# A Framework to Quantify Technical Flexibility in Power Systems Based on Reliability Certificates


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***Abstract*—Power systems are increasingly stressed by variable and unpredictable generation from various sources. We identify the qualitative framework of flexibility as an adequate tool to specify requirements that allow the system to handle this variability. An open problem is the quantification of technical flexibility that incorporates limitations from transmission system and component behavior in contrast to existing copper plate supply and demand balance approaches. We develop such a quantitative method for single components on the basis of a priori specified reliability criteria. Our framework bases on a combined static power flow and small signal stability analysis. In a perturbative approach we derive sensitivity-based formula for eigenvalue variations under nonlinear changes of steady power flow set points. To this end, we define rigorously the terms flexibility metric and technical flexibility of single components. We provide an algorithmic procedure for computation of tolerance ranges of individual system components such that the overall behavior remains reliable.**



***Index Terms*—Flexibility, Reliability, Power System Control**


## I. Introduction

In operation of electric power systems, difficulties are encountered in accommodating increasingly variable generation and demand. To adapt to unpredictable changes in power balance and to reduce their significance for reliable power provision, the deployed system architecture including operation schemes must provide a certain degree of tolerance to uncertainty and variability. This property is called flexibility [1]. Required flexibility to remain in reliable operation is a widely used term although at times without proper definition [2]. Qualitatively it is seen that flexibility of a power system is constituted by its components and dynamics at all time scales; moreover, it is affected by the overall loading level leading to trade-offs between efficiency in the sense of maximum asset utilization and inherent tolerance to loading variations [1]. Quantitative specification of flexibility requirements has been widely recognized as key for successful transitioning towards novel operational architectures, and flexibility metrics form a novel area of research, see [2], [3], [4], [5], [6] and references therein. Yet quantitative tools are missing that incorporate technical limitations stemming from the transmission system or control equipment, see [6], [7], [8].

In the cited studies, flexibility metrics are derived from capacity requirements to meet with changes in steady state power balance. A flexibility "trinity" as qualitative relation between power ramping, power, and energy was proposed in [2] in the so-called power node framework, and a methodology to assess technical available operational flexibility is derived. In [5], the insufficient ramping resources probability is proposed on the basis of generation adequacy metrics supporting long term planning of power systems. In [6], a metric is presented to quantify technical flexibility levels of individual generators and the whole system in order to accommodate power imbalance. None of the proposed methods handles technical limitations arising from power transfer via the transmission system or dynamic limitations. However, international experience shows that load ramp events on the scale of minutes to hours play a significant role in flexibility assignment [9], i.e. the interplay of changes in power flow set points and controller adjustments to meet with new steady state reliability requirements for dynamic operation. Moreover, the characterization of system-wide technical flexibility in a single parameter might be insufficient to guarantee reliable power provision for multiple disturbance scenarios that differ in their locational structure and in local magnitude of imbalance.

Here, we propose a method to quantify flexibility of single components in electric power systems on the basis of reliability specifications. We incorporate attributes of the transmission system in dispatching load imbalances and provide a sensitivity-based quantitative framework that accounts for relations between steady state power flow variations and reliability of controlled system component dynamics. As input we use dynamic generator models and an external power disturbance causing steady state power imbalance. Then, the output is a measure for the displacement of an a priori chosen eigenvalue. Technical flexibility is defined on the basis of reliability of operation being specified by the technical flexibility metric of choice.

*Notation:* The brackets $\langle \cdot, \cdot \rangle$ denote the inner product, the function $d(\cdot, \cdot)$ denotes distance, LHP denotes the open left half plane of the set of complex numbers $\mathbb{C}$, the vector $\boldsymbol{e}_i$ represents the $i$-th unit vector, and $\nabla_{\boldsymbol{x}} \boldsymbol{f} := \partial \boldsymbol{f} / \partial \boldsymbol{x}$ is the Jacobian matrix of partial derivatives of a vector valued function $\boldsymbol{f}$ w.r.t. elements of the vector $\boldsymbol{x}$. The asterisk $^*$ denotes the formal adjoint.

