Fun Examples for Teaching Linear and Nonlinear Circuits

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***Abstract*—Circuit theory is essentially a physically based theory of dynamical systems. To demonstrate basic phenomena example circuits have to be selected and designed. These circuits consist of a few essential elements, which makes it easier to explain the mathematics. But, the abstraction of these simplified circuits can be a hurdle for students. To give students with a strong interest in the application an access to the theory and to add more verve to the lectures, we gathered some circuit examples, which are easy to explain in detail and have an entertaining effect. In this paper, we focus on circuits with at least one reactive element.**

I. Introduction

There are two types of circuit theory lectures. The ones without circuits, which are basically dynamical systems theory lectures, and the ones without theory, which emphasize the description of important circuits [1]. It is a difficult task to combine the fundamental theory with meaningful examples, especially for first year students.

Our circuit theory lecture [2] is based on the book "Linear and Nonlinear Circuits" by Chua, Desoer and Kuh [3], where most circuits are abstractions of practical implementations. The content of this book allows an abstraction to dynamical systems theory, which is important for every civil, mechanical or electrical engineer. Our experience shows that students can have problems with this degree of abstraction. A student might get the impression that the topics are interesting mathematical problems, yet have no practical relevance.

We explain the practical relevance by discussing some essential circuits like a push-pull output stage and a phase locked loop. These explanations create a link between the theoretical foundation and their real-world applications. In this paper, we describe a collection of tutorial circuits we recently added to our lecture without compromising its theoretical content. To increase the enthusiasm of the students, we were looking for circuits which are easy to explain in detail and can be used for entertaining demonstrations. A complete list of the tutorials can be found in [4] (an English version will follow soon).

II. Electrical Engineering Students Community

The community of electrical engineering students is diverse. To offer a good education, the different learning types and interests need different attention. For the approximately five hundred first year students in electrical engineering at our university it is mandatory to pass circuit theory. To handle such a large class we give theory and tutorial lectures in front of the whole group. Additionally, we offer tutorial lectures in small groups with different teaching methods like collaboration between the students and assisted problem solving. The students can solve voluntary homework questions and visit a lab to get some practical experience with the circuits demonstrated in the lectures. Some extra time is reserved for questions and feedback from students, which need a direct contact.

We assume that electrical engineering students are interested in electronics, computers and mathematics. The interest in mathematics can be easily handled with a theoretical lecture. To give the students which like to work at a computer an entrance to circuit theory, we described how the circuits presented in the lecture can be simulated with openly available tools [5]. We could add a link to assembled electronics by integrating the examples in Section III as tutorials. The combination of simple circuits and fun applications is not conflicting with the previous, purely theoretical way of explaining the topics, but encourages the students to understand the lecture.

III. Tutorial Circuits

The examples are ordered according to our course, which starts with general reactive elements. Then, we cover the dynamic behavior of first-order circuits, second-order circuits and general nonlinear circuits in the time domain, respectively. The last topic is the analysis of *linear time invariant* (LTI) systems in the frequency domain. For the demonstrations, an electronics laboratory is set up in the lecture hall. In addition to blackboard and video projector, we use an oscilloscope, which is connected to the projector, power supplies, a wave generator and an active loudspeaker.

A. Reactive Elements: Capacitance Between two Humans

We ask two students to stand back to back on the stage to visualize $i = C\frac{\mathrm{d}v}{\mathrm{d}t}$, the current i through a capacitance C is proportional to the derivative of the voltage across the capacitance v with respect to the time t. The students are the two electrodes of a capacitor. One student has to hold on to the hot end of a wave generator and the other student to a resistor, which is connected to ground (Figure 1).

