Effect of different inspection strategies on the reliability of Daniels systems subjected to fatigue

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ABSTRACT: Inspections are an efficient means of enhancing the reliability of redundant structural systems subjected to fatigue. To investigate the effect of such inspections, we represent the deterioration state of a Daniels system by means of a probabilistic fatigue crack growth model of all elements, which considers stochastic dependence among element fatigue behavior. We include inspection results in the calculation of the system collapse probability through Bayesian updating of the system deterioration state. Based on this approach, we calculate the collapse probability of a deteriorating Daniels system conditional on different inspection strategies in terms of inspection coverage and inspection times. The acceptability of an inspection strategy is verified by comparing the calculated collapse probabilities with maximum acceptable system failure probabilities. This study is a step towards identifying optimal inspection strategies for redundant structural systems subjected to fatigue.

1 INTRODUCTION

Inspections are performed to reduce the uncertainty on the actual condition of deteriorating structures. In the context of fatigue deterioration, inspections are carried out to detect and measure the size of possible fatigue cracks. Inspection results form the basis for decisions on repair and maintenance actions.

In the past, reliability and risk-based methods have been developed to identify optimal inspection and maintenance strategies for structures subjected to fatigue, e.g. Skjong (1985), Madsen et al. (1987), Thoft-Christensen and Sørensen (1987), Fujita et al. (1989), Sørensen et al. (1991), Moan et al. (2000b), Faber et al. (2000) and Straub (2004). These methods have mainly focused on individual structural components neglecting the effect of stochastic dependence among element deterioration behavior because of computational limitations, see Straub and Faber (2005) for more details.

The effect of stochastic dependence among element deterioration behavior has been considered in some studies, e.g. Moan and Song (2000). Straub and Faber (2005) present a risk-based method for planning inspections on system level taking into account stochastic dependence among element deterioration behavior.

Recently, Straub and Der Kiureghian (2011) have proposed a method for computing the reliability of deteriorating structures accounting for the structural redundancy as well as the joint effect of deterioration failure of different elements. The proposed method determines the reliability of deteriorating redundant structures by easily computable idealized structural systems, which consist of a series system of Daniels subsystem.

Motivated by this model, the current study investigates the effect of different inspection strategies in terms of inspection coverage and inspection times on the reliability of Daniels systems. This represents a step towards determining risk-based optimal inspection strategies for redundant structural systems subjected to fatigue.

2 RELIABILITY OF DANIELS SYSTEMS SUBJECTED TO FATIGUE

A Daniels system, shown Figure 1, is an idealized representation of a redundant structural system (Daniels 1945). It consists of n elements with independent and identically distributed (iid) element capacities R_i, $i = 1, \ldots, n$, and perfect equal load sharing among the elements.

In agreement with Straub and Der Kiureghian (2011), we make two main assumptions to determine the reliability of a Daniels system subjected to fatigue. The first assumption is that, on a system level, we do not consider gradual degradation of element capacities, i.e. at any time t, an element has either its full capacity or has completely lost its capacity because of fatigue failure. At any time t there are

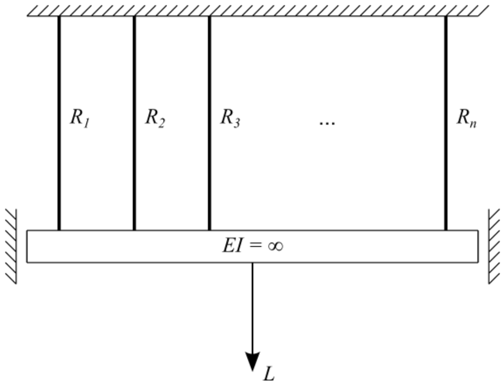

Figure 1. Daniels system with n elements.

$N_F(t)$ failed elements and $n - N_F(t)$ number of elements are available to resist the applied load. Consequently, $N_F(t)$ represents the deterioration state of a Daniels system.

The second assumption is that the deterioration state of a Daniels system is considered constant over a period $\Delta t = 1$ year. Conservatively, the system deterioration state in the period $[t - \Delta t, t]$ is set equal to the state at time t, $N_F(t)$. The event of collapse of a Daniels system in that period is denoted by $C(t)$. The probability of this event is given by the total probability theorem as

$$p_F(t) = \Pr\big(C(t)\big) = \sum_{j=0}^{n} \Pr(C(t)|N_F(t) = j)\Pr(N_F(t) = j) \quad (1)$$

where $\Pr(C(t)|N_F(t) = j)$ is the probability of collapse given that j elements have failed at time t due to fatigue and $\Pr(N_F(t) = j)$ is the probability that j elements have failed at time t due to fatigue.

