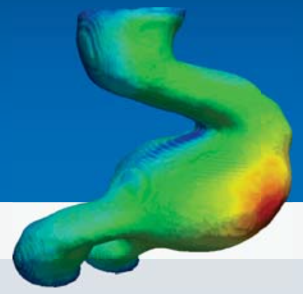
Detecting Growth of Abdominal Aortic Aneurysms using Variational Image Registration Techniques

Sebastian Kehl, Wolfgang A. Wall*, Michael W. Gee****

*Institute for Computational Mechanics, Technische Universität München, Germany

**Mechanics & High Performance Computing Group, Technische Universität München, Germany



Introduction

Prediction of AAA – growth using computational methods

- Computational models become increasingly capable of surpassing conventional methods of rupture risk prediction for abdominal aortic aneurysms (AAA), such as the diameter criterion[1].
- Incorporating arterial growth adds further accuracy to the predicting capabilities of computational approaches.

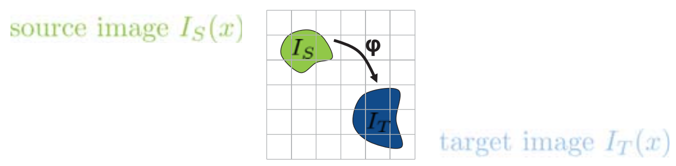
Reliable predictions need accurate spatial representations of model parameters

- The spatial distribution of material parameters is a priori unknown.
- Due to its non invasive character, image registration techniques play a key role in providing accurate information ("measurements of deformation") being used in parameter identification problems[2].

Mathematical Formulation

Variational formulation

A variational formulation of the image registration problem is used in combination with a multiresolutional grid approach to ease the computation.



A transformation $\varphi = x + u(x)$ relating two images defined on a domain Ω with coordinates x is computed as the minimizer of a functional $\mathcal{J}[u] = \mathcal{D}[u] + \alpha\mathcal{R}[u]$.

$$u^* = \arg\min_{u \in L^2} \mathcal{J}(I_T, I_S(\varphi)) \qquad (1)$$

$\mathcal{D}$ is a measure of similarity between the two images, e.g. the so called "sum of squared differences" (SSD) between pixel-wise image gray values

$$\mathcal{D}^{SSD}[u] = \tfrac{1}{2}\int_\Omega (I_T(x) - I_S(\varphi(x)))^2 dx$$

The regularization $\mathcal{R}$ is introduced to render the minimization problem "well-posed", e.g. by using the linear elastic potential energy

$$\mathcal{R}[u] = \int_\Omega \tfrac{\mu}{4} \sum_{j,k=1}^{d} (\partial_{x_j} u_k + \partial_{x_k} u_j)^2 + \tfrac{\lambda}{2}(\operatorname{div} u)^2 dx$$

Computational Approach

Computing the minimizer

The minimization of (1) is achieved by solving the Euler-Lagrange equation corresponding to:

$$d\mathcal{J}[u, v] = d\mathcal{D}[u, v] + \alpha d\mathcal{R}[u, v] \stackrel{!}{=} 0 \qquad (2)$$

Robust solution of (2) is obtained by a gradient flow formulation in combination with an implicit time integration scheme [3], resulting in

$$u_{n+1} = u_n - \tau(I + \tau(J^T J + A))^{-1}(J^T f - A[u]) \qquad (3)$$

$$f = I_T(x) - I_S(x + u)$$

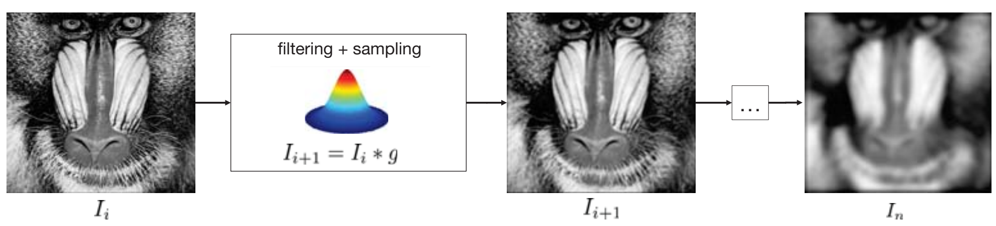
$$J = \nabla I_S(x + u)$$

$$A = \mu\Delta \bullet + (\lambda + \mu)\nabla \operatorname{div} \bullet$$

For $\tau = \infty$ (3) results in the Gauss-Newton approximation of (1). All computations are carried out using a Finite-Difference approximation of (3).

Multiresolutional framework

A multiresolutional framework is deployed to improve robustness and to speed up the computation. In an iterative process, the converged solution of a coarse grid representation of (3) is used as initial guess for the optimization on the next finer grid.



