Technische Universität München

Theoretische Elementarteilchenphysik

Flavour Alignment in physics beyond the Standard Model

Carolin Barbara Bräuninger

Vollständiger Abdruck der von der Fakultät für Physik der Technischen Universität München zur Erlangung des akademischen Grades eines

Doktors der Naturwissenschaften

genehmigten Dissertation.

Vorsitzender:	Univ.-Prof. Dr. Stefan Schönert
Prüfer der Dissertation:	1. Univ.-Prof. Dr. Alejandro Ibarra
	2. Univ.-Prof. Dr. Andrzej J. Buras, i.R.

Die Dissertation wurde am 23. Oktober 2012 bei der Technischen Universität München eingereicht und durch die Fakultät für Physik am 21. November 2012 angenommen.

A Jérémy

He insisted on being called Dr. Amelio.
That is always a bad sign.

Steve Jobs

Choisir la vie, c'est toujours choisir l'avenir.
Sans cet élan qui nous porte en avant nous ne serions rien
de plus qu'une moisissure à la surface de la terre.

Personne n'est plus arrogant envers les femmes, plus
agressif ou méprisant, qu'un homme inquiet pour sa virilité.

Simone de Beauvoir

Contents

Summary

There are numerous reasons to think that the Standard Model of physics is not the ultimate theory of nature on very small scales. However, attempts to construct theories that go beyond the Standard Model generically lead to high rates of flavour changing neutral processes that are in conflict with experiment:

Quarks are the fundamental constituents of protons and neutrons. Together with electrons they form the visible matter of the universe[1]. They come in three generations or "flavours". In interactions, quarks of different generations can mix, i.e. a quark of one flavour can transform into a quark of another flavour. In the Standard Model, at first order in perturbation theory, such processes occur only via the exchange of a charged particle. Flavour changing *neutral* processes can only arise in processes involving loops of charged particles. This is due to the fact that all couplings of two quarks to a neutral particle are diagonal in the basis of the mass eigenstates of the quarks. There is thus no mixing of quarks of different flavour at first order. Since the loop processes are suppressed by a loop factor, the Standard Model predicts very low rates for neutral processes that change the flavour of quarks. So far, this is in agreement with experiment.

In extensions of the Standard Model, new couplings to the quarks are usually introduced. In general there is no reason why the new coupling matrices should be diagonal in the mass basis of the quarks. These models therefore predict high rates for processes that mix quarks of different flavour.

Extensions of the Standard Model must therefore have a non-trivial flavour structure. A possibility to avoid flavour violation is to assume that the new couplings are *aligned* with the mass matrices of the quarks, i.e. diagonal in the same basis. This alignment could be due to a flavour symmetry. In this thesis, two extensions of the Standard Model with alignment are studied.

The first is a simple extension of the Standard Model where a second Higgs doublet is added. In such models, there are two Yukawa matrices for each fermion type. Going to the mass basis, one of them is diagonalized and together with the vacuum expectation

[1]Most of the matter in the universe is invisible Dark Matter, however.

value of the Higgs forms the mass matrix of the quarks. The other Yukawa matrix however is not diagonal. It couples two quarks and one of the mass eigenstates of the two Higgs doublets. Flavour violating processes can thus occur via the exchange of a neutral scalar. If the two Yukawa matrices were aligned for some reason this would not happen. However, the alignment can only be imposed at one energy scale and will be spoiled when evolving the couplings down to a lower scale. It is shown that in spite of this effect, alignment of the Yukawa couplings provides sufficient protection from flavour changing neutral currents to be in agreement with present experimental bounds.
Another, more ambitious, extension of the Standard Model are warped extra dimensions. In these models spacetime consists of a slice of five-dimensional Anti-de Sitter space (the "bulk") sandwiched in between two flat four-dimensional boundaries (the "branes"). The Higgs is assumed to live on one of the branes while all other particles are allowed to spread into the bulk. Particles that propagate in the bulk have a "KK tower" of heavier particles associated with them in the effective four-dimensional theory. In the bulk fermions have a vector-like mass term in addition to their Yukawa couplings to the Higgs. Via different localizations of the quarks' wave functions in the bulk, the huge differences in their masses can be explained. However, since the wave function profiles of the quarks are non-universal for the different flavours, so are the couplings to the KK excitations of gauge bosons. Rotating to the mass basis therefore introduces off-diagonal elements in these couplings and thus flavour changing neutral processes. Since the wave function profiles are a function of the eigenvalues of the vector-like masses, aligning these with the Yukawa couplings will suppress flavour violation. In this thesis a model that makes use of such an alignment mechanism is presented and is shown to be in agreement with experimental constraints.

Based on publications [1, 2].

Chapter 1

Challenges of Modern Particle Physics

What is fascinating about the Standard Model (SM) of particle physics is how interactions emerge as a consequence of symmetries imposed on the Lagrangian. Via the Noether theorem, these symmetries are furthermore equivalent to a conserved four-current.
In this sense, the SM is a $SU(3) \times SU(2) \times U(1)$ local gauge theory. It describes the electromagnetic, weak and strong interactions of quarks and leptons. Up to this day, it has seen tremendous successes: it predicted not only the existence of the W and Z bosons, of the gluon, and the top and charm quarks but also their masses to high precision. The confinement of quarks was correctly predicted. Up to today, all tests on flavour observables are in agreement with the SM prediction. This last point is a problem for most attempts to extend the SM, as we shall see in the following.
Despite these undeniable successes most physicists believe today that the SM is not the ultimate theory of the fundamental constituents of nature. I will outline some problems and open ends in sections 1.1, 1.2 and 1.3. Numerous attempts to extend the SM in order to address these issues exist and I will give a brief overview of them in section 1.4. Two of these extensions, the Two Higgs Doublet Model and the Randall-Sundrum Model will be investigated in this thesis.

1.1 The Hierarchy Problem

With the (probable) discovery of the Higgs boson [3, 4], the last particle of the SM was found. To understand the need for a Higgs boson we have to go a bit deeper into how forces emerge from the presence of symmetries in the SM. We will consider the weak force here, but the same principles hold for the strong and electromagnetic forces as well. The only difference is that their gauge bosons, i.e. the force carriers of strong and electromagnetic interactions are known to be massless. No breaking of these symmetries is therefore necessary as we shall see in the following.

1.1.1 The need for the Higgs

Local gauge invariance and electroweak interactions

The gauge group of the weak force is $SU(2)$. All fermions are doublets under this $SU(2)$ and transform as [5, 6, 7, 8]:

$$\psi = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} \rightarrow \exp\left(\imath\alpha^i(x)\frac{\sigma^i}{2}\right)\psi \tag{1.1}$$

where σ are the Pauli matrices. As the Pauli matrices do not commute this is a non-Abelian symmetry. It is also a local symmetry as α depends on the position in space-time x.
The equation of motion of fermions is the Dirac equation, which is for massless fermions:

$$\imath\partial\!\!\!/\psi(x) = 0 \tag{1.2}$$

From this we get the Lagrangian:

$$\mathcal{L} = \imath\bar{\psi}\partial\!\!\!/\psi \tag{1.3}$$

This is invariant under a global $SU(2)$ but not under the local $SU(2)$ transformation we are considering in eq. (1.1). We can make it invariant however by replacing the derivative ∂_μ by the *covariant* derivative D_μ defined as:

$$D_\mu = \partial_\mu - \imath g A^a_\mu \frac{\sigma^a}{2} \tag{1.4}$$

where we introduced a vector field A_μ that must transform as

$$A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{g}\partial_\mu\alpha(x) \tag{1.5}$$

for the Lagrangian to be invariant under the local $SU(2)$ symmetry.
This is where things get interesting: In the Lagrangian, replacing ∂_μ by D_μ, we get a term

$$\mathcal{L} \supset g\bar{\psi}\gamma^\mu A^a_\mu \frac{\sigma^a}{2}\psi \tag{1.6}$$

i.e. the fermions are interacting with the new vector field. All this from demanding local gauge invariance only! If we are to take this serious, we need to write all terms containing the fields ψ and A_μ that are Lorentz-invariant and invariant under the local

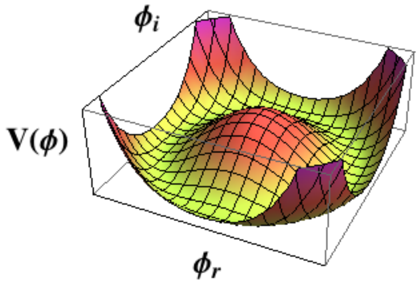


Figure 1.1: *The scalar potential of the Higgs field*

$SU(2)$ symmetry. There is only one other term and we get:

$$\mathcal{L} = \imath\bar{\psi}\gamma^\mu\partial_\mu\psi + g\bar{\psi}\gamma^\mu A^a_\mu\frac{\sigma^a}{2}\psi - \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} \tag{1.7}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu$. Note that a mass term for the vector field $\frac{1}{2}mA_\mu A^\mu$ is not allowed by the gauge symmetry. Including also the electromagnetic force (with symmetry group $U(1)$), the covariant derivative reads [5]:

$$D_\mu = \partial_\mu - \imath g A^a_\mu\frac{\sigma^a}{2} - \imath\frac{1}{2}g'B_\mu \tag{1.8}$$

where A_μ is the gauge boson of the $SU(2)$ symmetry and B_μ is the gauge boson of the $U(1)$. B_μ also has a kinetic term, of course:

$$\mathcal{L} \supset -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{1.9}$$

with $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.
We would like this gauge theory to describe the electromagnetic and weak forces we observe in nature. Photons are massless and hence the electromagnetic force is a long range force. The weak force, on the other hand, is known to be a short range force and its force carriers must therefore be massive. This has been confirmed by experiment. So we have a problem: We know the gauge bosons of the weak force must be massive but we cannot write a mass term for them!

Higgs to the rescue

Consider an $SU(2)$ doublet of complex scalar fields [5, 6, 7, 8]:

$$\phi = \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix} = \begin{pmatrix}\varphi_1 + \imath\varphi_2 \\ \varphi_3 + \imath\varphi_4\end{pmatrix} \tag{1.10}$$

The Lagrangian

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi) \tag{1.11}$$

with the potential

$$V(\phi^\dagger\phi) = -\mu^2\phi^\dagger\phi + \frac{1}{2}\lambda(\phi^\dagger\phi)^2 \tag{1.12}$$

is invariant under the gauge symmetry and the space-time symmetry. By doing an $SU(2)$ gauge transformation we can get rid off the upper component and write:

$$\phi = \begin{pmatrix} 0 \\ \varphi_r + \imath\varphi_i \end{pmatrix} \tag{1.13}$$

The potential is shown in fig. 1.1 as a function of φ_r, φ_i for $\mu^2 > 0$ and $\lambda > 0$. As can be seen it has a minimum at a radius v away from the origin:

$$\langle\varphi\rangle = v = \frac{\mu}{\sqrt{\lambda}} \tag{1.14}$$

This is the so-called *vacuum expectation value (vev)*. The important thing to note here is that the potential is manifestly invariant under $SU(2)$ rotations. For a particle sitting in the minimum of the potential however, the symmetry is hidden or *spontaneously broken* as one says. We do one further rotation to set $\varphi_i = 0$ and consider fluctuations of the scalar field around the vev:

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.15}$$

Plugging this into the covariant derivative term $(D_\mu\phi)^\dagger(D^\mu\phi)$ of the scalar Lagrangian we get a term [5]:

$$\mathcal{L} \supset \frac{1}{2}\left(\frac{gv}{2}\right)^2 A_\mu A^\mu \tag{1.16}$$

This is a mass term for A_μ! Our problem is solved. The field h is the physical Higgs field.

Fermion masses

We cannot write a direct mass term for fermions either, and they thus also need to acquire masses via the Higgs mechanism. To understand this we need to know that right- and left-handed fermions have different gauge quantum numbers: the left-handed fermions are doublets under $SU(2)$ while the right-handed ones are singlets. In total we have:

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u_R, \quad d_R, \quad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R \tag{1.17}$$

there might also be right-handed neutrinos but this shall not interest us here. It is then easy to see, that for ψ any of these fields a mass term such as

$$m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \tag{1.18}$$

would violate gauge invariance. We can however write *Yukawa-couplings* of the fermions to the Higgs [6]:

$$\mathcal{L}_{\text{Yukawa}} = (Y_u)_{ij}\bar{q}_{Li}u_{Rj}\tilde{\phi} + (Y_d)_{ij}\bar{q}_{Li}d_{Rj}\phi + (Y_e)_{ij}\bar{l}_{Li}e_{Rj}\phi + \text{h.c.} \tag{1.19}$$

where $\tilde{\phi} = i\tau_2\phi^*$. Inserting the vacuum expectation value of the Higgs field,

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{1.20}$$

we get mass terms for the fermion fields:

$$\mathcal{L}_{\text{Yukawa}} = \underbrace{(Y_u)_{ij}v}_{(M_u)_{ij}}\ \bar{u}_{Li}u_{Rj} + \underbrace{(Y_d)_{ij}v}_{(M_d)_{ij}}\ \bar{d}_{Li}d_{Rj} + \underbrace{(Y_e)_{ij}v}_{(M_e)_{ij}}\ \bar{e}_{Li}e_{Rj} + \text{h.c.} \tag{1.21}$$

Unitarity of the W_LW_L scattering amplitude

Furthermore, the Higgs field is also needed to restore unitarity of the W_LW_L scattering amplitude. Since this will not play a role in the following, we mention it here only for completeness.

1.1.2 Is nature fine-tuned?

The Higgs mechanism is for sure an impressive way to solve many problems of the SM at once, however it introduces a new and quite fundamental problem. This problem can be traced back to the fact that by introducing the Higgs, we introduce a scalar field into the theory. The bare mass of a scalar field receives radiative corrections that are ultraviolet divergent and thus need to be regularized by introducing a cut-off scale Λ, i.e. in the case of the Higgs [7]:

$$m_\phi^2 = \mu^2 + \underbrace{\frac{\lambda}{8\pi^2}\Lambda^2 - \frac{3}{8\pi^2}(Y_u)_{33}^2\Lambda^2 + \ldots}_{\Delta\mu^2} \tag{1.22}$$

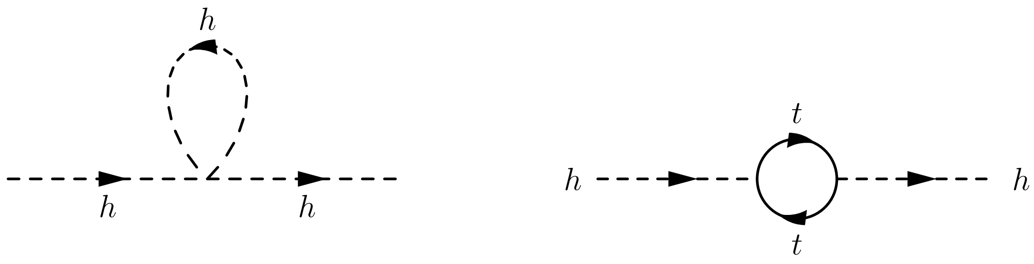


Figure 1.2: *Radiative corrections to the Higgs mass. The main contribution is due to the top quark in the loop.*

These corrections are due to Higgs and fermion loops as shown in fig. 1.2.

The SM can be seen as an effective theory of some more complete theory. Λ is then interpreted as the largest energy scale at which the theory is still valid. This could be the Planck scale since we know for sure that at this scale - where gravity becomes strong - quantum field theory (QFT) will break down.

From theory we can infer an approximate mass for the Higgs: from the masses of the gauge bosons we know the value of the vacuum expectation value $v = 246$ GeV. As λ cannot get too large if the theory is to stay perturbative, we must have $m_\phi^2 \sim (100 \text{ GeV})^2$. Indeed, this is in agreement with the new boson observed by ATLAS and CMS [3, 4]: Both experiments find $m_\phi \approx 126$ GeV. The bare mass parameter μ cannot possibly be know, so it is conceivable that it is very large and has opposite sign to $\Delta\mu^2$. Then it could cancel the large radiative corrections, up to the physical Higgs mass m_ϕ^2.

