Joint Opportunistic Scheduling and Selective Channel Feedback

Mehmet Karaca, Yunus Sarikaya, Ozgur Ercetin, Tansu Alpcan, and Holger Boche

***Abstract*—It is well known that Max-Weight type scheduling algorithms are throughput optimal since they achieve the maximum throughput while maintaining the network stability. However, the majority of existing works employing Max-Weight algorithm require the complete channel state information (CSI) at the scheduler without taking into account the associated overhead. In this work, we design a Scheduling and Selective Feedback algorithm (SSF) taking into account the overhead due to acquisition of CSI. SSF algorithm collects CSI from only those users with sufficiently good channel quality so that it always schedules the user with the highest queue backlog and channel rate product at every slot. We characterize the achievable rate region of SSF algorithm by showing that SSF supports $1+\epsilon$ fraction of the rate region when CSI from all users are collected. We also show that the value of ϵ depends on the expected number of users which do not send back their CSI to the base station. For homogenous and heterogeneous channel conditions, we determine the minimum number of users that must be present in the network so that the rate region is expanded, i.e., $\epsilon > 0$. We also demonstrate numerically in a realistic simulation setting that this rate region can be achieved by collecting CSI from only less than 50% of all users in a CDMA based cellular network utilizing high data rate (HDR) protocol.**

***Index Terms*—Opportunistic scheduling, limited channel feedback, queue stability, resource allocation.**

I. Introduction

SCHEDULING is an important problem for any shared resource scenario. The problem becomes more challenging in a dynamic setting such as wireless networks where the channel capacity is time varying due to multiple superimposed random effects such as mobility and multipath fading. Opportunistic scheduling has emerged as an attractive solution for improving the efficiency of these systems. The basic principle behind opportunistic scheduling is to exploit the independent variations of fading experienced different interfering links/users. In their seminal work, Tassiulas and Ephremides have shown that opportunistic *Max-Weight* algorithm which schedules the user with the highest queue backlog and transmission rate product at every time slot can stabilize the network, whenever this is possible [2]. This scheme has been generalized to wireless systems under various performance metrics such as fairness, delay and energy constraints [3], [4].

To exploit the advantages of multi-user diversity in the downlink the base station requires the instantaneous channel state information (CSI) of users. A common assumption in the literature is that the *exact* and *complete* channel state information of all users is available at every time slot. However, in general base station is unaware of the users' channel state information, which must be acquired by consuming a fraction of resource which is otherwise used for data transmission. To give an idea of how much resource needs to be allocated for channel state feedback, we consider a CDMA/HDR (High Data Rate) system [5], where the Signal-to-Noise Ratio (SNR) of each link is measured as channel state information. The value of the SNR is mapped to a value representing the maximum data rate that can support a given level of error performance. This value is then sent back to the base station via the reverse link data rate request channel (DRC). The channel state information is 4 bits long and it is sent back every 1.67 ms [1]. If there are 25 users in a cell, 100 bits of channel state information is sent back to the base station every 1.67 ms. This requires 60Kbps of channel rate dedicated only for reporting channel states. Comparing this with the minimum data rate of HDR system of 38.4Kbps, and the average data rate of 308Kbps, one can immediately appreciate the need for any method to reduce the amount of this feedback. The overhead due to channel state information feedback becomes even more significant in a multichannel communication system such as LTE.

There are several studies in the literature investigating scheduling under limited channel state feedback [6], [7], [8]. While these studies have provided structural properties of optimal policies, many of them do not take into account practical challenges such as queue backlogs of users and continuously varying wireless channel. Hence, there is still a need to develop limited feedback scheduling algorithms suitable for implementation in practical systems.

Our contributions are summarized as follows:

- We develop a *scheduling and selective feedback (SSF)* algorithm that schedules users according to the queue-lengths at the base station without full CSI. We show that SSF algorithm achieves a fraction $1+\epsilon$ of the achievable rate region of Max-Weight algorithm acquiring channel

Manuscript received September 25, 2012; revised January 7, 2013; accepted February 28, 2013. The associate editor coordinating the review of this paper and approving it for publication was M. L. Meran.

A preliminary version of the paper was presented at the International WDN Workshop on Cooperative and Heterogeneous Cellular Networks (WDN-CN2012) co-located with IEEE PIMRC in Sydney, Australia on Sep. 2012 [1].

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Digital Object Identifier 10.1109/TWC.2013.041713.121460

[1]The feedback is subject to delays. However, we assume that the feedback delay is small compared to the channel coherence time.

state information from all users.

- For homogeneous and heterogeneous channels, we identify the minimum number of users that must be present in the system to reap benefits of SSF algorithm.
- We evaluate the performance of the proposed SSF algorithm with numerical experiments, and demonstrate that SSF can support higher arrival rates than that of another prominent algorithm which was proposed in [8].

The rest of the paper is organized as follows. Section II summarizes the literature on opportunistic scheduling algorithms considering probing overhead. Section III presents the network model, the basic structure of the channel probing model. In Section IV, we present SSF algorithm, and characterize its achievable rate region. We demonstrate the efficacy of the proposed SSF algorithm as compared to Max-Weight with full channel state feedback and to the algorithm given in [8] in Section V. Finally, Section VI concludes the paper and presents possible future directions.

II. Related Work

There has been significant interest in developing joint feedback and scheduling algorithms for wireless systems. Opportunistic feedback has been proposed in [6], [9], [10], [11] where the system is designed primarily for exploiting multiuser diversity. In [6] users contend for the feedback channel if the channel state exceeds a pre-defined threshold. Similarly, in [10], multiple threshold levels are used to reduce the cost for obtaining CSI. For uplink scheduling, the authors in [9] propose an optimization framework in OFDM systems. A random access based feedback protocol for achieving multiuser diversity with limited feedback was proposed in [11]. More recently, similar idea was proposed in [12] where only the users with channels good enough are allowed to send feedback. We refer the readers to [13] and the references therein for more information on acquiring limited feedback. Most prior works study network capacity and feedback tradeoff by assuming infinitely backlogged user queues. However, when network stability problem, where the aim is to stabilize all users buffers, is considered this trade-off cannot be analyzed in the same way since queue size of each user should be taken into account.

Network stability problem with infrequent channel state measurements was investigated in [14] and it was shown that achievable rate region shrinks as the frequency of CSI feedback decreases. The impact of delayed CSI was investigated in [15]. In this work, based on these observations, we design an intelligent algorithm aiming to maximize throughput by acquiring CSI from as few number of users as possible. Our algorithm has low implementation complexity, and is shown to work under general channel conditions such as for time-correlated and non-stationary channels.

In [16], the authors proposed a joint scheduling and channel feedback algorithm stabilizing the network by allowing the base station to receive CSI from a subset of users. However, throughput-optimality of that algorithm can only be shown under certain conditions, i.e., when channel distributions are known. In [17], the authors proposed a variant of the algorithm in [16] to analyze queue-overflow performance with limited CSI. In [7], a feedback allocation algorithm was proposed for multi-channel system where only a limited number of CSI can be acquired at a time and channel distributions are known at the BS. Note that the algorithms in [16], [7], [17] require channel statistics to achieve throughput-optimality, which is impossible in practice. In [18], the authors proposed a scheduling algorithm under imperfect CSI in single-hop networks. In this work, the authors aimed to obtain a tradeoff between energy consumption and network capacity, when energy cost of acquiring CSI is taken into account.

