Modelling Human Gameplay at Pool and Countering it with an Anthropomorphic Robot

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Abstract. Interaction between robotic systems and humans becomes increasingly important in industry, the private and the public sector. A robot which plays pool against a human opponent involves challenges most human robot interaction scenarios have in common: planning in a hybrid state space, numerous uncertainties and a human counterpart with a different visual perception system. In most situations it is important that the robot predicts human decisions to react appropriately. In the following, an approach to model and counter the behavior of human pool players is described. The resulting model allows to predict the stroke a human chooses to perform as well as the outcome of that stroke. This model is combined with a probabilistic search algorithm and implemented to an anthropomorphic robot. By means of this approach the robot is able to defeat a player with better manipulation skills. Furthermore it is outlined how this approach can be applied to other non-deterministic games or to tasks in a continuous state space.

1 Introduction

Robots have been utilized for many years in manufacturing to handle repetitive tasks fast and precisely. Besides in industry, robotic systems are emerging in the private and public sector requiring human robot interaction (HRI). On a high level in HRI one is not only concerted of noticeable human actions but also of underlying intentions [1] in the context of collaborative [2], competitive [3] as well as severe tasks [4]. Regarding HRI these areas have the following challenges in common: (I) a continuous state space, (II) the necessity of being aware of the human. Concerning these two challenges competitive games represent attractive test scenarios because people encounter them in their daily life and furthermore they represent a controllable environment as well as a fixed set of rules. Requiring motor performance and planning capabilities the game of pool is chosen as representative game. Pool has a hybrid state space: the confined space of the

table presents a continuous state space whereas the turn-based character of the game represents a discrete one. In computational pool Monte-Carlo sampling approaches prove to be robust, [5, 6]. In contrast, an optimisation approach enables spectacular strokes, ball clusters are broken or multiple balls are sunk with one stroke [7, 8]. A fuzzy-logic planner together with a robotic system is considered in [9]. However, the behavior of the human and his level of expertise has thus far not been taken into account. Existing approaches use mathematical models to determine the difficulty of game situations and calculate the best stroke without analyzing the human. But considering the human can improve prediction and planning of the robotic system for two reasons: First, a lateral visual perspective limits the human perception which influences his behavior in contrast to the bird's eye camera perspective of the robot. Second, the conception and evaluation of the game situation of humans is different to mathematical models.
In this paper a new approach is presented that enables the robot to assess the difficulty of game situations from a human perspective. The play of the robot improves by predicting the future actions of its human opponent. In order to do so, the robot needs to be aware of its opponent and requires a model of human pool-playing behavior. Different to all approaches introduced above, this paper concentrates on understanding and modeling human gameplay. It presents the steps to develop such a model, whose core is: (I) the *subjective difficulty* representing how difficult humans perceive a game situation and (II) the *objective difficulty* which is defined as the probability of sinking a ball.
The remainder of this paper is organized as follows: Sec. 2 describes the experimental setup and the important methods and parameters. In Sec. 3 four experiments are presented with the goal of identifying and correlating *subjective* and *objective difficulty*. Finally, in Sec. 4 the results are discussed in context of their application and their transferability to other HRI tasks.

2 Setup, Methods and Parameters

Stroke Parameters in Pool Pool strokes can be described by five parameters: displacements a,b; slope γ; direction ϑ and stroke impulse p (see Fig. 1). For simplicity a, b=0 and $\gamma=\frac{\pi}{2}$ are chosen. The direction of the cue ball sinking an object ball centrally in the pocket is called ϑ_0 in the following. A detailed description of pool physics can be found in [10, 11] and is not include here.
Methods to Measure the Stroke Difficulty To gauge the component quantifying the stroke difficulty, respective methods from literature are considered. In contrast to the presented method, the following approaches focus on a mathematical optimal stroke and do not include human preferences. In method (I), a lookup table (LUT) is used to link multiple paradigm game situations with success rates, see [5]. Method II is described by three characteristic parameters: d_1/ d_2: the distances the cue ball/ object has to travel and Θ: the cutting angle between d_1 and d_2, see Fig. 2a. These three parameters are combined in