## II. Technical Flexibility in Power Systems

Assessment of technical flexibility is a tool that involves methods from power flow calculations and models describing power system component behavior. Technical flexibility is measured at hand of eigenvalue displacements in the complex plane w.r.t. a priori specified flexibility metrics. The mathematical framework and problem setting is given.

### A. Characterization of Flexibility and Technical Limitations

The authors of [5] define flexibility as the "ability of a system to deploy its resources to respond to change in net load, where net load is defined as the remaining system load not served by variable generation". In [6], "the term flexibility describes the ability of a power system to cope with variability and uncertainty in both generation and demand, while maintaining a satisfactory level of reliability at a reasonable cost, over different time horizons". In [6], the qualitative characterization is complemented by quantitative assessment of individual generators and system flexibility at hand of the relations

$$\mathrm{flex(i)} = \frac{0.5[P_{\max}(i) - P_{\min}(i) + \mathrm{Ramp(i)\Delta t}]}{P_{\max}(i)}, \tag{1a}$$

$$\mathrm{FLEX} = \sum_i \left[ \frac{P_{\max}(i)}{\sum_i P_{\max}(i)} \, \mathrm{flex(i)} \right]. \tag{1b}$$

The measures (1) yield capacity based numbers indicating the ability of the $i$-th unit or the system as a whole to tolerate steady state load variations within a time interval $\Delta t$ and within the operational ranges $P_{\max}(i) - P_{\min}(i)$.

Here, we highlight the fact that two systems can be compared in their respective system flexibility only with respect to a certain flexibility metric. The authors of [6] emphasize this fact, too, by denoting (1b) a "relative" concept. For instance, given same demand and identical amount of available wind generation one system is more flexible than another one, when it can accommodate more wind generation. In that case a system-wide wind accommodation (curtailment) factor would serve as (negative) flexibility metric.

When technical limitations are considered, a nodal capacity based system-level characterization of flexibility may fail, so that (1b) may not serve as metric in the sense of giving meaning to system comparison on the basis of FLEX. The reason for this are technical limitations such as congestion in the transmission system: a steady state power disturbance needs to be dispatched over lines during the accommodation process, so that system flexibility (1b) is in fact necessary but not a sufficient criterion for feasibility of the dispatch when other technical factors limit the process. In this context, paradoxically, congestion can even lead to the phenomenon that an increase in installed transmission lines might lead to a decrease in flexibility of some power system component(s), see [7], [10]. Therefore, technical flexibility associated with generation capacity should also encompass how and where power is generated, how it is transported (how it is traded and when consumed), as noted in [1].

Going beyond simple generation adequacy or nodal capacity calculations, tools for quantitative assessment of technical flexibility must methodologically take into account limitations stemming from transmission system and control equipment properties; that is, methods from power flow calculations must be combined with models describing controlled generator behavior. This methodological challenge has already been formulated in [8] and in the context of quantifying technical flexibility for power systems and components it is noted in [5], [7].

Methods for studies related to power flow solutions base on static optimization schemes and they are classically separated from methods to solve power system problems related to dynamic behavior in time [8]. A characterization of today's power systems control and planning schemes is illustrated in Fig. 1; the time scale characteristic is taken from [11] and we adapted the scheme by spatial scales. Technical flexibility assessment for reliable operation encompasses the gray shaded classical domains. Real-time balancing of power across the network couples dynamic components across various time scales. Assessing flexibility of dynamic components at time scale $\sim 10^{-1}$ sec necessarily involves more detailed generator models than technical flexibility quantification at a slower time scale. For instance, a generator can be described by the classical generator model including only phases and frequencies as states, or the more detailed 2-axis machine model with an IEEE Type 1 exciter including seven states, see [12]. The role of nonlocal coupling via power balance becomes significant when the grid becomes highly loaded. Then, inherent tolerance to local small variations reduces and the power system moves from being elastic to brittle [1].