$\Pr(C(t)|N_F(t) = j)$ can be determined from the Daniels system formulation. Assuming ductile element behavior and a constant distribution of the annual maximum load L, the solution is (Gollwitzer and Rackwitz 1990):

$$\Pr(C(t)|N_F(t) = j) = \Pr\left(\sum_{i=1}^{n-j} R_i - L \le 0\right) \quad (2)$$

which can be calculated by means of structural reliability methods, e.g. the first-order reliability method. The difference between ductile and brittle element behavior is shown in Straub and Der Kiureghian (2011).

$\Pr(N_F(t) = j)$ depends on the element fatigue reliability and the dependence among element fatigue behavior. We determine $\Pr(N_F(t) = j)$ as described in Section 4 by means of the probabilistic fatigue crack growth model presented in the next section.

3 PROBABILISTIC FATIGUE DETERIORATION MODELING

3.1 *Fatigue crack growth model*

In the current study we adopt a simple fatigue crack growth model based on Paris' law to describe fatigue deterioration of an element; Equation (3). This approach assumes that there is only one hot spot per element.

$$\frac{da}{dN} = C\left(B_{SIF}B_{\Delta S}\Delta S_e\sqrt{\pi a}\right)^m \quad (3)$$

a is the crack size, N is the number of stress cycles, ΔS_e is the equivalent stress range, C and m are empirical model parameters and $B_{\Delta S}$ and B_{SIF} are correction factors of the fatigue stress ranges and the stress intensity factor (SIF) range to account for the model uncertainty, discussed in Section 3.2. In the adopted formulation of Paris' law the geometry correction factor is one, which corresponds in theory to a through thickness crack in an infinite plate.

The equivalent stress range is $\Delta S_e = (E[\Delta S^m])^{1/m}$ where $E[\Delta S^m]$ is the expected value of the random fatigue stress ranges ΔS to the mth power; see e.g. (Madsen 1997). We assume that the long-term distribution of ΔS can be modeled by a Weibull distribution. ΔS_e is hence given by:

$$\Delta S_e = (E[\Delta S^m])^{1/m} = k\Gamma\left(1 + \frac{m}{\lambda}\right)^{1/m} \quad (4)$$

$\Gamma(.)$ denotes the Gamma function and k and λ are the Weibull scale and shape parameters.

Solving Equation (3) gives the crack size a as a function of the time t:

$$a(t) = \left[a_0^{\left(1-\frac{m}{2}\right)} + \left(1 - \frac{m}{2}\right) C B_{SIF}^m B_{\Delta S}^m \Delta S_e^m \pi^{\frac{m}{2}} \nu t\right]^{\left(1-\frac{m}{2}\right)^{-1}} \quad (5)$$

where a_0 is the initial crack size, ν is the cycle rate and νt is the number of stress cycles in the period $[0, t]$.

3.2 *Uncertainty in fatigue crack growth modeling*

The prediction of fatigue crack growth is subjected to significant uncertainty due to the simplistic representation of the actual physical phenomenon, the inherent variability of the underlying parameters and the limited information on those parameters.

To account for the uncertainties underlying the modeling of fatigue deterioration in each element i, the crack growth parameter C_i (uncertainty in material characteristics), the initial crack size $a_{0,i}$ (uncertainty in fabrication quality), the scale and shape parameters of the Weibull distributed stress ranges k_i and λ_i (statistical uncertainty in the calculation of the fatigue stress ranges), the correction factor $B_{\Delta S,i}$ (model uncertainty in the calculation of hot spot stress ranges) and the correction factor $B_{SIF,i}$ (model uncertainty in the calculation of the SIF ranges) are modeled as random variables.

The basic random variables that describe the fatigue deterioration of the n elements of a Daniels system are collected in the vector **X**:

$$\mathbf{X} = \left[\ln C_1, a_{0,1}, \ln k_1, \lambda_1, B_{\Delta S,1}, B_{SIF,1}, \dots, \ln C_n, a_{0,n}, \ln k_n, \lambda_n, B_{\Delta S,n}, B_{SIF,n}\right]^T$$

3.3 *Statistical dependence among element fatigue behavior*

The fatigue deterioration behavior of individual elements of a structural system is generally statistically dependent due to common uncertain influencing factors, such as environmental conditions and material characteristics. There is only limited information available on modeling stochastic dependence of element deterioration; e.g. for steel structures subject to fatigue, Vrouwenvelder (2004) estimated the dependence of fatigue deterioration among hot spots as a function of weld quality by comparing the scatter in fatigue performance within one production series to the scatter in the overall population. In general, however, it is required to estimate the stochastic dependence of element deterioration based on engineering judgment.

In the current study, the initial crack sizes are partly correlated among all elements with correlation coefficient ρ_a. This represents the statistical dependence due to common fabrication quality. The crack growth parameters are partly correlated with correlation coefficient $\rho_{\ln C}$, which reflects the statistical dependence due to common material characteristics. The scale parameters of the Weibull distributed stress ranges are partly correlated with correlation coefficient $\rho_{\ln k}$, representing the statistical dependence due to common loading characteristics. Finally, the correction factors $B_{\Delta S}$ and B_{SIF} are fully correlated among all elements. This approach is based on a correlation model suggested in Moan and Song (2000).

