. The latter depends on the number of channels K_{cj} the j^{th} PO receives, i.e. $K_{cj} = K_{j0} + \sum_{i=1}^{N} K_{ji}$. We assume that the CO knows the family of the valuation functions of POs, $V_k(\cdot)$, $k \in \mathcal{K}$ and of SOs, $U_k(\cdot)$, $k \in \mathcal{K}$, but not their exact types (p_j and α_{ji} respectively). Therefore the CO, in order to elicit this information, runs a Vickrey-Clarke-Groove (VCG) auction which is known to be efficient [9]. Every PO $j \in \mathcal{M}$ submits a vector bid $\mathbf{r}_j \in \mathcal{R}^{N+1}$, in order to declare its own type and the types of the SOs in its market. We assume that the first component of this vector $\mathbf{r}_j(1)$ represents the type of the PO, and the next N components the types of the SOs. The CO collects these bids, $\mathbf{r} = (\mathbf{r}_j : j = 1, 2, \ldots, M)$, and finds the channel allocation that maximizes the aggregate combined objective of all the POs by solving the **CO Spectrum Allocation Problem,** ($\mathbf{P_{co}}$):

$$\max_{\mathbf{K}_c} \sum_{j=1}^{M} J(\mathbf{r}_j, K_{j0}, \mathbf{K}_j) \quad (9)$$

s.t.

$$\sum_{j=1}^{M} K_{cj} \leq K, \ K_{cj} \in \{0, 1, 2, \ldots, K\} \quad (10)$$

$$K_{cj} = K_{j0} + \sum_{j=1}^{N} K_{ji}, \ j = 1, 2, \ldots, M \quad (11)$$

One simple method to find the solution $\mathbf{K}_c^*$ of problem $\mathbf{P_{co}}$, is to sort in decreasing order the valuations $V_k(\mathbf{r}_j(1))$ of all POs, $j = 1, 2, \ldots, M$, and the contributions $\pi_k(\mathbf{r}_j(i))$ $i = 1, 2, \ldots N$ of their SO clients, for each channel $k = 1, 2, \ldots K$. Then the CO allocates each channel to the operator with the highest valuation (for POs) or contribution (for SOs). Each PO gets the channels for itself and the underneath secondary market.

The payment imposed to each PO, according to the VCG payment rule [9], is equal to the externality it introduces:

$$Q(\mathbf{r}) = \sum_{m \neq j}^{M} J(\mathbf{r}_m, \tilde{K}_{m0}^*, \tilde{\mathbf{K}}_m^*) - \sum_{m \neq j}^{M} J(\mathbf{r}_m, K_{m0}^*, \mathbf{K}_m^*) \quad (12)$$

where $(K_{m0}^*, \mathbf{K}_m^*)$ are the channels allocated to each PO $m \in \mathcal{M} \backslash \{j\}$ and the respective SO market according to the solution of problem ($\mathbf{P_{co}}$), and $(\tilde{K}_{m0}^*, \tilde{\mathbf{K}}_m^*)$ the allocated channels, i.e. the solution of problem $\mathbf{P_{co}}$, when PO j does not participate in the auction, i.e. when $\mathbf{r}_j = \mathbf{0}$.

In this auction, the POs determine their bid by solving the following **PO Bidding Problem**, ($\mathbf{P_{po}^b}$) :

$$\mathbf{r}_j^* = \arg\max_{\mathbf{r}_j} \{J(p_j, \boldsymbol{\alpha}_j, K_{j0}^*, \mathbf{K}_j^*) - Q(\mathbf{r}_j, \mathbf{r}_{-j})\} \quad (13)$$

Since VCG auctions are incentive compatible [9], each PO $j \in \mathcal{M}$ will reveal its actual type, $\mathbf{r}_j^*(1) = p_j$, and the true types of its SOs $\mathbf{r}_j^*(i+1) = \alpha_{ji}$, $i = 1, 2, \ldots, N$ which it learned in the first stage of this hierarchical spectrum allocation.