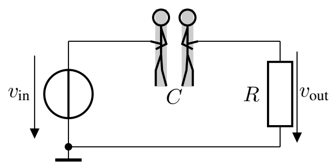

Figure 1. Capacitance between two humans

The resistance is chosen so small that its influence on the current can be neglected for the used frequency and so big that the voltage has a reasonable amplitude for the measurement ($1\,\mathrm{k\Omega}$ at $10\,\mathrm{kHz}$). If a sine wave is used as input signal

(20 V peak-to-peak), the current will be a sine wave, which is shifted by $\frac{\pi}{2}$, as the derivative demands. The voltage of the wave generator v_{in} and the voltage across the resistor v_{out}, which is proportional to the current, are made visible for everyone with the oscilloscope. The students can stand closer or step apart to influence their capacitance and, therefore, the amplitude. If they shake hands, they will turn resistive, the phase difference will disappear and the amplitude will increase dramatically. By sending different functions, like a triangular or rectangular wave, the derivations of these signals are shown on the oscilloscope. To give the students a link to real applications, we briefly discuss the behavior of capacitive touch screens, which are used in smartphones.

The voltage across the capacitor depends on the history of the current and, therefore, has a memory. By showing that the students build a capacitor, we can show that they have a memory.

B. RC Circuits: Voltage Multiplier

A typical introduction to first-order circuits is to discuss the response of a circuit built from one resistance and one capacitance with initial charge (RC Circuit). These circuits can then be extended with sources, additional linear and nonlinear resistances and switches.

We discuss the behavior of an initially uncharged RC circuit with an ideal diode and a voltage source v_{in}, which produces a rectangular wave v_{in} alternating between zero and some positive voltage V (Figure 2). During the timeframes with V at the source, the diode is conductive and the voltage across the capacitance v_C converges to V exponentially. This is the exact behavior as if there was a constant voltage source V and no diode. Whenever the voltage of the source is zero, there is a non positive voltage drop across the diode. The diode does not conduct any current and the voltage across the capacitance stays constant during that time. After some periods, the system approaches a state, where the voltage across the capacitance is V and the voltage across the diode v_{out} is a rectangular wave alternating between $-V$ and zero (Figure 2).

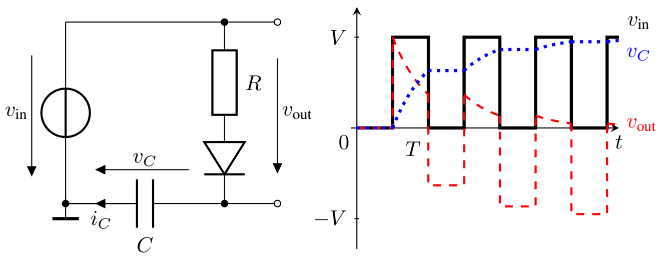

Figure 2. RC circuit

After discussing this RC circuit, it is only a small step to explain the half-wave series multiplier. To approximate the behavior, we regard the stages of the multiplier cascade independently and reuse the output voltage v_{out} of the previous stage as source v_{in} in the next stage. The extension to a joint treatment of the capacitors and a sinusoidal input is discussed in a few words without going beyond the scope of first-order circuits. Finally, we demonstrate the high voltage multiplier in Figure 3, which produces sparks of roughly 5 cm length.

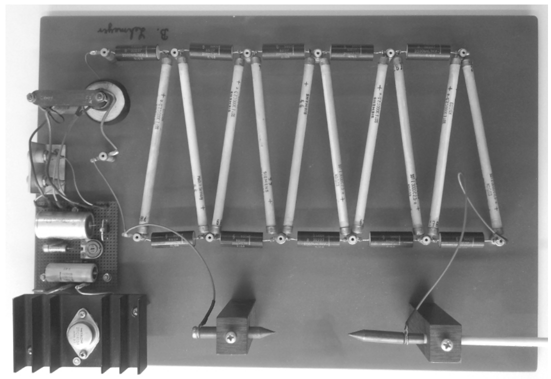

Figure 3. High voltage multiplier

C. Dynamic Route: Joule Thief

The dynamic route of first-order circuits is a methodology to analyze two-terminal resistive networks which are connected to one capacitance or inductance [3]. With an inductance L, the voltage from terminal one to terminal two must follow $v = -L\frac{\mathrm{d}i}{\mathrm{d}t}$, where i is the current into terminal one (Figure 4). From $v > 0$ or $v < 0$ follows $\frac{\mathrm{d}i}{\mathrm{d}t} < 0$ or $\frac{\mathrm{d}i}{\mathrm{d}t} > 0$, the current i has to drop or rise, respectively. For $v = 0$ the circuit is in an equilibrium and depending on the dynamic route around this point, the stability of the equilibrium can be determined. With nonlinear driving-point characteristics of the resistive network, it is possible that the circuit reaches an impasse point. It might be that i has to drop or rise, but the driving-point characteristic does not allow this change. The impasse points result from poor modelling, more reactive elements are needed to describe the behavior of the circuit. But, the increased order can be avoided by introducing the “jump rule”: If a first-order circuit reaches an impasse point, the dynamic route can be continued by jumping instantaneously to another point on the characteristic of $\mathcal{N}$ such that the state variable is continuous.

