The Generalized Scattering Transformaton for Stable Teleoperation with Communication Unreliabilities

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A teleoperation system allows the human to manipulate in remote/inaccessible/dangerous or scaled environments. From a control point of view, the haptic control loop, where motion and force data are exchanged between the master and the slave manipulator, is very challenging as it is closed over a communication network, e.g. the Internet. The communication network introduces unreliabilities such as (time-varying) time delay and packet loss, which are not only distorting the human haptic perception of the remote environment but can destabilize the overall system. In recent years control approaches based on the passivity framework and the scattering transformation have been developed in order to stabilize the teleoperation system in the presence of such communication unreliabilities, see e.g. [1,2]. The major reason for the success of the passivity formalism in teleoperation is that it can cope with the largely unknown, nonlinear and time-varying human arm and environment dynamics, which, however, can assumed to be passive. The scattering transformation transforms passive systems into finite gain $\mathcal{L}_2$-stable systems with a $\mathcal{L}_2$ gain $\gamma \leq 1$ and guarantees the finite gain $\mathcal{L}_2$-stability of the overall teleoperation system for all communication network operators that are small gain, e.g. constant time delay but also appropriately handled packet loss [2]. However, the passivity framework is known to be conservative resulting in a distorted display of the remote environment properties. Current research is concentrating on relaxing this conservatism. In this work we will use the approximate knowledge on damping properties of the human arm, the controlled manipulators and the environment. In fact, we can show that these subsystems are QSR-dissipative [4], which will be exploited to the benefit of transparency in teleoperation architectures. The approach is based on the *generalized* scattering transformation [3] which applies to QSR-dissipative systems, ensuring finite gain $\mathcal{L}_2$-stability for arbitrary small gain network operators in the closed loop, analogously to the standard scattering transformation.

A dynamical system $\Sigma : \dot{x} = f(x,u),\ y = h(x,u),\ x \in \Re^n,\ u, y \in \Re^p$ is called QSR-dissipative if there exist a positive semi-definite function $V : \Re^n \to \Re_+$ such that for each admissible u and each $t \geq 0$

$$V(x(t)) - V(x(0)) \leq \int_0^t \begin{bmatrix} u & y \end{bmatrix}^T P \begin{bmatrix} u \\ y \end{bmatrix} d\tau, \text{ with } P = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}.$$

where the dependencies of u, y from τ are not explicitly written for ease of notation. The uncertain environment and human arm subsystems can be represented as input-feedforward-output-feedback passive systems (IF-OFP), a subclass of QSR-dissipative systems, which are represented by the choice $Q = -\delta I,\ R = -\epsilon I,\ S = \eta I,\ \delta, \epsilon \in \Re,\ \eta = \frac{1}{2}$ with I the unity matrix. The system is called passive if $\delta = \epsilon = 0$, output-feedback strictly passive (OFP(ϵ)) if $\delta = 0$ and $\epsilon > 0$, and input-feedforward strictly passive (IFP(δ)) if $\delta > 0$ and $\epsilon = 0$. If one or both of the values δ, ϵ are negative there is a shortage of passivity. The damping force term $f_d(x, \dot{x}, t)$ of environment and human arm is assumed to be continuous, potentially time-varying and nonlinear function for which $f_d(x, \dot{x}, t) \geq d_{min}\dot{x}$ holds. A nonlinear mass-spring-damper system can be shown to be IFP(δ) with the input/output pair velocity/force and $\delta > 0$ depending on the minimum linear damping coefficient d_{min}, and OFP(ϵ) with the input/output pair force/velocity and $\epsilon > 0$ also depending on it. In Fig. 1 the feedback interconnection structure is illustrated for a force-velocity architecture: the human arm and the environment are IFP(δ_h) and IFP(δ_e), respectively; the impedance controlled manipulators, with force and gravity compensation are OFP(ϵ_m) and OFP(ϵ_s), respectively. Feedback interconnected IFP and OFP systems still exhibit IF-OFP properties. Particularly, it can be

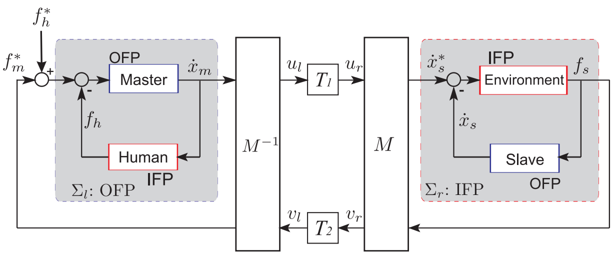

Figure 1: Teleoperation system with time delay and generalized scattering transformation