C. Inefficiency of the Unregulated Hierarchical Allocation

From the previous analysis it is evident that the main reason that renders this hierarchical spectrum allocation inefficient is the conflicting objectives problem. The POs act so as to maximize their valuation and expected revenue while the CO would like to increase the allocative efficiency of the channels. Clearly, POs allocate their channels to the SOs that pay higher and not to those with the highest valuations. Moreover, a PO may reserve a channel for itself, although there is a SO with a higher valuation for it, if selling it does not yield high enough revenue. Additionally, due to this misalignment of the objectives the CO may allocate too many channels to a PO who has low secondary demand and less channels to a PO with higher secondary demand.

There is an *important* observation to make here. Although that the controller learns the actual demand of the primary and secondary spectrum markets, through the VCG auction it runs, it cannot allocate the channels efficiently. Namely, if the CO decides to maximize another function, e.g. the sum of POs and SOs valuations, and not the combined objectives of the POs, then the auction would not be incentive compatible anymore. The POs are free to select their objective function (revenue maximization) and the CO has to comply with this and run an auction with the same objective.

IV. Regulated Hierarchical Spectrum Allocation

A. Incentive Mechanism $\mathcal{M}_R$

The goal of the controller is to induce the channel allocation $\mathbf{K}_c^\beta = \{\{K_{j0}^*, \mathbf{K}_j^*\} : j = 1, 2, \ldots, M\}$ for each PO $j \in \mathcal{M}$ and the respective secondary market that maximizes its objective $C(\beta)$, given by eq. (2). This allocation stems from the solution of the **CO Balanced Spectrum Allocation Problem,** ($\mathbf{P_{co}^{bal}}$):

$$\max_{\mathbf{K}_c} \sum_{j=1}^{M} \left[J(p_j, \boldsymbol{\alpha}_j, K_{j0}, \mathbf{K}_j) + \beta \sum_{i=1}^{N} \sum_{k=1}^{K_{ji}} U_k(\alpha_{ji}) \right] \quad (14)$$

s.t.

$$\sum_{j=1}^{M} (K_{j0} + \sum_{i=1}^{N} K_{ji}) \leq K_c, \; K_{j0}, K_{ji} \in \{0, 1, \ldots, K_c\} \quad (15)$$

parameter $\beta \in R^+$ is determined by the CO and defines implicitly the revenue of the POs and the welfare of the SOs.

The difficulties the controller encounters to achieve its goal are: (i) the CO is not aware of the types of the POs, p_j, $j \in \mathcal{M}$, (ii) it does not know the types of the SOs in each secondary market a_{ji}, $i \in \mathcal{N}_j$, $j \in \mathcal{M}$, and (iii) it cannot directly dictate the POs how to redistribute the channels they acquired nor it can observe how they did allocated them. The introduced incentive mechanism, which we call *Mechanism* $\mathcal{M}_\mathcal{R}$, addresses these issues and achieves the desirable spectrum allocation.

The proposed scheme is based on pricing and the underlying idea is that the controller creates a coupling between the spectrum allocation decisions of the POs and their cost for acquiring the spectrum in order to bias their revenue maximizing strategy. Namely, we suggest that the CO should reimburse the PO $j \in \mathcal{M}$ with the following price:

$$L_j(\boldsymbol{\alpha}_j, \mathbf{K}_j, \beta) = \beta \sum_{i=1}^{N} \sum_{k=1}^{K_{ji}} U_k(\alpha_{ji}) \quad (16)$$

This modifies the objective function of the PO as follows:

$$J_R(\cdot) = J(p_j, \boldsymbol{\alpha}_j, K_{j0}, \mathbf{K}_j) + \beta \sum_{i=1}^{N} \sum_{k=1}^{K_{ji}} U_k(\alpha_{ji}) \quad (17)$$

$J_R(p_j, \boldsymbol{\alpha}_j, K_{j0}, \mathbf{K}_j, \beta)$ is the regulated new combined objective of each PO which depends on parameter β and is aligned with the balanced objective of the CO, eq. (2). In the sequel, we explain how this modification impacts the interaction of POs and SOs.