The correlation coefficients of the basic random variables **X** are summarized in the correlation matrix $\mathbf{R_{XX}} = \left[\rho_{X_k X_l}\right]_{r\times r}$ where r is the length of **X**.

3.4 *Fatigue failure event of an individual structural element*

The event of fatigue failure of element i at time t, $F_i(t)$, is defined by the limit state function $g_i(\mathbf{x},t)$ as $F_i(t) = \{g_i(\mathbf{x},t) \le 0\}$, where **x** is a realization of **X**. We assume that fatigue failure of element i occurs if the crack size a_i exceeds a critical crack size $a_{c,i}$, see e.g. (Madsen 1997). The limit state function $g_i(\mathbf{x},t)$ can in this case be written as:

$$g_i(\mathbf{x},t) = a_{c,i} - a_i(\mathbf{x},t) \tag{6}$$

$a_i(\mathbf{x},t)$ is defined according to Equation (5).

4 ESTIMATION OF THE PROBABILITY OF THE NUMBER OF ELEMENTS FAILED DUE TO FATIGUE DETERIORATION

In the current study we determine the probability that $j = 0, \dots, n$ elements have failed at time t due to fatigue deterioration, $\Pr(N_F(t) = j)$, indirectly based on the exceedance probabilities $\Pr(N_F(t) \ge j)$, $j = 1, \dots, n$. The event $\{N_F(t) \ge j\}$ is defined by the limit state function $g_j(\mathbf{x},t)$ so that $\{N_F(t) \ge j\} = \{g_j(\mathbf{x},t) \le 0\}$. The limit state function $g_j(\mathbf{x},t)$ can be written as:

$$g_j(\mathbf{x},t) = j - \sum_{i=1}^{n} I_i(\mathbf{x},t) \tag{7}$$

$I_i(\mathbf{x},t)$ is the indicator function: $I_i(\mathbf{x},t) = 1$ if $g_j(\mathbf{x},t) \le 0$ and $I_i(\mathbf{x},t) = 0$ otherwise.

The probability of the event $\{N_F(t) \ge j\}$ can be computed by integrating the joint PDF $f_{\mathbf{X}}(\mathbf{x})$ over the domain $\{g_j(\mathbf{x},t) \le 0\}$.

$$\Pr(N_F(t) \ge j) = \int_{g_j(\mathbf{x},t)\le 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{8}$$

In the current study we evaluate the integral of Equation (8) using subset simulation, which is an adaptive Monte Carlo method proposed by Au and Beck (2001).

For the purpose of applying subset simulation the basic random variables **X** are transformed to uncorrelated standard normal variables **U** using a bijective transformation T, i.e. $\mathbf{U} = T(\mathbf{X})$. Following Liu and Der Kiureghian (1986), the transformation T can be performed utilizing the Nataf model which approximates the joint PDF $f_{\mathbf{X}}(\mathbf{x})$ based on the marginal distributions of the elements of **X** and the correlation matrix $\mathbf{R_{XX}}$.

The limit state function $g_j(\mathbf{x},t)$ is replaced by the corresponding limit state function $G_j(\mathbf{u},t)$ defined as

$$G_j(\mathbf{u},t) = g_j(T^{-1}(\mathbf{u}),t) \tag{9}$$

The integral of Equation (8) can now be expressed in standard normal space

$$\Pr(N_F(t) \ge j) = \int_{G_j(\mathbf{u},t)\le 0} \varphi_{\mathbf{U}}(\mathbf{u}) d\mathbf{u} \tag{10}$$

where $\varphi_{\mathbf{U}}(.)$ is the multivariate uncorrelated standard normal joint PDF.

When applying subset simulation to compute the probability $\Pr(N_F(t) \geq j)$, intermediate events $E_1, \ldots, E_m$ are defined as:

$$E_k = \{G_j(\mathbf{u}, t) \leq c_k\}, \qquad k = 1, \ldots, m \tag{11}$$

where c_k are defined such that $c_1 > \cdots > c_m = 0$. It follows that $E_1 \supset \cdots \supset E_m = \{N_F(t) \geq j\}$. The probability $\Pr(N_F(t) \geq j)$ is then obtained as:

$$\begin{aligned}\Pr(N_F(t) \geq j) &= \Pr\left(\bigcap_{k=1}^{m} E_k\right) \\ &= \Pr(E_1) \prod_{k=2}^{m} \Pr(E_k | E_{k-1})\end{aligned} \tag{12}$$

where $\Pr(E_k|E_{k-1})$ is the conditional probability of E_k given E_{k-1}. With a suitable choice of the constants c_k, $k = 1, \ldots, m$, the conditional probabilities $\Pr(E_k|E_{k-1})$ can be made sufficiently large so that they can be estimated using crude MCS. This means that the original problem of evaluating the small probability of the rare event $\{N_F(t) \geq j\}$ reduces to computing a sequence of m larger conditional probabilities.