In [19] and [20], the authors proposed joint channel estimation and opportunistic scheduling algorithm by assuming independent and identically distributed (iid) and time-correlated channel processes, respectively. In [21], the authors proposed to estimate the channel statistics by using some portion of the time slots for observation slot with some probability over iid channels. Unlike [19] and [21], we do not estimate channel statistics at all. Hence, our algorithm is robust under more general channel conditions such as time-correlated or non-stationary channels. In [22], it was assumed that wireless channels evolve as Markov-modulated ON/OFF processes. With this assumption, an exploitation-exploration trade-off was investigated. Similarly, in [23], a two state discrete time Markov chain with a bad state which yields no reward and a good state which yields reward was considered. The performance of [22] and [23] depends on the underlaying stochastic process of the channel evolves according to a fixed stationary process such ergodic Markov chain. In practice, such an assumption does not hold most of the time. For instance, the measurement study in [24] shows that the wireless channel exhibits time-correlated and non-stationary behavior.

Unlike the works above, we assume a more flexible channel acquiring model where receiving CSI from a single user consumes a certain portion of resource which is otherwise used for transmission of data (e.g., β fraction of a time slot). A similar model was also considered in [25]. However, in that work the network stability problem was not investigated. In this sense, the most related work is [8], where the optimal feedback and scheduling scheme for a single-channel downlink is determined. Specifically, in [8], the server has a cost for obtaining CSI and it gains a reward (defined as queue weighted throughput) for each scheduled user. The problem of finding optimal set of users from which CSI is obtained. Then, the user scheduled in each slot is transformed into an optimal stopping time problem which is solved as a Markov Decision Process (MDP) given a priori channel distributions of all users. Unlike our algorithm, the optimal algorithm in [8] has high computational complexity due to its MDP formulation especially when the number of channel states is large. However, the performance of our algorithm does not suffer from larger number of channel states.

III. System Model

We consider a multiuser downlink network with N users and a single base station (BS), where users wish to receive data from the BS via the downlink channel as shown in Figure 1. We consider a time-slotted system where the time slot is the resource to be shared among different nodes. We adopt a

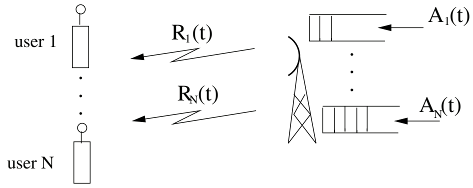

Fig. 1. Cellular network model.

non-interference model where only one node is receiving data at any given time. Random channel gains between base station and other nodes in the network are assumed to be iid across time according to a general distribution and independent across nodes with values taken from a finite set. Moreover, we assume that channel gains are time-varying, but fixed over the time slot duration. Practical systems, such as CDMA/HDR system [5], implement adaptive modulation schemes which adjust signal constellation, coding rate/scheme, etc., relative to the channel fading. When the instantaneous Signal-to-Noise-Ratio (SNR) falls within a given range, the associated signal constellation is transmitted. Since only a discrete finite set of L constellations is available, only a fixed set of data rates $\mathcal{R} = \{r_1, r_2, \ldots, r_L\}$ can be supported. Assume that without loss of generality, $r_k > r_l$ if $l > k$. Let $R_n(t) \in \mathcal{R}$ be the supported data rate on the downlink channel to node n at time slot t. The steady state distribution of supported channel rates for node n is given as $\Pr[R_n(t) = r_l] = p_n^l$, for all $l = 1, \ldots, L$, where all rates are observed with non-zero probability, i.e., $p^{min} = \min_{1 \le n \le N, 1 \le l \le L}\{p_n^l\} > 0$.

The base station does not have the knowledge of channel states of the receivers at the beginning of the slot, but it has to acquire this information. At the beginning of each time slot, t, base station broadcasts a pilot signal with a fixed and known power. Each node n determines its supportable data rate $R_n(t)$ by measuring the received SNR. Acquiring the supportable data rates from the receivers consumes resources. In this work, we adopt the same model as in [8] and assume that acquiring channel state information from a receiver consumes a fraction β of a slot. Let $\mathcal{S}(t)$ be the set of users from which channel state information is acquired, where $S(t)$ is the cardinality of set $\mathcal{S}(t)$. Note that $0 < S(t) \le N$. Hence, if base station decides to transmit to node n, then it can transmit only $(1 - \beta S(t))R_n(t)$ bits. Clearly, we assume that the number of users is upper bounded by $N < \frac{1}{\beta}$. Since the transmitter can transmit to a receiver only after knowing its channel state, there is a trade off between the number of nodes from which channel state information is acquired and the fraction of time left for actual data transmission.

We use indicator variable $\mathcal{I}_n(t)$ to represent the scheduler decision, where $\mathcal{I}_n(t) = 1$ if node $n \in \mathcal{S}(t)$ is scheduled for transmission in slot t, and $\mathcal{I}_n(t) = 0$ otherwise. By definition, at most one user can be served at a time slot, i.e., $\sum_{n=1}^{N} \mathcal{I}_n(t) = 1$, $\forall t$. Assuming unit slot length, the amount of data that can be transmitted to user n when channel states of $S(t)$ users are acquired at slot t is,

$$D_n(t) = (1 - \beta S(t))R_n(t)\mathcal{I}_n(t). \tag{1}$$

Base station maintains a separate queue for each node n. Packets arrive according a stationary arrival process that is independent across users and time slots. Let $A_n(t)$ be the amount of data arriving into the queue of user n at time slot t. Let $\mathbf{Q}(t) = (Q_1(t), Q_2(t), \ldots, Q_N(t))$ denote the queue length vector. The dynamics of queue length for node n is given as

$$Q_n(t+1) = [Q_n(t) + A_n(t) - D_n(t)]^+, \tag{2}$$

where $[x]^+ = \max(x, 0)$. We say that the system is stable if the mean queue length for all the receiver nodes is finite. In their seminal paper, Tassiulas and Ephremides [2] have shown that Max-Weight algorithm could ensure stability of the user buffers whenever this is at all possible. Max-Weight scheduling policy schedules the user n^* for which the transmission rate weighted by the queue length is the maximum, i.e.,

$$n^* = \operatorname*{argmax}_n W_n(t) = \operatorname*{argmax}_n Q_n(t)R_n(t), \tag{3}$$

However, Max-Weight policy neglects the cost of channel state acquisition and thus, assumes perfect knowledge of channel states at the beginning of time slot. In this paper, we develop an algorithm that can find the user with maximum weighted rate, i.e., $n^* = \operatorname{argmax}_n W_n(t)$ without acquiring the channel states from all users. Consequently, in the following section, we characterize the stability region of the system defined by the set of arrival rates $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_N)$, where $\lambda_n = \mathbb{E}[A_n(t)]$ such that our proposed selective channel state acquisition and scheduling policy can stabilize.

Discussion: The convex hull of the set of all arrival rate vectors $\boldsymbol{\lambda}$ for which there exists an appropriate scheduling policy that stabilizes the network is called *achievable rate region*. When the cost of acquiring channel state information is neglected, i.e., $\beta = 0$, the achievable rate region is the largest, since the resources are used completely for only actual data transmission. Let Λ_h denote this *hypothetical rate region*, the boundary of which can never be achieved in real systems [26]. On the other extreme, let Λ_f denote the achievable rate region achieved when channel state information is acquired from *all* users with non-zero acquisition cost, i.e., $\beta > 0$. Clearly, $\Lambda_f \subset \Lambda_h$. Finally, let Λ denote the achievable rate region of any other scheduling algorithm that selectively acquires channel state information from only a portion of users. In the next section, we propose a scheduling and selective feedback algorithm, and show that its achievable rate region satisfies $\Lambda_f \subset \Lambda \subseteq \Lambda_h$. Hence, our proposed algorithm is more efficient than Max-Weight algorithm acquiring channel state information from all users.