B. The β-Optimal Auction Mechanism

Each PO maximizes $J_R(\cdot)$ by solving a new allocation problem $\mathbf{P}_{\mathbf{po}}^{\beta}$ which differs from the respective $\mathbf{P_{po}}$ problem in the objective function that is given now by eq. (17). The primary operator runs again an auction to elicit the SOs types missing information. However, this is neither an efficient nor an optimal auction and hence it cannot employ any of the known auction schemes. To address this problem, we introduce a new multi-item auction mechanism, the β-**optimal** auction, which ensures the maximization of the balanced objective defined in eq. (17). This mechanism is similar to the optimal auction discussed in section III-A with the difference that the allocation rule is biased by parameter β. This modification affects the allocation of the channels and results in reduced payments from the bidders to the auctioneer and improved efficiency in channels allocation. The combination of optimal and efficient auctions has been also suggested in [22] for single item allocation where the authors proposed an efficient auction with a lower bound on the sellers revenue.

Let us now explain the rationale and machinery of the β-optimal auction. First we define the β-contribution for each SO $i \in \mathcal{N}_j$ under a certain PO $j \in \mathcal{M}$, as follows:

$$\pi_k^\beta(b_i) = \pi_k(b_i) + \beta U_k(b_i) \quad (18)$$

which can be written:

$$\pi_k^\beta(b_i) = (1+\beta) U_k(b_i) - \frac{dU_k(\alpha)}{d\alpha}\bigg|_{\alpha=b_i} \frac{1-F(b_i)}{f(b_i)} \quad (19)$$

Since $\beta \geq 0$ it is $\pi_k^\beta(\alpha_{ji}) \geq \pi_k(\alpha_{ji})$ for all the SOs and all the channels. The reason we selected this particular expression for $\pi_k^\beta(\cdot)$ can be understood if one compares $J(\cdot)$ with $J_R(\cdot)$. Also, it will be explained in the sequel in the proof of Proposition 1. Finally, notice that if the initial *contributions* satisfy the *regularity* conditions, [6], then the β-contributions will also satisfy them, and hence problem $\mathbf{P}_{\mathbf{po}}^{\beta}$ will be *regular*.

β-Optimal Auction Allocation Rule: Similarly to the allocation rule of the optimal auction, the j^{th} PO calculates the $\pi_k^\beta(b_i)$ for all SOs $i \in \mathcal{N}_j$ and all channels $k = 1, \ldots, K_{cj}$ and compares them with its own valuations in order to construct the contribution-valuation vector $\mathbf{X}_j^\beta$:

$$\mathbf{X}_j^\beta = (x_{(l)}^\beta : x_{(l)}^\beta > x_{(l+1)}^\beta, \, l = 1, \ldots, K_{cj}^\beta) \quad (20)$$

Using $\mathbf{X}_j^\beta$, the PO allocates its channels to the respective K_{cj} highest contributions and valuations. The resulting channel allocation $(K_{j0}^\beta, \mathbf{K}_j^\beta)$ solves problem $\mathbf{P}_{\mathbf{po}}^{\beta}$ and maximizes the new objective $J_R(\cdot)$. Again, this allocation rule is monotone increasing in the types of the SOs.

β-Optimal Auction Payment Rule: The payment rule changes in order to comply with the new allocation rule. Namely, the minimum bid that the i^{th} SO needs to submit in order to acquire the k^{th} channel is:

$$z_k^\beta(b_{-i}) = \inf\{\hat{\alpha}_{ji} \in \mathcal{A} : \pi_k^\beta(\hat{\alpha}_{ji}) \geq \max\{0, x^\beta_{(K_{cj}^\bullet+1)}\}\} \tag{21}$$

and, similarly to the previous mechanism, the total payment for this SO is :

$$h^\beta(b_i, b_{-i}) = \sum_{k=1}^{K_{ji}^\beta(b_i,b_{-i})} U_k(z_k^\beta(b_{-i})) \tag{22}$$