In such situations an ill-conditioned power flow Jacobian causes sensitive entanglements of components at far distant places in the network across several time scales, see [8], [13]. Thus, intertwining static power flow methods with dynamic models may be accomplished at hand of the power flow Jacobian. Investigations where steady state power flow is combined with analysis of system dynamics is reported for instance in [14] or [15]. There, the power flow Jacobian determines real-time operational properties because it is integral part of the linearized system matrix determining linear behavior. The cited studies relate dynamics to power flow in situations where the Jacobian becomes singular, i.e. the system is at a voltage collapse point.

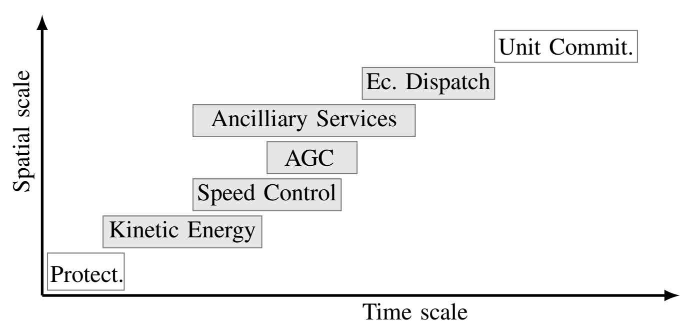


Fig. 1. Temporal and spatial scales of operational and planning schemes

### B. Mathematical Setting for Technical Flexibility

The controlled dynamics of an electric power system $\Sigma_{\mathrm{PS}}$ are in general represented by a system of nonlinear differential algebraic equations

$$\Sigma_{\mathrm{PS}} \begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y}) \\ \boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \end{cases}, \tag{2}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ represents the controlled dynamical variables (depending on the chosen generator model), and the vector $\boldsymbol{y} \in \mathbb{R}^q$ represents the algebraic variables (e.g. voltage magnitudes and angles at buses, stator variables); the nonlinear relation $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{0}$ refers to algebraic network equations, i.e. load flow at $PQ$ buses from where also power balance including $PV$ buses is determined, see [15]. Dynamic stability analysis and operation rely on linearization of dynamics about a stationary power flow solution as operational set point. Define $\boldsymbol{z}^T = (\boldsymbol{x}^T, \boldsymbol{y}^T) \in \mathbb{R}^{n+q}$. Then, the unique operational set point is a vector, denoted by $\boldsymbol{z}_{\mathrm{opt}}$, defined via a residual $\mathcal{R}(\boldsymbol{z})$ as

$$\mathcal{R}(\boldsymbol{z}_{\mathrm{opt}}) := \begin{bmatrix} \boldsymbol{f}(\boldsymbol{x}_{\mathrm{opt}}, \boldsymbol{y}_{\mathrm{opt}}) \\ \boldsymbol{g}(\boldsymbol{x}_{\mathrm{opt}}, \boldsymbol{y}_{\mathrm{opt}}) \end{bmatrix} = \boldsymbol{0}, \tag{3}$$

and it can be computed using for example static optimization as $\boldsymbol{z}_{\mathrm{opt}} = \arg\min ||\mathcal{R}(\boldsymbol{z})||_2$.

The dynamics of small perturbations $\Delta \boldsymbol{z}^T = (\Delta \boldsymbol{x}^T, \Delta \boldsymbol{y}^T)$ evolving about $\boldsymbol{z}_{\mathrm{opt}}$ are derived from linearization of (2), so that