The probability $\Pr(E_1)$ is computed using crude MCS through sampling from $\varphi_{\mathbf{U}}(\mathbf{u})$. The probabilities $\Pr(E_k|E_{k-1})$ are also estimated using MCS but the samples required for estimating the probabilities $\Pr(E_k|E_{k-1})$, $k = 2, \ldots, m$, are generated from conditional PDFs $\varphi_{\mathbf{U}}(\mathbf{u}|E_{k-1})$, $k = 2, \ldots, m$. This is achieved by means of the Markov Chain Monte Carlo (MCMC) method. In the current study we apply an adaptive MCMC algorithm proposed by Papaioannou et al. (2012).

For estimating the probabilities $\Pr(N_F(t) \geq j)$, $j = 1, \ldots, n$, we make use of the fact that $\{N_F(t) \geq j\} \supset \{N_F(t) \geq j-1\}$. We start by calculating $\Pr(N_F(t) \geq 1)$ using subset simulation. Subsequently, we use the samples conditional on the event $\{N_F(t) \geq 1\}$ as seeds for generating the initial samples for computing the conditional probability $\Pr(N_F(t) \geq 2 | N_F(t) > 1)$ using the MCMC sampling approach. Finally, it is $\Pr(N_F(t) \geq 2) = \Pr(N_F(t) \geq 2 | N_F(t) \geq 1) \Pr(N_F(t) \geq 1)$. We continue this process until all probabilities $\Pr(N_F(t) \geq j)$, $j = 1, \ldots, n$, are estimated.

5 SYSTEM RELIABILTY UPDATING WITH INSPECTION DATA

5.1 *Inspection modeling*

In the context of fatigue deterioration, the relevant inspection events are detection/no-detection of a fatigue crack and measurement of the size of a detected fatigue crack (Straub 2004). In the current study, we assume that any inspection of any element i results in a no-detection event. (Otherwise the element would be repaired or replaced.)

The event of no-detection of a fatigue crack with size a at element i during an inspection at time t_{insp} is denoted as $\overline{D}_i(t_{insp})$. It is expressed in terms of a limit state function $g_{\overline{D},i}(\mathbf{x}_+, t_{insp})$ such that $\overline{D}_i(t_{insp}) = \{g_{\overline{D},i}(\mathbf{x}_+, t_{insp}) \leq 0\}$. $g_{\overline{D},i}(\mathbf{x}_+, t_{insp})$ is defined in accordance with Hong (1997):

$$g_{\overline{D},i}(\mathbf{x}_+, t_{insp}) = PoD\left(a_i(\mathbf{x}, t_{insp})\right) - p_{i,insp} \tag{13}$$

$p_{i,insp}$ is a realization of the random variable $P_{i,insp}$, which is uniformly distributed in the interval [0,1], $\mathbf{x}_+$ is a realization of the augmented vector of basic random variables $\mathbf{X}_+ = [\mathbf{X}, P_{i,insp}]^T$, $PoD(a)$ is the probability of detection of a fatigue crack of size a.

PoD curves model the quality of inspection methods, which are generally not exact due to uncertain factors such as measurement errors, inspector performance and environmental conditions. In the current study we adopt an exponential PoD model in accordance with Moan et al. (2000a):

$$PoD(a) = 1 - \exp(-a/\lambda_D) \tag{14}$$

Since fatigue deterioration of individual elements is generally interdependent due to common uncertain influencing factors, it is possible to infer the condition of uninspected elements from inspection results obtained at other elements. In the following we refer to the set of elements inspected at inspection time t_{insp} as the inspection sample $S(t_{insp})$.

All inspection results obtained within the period $[0, t]$ are expressed by an event $Z(t)$ as follows:

$$Z(t) = \bigcap_{insp=1}^{n_{insp}} \left(\bigcap_{i \in S(t_{insp})} \overline{D}_i(t_{insp}) \right) \tag{15}$$

n_{insp} is the number of inspections that are performed within the period $[0, t]$. The event $Z(t)$ is described in terms of a limit state function $g_Z(\mathbf{x}_+, t)$ such that $Z(t) = \{g_Z(\mathbf{x}_+, t) \leq 0\}$. The limit state function $g_Z(\mathbf{x}_+, t)$ is defined as a combination of the individual limit state functions $g_{\overline{D},i}(\mathbf{x}_+, t_{insp})$, which follows from system reliability theory; see e.g. (Madsen et al. 1986).

$$g_Z(\mathbf{x}_+, t) = \max_{1 \leq insp \leq n_{insp}} \left(\max_{i \in S(t_{insp})} g_{\overline{D},i}(\mathbf{x}_+, t_{insp}) \right) \tag{16}$$

Note that for every inspected element i a new uniformly distributed variable $P_{i,insp}$ is added to the augmented vector of basic random variables $\mathbf{X}_+$ following Equation (13).