Under this new auction mechanism, each SO $i \in \mathcal{N}_j$ selects its bid so as to maximize its new payoff, (**SO β-Bidding Problem, $\mathbf{P}^\beta_{\mathbf{so}}$**):

$$b_i^* = \arg\max_{b_i}\{ \sum_{k=1}^{K_{ji}^\bullet(b_i,b_{-i})} U_k(\alpha_{ji}) - h^\beta(b_i, b_{-i})\} \tag{23}$$

This new auction mechanism improves the efficiency of the POs - SOs interaction and at the same time retains the required properties of the optimal auctions as we explain with the following proposition.

Proposition 1. *The β-optimal auction mechanism preserves the incentive compatibility and the individual rationality properties of optimal multi-unit auction introduced in [6].*

Proof: We focus on PO $j \in \mathcal{M}$ with K_{cj} channels. We denote s_{ik} the probability of SO i for receiving the k^{th} channel which depends on the types of all the SOs. Additionally, $c_i(\alpha_{ji})$ is the payment of each SO i for all the channels it acquired. *Definition* 2 and *Lemma* 1 in [6] give the necessary conditions for the structure of the bidders (SOs) valuation functions in order to ensure the (IC) and (IR) properties. These conditions hold independently of the objective of the auctioneer (PO) and hence they are not affected by the incorporation of the linear term of the SOs valuation.

The objective of the PO w.r.t. the expected types of the SOs is:

$$E_{\mathcal{A}}[J_R] = \sum_{i=1}^{N} E_{\mathcal{A}}[c_i(\alpha_{ji})] + \beta \sum_{i=1}^{N} E_{\mathcal{A}}[\sum_{k=1}^{K_{cj}} U_k(\alpha_{ji}) s_{ik}] + + E_{\mathcal{A}}[\sum_{k=1}^{K_{cj}} V_k(p_j)(1 - \sum_{i=1}^{N} s_{ik})]$$

The first term is the payment by the SOs, the second is the pricing and the third the valuation for the channels that are not sold. After some algebraic manipulations and following the proof of *Proposition* 1 in [6], we get:

$$E_{\mathcal{A}}[J_R] = \sum_{i=1}^{N} E_{\mathcal{A}}\left[\sum_{k=1}^{K_{cj}} [(1+\beta)U_k(\alpha_{ji}) - V_k(p_j) - \frac{dU_k(\alpha_{ji})}{d\alpha} \frac{1 - F(\alpha_{ji})}{f(\alpha_{ji})}] s_{ik} \right] - \sum_{i=1}^{N}[\sum_{k=1}^{K_{cj}} U_k(0) - c_i(0)] + E_{\mathcal{A}}[\sum_{k=1}^{K_{cj}} V_k(p_j)]$$

Notice that the first term of contains the expression of $\pi_k^\beta(\cdot)$. Using the necessary (IC) and (IR) conditions from *Lemma* 1 in [6], it stems that the β-optimal payment rule is given again by equation (10) of [6]:

$$c_i^*(\alpha_{ji}) = E_{\mathcal{A}}\left[\sum_{k=1}^{K_{cj}} U_k(\alpha_{ji}) s_{ik} - \int_0^{\alpha_{ji}} \frac{dU_k(\alpha)}{d\alpha} s_{ik} d\alpha \right] \tag{24}$$

where the probabilities of allocation are selected so as to maximize the new objective of the auctioneer (instead of revenue only maximization as in [6]). The optimal payment rule is the one that yields zero payment and zero channel allocation for SOs with zero type.

If the problem is *regular* then the payment is as we described in section IV and the first term in the PO's objective is maximized by using the β-optimal allocation rule. This can be easily derived following the proof of the respective *Proposition* 2 in [6]. Notice that if the original respective problem in [6] is *regular* then also this modified problem is *regular*. Apparently, the inclusion of the SOs buyers valuations does not affect the monotonicity of the allocation rule nor the critical value property of the payment rule, [20]. Hence, the modified auction is still truthful. ■

This new type of auction yields a more efficient allocation than the typical optimal auction of Myerson, [5] as the following proposition states.