However, look at the numbers: $\Lambda^2 = M_{\text{Planck}}^2 \sim (10^{19} \text{ GeV})^2 \sim 10^{38} \text{ GeV}^2$ and $m_\phi^2 \sim (100 \text{ GeV})^2 \sim 10^4 \text{ GeV}^2$. This means that the cancellation would need to be as precise as $1 : 10^{34}$. Most people think that this is not a good solution to the problem. How can the bare mass know to such high precision the value of the radiative corrections? This is what is called the *hierarchy problem*.

Another solution is of course to choose a smaller cut-off Λ. But then we need a new theory already at this scale. And this scale should not be much higher than the electroweak scale (i.e. the scale of the Higgs vacuum expectation value). So experiments, especially those at the LHC will tell soon. Ideas about what this new theory could be include supersymmetry and new strongly coupled dynamics.

Another way to phrase the hierarchy problem is to ask: Why is the scale of electroweak symmetry breaking (EWSB) so much lower than the Planck scale? Or: Why is gravity so much weaker that the other forces?

1.2 The Flavour Puzzle

In nature, fermions come in three generations or flavours, i.e. for each particle there are two other particles that have exactly the same quantum numbers but a different mass. Therefore, the Yukawa couplings in eq. (1.19) are 3×3 matrices. A priori, these matrices are not diagonal and to go to the so-called mass basis we must therefore make unitary transformations on the fermion fields [6, 8]:

$$u_R = V_{uR} u_R \tag{1.23}$$

$$u_L = V_{uL} u_L \tag{1.24}$$

and analogously for the down quarks and charged leptons. The mass matrices

$$M_u = v \cdot V_{uL} Y_u V_{uR}^\dagger = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \tag{1.25}$$

etc. are then diagonal. However, look at the couplings of the fermions to the charged weak gauge bosons $W^\pm$:

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}} \left(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L \right) W_\mu^+ + \text{h.c.} \tag{1.26}$$

since V_{uL} and V_{dL} are not identical, the coupling of up and down quarks to the W is no longer diagonal but proportional to a matrix

$$V_{\text{CKM}} = V_{uL}^\dagger V_{dL} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1.27}$$

called the *Cabibbo-Kobayashi-Maskawa (CKM) matrix*. Being a unitary matrix, we can parameterize the CKM matrix as a product of rotation matrices:

$$V_{\text{CKM}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-\imath \delta_{\text{KM}}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{\imath \delta_{\text{KM}}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{\imath \delta_{\text{KM}}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{\imath \delta_{\text{KM}}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{\imath \delta_{\text{KM}}} & c_{23} c_{13} \end{pmatrix} \tag{1.28}$$

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. We will learn later about the significance of the phase δ_{KM}. In other words, quarks of different generations can mix. It is very important to note that this does not happen in the case of the exchange of a neutral Z boson or a photon:

$$\mathcal{L} \supset -e \sum_{\substack{i=u,d,c,s,t,b,\\ e,\mu,\tau}} \bar{\psi}_i \gamma^\mu Q_i \psi_i A_\mu - \frac{e}{\sin 2\theta_W} \sum_{\substack{i=u,d,c,s,t,b\\ e,\mu,\tau,\nu_e,\nu_\mu,\nu_\tau}} \bar{\psi}_i \gamma^\mu (v_i - a_i \gamma^5) \psi_i Z_\mu \tag{1.29}$$

where $\psi_u = u_L + u_R$ etc. this is due to the unitarity of the matrices used to diagonalize the Yukawas:

$$V_{uL}^\dagger V_{uL} = \mathbb{1}, \text{ etc.} \tag{1.30}$$

In the SM, at tree level there are no flavour changing neutral currents (FCNCs). This will become very important later.

The experimental values for the CKM matrix and the eigenvalues of the quark mass matrix are [9]:

up	**charm**	**top**
$2.3^{+0.7}_{-0.5}$ MeV	$1.275^{+0.025}_{-0.025}$ GeV	$173.5 \pm 0.6 \pm 0.8$ GeV
down	**strange**	**bottom**
$4.8^{+0.7}_{-0.3}$ MeV	95^{+5}_{-5} MeV	$4.18^{+0.03}_{-0.03}$ GeV

$$V_{\text{CKM}} = \begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & (3.89 \pm 0.44) \cdot 10^{-3} \\ 0.230 \pm 0.011 & 1.023 \pm 0.036 & (40.6 \pm 1.3) \cdot 10^{-3} \\ (8.4 \pm 0.6) \cdot 10^{-3} & (38.7 \pm 2.1) \cdot 10^{-3} & 0.88 \pm 0.07 \end{pmatrix}$$

These can be measured by assuming that NP effects can be neglected in all cases where the process can occur at tree level in the SM. Several questions come to mind:

- Why are there three generations? Why not only one? Or more than three? (Are there only three?)
- Why are the mixing angles so small?
- Why is there such a strong hierarchy within the quark masses?

These open questions are usually referred to as the *flavour puzzle*.

1.3 Other Open Questions: Dark Matter, Neutrino Masses, Cosmology

There are other open questions in modern particle physics which we will mention here briefly for the sake of completeness.

Dark Matter

The universe seems to be permeated by a yet unknown form of matter called *Dark Matter*. This can be deduced from the rotation of stars in galaxies, from simulations of galaxy formation and from galaxy cluster dynamics (via gravitational lensing).

Although these findings could also be explained by modifying Newtonian dynamics (so-called MOND theories [10, 11, 12]) most physicists nowadays suppose that Dark Matter is composed of yet unknown particles. These particles do not interact via the electromagnetic or strong forces but could interact via the weak force. Some Beyond the Standard Model (BSM) theories have a Dark Matter candidate, which makes them more compelling in the opinion of some people.

Neutrino Masses

The experimental result that neutrinos have non-zero but tiny masses came as quite a surprise.
Given that neutrinos do not have any conserved charge, we can write a *Majorana mass term* for them [13, 14]:

$$\mathcal{L} \supset -m\psi^T C^{-1}\psi \tag{1.31}$$

where C is the charge conjugation operator. In addition, we can also write a *Dirac mass term* as for the other fermions:

$$\mathcal{L} \supset -m\bar{\psi}\psi = -m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right) \tag{1.32}$$

We note the following: A Dirac term as in eq. (1.32) can only exist if there are also right-handed neutrinos ν_R in addition to the particle content of the SM. A Majorana mass term as in (1.31) is not possible for the left-handed neutrino ν_L because it has weak isospin projection $I_3 = 1/2$ and the Majorana mass term is thus not invariant under isospin transformations. Such a term is possible for right-handed neutrinos being singlets under all transformations, however.
The presence of heavy right-handed neutrinos could explain the smallness of the masses

of the left-handed neutrino. This is the famous *see-saw mechanism*. Basically, a mass term for the left-handed neutrinos is generated by integrating out the heavy right-handed neutrinos: The full Lagrangian with Majorana and Dirac mass terms reads:

$$\mathcal{L} \supset (m_D)_{ij}\bar{\nu}_{Li}\nu_{Rj} + \frac{1}{2}M_{ij}\bar{\nu}^c_{Ri}\nu_{Rj} + \text{h.c.} \tag{1.33}$$

Using $\bar{\nu}_L m_D \nu_R = \bar{\nu}^c_R m^T_D \nu^c_L$ we can write this in matrix form:

$$\mathcal{L} \supset \frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \bar{\nu}^c_R\end{pmatrix}\begin{pmatrix}0 & m_D \\ m^T_D & M\end{pmatrix}\begin{pmatrix}\nu^c_L \\ \nu_R\end{pmatrix} + \text{h.c.} \tag{1.34}$$

Diagonalizing this, one finds the mass eigenvalues M and $-m_D\frac{1}{M}m^T_D$ (these "eigenvalues" are matrices of course since neutrinos also come in three generations). We see that the masses of the light, left-handed neutrinos are indeed suppressed by the heavy mass scale of the right-handed neutrinos. However, up to this day nothing of this is confirmed experimentally.

Cosmology

In cosmology there are also some fundamental questions which could not yet be answered. One of them is why there is more matter than anti-matter in the universe. Such an asymmetry, also called baryon asymmetry, can be generated only if the three Sakharov criteria are fulfilled [15]:

1. Baryon number violation. (This is obvious).

2. C and CP violation. (This is necessary because one can prove that, if C and CP are conserved, the rate of any process that produces an excess of baryons is equal to the rate of the complementary process that produces an equal excess of anti-baryons. Thus no net baryon asymmetry can be produced without C and CP violation.)

3. Departure from thermal equilibrium. (The equilibrium average of the baryon number is zero.)

There are mainly two ideas around how the baryon asymmetry could have been produced [16, 17, 18, 19]:

- **Baryogenesis**: A baryon asymmetry is generated directly by processes fulfilling the Sakharov conditions.

- *GUT baryogenesis*: Since quarks and leptons are in the same representation, there is always baryon number violation in *grand unified theories (GUTs)*. Moreover, there are many possible complex phases to generate the necessary CP violation (explicit CP breaking). A departure from thermal equilibrium is also guaranteed as the time scales of the decays of heavy scalars or gauge bosons are slow compared to the expansion of the universe. The problem of GUT baryogenesis is that it is not accessible to experiments and thus cannot be verified.
- *Electroweak Baryogenesis*: The SM Lagrangian has an accidental symmetry that implies the conservation of baryon number. It is thus not possible to violate baryon number at any order of perturbation theory. There are non-perturbative processes called sphalerons however that violate the sum of baryon and lepton number, $B+L$ but conserve $B-L$. The probability for these processes to occur today is exponentially suppressed, but in the early universe these processes could happen at high rates. The out-of-equilibrium condition is fulfilled if there is a strong first order electroweak phase transition in the early universe. The problem is that in the SM there is not enough CP violation to make EW baryogenesis work. In the *Minimal Supersymmetric Model (MSSM)* there are additional sources of CP violation, so this was long time thought to be a good way to generate the observed baryon asymmetry, especially because this is a scenario that is falsifiable at colliders [20]. And since it is falsifiable it was falsified. The requirement of a strong first order phase transition translates into a bound on the Higgs mass of $m_H \lesssim 120$ GeV. Given the latest development on the Higgs frontier, cf. sec. 1.1, this looks rather bad.

- **Leptogenesis**: We have mentioned the possibility that the smallness of the masses of left-handed neutrinos is a hint to the existence of heavy right-handed neutrinos in the last paragraph. Now, the decay of heavy right-handed neutrinos could also generate a lepton asymmetry that via sphaleron processes could be converted into a baryon asymmetry. CP violation is generated in these decays by the interference of tree level and one-loop diagrams. The decay is out of equilibrium if its width is small compared to the expansion rate of the universe. The disadvantage of this model is the same as for GUT baryogenesis: It is not testable at colliders in the not-too-distant future.

Another yet unsolved problem in cosmology is why the cosmological constant is so much smaller than the vacuum energy predicted by QFT: Assuming a cut-off at the

Planck-scale, the vacuum energy should be of the order of

$$\rho_\Lambda^{\text{QFT}} \sim (10^{18}\ \text{GeV})^4 \sim 2 \cdot 10^{110}\,\frac{\text{erg}}{\text{cm}^3} \tag{1.35}$$

while the observed cosmological constant (that can be interpreted as the vacuum energy) is:

$$|\rho_\Lambda^{\text{obs.}}| \leq (10^{-12}\ \text{GeV})^4 \sim 2 \cdot 10^{-10}\,\frac{\text{erg}}{\text{cm}^3} \tag{1.36}$$

This can be concluded from the measurement of the redshift of distant supernovae Ia and from measurements of the density fluctuations of the *cosmic microwave background (CMB)*. Of course, the vacuum energy needs to be renormalized but if we are to solve the *cosmological constant problem* by renormalization only, the fine-tuning is a lot worse than for the Higgs mass even.

Other open questions in cosmology include the question what drives inflation and, of course, at the end of the day general relativity needs to be reconciled with quantum field theory.

1.4 Physics Beyond the Standard Model

In order to resolve one or several of the problems exposed in section 1 several "Beyond the Standard Model" theories have been proposed and we will briefly sketch the main ideas of the more famous ones of them here. Two BSM theories, the *Two Higgs Doublet Model (2HDM)* and *warped extra-dimensions* will be treated in more detail in chapters 3 and 4, respectively, and will therefore not be mentioned here.

1.4.1 Supersymmetry

Supersymmetry (SUSY) is a space-time symmetry that relates fermions and bosons. The supersymmetry generator Q transforms a fermion into a boson and vice versa [7]:

$$Q\,|\text{boson}\rangle = |\text{fermion}\rangle \qquad\qquad Q\,|\text{fermion}\rangle = |\text{boson}\rangle \tag{1.37}$$

The irreducible representations of the supersymmetry are called *supermultiplets*. They contain fermions and bosons, called *superpartners*. Since the supersymmetry generator Q commutes with the generators of the gauge group, fermions and bosons in the same supermultiplet have the same electric charge, weak isospin and color degree of freedom. The simplest way to assign the particles of the SM to supermultiplets is as follows: All the SM fermions must be in so called *chiral supermultiplets* consisting of a single Weyl fermion and a complex scalar. The superpartners of the fermions are called *sfermions*: *squarks* and *sleptons*, see table 1.1.

The gauge bosons are in *gauge supermultiplets*, paired with spin 1/2 fermions called *gauginos*: *gluinos*, *winos*, *binos*, see table 1.2. It is not possible to pair the gauge bosons with the quarks and leptons of the SM because the fermionic superpartners of the gauge bosons must have the same gauge transformation properties for left- and right-handed states, which is not the case for the fermions of the SM.

The Higgs boson resides in a chiral supermultiplet, see table 1.1. Actually there are two Higgs supermutliplets ϕ_u and ϕ_d. This is necessary since the condition for the gauge anomaly of the electroweak symmetry to vanish contains $\mathrm{Tr}(Y^3) = 0$ where Y is the weak hypercharge. In the SM this is miraculously satisfied. However, if we introduce additional fermions, as is done in supersymmetry, since the trace runs over all fermions, the condition is no longer necessarily satisfied. A second Higgs is thus needed in order to cancel the contribution to $\mathrm{Tr}(Y^3)$ coming from the superpartner of the first Higgs (called *Higgsino*). Another reason to introduce the second Higgs is that only a $Y = 1/2$ Higgs can have Yukawa couplings to up-type quarks and only a $Y = -1/2$ Higgs can have Yukawa couplings to down-type quarks and charged leptons: In the SM we have:

$$\mathcal{L}_{\text{Yukawa}} = \bar{q}_L Y_u u_R \tilde{\phi} + \bar{q}_L Y_d d_R \phi + \bar{l}_L Y_e e_R \phi + \text{h.c.} \tag{1.38}$$

where $\tilde{\phi} = i\tau_2\phi^*$, see eq. (1.19). Supersymmetric Lagrangians however need to be analytic and thus cannot contain a complex conjugate field ϕ^*. Thus we need two Higgses: one that couples to the up-type quarks and one that couples to the down-type quarks and charged leptons:

$$\mathcal{L}_{\text{Yukawa}} = \bar{q}_L Y_u u_R \phi_u - \bar{q}_L Y_d d_R \phi_d - \bar{l}_L Y_e e_R \phi_d \tag{1.39}$$

The model described thus far is called the *minimal supersymmetric model (MSSM)*. There are other supersymmetric models, containing additional scalars or several copies of the supersymmetry generator Q. Some of them are motivated by phenomenology, while others are merely interesting from a theoretical point of view. Here, we will deal with the MSSM only.