5.2 *Updating of the system collapse probability with inspection data*

We include information on deteriorating elements obtained through inspections in the calculation of the system collapse probability $p_F(t)$ through Bayesian updating of $\Pr(N_F(t) = j)$. The updated probability that j elements have failed at time t due to fatigue deterioration given the inspection event $Z(t)$ is defined as

$$\Pr(N_F(t) = j|Z(t)) = \frac{\Pr(N_F(t) = j \cap Z(t))}{\Pr(Z(t))} \quad (17)$$

For estimating the conditional probability $\Pr(N_F(t) = j|Z(t))$ we adopt the following procedure: After every inspection performed at t_{insp} the probability of the inspection event $\Pr(Z(t))$ is calculated using subset simulation. The samples conditional on the event $Z(t)$ are subsequently used as seeds for generating the initial samples for estimating the exceedance probabilities $\Pr(N_F(t) \geq j|Z(t))$, $j = 1, \dots, n$, for every year $t > t_{insp}$ also using subset simulation as described in Section 4. This procedure is repeated every time new inspection data are available.

The probability of collapse of a Daniels system is conditionally independent of the inspection event $Z(t)$ given the number of elements with fatigue failure $N_F(t)$. It follows that the updated probability of collapse of a Daniels system is given by

$$\begin{aligned} p_F(t) &= \Pr(C(t)|Z(t)) \\ &= \sum_{j=0}^{n} \Pr(C(t)|N_F(t) = j) \Pr(N_F(t) = j|Z(t)) \end{aligned} \quad (18)$$

6 CASE STUDY

In the following we consider a Daniels system with $n = 4$ elements with ductile element behavior. In accordance with Straub and Der Kiureghian (2011) the applied maximum annual load L is modeled by a log-normal distribution with coefficient of variation (CoV) $\delta_L = 0.25$ and the element capacities R_i are modeled as iid normal distributions with CoV $\delta_R = 0.15$. We select the ratio of the mean values of nR_i and L such that in its undamaged state the Daniels system has a probability of failure $\Pr(C(t)|N_F(t) = 0) = 5 \cdot 10^{-6}$. This value has a reference period $\Delta t = 1$ year but it is independent of time t. The resulting ratio is $n\mu_{R_i}/\mu_L = 3.04$. $\Pr(C(t)|N_F(t) = j)$ is calculated using Equation (2) and shown in Figure 2.

The probabilistic models of the fatigue parameters are listed in Table 1 for all four elements. These values are in accordance with Moan and Song (2000). Table 1 also lists the value of the PoD parameter λ_D (see Equation (14)), which is selected so that the PoD model is representative for magnetic particle inspection; see (Moan et al. 2000a).

To investigate the influence of stochastic dependence among element fatigue behavior we consider three dependence cases (low, medium, high), which are defined in terms of the correlation coefficients

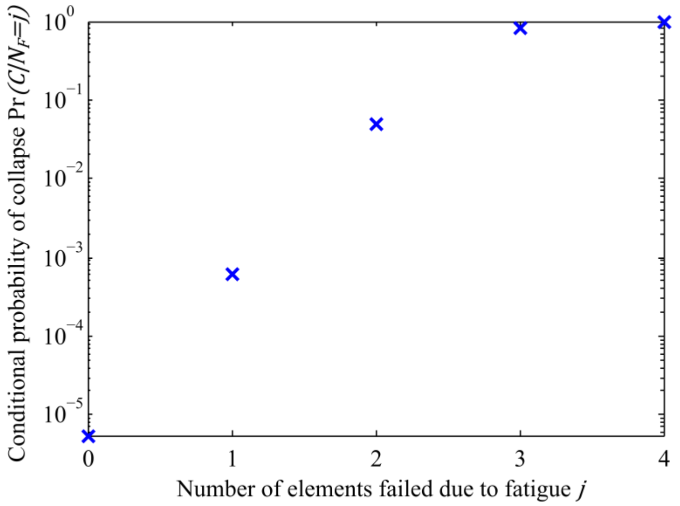

Figure 2. Conditional probability $\Pr(C(t)|N_F(t) = j)$ for a Daniels system with $n = 4$ elements.

Table 1. Parameters of the fatigue deterioration model and PoD model.