Proposition 2. *The β-optimal auction is more efficient than the optimal auction.*

Proof: In β-optimal auction, the allocation of items is accomplished with regard to the modified contributions $\pi_k^\beta(\cdot)$ which are larger than the respective contributions of the optimal auction $\pi_k(\cdot)$. Notice that it holds $U_k(\cdot) \geq \pi_k^\beta(\cdot) \geq \pi_k(\cdot)$, for all $k = 1, 2, \ldots, K$. This means that the β-optimal auction induces a channel allocation that is more close to the efficient allocation that is produced if the auctioneer considers the actual valuations $U_k(\cdot)$ of the bidders and not their contributions $\pi_k(\cdot)$. ■

C. Efficacy and Requirements of Mechanism $\mathcal{M}_R$

The pricing that is imposed by the CO, eq. (16), changes the bidding strategy of the POs. Firstly, it is important to emphasize that due to the reimbursement component $L_j(\cdot)$, the POs do not have an incentive to report truthfully the types of the SOs, as they did in the unregulated allocation scenario. Unfortunately, the VCG payment rule does not ensure truthful bidding in this case. Notice that the reimbursement, which is not part of the POs actual (initial) utility function, depends on the reported type values. Therefore, the primary operators

Algorithm 1 (Mechanism $\mathcal{M}_R$)

1st Stage: *Channel Allocation (β is announced by the CO).*
(1.1:) Each SO i bids to the respective PO, according to problem $\mathbf{P}_{\mathbf{so}}^{\beta}$, eq. (23).
(1.2:) Each PO collects the bids from the SOs, and participates in the VCG auction organized by the CO by bidding its type, eq. (25).
(1.3:) The CO solves problem $\mathbf{P}_{\mathbf{co}}^{\mathbf{bal}}$, and allocates $K_{cj}^{*} = K_{j0}^{*} + \sum_{i=1}^{N} K_{ji}^{*}$ channels to each PO $j \in \mathcal{M}$.
(1.4:) Each PO redistributes its channels according to the *β-Optimal Allocation* Rule.
2nd Stage: *Payments and Feedback.*
(2.1:) Each SO i pays the respective PO an amount of $h^{\beta}(b_i, b_{-i})$ monetary units, eq. (22).
(2.2:) Each SO i reveals to the CO the allocation decisions of the respective PO and its own type (*feedback* for K_{ji}^{*} and α_{ij}).
(2.3:) The CO collects the feedback and calculates $L_j(\alpha_j, \mathbf{K}_j, \beta)$, eq. (16), and the total price Λ_j each PO j has to pay:

$$\Lambda_j = Q_R(\mathbf{r}_j, \mathbf{r}_{-j}) - L_j(\alpha_j, \mathbf{K}_j(\mathbf{r}_j, \mathbf{r}_{-j}), \beta)$$

where $Q_R(\cdot)$ is the VCG price for problem $\mathbf{P}_{\mathbf{co}}^{\mathbf{bal}}$, eq. (26).

will most probably over-report the types of the respective secondary market. This means that the CO has to acquire the SOs type information by other means. One way is to overhear the auction results among the POs and the SOs or being informed directly from the SOs. We represent all this kind of methods with a feedback loop between the SOs and the controller.