IV. Scheduling and Selective Feedback (SSF) Algorithm

In this section, we propose a joint scheduling and selective feedback algorithm that determines at each time slot the user with the maximum weighted rate without acquiring channel states from every user. The key idea is to identify and eliminate those users which has no possibility of having the maximum weighted rate, and acquire channel states from the rest of the users. For this purpose, algorithm follows a two step procedure. In the first step, base station identifies the user with the largest downlink queue length, say user n, and acquires its channel state, i.e., $R_n(t)$. Base station broadcasts $R_n(t)$ to all users, and asks them to forward their channel state only if their rates are larger than $R_n(t)$. The procedure is explained in detail in Algorithm 1.

Algorithm 1: SSF Algorithm

At every time slot, do:
(1) Selective Feedback:

Step 1: Determine the user which has the maximum queue length,

$$i^* \triangleq \underset{1\leq i\leq N}{\operatorname{argmax}}\{Q_i(t)\}$$

Step 2: Acquire the channel state of user i^*, i.e., $R_{i^*}(t)$.
Step 3: Broadcast the value of $R_{i^*}(t)$.
Step 4: The users with rates higher than $R_{i^*}(t)$ report their channel states to base station.

$$\mathcal{S}(t) \triangleq \{1 \leq j \leq N : R_j(t) > R_{i^*}(t)\}.$$

(2) scheduling decision:
Base station schedules the user n^* with the maximum weighted rate:

$$n^* = \underset{n\in\mathcal{S}(t)\cup\{i^*\}}{\operatorname{argmax}} W_n(t), \quad (4)$$

i.e., $\mathcal{I}_{n^*}(t) = 1$, and updates queue lengths according to (2).

The following Theorem shows that SSF and Max-Weight algorithms make the same scheduling decision at every time slot.

Theorem 1: SSF algorithm schedules the user with the maximum weighted rate at every time slot.

Proof: Let $W_{i^*}(t)$ be the weighted rate of the user with the maximum queue size at time slot t, i.e., $i^* = \operatorname{argmax}_{1\leq n\leq N} Q_n(t)$. Also let $W^*(t)$ be the maximum weighted rate among all users, i.e., $W^*(t) = \max_{1\leq n\leq N} W_n(t)$. SSF algorithm schedules the user with the maximum weighted rate from set of users $\mathcal{S}(t) = \{j : R_j(t) \geq R_{i^*}(t)\}$. For any user $k \notin \mathcal{S}(t)$ $R_k(t) < R_{i^*}(t)$, and since $Q_{i^*}(t) > Q_k(t)$, then $W_k(t) < W_{i^*}(t)$. Hence, $\max_{n\in\mathcal{S}(t)} W_n(t) = W^*(t)$. ∎

A. Achievable Rate Region of SSF Algorithm

Let $M(t)$ denote the number of users which *do not send* their channel state information at time t. The total fraction of time slot consumed for acquisition of channel state information is calculated as follows: A fraction β of time slot is used to acquire channel state information of user i^*, and another fraction β of time slot is used to broadcast $R_{i^*}(t)$. Then, those users with rates higher than $R_{i^*}(t)$ forward their channel state information, where each such feedback also consumes a fraction β of time slot. Hence, in total a fraction $(N - M(t) + 1)\beta$ of a time slot is consumed for acquisition of channel state information. The following theorem characterizes the achievable rate region of SSF algorithm, Λ, as compared to that of Max-Weight algorithm, i.e., Λ_f.

Theorem 2: SSF algorithm can support a fraction $(1 + \epsilon)$ of the rate region Λ_f, i.e., $\Lambda \subseteq (1 + \epsilon)\Lambda_f$, where

$$\epsilon = \frac{\beta\left(\mathbb{E}[M(t)] - 1]\right)}{1 - \beta N}. \quad (5)$$

Proof: The proof relies on a theorem given in [27], and the calculation of expected weighted rate obtained by Max-Weight and SSF algorithms for any given $\mathbf{Q}(t)$. Details of the proof are given in Appendix A. ∎

Corollary 1: The upper bound on ϵ is given by,

$$\epsilon_{up} = \frac{\beta N}{1 - \beta N} \quad (6)$$

According to Theorem 2, the performance of SSF algorithm depends on the expected number of users from which channel state information is not acquired, i.e., $\mathbb{E}[M(t)]$. Clearly, if $\mathbb{E}[M(t)] > 1$, then $\epsilon > 0$. Next, we calculate $\mathbb{E}[M(t)]$ under homogeneous and heterogeneous channel models.

B. Performance of SSF with Homogenous Channels

We first calculate the performance of SSF algorithm by considering a homogenous channel model, where the channel state probability distributions are the same for all users, i.e.,

$$p_n^k = p_k, 1 \leq n \leq N, \quad 1 \leq k \leq L$$

Lemma 1: When channels are homogenous, $\mathbb{E}[M(t)]$ is given as follows:

$$\mathbb{E}[M(t)] = \left[p_1 + p_2\sum_{k=2}^{L} p_k + p_3\sum_{k=3}^{L} p_k + \cdots + p_L^2\right](N - 1). \quad (7)$$

Proof: The proof is provided in Appendix B. ∎

Next, we consider the special case when the channels are homogenous and channel state probabilities are *uniformly* distributed, i.e., $p_k = \frac{1}{L}$ for all $k \in \{1, 2, \cdots, L\}$.

Lemma 2: When channels are homogenous and uniformly distributed, $\mathbb{E}[M(t)]$ is given as follows:

$$\mathbb{E}[M(t)] = \left[\frac{1}{2} + \frac{1}{2L}\right](N - 1). \quad (8)$$

Proof: The proof is provided in Appendix C. ∎

Corollary 2: For homogenous channels with uniform channel state distribution, ϵ is given as:

$$\epsilon = \frac{\beta\left(\left[\frac{1}{2} + \frac{1}{2L}\right](N - 1) - 1\right)}{1 - \beta N}. \quad (9)$$

The value of ϵ depends on the channel statistics, the number of users and the time needed for acquiring CSI feedback. As an example, consider a HDR system, where all users have uniform channel distributions. In a typical HDR system, there are 11 SNR levels (i.e., $L = 11$). Also assume that there are $N = 15$ users and $\beta = 0.02$. According to Corollary 3 the value of ϵ is thus 0.19. Hence, by using SSF we can support a user arrival rate which is at least 19% higher than that can be supported by Max-Weight algorithm acquiring CSI from all users. Note that the minimum value of ϵ is zero which occurs at $\beta = 0$. As β increases, ϵ increases as well, and hence higher the probing cost (i.e., higher β), the more advantageous it is using SSF algorithm.

We now give our main result for homogenous channels.

Theorem 3: For homogeneous channels, SSF algorithm guarantees to achieve a larger rate region than Λ_f, i.e., $\epsilon > 0$, if the number of users is greater that 3, i.e., $N > 3$.

Proof: The proof proceeds in two parts. In the first part, we show that $\mathbb{E}[M(t)]$ is minimum when channels are uniform. In the second part, we show that if $N > 3$, then $\epsilon > 0$ when channels are uniform. Details of the proof are provided in Appendix D. ■

Note that the exact value of ϵ for homogeneous channels can be calculated by Theorem 2 and Lemma 1. Also note that Theorem 3 holds if $\frac{1}{\beta} > 3$.