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \Delta \boldsymbol{x} \\ \boldsymbol{0} \end{bmatrix} = \underbrace{\begin{bmatrix} \nabla_{\boldsymbol{x}} \boldsymbol{f}|_{\boldsymbol{z}_{\mathrm{opt}}} & \nabla_{\boldsymbol{y}} \boldsymbol{f}|_{\boldsymbol{z}_{\mathrm{opt}}} \\ \nabla_{\boldsymbol{x}} \boldsymbol{g}|_{\boldsymbol{z}_{\mathrm{opt}}} & \nabla_{\boldsymbol{y}} \boldsymbol{g}|_{\boldsymbol{z}_{\mathrm{opt}}} \end{bmatrix}}_{=:\boldsymbol{A}(\boldsymbol{z}_{\mathrm{opt}})} \begin{bmatrix} \Delta \boldsymbol{x} \\ \Delta \boldsymbol{y} \end{bmatrix}. \tag{4}$$

Here, the linearized system matrix $\boldsymbol{A}(\boldsymbol{z}_{\mathrm{opt}}) \triangleq \nabla_{\boldsymbol{z}} \mathcal{R}(\boldsymbol{z})|_{\boldsymbol{z}_{\mathrm{opt}}}$ contains the Jacobian of the algebraic network equations $\nabla_{\boldsymbol{y}} \boldsymbol{g}$, see [12]. Small signal stability of the controlled system can be determined by means of the generalized eigenvalue problem

$$\boldsymbol{A}(\boldsymbol{z}_{\mathrm{opt}}) \boldsymbol{v}_i = \lambda_i \boldsymbol{B} \boldsymbol{v}_i, \quad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \tag{5}$$

where $\lambda_i = \sigma_i + j\omega_i$ is a complex eigenvalue, and $\boldsymbol{v}_i$ the associated right-eigenvector. A negative growth rate $\sigma_i$ yields an exponentially decaying motion in the direction of the eigenvector $\boldsymbol{v}_i$ whereat the frequency $\omega_i$ characterizes the oscillation while decaying in time.

*Problem Setting:* When stationary power flow from generation or demand changes, the steady state $\boldsymbol{z}_{\mathrm{opt}}$ for the power system will change to a new one, denoted by $\boldsymbol{z}^+_{\mathrm{opt}}$. Denote by $\boldsymbol{d} \in \mathbb{R}^{n+q}$ a large, steady, external disturbance of unpredicted power infeed or outflow affecting the system (4). The new operational set point is determined from the forced residual equation satisfying

$$\mathcal{R}(\boldsymbol{z}^+_{\mathrm{opt}}) + \boldsymbol{d} = \boldsymbol{0}. \tag{6}$$

By that, the small signal assumption made above is no more valid. In these situations local generator controller settings are adapted to $\boldsymbol{z}_{\mathrm{opt}}$, but large enough power flow changes within the network require controller parameters derived on the basis of $\boldsymbol{z}^+_{\mathrm{opt}}$ to guarantee reliable operational behavior.

The problem under consideration is to assess technical flexibility under externally forced steady state power flow changes $\delta \boldsymbol{z}_{\mathrm{opt}}(\boldsymbol{d}) = \boldsymbol{z}^+_{\mathrm{opt}}(\boldsymbol{d}) - \boldsymbol{z}_{\mathrm{opt}}(\boldsymbol{d} = \boldsymbol{0})$. We seek to quantify technical flexibility by means of the displacement of a characteristic eigenvalue $\delta\lambda = \lambda^+ - \lambda = \delta\sigma + j\delta\omega$ due to changes in behavior induced by the set point variation $\delta \boldsymbol{z}(\boldsymbol{d})$. Technical flexibility is then quantified on the basis of acceptable displacements of the chosen eigenvalue characterizing the changed linear dynamics. What "acceptable" means, needs to be defined a priori leading to the concept of reliability/flexibility metrics of interest.

*Remark 1:* The dominant eigenvalue, i.e. the one with the largest growth rate, could be a suitable choice for the characteristic eigenvalue, because it usually is most important for stabilization.

In the following we assume $\mathcal{R}$ to be twice continuously differentiable w.r.t. $\boldsymbol{z}$. Moreover, the multiplicities of the eigenvalues of $\boldsymbol{A}$ are maintained after the system matrix changes due to the effect of the steady forcing with $\boldsymbol{d}$.