Parameter	Dimension	Distribution	Values
$\ln k$: scale parameter of Weibull distributed stress range ΔS	corresponding to N/mm^2	normal	$\mu = 1.6$ $\sigma = 0.22$
λ: shape parameter of Weibull distributed stress range ΔS	-	normal	$\mu = 0.8$ $\sigma = 0.08$
ν: stress cycle rate	stress cycles per year	determin.	$\nu = 5 \cdot 10^6$
t_{SL}: service life	years	determin.	$t_{SL} = 50$
a_c: critical crack size	mm	determin.	$a_c = 10$
a_0: initial crack size	mm	exponential	$\mu = 0.11$
m: crack growth parameter	-	determin.	$m = 3.1$
$\ln C$: crack growth parameter	corresponding to N und mm	normal	$\mu = -29.97$ $\sigma = 0.5095$
$B_{\Delta S}$: stress range correction factor (model uncertainty)	-	log-normal	$\mu = 1.0$ $\sigma = 0.1$
B_{SIF}: SIF range correction factor (model uncertainty)	-	log-normal	$\mu = 1.0$ $\sigma = 0.1$
λ_D: Parameter of PoD model	mm	determin.	$\lambda_D = 1.95$

Table 2. Correlation coefficients among the parameters of the element fatigue models.

	"low dependence"	"medium dependence"	"high dependence"
ρ_a	0.0	0.3	0.8
$\rho_{\ln C}$	0.0	0.3	0.8
$\rho_{\ln k}$	0.0	0.3	0.8

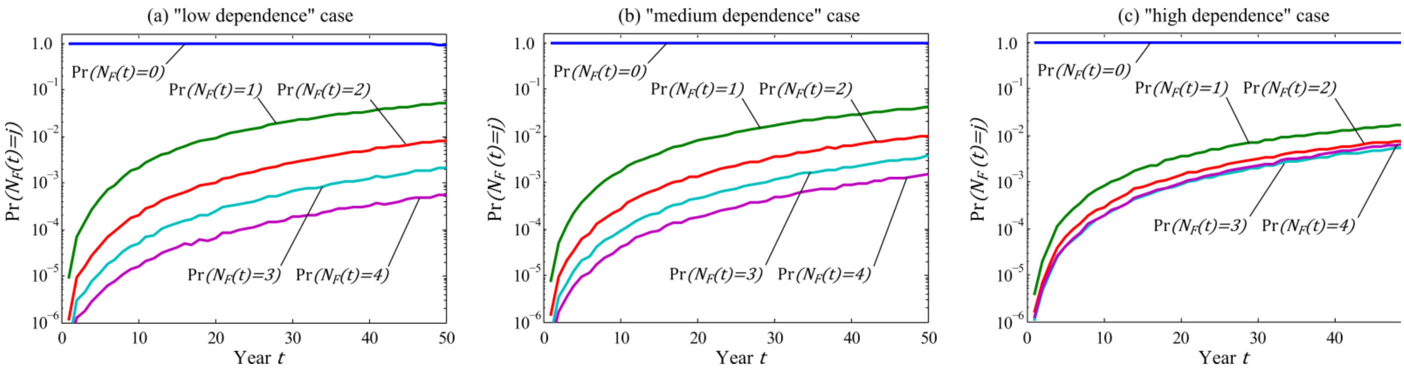

Figure 3. Probability that j elements have failed at time t due to fatigue deterioration, $\Pr(N_F(t) = j)$, as a function different degrees of dependence among element fatigue behavior.

ρ_a, $\rho_{\ln C}$ and $\rho_{\ln k}$ as listed in Table 2. Note that the stress range correction factor $B_{\Delta S}$ and the SIF range correction factor B_{SIF} are considered as fully correlated among all elements (see Section 3.3).

Subset simulation is implemented with 10^4 samples and $\Pr(E_k|E_{k-1}) = 0.1$. The results are presented in Figure 3.

As expected, the probability $\Pr(N_F(t) = 0)$ is close to one for all three dependence cases, but decreases with time t. The influence of the dependence among element deterioration can also be seen. The results show that the probability $\Pr(N_F(t) = 1)$ reduces and the probabilities $\Pr(N_F(t) = 3)$ and $\Pr(N_F(t) = 4)$ increase with increasing degree of dependence among element deterioration. This is explained with an increase of the probability of joint occurrence of several element fatigue failures with increasing degree of dependence. These findings are supported by the values listed in Table 3, which summarizes the probabilities that j elements have failed at time $t = 50$ years.

The calculated probabilities $\Pr(C(t)|N_F(t) = j)$ and $\Pr(N_F(t) = j)$, $j = 0, \ldots, n$, are utilized to compute the collapse probability $p_F(t)$ using Equation (1). The results are illustrated in Figure 4.

As expected, the collapse probability $p_F(t)$ increases with time t. The collapse probability $p_F(t)$ increases with increasing degree of dependence among element fatigue behavior. At the end of the service life, it is $p_F(t = 50yr) = 2.6 \cdot 10^{-3}$ for the "low dependence" case, $p_F(t = 50yr) = 5.1 \cdot 10^{-3}$ for the "medium dependence" case and $p_F(t = 50yr) = 1.2 \cdot 10^{-2}$ for the "high dependence" case. The increase of the collapse probability with increasing degree of dependence among element fatigue behavior is expected in a redundant system.