$$\kappa_{\mathrm{Lit}}{}^{-1} = \kappa_c = \frac{\cos\Theta}{d_1\, d_2}, \tag{1}$$

to calculate a stroke difficulty measurement [8]. Method III is based on the allowed angular deviation (AAD) which is the maximal angle deviation from ϑ_0 for a successful stroke [12]. Following this, the cumulated allowed angular deviation (Δ_{CAAD}) is defined as the sum of left and right AAD, see Fig. 2b. Since method I and III have a similar approach of using the required precision as a difficulty measure, method III is not dependent on a simulator or the introduction of noise. Thus method II and III are chosen to benchmark the new human difficulty quantification model (DQM) introduced in this paper, see Sec. 3.2.

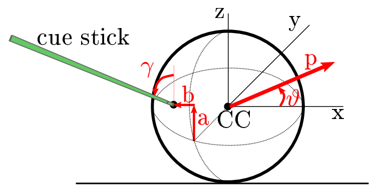

Fig. 1. `CC`: cue ball center; `a`, `b`: displacements of the contact point between cue ball and cue tip to the cue ball center; γ: slope between cue stick and pool table normal; ϑ: direction of the cue ball; `p`: impulse magnitude of the stroke

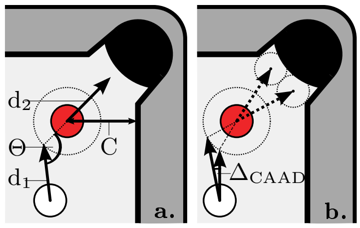

Fig. 2. Parametric description of difficulty methods. a) Geometric parameters. b) Cumulated AAD (Δ_{CAAD}).

Pool Setup: Visual System, Pool Table, Robot For the three experiments performed in a real world environment, a pool table with standard 8-ball dimensions (2.24 m×1.12 m) was used. A projector, located approximately 2.5 m above the table, is used to project game situations on the table. Parallel to the projector there is a camera, with a resolution of 1280×960 pixel.The image processing algorithm: (I) locates the pool balls on the table and (II) determines their color, both with a rate of 30 Hz. How ball color and position is extracted from the video is described in [12]. The anthropomorphic robot consists of an omni-directional platform and two seven DOF manipulators with two special end-effectors to perform strokes properly, see [10].

Probabilistic Search Algorithm To calculate the stroke parameters for the robot the probabilistic search algorithm described in [10] is applied with the following adjustments: (I) optimization is extended to both the ϑ- and `p`-parameter; (II) a new variable *tactic* is introduced which depends on the game situation: if the robot leads, *tactic* is set to "offensive", while if the robot is behind it is set to "defensive"; (III) a clustering step is introduced after the simulation in order to reduce the search space. For a detailed description and especially for a description of the evaluation process, see [10].

3 Human Difficulty Quantification Model (HDQM)

In the following, four experiments are described which: (I) identify the important parameters humans include into their decision process; (II) identify the decisiveness of the important parameters and determine an equation for the human perceived difficulty (*subjective difficulty*); (III) examination of the correlation between the *subjective difficulty* and the probability for humans to sink a ball (*objective difficulty*) and (IV) comparison of the competitiveness of the robot using (a) the HDQM integrated into a probabilistic search algorithm and (b) performing the easiest stroke according to the Δ_{CAAD}-method.

3.1 Exp. 1: Factors Influencing Subjective Difficulty

According to experts, [13]: Θ, d_1, d_2 are the three parameters to determine the difficulty of a game situation, see Fig. 2a. Since this assumption is not verified by experiments yet, an experiment on a real world pool table is performed.

Participants In total 25 people (15 male, age: 21-31 years, M = 25) participated in the experiment.