## III. Quantification of Flexibility Based on Reliability Certificates

Rigorous definitions of reliability, the associated metrics, and technical flexibility of individual components are given. For this purpose, a formula for estimates of eigenvalue variations under nonlinear power flow changes is derived and we state an algorithmic procedure for quantification. We highlight problems towards multiple component and system-wide technical flexibility assessment.

### A. Reliability, Certificates, and Metrics

As emphasized in Section II, measuring and comparison of component flexibilities, here at hand of $\delta\lambda$, requires a metric to give a technical meaning to the distance $d(\lambda^+, \lambda)$. In the context of technical flexibility, as proposed in this paper, such a metric is determined by reliability; reliability is defined by a set of points $\lambda^+ \in \mathbb{C}$ that are admissible after steady state changes occur.

*Definition 1 (Reliability region, and reliability certificates):* Consider a steady state operational behavior of $\Sigma_{\mathrm{PS}}$ characterized by $\lambda \in \mathrm{LHP}$. A reliability region is given as set

$$\mathcal{C}_\alpha(\lambda) := \{s \in \mathbb{C} : \, d(\lambda, s) < \alpha, \quad \alpha > 0\}, \tag{7}$$

where the distance function $d(\lambda, s)$ is called reliability specification w.r.t. a steady state behavior, and the constant $\alpha$ is a level set value.

The small perturbation behavior of $\Sigma_{\mathrm{PS}}$ is called $\alpha$-reliable when the dominant eigenvalue $\lambda^+ \in \mathbb{C}$ is contained in $\mathcal{C}_\alpha(\lambda)$ after a (large enough) disturbance has induced a change of steady state. Then $\lambda^+ \in \mathcal{C}_\alpha(\lambda)$ is called reliability certificate.

That is, the boundary of a reliability region, denoted by $\partial\mathcal{C}_\alpha(\lambda)$, quantifies what we mean by reliable in terms of giving a quantitative sense to tolerated deviations $\delta\sigma$ and $\delta\omega$. For example, a reliability specification as in Fig. 2(a), tolerates only little deviations in the direction of the unstable region, but exhibits greater tolerance in the negative direction. This has the physical interpretation of having lower tolerance in those directions in the LHP, where the electromechanical mode becomes poorly dampened, see [12] and [16], and critical (driven) system resonances or even dynamic instability might occur, see [16] and [17].

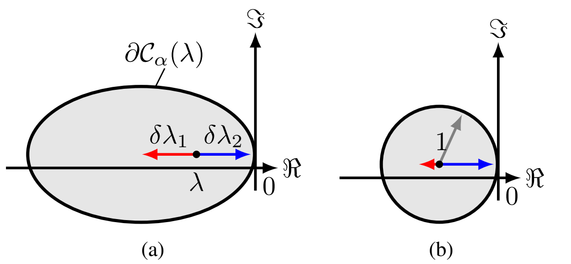


Fig. 2. Reliability regions and two eigenvalue deviations before (a) and after (b) scaling with $\boldsymbol{M}(\lambda_c)$; only the upper complex half plane is depicted due to the symmetry of the spectrum.

Specifying the boundary $\partial\mathcal{C}_\alpha(\lambda)$ as constant value reliability gives rise to a metric that characterizes $\partial\mathcal{C}_\alpha(\lambda)$.

*Definition 2 (Reliability/Flexibility metric):* Denote by $\lambda_c \in \partial\mathcal{C}_\alpha(\lambda)$ the points lying on a reliability boundary. A metric specifying constant levels of reliability $\partial\mathcal{C}_\alpha(\lambda)$ is a distance function $d(\cdot,\lambda)$ given by the inner product parameterized by weighting matrices $\boldsymbol{M}(\lambda_c) \succ 0$ such that for each $\lambda_c \in \partial\mathcal{C}_\alpha(\lambda)$ the complex number $\lambda_c - \lambda =: \delta\lambda_c$ satisfies

$$d(\lambda_c,\lambda) \triangleq ||\delta\lambda_c||_{\boldsymbol{M}} := \sqrt{\begin{pmatrix}\delta\sigma_c\\ \delta\omega_c\end{pmatrix}^T \begin{bmatrix} m_\sigma & 0\\ 0 & m_\omega\end{bmatrix}\begin{pmatrix}\delta\sigma_c\\ \delta\omega_c\end{pmatrix}} = 1. \tag{8}$$

