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Operational transfer path analysis predicting contributions to the vehicle interior noise for different excitations from the same sound source

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For a quantitative ranking of contributions from several sources via different transfer paths to a response, in this example the vehicle interior sound, Transfer Path Analysis is applied. The method used in this experiment is the Operational Transfer Path Analysis, which calculates linearized transfer functions between selected source and response channels using singular value decomposition. For the analysis, several multichannel measurements, containing synchronous data for structure- and air-borne sound, in different operating conditions were performed. The transfer functions calculated from an engine run-up measurement are used to analyze the contributions of the main sound sources over the dominant transfer paths to the vehicle interior noise. In addition, the excitation from the main source was altered while all other aspects of source and structure were kept the same. In this experiment the engine excitation was altered by changing parameters of the engine control unit. Using the calculated transfer functions, predictions of the vehicle interior noise for the altered excitations are evaluated. The resulting sound pressure level of the calculated vehicle interior sound is in accordance with the values from the original measurements.

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1 INTRODUCTION

Contributions from different sources, propagating over various transfer paths, to response signals are, in the field of noise, vibration, harshness (NVH), examined with transfer path analysis (TPA). The method chosen for this study is the so called operational transfer path analysis (OTPA), which works with measurements performed when the structure is in typical operating conditions. The applied OTPA method determines linear relations between sources and receivers regarding coherent sources and is also known as cross talk cancellation (CTC). An overview of the OTPA method and a discussion of some crucial characteristics will be given in section 2.

Although TPA can be used to assess noise and vibration problems in a variety of fields, it is most commonly used in automotive engineering, which is also presented in this paper. For this application example, an extensive measurement campaign with a passenger car on an acoustic roller dynamometer was performed. The main focus of the study is noise emitted by the drivetrain, resulting in a close sensor placement at the drivetrain, without disregarding other dominant sources. In this paper, we concentrate on the interior noise of the car so the response sensor is placed inside the passenger cabin. The details of the measurement setup will be given in section 3.

The conducted measurements are then analyzed with the introduced OTPA method. The results are validated by comparison of the simulated overall signal and the measured response signal for the measurements used for the TF estimate, but also with measurements for which the sources where altered using different settings of the engine control unit (ECU). The reliability of the analysis results, and some characteristics which should be taken into account, are discussed. An excerpt of the analyzed contributions to the interior noise will be given as objective of the OTPA analysis with the validation in section 4.

The paper will be concluded with a summary in section 5.

2 OPERATIONAL TRANSFER PATH ANALYSIS METHOD

The idea of the OTPA method applied here goes back to the multiple input, multiple output (MIMO) method by Bendat[1]. However, the application of the singular value decomposition (SVD), to find the linearized transfer functions (TF), was introduced by Noumura and Yoshida[2]. These TFs are also referred to as transmissibilities, which were introduced by Ribeiro et al.[3]. A detailed description of differences and similarities of the MIMO and OTPA methods is given by de Klerk and Ossipov[4]. A basic understanding of the OTPA principles is helpful in order to plan the measurements and interpret the results. Therefore the basic principles will be described in the following section.

2.1 Basic principles

As already mentioned, the objective of the TPA is the estimation of the contributions from different sources, transferred over various path, to a receiver. The sources or excitations of the system are described by sensor signals at the corresponding reference positions, and can be seen as the input degrees of freedom (DoF) for the analysis. The receivers, described by sensor signals at the corresponding response positions are the output DoF. The principle of the OTPA is to simultaneously measure source and response signals, while the analyzed system is in typical operation conditions, in order to determine linear relations between input DoF and output DoF.

In a linear system the relation between excitations and response can be described by

$$\mathbf{x}(j\omega)\mathbf{H}(j\omega) = \mathbf{y}(j\omega), \tag{1}$$

where $\mathbf{H}(j\omega)$ is the transfer function matrix, $\mathbf{x}(j\omega)$ the reference vector and $\mathbf{y}(j\omega)$ the response vector. The frequency dependency is denoted by $(j\omega)$. The dimension of the excitation vector corresponds to the number of input DoFs m, i.e. the number of excitation signals. The dimension of the response vector corresponds to the number of output DoFs n, i.e. the number of response signals. A brief insight on the selection of reference and response positions for OTPA will be given in subsection 2.2. In the conducted measurements the excitation signals of the drivetrain and other dominant sources were measured with acceleration and sound pressure sensors and the response signal with a sound pressure sensor inside the cabin.