Setup and Procedure For the experiment 24 scenarios are designed. For each scenario two playable object balls are placed on the pool table. Circles are projected on the cloth to position the balls precisely and to create comparable game situations for all participants. The camera is used to prove correct positioning. In the first 18 scenarios one of the parameters: Θ, d_1, d_2 and Δ_{CAAD} are kept constant for both object balls. In the remaining six scenarios one of the two object balls is easier to play according to its Δ_{CAAD}-value but simultaneously either more difficult to reach or partially blocked by an obstacle ball. For each scenario participants are asked to freely state the easier ball to sink (descision δ_{exp}) and the reasons of their decisions. Every participant judges each scenario once. For data analysis all answers are recorded and equal decision factors were accumulated. Note that sometimes more than one reason is mentioned for choosing one scenario which indicates that the reasons might be interconnected.

Results and Discussion Results of the first 18 scenarios show that the reason for deciding which is the easier ball to sink is most often a small cutting angle Θ (264 times). Also important for the decision appear to be the distances d_1 (31 times) and d_2 (75 times). Other factors are mentioned fewer times and not consistent over participants and are therefore not considered for further analysis. Thus, in line with the literature (see [13]), the three parameters Θ, d_2 and d_1 are -in this order- most crucial. Regarding the remaining six scenarios in 70% of the cases the ball with the higher Δ_{CAAD} is chosen when the object ball is difficult to reach, while when there is an obstacle ball, in 60% of cases the easier ball is chosen. Therefore it can be assumed that the Δ_{CAAD} has a higher impact on players' decisions than the disturbance by a obstacle. For that reason, both disturbance factors will not be considered in the following any more.

3.2 Exp. 2: Decisiveness of Parameters

In a second experiment the decisiveness of each of the crucial parameters is determined to find an algebraic description.

Participants In total 23 people (13 male, age: 18-51 years, M = 25) participated in this experiment.

Setup and Procedure For the experiment, twelve images of pool scenarios, depicting one cue ball and one object ball are created. The scenarios are chosen defining six κ_{Lit}-levels. For each level two parameter combinations are selected: (I) low Θ and high distance (d_1, d_2) values, (II) high Θ and low distance (d_1, d_2) values. To compare every image with every other, six sets of six images are shown to each participant, whereas the images per set are varied systematically. To get an overall ranking the six set-ranking are broken down to multiple pairwise comparisons and then subsequently combined to one overall ranking.
Results and Discussion For statistical analysis a Friedmann ANOVA is performed which reveals a significant main effect between ranks, $\chi^s_F(11) = 206.31$, $p < 0.001$. Additionally, the rankings are used to calculate a 25% trimmed mean, $\bar{\rho}_{25\%}$ (interquartile mean), representing an average rank for each scenario. These $\bar{\rho}_{25\%}$ are sorted according to their value resulting in an overall ranking R_{exp}. In summary, an order of difficulty (R_{exp}) and an average rank ($\bar{\rho}_{25\%}$) are obtained to ascertain an equation describing the *subjective difficulty* in the following section, see Fig. 3.

3.3 Equation of Human Subjective Difficulty

The results of Exp. 1 and Exp. 2 are used in a double stage optimization process to determine an equation for human subjective difficulty. Results are: (I) the decision ($\delta_{\text{exp}}(m)$, m $= 1\ldots18$) according to Exp. 1, (II) the overall ranking ($R_{\text{exp}}(n)$, n $= 1\ldots12$) and (III) the average ranking ($\bar{\rho}_{25\%}(n)$, n $= 1\ldots12$) both taken from Exp. 2. To find the most appropriate algebraic description, three template functions are used. The template functions are:

$$\kappa_{\text{I}} = a_1 d_1^{c_1(d_1)} + a_2 d_2^{c_2(d_2)} + a_3 \cos(\Theta)^{c_3(\Theta)}\,, \quad (2)$$

$$\kappa_{\text{II}} = \kappa_{\text{Lit}} + a_1 d_1^{c_1(d_1)} + a_2 d_2^{c_2(d_2)} + a_3 \cos(\Theta)^{c_3(\Theta)}\,, \quad (3)$$

$$\kappa_{\text{III}} = \frac{d_1^{c_1(d_1)}\, d_2^{c_2(d_2)}}{\cos(\Theta)^{c_3(\Theta)}}. \quad (4)$$