Thus, defining in each direction how far a dominant eigenvalue is allowed to deviate determines a set of weighting matrices that set these distances point-wise to unity. Such a mapping, as depicted in Fig. 2(a) and Fig. 2(b), allows to compare deviations $\delta\lambda$ w.r.t. a reliability boundary/metric in terms of ratios $||\delta\lambda||/||\delta\lambda_c||$.

*Example:* Consider a disturbance that has effect $\delta\lambda_1$ when forcing the system at a component “1”, and the effect $\delta\lambda_2$ when forcing the system at a different component “2”, as illustrated in Fig. 2(a). Both deviations are of equal size when measured by the standard 2-norm, i.e. $||\delta\lambda_1|| = ||\delta\lambda_2||$. However, w.r.t. the reliability specification $\delta\mathcal{C}_\alpha(\lambda)$, which determines unique weighting matrices $\boldsymbol{M}_1, \boldsymbol{M}_2$, comparison of the two components in their technical flexibility yields $||\delta\lambda_1||_{\boldsymbol{M}_1} > ||\delta\lambda_2||_{\boldsymbol{M}_2}$, as depicted in Fig. 2(b). We therefore have less technical flexibility in component “1” than in component “2”, because the eigenvalue deviation is greater w.r.t. the specified metric.

### B. Estimating Eigenvalue Displacements

The difference $\delta\lambda$ can be approximated to first order by the first variation of the function $\lambda$ at point $\boldsymbol{z}_{\text{opt}}$ with respect to the input $\boldsymbol{d}$. The first variation is mathematically represented by the Gâteaux differential

$$\delta\lambda(\boldsymbol{z}_{\text{opt}};\boldsymbol{d}) = \lim_{\tau\to 0}\frac{\lambda^+(\boldsymbol{z}_{\text{opt}} + \tau\delta\boldsymbol{z}_{\text{opt}}(\boldsymbol{d})) - \lambda(\boldsymbol{z}_{\text{opt}})}{\tau} \tag{9a}$$
$$= \tau\,\langle \boldsymbol{S}_{\boldsymbol{d}}(\lambda), \boldsymbol{d}\rangle \tag{9b}$$

where $\boldsymbol{S}_{\boldsymbol{d}}$ is a sensitivity vector for $\lambda$ called Gâteaux derivative. It represents the direction of the steady system response measured by $\delta\lambda$ with respect to the steady external disturbance $\boldsymbol{d}$. The estimate of the eigenvalue displacement (9a) has a linear dependence in $\boldsymbol{d}$ via $\boldsymbol{S}_{\boldsymbol{d}}$. The nonlinearity of $\delta\lambda$, which originates in the change of set point, is contained in the Gâteaux derivative. The sensitivity vector can be analytically obtained from a Lagrangian approach known in constrained optimization. Given the pre-disturbance eigenvalue $\lambda$, the objective is to find the value $\lambda(\boldsymbol{A}(\boldsymbol{z}^+_{\text{opt}}))$ that is most distant to the initial eigenvalue, i.e. $||\delta\lambda||$ is maximized, whereat we require that $\lambda(\boldsymbol{A}(\boldsymbol{z}^+_{\text{opt}}))$ satisfies the post-disturbance eigenvalue problem (5); this constraint optimization problem yields a possible Lagrangian function of the form

$$\mathcal{L} = ||\delta\lambda|| - \left\langle \boldsymbol{w}, \left[\lambda^+\boldsymbol{B} - \boldsymbol{A}(\boldsymbol{z}^+_{\text{opt}}(\boldsymbol{d}))\right]\boldsymbol{v}\right\rangle, \tag{10}$$

where $\boldsymbol{w}$ is a vector of Lagrange multipliers which coincides with the left-eigenvector associated to $\lambda$.