One of the main arguments in favor of supersymmetry is that it solves the hierarchy problem: Going back to eq. (1.22) we see that contributions coming from scalars (in this case the Higgs) have opposite sign than contributions due to fermions (in this case the top quark). Now in supersymmetry we have a boson for every fermion and their couplings are related in a way such that the radiative corrections to the Higgs mass exactly cancel.

supermultiplet	**bosons (spin 0)**	**fermions (spin 1/2)**
q	$\left(\widetilde{u}_L\ \widetilde{d}_L\right)$	$(u_L\ d_L)$
$\bar{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$
$\bar{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$
l	$(\widetilde{\nu}\ \widetilde{e}_L)$	$(\nu\ e_L)$
$\bar{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$
ϕ_u	$(\phi^+_u\ \phi^0_u)$	$\left(\widetilde{\phi}_u\ \widetilde{\phi}^0_u\right)$
ϕ_d	$\left(\phi^0_d\ \phi^-_d\right)$	$\left(\widetilde{\phi}^0_u\ \widetilde{\phi}^-_u\right)$

Table 1.1: *Chiral supermultiplets in the MSSM. Note that left- and right-handed fermions have each their own superpartner (The subscripts "L" and "R" for the bosons are just part of the name, they do not indicate the chirality of the particle - they are scalars, after all!)*

fermions (spin 1/2)	**bosons (spin 1)**
$\widetilde{g}$	g
$\widetilde{W}^\pm\ \widetilde{W}^0$	$W^\pm\ W^0$
$\widetilde{B}^0$	B^0

Table 1.2: *Gauge supermultiplets in the MSSM. After electroweak symmetry breaking the gauge eigenstates W^0 and B^0 mix to give the mass eigenstates Z^0 and γ. Correspondingly, their superpartners mix to give the mass eigenstates $\widetilde{Z}^0$ and $\widetilde{\gamma}$, the* zino *and the* photino.

Life ain't that good, of course. Nobody has ever observed any of the superpartners of the SM particles. If supersymmetry were unbroken, they should have exactly the same masses as the SM particles. Since they are yet undiscovered, they must be heavier and supersymmetry must be broken.

Soft SUSY breaking scenarios ensure that the relations in between the fermion and boson couplings hold also in broken SUSY. There are no thus no quadratically divergent contributions to the Higgs mass such as:

$$\Delta\mu^2 = \frac{1}{8\pi^2}\left(\lambda_s - |\lambda_f|^2\right)\Lambda^2 \tag{1.40}$$

where λ_s and λ_f represent the scalar and fermion couplings, respectively. The problem is that soft SUSY breaking introduces a lot of undetermined new parameters, the *soft masses*. The MSSM with soft SUSY breaking still has some amount of fine-tuning [21, 22]: For the Higgs mass at one-loop level you get:

$$m_h^2 = m_Z^2 + \frac{3G_F m_t^4}{\sqrt{2}\pi^2}\left[\log\frac{m_{\tilde{t}}^2}{m_t^2} + X_t^2\left(1 - \frac{X_t^2}{12}\right)\right] \tag{1.41}$$

where $X_t^2 = |A_t|^2/m_{\tilde{t}}^2$ and A_t is the scalar[3]-coupling of the stop. Already LEP set the lower bound of the Higgs mass at 114.4 GeV [23]. To satisfy this bound we need a stop mass $m_{\tilde{t}} \geq 1$ TeV. All scalar masses at the high energy scale should be of the same order for magnitude. Look at the mass of the Z however. In the MSSM it can be calculated to be:

$$m_Z^2 \approx -2m_{\phi_u}^2(M_Z) - 2\mu^2 \tag{1.42}$$

where m_{ϕ_u} is the soft mass of ϕ_u. Since we have $m_{\phi_u}^2 \sim \mu^2 \sim 1$ TeV, we need a fine-tuning of about 1 % to get the correct Z mass of $m_Z = 91$ GeV. This is called the *little hierarchy problem*.

Another problem with supersymmetry is that it can lead to proton decay: The supersymmetric Lagrangian is constructed from the *superpotential* which a priori contains terms that violate baryon or lepton number:

$$W_{\Delta L=1} = \frac{1}{2}\lambda_{ijk} l_i l_j \bar{e}_k + \lambda'_{ijk} l_i q_j \bar{d}_k + \mu'_i l_i \phi_u \tag{1.43}$$

$$W_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk}\bar{u}_i \bar{d}_j \bar{d}_k \tag{1.44}$$

Due to processes as the one shown in fig. 1.3 the lifetime of the proton would be extremely short if these couplings were present and unsupressed. To get rid off these unpleasant terms a new symmetry called *matter parity* that forbids them is imposed

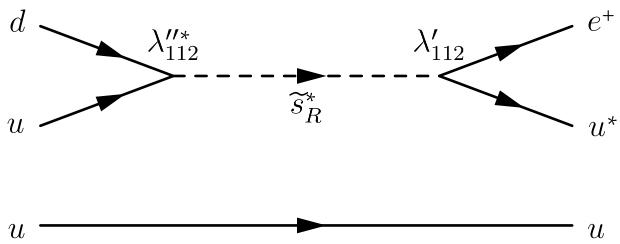


Figure 1.3: *Proton decay in the MSSM without R-parity: $p \to \pi^0 e^+$.*

on the Lagrangian. It is defined as:

$$P_M = (-1)^{3(B-L)} \tag{1.45}$$

Only terms with matter parity +1 are allowed in the Lagrangian. Matter parity can be recast as *R-parity*:

$$P_R = (-1)^{3(B-L)+2s} \tag{1.46}$$

Matter parity and R-parity conservation are precisely equivalent and we notice the following fact: The SM particles have all R-parity +1 while the new, yet undiscovered superpartners (*sparticles*) have R-parity -1. Therefore every interaction vertex must contain an even number of sparticles and the *lightest supersymmetric particle (LSP)* is stable! It is therefore a good Dark Matter candidate, provided that it is electrically neutral and colorless.
A third motivation for supersymmetry is that the running gauge couplings of the weak, electromagnetic and strong forces meet in one point only if supersymmetry is assumed. This is interesting in the view of *grand unified theories (GUTs)* that unify all three forces.

1.4.2 Extra Dimensions

At the turn of the millennium two main ideas of how to solve the hierarchy problem using extra dimensions of space-time were put forward: *warped extra dimensions*, also known as the *Randall-Sundrum (RS) model* [24] and *large extra dimensions* also known as the *Arkani-Hamed Dimopoulos Dvali (ADD) model* [25, 26, 27]. Since the former will be treated in detail in chapter 4 we will briefly sketch only the latter here.

Kaluza-Klein states

To start with however let us consider a 5D scalar field ϕ that lives in a space-time where the 5th dimension is compactified on a circle of radius R, i.e. the points $y = -\pi R$

and $y = \pi R$ are identified (y is the coordinate along the fifth dimension). Due to these periodic boundary conditions we can Fourier-expand the field ϕ with respect to the coordinate y [28]:

$$\phi(x, y) = \sum_{n=-\infty}^{\infty} \phi_n(x) e^{i\frac{ny}{R}} \tag{1.47}$$

Plugging this into the Lagrangian of a massless scalar field in five dimensions, $\mathcal{L} = \frac{1}{2}\partial_A\phi\partial^A\phi$ (A runs over all five dimensions) we get:

$$\mathcal{L} = \frac{1}{2}\sum_n \left[\partial_\mu\phi_n\partial^\mu\phi_n - m_n^2\phi_n^2\right] \tag{1.48}$$

where $m_n = n/R$ and μ runs only over the four non-compact dimensions. We see that our scalar field has a *Kaluza-Klein (KK) tower* of heavier particles with masses m_n. Fermion, vector and tensor fields have similar KK towers of heavier particles. Since none of these KK states has ever been discovered, all of them must be heavier than the discovery reach of past colliders. The massless $n = 0$ field is called the *zero-mode*. The zero-modes correspond to the particles of the SM.

Large extra dimensions

In ADD it is assumed that space time has n compact extra dimensions. Gauß' law then tells us that the Planck scale $M_{\mathrm{Pl}}^{\mathrm{4D}}$ measured in the effective 4D theory is related to the fundamental Planck scale M_{Pl} by [29, 28, 30]:

$$(M_{\mathrm{Pl}}^{\mathrm{4D}})^2 = V_n M_{\mathrm{Pl}}^{2+n} \tag{1.49}$$

where V_n is the volume of the extra dimensions. In other words, depending on the size of the volume of the extra dimension, the fundamental Planck scale might be a lot smaller than what was previously thought. The enormousness of the Planck scale would be only a phantasm of our 4D viewpoint! The hierarchy problem would be solved! A radical revision of quantum physics in order to include gravity would be needed already just above the electroweak scale!

As an example, consider the extra dimensions to be compactified on an n-Torus of radius R (for an example of a 2-torus see fig. 1.4). The volume of a torus is $V_n = (2\pi R)^n$. Putting in the value for the 4D Planck scale and requesting the fundamental Planck scale to be $M_{Pl} \sim 1$ TeV, we can calculate R as a function of the number of extra dimensions. If there were only one extra-dimension it would need to be almost as big as the solar system, which is of course ruled out. For two extra-dimensions we get $R \sim 200\ \mu$m, at the brink of what is still allowed by experiments searching for

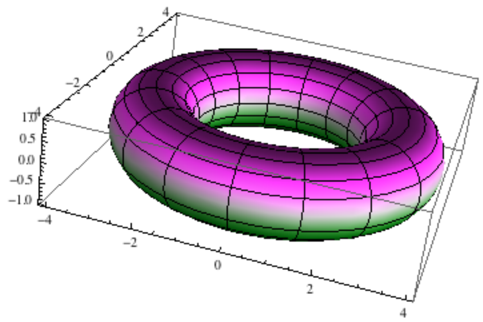


Figure 1.4: *A two-dimensional Torus*

deviations from Newton's law of gravity.

We have seen in the previous paragraph that particles that propagate in compact extra-dimensions of space-time have a KK tower of heavier particles associated with them. Since no KK states of SM particles have been observed so far, this means that their masses must be $\gtrsim 100$ GeV. The mass of the lightest KK particle is $\sim 1/R$ and therefore we must have $1/R \geq 100$ GeV. To get a fundamental Planck scale of 1 TeV, we would then need $n \geq 10$. This would exclude superstring theory, since it needs $n = 6$ or $n = 7$. There is a way out however: particles that do not propagate in the extra-dimensions do not have a KK tower associated with them. One therefore usually assumes that the SM particles are confined to the four dimensional boundary, the *brane* and that only the graviton propagates into the extra dimensions.

We have seen, eq. (1.49), that ADD can bring down the fundamental Planck scale to the weak scale. However, this is more of a reformulation than a solution to the hierarchy problem. The reason is that instead of having to explain why the weak scale is so much smaller than the Planck scale we now need to explain why the volume of the extra dimensions V_n is so much bigger than $1/M_{\text{Pl}}^n$. Supersymmetry could help solve this problem as we shall see in the next paragraph.

The radion

According to General Relativity, the size and shape of the extra-dimensions is dynamical as is all of spacetime. Consider the metric of a five-dimensional spacetime [30]:

$$
\begin{aligned}
\mathrm{d}s^2 &= G_{AB}(X)\mathrm{d}X^A\mathrm{d}X^B \\
&= G_{AB}(X(X'))\frac{\partial X^A}{\partial X'^{A'}}\mathrm{d}X'^{A'}\frac{\partial X^B}{\partial X'^{B'}}\mathrm{d}X'^{B'} \qquad (1.50)\\
&= G_{A'B'}(X')\mathrm{d}X'^{A'}\mathrm{d}X'^{B'} \qquad (1.51)
\end{aligned}
$$

where $X = (x, y)$ and obviously:

$$G_{A'B'}(X') = G_{AB}(X(X')) \frac{\partial X^A}{\partial X'^{A'}} \frac{\partial X^B}{\partial X'^{B'}} \tag{1.52}$$

Expanding the metric around flat spacetime,

$$G_{AB}(X) = \eta_{AB} + h_{AB}(X) \tag{1.53}$$

we get in almost axial gauge[1]:

$$h_{\mu 5}(x, y) = h^{(0)}_{\mu 5}(x) \tag{1.54}$$

$$h_{55}(x, y) = h^{(0)}_{55}(x) \tag{1.55}$$

$$h_{\mu\nu}(x, y) = h^{(0)}_{\mu\nu}(x) + \sum_{n=1}^{\infty} (h^{(n)}_{\mu\nu}(x) e^{\frac{iny}{R}} + \text{c.c.}) \tag{1.56}$$

Since the Einstein action written in terms of the 5D Ricci scalar contains only terms with derivatives, the $h^{(0)}_{AB}$ are massless 4D fields without any potential at all. The KK states $h^{(n)}_{\mu\nu}$ have masses $m_n = n/R$ as we have seen above. The fields $h^{(0)}_{\mu 5}$ can be gauged away but the massless field $h^{(0)}_{55}$ remains. What is the role of this field? Since it is independent of the extra dimensional coordinate y and doesn't have a potential, it can have any vev:

$$\langle h^{(0)}_{55} \rangle = \xi \tag{1.57}$$

Assuming $\langle h^{(0)}_{\mu\nu} \rangle = 0$ and $\langle h^{(0)}_{\mu 5} \rangle = 0$ and that the extra dimension is compactified on a circle with radius R, we get for the vev of the line element:

$$\langle \mathrm{d}s^2 \rangle = \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu - (1 + \xi) \mathrm{d}y \mathrm{d}y \tag{1.58}$$

$$= \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu - (1 + \xi) R^2 \mathrm{d}\phi^2 \tag{1.59}$$

In other words, the vev of the radius of the extra dimension is $R' = \sqrt{1 + \xi} R$. Fluctuations of the scalar field $h^{(0)}_{55}$ thus correspond to fluctuations of the radius of the extra dimension. It is therefore called the *radion*.

Now back to the problem that we need to explain why V_n is so big in ADD. In the case of an extra-dimension compactified on a circle a big V_n means a large vev for the radius, $\langle R' \rangle$. Fermion loops induce an effective potential for the radion. Supersymmetry will then cause this potential to be flat at large values of R', thus making $\langle R' \rangle$ big.

[1] Almost axial gauge is the gauge where the fifth component of a field is independent from the coordinate along the extra-dimensions, in this case y.

Universal Extra Dimensions

In *Universal Extra Dimensions (UED)* [31] all fields are allowed to propagate in the extra-dimensions. These extra-dimensions (often there is only one) have radii of $\sim 10^{-18}$ m and are compactified on an orbifold in order to reproduce the SM at low energy. These models do not address the hierarchy problem, but they can provide a DM candidate as we shall see in the following.

KK parity The lightest KK particle (called *LKP*) cannot decay to its zero mode or to other SM particles. This is due to the so-called *Kaluza-Klein parity*, a conserved multiplicative quantum number: The number n of the Kaluza-Klein level of a particle is a measure of its (quantized) momentum in the extra dimension. Since in UED no location along the extra-dimension is exceptional, i.e. there is translational invariance, extra-dimensional momentum conservation and thus KK-number conservation holds, according to Noether's theorem. The orbifolding however breaks the translational invariance. It can be shown however that the KK-parity, defined as $(-1)^n$, is still conserved [32, 33]. This is enough to have the LKP stable. If it is neutral it can then be Dark Matter.

Chapter 2

The Problem with Flavour

2.1 Flavour and the Standard Model

We already said in sec. 1.2, that in the SM there are no flavour changing neutral currents at tree level. Let's pause here for a moment and reflect on why this is the case. Among the particles of the SM there are four neutral bosons, that could mediate FCNCs at tree level: the gluon, the photon, the Higgs and the Z^0 boson [34, 35, 36, 37]. The gluon and the photon correspond to exact gauge symmetries and their couplings to the fermions arise from the kinetic terms. Their couplings are thus *universal*, i.e. proportional to the identity matrix. However you rotate a universal matrix, it will always stay universal and thus diagonal. Gluons and photons therefore cannot mediate FCNCs.