Table 3. Probability that j elements have failed at time $t = 50$ years due to fatigue deterioration as a function of different degrees of dependence among element fatigue behavior.

j	$\Pr(N_F(t = 50yr) = j)$		
	"low dependence" case	"medium dependence" case	"high dependence" case
0	0.939	0.944	0.962
1	5.1E-02	4.1E-02	1.7E-02
2	8.2E-03	9.9E-03	8.2E-03
3	2.0E-03	3.7E-03	5.6E-03
4	5.4E-04	1.5E-04	6.7E-03

To investigate the effect of different inspection strategies on the reliability of Daniels systems subjected to fatigue, we first consider the case where inspections are performed at a constant interval $\Delta t_{insp} = 10$ years. We consider different cases of inspection coverage such that the inspection sample is $S = \{1\}$, $\{1,2\}$, $\{1,2,3\}$ or $\{1,2,3,4\}$ at every inspection.

The collapse probability of the Daniels system $p_F(t)$ conditional on the different inspection strategies in terms of inspection times and inspection coverage are calculated according to Section 5. The results are presented in Figure 5. The fact that the collapse probability $p_F(t)$ reduces after every inspection is due to the assumption that every inspection results in a no-detection event (see Section 5.1).

As discussed in Section 5.1, due to the dependence among element fatigue behavior it is possible to infer the condition of uninspected elements from inspections results obtained at other elements. The indirect information of an inspection about the condition of uninspected elements increases not only with increasing number of inspected elements but also with increasing dependence among element fatigue behavior. When comparing the different degrees of dependence among element deterioration behavior in Figure 5, it can be seen that the relative reduction in collapse probability is largest for the

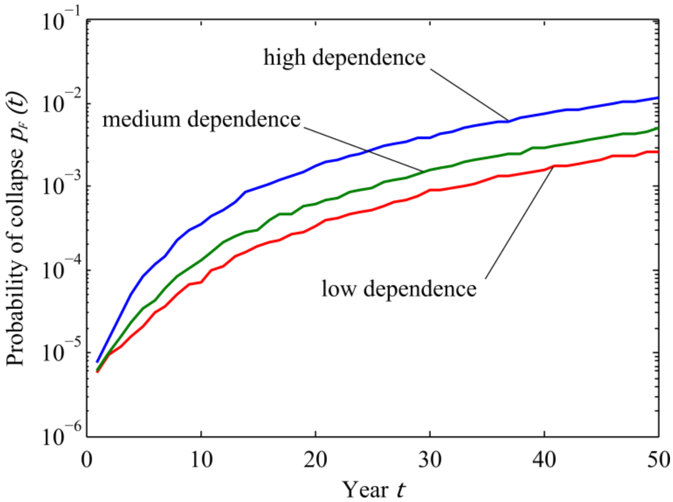

Figure 4. Probability of collapse $p_F(t)$ as a function of different degrees of dependence among element fatigue behavior.

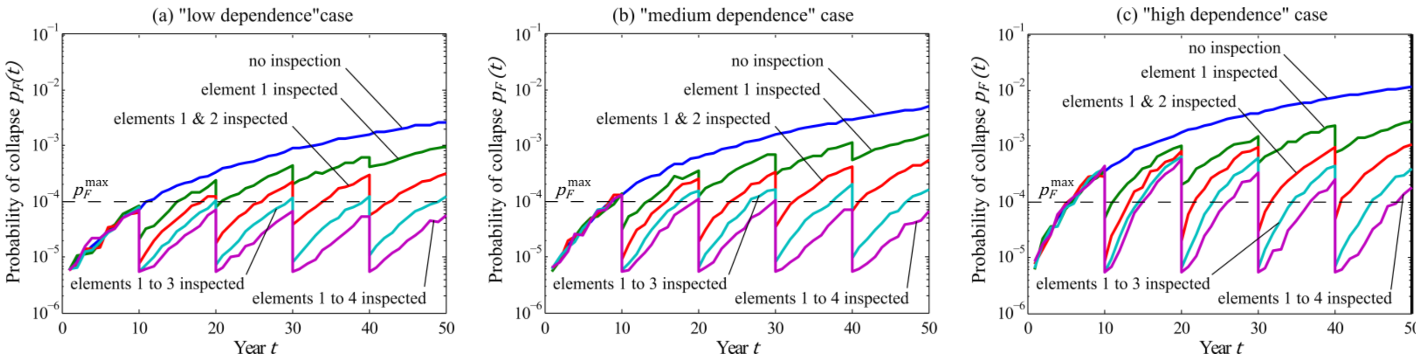

Figure 5. System collapse probability $p_F(t)$ as a function of inspection coverage and degree of dependence among element fatigue behavior. Inspections are performed at a constant interval $\Delta t_{insp} = 10$ years (constant interval approach).