To show how the TFs are estimated from operational measurement, Eqn. (1) is expanded to

$$\begin{bmatrix} x_1 & \dots & x_m \end{bmatrix} \begin{bmatrix} H_{11} & \cdots & H_{1n} \\ \vdots & \ddots & \vdots \\ H_{m1} & \cdots & H_{mn} \end{bmatrix} = \begin{bmatrix} y_1 & \dots & y_n \end{bmatrix}. \tag{2}$$

The frequency dependency of Eqn. (1) is omitted for a clear representation. However Eqn. (2) and following have to be evaluated for each frequency line. For the frequency analysis the operational measurement is spilt into p blocks, for which synchronous frequency spectra for each reference and response channel are calculated, and Eqn. (2) is hence expanded to

$$\begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{p1} & \cdots & x_{pm} \end{bmatrix} \begin{bmatrix} H_{11} & \cdots & H_{1n} \\ \vdots & \ddots & \vdots \\ H_{m1} & \cdots & H_{mn} \end{bmatrix} = \begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{p1} & \cdots & y_{pn} \end{bmatrix}, \tag{3}$$

or expressed in a more compact form with the reference matrix $\mathbf{X}$ and the response matrix $\mathbf{Y}$

$$\mathbf{X}\mathbf{H} = \mathbf{Y}. \tag{4}$$

The m measured reference signals will most likely be coherent to each other to some degree since there is the possibility of cross talk between the reference positions. Although the p frequency spectra will be coherent to some point, they contribute to an overdetermination of the equation system. Details on the selection of operating conditions will be given in subsection 2.3. For the following solution it is assumed that the number of frequency spectra is greater than the number of reference signals $p > m$. Equation (4) can therefore be solved for $\mathbf{H}$

$$\mathbf{H} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{Y} = \mathbf{X}^+\mathbf{Y}, \tag{5}$$

where $\mathbf{X}^+$ is “the pseudoinverse” of $\mathbf{X}$, providing a least squares solution to the linear equation system[5]

$$\mathbf{X}^+ = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T. \tag{6}$$

Since matrix $\mathbf{X}$ has more rows than columns, i.e. $p > m$, the calculation of the pseudoinverse is an overdetermined least squares problem, to which a common and accurate solution is given by singular value decomposition (SVD).[6] If the SVD of $\mathbf{X}$ is

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T, \tag{7}$$

the pseudoinverse of $\mathbf{X}$ is

$$\mathbf{X}^+ = \mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^T, \tag{8}$$

where $\mathbf{U}$ is an $p \times p$ unitary matrix, $\mathbf{V}$ an $m \times m$ unitary matrix, and $\mathbf{\Sigma}$ an $p \times m$ matrix with nonnegative real numbers σ_r on the diagonal. All $\sigma_r > 0$ are the singular values of $\mathbf{X}$. In numerical mathematics it is common practice to treat small singular values below a prescribed tolerance as zero.[7] In application scenarios, small singular values are often caused by noise and other disturbances, and should therefore be rejected.[2,4]

With the results from the SVD an estimate for the linearized TF can be calculated by

$$\tilde{\mathbf{H}} = \mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{Y}. \tag{9}$$

Note that by applying the SVD, the aforementioned coherence problems are also addressed.[2,4,8]

The synthesized responses $\tilde{\mathbf{Y}}$ can be calculated with measured references $\mathbf{X}$, which do not have to be from the measurement used to estimate the TF, according to

$$\mathbf{X}\tilde{\mathbf{H}} = \tilde{\mathbf{Y}}. \tag{10}$$

These can be used to validate the TF estimate, by comparing the synthesized responses $\tilde{\mathbf{Y}}$ with the measured responses $\mathbf{Y}$ which corresponds to the references $\mathbf{X}$ used for the synthesis.

To calculate time signals of the contribution of each reference signal to each response signal, the estimated TFs approximated by FIR filters, which are applied to the measured reference time signal. The overall synthesized time signal for one response channel is the sum of the contributions from all reference points, which can also be used to validate estimated transfer functions, by comparison to the measured time signal of that response channel.