Equation (2) is linear and (3), (4) are derived from (1), introduced in [8]. The exponents c_1, c_2 and c_3 ($\underline{c} = \{c_1, c_2, c_3\}$) are chosen as polynomials of degree one. The factors a_1, a_2 and a_3 ($\underline{a} = \{a_1, a_2, a_3\}$) are chosen as coefficients. These template functions are then fitted to the experimental data using two cost-functions and the Nelder-Mead algorithm. The underlying approach of the cost-functions is the least-square-method. Applying the difficulty equations (2-4) to the scenario parameters (Θ, d_1, d_2) from Exp. 1 and Exp. 2, the rankings $R_{\text{eq}}(\underline{a}, \underline{c})$, $\hat{\kappa}_{\text{eq}}(\underline{a}, \underline{c})$ and decisions $\delta_{\text{eq}}(\underline{a}, \underline{c})$ are obtained. The two cost functions

$$\Gamma_{\text{I}} = \sum_{n=1}^{12} [R_{\text{eq}}(n, \underline{a}, \underline{c}) - R_{\text{exp}}(n)]^2 + \sum_{m=1}^{18} [\delta_{\text{diff}}(m, \underline{a}, \underline{c})]^2, \quad (5)$$

$$\text{where } \delta_{\text{diff}}(m, \underline{a}, \underline{c}) = \begin{cases} 2, \text{ if } \delta_{\text{eq}}(m, \underline{a}, \underline{c}) \neq \delta_{\text{exp}}(m) \\ 0, \text{ else} \end{cases}$$

$$\text{and } \Gamma_{\text{II}} = \sum_{n=1}^{12} [\hat{\bar{\rho}}_{25\%}(n) - \hat{\kappa}_{\text{eq}}(n, \underline{a}, \underline{c})]^2, \tag{6}$$

compare the three experimental results with the rankings and the decision using one of the difficulty equations (2-4). The summation is done over all scenarios in Exp. 1: 18 and Exp. 2: 12. In order to compare the average ranking ($\bar{\rho}_{25\%}$) with the equation value (κ_{eq}), it is necessary to normalize the values (using L^1-norm) which result in $\hat{\bar{\rho}}_{25\%}$ and $\hat{\kappa}_{\text{eq}}$ in (6). The optimization task

$$\min_{\underline{a}, \underline{c}} (\Gamma_{\text{I}}), \tag{7}$$

minimizes the value of (5) and ensures that (2-4) best match the rankings and decisions of the participants. However, equation (5) compares the numerical values of R_{eq} and R_{exp} as well as the decisions δ_{eq} and δ_{exp}. Since rankings and decisions in (5) are discrete, parameter intervals for optimal solutions using (7) are found. Hence a second optimization step

$$\min_{\underline{a}, \underline{c}} (\Gamma_{\text{II}}), \tag{8}$$

using cost function (6) is performed matching continuous numerical values of the rankings ($\hat{\bar{\rho}}_{25\%}$, $\hat{\kappa}_{\text{eq}}$). In (8), only those $\underline{a}$- and $\underline{c}$-intervals are considered which are obtained from (7). The result of (8) are an optimal fitted equation κ_{I}, κ_{II} or κ_{III} (2-a4). Preliminary optimization results show: c_1, c_2 can be chosen as constants and c_3 has to be linear. Final results, show that κ_{III} (4) leads to the best approximation, due to its lowest Γ_{I} (5)- and Γ_{II}-value (6), see Table 1t.