Each PO $j \in \mathcal{M}$ after receiving the bids of its SOs, determines its optimal bid, representing its own type, by solving the **PO β-Bidding Problem, ($\mathbf{P}_{\mathbf{po}}^{\mathbf{b}}$)**:

$$\hat{p}_j^{*} = \arg\max\{J_R(p_j, \boldsymbol{\alpha}_j, K_{j0}^{\beta}, \mathbf{K}_j^{\beta}) - Q_R(r_j, r_{-j})\} \quad (25)$$

where $\hat{p}_j \in [0, P_{max}]$. Also, $Q_R(\cdot)$ is the new price charged by the controller when it employs mechanism $\mathcal{M}_R$. Specifically, the CO determines the channel allocation by solving problem $\mathbf{P}_{\mathbf{co}}^{\mathbf{bal}}$, eq. (14) - (15), and calculates the new VCG prices as follows:

$$Q_R(\hat{\mathbf{p}}) = \sum_{m \neq j}^{M} J_R(r_m, \tilde{K}_{m0}^{\beta}, \tilde{\mathbf{K}}_m^{\beta}) - \sum_{m \neq j}^{M} J_R(r_m, K_{m0}^{\beta}, \mathbf{K}_m^{\beta}) \quad (26)$$

where $\hat{\mathbf{p}} = (p_j, j \in \mathcal{M})$. Again, the number of channels $(K_{m0}^{\beta}, \mathbf{K}_m^{\beta})$ allocated to each PO m depends on bids submitted by all the POs. Also, $(\tilde{K}_{m0}^{\beta}, \tilde{\mathbf{K}}_m^{\beta})$ is the optimal channel allocation, i.e. the solution of problem $\mathbf{P}_{\mathbf{co}}^{\mathbf{bal}}$ for $\hat{p}_j = 0$. Therefore, the POs are induced to bid truthfully, $\hat{p}_j^{*} = p_j$, $\forall j \in \mathcal{M}$. We summarize mechanism $\mathcal{M}_R$ in Algorithm **1**.

In order to calculate the prices $\Lambda_j(\cdot)$, the CO needs to know additionally the amount of spectrum that is allocated to them by the PO. There are many different methods and scenarios about how the CO can acquire this information. First, the SOs may directly provide it through a feedback loop, along with their types. Equivalently, the CO may be able to observe the interaction of the SOs with the respective PO. Finally, the CO may be able to observe how many channels the POs reserve for them and this way infer how many they reallocate to their clients (secondary operators).

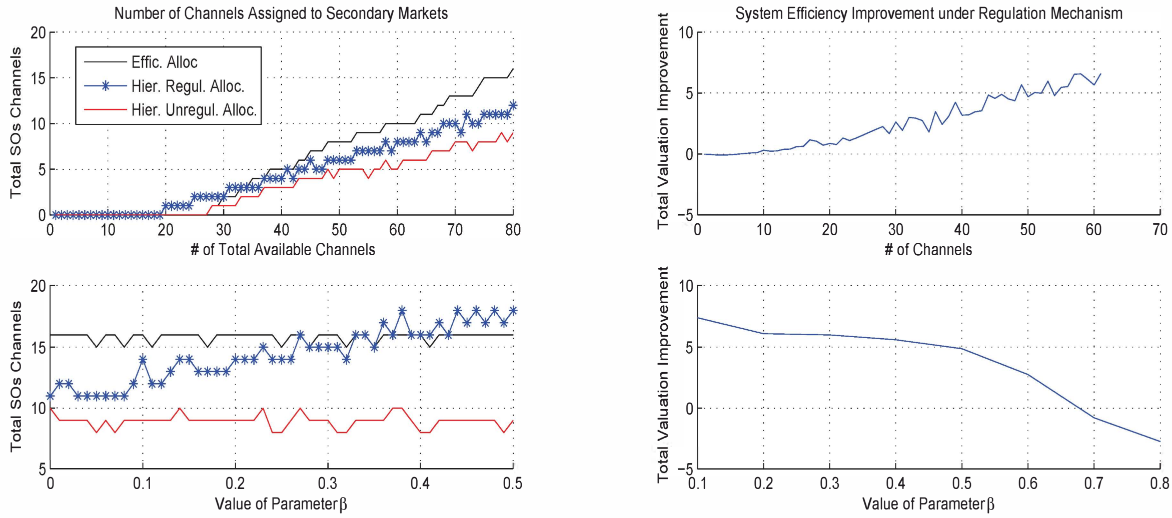
Since the controller is on top of this hierarchy and manages the spectrum, we can easily consider many similar methods that will allow him to receive direct or indirect feedback about the SO - PO interaction. Clearly, this assumption is necessary in order to increase the efficiency of the market. As it was made clear from the previous analysis, if the CO cannot impose the transaction rules of the lower level market and at the same time cannot observe the channel allocation decisions of POs, then there does not exist a realizable method for increasing the allocative efficiency.