2.2 Sensor Positions

The interpretation of the OTPA results requires a basic understanding of the sound propagation in the analyzed system and the placement of the sensors used to characterize sources and receivers is a crucial point for the reliability. The selection of the receiver sensor positions is in most cases quite evident, since these sensors are typically placed where a person would feel the vibration or hear the sound. Choosing the reference positions to characterize the sources is not that obvious. The sensors should be placed in general as close as possible to each source, for a good separation from influences of other sources, even though the OTPA cancels certain amounts of cross talk. If a dominant source is unattended, the OTPA results can be deceptive. If the unattended source and others sources where sensors are placed are coherent, the contribution of the neglected source will be distributed to the contributions of the coherent sources. This leads to an overestimation of those contributions and will therefore not show as a deviation in the estimated response signals. If the unattended source is not coherent to other sources, the contribution of that source will be missing, which will lead to lower overall response signals.[8,9]

Not only contributions of dominant sources can be estimated by OTPA, but also noise propagation through the components of the structure. A transfer path can be separated in several levels, e.g. on the active and the passive side of an engine mount. If reference sensors are placed on different levels in the transfer path, the separation of the different levels is essential for the analysis. Otherwise the OTPA results can be deceptive, since the excitation of a source is represented at several levels of the structure, which leads to not separable contributions during the interpretation. Nevertheless if the structure equipped with sensors at different levels, separate analyses can be set up, taking only reference signals of one level into account. The results of the separate analyses help to understand noise propagation throughout the structure.[4]

Different physical quantities, likewise for reference and response positions can be combined in a single OTPA setup as long as they originate from a synchronous measurement and the frequency analysis is performed with the same parameters. Different sensor types can be applied to the structure, e.g. accelerometers to measure structure-borne sound or microphones to measure airborne sound. The signals for the different physical quantities will most likely have different numeric ranges. Applying the SVD to an input matrix $\mathbf{X}$ of that kind, would result in underestimation or complete disregard of the contributions of a weaker group of inputs. Therefore the signals of each physical quantity have to be normalized before the SVD.[2,9]

2.3 Operating conditions

The selection of the operating conditions for measurements required to estimate the TFs is another critical point for OTPA. The objectives are to represent all typical operating conditions, and to achieve low coherence between the frequency spectra used for the calculation of the TFs, in order to get quality OTPA results. This means that for the estimation of the TFs, the measurements should be performed while the operating conditions are varied over the whole typical operating range, in either a single or multiple measurements. As mentioned before, a minimum requirement is that the number of frequency spectra p is greater than the number of reference signals m, i.e. $p > m$.

In automotive engineering typical possibilities to vary the operating conditions are to change the engine torque or the engine rotational speed which also leads to changes in driving speed and changes in rotational speeds in the whole drivetrain and wheels. E. g. during an engine run-up, the engine orders will shift from low to high frequencies, thus exciting the system over a wide frequency range. Frequency spectra calculated at discrete time steps during the run-up are likely to show low coherence since the frequencies constantly change. This reasoning does not only hold for reference signals representing the engine as source but for all other sources in the drivetrain including the wheels. Another standard test case in automotive NVH engineering is the constant speed test, in which e.g. the engine load could be varied to change the operating conditions.[4,8,9]

3 MEASUREMENT SETUP

The objective of the field campaign was to analyze contributions of various sources to the interior noise of a diesel car, with a main focus on the sources of the drivetrain. The examined car has a three liter straight six-cylinder diesel engine with two-phase turbocharging, an eight-speed automatic transmission and rear-wheel drive. Throughout the measurements presented here, slick tires were mounted on the car. The gears of the transmission can be selected manually, to allow measurements over the whole rotational-speed (rpm) range without gear change. To alter the excitation of the system, engine control units (ECU), with different parameter sets, were available for the measurements.

In order to provide defined conditions for repeatable measurements, all tests were carried out on a vehicle test bench, which is described in detail by Finsterhölzl et al.[10]. In a semi-anechoic room, the car is fixed on an acoustic chassis dynamometer, which assured that the different operating conditions could be reproduced. The roller surface used was optimized to produce little noise from the tires. Although it was an all-wheel chassis dynamometer, only the rollers for the driven wheels were active, since wheel noise and excitation from the wheel suspension were not in the focus of the study.