Table 1. Best parameters of κ-functions obtained from double stage optimization

EQ.	a_1	a_2	a_3	c_1	c_2	c_3	Γ_{I}	Γ_{II}
κ_{I} (2)	-0.5	1.3	-4.9	-1.08	4.64	0.2+2.2 Θ	24	6E-2
κ_{II} (3)	0.8	-1.4	-4.9	1.94	-0.75	2.8+1.9 Θ	12	8E-2
κ_{III} (4)	–	–	–	0.33	0.38	4.1-2.7 Θ	8	2E-3

The *subjective difficulty* is therefore defined as

$$\kappa_{\text{Exp}} = \frac{\cos(\Theta)^{4.1 - 2.7\,\Theta}}{d_1^{0.33}\, d_2^{0.38}}, \Theta \in [0, 90\,°]\,. \tag{9}$$

In line with the impressions from Exp. 1, in (9) the cutting angle of a pool situation is more decisive than the two travel distances, due to the larger exponent. Note regarding (9): Since the exponent of the cos-term is: (I) smaller than the exponent of d_2-term for $\Theta > 79\,°$ and (II) negative for $\Theta > 87\,°$, (9) is valid for $\Theta \in [0\,°, 79\,°]$. In Fig. 3, the *subjective difficulty* κ_{Exp} is compared with the average ranking of Exp. 2 and with the DQM κ_{Lit} (1) and Δ_{CAAD} (Fig. 2b), which are used in computational and robotic pool. In all scenarios, except number 8, the difficulty perception of participants is described best by κ_{Exp} (9), see Fig. 3. Since it is not guaranteed that a player is able to correctly assess the difficulty of a stroke, a method representing the *objective difficulty* is necessary.

3.4 Exp. 3: Human Objective Difficulty

The *objective difficulty* of strokes is examined by an experiment in which pool players of similar experience played real pool games against each other.

Participants Two experienced participants (P1 and P2) who play a few times a week and two amateur participants (P3 and P4) who play a few times a year took part in the experiment.

Setup and Procedure The participants are asked to play multiple 8-ball games against each other, to avoid bank-shots and kick-shots and to hit the cue ball centrally. Each participant performs 210-270 strokes. The setup described in Sec. 2 is used to determine the trajectories of all balls and strokes.

Data Analysis For each performed stroke the parameters Θ, d_1, d_2 are determined and Δ_{CAAD}, κ_{Exp} (9), κ_{Lit} (1) are calculated. The resulting difficulty values (Δ_{CAAD}, κ_{Exp}, κ_{Lit}) of each player are divided into eleven intervals chosen to represent approximately a constant amount of values. A stroke success rate (*objective difficulty*) for each interval is calculated and correlation between the three DQM and the *objective difficulty* is obtained.

Results and Discussion The resulting correlation coefficients, depicted in Table 2, show the degree of linearity between the *objective difficulty* and each DQM, see also Fig. 4. The correlation coefficients between *objective difficulty* and Δ_{CAAD} or κ_{Lit} are lower in comparison to the coefficients between *objective difficulty* and κ_{Exp}.

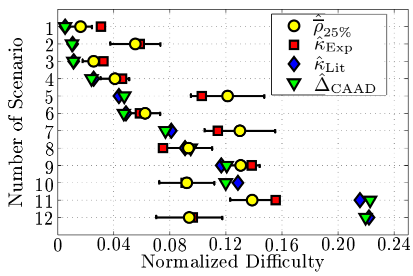

Fig. 3. Comparison of scenario difficulties according to experiment and DQM.

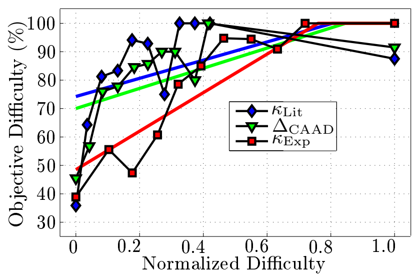

Fig. 4. Player P1: Correlation between objective stroke difficulty, and DQM.