Applying necessary optimality conditions, see [18] for full details, one finds that the displacement $\delta\lambda$ w.r.t. changes in the stationary power flow solution $\delta\boldsymbol{z}_{\text{opt}}$ computes via an eigenvalue sensitivity vector $\boldsymbol{\mathcal{S}}_{\boldsymbol{z}}$ as

$$\delta\lambda = \langle \boldsymbol{\mathcal{S}}_{\boldsymbol{z}}, \delta\boldsymbol{z}_{\text{opt}}\rangle, \quad \boldsymbol{S}_{\boldsymbol{z}} = \left[\nabla_{\boldsymbol{z}}\left[\boldsymbol{A}(\boldsymbol{z}_{\text{opt}})\boldsymbol{v}\right]\big|_{\boldsymbol{z}_{\text{opt}}}\right]^*\boldsymbol{w}. \tag{11}$$

Here one has to use the normalization condition $\langle\boldsymbol{w},\boldsymbol{B}\boldsymbol{v}\rangle = 1$, see [18] for proof and detailed derivation.

Based on (6) we can directly relate $\delta\lambda$ to a disturbance $\boldsymbol{d}$. Assuming the disturbance to be small and writing it as $\kappa\boldsymbol{d}, \kappa > 0, ||\boldsymbol{d}||_2 = 1$, the relation between $\delta\boldsymbol{z}_{\text{opt}}$ and $\kappa\boldsymbol{d}$ computes from (6) by taking differentials, i.e.

$$\nabla_{\boldsymbol{z}}\mathcal{R}(\boldsymbol{z})\big|_{\boldsymbol{z}_{\text{opt}}}\delta\boldsymbol{z}_{\text{opt}} + \kappa\boldsymbol{d} = \boldsymbol{0} \tag{12a}$$
$$\Rightarrow\quad \delta\boldsymbol{z}_{\text{opt}} = -\left[\nabla_{\boldsymbol{z}}\mathcal{R}(\boldsymbol{z})\big|_{\boldsymbol{z}_{\text{opt}}}\right]^{-1}\kappa\boldsymbol{d}. \tag{12b}$$

Then, substitution of $\delta\boldsymbol{z}_{\text{opt}}$ in (11) using expression (12b) yields

$$\delta\lambda = \left\langle \boldsymbol{S}_{\boldsymbol{z}}(\lambda), -\left[\nabla_{\boldsymbol{z}}\mathcal{R}(\boldsymbol{z})\big|_{\boldsymbol{z}_{\text{opt}}}\right]^{-1}\kappa\boldsymbol{d}\right\rangle \tag{13a}$$
$$= \kappa\left\langle -\left(\left[\nabla_{\boldsymbol{z}}\mathcal{R}(\boldsymbol{z})\big|_{\boldsymbol{z}_{\text{opt}}}\right]^{-1}\right)^*\boldsymbol{S}_{\boldsymbol{z}}(\lambda), \boldsymbol{d}\right\rangle \tag{13b}$$

from where we obtain the sought eigenvalue sensitivity with respect to the disturbance vector $\boldsymbol{d}$ as

$$\boldsymbol{S}_{\boldsymbol{d}} = -\left(\left[\nabla_{\boldsymbol{z}}\mathcal{R}(\boldsymbol{z})\big|_{\boldsymbol{z}_{\text{opt}}}\right]^{-1}\right)^*\boldsymbol{S}_{\boldsymbol{z}}(\lambda). \tag{14}$$

### C. Defining Technical Flexibility and Algorithmic Procedure

Reliability can be described by specifying the region of interest $\mathcal{C}_\alpha$. We quantify technical flexibility of a component according to the allowable disturbance magnitude at the component of choice.