From the numerous measurement sensors used, most were, yet carefully selected, standard, piezoelectric accelerometers for the measurement of structure-borne sound and free-field measurement microphones for the measurement of airborne sound. Measurements were performed and analyzed with PAK software from Müller-BBM VibroAkustik Systeme GmbH and corresponding, synchronized PAK-Mobile MK II frontends.

For the OTPA presented here, only reference sensor positions close to the noise sources were chosen, e.g. ahead of engine, gearbox or rear axle carrier mounts. A total count of 44 reference signals is used for the analysis, recorded at the positions given in Table 1. As some of the sensors for structure-borne sound are tri-axial accelerometers, the number of reference positions does not add up to the number of reference signals. The response signal was measured, with a free-field measurement microphone, at the position of the drivers head. All signals were measured with a sampling rate of 52 kHz and a resolution of 24 bit.

Table 1 – Reference sensor positions.

Source	Position	Number of
Structure-borne	Engine	3
	Gearbox	3
	Rear axle	3
	Wheel carriers	4
Airborne	Engine	8
	Exhaust	4
	Gearbox	1
	Intake	1
	Rear axle	1
	Rear tires	4

As mentioned in subsection 2.3, a variety of operating conditions should be measured, in order to get reliable OTPA results. For each ECU several measurements were performed, engine run-ups with half and wide open throttle and an approximate duration of 25 seconds, and constant engine speed measurements with 1500, 2000, 3000 rpm and simulated slopes of 0, 30 and 50 percent, of which 2 second sections are used.

All of the mentioned measurement conditions are used to estimate the TFs from each reference position to the response position. For the fast Fourier transform (FFT), ahead of the OTPA calculation, a block size of 16384 samples is used. The singular values used for the estimation, were chosen by the cumulative percentage of the total variation they contribute.[7] By comparison of measured and synthesized response signal, a cumulative percentage of 95 % was chosen, and is used for all following OTPA.

4 RESULTS

In a complex structure like a car, with numerous sources and transfer paths, the reliability of the estimated TFs, respectively the quality of the OTPA, should be evaluated prior to further analysis of the results. An obvious and common choice to validate the TF estimates, is by comparing measured and synthesized response signal.[4,11] The fact that the signals used to estimate the TF, or at least quite similar signals, are also used to validate the results is sometimes considered as a problem.[9] Since the sound of one of the main sound sources, the engine, was specifically changed, using ECU with different parameter sets, over a broad range, some aspects of that issue can be addressed. Therefore TF estimates, calculated with a whole measurement set of ECU1 will be used to synthesize response signals for ECU1-3, which will be compared with the corresponding measured response signal.

4.1 Comparison with engine control unit 1

In the first step of the validation the synthesized and measured response signals for ECU1 are compared. Although not the same measurements were used to synthesize the response signal and to estimate the TF, the measurements were quite similar, since every measured operating condition was included in the TF estimate. Since the excitation from the engine varies over the rpm range, two constant engine speed measurements, with 1500 and 3000 rpm were selected for the validation.