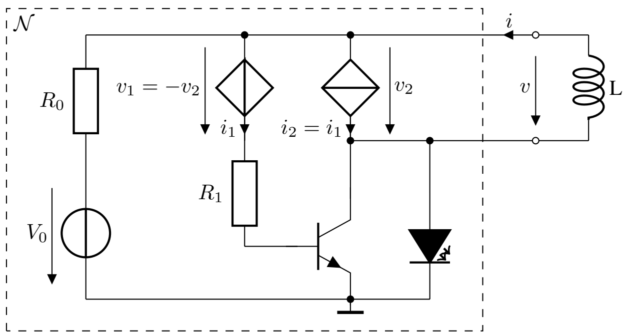

Figure 4. Joule thief

In the lecture, we discuss the so called *joule thief*, which is a Meißner or Armstrong oscillator with a light emitting diode as load (Figure 4) [6]. The controlled sources model together with the inductance a transformer. The driving-point characteristic of the resistive part of the joule thief and the dynamic route are shown in Figure 5. The equilibrium (EQ) is unstable and the dynamic route describes an oscillation with jumps at the impasse points (IP). The diode emits light, even if the supply voltage is far below the required forward voltage.

We motivated the students to build joule thieves at home, where they can steal the last joules from almost discharged batteries.

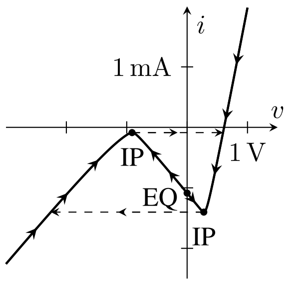

Figure 5. v-i characteristic of $\mathcal{N}$ with dynamic route

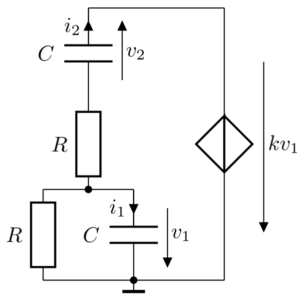

Figure 6. Wien bridge oscillator

D. Second-Order Circuits: Phase Portrait and Synthesizer

In general, autonomous second-order circuits can be described by two coupled first-order differential equations. To solve the equations in the time domain, we write them in homogeneous form

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{A}\boldsymbol{x},$$

where $\boldsymbol{x}$ is a vector containing the two state variables and $\boldsymbol{A} \in \mathbb{R}^{2\times 2}$ is the coefficient matrix. If $\boldsymbol{A}$ is not defective, we can use the substitution $\boldsymbol{x} = \boldsymbol{Q} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$, where $\boldsymbol{Q}$ is the eigenbasis of $\boldsymbol{A}$. Now, we get the two decoupled first-order differential equations

$$\frac{\mathrm{d}\xi_1}{\mathrm{d}t} = \lambda_1\xi_1, \quad \text{and} \quad \frac{\mathrm{d}\xi_2}{\mathrm{d}t} = \lambda_2\xi_2,$$

where $\lambda_{1,2}$ are the two eigenvalues of $\boldsymbol{A}$. These equations can be solved with the methods learned for first-order circuits. Depending on the eigenvalues, the solution for the state variables can be a superposition of two exponential functions or a positively or negatively damped sine wave.

The phase portrait of a second-order circuit is generated by plotting the two state variables against each other for different suitable initial values. A phase portrait shows the equilibria of a circuit and how the state variables evolve over the time, which makes it an invaluable tool for analysing nonlinear dynamic circuits. Depending on the sign of the real and imaginary part of $\lambda_{1,2}$, typical phase portraits can be distinguished.