C. Performance of SSF with Heterogenous channel

Next, we consider the case when user channels are not identical, i.e., $p_n^k \neq p_m^k$, for all $n \neq m$. First, in the following Lemma, we give a lower bound for $\mathbb{E}[M(t)]$.

Lemma 3: When channels are heterogenous, $\mathbb{E}[M(t)]$ is lower bounded as follows:

$$\mathbb{E}[M(t)] \geq (N-1)\left(p^{min} + (p^{min})^2\left[\frac{L(L-1)}{2}\right]\right) \quad (10)$$

Proof: The proof is provided in Appendix E. ■

The following theorem gives our main result for heterogeneous channels.

Theorem 4: For heterogeneous channels SSF algorithm guarantees to achieve a larger rate region than Λ_f, i.e., $\epsilon > 0$, if the number of users satisfies

$$N > \frac{1}{\left(p^{min} + (p^{min})^2\left[\frac{L(L-1)}{2}\right]\right)} + 1. \quad (11)$$

Proof: Recall that in order to achieve larger rate region, $\mathbb{E}[M(t)]$ should be larger than 1. Lemma 3 gives a lower bound for $\mathbb{E}[M(t)]$. Hence, if the right hand side of inequality (10) is greater than 1, then $\mathbb{E}[M(t)] > 1$ and SSF guarantees to achieve larger rate region. ■

In an HDR system, there are 11 SNR levels which correspond to 11 different transmission rates, i.e., $L = 11$ [5]. In addition, as given in [5], almost all data rates occur with a probability higher than $p^{min} = 0.05$ in a typical embedded sector. Hence, for a typical HDR system, the number of users in the system should be at least 7, in order to reap the benefits of SSF algorithm. If the number of users in the system is lower than 7, then it is optimal to acquire channel state information from all users in the system.

Corollary 3: For heterogenous channels, ϵ is given by,

$$\epsilon \geq \frac{\beta\left((N-1)\left(p^{min} + (p^{min})^2\left[\frac{L(L-1)}{2}\right]\right) - 1\right)}{1-\beta N} \quad (12)$$

D. Implementation Issues

Here, we mention some implementation issues arising when SSF algorithm is applied. In practice, CSI is sent back to the base station at a basic rate specified by the system so that this information can be decoded at the base station without any error. In HDR system, the minimum supportable rate is 38.4kb/s, and a single CSI is coded by using 4 bits. Hence, the required time for the transmission of a single CSI is equal to 0.1 millisecond (ms). In HDR system the duration of time slot is 1.67 ms, so 6.25% fraction of the time slot is consumed for each CSI acquisition. Hence, for a typical HDR system $\beta = 0.0625$.

Note that when all users send feedback, the base station can inform each user about its specific time slot to send feedback in the beginning and it does not change after that. For instance, at every time slot the base station first acquires CSI from user 1, then from user 2 and so on. However, with SSF a user will not know the other users who will report their channel information, and hence it cannot predict its order in sending feedback. In dynamic CSI feedback, there should be a mechanism to control the users' feedback order which resolves possible collisions during CSI transmissions on the uplink channel. Developing a mechanism to acquire CSI from a limited number of users remains as a research challenge. Note that any mechanism that is proposed to solve this problem brings some additional timing cost. Here, we propose a simple heuristic to prevent collision during the transmission of CSI: the base station always starts collecting CSI with a specific user as in the static case. If a user has a data rate greater than that of user i^*, it first announces it to the base station by sending 1 bit signal. Then, the base station waits until the user transmits its CSI, and then switches to the next user. Otherwise, it sends 0 bit signal, and the base station switches to the next user. This mechanism effectively controls the feedback order and prevents collisions but it brings some additional timing cost due to transmission of additional one bit length signals. Let us denote κ as the fraction of time slot consumed for all probing processes within SSF algorithm including acquisition of CSI, broadcasting ID, switching, etc. We next determine the effect of timing cost on the rate region of SSF algorithm.

Corollary 4: For homogenous channels, when in total a fraction $(N - M(t) + \kappa)\beta$ of a time slot is consumed where $\kappa \geq 1$ then SSF algorithm guarantees to achieve a larger rate region than Λ_f, i.e., $\epsilon > 0$, if the number of users satisfies

$$N > \frac{\kappa}{\frac{1}{2} + \frac{1}{2L}} + 1, \quad (13)$$

Proof: Note that $\mathbb{E}[M(t)]$ depends only on the channel statistics and hence additional probing cost does not effect $\mathbb{E}[M(t)]$. Thus, Lemma 2 is still valid and, $\mathbb{E}[M(t)] = [\frac{1}{2} + \frac{1}{2L}](N-1)$. By following the same lines of the proof of Theorem 2, one can determine that

$$\epsilon = \frac{\beta\left(\mathbb{E}[M(t)] - \kappa\right)}{1-\beta N}. \quad (14)$$

Clearly, when $\mathbb{E}[M(t)] > \kappa$ then, $\epsilon > 0$. Thus,

$$\mathbb{E}[M(t)] = [\frac{1}{2} + \frac{1}{2L}](N-1) > \kappa,$$

and $N > \frac{\kappa}{\frac{1}{2}+\frac{1}{2L}} + 1$. ■

Hence, there must exist at least $\left\lceil \frac{\kappa}{\frac{1}{2}+\frac{1}{2L}} + 1 \right\rceil$ users to achieve a larger rate region compared to Max-Weight algorithm with complete CSI, where $\lceil x \rceil$ is the smallest integer that is greater than x. In other words, when additional probing cost is considered in terms of β the minimum number of users that is required to achieve larger rate region increases.

V. Numerical Results

In our simulations, we model a single cell CDMA downlink transmission utilizing high data rate (HDR) [5]. The base station serves 15 users and keeps a separate queue for each user. Time is slotted with length $T = 2$ ms. Packets arrive at each slot according to Poisson distribution for each users with mean λ_n. The size of a packet is set to 128 bytes which corresponds to the size of an HDR packet. Each channel has 11 possible states with rates as given in [5].

We compare SSF algorithm with the algorithm in [8], which we call Cha. Specifically, for homogenous channels, an optimal acquisition strategy is to receive CSI from users in the decreasing order of their queue lengths as shown in Corollary 1 in [8]. However, for heterogenous channels, an optimal strategy is not characterized. Hence, we compare Cha and SSF only when channels are homogenous.

A. Homogenous Channels

First, we evaluate the performance of SSF algorithm when channels are homogenous and uniformly distributed.

1) Uniform Channels: In uniform case, $p_n = 1/11$ for all users since there are 11 channel states and the channel state distributions are identical. Figure 2 depicts the average total queue sizes in terms of packets vs. the overall arrival rate when $\beta = 0.02$, $\beta = 0.04$, $\beta = 0.06$ and $\beta = 0.08$. The maximum supportable arrival rate is achieved by a *hypothetical* Max-Weight algorithm where the feedback cost is zero, i.e., $\beta = 0$. Note that for $\beta = 0.02$ the lowest supportable rate is achieved by Cha, and it is achieved by Max-Weight acquiring channel states from all users for $\beta = 0.08$. The reason that Max-Weight is worse for larger values of β is that the decrease in throughput due to feedback costs becomes more prominent. SSF algorithm achieves a very similar performance as compared to the hypothetical Max-Weight algorithm for $\beta = 0.02$, and it outperforms both of the other algorithms for all values of β. The main reason that SSF outperforms Cha is *not* because it acquires feedback from fewer average number of users. On the contrary, in our numerical simulations, we observe that SSF has $\mathbb{E}[M(t)] = 7.63$, whereas Cha has a larger value of $\mathbb{E}[M(t)]$. It is because SSF *always* schedules the user with maximum weighted rate at every slot, while Cha does not. Note that the performance of Cha becomes similar to that of SSF as the condition on β is violated, and it may become better than that of SSF for increasing values of β. However, Figure 2 shows that for practical values of β that we determined in Section 4.D SSF outperforms Cha.