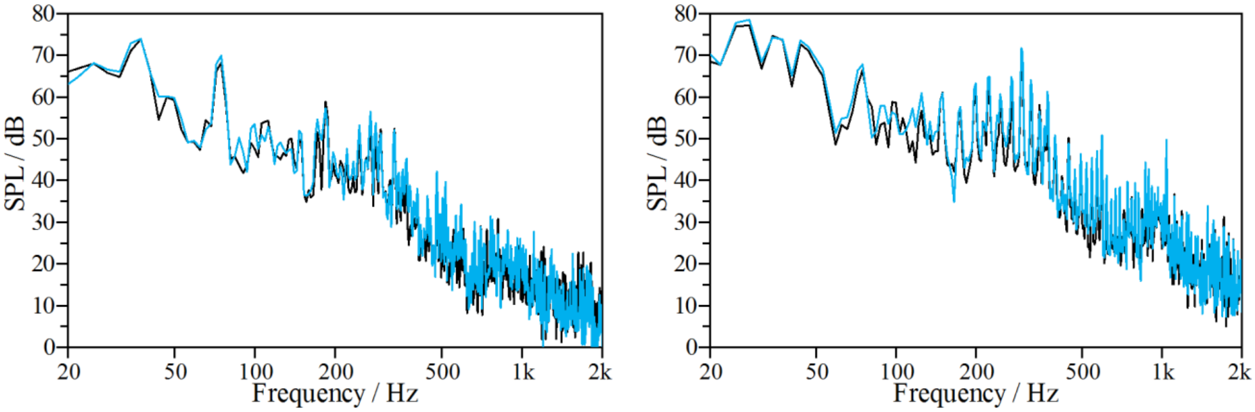

Fig. 1 - Measured (black) and synthesized (blue) interior noise sound pressure level (SPL) for a constant engine speed of 1500 rpm (left) and 3000 rpm (right) with engine control unit 1.

The maximum sound pressure level (SPL) $L_{F,max}$ of measured and synthesized response differ, by 1.9 dB for the 1500 rpm measurement condition and by 0.7 dB for the 3000 rpm measurement condition. Positive differences indicate that the OTPA overestimates the response sound pressure level. The amplitude spectra in Fig. 1 show good accordance between measured and synthesized response. For a more detailed display, a frequency range from 20 Hz to 2 kHz is shown. Above 2 kHz the SPL is in general low.

The good accordance between measured and synthesized response signals leads to the conclusion that the OTPA result is of good quality and neither noise nor other disturbances did influence the result considerably. It also seems, with the chosen sensor placement, no major source was missed. There is as already mentioned still the possibility that the contribution of a source which was not measured is distributed to nearby coherent sources. If unaccountable contributions occur during the interpretation of the OTPA results, this is a possible reason.

4.2 Comparison with engine control unit 2

A second set of measurements was conducted with another engine control unit (ECU2) with a different parameter set. The SPL of the reference signals is up to 3 dB higher compared to ECU1. For the same operating conditions the acceleration level is up to 5 dB higher, in particular for the engine reference points.

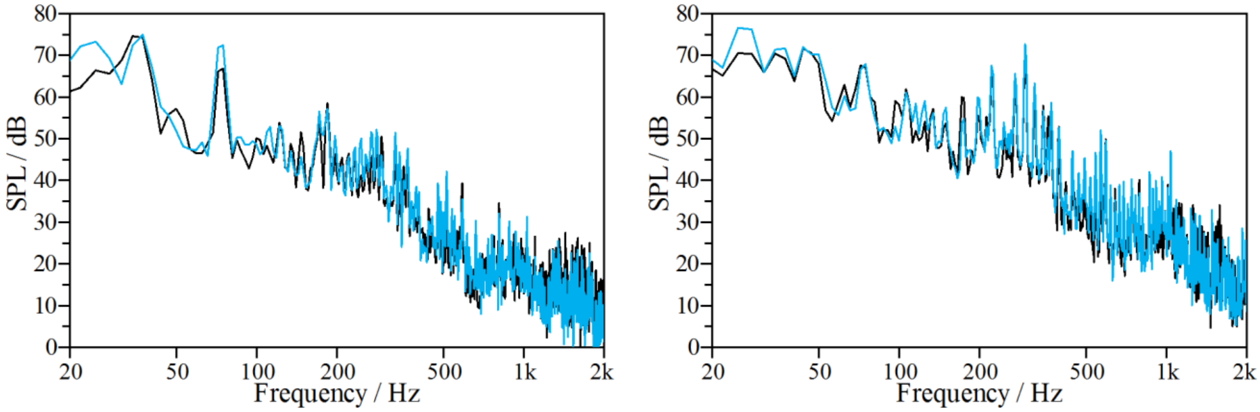

Fig. 2 - Measured (black) and synthesized (blue) interior noise sound pressure level (SPL) for a constant engine speed of 1500 rpm (left) and 3000 rpm (right) with engine control unit 2.

