Feasibility Conditions for EDF

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Abstract

We present two feasibility conditions for EDF when deadlines are less than or equal to periods. The complexity of these conditions is $O(n)$ and $O(n \cdot \log n)$ respectively. For $O(n)$, we show that our feasibility condition is better than the one of Devi if the difference between the maximum and minimum deadline is less than $\frac{1}{1-U}$. For $O(n \cdot \log n)$, which is also the complexity of Devi's condition, we show that our condition is always better than Devi's condition.

1. Introduction

Feasibility conditions are not exact, however, they are less complex than exact solutions. This latter is the reason why they are eligible for allocation algorithms where the complexity of allocation heuristic itself makes the use of exact feasibility tests almost impractical.

Additionally, feasibility conditions are useful to perform on-line acceptance tests on dynamic real-time systems, where a trade-off between exactness and running time is required.

For this technical report, we assume all task parameter to be integers.

1.1. Known feasibility conditions

Density condition: The density condition [2] presents a complexity $O(n)$, but it is too pessimistic. Its expression follows:

$$\sum_{i=1}^{n} \frac{e_i}{d_i} \leq 1. \tag{1}$$

Where d_i is the relative deadline of the i-th task, p_i represents its period, e_i the corresponding worst-case execution demand, and n the number of tasks.

Devi's condition: Devi's condition [1] is less pessimistic than the density condition, but it presents higher complexity $O(n \cdot \log n)$. This feasibility condition consists in verifying the following inequality for all k for which $1 \leq k \leq n$ and $d_i \leq p_i$ hold:

$$\sum_{i=1}^{k} \frac{e_i}{p_i} + \frac{1}{d_k} \sum_{i=1}^{k} \left(\frac{p_i - d_i}{p_i} \right) \cdot e_i \leq 1. \tag{2}$$

Devi's condition requires all tasks to be sorted in non-decreasing order of relative deadlines. So if index i is less than j, the relative deadline d_i will be less than or equal to d_j. This prerequisite makes it useless for on-line allocation algorithms.

1.2. A better condition with $O(n)$

We assume that $e_{min} \leq d_{min}$ holds, where index min denotes the task with the shortest deadline. The main idea is to calculate the feasibility bound for the task set as stated in [3]; if this bound is less than or equal to the minimum deadline of the task set, it will be not necessary to verify any deadline and the feasibility of the task set will be guaranteed. This condition can be mathematically expressed as follows:

$$\frac{\sum_{i=1}^{n}(1 - \frac{d_i}{p_i}) \cdot e_i}{1-U} - \frac{1}{1-U} \leq d_{min}.$$

Recalling that $U = \sum_{i=1}^{n} \frac{e_i}{p_i}$ and rearranging terms we obtain:

$$\sum_{i=1}^{n} \frac{e_i}{p_i} + \frac{1}{d_{min}} \sum_{i=1}^{n} \left(\frac{p_i - d_i}{p_i} \right) \cdot e_i - \frac{1}{d_{min}} \leq 1. \tag{3}$$

LEMMA 1 *The feasibility condition given in 3 performs always better than the density condition.*

Proof: We proceed rearranging equation 3:

$$\sum_{i=1}^{n} \frac{e_i}{p_i} \cdot \left(1 - \frac{d_i}{d_{min}}\right) + \frac{\sum_{i=1}^{n} e_i}{d_{min}} - \frac{1}{d_{min}},$$

$$
\begin{aligned}
&\leq \sum_{i=1}^{n} \frac{e_i}{d_i} \cdot \left(1 - \frac{d_i}{d_{min}}\right) + \frac{\sum_{i=1}^{n} e_i}{d_{min}} \\
&\quad - \frac{1}{d_{min}}, \\
&= \sum_{i=1}^{n} \frac{e_i}{d_i} - \frac{\sum_{i=1}^{n} e_i}{d_{min}} + \frac{\sum_{i=1}^{n} e_i}{d_{min}} - \frac{1}{d_{min}}, \\
&= \sum_{i=1}^{n} \frac{e_i}{d_i} - \frac{1}{d_{min}}.
\end{aligned}
$$

This is the density condition 1 minus the factor $\frac{1}{d_{min}}$. Consequently, the thesis holds true. □

LEMMA 2 *The feasibility condition given in 3 performs better than Devi's condition if* $d_{max} - d_{min} < \frac{1}{1-U}$.