Now, consider the Higgs' couplings: The Higgs couples to the fermions via Yukawa couplings, see eq. (1.19). Inserting

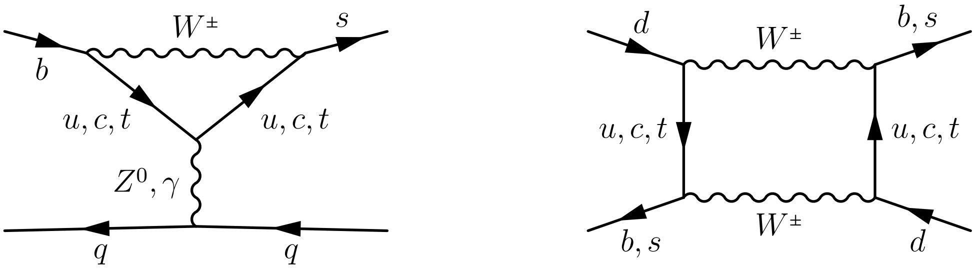
$$\phi = \begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{2.1}$$

where h are the fluctuations of the Higgs field around its vacuum expectation value, we get:

$$\mathcal{L}_{\text{Yukawa}} = \bar{u}_L \underbrace{vY_u}_{M_u} u_R + \bar{u}_L Y_u u_R h + \bar{d}_L \underbrace{vY_d}_{M_d} d_R + \bar{d}_L Y_d d_R h \tag{2.2}$$

Obviously, the Higgs' couplings Y_u, Y_d are diagonal in the same basis as the mass matrices $M_u = vY_u, M_d = vY_d$. One says that the Yukawa couplings and the mass matrices are *aligned*.

Finally, we already saw in section 1.2 that the Z^0 couplings are also diagonal in the



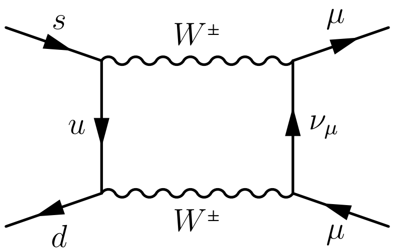
(a) penguin diagram (b) box diagram

Figure 2.1: *Examples of FCNCs in the SM at one-loop level.*

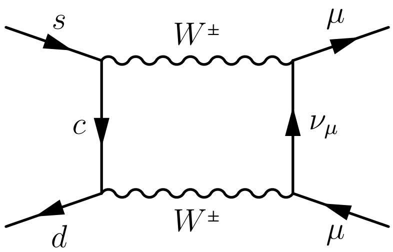
mass basis. This is due to the unitarity of the transformation matrices V_{uL}, V_{dL}. FCNCs can occur via $W^{\pm}$ loops, however, as is shown in fig. 2.1. Since these are loop processes, their rate is highly suppressed, a fact that is confirmed by experiment and poses considerable problems when one tries to extend the SM.

The GIM mechanism

In the past, it was not understood why the decay $K_L \to \mu^+\mu^-$ was not observed. At that time, the charm quark had not yet been discovered and the decay was thought to occur via:



However, the existence of a fourth quark was predicted by Lee Glashow, Jean Illiopoulos and Luciano Maiani [38]. It would contribute a second Feynman diagram:



The amplitude is then proportional to the sum of these diagrams while the diagrams depend on a combination of matrix elements of the CKM matrix and the mass of the quark in the loop:

$$\mathcal{A}(K_L \to \mu^+\mu^-) \propto f(m_u)V_{us}V_{ud}^* + f(m_c)V_{cs}V_{cd}^* \tag{2.3}$$

On the other hand, the CKM matrix is unitary and we must therefore have (for two generations of quarks):

$$V_{us}V_{ud}^* + V_{cs}V_{cd}^* = 0 \tag{2.4}$$

That means that the parts independent of $m_{u,c}$ in 2.3 will cancel. Explicit calculation shows that the leading term in $f(m_{u,c})$ is $\propto m_{u,c}^2/m_W^2$. This explains why the decay $K_L \to \mu^+\mu^-$ is not observed: Even the leading contribution to its amplitude is still suppressed by m_c^2/m_W^2 so its rate is very small. This allowed to set an upper bound on the charm quark's mass.

CP violation

Let us turn to one other important aspect of flavour physics in the SM before we will consider new physics models in the light of flavour. How many parameters are there in the SM quark flavour sector? There are two Yukawa matrices in the Lagrangian, so this would make up for $2 \times 9 = 18$ real parameters and $2 \times 9 = 18$ imaginary ones. This is without taking possible field redefinintions that absorb some of these parameters, however. The transformations

$$q_L \to V_q q_L \qquad u_R \to V_u u_R \qquad d_R \to V_d d_R \tag{2.5}$$

leave the Lagrangian invariant, apart from a rotation of the Yukawas:

$$Y_u \to V_q Y_u V_u^\dagger \qquad Y_d \to V_q Y_d V_d^\dagger \tag{2.6}$$

Each of the rotation matrices V_q, V_u, V_d contain three real parameters and six imaginary ones. We can use these rotations to remove unphysical parameters. Out of the 18 real parameters we can remove $3 \times 3 = 9$ and out of the 18 imaginary parameters we can remove 17. We cannot remove the full 18 imaginary parameters since the Lagrangian has a $U(1)_{B-L}$ symmetry. This leaves us with 9 real parameters, the 6 quark masses and the three angles of the CKM matrix. The left-over imaginary parameter corresponds to the phase δ_{KM} in the CKM matrix. A complex Yukawa coupling is related to CP violation, as can be seen as follows: The hermiticity of the Lagrangian implies that it has Yukawa couplings of the form:

$$\bar{\psi}_L Y \psi_R \phi + \bar{\psi}_R Y^* \psi_L \phi^\dagger \tag{2.7}$$

If we perform a CP transformation we get:

$$\bar{\psi}_L Y \psi_R \phi + \bar{\psi}_R Y^* \psi_L \phi^\dagger \xrightarrow{CP} \bar{\psi}_R Y \psi_L \phi^\dagger + \bar{\psi}_L Y^* \psi_R \phi \tag{2.8}$$

We thus have CP conservation if $Y = Y^*$. The phase of the CKM matrix is the only source of CP violation in the SM.

2.2 Flavour in New Physics Models

Flavour in new physics models can be studied in a model-independent manner using an *effective field theory (EFT)* approach. In EFT one integrates out fields that are heavier than the energy scale one is interested in. This leaves us with terms in the Lagrangian that have negative mass dimension, i.e. that are suppressed by a mass scale M, the scale where new physics kicks in. Of course, such a theory is non-renormalizable. This is not a problem however, since we do not claim that it is valid at all energy scales. At the scale M the new physics needs to be taken into account and the complete theory should then be renormalizable. Summing up, we have [36]:

$$\mathcal{L}_{\text{eff.}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(d)}}{M^{d-4}} \mathcal{O}_i^{(d)} \tag{2.9}$$

where $\mathcal{O}^{(d)}$ are operators made up of SM fields and the coefficients $C_i^{(d)}$, called *Wilson coefficients* are of course unknown. For a given NP model they can be calculated by integrating out the new particles at the NP scale M and evolving these couplings down to the scale where the experiments are performed, i.e. for example the mass scale of the meson that is studied: 4.6 GeV for bottom mesons, 2.8 GeV for charmed mesons and 2 GeV for kaons. In addition, the hadronic matrix element of the meson has also to be calculated in order to compare to experiment.

A complete set of $\Delta F = 2$ operators (i.e. processes that change flavour by two units)

Operator	Bounds on M in TeV ($C_i^{(d)} = 1$)		Bounds on $C_i^{(d)}$ ($M = 1$ TeV)	
	Re	Im	Re	Im
$(\bar{s}_L\gamma^\mu d_L)^2$	$9.8 \cdot 10^2$	$1.6 \cdot 10^4$	$9.0 \cdot 10^{-7}$	$3.4 \cdot 10^{-9}$
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	$1.8 \cdot 10^4$	$3.2 \cdot 10^5$	$6.9 \cdot 10^{-9}$	$2.6 \cdot 10^{-11}$
$(\bar{c}_L\gamma^\mu u_L)^2$	$1.2 \cdot 10^3$	$2.9 \cdot 10^3$	$5.6 \cdot 10^{-7}$	$1.0 \cdot 10^{-7}$
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	$6.2 \cdot 10^3$	$1.5 \cdot 10^4$	$5.7 \cdot 10^{-8}$	$1.1 \cdot 10^{-8}$
$(\bar{b}_L\gamma^\mu d_L)^2$	$5.1 \cdot 10^2$	$9.3 \cdot 10^2$	$3.3 \cdot 10^{-6}$	$1.0 \cdot 10^{-6}$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	$1.9 \cdot 10^3$	$3.6 \cdot 10^3$	$5.6 \cdot 10^{-7}$	$1.7 \cdot 10^{-7}$
$(\bar{b}_L\gamma^\mu s_L)^2$	$1.1 \cdot 10^2$		$7.6 \cdot 10^{-5}$	
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	$3.7 \cdot 10^2$		$1.3 \cdot 10^{-5}$	
$(\bar{t}_L\gamma^\mu u_L)^2$	12		$7.1 \cdot 10^{-3}$	

Table 2.1: *Bounds on dimension six $\Delta F = 2$ operators (numbers from [39, 40, 41])*

is given by five operators of mass dimension six (i.e. suppressed by M^{-2})[1]:

$$\mathcal{O}_1^{q_i q_j} = \bar{q}_{jL}^\alpha \gamma_\mu q_{iL}^\alpha \bar{q}_{jL}^\beta \gamma^\mu q_{iL}^\beta \tag{2.10}$$

$$\mathcal{O}_2^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\alpha \bar{q}_{jR}^\beta q_{iL}^\beta \tag{2.11}$$

$$\mathcal{O}_3^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\beta \bar{q}_{jR}^\beta q_{iL}^\alpha \tag{2.12}$$

$$\mathcal{O}_4^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\alpha \bar{q}_{jL}^\beta q_{iR}^\beta \tag{2.13}$$

$$\mathcal{O}_5^{q_i q_j} = \bar{q}_{jR}^\alpha q_{iL}^\beta \bar{q}_{jL}^\beta q_{iR}^\alpha \tag{2.14}$$

where i, j are flavour and α, β are colour indices. Experimental bounds on the NP scale or alternatively on the Wilson coefficient are given in table 2.1. As you can see, we either need a rather high new physics scale M or the Wilson coefficients must be very small. This is usually referred to as the *new physics flavour problem.* A high NP scale would leave us with corrections to the Higgs' mass that are still quite considerable and thus with substantial fine-tuning, i.e. a little hierarchy problem. Most theorists thus prefer the latter solution (not to speak of the experimentalists who also prefer NP at the electroweak scale in order to make it accessible to the LHC). Such small Wilson coefficients hint at a nontrivial flavour structure of NP. Before we will attack this topic however, let's pause and consider flavour in the SM in the light of symmetries.

[1]There are no dimension five operators relevant for flavour physics

2.3 Flavour symmetries in the SM

In the absence of the Yukawa couplings the SM is invariant under the flavour symmetry [36]:

$$\mathcal{G}_{\text{SM}} = U(3)_{q_L} \times U(3)_{u_R} \times U(3)_{d_R} \tag{2.15}$$

Under this symmetry the quark fields transform as:

$$q_L(\mathbf{3}, 1, 1) \qquad u_R(1, \mathbf{3}, 1) \qquad d_R(1, 1, \mathbf{3}) \tag{2.16}$$

To study flavour in the SM and beyond one promotes the Yukawa couplings to so-called *spurion* fields[2] that transform under the flavour symmetry as bi-fundamentals:

$$Y_u(\mathbf{3}, \bar{\mathbf{3}}, 1) \qquad Y_d(\mathbf{3}, 1, \bar{\mathbf{3}}) \tag{2.17}$$

The Lagrangian is then formally invariant under the flavour symmetry. The spurion fields acquire background values that break the flavour symmetry.
Consider the objects

$$A_{u_R,d_R} = Y^\dagger_{u,d} Y_{u,d} - \frac{1}{3}\text{Tr}(Y^\dagger_{u,d} Y_{u,d})\mathbb{1} \tag{2.18}$$

and

$$A_{u_L,d_L} = Y_{u,d} Y^\dagger_{u,d} - \frac{1}{3}\text{Tr}(Y_{u,d} Y^\dagger_{u,d})\mathbb{1} \tag{2.19}$$

which transform as:

$$A_{u_R,d_R} \to V_{uR,dR} A_{u_R,d_R} V^\dagger_{uR,dR} \tag{2.20}$$

$$A_{u_L,d_L} \to V_{uL,dL} A_{u_L,d_L} V^\dagger_{uL,dL} \tag{2.21}$$

i.e. they are adjoints of $U(3)_{uR,dR}$ and $U(3)_{qL}$, respectively. We can even further enhance the flavour group: Only the $W^\pm$ couplings g link the up and down quarks. Thus, in the absence of $W^\pm$ couplings the flavour symmetry is:

$$\mathcal{G}_{\text{SM without } W^\pm} = U(3)_{u_L} \times U(3)_{d_L} \times U(3)_{u_R} \times U(3)_{d_R} \tag{2.22}$$

Flavour conversion is then forbidden, since, as we have already seen in 1.2, only the charged currents link in between the different flavours. By promoting the coupling g to a spurion, the SM Lagrangian is formally invariant under $\mathcal{G}_{\text{SM without } W^\pm}$ and the transformation properties of the fields and spurions are:

$$u_L(\mathbf{3}, 1, 1, 1) \qquad d_L(1, \mathbf{3}, 1, 1) \qquad u_R(1, 1, \mathbf{3}, 1) \qquad d_R(1, 1, 1, \mathbf{3}) \tag{2.23}$$

[2] These are just auxiliary fields however: they have no kinetic terms and are dimensionless.

$$g(\mathbf{3},\bar{\mathbf{3}},1,1) \qquad Y_u(\mathbf{3},1,\bar{\mathbf{3}},1) \qquad Y_d(1,\mathbf{3},1,\bar{\mathbf{3}}) \tag{2.24}$$

$\mathcal{G}_{\text{SM without } W^\pm}$ is fully broken via the background values of the Yukawas and g. In the weak interaction basis the background value of g is proportional to the identity matrix:

$$\langle g \rangle_{\text{int.}} \propto \mathbb{1} \tag{2.25}$$

while in the mass basis its background value is the CKM matrix:

$$\langle g \rangle_{\text{mass}} = V_{\text{CKM}} \tag{2.26}$$

Flavour conversion occurs because we cannot simultaneously diagonalize A_{u_L}, A_{d_L} and g. The misalignement between A_{u_L} and A_{d_L} is characterized by the CKM matrix. Parameterizing the CKM matrix as (so-called *Wolfenstein parametrization* [42]):

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - \imath\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-\imath\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \tag{2.27}$$

with $\lambda = 0.22535 \pm 0.00065$, $A = 0.811^{+0.022}_{-0.012}$, $\rho = 0.131^{+0.026}_{-0.013}$ and $\eta = 0.345^{+0.013}_{0.014}$ [9], we can write the spurions as [43]:

$$A_{u_R,d_R} = \text{diag.}(0,0,y_{t,b}^2) - \frac{y_{t,b}^2}{3}\mathbb{1} + \mathcal{O}\left(\frac{m_{c,s}^2}{m_{t,b}^2}\right) \tag{2.28}$$

$$A_{u_L,d_L} = \text{diag.}(0,0,y_{t,b}^2) - \frac{y_{t,b}^2}{3}\mathbb{1} + \mathcal{O}\left(\frac{m_{c,s}^2}{m_{t,b}^2}\right) + \mathcal{O}(\lambda^2) \tag{2.29}$$

We see that the four spurions are approximately aligned or in other words that there is an approximate residual $U(2)_{u_R} \times U(2)_{d_R}$ flavour symmetry (the left-handed flavour symmetry is broken by the background value of g). This means that flavour-changing RH currents which involve light quarks are very small. Furthermore, we see that to leading order, flavour conversion is due only to the large top Yukawa coupling.