The measured and synthesized interior noise SPL show good agreement. At low frequencies the OTPA tends to overestimate the SPL, which can be seen in Fig. 2. Maximal differences between SPL of the synthesized and the measured response signal are 2.2 dB for the 1500 rpm measurement condition and 3.2 dB for the 3000 rpm measurement condition. Although different measurements were used to estimate the TFs and to synthesize the response signal, the match between measured and synthesized response is still good, which speaks for the quality of the OTPA result.

4.3 Comparison with engine control unit 3

The third ECU used during the measurements leads to rather high level differences in comparison to ECU1. For the same operating conditions the SPLs differed up to 8 dB and the acceleration levels changed up to 10 dB, especially for the engine reference positions.

Measured and synthesized response signals differed by 22.7 dB for the 1500 rpm constant engine speed measurement and by -0.2 dB for the 3000 rpm constant engine speed measurement. Even though the reference signals showed large variation, a disagreement this large is unexpected. The difference between the 3000 rpm measurement condition, which is in good agreement with the measured signal, and the 1500 rpm measurement condition shows the necessity of analyzing different operating conditions. Note that the comparison of measured and synthesized interior noise for a constant engine speed of 1500 rpm in Fig. 3. has a different axis scaling.

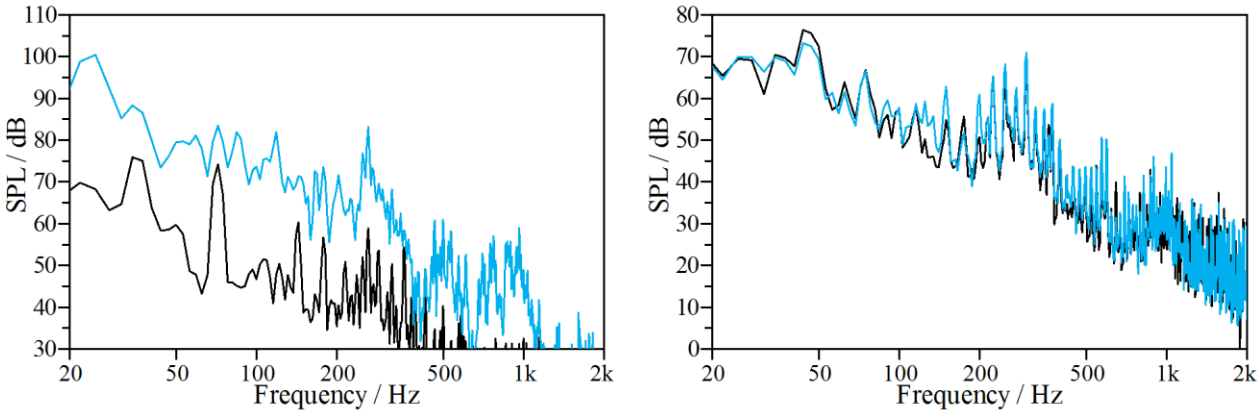

Fig. 3 - Measured (black) and synthesized (blue) interior noise sound pressure level (SPL) for a constant engine speed of 1500 rpm (left) and 3000 rpm (right) with engine control unit 3.

A possible explanation of the large disagreement is nonlinear behavior of the vehicle structure, which is not accounted for by the OTPA. This problem could have been missed in the validation with ECU2 since the variation of the reference signals is by far smaller than with ECU3. If measurements with ECU3 are included in the OTPA and nonlinear system behavior is the problem, the estimated TFs are averaged and the expectation is that while estimates for the 1500 rpm measurement condition for ECU3 get better, the other estimates get worse. To examine this point new TFs were estimated, adding a set of measurements of ECU3 similar to the one already used.