Regarding κ_{Exp} (9) a high correlation coefficient and a nearly monotonic behavior makes it possible to approximate the correlation with a linear relationship as depicted in Fig. 4. However it is necessary to determine an approximation for every human opponent due to different skill levels. Nevertheless these results show that it is possible to transform the *subjective difficulty* into an *objective difficulty* using a linear approximation. Additionally the *objective difficulty* can be considered as sinking probability and hence can be used in an probabilistic best shot search algorithm.

3.5 Human Decisions, Abilities to Plan Ahead

To predict the stroke a human will perform, the *subjective* and *objective difficulty* of a given situation are not sufficient. In addition, a method to acknowledge the planning capabilities is required. Humans make their choice according to the difficulty of a stroke, but especially experienced players plan ahead and consider future game situations. To counter human gameplay, predicting the opponent's next stroke or at least narrowing the choices to few strokes is advantageous. The goal is therefore to increase the rate of correct prediction (CP_R) and to examine the difference in planning between experts and novices. Thus, all possible strokes in a given game situation are simulated and the difficulty value of the current stroke $\kappa_{\mathrm{Exp},0}$ (9) is combined with the difficulty value of the easiest stroke of the simulated game situation $\kappa_{\mathrm{Exp},1}$, using a discounted finite-horizon return function [14]. To combine these values,

$$\kappa_{\mathrm{n}}^{\mathrm{total}} = \sum_{i=0}^{n} \delta^i \, \kappa_{\mathrm{Exp},i} \ , \tag{10}$$

is used where, δ represents the importance of future situations. It is assumed that (I) the current game situation is most important for a pool player and that (II) an easy game situation is preferred. Hence: $\delta \in [0, 1]$. The n-parameter in (10) represents the amount of simulations into the future and is set to $n = 1$. $\kappa_{\mathrm{n}}^{\mathrm{total}}$ (10) calculated for each currently possible stroke is used to determine the human's most probable next stroke. After playing, the performed stroke is compared with the predicted stroke to obtain the rate of correct prediction (CP_R). CP_R is also evaluated based on the two other presented DQMs (Δ_{CAAD} and κ_{Lit}) as shown in Table 3. Here, only the decisions of amateur player three are better described by the Δ_{CAAD} method. It shows that the new developed equation κ_{Exp} (9) predicts the human decision better than Δ_{CAAD} and κ_{Lit} (1), while κ_{Lit} has in general the lowest CP_R. Fig. 5 shows that simulations can increase the CP_R. The CP_R of the experienced player has its maximum at $\delta \approx 0.5$, which means the player considers the actual situation twice as important as the future situation. In comparison, results of the amateur player show a flat, convex CP_R function, thus considering future strokes result in a loss of accuracy of the prediction. These results coincide with the amateurs' statement, only to consider the current game situation. Summarizing, amateurs - in contrast to experts - do not plan ahead.

Table 2. Correlation coefficients between objective difficulty and three DQMs

Player	Strokes	Correlation Coefficient Δ_{CAAD}	κ_{Lit}	κ_{Exp}
One	216	0.37	0.21	0.79
Two	182	0.52	0.42	0.75
Three	164	0.47	0.59	0.85
Four	152	0.33	0.29	0.77

Table 3. Rates of correct predictions using different DQM (max|mean value of CP_R)

Player	Rate of Correct Prediction (%) Δ_{CAAD}	κ_{Lit}	κ_{Exp}
One	71.6|70.8	69.6|69.0	74.9|73.1
Two	74.1|70.6	70.3|68.6	74.3|70.7
Three	70.6|70.1	68.0|66.5	69.7|68.0
Four	67.2|66.4	69.0|67.7	71.7|69.3

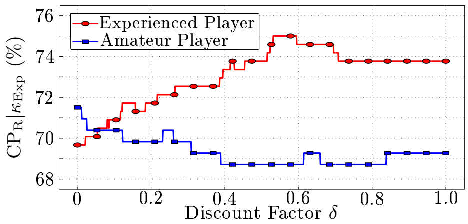

Fig. 5. $\mathrm{CP_R}$ using method κ_{Exp}. A maximum rate of 75% shows, actions of human player cannot be exactly predicted.