2.4 Minimal Flavour Violation

We have seen in 2.2 that new physics at the TeV scale must have a non-trivial flavour structure in order to be in agreement with experimental constraints on FCNCs. It wouldn't make sense to impose the flavour group $\mathcal{G}_{\text{SM}}$ on a NP model however, since it is already broken in the SM. But we can do the following: treating the Yukawa couplings as spurions, just as in 2.3, we request that these are the *only* sources of flavour violation and require that all terms in the Lagrangian are (formally) invariant under $\mathcal{G}_{\text{SM}}$. This principle, called *Minimal Flavour Violation (MFV)* [43, 44, 45], guarantees that low-energy flavour violating processes differ only very little from SM predictions. This is because the approximate $U(2)$ flavour symmetry of the SM is conserved when enlarging the SM to include NP at the TeV scale. To study the effects of NP in a model-independent way, we can use an EFT and find that for processes involving external down-type quarks[3], the basic bilinear FCNC structures are:

$$\bar{q}_L Y_u Y_u^\dagger q_L \qquad \bar{d}_R Y_d^\dagger Y_u Y_u^\dagger q_L \qquad \bar{d}_R Y_d^\dagger Y_u Y_u^\dagger Y_d d_R \tag{2.30}$$

We can always rotate the background value of the spurion fields to a basis where Y_d is diagonal:

$$Y_d^{\text{diag.}} = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix} \qquad Y_u = V_{\text{CKM}}^\dagger Y_u^{\text{diag.}} = V_{\text{CKM}}^\dagger \begin{pmatrix} y_u & & \\ & y_c & \\ & & y_t \end{pmatrix} \tag{2.31}$$

All but the top Yukawa coupling y_t are very small (since the other quarks are light) and we thus get:

$$(Y_u Y_u^\dagger)_{ij} \sim A_{\text{FC}} \equiv y_t^2 \times \begin{cases} V_{ti}^* V_{tj} & \text{for } i \neq j \\ 0 & \text{for } i = j \end{cases} \tag{2.32}$$

The only relevant bilinear FCNC structures are then:

$$\bar{q}_L A_{\text{FC}} q_L \qquad \bar{d}_R Y_d^{\text{diag.}} A_{\text{FC}} q_L \tag{2.33}$$

From these, a bunch of FCNC dimension-six operators can be constructed (see [43] for a complete list). Comparing to experimental results, bounds on the scale of NP in MFV models can be inferred. We get a NP scale of $M \sim$ a few TeV, a significant improvement to what we found in table 2.1.

You may ask whether there are any concrete NP models that exhibit MFV and indeed there are: Supersymmetry with gauge-mediated SUSY breaking is an example [46]. In order to achieve gauge-mediated SUSY breaking some new chiral supermultiplets, the messengers, are introduced. They couple to the source of SUSY breaking, but also to

[3] We focus on external down-type quarks since experimental bounds on processes with external up-type quarks are less stringent.

the fields of the MSSM via the ordinary gauge bosons of the SM. Soft SUSY breaking terms are thus induced by loops in a way that the flavour sector respects MFV. We will get to know another model incorporating MFV in section 3.4.
One last word on MFV in supersymmetry: In the limit where the ratio of the expectation values of the two Higgs fields of the MSSM, usually denoted $\tan\beta$, is large, the bottom Yukawa coupling can become comparable to the top Yukawa and there is a second non-negligible flavour-violating structure in addition to 2.32
A variation of MFV is *General Minimal Flavour Violation (GMFV)* [36, 47] where flavour diagonal CP violating phases are added to generic MFV models. The experimental constraints on the NP scale are then substantially higher than in the case where only the SM Yukawas are responsible for the breaking of flavour *and* CP. This is mainly due to bounds coming from electric dipole moments.

Chapter 3

Two Higgs Doublet Models

3.1 Extending the Higgs Sector

Even after the (probable) discovery of the Higgs at the LHC it is still conceivable that the Higgs sector differs from its simplest form that is realized in the SM. The easiest possibility to extend it is to simply add a second Higgs doublet, i.e. a second scalar $SU(2)$ doublet with hypercharge $Y = 1/2$. This is what is called a *Two Higgs Doublet Model (2HDM).*

3.1.1 EWSB in the presence of a second Higgs

The most general potential we can write for two identical scalars ϕ_1 and ϕ_2 is [48]:

$$\begin{aligned} V_{\rm 2HDM} &= m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 - (m_{12}^2\phi_1^\dagger\phi_2 + \text{h.c.}) \\ &+\frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+\left\{\frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right]\phi_1^\dagger\phi_2 + \text{h.c.}\right\} \end{aligned} \tag{3.1}$$

where $m_{12}^2, \lambda_5, \lambda_6$ and λ_7 can be complex (that would mean that the potential is explicitly CP violating). This potential must be bounded from below, that is there must be no directions in field space where $V_{\rm 2HDM} \to \infty$. The 2HDM potential is bounded from below if and only if [49, 50]

$$\lambda_1 > 0 \qquad \lambda_3 > -\sqrt{\lambda_1\lambda_2} \tag{3.2}$$

$$\lambda_2 > 0 \qquad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2} \tag{3.3}$$

For λ_6 and λ_7 only necessary conditions can be derived [49, 51, 52]:

$$2\,|\lambda_6 + \lambda_7| < \frac{\lambda_1 + \lambda_2}{2} + \lambda_3 + \lambda_4 + \lambda_5 \tag{3.4}$$

All these conditions only ensure that the vacuum is bounded from below at tree level. One must also take into account radiative corrections, i.e. the dependence of the λ's on the renormalization scale μ. Ensuring that the potential is bounded from below will then put lower bounds on the values of the λ's. On the other hand perturbativity must not break down either and thus there will be upper bounds on λ as well.

3.1.2 Phenomenology of the 2HDM

As long as we only have one Higgs boson we can always perform an $SU(2)$ rotation on its vacuum expectation value such that it reads:

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{3.5}$$

Then, the electric charge operator Q always annihilates the vacuum:

$$Q\,\langle\phi\rangle = 0 \tag{3.6}$$

In other words, $U(1)_{\text{em.}}$ is unbroken and the photon remains massless as it should be. In a 2HDM on the other hand, we can make use of the $SU(2)\times U(1)$ symmetry to reduce the most general vev to the form [53]

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad\qquad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 e^{i\xi} \end{pmatrix} \tag{3.7}$$

Then, only for $u = 0$ the photon will remain massless [48]. It is certainly a drawback of the 2HDM that the photon is not automatically massless. Let us however assume that we are in a part of the parameter space where $u = 0$ and thus the photon is massless and carry on. We can furthermore shift the phase ξ to the Yukawa couplings and the potential. We can then rotate the the Higgs fields to a basis where only one of them, say Φ_1 acquires a vev (sometimes called Higgs basis):

$$\begin{pmatrix} \Phi_1 \\ -\Phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ \sin\beta & -\cos\beta \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{3.8}$$

where $\tan\beta = v_2/v_1$, i.e. $\tan\beta$ is the ratio of the vev's in the original basis. To find the mass eigenstates of the Higgs doublets, we start by writing the two complex doublets in terms of eight real fields [54]:

$$\Phi_1 = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} \varphi_5 + i\varphi_6 \\ \varphi_7 + i\varphi_8 \end{pmatrix} \tag{3.9}$$

For

$$\Phi_1 = \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.10}$$

to be the minimum of the potential, we must require

$$\left.\frac{\partial V}{\partial \varphi_i}\right|_{\begin{cases} \varphi_k = 0 & \text{for } k \neq 3 \\ \varphi_k = v & \text{for } k = 3 \end{cases}} = 0 \tag{3.11}$$

This yields the minimum conditions[1]:

$$m_{11}^2 = -\lambda_1 v^2 \qquad m_{12}^2 = \lambda_6 v^2 \tag{3.12}$$

We now calculate the mass (squared) matrix

$$M_{ij}^2 = \frac{1}{2} \left.\frac{\partial V}{\partial \varphi_i \partial \varphi_j}\right|_{\begin{cases} \varphi_k = 0 & \text{for } k \neq 3 \\ \varphi_k = v & \text{for } k = 3 \end{cases}} \tag{3.13}$$

$$M^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\lambda_1 v^2 & 0 & 0 & 0 & 2\lambda_6 v^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{22}^2 + \lambda_3 v^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{22}^2 + \lambda_3 v^2 & 0 & 0 \\ 0 & 0 & 2\lambda_6 v^2 & 0 & 0 & 0 & m_{22}^2 + \eta v^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{22}^2 + \eta' v^2 \end{pmatrix} \tag{3.14}$$

[1] I assume here that λ_6 and m_{12} are real, i.e. that there are no new sources of direct CP violation in the Higgs sector

where we defined

$$\eta = \lambda_3 + \lambda_4 + \lambda_5 \tag{3.15}$$

$$\eta' = \lambda_3 + \lambda_4 - \lambda_5 \tag{3.16}$$

The eigenvalues of the matrix describing the $\varphi_1 - \varphi_5$ (or $\varphi_2 - \varphi_6$) mixing give us the masses of the charged Higgses $H^{\pm}$ and the charged Goldstone bosons $G^{\pm}$ (these are massless of course):

$$M^2_{\varphi_1\varphi_5} = M^2_{\varphi_2\varphi_6} = \begin{pmatrix} 0 & 0 \\ 0 & m_{22}^2 + \lambda_3 v^2 \end{pmatrix} \tag{3.17}$$

$$m^2_{G^{\pm}} = 0 \qquad m^2_{H^{\pm}} = m^2_{22} + \lambda_3 v^2 \tag{3.18}$$

The $\varphi_4 - \varphi_8$ mixing yields the masses of the CP odd neutral Higgs A and the neutral Goldstone boson G:

$$M^2_{\varphi_4\varphi_8} = \begin{pmatrix} 0 & 0 \\ 0 & m_{22}^2 + \eta' v^2 \end{pmatrix} \tag{3.19}$$

$$m^2_{G} = 0 \qquad m^2_{A} = m^2_{22} + \eta' v^2 \tag{3.20}$$

Finally, we have the $\varphi_3 - \varphi_7$ mixing yielding the CP even mass eigenstates h and H:

$$M^2_{\varphi_3\varphi_7} = \begin{pmatrix} 2\lambda_1 v^2 & 2\lambda_6 v^2 \\ 2\lambda_6 v^2 & m_{22}^2 + \eta v^2 \end{pmatrix} \tag{3.21}$$

Here things get more interesting since this matrix is not yet diagonal. As the matrix is symmetric it is diagonalized by an orthogonal matrix which we choose to parameterize as:

$$O = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \tag{3.22}$$

The mass eigenstates h and H are then given in terms of φ_3 and φ_7 as:

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \varphi_3 \\ \varphi_7 \end{pmatrix} \tag{3.23}$$

From

$$O^T M^2_{\varphi_3\varphi_7} O = \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix} \tag{3.24}$$

we get:

$$m_H^2 = \frac{1}{2}\left(m_{22}^2 + \eta v^2 + 2\lambda_1 v^2 + \sqrt{(m_{22}^2 + \eta v^2 - 2\lambda_1 v^2)^2 + 16\lambda_6^2 v^4}\right) \tag{3.25}$$

$$m_h^2 = \frac{1}{2}\left(m_{22}^2 + \eta v^2 + 2\lambda_1 v^2 - \sqrt{(m_{22}^2 + \eta v^2 - 2\lambda_1 v^2)^2 + 16\lambda_6^2 v^4}\right) \tag{3.26}$$

$$\cos(2\alpha) = -\frac{m_{22}^2 + \eta v^2 - 2\lambda_1 v^2}{\sqrt{(m_{22}^2 + \eta v^2 - 2\lambda_1 v^2)^2 + 16\lambda_6^2 v^4}} \tag{3.27}$$

$$\sin(2\alpha) = -\frac{4\lambda_6 v^2}{\sqrt{(m_{22}^2 + \eta v^2 - 2\lambda_1 v^2)^2 + 16\lambda_6^2 v^4}} \tag{3.28}$$

In conclusion, in a 2HDM one can expect to find a heavy and a light CP even neutral boson, H and h, a CP odd neutral boson, A, and a two charged bosons, $H^\pm$. The Goldstone bosons $G^\pm$ and G are eaten by the $W^\pm$ and Z gauge bosons, respectively.

3.2 FCNCs in Two Higgs Doublet Models

A priori, both Higgs doublets can couple to all quarks and the Yukawa Lagrangian thus reads for the quark sector[2]:

$$\mathcal{L} \supset (Y_u^{(1)})_{ij}\bar{q}'_{Li}u'_{Rj}\tilde{\phi}_1 + (Y_d^{(1)})_{ij}\bar{q}'_{Li}d'_{Rj}\phi_1 + (Y_u^{(2)})_{ij}\bar{q}'_{Li}u'_{Rj}\tilde{\phi}_2 + (Y_d^{(2)})_{ij}\bar{q}'_{Li}d'_{Rj}\phi_2 \tag{3.29}$$

There is no reason to expect that both Yukawa couplings $Y_u^{(1)}, Y_u^{(2)}$ or $Y_d^{(1)}, Y_d^{(2)}$ are diagonal in the same basis. In general they are not and this will induce FCNCs at tree level via the interchange of a neutral Higgs. Such processes include $B_s - \bar{B}_s$-mixing and the rare decay $\bar{B}_s \to \mu^+\mu^-$, see figure 3.1. Since there are stringent experimental bounds on these processes, this eliminates the possibility of throwing a second Higgs into the theory without taking any further precautions.
One possibility of avoiding too high rates for FCNC processes is of course to make one of the two Higgses very heavy such that it can be integrated out at the weak scale, leaving us with an effective theory that resembles closely to the SM. This is called the *decoupling limit* [55]. From equations (3.18), (3.20), (3.26) and (3.28), we see that for

[2]The quark fields are primed here because we will later rotate them in order to diagonalize the mass matrices and the rotated fields will have no primes then.

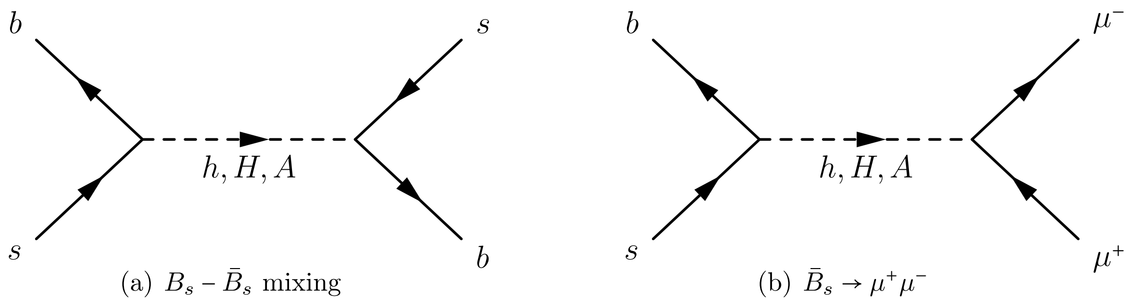


Figure 3.1: *Feynman diagrams for tree level flavour changing processes in a general 2HDM*

$m_{22}^2 \gg m_{11}^2 \sim v^2$ we get $m_{H^\pm}^2 \sim m_A^2 \sim m_H^2 = m_{22}^2 + \mathcal{O}(v^2)$ and $m_h^2 \sim v^2$. However, this is not a basis-independent characterization of the decoupling limit, because to deduce (3.18), (3.20), (3.26) and (3.28), we went to a certain basis, see eq.s (3.8) and (3.10). A basis-independent characterization is given by the following: Form the matrix m_{ij}^2, containing the quadratic terms of the potential, eq. (**??**). Label its eigenvalues m_a^2 and m_b^2 where by convention $|m_a^2| \leq |m_b^2|$. The decoupling limit is then given as $m_a^2 < 0$, $m_b^2 > 0$ such that $|m_b^2| \gg |m_a^2| \sim v^2$.