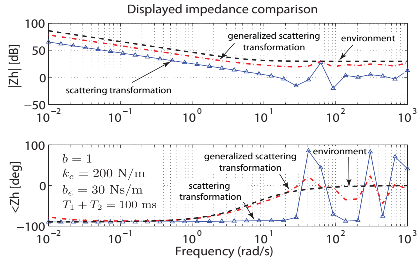

Figure 2: Comparison of displayed with environment impedance

shown that an OFP(ϵ_1) with an IFP(δ_2) system in its feedback is OFP($\epsilon_1+\delta_2$) whereas an IFP(δ_1) with an OFP(ϵ_2) system in its feedback is IFP($\delta_1+\epsilon_2$). Accordingly, the lefthand subsystem Σ_l in Fig. 1 with $f_m^*+f_h^*$ as input and $\dot{x}_m$ as output is OFP(ϵ_l) with $\epsilon_l=\epsilon_m+\delta_h>0$, and the righthand subsystem Σ_r with $\dot{x}_s^*$ as input and f_s as output is IFP(δ_r) with $\delta_r=\epsilon_s+\delta_e>0$. In general, from now on we consider the networked interconnection of an OFP(ϵ_l) and an IFP(δ_r) system with $\epsilon_l,\delta_r>0$.

The generalized scattering transformation is a linear input/output transformation represented by the matrix M in Fig. 1. Instead of the lefthand output variable $\dot{x}_m$ the variable u_l is transmitted. Analogously, v_r is transmitted instead of f_s where

$$\begin{bmatrix} u_l \\ v_l \end{bmatrix} = M \begin{bmatrix} \dot{x}_m \\ f_m^* \end{bmatrix}, \begin{bmatrix} u_r \\ v_r \end{bmatrix} = M \begin{bmatrix} \dot{x}_s^* \\ f_s \end{bmatrix}, M = \begin{bmatrix} \cos\theta I & \sin\theta I \\ -\sin\theta I & \cos\theta I \end{bmatrix} \begin{bmatrix} \sqrt{b} I & 0 \\ 0 & \frac{1}{\sqrt{b}} I \end{bmatrix}$$

where $\theta \in [-\frac{\pi}{2},\frac{\pi}{2}]$ represents the rotation angle crucial for the stability result and $b>0$ is a free tuning parameter. The choice of the transformation angle θ is based on the IFP- and OFP-properties of each side.

The main result of this work is the following: Finite gain $\mathcal{L}_2$-stability of the overall system consisting of networked interconnection of an OFP(ϵ_l) and an IFP(δ_r) system with $\epsilon_l,\delta_r>0$ is ensured for any small gain operator in the network, if $\theta\in[\theta_l,\theta_r]$. Here θ_l is one of the two solutions of $\cot 2\theta_l=\epsilon_l b$, which simultaneously satisfies $\sin(\theta_l)\cos(\theta_l)-\epsilon_l\sin^2(\theta_l)>0$; and θ_r is one of the two solutions of $\cot 2\theta_r=-\delta_r b^{-1}$, which simultaneously satisfies $\sin(\theta_r)\cos(\theta_r)-\delta_r\cos^2(\theta_r)>0$. Hence, instead of choosing $\theta=45°$ as for standard scattering transformation [1], here θ can be chosen out of an interval. We can show that transparency can be improved, i.e. the displayed mechanical properties at the master side come closer to the real environment properties, by proper choice of $\theta\neq 45°$. This is exemplarily demonstrated in simulation where the proposed approach is applied for negligible slave dynamics and a linear time-invariant spring-damper environment with a $\delta_r=d_{min}=30$ and passive lefthandside system, i.e. $\epsilon_l=0$. The resulting system is delay-independently stable for all $\theta\in[45°,89°]$. Furthermore, the displayed stiffness for $\theta=89°$ with the generalized scattering approach is with 85 N/m by far closer to the environment stiffness of 100 N/m than with the scattering transformation $\theta=45°$, where it is only 18 N/m, see Fig. 2.

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