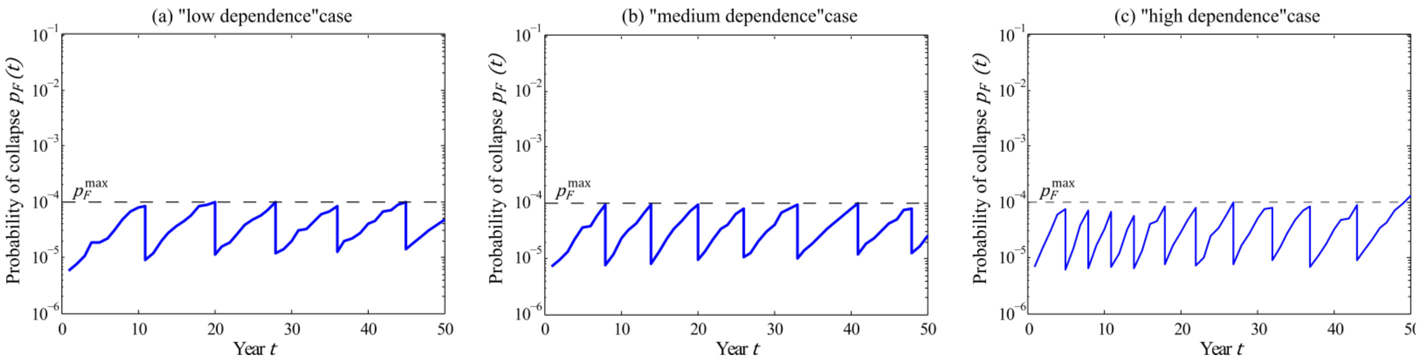

Figure 6. System collapse probability $p_F(t)$ as a function of degree of dependency among element fatigue behavior. Element sets $S_1 = \{1,2\}$ and $S_2 = \{3,4\}$ are alternately inspected. Inspections are performed one year before $p_F(t)$ exceeds the threshold probability $p_F^{max} = 10^{-4}$ (threshold approach).

case where the elements have a high dependence of deterioration behavior.

Figure 5 also includes a threshold for the maximum allowable collapse probability $p_F^{max} = 10^{-4}$, which corresponds to a system target reliability index $\beta^{max} = -\Phi^{-1}(p_F^{max}) = 3.7$ as recommended in the Probabilistic Model Code of the JCSS (2006) for the ultimate limit state when the consequences of element failure and the relative cost of a safety measure are large. This value has a 1-year reference period. A certain inspection strategy can be considered acceptable if $p_F(t) \leq p_F^{max}$ at all times t.

The results in Figure 5(a) show that it is sufficient to only inspect elements 1 to 3 such that the given acceptance criterion is met for the "low dependence" case. From Figure 5(b) it can be seen that it is necessary to inspect all elements in order to comply with the acceptance criterion for the "medium dependence" case. Figure 5(c) demonstrates that none of the considered inspection strategies comply with the given acceptance criterion for the "high dependence" case. The number of elements that are required to be inspected reduces with decreasing degree of dependency among element fatigue behavior, since the a-priori system collapse probability $p_F(t)$ reduces with decreasing degree of dependency among element fatigue behavior (see also Figure 4).

Finally, we consider the case where inspection samples $S_1 = \{1,2\}$ and $S_2 = \{3,4\}$ are alternately inspected. This strategy ensures that all nodes are inspected at some point during the service life. Inspections are performed one year before the system collapse probability $p_F(t)$ exceeds the threshold probability $p_F^{max} = 10^{-4}$. This approach makes it possible to determine the minimum number of required inspections as a function of the given inspection coverage and the given acceptance criterion. The results are illustrated in Figure 6.

From Figure 6(a) it can be seen that the smallest number of inspections is again required for the "low dependence" case (5 inspections). For the "medium dependence" case, 7 inspections are required (see Figure 6(b)). 10 inspections are required for the "high dependence" case (see Figure 6(c)).

7 CONCLUDING REMARKS

The case study in this paper illustrated that the stochastic dependence among element fatigue behavior is relevant when determining the reliability of a redundant structural system. Hence, the stochastic dependence among element deterioration behavior should be carefully considered when assessing the reliability of deteriorating redundant structural systems.

The presented work demonstrates that it is possible to infer the condition of uninspected elements from inspection results obtained at other elements (inspection sample) due to the stochastic dependence among the element fatigue behavior. Furthermore, all information obtained through inspection samples throughout the service life can be utilized to update the system collapse reliability. Consequently, the collapse probability conditional on an inspection strategy in terms of inspection coverage and inspection times can be computed. The acceptability of a given inspection strategy can be verified by comparing the computed system collapse probability with maximum allowable system failure probabilities, e.g. as recommended in the Probabilistic Model Code of the JCSS (2006). On this basis it is possible to determine optimal inspection strategies in terms of inspection coverage and inspection times for Daniels systems.

This approach may be extended to determine risk-based optimal inspection strategies for actual redundant structural systems based on the method proposed by Straub and Der Kiureghian (2011), which represents the structural system through a series system of Daniels subsystems.

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