Treatment of boundary conditions

Generally image boundaries don't coincide with physical constraints on the deformation. This is accounted for by applying "zero-traction" Neumann boundary conditions to the regularization operator:

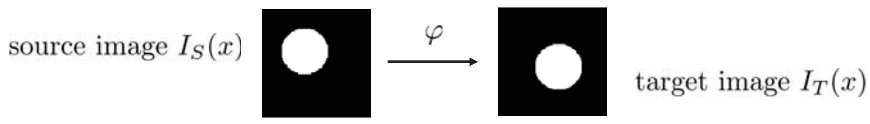
$$\left.\begin{aligned} \langle \nabla u_k + \partial_{x_k} u, n \rangle &= 0 \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}\right\} \partial\Omega$$

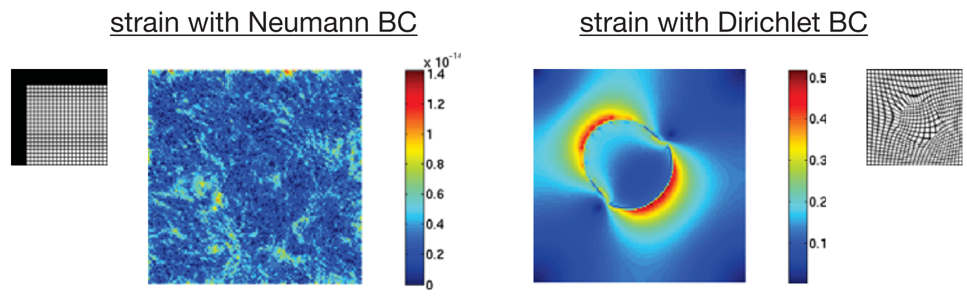
This effectively reduces the artificial stiffness of the optimization problem introduced by too restrictive Dirichlet boundary conditions.

Results

Influence of boundary conditions

The influence of different boundary conditions is analyzed by registering a pure translational motion in 2D:





The visualization of the Green-Lagrange strain resulting from the respective deformation reveals the improved reproduction (up to machine accuracy) of the "true" motion (a rigid body mode, free of strain) by using the "zero traction" boundary condition.

Application to patient specific data

CT-Data of a patient suffering from AAA, recorded with a time lag of one year, is processed such that resulting binary masks (Fig. 1) can be registered against each other. The visualization of the deformation field shows increased growth with a maximum deformation of 4.17*mm* in the region of the aneurysm sac.

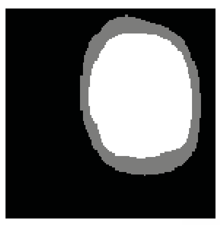
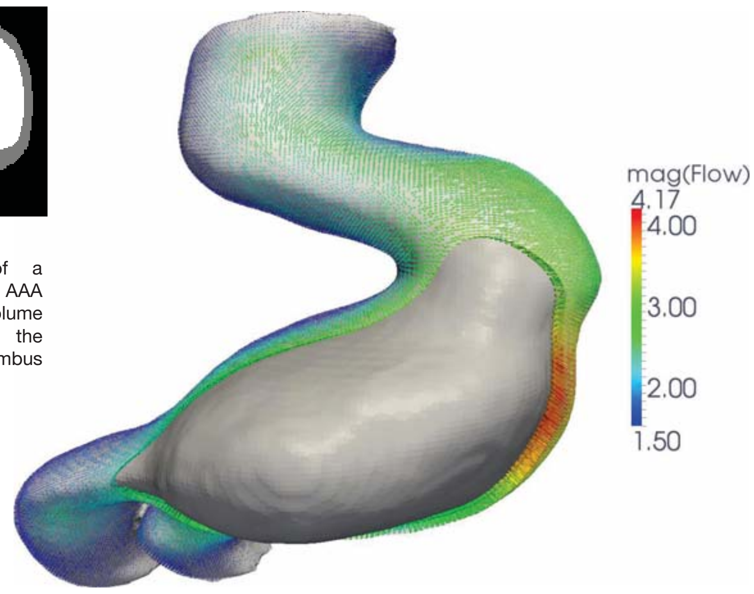


Figure 1:
Binary mask of a patient specific AAA with the luminal volume in white and the intraluminal Thrombus in gray



Conclusion & Outlook

- The presented image registration framework is capable of detecting deformations from image data. Especially the "physically motivated" boundary condition contributes efficiently to an increase in accuracy.
- The accuracy of the results needs to be validated for patient specific applications in the future. Comparison against marker tracked motion might result in a quantitative evaluation.
- The application of the deformation detected via image registration can then be applied in an inverse setup for parameter identification.

References

[1] Maier A, Gee MW, Reeps C, Pongratz J, Eckstein HH, Wall WA. A Comparison of Diameter, Wall Stress, and Rupture Potential Index for Abdominal Aortic Aneurysm Rupture Risk Prediction. *Annals of Biomedical Engineering* May 2010; 38(10):3124–3134.

[2] Gokhale, Nachiket H., Paul E. Barbone, and Assad A. Oberai. "Solution of the nonlinear elasticity imaging inverse problem: the compressible case." Inverse Problems 24.4 (2008): 045010.

[3] Fowler, K. R., and C. T. Kelley. "Pseudo-transient continuation for nonsmooth nonlinear equations." SIAM journal on numerical analysis 43.4 (2005): 1385-1406.