V. Numerical Results

In order to obtain insights about the proposed mechanism $\mathcal{M}_R$, we simulate a representative three-layer hierarchical market with one CO, $M = 2$ POs and $N = 10$ SOs under each PO. We assume that the POs valuation functions for the k^{th} channel are $V_k(p_j) = p_j/k$, where the types p_j are uniformly distributed in the interval $[5, 6]$. Similarly, the SOs valuations are $U_k(\alpha_{ji}) = 0.1\alpha_{ji}/k$, and their types follow a uniform distribution $F(x) = x/4$ on the interval $(0, 4]$. The SOs contributions are $\pi_k(\alpha_{ji}) = (0.2\alpha_{ji} - 0.4)/k$ and the respective β-contributions are $\pi_k^{\beta}(\alpha_{ji}) = [(0.2+\beta)\alpha_{ji} - 0.4]/k$. For each random realization of the SOs and POs types, the results are averaged over 40 runs in order to capture the variance on the spectrum demand.

For our study we use as a benchmark the *efficient* channel allocation to the SOs. This allocation corresponds to the hypothetical scenario where the CO would be able to assign directly the channels to both the POs and the SOs and maximize the aggregate spectrum valuations. In the upper plot of Figure 3(a) we show that in hierarchical unregulated market the number of total channels assigned to the SOs is less than the channels in the *efficient* allocation. Mechanism $\mathcal{M}_R$ with $\beta = 0.1$ reduces this difference and increases the SOs channels. Notice that the number of SOs channels is stil less than in the *efficient* allocation scenario, since the goal of the CO is the combined revenue-efficiency balanced allocation.

In the same Figure we show that the number of channels assigned to SOs vary with the value of β. Namely, when $\beta = 0$ the allocation is identical with the unregulated case while for $\beta \approx 0.35$ it reaches the *efficient* allocation. Notice that for larger values of $\beta > 0.37$ the SOs receive even more channels. This means that the CO favors the SOs too much and render the channel allocation inefficient. The impact of β is depicted also in the lower plot of Figure, 3(b) where we see that for large values the improvement in the aggregate valuation of the POs and SOs becomes negative. For this plot, the number of SOs is $N = 20$ and the system welfare is maximized for



(a) *Upper Plot*: For $\beta = 0.1$ the regulation mechanism $\mathcal{M}_R$ increases the number of channels assigned to the SOs. *Lower Plot*: The SOs receive more channels for larger values of β ($K_c = 80$)

(b) *Upper Plot*: The aggregate network efficiency (POs and SOs valuations) increases with the $\mathcal{M}_R$, $\beta = 0.1$, $N = 20$, $K_c = 1 : 60$. *Lower Plot*: For large values of β the network efficiency decreases since the SOs are favored more than the POs.

Fig. 3. Channel allocation in a market with $M = 2$ POs and $N = 10$ SOs under each PO.

$\beta = 0.1$. If β is further increased, the welfare improvement decreases and eventually becomes negative.

VI. Conclusions

We explained that the emerging hierarchical spectrum markets will fail to allocate channels efficiently due to the revenue maximizing strategy of the primary operators who act as intermediaries. In order to solve this problem, we proposed an incentive mechanism that can be used by the controller so as to regulate the interaction between the primary and secondary operators and to induce a new market equilibrium. This equilibrium can be parameterized by a scalar parameter defined by the controller and determines the efficiency of the secondary markets by adjusting the number of channels allocated to the SOs. The mechanism is based on a novel auction scheme which has a revenue-welfare balanced objective.

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