As is shown in [55], in the decoupling limit FCNCs generated by tree level Higgs exchanges are suppressed by a factor of order $\mathcal{O}(v^2/\Lambda_{2HDM}^2)$ where Λ_{2HDM} is the mass scale of the heavier Higgses $H, H^\pm, A$, i.e. the scale at which the model truly becomes a 2HDM. Furthermore, the couplings of the light Higgs h to fermions and gauge bosons and its self-couplings also deviate by terms of $\mathcal{O}(v^2/\Lambda_{2HDM}^2)$ only from their SM values.

3.2.1 The usual way out: Discrete Symmetries

The catch with the decoupling limit is that at low energies it mimics the SM and therefore no new physics is to be expected at the LHC if a 2HDM in the decoupling limit is really what nature has concocted for us. Physicists have thus come up with 2HDMs that do not induce FCNCs at tree level while having an interesting phenomenology at the electroweak scale.

Looking back at eq. (3.29) we see that FCNCs wouldn't occur (well, at least at tree level, that is) if each of the two Higgs doublets could couple to only one of the quark fields. Then, there would be only one Yukawa matrix per quark species which could be diagonalized to go to the mass basis. More generally, according to the Paschos-Glashow-Weinberg theorem [56, 57], the necessary and sufficient conditions for the absence of FCNCs at tree level is that all fermions of a given charge and helicity transform according to the same irreducible representation of $SU(2)$, correspond to

the same eigenvalue of T_3 and that a basis exists in which they receive their mass from a single source.
This condition can be ensured by introducing a discrete symmetry $\mathcal{Z}_2$. One distinguishes several types of 2HDMs:

- In type I the second Higgs doublet transforms as $\phi_2 \to -\phi_2$, while all the other fields are even under this symmetry. ϕ_2 can therefore not couple to the quark fields [58, 59].

- In type II models the ϕ_1 and down-type quark fields transform as $\phi_1 \to -\phi_1$, $d_R \to -d_R$ and the other fields are even under the discrete symmetry[3]. This means that ϕ_1 can only couple to down type quarks, while ϕ_2 only couples to up-type quarks[4] [59, 60].

- Another possibility is to have one Higgs couple only to fermions and one Higgs couple only to the quarks. This can be realized by the symmetry transformations $\phi_1 \to \phi_1$, $\phi_2 \to -\phi_2$, $q_L \to q_L$, $l_L \to l_L$, $u_R \to -u_R$, $d_R \to -d_R$ and $e_R \to e_R$. This is known by the names of *leptophilic 2HDM, lepton specific 2HDM, type X 2HDM* and *leptonic Higgs* in the literature [61, 62, 63, 64, 65, 66, 67, 68, 69].

- Furthermore, there is also the possibility to have the Higgs couplings "switched", such that, unlike in the MSSM, the leptons couple to the same Higgs as the up-type quarks. (Symmetry transformations: $\phi_1 \to \phi_1$, $\phi_2 \to -\phi_2$, $q_L \to q_L$, $l_L \to l_L$, $u_R \to -u_R$, $d_R \to d_R$ and $e_R \to -e_R$) [62, 66, 67, 68, 69].

Note that in all cases the terms proportional to m_{12}^2, λ_6 and λ_7 in the potential (??) are absent. These 2HDMs do not have a decoupling limit. This is because with $m_{12}^2 = \lambda_6 = \lambda_7 = 0$ the minimum conditions of the potential are [55]:

$$m_{11}^2 = -\frac{1}{2}v^2\left[\lambda_1 c_\beta^2 + (\lambda_3 + \lambda_4 + \lambda_5)s_\beta^2\right] \tag{3.30}$$

$$m_{22}^2 = -\frac{1}{2}v^2\left[\lambda_2 s_\beta^2 + (\lambda_3 + \lambda_4 + \lambda_5)c_\beta^2\right] \tag{3.31}$$

where $c_\beta = \cos\beta$ and $s_\beta = \sin\beta$. But then $m_{11}^2 \sim m_{22}^2 \sim \mathcal{O}(v^2)$ and there is no decoupling limit.
The discrete symmetry can be softly broken by having a non-zero m_{12}^2-term in the potential (??). This would only introduce finite Higgs mediated FCNCs at one loop level [55].

[3] Apart from e_R, which is also odd.
[4] And charged leptons since we also have $e_R \to -e_R$, but we are not considering leptons here.

Note that all these models can also be obtained by imposing a Peccei-Quinn symmetry $\mathrm{U}(1)_{\mathrm{PQ}}$ instead of the discrete symmetry. For example for type II 2HDMs: d_R and ϕ_1 must have opposite charge under $\mathrm{U}(1)_{\mathrm{PQ}}$, while all the other fields are neutral.

3.3 Benefits of having a second Higgs

Since we got into so much trouble by introducing a second Higgs you may ask: Why introduce a second Higgs, if all we get is problems? Indeed, apart from the nobody-else-has-done-it-thus-far argument there are some interesting effects that may occur when you introduce a second or even more Higgs bosons.

3.3.1 The MSSM as a 2HDM

First it has to be noted that the MSSM is a 2HDM. As we have seen in 1.4.1, the superpotential must be analytic in the superfields and thus we cannot have a term $\bar{q}_{Li} Y_u u_{Rj} \tilde{\phi}$ as in the SM Lagrangian. We thus need two Higgses: one for the up- and one for the down- and charged lepton sectors. The Yukawa Lagrangian then reads:

$$\mathcal{L}_{\text{Yukawa}} = \bar{q}_L Y_u u_R \phi_u - \bar{q}_L Y_d d_R \phi_d - \bar{l}_L Y_e e_R \phi_d \tag{3.32}$$

The second argument is that with only one Higgs, the electroweak symmetry would have a gauge anomaly in the MSSM. The superfields have the following quantum numbers under the gauge group:

$$Q(\mathbf{3},\mathbf{2},1/6) \quad \bar{u}(\bar{\mathbf{3}},\mathbf{1},-2/3) \quad \bar{d}(\bar{\mathbf{3}},\mathbf{1},1/3) \quad H_u(\mathbf{1},\mathbf{2},1/2) \quad H_d(\mathbf{1},\mathbf{2},-1/2) \tag{3.33}$$

H_u can thus not have a Yukawa coupling to the down-type quarks and H_d cannot have a Yukawa coupling to the up-type quarks. We see that the MSSM is a type II 2HDM.

3.3.2 The inert doublet model

Electroweak precision tests (EWPT) at LEP showed that the Higgs must be light ($m_\phi < 186$ GeV at 95 % C.L.). On the other hand in order for contributions from higher dimensional operators not to exceed experimental EWPT limits, the cut-off of the SM must be high, $\Lambda \geq 5$ TeV. These two facts lead to the so-called *LEP paradox* [70]: if the radiative corrections to the Higgs mass, coming notably from the top loop, are cut-off only at $\Lambda \sim 5$ TeV, δm_ϕ largely exceeds the value of m_ϕ preferred by the EWPT. We thus have a naturalness problem.

Introducing a second, "inert" Higgs could allow for a heavy Higgs (the one that is responsible for EW symmetry breaking and fermion masses), thus improving the naturalness of the effective theory below Λ [71, 72]. In the inert doublet model there is a second Higgs that - like in type I 2HDMs - possesses the parity:

$$\phi_2 \to -\phi_2 \tag{3.34}$$

while all other fields are even under this transformation. Unlike in the conventional type I 2HDM this parity is not spontaneously broken by doublet vevs: only ϕ_1 acquires a vev. The particle spectrum of this model is the following: there is a neutral Higgs boson of 400 - 600 GeV that gives masses to the W and Z bosons and the fermions. Additionally, there are "inert" particles that do not couple to the fermions: a charged scalar, a CP odd and a CP even neutral scalar. These particles do have electroweak and quartic interactions, of course, so the name "inert" might be a bit misleading.
Now how does this help us with avoiding constraints on the Higgs mass coming from EWPT? The Higgs mass influences EWPT via logarithmic contributions to the parameters T and S:

$$T \approx \frac{3}{8\pi c^2} \ln \frac{m_\phi}{m_Z} \tag{3.35}$$

$$S \approx \frac{1}{6\pi} \ln \frac{m_\phi}{m_Z} \tag{3.36}$$

The inert doublet produces a compensating ΔT and higher masses m_ϕ become possible. This allows the theory to be "natural" up to $\Lambda \sim 1.5$ TeV.
With the recent discovery of a light Higgs, these considerations have however become obsolete. But there is another interesting aspect of the inert doublet model: The lightest one of the inert scalars is necessarily stable and thus a Dark Matter candidate[5] [73, 74, 75].

3.3.3 Baryogenesis in 2HDMs

A further benefit of 2HDMs is that they might reopen the window to electroweak baryogenesis. In the SM there are two reasons why electroweak baryogenesis doesn't work: 1) there is not enough CP violation and 2) for Higgs masses above 20 GeV, the Higgs vev is small compared to the critical temperature at the time of the electroweak phase transition: $v_c/T_c < 1$. During the electroweak phase transition, bubbles of the broken phase (i.e. non-zero Higgs vev) appear which expand until they fill all of space.

[5] This implies of course that the lightest inert scalar must be neutral.

Inside the bubbles, the baryon number violating sphalerons must come to a halt or the created baryon excess will be equilibrated to zero. This condition is fulfilled only if $v_c/T_c \gtrsim 1$.
In [76] it is shown that a 2HDM with a discrete symmetry $\phi_2 \to -\phi_2$ that is softly broken by a term $m_{12}^2\phi_1^\dagger\phi_2$ (i.e. a type I 2HDM) can have $v_c/T_c \gtrsim 1$ for a Higgs mass of up to 300 GeV. This is due to the new scalars' contribution to the cubic thermal potential $\phi^3 T$. Note that this is not possible in SUSY however, as m_{12}^2 is always tuned in such a way as to minimize the impact of the extra scalars no matter how heavy they are. Also, new sources of CP violation can be present in 2HDMs.
Another possibility is to consider baryogenesis in the inert doublet model [77, 78]. It turns out that in these kinds of models it is possible to have baryogenesis and a viable DM (i.e. a neutral "inert" scalar) candidate at the same time! An SU(2) singlet added to the SM could not do this job: although it can help with the phase transition it cannot at the same time account for DM.
2HDMs without a discrete symmetry where considered in [79]. Here, Minimal Flavour Violation (see sec. 2.4) is used in order to suppress the FCNCs. The Yukawa couplings of the second Higgs are thus a function of the Yukawa couplings of the first Higgs:

$$Y_{u,d}^{(2)} = \eta_{u,d}Y_{u,d}^{(1)} + \eta'_{u,d}Y_{u,d}^{(1)}Y_{u,d}^{(1)\dagger}Y_{u,d}^{(1)} + \dots \tag{3.37}$$

The scalar potential is the same as in eq. (3.1). There are several sources of CP violation in this model: The phases of the Yukawa couplings of the second Higgs boson ϕ_2 cannot be removed by field redefinitions since these were already used up in order to push the phases of the Yukawa coupling of the first Higgs into the CKM matrix. Moreover, as already noted in 3.1.1, the couplings $m_{12}^2, \lambda_5, \lambda_6$ and λ_7 can be complex. Two phases can be removed however: going to a basis where ϕ_2 doesn't get a vev (this is always possible in this kind of models), m_{12}^2 and λ_6 are linearly related thus removing one of their phases. In addition we can perform a phase transition on ϕ_2, removing another phase. In this model, it is found that there is enough baryogenesis in a small subset of parameter space only.

3.3.4 Neutrino masses in 2HDMs

2HDMs are also interesting for the study of neutrino masses. Recall from sec. 1.3 that the smallness of neutrino masses is an unsolved puzzle in the SM. The proposed solution of having the small masses of the left-handed neutrinos induced by heavy right-handed neutrinos via the seesaw mechanism has the downside that the new physics involved in the neutrino mass generation will not be measurable in the near future, for example

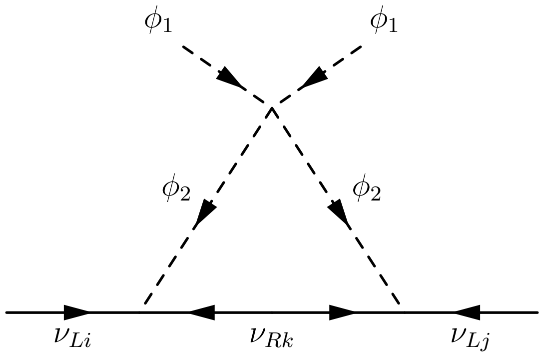


Figure 3.2: *Radiative generation of neutrino masse in the model by [73]*

at the LHC.

An alternative idea is to produce the masses of the left-handed neutrinos radiatively [73]. This is possible if we add to the SM a second Higgs doublet $\phi_2 = (\varphi^+, \varphi^0)$ and three right-handed neutrinos ν_{Ri}. The new particles are supposed to be all odd under a Z_2 while all other particles are even under this same Z_2. The new particles have the Yukawa coupling

$$\mathcal{L} \supset (Y_e^{(2)})_{ij}(\bar{\nu}_{Li}\varphi^0 - \bar{e}_{Li}\varphi^+)\nu_{Rj} + \text{h.c.} \tag{3.38}$$

Majorana mass term

$$\frac{1}{2}M_{ij}\bar{\nu}_{Ri}^C\nu_{Rj} + \text{h.c.} \tag{3.39}$$

and the scalar quartic term

$$\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.} \tag{3.40}$$

Note that, since ϕ_2 does not have a vev, (3.38) does not generate a Dirac mass term for the neutrinos. Masses for the right-handed neutrinos are then induced at one loop level, via the Feynman diagram in fig. 3.2.

Moreover, since the Z_2 symmetry is exact, the lightest one of the new particles will be stable and is therefore a DM candidate. This can be either one of the additional physical scalar fields or the lightest right-handed neutrino. Due to the loop, the seesaw scale is reduced by a factor of roughly $\lambda_5/16\pi^2$. For $\lambda_5 \sim 10^{-4}$ the new particles could then be light enough to be observed at the LHC.

Another idea [80] is to have the second Higgs boson generate a *Dirac* mass term for the neutrinos. If the second Higgs boson has a tiny vev v_2, the smallness of the neutrino masses would be explained without invoking any physics above the TeV scale. In the model of [80] a global U(1) symmetry is imposed under which the ϕ_2 and ν_R carry charge $+1$ and all the other fields are uncharged. ϕ_2 thus only couples to ν_R and there is no Majorana mass term. The U(1) symmetry is broken explicitly by the term

$m_{12}^2\phi_1^\dagger\phi_2$. This leaves ϕ_2 with the vev

$$v_2 = \frac{m_{12}^2 v_1}{M_A^2} \tag{3.41}$$

We can thus achieve $v_2 \sim$ eV for $M_A \sim 100$ GeV and $m_{12}^2 \sim (\mathcal{O}(100 \text{ keV}))^2$. The smallness of m_{12}^2 could be explained by some high-scale physics, for example a spontaneous breaking of the U(1) in a hidden sector that is then communicated to the Higgs sector via loops or heavy messenger particles. Radiative corrections will not drive up m_{12}^2 since they are proportional to m_{12}^2 itself and only logarithmically dependent on the cut-off.