Note that one may obtain the value of ϵ by observing the simulation results. For example, for the scenario given in Figure 2 (a), the value of ϵ can be calculated as 0.19 by using (9), and noting that $N = 15$, $L = 11$ and $\beta = 0.02$. In other words, an arrival rate which is 19% higher than that of Max-Weight algorithm acquiring full CSI can be supported by using SSF. From Figure 2 (a), the maximum supportable rate by Max-Weight with full CSI is approximately 3 packets/slot whereas SSF can support up to 3.57 packets/slot, which is

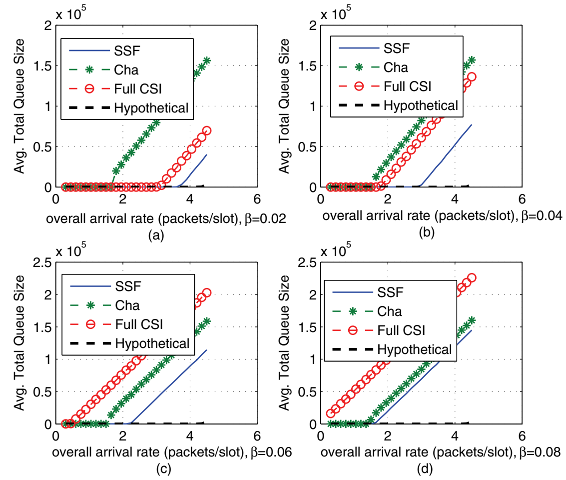

Fig. 2. Performance of SSF algorithm with homogenous and uniform channels.

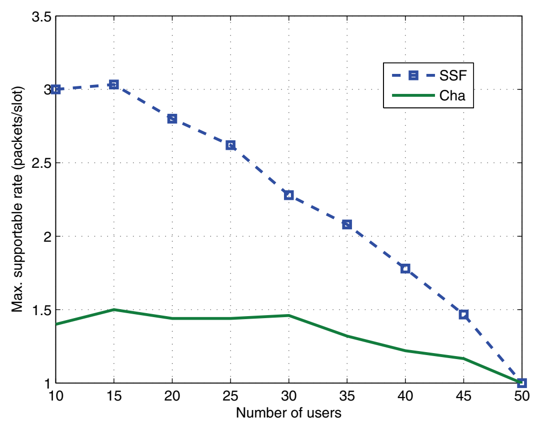

Fig. 3. Maximum supportable rate vs. number of users.

fact 19% higher than that of Max-Weight algorithm with full CSI, confirming our analytical calculation from (9). We have performed additional numerical studies to understand the performance of Cha and SSF as the number of users in the network increases. We choose $\beta = 0.03$, and vary N between 10 and 50 users. Note that for large values of N, there may not be sufficient time for data transmission since the entire slot is used for probing. In such a case, data transmission rate equals to zero. Figure 3 depicts the maximum supportable arrival rate vs. the number of users in the network. As N increases, the maximum supportable rate of both SSF and Cha decreases since the remaining time for data transmission decreases as well. However, SSF outperforms Cha for all values of N.

2) Non-Uniform Channels: Here, we investigate the performance of SSF algorithm when channels are identical but the channel state distributions are not uniform. In this case, the channel state distribution p_l, $l = 1, \ldots, 11$ is given as $\{0.01, 0.01, 0.03, 0.08, 0.15, 0.24, 0.18, 0.09, 0.12, 0.05, 0.04\}$.

Figure 4 depicts the average total queue sizes in terms of packets vs. the overall arrival rate when $\beta = 0.02$ and $\beta = 0.05$. The maximum supportable arrival rate is achieved with hypothetical algorithm, and the lowest supportable rate is achieved by Max-Weight algorithm acquiring channel states

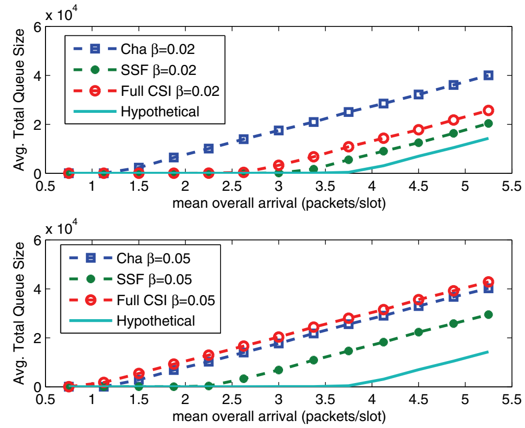

Fig. 4. Performance of SSF algorithm with homogenous and non-uniform channels.

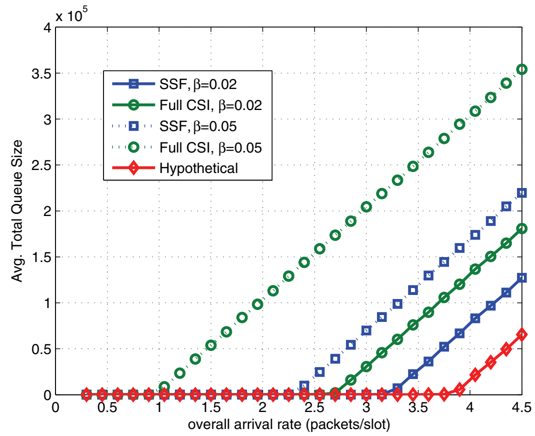

Fig. 5. Performance of SSF algorithm with heterogenous channels.

from all users. We can again observe that SSF outperforms Cha when the channels have non-uniform distribution.

B. Heterogenous Channels

For the case of heterogeneous channels, we divide the users into three groups where there are five users in each group. The channel state distributions of each group are given as follows: for the first group, $n = \{1, 2, \ldots, 5\}$, $p_l^n, l = \{1, \ldots, 11\}$ is given as $\{0.01, 0.01, 0.03, 0.08, 0.15, 0.24, 0.18, 0.09, 0.12, 0.05, 0.04\}$, for the second group, $n = \{6, 7, \ldots, 10\}$, $p_l^n = \{0.01, 0.02, 0.02, 0.07, 0.2, 0.24, 0.15, 0.05, 0.12, 0.08, 0.04\}$ and for the last group, $n = \{11, 12, \ldots, 15\}$, $p_l^n = \{0.02, 0.01, 0.02, 0.06, 0.21, 0.25, 0.18, 0.05, 0.11, 0.06, 0.03\}$. Figure 5 depicts the average total queue sizes in terms of packets vs. the overall arrival rate for $\beta = 0.02$ and $\beta = 0.05$. The maximum supportable arrival rate is still achieved by hypothetical algorithm and the lowest supportable rate is achieved by Max-Weight algorithm acquiring channel states from all users. when $\beta = 0.05$ the supportable rate achieved by SSF algorithm is 2.25 packets/second whereas it is approximately 1.12 packets/second with full CSI. Hence, SSF achieves larger rate region than Max-Weight with full CSI has. Also, from Figure 5, when $\beta = 0.05$, the maximum supportable arrival rate of Max-Weight acquiring complete CSI is around 1.1 packets/slot whereas it is approximately 2.4 packets/slot with SSF. Hence, the gain with SSF is approximately 118% (i.e., $\epsilon = 1.18$).