The state variables of the Wien bridge oscillator in Figure 6 are the voltages v_1 and v_2 across the two capacitances, respectively. The eigenvalues of the system can be calculated as

$$\lambda_{1,2} = \frac{1}{2RC}\left(k - 3 \pm \sqrt{(k-3)^2 - 4}\right).$$

The amplification factor k will determine, if the eigenvalues are complex or real valued and if they have a positive or negative real part. The equilibrium of this circuit is always the origin of the phase plane. With complex eigenvalues, which have a negative real part, the phase portrait is a logarithmic spiral, which is attracted by the origin. With a positive real part, the origin becomes a repeller.

The voltage controlled voltage source can be realized with a non-inverting amplifier, where the amplification k can be adjusted with a potentiometer. Now, we can visualize the logarithmic spiral by connecting the circuit to an oscilloscope over a network which calculates ξ_1 and ξ_2, as long as the amplifier does not saturate. Playing with the potentiometer, which controls k, alternates the origin between repelling and attracting. The frequency of the oscillation and the speed of the logarithmic decay can be changed continuously. If we measure v_1 and v_2 with the oscilloscope, we can demonstrate the effect of the transformation $\boldsymbol{x} = \boldsymbol{Q} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$.

To give an example for the use of oscillators, we connected the ouput of the Wien bridge oscillator to a loudspeaker and played some music by switching the resistances R between different values.

E. Nonlinear Circuits: FM Transmitter

The focus of our nonlinear circuits discussion is on oscillators and bistable circuits. We already introduced two oscillator circuits, the joule thief and the Wien bridge oscillator, but analyzed them with simplified methods. We regarded the joule thief with only one reactance and the Wien bridge oscillator without saturation. A circuit with bounded state variables has to have only one repelling equilibrium to be an oscillator. Additional virtual but stable equilibria bound the state variables.

To demonstrate the behavior of oscillators, we use the frequency modulation transmitter in Figure 8. We approximate the behavior of the bipolar transistor with three different linear models. First, if both, the base-emitter and the base-collector junction are reverse biased, there will be no current through the transistor and there are open circuits between all three terminals. Calculating the equilibrium with this model is done by replacing all capacitors by open circuits and the inductor by a short circuit. This equilibrium will be a stable one, but it is only virtual, because the base-emitter junction will be forward biased, which is contradicting our assumptions. Second, if both p-n junctions are forward biased, the transistor can be modeled with two voltage sources, one between base and emitter, the other one between base and collector. Again we calculate the equilibrium point as before and find it is again stable but virtual, because the base-collector junction will be reverse biased at this point.

Finally, we analyze the circuit with the transistor in the forward mode, i.e. the base-emitter junction is forward biased and the base-collector junction is reverse biased. The equilibrium now is unstable but real, it is not contradicting the assumptions. The low frequency modulating voltage is changing the actual values of the nonlinear capacitors changing the eigenvalues of the circuit within the linearized region around this equilibrium. The oscillation frequency of this third order circuit is determined as $f \approx \frac{1}{2\pi\sqrt{L(C_1+C_c)}}$, where C_c is the linearized value of the nonlinear base-collector junction capacitor. C_c is controlled in turn by the low frequency modulating source v_{lf}. The circuit generates a frequency modulated signal transmitted from the coil.

The explanation of this circuit is more laborious than the rest of the tutorials and legal advice on building transmitters has to be given, but the fascination of the students for the application is enormous. Choosing the right song strongly influences the success of the demonstration.