Apart from the smallness of their masses, neutrinos are distinct from the other SM fermions in one other aspect: the hierarchy in between their masses is much smaller than the hierarchy in between the masses of the quarks and charged leptons[6]:

$m_u \approx 2.5$ MeV	$m_c \approx 1.3$ GeV	$m_t \approx 180$ GeV	$\sim 1:500:70000$
$m_d \approx 5$ MeV	$m_s \approx 100$ MeV	$m_b \approx 5$ GeV	$\sim 1:20:1000$
$m_e \approx 0.5$ MeV	$m_\mu \approx 105$ MeV	$m_\tau \approx 1.8$ GeV	$\sim 1:200:3500$
$\Delta m^2_{\text{sol.}} \approx 7.6 \cdot 10^{-5}$ eV^2		$\Delta m^2_{\text{atm.}} \approx 2.4 \cdot 10^{-3}$ eV^2	

This observation could in fact be explained by a 2HDM [81]: Add to the SM a second Higgs boson and at least one right-handed neutrino. The Lagrangian involving the right-handed neutrinos then reads:

$$\mathcal{L} \supset -(Y_\nu^{(1)})_{ij}\bar{l}_{Li}\nu_{Rj}\tilde{\phi}_1 - (Y_\nu^{(2)})_{ij}\bar{l}_{Li}\nu_{Rj}\tilde{\phi}_2 + \frac{1}{2}M_{ij}\bar{\nu}_{Ri}^C\nu_{Rj} + \text{h.c.} \tag{3.42}$$

Assuming that the mass scale of the right-handed neutrinos as well as the mass scale of the extra Higgs mass eigenstates $H, H^\pm$ and A is much larger than the electroweak scale (decoupling limit, see sec. 3.2), this model is described at low energies by an effective theory with the following operator for the left-handed neutrinos:

$$\mathcal{L}^{\text{eff.}} \supset -\frac{1}{2}(Y_\nu^{(a)}M^{-1}Y_\nu^{(b)\text{T}})_{ij}(\bar{l}_{Li}\tilde{\phi}_a)(\tilde{\phi}_b^{\text{T}} l_{Lj}^{\text{C}}) + \text{h.c.} \tag{3.43}$$

One can always go to a basis where only one of the Higgses, say ϕ_1, gets a vev. If only one right-handed neutrino is added, $Y_\nu^{(a)}$ are 3-vectors and the resulting mass matrix is then at tree level:

$$(M_{\nu_L}^{\text{tree}})_{ij} = \frac{Y_\nu^{(1)}Y_\nu^{(1)\text{T}}}{M}v^2 \tag{3.44}$$

[6]In the case of neutrinos only the mass differences squared are known.

where M is just the mass of the heavy neutrino. As in the standard seesaw mechanism, the masses of the left-handed neutrinos are suppressed by the huge mass of the right-handed neutrino, thus explaining their smallness. This is a matrix of rank 1 and thus has only one non-vanishing eigenvalue. However, there are radiative corrections and a second mass eigenvalue will be generated radiatively. Thus, in this model, one right-handed neutrino is enough to generate two different mass eigenvalues for the left-handed neutrinos. Now to the mild hierarchies in the neutrino masses. The problem is that one expects the neutrino Yukawa couplings to be hierarchical, since the other fermions also must have hierarchical Yukawas in order to explain their hierarchical masses[7]. In the common seesaw model with one Higgs and (at least) two right-handed neutrinos, the mass hierarchy for the left-handed neutrinos is given by:

$$\frac{m_{\nu 3}}{m_{\nu 2}} \sim \frac{|Y_2|^2}{|Y_1|^2}\frac{M_1}{M_2} \tag{3.45}$$

This means that if $|Y_2| \gg |Y_1|$, as is expected, we must have a huge hierarchy in between the masses M_1 and M_2 of the right-handed neutrinos in order to compensate for the hierarchy of the Yukawas and render a mild hierarchy for the left-handed neutrinos. This is not impossible but weird, since the hierarchy of the right-handed neutrino masses must almost exactly cancel the hierarchy of the Yukawas. In a 2HDM with one or more right-handed neutrinos this problem does not occur since the second mass eigenvalue is generated radiatively and is thus suppressed only by a loop factor which is partly compensated for by a large logarithm (notably the logarithm of the mass scale of the right handed neutrinos divided by the mass scale of the new scalars). Note that for this model to work, there must be a misalignment in between the Yukawa couplings of the two Higgses to the neutrinos. This will lead to lepton flavour violation. These processes are suppressed however by going to the decoupling limit of the additional scalars and are thus consistent with experimental bounds.

3.3.5 Further ideas

Another idea is to have *private* Higgs fields that couple to only one fermion type and generation (i.e. up, down, charm, strange, top bottom) each [82]. This could help explain the huge hierarchies within fermion masses. FCNCs do occur in this model, but experimental bounds can be met by making the new fields sufficiently heavy.

Moreover, 2HDMs can always be seen as effective theories of some more complete high energy theory.

[7]See however sec. 4.7.2 for a model with anarchic couplings for all the fermions.

3.4 A General 2HDM with Yukawa Alignment

We are now turning to 2HDMs without any discrete symmetry, so-called type III 2HDMs [83]. In a general model with two $Y = 1/2$ Higgs doublets, both Higgses can couple to all fermions and the Yukawa part of the Lagrangian reads:

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} = (Y_u^{(1)})_{ij}\bar{q}'_{Li}u'_{Rj}\tilde{\phi}_1 + (Y_d^{(1)})_{ij}\bar{q}'_{Li}d'_{Rj}\phi_1 + (Y_e^{(1)})_{ij}\bar{l}'_{Li}e'_{Rj}\phi_1 \\ +(Y_u^{(2)})_{ij}\bar{q}'_{Li}u'_{Rj}\tilde{\phi}_2 + (Y_d^{(2)})_{ij}\bar{q}'_{Li}d'_{Rj}\phi_2 + (Y_e^{(2)})_{ij}\bar{l}'_{Li}e'_{Rj}\phi_2 + \text{h.c.}\end{aligned} \tag{3.46}$$

The neutral components of the two Higgs doublets acquire vacuum expectation values (vevs) during electroweak symmetry breaking (EWSB) which in general can be complex. While one phase can be rotated away, the phase difference is physical. Nevertheless we can choose to work in a basis where both vevs are real, shifting the phase to the potential and the Yukawas:

$$\langle\phi_a\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_a \end{pmatrix}. \tag{3.47}$$

As we have seen in sec. 3.2, without any further protection such a model would lead to unacceptably high FCNCs. In order to minimize these, without imposing *ad hoc* discrete symmetries we postulate that at a high energy cut-off scale, Λ, the Yukawa couplings of the same fermion type are aligned [84, 1]. We parameterize this condition as:

$$Y_u^{(1)}(\Lambda) = \cos\psi_u Y_u, \qquad Y_u^{(2)}(\Lambda) = \sin\psi_u Y_u, \tag{3.48}$$

$$Y_d^{(1)}(\Lambda) = \cos\psi_d Y_d, \qquad Y_d^{(2)}(\Lambda) = \sin\psi_d Y_d, \tag{3.49}$$

$$Y_e^{(1)}(\Lambda) = \cos\psi_e Y_e, \qquad Y_e^{(2)}(\Lambda) = \sin\psi_e Y_e; \tag{3.50}$$

Note that type I 2HDMs are contained in this parameterization as the special case $\psi_u = \psi_d = \psi_e = 0$, type II as $\psi_u = 0,\ \psi_d = \psi_e = \frac{\pi}{2}$, leptophilic as $\psi_u = \psi_d = \frac{\pi}{2},\ \psi_e = 0$ and "switched" as $\psi_u = \psi_e = \frac{\pi}{2},\ \psi_d = 0$.

3.4.1 Radiative corrections to the alignment Yukawa couplings

In a 2HDM with Yukawa alignment there are a priori no FCNCs at tree level. However, the alignment can only be imposed at *one* energy scale. When considering particle physics processes at another energy scale, we need to evolve all couplings using the renormalization group equations (RGE) in order to calculate the actual couplings at the energy scale we are interested in. The RGEs for a general 2HDM have been derived in [85]. We reproduce them in our notation in appendix A.3. The radiative corrections introduce a misalignment of the Yukawa couplings at low energy. To see whether this leads to unacceptably large FCNCs, we solved the RGEs numerically and analytically using the so-called "leading log approximation" which estimates the down-type quark couplings at the electroweak scale as:

$$Y_d^{(k)}(m_Z) \approx Y_d^{(k)}(\Lambda) + \frac{1}{16\pi^2}\beta_{Y_d^{(k)}}(\Lambda)\log\left(\frac{m_Z}{\Lambda}\right), \tag{3.51}$$

and similarly for the Yukawa matrices of the up-type quarks and leptons. Inserting the β-function (A.7), the coupling at the EW scale takes the form:

$$Y_d^{(k)}(m_Z) \approx k_d^{(k)} Y_d + \epsilon_d^{(k)} Y_u Y_u^\dagger Y_d + \delta_d^{(k)} Y_d Y_d^\dagger Y_d, \tag{3.52}$$

where the coefficients $k_d^{(k)}$, $\epsilon_d^{(k)}$ and $\delta_d^{(k)}$ can be found in appendix A.4, as well as the corresponding formulae for up-type quarks and leptons.

3.4.2 Flavour violating neutral Higgs couplings

To derive the low energy Lagrangian it is convenient to rotate the Higgs fields to the Higgs basis, cf. eq. (3.8). In this basis the Lagrangian can be written in the following form (quark sector):

$$\mathcal{L}_{\text{Yukawa}} = \frac{\sqrt{2}}{v}\left\{\bar{q}_L'\left(M_u\tilde{\Phi}_1 + \Gamma_u\tilde{\Phi}_2\right)u_R' + \bar{q}_L'\left(M_d\Phi_1 + \Gamma_d\Phi_2\right)d_R' + \text{h.c.}\right\}, \tag{3.53}$$

where $v^2 = v_1^2 + v_2^2 = (246\text{ GeV})^2$ and the couplings $M_{u,d}$ and $\Gamma_{u,d}$ are evaluated at the scale m_Z. Their expression in terms of the original couplings in the basis $\{\phi_1, \phi_2\}$ is:

$$M_{d,u}(m_Z) = \frac{v}{\sqrt{2}}\left(\cos\beta\ Y_{d,u}^{(1)}(m_Z) + \sin\beta\ Y_{d,u}^{(2)}(m_Z)\right), \tag{3.54}$$

$$\Gamma_{d,u}(m_Z) = \frac{v}{\sqrt{2}}\left(-\sin\beta\ Y_{d,u}^{(1)}(m_Z) + \cos\beta\ Y_{d,u}^{(2)}(m_Z)\right). \tag{3.55}$$

In order to rewrite the Lagrangian eq. (3.53) in terms of the mass eigenstates, we first express the Higgs doublets Φ_1, Φ_2 in terms of the physical Higgs fields $h, H, A, H^\pm$ and the Goldstone bosons $G^0, G^\pm$:

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \cos(\alpha-\beta)H - \sin(\alpha-\beta)h + iG^0) \end{pmatrix}, \tag{3.56}$$

$$\Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\sin(\alpha-\beta)H + \cos(\alpha-\beta)h + iA) \end{pmatrix}, \tag{3.57}$$

for a CP conserving Higgs potential and where α is the mixing angle of the mass eigenstates [53]. Note that in a general 2HDM the ratio of the expectation values $\tan\beta$ has no well defined meaning. The basis of the Higgs fields can be freely chosen and we could just have started in the basis Φ_1, Φ_2 instead of ϕ_1, ϕ_2 (thus setting $\beta = 0$). The only relevant mixing angle is therefore $\alpha - \beta$. This is in contrast to 2HDM type I and II where there is a clear distinction of the two Higgs doublets by the way they couple to the fermions. The ratio of the two vevs then gets a real, physical meaning.

Finally, we perform unitary transformations of the quark fields in flavour space

$$u'_L = V_u^L(m_Z)\; u_L, \qquad u'_R = V_u^R(m_Z)\; u_R, \tag{3.58}$$
$$d'_L = V_d^L(m_Z)\; d_L, \qquad d'_R = V_d^R(m_Z)\; d_R, \tag{3.59}$$

in order to diagonalize the quark mass matrices: $M_u^{\text{diag.}} = V_u^L M_u V_u^{R\dagger}$, $M_d^{\text{diag.}} = V_d^L M_d V_d^{R\dagger}$. In this new basis, where the Higgs and quark mass matrices are all diagonal, $\Gamma_u(m_Z)$ and $\Gamma_d(m_Z)$ are not diagonal and thus give rise to the following flavour violating neutral Higgs couplings:

$$\begin{aligned} \mathcal{L} \supset\; & \bar{u}_L \Delta_u \left[\cos(\alpha-\beta)h + \sin(\alpha-\beta)H - iA\right] u_R \\ & + \bar{d}_L \Delta_d \left[\cos(\alpha-\beta)h + \sin(\alpha-\beta)H + iA\right] d_R, \end{aligned} \tag{3.60}$$

where:

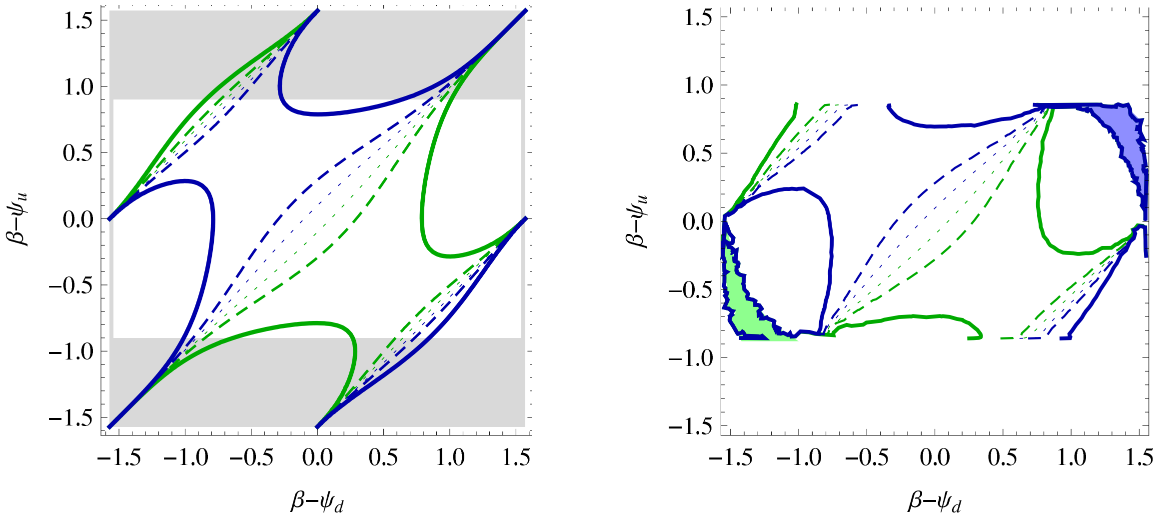
$$\Delta_u = \frac{1}{v} V_u^{L\dagger}(m_Z)\Gamma_u(m_Z)V_u^R(m_Z), \tag{3.61}$$
$$\Delta_d = \frac{1}{v} V_d^{L\dagger}(m_Z)\Gamma_d(m_Z)V_d^R(m_Z). \tag{3.62}$$

It is possible to calculate approximate expressions for Δ_u, Δ_d noting that:

$$\Delta_u = \frac{1}{v}(V_u^{L\dagger}\Gamma_u M_u^{-1} V_u^L)(V_u^{L\dagger} M_u V_u^R), \tag{3.63}$$

and analogously for Δ_d. Substituting eqs. (3.52), (3.54) and (3.55) and keeping the lower order terms in ϵ_u, δ_u we find that the off-diagonal couplings read:

$$\Delta_u^{\text{off-diag.}} = E_u Q_u, \tag{3.64}$$
$$\Delta_d^{\text{off-diag.}} = E_d Q_d, \tag{3.65}$$



(a) Analytical approximation of E_d (b) Numerical result for $2.5E_d$

Figure 3.3: *Contour plots of E_d for $\Lambda = 10^{19}$ GeV. The left figure corresponds to the analytic formula, eq. (3.69). Solid/dashed/dotted lines correspond to the absolute values of 1/0.3/0.1, blue lines correspond to negative values, green lines to positive ones. The right figure shows $2.5\Delta_{d,23}/Q_{d,23}$ where $\Delta_{d,23}$ has been obtained by numerically solving the RGEs. The rescaling was done in order to make the comparison to the analytical result easier.*

where, assuming real ψ_u, ψ_d:

$$Q_u \equiv \frac{1}{v^3}\left(V_{CKM}\left(M_d^{diag.}\right)^2 V_{CKM}^\dagger M_u^{diag.}\right)^{\text{off-diag.}}, \tag{3.66}$$

$$E_u \equiv \frac{1}{8\pi^2}\frac{\sin(2(\psi_u-\psi_d))}{\cos^2(\beta-\psi_u)\cos^2(\beta-\psi_d)}\log\left(\frac{m_Z}{\Lambda}\right), \tag{3.67}$$

$$Q_d \equiv \frac{1}{v^3}\left(V_{CKM}^\dagger\left(M_u^{diag.}\right)^2 V_{CKM} M_d^{diag.}\right)^{\text{off-diag.}}, \tag{3.68}$$

$$E_d \equiv -E_u. \tag{3.69}$$

Thus, the off-diagonal elements of the flavour violating Higgs couplings $\Delta_{u,d}$ can be factorized in two parts: $Q_{u,d}$ are determined by the experimental values of the entries of the CKM matrix and the quark masses, whereas $E_{u,d}$ depend on the unknown details of the 2HDM and the scale Λ at which the alignment condition is imposed. It is apparent from (3.67) and (3.69) that $E_{u,d}$ depend just on two parameters: $\beta-\psi_u$ and $\beta-\psi_d$. Moreover, for $\psi_u = \psi_d$ and $\psi_u = \psi_d \pm \pi/2$ the flavour violating Higgs couplings vanish, since $E_u = E_d = 0$. This choice includes as special cases all the 2HDMs with discrete symmetries we looked at in 3.2.1.