VI. Conclusion

We have developed joint scheduling channel feedback algorithm in a single channel wireless downlink network. We have assumed that acquiring CSI of a user consumes a certain fraction of data slot, hence, decreases the achievable throughput. The set of channels is determined by considering the channel gain of the user with maximum queue size. We have shown that the proposed joint algorithm can support $1+\epsilon$ fraction of full rate region achieved when all CSIs are available at the scheduler. Then, we have proved the sufficient condition for $\epsilon > 0$ for both homogenous and heterogenous channels by determining the expected number of reported CSI within our algorithm. In simulation results, we show that by applying our algorithm the base station can stabilize the network and achieves larger rate region with collecting CSI from less than the half of users instead of full CSI. In this work, the analytical and simulation results are given by assuming channels are iid. As a future work, we will provide the result by considering more general channel models, i.e., correlated or Markovian channels. Also, we want to extend our work for the multichannel wireless system, i.e., OFDM networks with considering fairness among users. Another future direction is to investigate the design of a new dynamic feedback algorithm further reducing the probing overhead by using the knowledge of channel statistics.

Appendix A
Proof of Theorem 2

We first define the following two functions:

$$f_s(\boldsymbol{Q}(t)) = \mathbb{E}\left[\sum_{n\in\mathcal{S}(t)} (1-\beta S(t))W_n(t)I_n(t)|\boldsymbol{Q}(t)\right],$$

$$f_m(\boldsymbol{Q}(t)) = \mathbb{E}\left[\sum_{n\in\mathcal{N}} (1-\beta N)W_n(t)I_n(t)|\boldsymbol{Q}(t)\right],$$

where the expectation is taken with respect to the randomness of channel variations and scheduling decisions. Given $\boldsymbol{Q}(t)$, both Max-Weight algorithm with full CSI and SSF schedules the same user with the maximum weighted rate at every time slot. Hence, the value of $W_n(t)I_n(t)$ is the same for both functions, and the only difference between $f_s(\boldsymbol{Q}(t))$ and $f_m(\boldsymbol{Q}(t))$ appears in the number of reported CSI. The performance of SSF algorithm is determined by using the following theorem proven in [27].

Theorem 5: [27] If for some $\epsilon > 0$ SSF algorithm guarantees

$$f_s(\boldsymbol{Q}(t)) \geq (1+\epsilon) f_m(\boldsymbol{Q}(t))$$

for all $\boldsymbol{Q}(t)$, then SSF can achieve $1+\epsilon$ fraction of the rate region Λ_f, i.e., $\Lambda \subseteq (1+\epsilon)\Lambda_f$.

Note that $f_s(\boldsymbol{Q}(t))$ can be rewritten as follows:

$$\begin{aligned} &f_s(\boldsymbol{Q}(t)) = \\ &\mathbb{E}\left[\sum_n (1-\beta(N+1-M(t)))W_n(t)I_n(t)|\boldsymbol{Q}(t)\right] \\ &= f_m(\boldsymbol{Q}(t)) + \mathbb{E}\left[\sum_n (\beta M(t)-\beta)W_n(t)I_n(t)|\boldsymbol{Q}(t)\right] \end{aligned}$$

Now, we consider the value of $f_s(\boldsymbol{Q}(t))/f_m(\boldsymbol{Q}(t))$ such that

$$\begin{aligned} &f_s(\boldsymbol{Q}(t))/f_m(\boldsymbol{Q}(t)) = \\ &\frac{f_m(\boldsymbol{Q}(t)) + \mathbb{E}\left[\sum_{n\in\mathcal{S}(t)}(\beta M(t)-\beta)W_n(t)I_n(t)|\boldsymbol{Q}(t)\right]}{f_m(\boldsymbol{Q}(t))} \\ &= 1 + \frac{\mathbb{E}\left[\sum_n (\beta M(t)-\beta)W_n(t)I_n(t)|\boldsymbol{Q}(t)\right]}{f_m(\boldsymbol{Q}(t))}, \end{aligned}$$

where

$$\begin{aligned} &\mathbb{E}\left[\sum_{n\in\mathcal{S}(t)}(\beta M(t)-\beta)W_n(t)I_n(t)|\boldsymbol{Q}(t)\right] = \\ &\sum_{m=0}^{N-1}(\beta m-\beta)\mathbb{E}\left[\sum_{n\in\mathcal{S}(t)} W_n(t)I_n(t)|\boldsymbol{Q}(t), M(t)=m\right] \\ &\times \Pr[M(t)=m] \end{aligned} \tag{15}$$

Note that

$$\mathbb{E}\left[\sum_{n\in\mathcal{S}(t)} W_n(t)I_n(t)|\boldsymbol{Q}(t), M(t)=m\right] = \frac{f_m(\boldsymbol{Q}(t))}{1-\beta N}$$

Hence, (15) can be rewritten as follows:

$$\begin{aligned} (15) &= \frac{f_m(\boldsymbol{Q}(t))}{1-\beta N}\sum_{m=0}^{N-1}[(\beta m-\beta)]\Pr[M(t)=m] \\ &= \frac{\beta f_m(\boldsymbol{Q}(t))\left(\mathbb{E}[M(t)]-1]\right)}{1-\beta N} \end{aligned}$$

Thus we have,

$$f_s(\boldsymbol{Q}(t))/f_m(\boldsymbol{Q}(t)) \geq 1 + \frac{\beta\left(\mathbb{E}[M(t)]-1\right)}{1-\beta N}$$

Hence, the proposed algorithm can support $(1+\epsilon)$ fraction of the rate region Λ_f where

$$\epsilon = \frac{\beta\left(\mathbb{E}[M(t)]-1\right)}{1-\beta N}.$$

This completes the proof.

Appendix B
Proof of Lemma 1

Let $\mathbb{E}[M(t)|R_{i^*}(t) = r_k, n = i^*]$ be the conditional expectation of number of users from which channel state information is *not* acquired when the user with the maximum queue length and its channel state are given. Note that for homogenous channels, the following equality holds,

$$\mathbb{E}[M(t)|R_{i^*}(t) = r_k, n = i^*] = \mathbb{E}[M(t)|R_{i^*}(t) = r_k]. \tag{16}$$

The value of $\mathbb{E}[M(t)]$ is simply the expectation of (16) over the channel state distribution:

$$\mathbb{E}[M(t)] = \sum_{k=1}^{L}\mathbb{E}[M(t)|R_{i^*}(t) = r_k]\Pr[R_{i^*}(t) = r_k].$$

Note that if $R_{i^*}(t) = r_1$, then SSF does not acquire channel state information from other users in the network, since user i^* is the user with the maximum weight. Hence with probability, $\Pr[R_{i^*}(t) = r_k] = p_1$, the remaining $N-1$ users do not report their channel states, i.e., $\mathbb{E}[M(t)|R_{i^*}(t) = r_1] = (N-1)$. Meanwhile, if $R_{i^*}(t) = r_2$, user $j \neq i^*$ does not report its channel state if $R_j(t) \leq r_2$, which occurs with probability $\Pr[R_j(t) \leq r_2] = \sum_{k=2}^{L} p_k$. Thus,

$$\mathbb{E}[M(t)|R_{i^*}(t) = r_2] = (N-1)\sum_{k=2}^{L} p_k.$$

In general, for homogeneous channels, the conditional distribution of random variable $M(t)$ given that the channel state of the user with the maximum queue length is r_l, is a binomial distribution with probability $\sum_{k=l}^{L} p_k$. Then, $\mathbb{E}[M(t)]$ is as follows,

$$\begin{aligned} &\mathbb{E}[M(t)] = \\ &\left[p_1 + p_2\sum_{k=2}^{L}p_k + p_3\sum_{k=3}^{L}p_k + \ldots + p_L^2\right](N-1) \end{aligned}$$