*Definition 3 (Technical flexibility of one component):* Consider a reliability specification in terms of a specified set $\mathcal{C}_\alpha(\lambda)$, and the index $i$ for the component of interest described by $z_i$. The technical flexibility of the $i$-th component w.r.t. a power injection $\boldsymbol{d} = \kappa \boldsymbol{e}_i, \kappa > 0$ is the value $\mathrm{flex}_\mathrm{i}^+$ obtained as

$$\mathrm{flex}_\mathrm{i}^+ = \arg\max_{\kappa} \kappa \boldsymbol{e}_\mathrm{i} \quad \text{s.t.} \quad \lambda + \delta\lambda \in \mathcal{C}_\alpha(\lambda). \tag{15}$$

The technical flexibility of the $i$-th component w.r.t. a power drop $\boldsymbol{d} = \kappa \boldsymbol{e}_i, \kappa < 0$ is the value $\mathrm{flex}_\mathrm{i}^-$ obtained as

$$\mathrm{flex}_\mathrm{i}^- = \arg\min_{\kappa} \kappa \boldsymbol{e}_\mathrm{i} \quad \text{s.t.} \quad \lambda + \delta\lambda \in \mathcal{C}_\alpha(\lambda). \tag{16}$$

The respective deviation $\delta\lambda$ is as defined in (13).

The maximal component flexibilities $\mathrm{flex}_\mathrm{i}^\pm$ can directly be compared against each other, because they are obtained as those values where the eigenvalue deviation first hits the constant-value reliability boundary. An algorithm to compute these quantities is as follows:

1) Choose a detailed enough generator model, and compute a steady state operational point $\boldsymbol{z}_\mathrm{opt}$ according to (3).
2) Setup the system matrix $\boldsymbol{A}(\boldsymbol{z}_\mathrm{opt})$ of the linearized dynamics according to (4).
3) Compute the eigendata of interest $(\lambda, \boldsymbol{v}, \boldsymbol{w})$ by solving the generalized eigenvalue problem (5) and its adjoint version.
4) Define a reliability region $\mathcal{C}_\alpha(\lambda) \Leftrightarrow$ metric $\boldsymbol{M}(\lambda_c)$.
5) Compute the sensitivity field $\boldsymbol{S_d}$ as in (14) via the sensitivity $\boldsymbol{S_z}$ given in (11).
6) Choose a component $i$ of interest, set $\boldsymbol{d} = \kappa \boldsymbol{e}_i$ and compute $\mathrm{flex}_\mathrm{i}^\pm$ according to Def. 3.

### D. Towards Multiple Component and Whole System Flexibility

Multiple component or system flexibility could be defined using a vector of component flexibilities. Then, to obtain a single measure similar to FLEX, one has to apply again a norm on this vector. It is not clear what a suitable norm with associated metric could be. Moreover, the issue of directionality arises, since there might be several combinations of differing component flexibilities that lead to the same value of system flexibility. In that context system flexibility is best characterized by a volume in parameter space of component flexibilities, where certain directions might be more sensitive than others. These phenomena are known in multivariable systems and lead to problems in multi-criteria optimization and control assignment, where useful trade-offs have to be explored.

## IV. Conclusion

In this paper we develop a sensitivity-based framework to quantify technical flexibility which has the power to include limitations from transmission system and component dynamics in flexibility studies. By that, we close a methodological gap between existing operational schemes considering either dynamics or static dispatch planning. Our method relies on eigenvalue computations and on the information contained in the Jacobian of the residual being related to set point changes. We raised the issue of defining suitable metrics and put it on mathematical grounds in contrast to existing works where these terms are used in a rather casual sense. This framework is to be understood as a first step in the direction of technical flexibility assignment. Crucial extensions are to overcome smallness of changes, or to define a similar framework using different dynamic stability methods, for instance based on energy function methods. A subsequent work is in progress where this method is applied to an IEEE test case. For verification and analysis of limitations of the proposed framework, application to real data from stressed power system situations should be considered, as well as testing in large-scale numerical simulations.

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