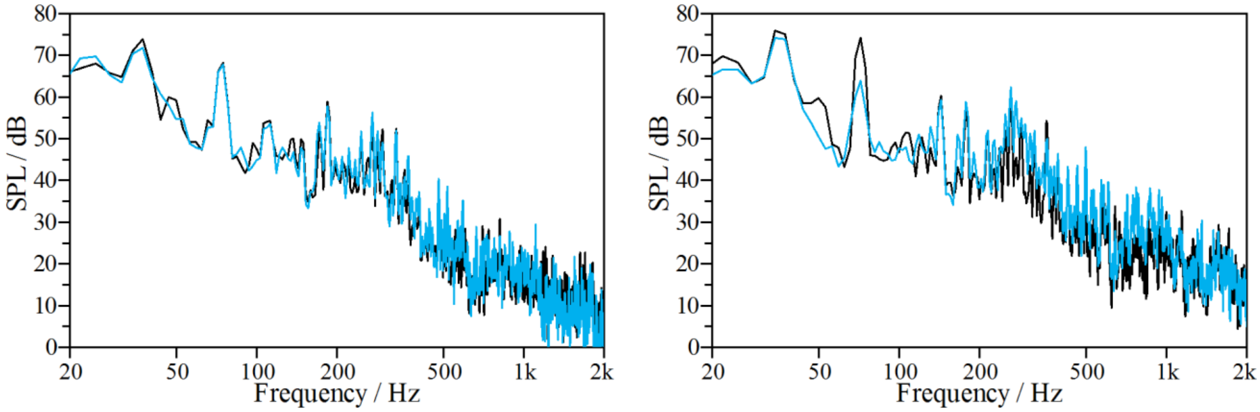

Fig. 4 - Measured (black) and synthesized (blue) interior noise sound pressure level (SPL) for a constant engine speed of 1500 rpm with engine control unit (ECU) 1 (left) and ECU3 (right).

If analyzed with TFs estimated from the combined measurement set the difference for the 1500 rpm operating condition for ECU1 is -1.2 dB, which is smaller than the difference when only measurements of ECU1 are used. The difference for ECU3 is reduced from 22.7 dB to -2.3 dB. If evaluated over frequency, as displayed in Fig. 4, also very good agreement between measured and synthesized response signals shows for both ECUs. The better estimate of the interior noise estimate for each ECU, leads to the conclusion that the additional measurements provide new operating conditions, which frequency spectra are incoherent to the previous, that improve the quality of the OTPA results especially at low frequencies.

4.4 Contribution analysis

Since the objective of OTPA is to analyze contributions from dominant sources, a brief insight to contributions of the main sources to the vehicle interior noise for the described measurement setup is given here. For the analysis presented here the TFs were estimated from excitation signals measured with ECU1 and ECU3, since this setting showed the best results in the validation.

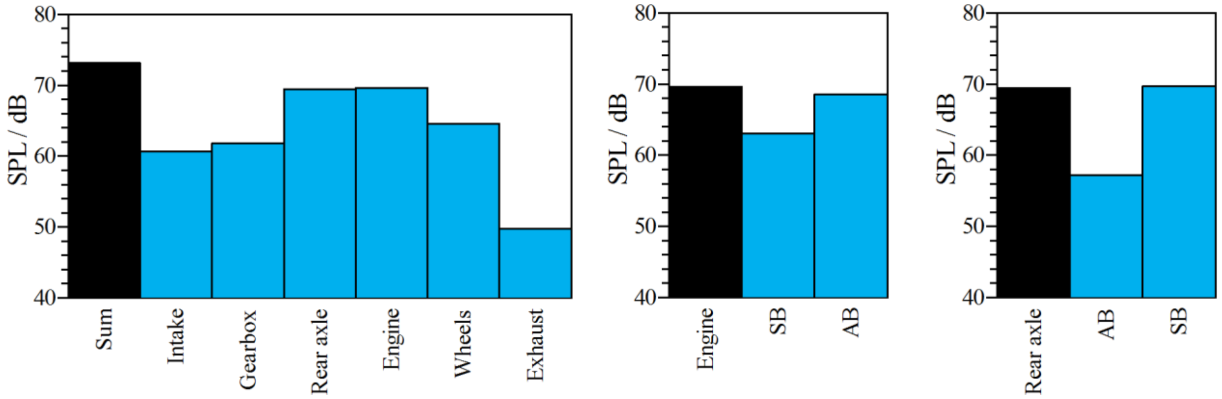

Fig. 5 – Contributions to the interior noise (Sum) at a constant engine speed of 1500 rpm with engine control unit 1, of all dominant noise sources (left), of the engine, split into airborne (AB) and structure-borne (SB) noise sources (middle), of the rear axle split into AB and SB noise sources (right).