Appendix C
Proof of Lemma 2

If all channels are uniformly distributed, then $p_k = \frac{1}{L}$. Hence, by (7) $\mathbb{E}[M(t)]$ is obtained as

$$\begin{aligned} &\mathbb{E}[M(t)] \\ &= \left[\frac{1}{L} + \frac{1}{L}\sum_{k=2}^{L}\frac{1}{L} + \frac{1}{L}\sum_{k=3}^{L}\frac{1}{L} + \ldots + \frac{1}{L^2}\right](N-1) \\ &= \left[\frac{1}{L} + \frac{1}{L^2}(L-1) + \frac{1}{L^2}(L-2) + \ldots + \frac{1}{L^2}\right](N-1) \end{aligned} \tag{17}$$

Re-arranging (17), we obtain

$$\begin{aligned} \mathbb{E}[M(t)] &= \left[\frac{1}{L} + \frac{L(L-1)}{2L^2}\right](N-1) \\ &= \left[\frac{1}{2} + \frac{1}{2L}\right](N-1) \end{aligned}$$

Appendix D
Proof of Theorem 3

We prove the theorem by showing that $\mathbb{E}[M(t)]$ is a jointly convex function of $(p_1, p_2, \ldots, p_L)$, and the minimum of this convex function is achieved when channels are uniformly distributed.

Lemma 4: $\mathbb{E}[M(t)]$ is a jointly convex function of $(p_1, p_2, \ldots, p_L)$.

Proof: The Hessian of $\mathbb{E}[M(t)]$ in (7) can be given as follows,

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 2 & 1 & 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 2 & 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 1 & 2 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 1 & 1 & 2 & 1 & \cdots & 1 \\ 0 & 1 & 1 & 1 & 1 & 2 & \cdots & 1 \\ 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 1 & 1 & 1 & 1 & \cdots & 2 \end{pmatrix} (N-1)$$

Now, we show that H is positive definite matrix. Let $\mathbf{x} = [x_1\ x_2\ x_3 \ldots\ x_L]$ be any vector and $\mathbf{x} \in \mathbb{R}^{L-1}$. If $\mathbf{x}H\mathbf{x}^T > 0$ then, H is positive definite matrix and $\mathbb{E}[M(t)]$ is convex function of $(p_1, p_2, \ldots, p_L)$ [28]. By simple manipulations, we obtain

$$\mathbf{x}H\mathbf{x}^T = \left[\sum_{l=2}^{L} x_l^2 + \left(\sum_{l=2}^{L} x_l\right)^2\right](N-1) > 0. \tag{18}$$

■

Lemma 5: $\mathbb{E}[M(t)]$ has the minimum value when channels are uniformly distributed.

Proof: We already showed that $\mathbb{E}[M(t)]$ is jointly convex function of $(p_1, p_2, \ldots, p_L)$. Hence, the first order conditions can be obtained as follows by noting that $p_1 = 1 - \sum_{k=2}^{L} p_k$

$$\frac{\delta\mathbb{E}[M(t)]}{\delta p_2} = -1 + 2p_2 + (p_3 + p_4 + \ldots + p_L) = 0$$
$$\frac{\delta\mathbb{E}[M(t)]}{\delta p_3} = -1 + 2p_3 + (p_2 + p_4 + \ldots + p_L) = 0$$
$$\frac{\delta\mathbb{E}[M(t)]}{\delta p_4} = -1 + 2p_4 + (p_2 + p_3 + p_5 + \ldots + p_L) = 0$$
$$\vdots$$
$$\frac{\delta\mathbb{E}[M(t)]}{\delta p_L} = -1 + 2p_L + (p_2 + p_3 + p_5 + \ldots + p_{L-1}) = 0$$

In matrix notation, these equations can be represented as

$$\begin{pmatrix} 2 & 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 2 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & 2 & 1 & \cdots & 1 \\ 1 & 1 & 1 & 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & 1 & 1 & \cdots & 2 \end{pmatrix} \begin{pmatrix} p_2 \\ p_3 \\ p_4 \\ p_5 \\ \vdots \\ p_L \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \tag{19}$$

Solving this linear system, we have,

$$p_k - p_l = 0, \forall k, l\ k \neq l \tag{20}$$

Hence,

$$p_k = p_l, \forall k, l\ k \neq l \tag{21}$$

Thus,

$$p_k = \frac{1}{L}, \forall k. \tag{22}$$

Thus, when the channel distributions are uniform, $\mathbb{E}[M(t)]$ has the minimum value. ■

We now prove the second part of the theorem. We show that when channels are uniform, $\mathbb{E}[M(t)]$ is a decreasing function of L.

Lemma 6: $\mathbb{E}[M(t)]$ is a decreasing function of L.

Proof: From (8),

$$\mathbb{E}[M(t)] = \left[\frac{1}{2} + \frac{1}{2L}\right](N-1) \tag{23}$$

Taking the derivative of $\mathbb{E}[M(t)]$ with respect to L yields that,

$$\frac{d\mathbb{E}[M(t)]}{dL} = \left[\frac{-1}{2L^2}\right](N-1) < 0. \tag{24}$$

Thus, $\mathbb{E}[M(t)]$ is a decreasing function of L. ■

Now, it is easy to see that in (8), taking $L \to \infty$ yields

$$\begin{aligned}\lim_{L\to\infty} \mathbb{E}[M(t)] &= \lim_{L\to\infty} \left[\frac{1}{2} + \frac{1}{2L}\right](N-1) \\ &= \frac{N-1}{2}.\end{aligned} \tag{25}$$

In the limiting case, when $N > 3$, $\mathbb{E}[M(t)] > 1$. In addition, according to Lemma 6 if L is finite, $\mathbb{E}[M(t)]$ is still greater than 1 whenever $N > 3$ since $\mathbb{E}[M(t)]$ decreases as L increases. We can conclude that when all channels are uniformly distributed, and when $N > 3$, then $\mathbb{E}[M(t)] > 1$. As a result, we guarantee to expand the rate region, i.e., $\epsilon > 0$. In addition, according to Lemma 4 and Lemma 5, $\mathbb{E}[M(t)]$ has its minimum value when channels are uniform. Therefore, for homogenous channels, when $N > 3$, then $\epsilon > 0$ and rate region is expanded. This completes the proof.