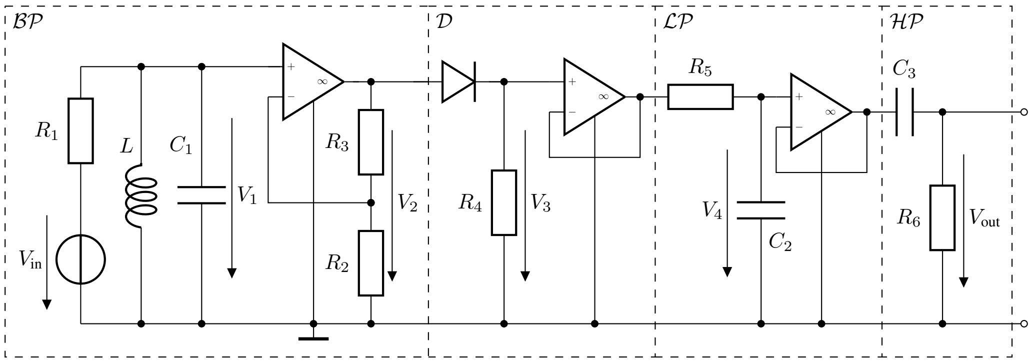

Figure 7. Crystal radio with decoupled stages

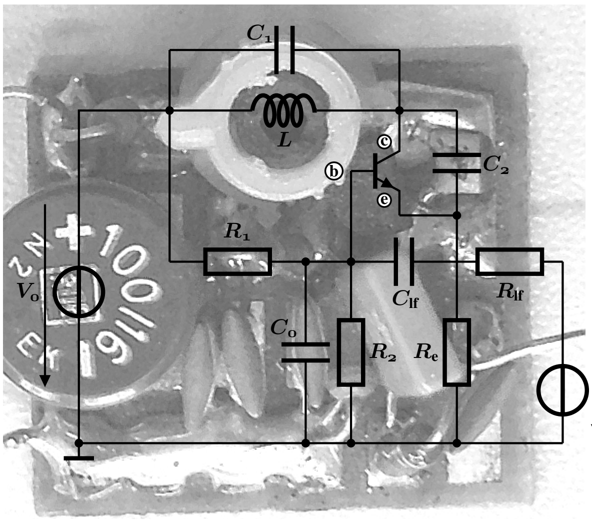

Figure 8. FM transmitter

F. Network Functions: Crystal Radio

A great share of the curriculum is reserved for discussing LTI systems in the frequency domain. Typically presented circuits are low pass, high pass and band pass filters. All of these filters can be found in a crystal radio (including the filtering characteristic of the speaker), although, the radio itself is not a linear system because of the diode (or crystal detector). This gives us the opportunity to discuss the properties of LTI systems. To keep the equations for the crystal radio simple, we add buffer or non-inverting amplifiers after each stage, respectively (Figure 7). The network functions of the different stages are now independent of each other and can be calculated without a load, respectively. By discussing their behavior for different frequencies, we inherently describe the operation of the crystal radio. Only the influence of the diode and the propagation of a signal through all stages needs to be amended.

In Figure 7, the signal from the antenna is called V_{in}. The band pass filter $\mathcal{BP}$ rejects all signals with a frequency outside of a small band around the resonance frequency $\frac{1}{2\pi\sqrt{LC_1}}$, which we tune to roughly $1\,\text{MHz}$. All passed signals V_1 are amplified and the diode $\mathcal{D}$ cuts off their negative part. The low pass filter $\mathcal{LP}$ damps the high frequencies and the output voltage V_4 is the envelope of the input signal V_3. Finally, the high pass filter $\mathcal{HP}$ removes the direct current component in V_4.

Again, this circuit can be easily assembled and the mode of operation can be visualized on the oscilloscope by measuring the signals after each stage, respectively. We transmitted a song from the frequency generator, which is capable of amplitude modulation, to the crystal radio with a loudspeaker as a load. As transmission path we use the capacitive coupling between two students (Section III-A).

IV. Student Feedback

The student feedback was very positive. After demonstrating the first live circuit, where we played music on the Wien bridge oscillator, the students instantaneously asked for more examples. We could improve our annual poll ratings in motivation, comprehensibility and visualization. Around a third of the students mentioned the example circuits in the free text field “What did you like about the lecture?”. Many students told us that they rebuild the devices, which helped them to understand the circuits.

V. Conclusion

In this paper, we presented a collection of easy to analyze circuits, which support the teaching and understanding of linear and nonlinear circuits. We did not deviate from the strictly theoretical lecture in the tutorials, but, added some extra examples, applications and links to follow-up lectures. With this step, we could pierce the layer of abstraction and help the students to see the context of the lecture. By using fun examples, we could increase the motivation of the students and diversify our lectures, which was very much appreciated by the students.

Acknowledgement

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