An overview of the contributions from the different sources, which are described by the sensors at the corresponding measurement positions for this analysis, shows that the highest estimated contributions originate from the engine and the rear axle, if compared by the linear SPL. Since the measurements were setup to specifically analyze the engine noise, it is not surprising that the engine contributes most to the interior noise. A contribution of the rear axle this high was not expected and has to be investigated. Both airborne and structure-borne noise sources of the engine contribute to the vehicle interior noise. For the rear axle contribution the dominant noise is structure-borne noise.

The high contribution to the vehicle interior noise at the driver position was, from personal listening experience, quite surprising since the engine noise was clearly prominent. The amplitude frequency spectrum of the rear axle contribution shows that almost only frequencies below 100 Hz contribute to the vehicle interior noise and for higher frequencies the engine contribution is dominant, which explains the listening experience.

The amplitude frequency spectrum of the engine contribution shows the dominant engine orders for both airborne and structure-borne sound. Further analysis can provide information about which of the sources in the engine compartment, represented by the corresponding sensors, contribute most at a specific frequency range.

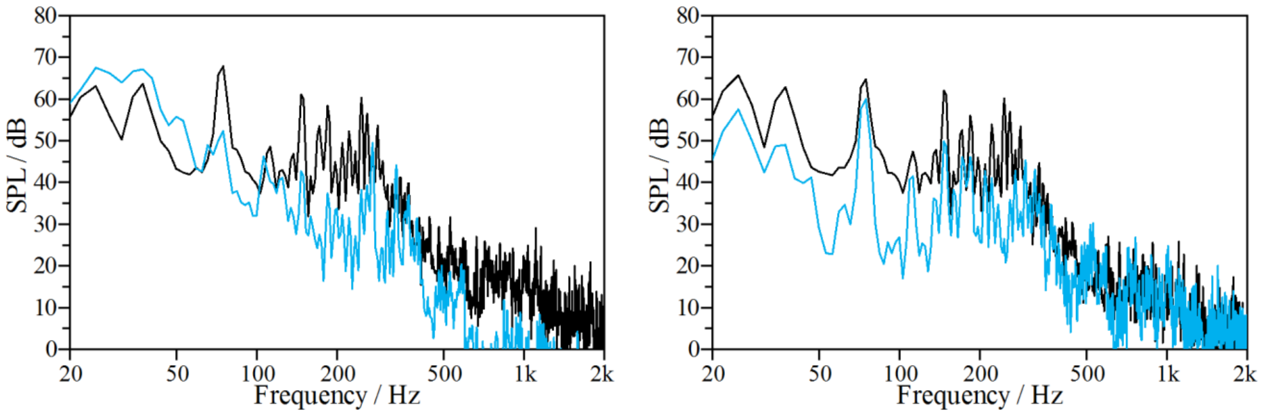

Fig. 6 - Contributions to the interior noise at a constant engine speed of 1500 rpm with engine control unit 1, of the two most dominant sources (left) the engine (black) and the rear axle (blue), of the engine (right) split into airborne (black) and structure-borne (blue) noise sources.

5 CONCLUSIONS

In order to analyze the contributions of the various sound sources in a vehicle over different transfer paths to a response in the vehicle interior, the operational transfer path analysis (OTPA) method was chosen. The basic principles of the OTPA were discussed in order to show which considerations have to be made during the planning of OTPA measurements. A basic understanding of the analyzed structure is necessary to select the reference positions of the sensors and to draw reliable conclusions about the contributions to the response signal. It is also important to perform measurements in all possible operating conditions in order to estimate a reliable model of the analyzed structure which has also been shown during the validation of the method.

An excerpt of an extensive field campaign has been presented, which provided the opportunity to validate the method and its results not only with the excitation measurements used for estimation of the linearized transfer functions, but also with measurements where the vehicles engine, one of the main sound sources, has been altered with different engine control unit parameters. The results of the OTPA were validated by comparing the measured and synthesized response signals, i.e. the vehicle interior noise. The validation showed that the OTPA is a reliable method to analyze contributions to a sound, using measurements in different, real operating conditions, with a single measurement setup.

A brief insight to the analyzed contributions from different sources to the vehicle interior noise was given. It showed that the OTPA provides reasonable estimates for the different contributions and the potential of analyzing the contributions not only on basis of sound pressure levels.

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