Appendix E
Proof of Lemma 3

Now, we consider the case that the channels are not identical, i.e., $p_n^k \neq p_m^k$, where $n \neq m$ and $\forall k$. The main difference between homogenous and heterogenous cases is that (16) does not hold for heterogenous channels. Let $\chi_n(t)$ be the event that user n is the user with maximum queue size at time t. Also, let $\varphi_k(t)$ be the event that the user with maximum queue size has channel state k at time t, i.e., $R_{i^*}(t) = r_k$. The probabilities of these events are denoted by $\Pr[\chi_n(t)]$ and $\Pr[\varphi_k(t)]$, respectively. Then $\mathbb{E}[M(t)]$ can be found as follows:

$$\mathbb{E}[M(t)] = \sum_{n=1}^{N}\sum_{k=1}^{L} \mathbb{E}[M(t)|\chi_n(t), \varphi_k(t)] \Pr[\chi_n(t), \varphi_k(t)].$$

Hence,

$$\begin{aligned}
&\mathbb{E}[M(t)] = \\
&\Pr[\chi_1(t)]\left(p_1^1(N-1) + p_1^2\sum_{n=2}^{N}\sum_{k=2}^{L}p_n^k + \ldots + p_1^L\sum_{n=2}^{N}p_n^L\right) \\
&+\Pr[\chi_2(t)]\left(p_2^1(N-1) + p_2^2\sum_{\substack{n=1\\n\neq 2}}^{N}\sum_{k=2}^{L}p_n^k + \ldots + p_2^L\sum_{\substack{n=1\\n\neq 2}}^{N}p_n^L\right) \\
&+\Pr[\chi_3(t)]\left(p_3^1(N-1) + p_3^2\sum_{\substack{n=1\\n\neq 3}}^{N}\sum_{k=2}^{L}p_n^k + \ldots + p_3^L\sum_{\substack{n=1\\n\neq 3}}^{N}p_n^L\right) \\
&\vdots \\
&+\Pr[\chi_N(t)]\left(p_N^1(N-1) + p_N^2\sum_{\substack{n=1\\n\neq N}}^{N}\sum_{k=2}^{L}p_n^k + \ldots + p_N^L\sum_{\substack{n=1\\n\neq N}}^{N}p_n^L\right).
\end{aligned}$$

Note that $p_n^k \geq p^{min}$ for all n, k. Hence, a lower bound on $\mathbb{E}[M(t)]$ can be given as follow,

$$\begin{aligned}
\mathbb{E}[M(t)] \geq [&p^{min}(N-1) + p^{min}(p^{min}(L-1)(N-1) \\
&+ p^{min}(p^{min}(L-2)(N-1) + \ldots \\
&+ p^{min}(p^{min}(N-1)]
\end{aligned}$$

By rearranging, we have,

$$\mathbb{E}[M(t)] \geq (N-1)\left(p^{min} + (p^{min})^2\left[\frac{L(L-1)}{2}\right]\right)$$

This completes the proof.

REFERENCES

[1] M. Karaca, Y. Sarikaya, O. Ercetin, T. Alpcan, and H. Boche, "Efficient wireless scheduling with limited channel feedback and performance guarantees," in *2012 PIMRC Workshop-WDN*.

[2] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Trans. Autom. Control*, vol. 37, no. 12, pp. 1936–1948, Dec. 1992.

[3] X. Liu, E. K. P. Chong, and N. B. Shroff, "A framework for opportunistic scheduling in wireless networks," *Comput. Netw.*, vol. 41, 2003.

[4] M. J. Neely, "Energy optimal control for time varying wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 7, pp. 2915–2934, 2006.

[5] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushyana, and S. Viterbi, "CDMA/HDR: a bandwidth efficient high speed wireless data service for nomadic users," *IEEE Commun. Mag.*, vol. 38, no. 7, pp. 70–77, 2000.

[6] X. Qin and R. Berry, "Opportunistic splitting algorithms for wireless networks," in *Proc. 2004 IEEE INFOCOM*, vol. 6, pp. 1662–1672.

[7] M. Ouyang and L. Ying, "On optimal feedback allocation in multichannel wireless downlinks," in *2011 Mobihoc*.

[8] P. Chaporkar, A. Proutiere, H. Asnani, and A. Karandikar, "Scheduling with limited information in wireless systems," in *2009 Mobihoc*.

[9] J. Huang, V. Subramanian, R. Agrawal, and R. Berry, "Joint scheduling and resource allocation in uplink OFDM systems for broadband wireless access networks," *IEEE J. Sel. Areas Commun.*, vol. 27, pp. 226–234, Feb. 2009.

[10] V. Hassel, D. Gesbert, M.-S. Alouini, and G. Oien, "A threshold-based channel state feedback algorithm for modern cellular systems," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2422–2426, 2007.

[11] T. Tang and R. Heath, "Opportunistic feedback for downlink multiuser diversity," *IEEE Commun. Lett.*, vol. 9, no. 10, pp. 948–950, 2005.

[12] T. Samarasinghe, H. Inaltekin, and J. Evans, "The feedback-capacity tradeoff for opportunistic beamforming," in *2011 IEEE ICC*.

[13] D. Love, R. Heath, V. Lau, D. Gesbert, B. Rao, and M. Andrews, "An overview of limited feedback in wireless communication systems," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341–1365, Oct. 2008.

[14] K. Kar, X. Luo, and S. Sarkar, "Throughput-optimal scheduling in multichannel access point networks under infrequent channel measurements," in *2007 IEEE INFOCOM*.

[15] L. Ying and S. Shakkottai, "On throughput optimality with delayed network-state information,," in *Proc. 2008 Inf. Theory Applications Workshop*.

[16] A. Gopalan, C. Caramanis, and S. Shakkottai, "On wireless scheduling with partial channel-state information," in *Proc. 2007 Allerton Conf.*

[17] ——, "Low-delay wireless scheduling with partial channel-state information," in *2012 IEEE INFOCOM*.

[18] C.-P. Li and M. J. Neely, "Energy-optimal scheduling with dynamic channel acquisition in wireless downlinks," *IEEE Trans. Mobile Comput.*, vol. 9, no. 4, pp. 527–539, Apr. 2010.

[19] M. J. Neely, "Max weight learning algorithms with application to scheduling in unknown environments," in arXiv:0902.0630v1, Feb. 2009.

[20] M. Karaca, T. Alpcan, and O. Ercetin, "Smart scheduling and feedback allocation over non-stationary wireless channels," in *2012 IEEE ICC WS – SCPA*.

[21] W. Ouyang, S. Murugesan, A. Eryilmaz, and N. B. Shroff, "Scheduling with rate adaptation under incomplete knowledge of channel/estimator statistics," in *2010 Allerton Conf. Commun., Control, Comput.*

[22] ——, "Exploiting channel memory for joint estimation and scheduling in downlink networks," in *2011 IEEE INFOCOM*.

[23] C. Tekin and M. Liu, "Approximately optimal adaptive learning in opportunistic spectrum access," in *2012 IEEE INCOFOM*.

[24] B. Radunovic, A. Proutiere, D. Gunawardena, and P. Key, "Exploiting channel diversity in white spaces," Technical Report MSR-TR-2011-53, 2011.

[25] N. Chang and M. Liu, "Optimal channel probing and transmission scheduling for opportunistic spectrum access." in *2007 Mobicom*.

[26] J. G. Andrews, S. Shakkottai, R. Heath, N. Jindal, M. Haenggi, R. Berry, D. Guo, M. Neely, S. Weber, S. Jafar, and A. Yener, "Rethinking information theory for mobile ad hoc networks," *IEEE Commun. Mag.*, vol. 46, no. 12, pp. 94–101, Dec. 2008.

[27] A. Eryilmaz, R. Srikant, and J. Perkins, "Stable scheduling policies for fading wireless channels," *IEEE/ACM Trans. Netw.*, vol. 13, pp. 411–424, 2005.

[28] S. Boyd and L. Vandenbeghe, *Convex Optimization*. Cambridge University Press, 2004.

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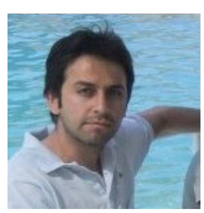

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