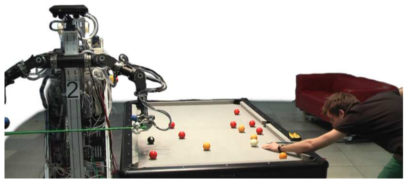

Fig. 6. Game situation between human pool player and anthropomorphic robot.

3.6 Exp. 4: Robot Applying Human Pool Model

In Exp. 4 a robot has to play thirteen 8-ball pool games against a human (Fig. 6), to gauge the competitiveness of the HDQM integrated into the above described probabilistic search algorithm, [10]. The setup is as described in Sec. 2. Two strategies for choosing the robotic stroke are evaluated: (I) human adapted search algorithm strategy (HAS) using the previously described HDQM and probabilistic search algorithm and (II) easiest shot strategy (ES) according to the Δ_{CAAD}. The robot plays seven games using the ES strategy and six games using the HAS strategy.

Participant One player (male, age: 24), manipulation skills: σ_p : 0.01 Ns, σ_ϑ : 0.007 rad. The participant is not specifically instructed.

Robot Behavior For the experiment the same robot as described in Sec. 2 is used (σ_p : 0.03 Ns, σ_ϑ : 0.012 rad), which has a restricted workspace. In situations in which the cue ball is unplayable for the robot, it is moved towards or away from the cushion. The HDQM is used within the search algorithm whenever it is the human's turn. Furthermore the simulator and the evaluation function consider the pool skills of the human opponent ($\delta = 0.5$) such as the precision of performing a stroke. The algorithm has a search depth of two, meaning it plans two strokes ahead.

Evaluation Since the number of games is limited the evaluation is based on the strokes' success rate. Using the same robot with both strategies the success rates represents how good the respective strategy prepares future strokes.

Results and Discussion In games using the HAS strategy, the robot sinks 51% of 67 strokes. In comparison using the ES strategy the robot sinks 28% of 89 strokes. The evaluation shows that the robot plays significantly (two-sample t-test, p=0.0037) better when using the HAS. In addition, the robot adapting to the introduced human pool model while using a probabilistic search algorithm can beat a player of better-than-beginner level.

4 General Discussion and Conclusion

This work presents a human pool player model which determines the *subjective* and *objective difficulty* of game scenarios from a human perspective. Although Smith [15] states that it is not possible to consider the opponent's move, the presented approach is capable of narrowing the amount of resulting situations after a human stroke. Integrated in a probabilistic search algorithm it enables a robot to predict human choices of pool strokes and to approximate their outcome. Although the contribution of the HDQM in the HAS is not examined explicitly it is shown that the complete algorithm improves pool playing abilities of a robot by improving its own stroke preparation and by creating more difficult situations for the opponent. Furthermore, model and algorithm, are not developed for specific pool rules and can therefore easily be transformed to play other pool variants. Varying the parameters in the presented HDQM would change the game behavior and competitiveness of the robot and hence allow the building of a training environment. The method of determining between subjective and objective difficulty can be applied to other non-deterministic games (e.g. carrom, bowls) in which the human behavior should be considered. Furthermore, the approach is transferable to non game related areas. For instance the difficulty of reaching objects (e.g. a cup) in a given situation might be assessable by using geometric parameters such as distances and orientations (e.g. handle orientation) and thus a robot could predict and prevent difficult to reach objects.

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