Proof: Assuming that Devi's condition holds for all $k \leq n-1$, we proceed verifying if it also holds for last task $k = n$. So, replacing k by n in 2, $\sum_{i=1}^{n} \frac{e_i}{p_i}$ by U and d_k by d_{max}—recall that tasks are sorted in non-decreasing order of deadlines so $d_n = d_{max}$ is the longest deadline, we get:

$$
U + \frac{1}{d_{max}} \sum_{i=1}^{n} \left(\frac{p_i - d_i}{p_i}\right) \cdot e_i \leq 1.
$$

Rearranging terms:

$$
\begin{aligned}
\frac{\sum_{i=1}^{n} \left(\frac{p_i - d_i}{p_i}\right) \cdot e_i}{d_{max}} &\leq 1 - U, \\
\frac{\sum_{i=1}^{n} \left(\frac{p_i - d_i}{p_i}\right) \cdot e_i}{1 - U} &\leq d_{max}. \qquad (4)
\end{aligned}
$$

Inequality 3 can also be expressed as:

$$
\begin{aligned}
\frac{\sum_{i=1}^{n} \left(\frac{p_i - d_i}{p_i}\right) \cdot e_i - 1}{d_{min}} &\leq 1 - U, \\
\frac{\sum_{i=1}^{n} \left(\frac{p_i - d_i}{p_i}\right) \cdot e_i}{1 - U} &\leq d_{min} + \frac{1}{1 - U}. \qquad (5)
\end{aligned}
$$

As the left members of both 4 and 5 are identical. Inequality 3 will perform better than Devi's condition whenever the following holds:

$$
\begin{aligned}
d_{max} &< d_{min} + \frac{1}{1 - U}, \\
d_{max} - d_{min} &< \frac{1}{1 - U}.
\end{aligned}
$$

□

1.3. A better condition with $O(n \cdot \log n)$

The idea remains the same, but this time, we consider tasks to be sorted in non-decreasing order of deadlines (like for Devi's condition). So, we must calculate the feasibility bound for all k for which $1 \leq k \leq n$ holds:

$$
\frac{\sum_{i=1}^{k} (1 - \frac{d_i}{p_i}) \cdot e_i}{1 - U_k} - \frac{1}{1 - U_k} \leq d_k. \qquad (6)
$$

Where U_k is the processor utilization of the first k tasks.

LEMMA 3 *The feasibility condition given in 6 performs always better than Devi's condition.*

Proof: Proceeding analogously to lemma 2, we get the expression of Devi's condition for the k-th Task:

$$
U_k + \frac{1}{d_k} \sum_{i=1}^{k} \left(\frac{p_i - d_i}{p_i}\right) \cdot e_i \leq 1.
$$

Rearranging terms:

$$
\frac{\sum_{i=1}^{k} \left(\frac{p_i - d_i}{p_i}\right) \cdot e_i}{1 - U_k} \leq d_k. \qquad (7)
$$

Similarly, inequality 6 can be expressed as:

$$
\frac{\sum_{i=1}^{k} \left(\frac{p_i - d_i}{p_i}\right) \cdot e_i}{1 - U_k} \leq d_k + \frac{1}{1 - U_k}. \qquad (8)
$$

Here again, as the left members of both 7 and 8 are identical. Inequality 6 will always outperform Devi's condition, because the following is always true:

$$
\begin{aligned}
d_k &< d_k + \frac{1}{1 - U_k}, \\
0 &< \frac{1}{1 - U_k}.
\end{aligned}
$$

□

2. Conclusions

We presented two feasibility conditions for EDF when deadlines are less than or equal to periods and assuming integer parameters. Both proposed feasibility conditions were analytically proven to be better than the existing ones, however, practical experiments show that improvement over existing feasibility conditions decays as periods become much more greater than 1.

References

[1] M. Devi. An improved schedulability test for uniprocessor periodic task systems. *Proceedings of the 15th Euromicro Conference on Real-Time Systems*, 2003.

[2] J. Liu. *Real-Time Systems*. Prentice Hall, 2000.

[3] A. Masrur and G. Färber. Ideas to improve the performance in feasibility testing for edf. *Proceedings of the 18th Euromicro's WiP Session*, 2006.