In fig. 3.3 contours of E_d are plotted. The parameter range $-\frac{\pi}{2} < \beta-\psi_{d,u} < \frac{\pi}{2}$ is sufficient as E_d is invariant under the shift $\beta-\psi_{d,u} \to \beta-\psi_{d,u}+\pi$. As cut-off $\Lambda = 10^{19}$ GeV has been chosen as it is the scenario where maximal FCNCs can be expected and largest deviations of the leading log approximation. For this cut-off the grey shaded rectangles are not accessible as some Yukawa couplings become non-perturbative below $\Lambda = 10^{19}$ GeV. As will be shown in section 3.4.3 E_d can easily be $\mathcal{O}(1)$ while still satisfying the bounds on exotic contributions to the FCNCs.

To evaluate the accuracy of the analytical formulae we have calculated the flavour violating Higgs coupling Δ_{d23} solving numerically the full one-loop RGEs in the appendix.[8] We show in fig. 3.3(b) Δ_{d23}/Q_{d23} and we find a very good agreement with the value E_d calculated analytically in eqs. (3.67) and (3.69) up to an overall factor of 2.5 (due to the large, flavour independent, effects in the running of the strong coupling constant and the top Yukawa coupling, which are not contemplated by the leading log approximation). We find, nevertheless, a new feature: There are regions with flipped sign at the top right and bottom left of the figure which are shown shaded. In these regions, differences in the running of $Y_d^{(1)}$ compared to $Y_d^{(2)}$, not present in the leading log approximation, lead to a change of the sign of M_d, cf. eq. (3.54). Diagonalizing the quark masses, eqs. (3.59), this sign is transferred to Δ_d.

Before deriving upper bounds on the flavour off-diagonal couplings we will present our approximate formulae in two parameterizations widely used in the literature: the Wolfenstein parameterization of the CKM matrix and the Cheng & Sher parameterization of flavour violating couplings in a general 2HDM.

In the Wolfenstein parameterization

Using the Wolfenstein parameterization of the CKM matrix, eq. (2.27) and the approximate expressions

$$M_u^{diag.} \sim \frac{v}{\sqrt{2}}\mathrm{diag}(\lambda^6, \lambda^3, 1), \quad M_d^{diag.} \sim \frac{v}{\sqrt{2}}\mathrm{diag}(\lambda^6, \lambda^4, \lambda^2), \tag{3.70}$$

we get the following estimates:

$$[Q_u]_{12} \sim \lambda^{12}, \qquad [Q_u]_{13} \sim \lambda^8, \qquad [Q_u]_{23} \sim \lambda^6, \tag{3.71}$$

$$[Q_d]_{12} \sim \lambda^9, \qquad [Q_d]_{13} \sim \lambda^5, \qquad [Q_d]_{23} \sim \lambda^4, \tag{3.72}$$

and smaller values for the $(21), (31), (32)$ entries.

[8] We have used $\beta - \psi_e = 0$ but the result is independent unless $\cos(\beta - \psi_e) \to 0$.

In the Cheng & Sher parameterization

The non-diagonal couplings of 2HDM are often parameterized as [86]:

$$(\Delta_u)_{ij} = \lambda^u_{ij}\frac{\sqrt{m_{ui}m_{uj}}}{v}, \qquad (\Delta_d)_{ij} = \lambda^d_{ij}\frac{\sqrt{m_{di}m_{dj}}}{v}. \tag{3.73}$$

Bounds on the coefficients $\lambda^{u,d}_{ij}$ have been derived from experimental results e.g. in [83]. These bounds depend on the masses of the Higgs bosons and on whether the parameters $\lambda^{u,d}_{ij}$ are assumed to be universal or to posses some kind of hierarchy. We do *not* assume universality. Instead the Yukawa alignment condition leads to:

$$|\lambda^u_{12}| \sim 6\times 10^{-7}\ E_u, \qquad |\lambda^u_{13}| \sim 10^{-4}\ E_u, \qquad |\lambda^u_{23}| \sim 7\times 10^{-5}\ E_u, \tag{3.74}$$

$$|\lambda^d_{12}| \sim 5\times 10^{-4}\ E_d, \qquad |\lambda^d_{13}| \sim 6\times 10^{-2}\ E_d, \qquad |\lambda^d_{23}| \sim 0.1\ E_d. \tag{3.75}$$

3.4.3 Experimental Bounds

In general, there are numerous experimental bounds on the parameters of 2HDMs. Constraints derived for the type I or type II apply also in our scenario as it is more general. In [84] bounds on the aligned 2HDM have been studied explicitly. As the alignment condition is broken by radiative corrections, in addition tree-level FCNCs are present. This leads to further constraints on the parameters of this type of 2HDMs, as FCNCs are known from experiment to be highly suppressed.

Meson-antimeson mixing

Stringent experimental bounds on FCNCs come from meson-antimeson mixing. In the SM this mixing can occur only at loop level while in a general 2HDM there is also a tree level mediation, see fig. 3.1(a).

A consequence of meson-antimeson mixing - whether at tree level or in a loop process - is that the flavour eigenstates are not mass eigenstates. This means that there are tiny mass differences in between mesons and antimesons that have been determined experimentally for B^0_d, B^0_s, D^0 and K^0 mesons. Here, we treat only the $B^0_s - \bar{B}^0_s$ system as it gives the strongest constraints (see e.g. eqs. (3.74) and (3.75)). The effective Hamiltonian of the $\Delta B = 2$ transition $B^0_s \leftrightarrow \bar{B}^0_s$ is at scale $\sim m_Z$:

$$H^{\Delta B=2}_{\text{eff.}} = \sum_{i,a} C^a_i(m_Z) Q^a_i(m_Z), \tag{3.76}$$

where in a 2HDM with flavour violation at tree level the relevant operators are:

$$Q_1^{SLL} = (\bar{b}_R s_L)(\bar{b}_R s_L), \quad Q_1^{SRR} = (\bar{b}_L s_R)(\bar{b}_L s_R), \quad Q_2^{LR} = (\bar{b}_R s_L)(\bar{b}_L s_R). \tag{3.77}$$

The corresponding Wilson coefficients can be read off the effective Hamiltonian to be:

$$C_1^{SLL} = -\frac{(\Delta^*_{d23})^2}{2}\left(\frac{s^2_{\alpha-\beta}}{m_H^2} + \frac{c^2_{\alpha-\beta}}{m_h^2} - \frac{1}{m_A^2}\right) \quad C_1^{SRR} = -\frac{(\Delta_{d32})^2}{2}\left(\frac{s^2_{\alpha-\beta}}{m_H^2} + \frac{c^2_{\alpha-\beta}}{m_h^2} - \frac{1}{m_A^2}\right) \tag{3.78}$$

$$C_2^{LR} = -\Delta^*_{d23}\Delta_{d32}\left(\frac{s^2_{\alpha-\beta}}{m_H^2} + \frac{c^2_{\alpha-\beta}}{m_h^2} + \frac{1}{m_A^2}\right) \tag{3.79}$$

and the meson-antimeson mass difference can be calculated as:

$$\Delta m_{B_s} = \left|\Delta m^{\rm SM}_{B_s} + \frac{2}{3} m_{B_s} F^2_{B_s}\left[P_2^{LR} C_2^{LR}(m_Z) + P_1^{SLL}\left(C_1^{SLL}(m_Z) + C_1^{SRR}(m_Z)\right)\right]\right| \tag{3.80}$$

where the coefficients P_i^a include both the renormalization group evolution from the high scale m_Z down to low energy $\sim m_{B_s}$ and the hadronization of the quarks to mesons. They can be calculated using the formulae in [87] and lattice QCD results from [88]. For the $B_s^0 - \bar{B}_s^0$-system we get $P_2^{LR} = 3.0$ and $P_1^{SLL} = -1.9$. As $[\Delta_d]_{ij} \gg [\Delta_d]_{ji}$ for $j > i$ the term involving C_1^{SRR} is always neglible, whereas C_2^{LR} dominates only for degenerate Higgs masses or in the decoupling limit ($c_{\alpha-\beta} \to 0,\ m_H \approx m_A$). Therefore:

$$\Delta m_{B_s} \simeq \left|\Delta m^{\rm SM}_{B_s} + \frac{1}{3} m_{B_s} F^2_{B_s} P_1^{SLL} \Delta^{*2}_{d23}\left(\frac{s^2_{\alpha-\beta}}{m_H^2} + \frac{c^2_{\alpha-\beta}}{m_h^2} - \frac{1}{m_A^2}\right)\right|, \tag{3.81}$$

$$\Delta m_{B_s} \simeq \left|\Delta m^{\rm SM}_{B_s} + \frac{2}{3} m_{B_s} F^2_{B_s} P_2^{LR} \Delta^*_{d23}\Delta_{d32}\left(\frac{s^2_{\alpha-\beta}}{m_H^2} + \frac{c^2_{\alpha-\beta}}{m_h^2} + \frac{1}{m_A^2}\right)\right| \quad \text{degenerate masses or decoupling limit} \tag{3.82}$$

The SM prediction for the mass difference $\Delta m^{\rm SM}_{B_s} = (135\pm 20)\cdot 10^{-13}$ GeV [89] agrees quite well with the experimental value $\Delta m^{\rm exp.}_{B_s} = (116.4 \pm 0.5) \cdot 10^{-13}$ GeV [90]. Nevertheless there is room for new physics as long as the new contribution to the mass difference is smaller than the theoretical uncertainty, i.e. $20 \cdot 10^{-13}$ GeV. Using eq. (3.65) this leads to the approximate bound (we take $F_{B_s} = 238.8 \pm 9.5$ MeV [88], $m_{B_s} = 5.37$ GeV [90] and values for the quark masses at m_Z from [91]):

$$\left|\frac{s^2_{\alpha-\beta}}{m_H^2} + \frac{c^2_{\alpha-\beta}}{m_h^2} - \frac{1}{m_A^2}\right| |E_d|^2 \lesssim \frac{1}{(80\ \text{Gev})^2}. \tag{3.83}$$

Thus, even for light Higgs masses, $\mathcal{O}(100$ GeV$)$, the present experimental constraints from meson-antimeson mixing allow E_d to be of $\mathcal{O}(1)$. In the case of degenerate masses or the decoupling limit the bound is even weaker:

$$\left|\frac{s^2_{\alpha-\beta}}{m_H^2}+\frac{c^2_{\alpha-\beta}}{m_h^2}+\frac{1}{m_A^2}\right||E_d|^2 \lesssim \frac{1}{(20\text{ Gev})^2}. \tag{3.84}$$

3.4.4 Leptonic B decays

The decay $\bar{B}_s \to l^+l^-$ ($l = e, \mu, \tau$), based on the flavour transition $b \to s$, is another example of a process that can be mediated at tree level in a general 2HDM (see fig. 3.1(b)) but neither in the SM nor in an aligned 2HDM. As the branching ratio depends on the lepton Yukawa coupling one expects the decay $\bar{B}_s \to \tau^+\tau^-$ to be favoured. However, the produced τ's decay immediately to jets and leptons whose observed invariant mass will not reconstruct back to the mass of the B meson, so that these decays cannot be tagged in detectors [92]. In contrast, tagging $\bar{B}_s \to \mu^+\mu^-$ is rather easy leading to an unparalleled bound on the branching ratio. Integrating out the Higgs boson, the matrix element for the h exchange is, assuming real $\Delta_{e\,22}$ (see Feynman rule (A.32)):

$$\mathcal{M}^h_{\bar{B}_s\to\mu^+\mu^-} = \frac{1}{4}c_{\alpha-\beta}(\Delta_{d\,23}-\Delta^*_{d\,32})\,\langle\bar{B}_s|\,\bar{s}\gamma^5 b\,|0\rangle\,\frac{1}{m_h^2}\left(-\frac{m_\mu}{v}s_{\alpha-\beta}+c_{\alpha-\beta}\Delta_{e\,22}\right)\bar{\mu}\mu, \tag{3.85}$$

since $\langle\bar{B}_s|\,\bar{s}b\,|0\rangle$ is zero, as $\bar{B}_s$ is parity-odd, whereas $\bar{s}b$ is parity-even (see e.g. [93]). The matrix element for H exchange is of the same form as eq. (3.85), with the replacements $c_{\alpha-\beta} \to s_{\alpha-\beta}$, $s_{\alpha-\beta} \to -c_{\alpha-\beta}$, $m_h \to m_H$. Lastly, the invariant amplitude for the exchange of a pseudo-scalar A can be inferred from eq. (A.33) to be:

$$\mathcal{M}^A_{\bar{B}_s\to\mu^+\mu^-} = \frac{1}{4}(\Delta_{d\,23}+\Delta^*_{d\,32})\,\langle\bar{B}_s|\,\bar{s}\gamma^5 b\,|0\rangle\,\frac{1}{m_A^2}\Delta_{e\,22}\;\bar{\mu}\gamma^5\mu. \tag{3.86}$$

The decay rate can now be straightforwardly calculated from the decay amplitudes. Using

$$\langle\bar{B}_s|\,\bar{s}\gamma^5 b\,|0\rangle \approx i f_B m_{B_s}, \tag{3.87}$$

and neglecting again terms proportional to $[\Delta_d]_{ji,j>i}$ we obtain:

$$\Gamma_{\bar{B}_s\to\mu^+\mu^-} = \frac{f^2_{B_s}m^3_{B_s}}{64\pi}|\Delta_{d\,23}|^2 \times \left\{\frac{\Delta^2_{e\,22}}{m_A^4}+\left|\frac{s_{\alpha-\beta}}{m_H^2}\left(\frac{m_\mu}{v}c_{\alpha-\beta}+\Delta_{e\,22}s_{\alpha-\beta}\right)+\frac{c_{\alpha-\beta}}{m_h^2}\left(-\frac{m_\mu}{v}s_{\alpha-\beta}+\Delta_{e\,22}c_{\alpha-\beta}\right)\right|^2\right\}. \tag{3.88}$$

Requiring that the tree level alone does not exceed the present experimental bound $\mathrm{BR}(\bar{B}_s \to \mu^+\mu^-) < 6.4 \cdot 10^{-9}$ [90] we find for $\Delta_{e\,22} \gg \frac{m_\mu}{v}$:

$$\sqrt{\frac{1}{m_A^4} + \left| \frac{s^2_{\alpha-\beta}}{m_H^2} + \frac{c^2_{\alpha-\beta}}{m_h^2} \right|^2} \Delta_{e\,22} |E_d| \lesssim \frac{1}{(1000\ \mathrm{Gev})^2}. \tag{3.89}$$

Hence for Higgs masses of $\mathcal{O}(100\ \mathrm{GeV})$ and E_d of $\mathcal{O}(1)$ there is only a conflict for $\tan(\psi_e - \beta) \sim \Delta_{e\,22}\frac{v}{m_\mu} \gtrsim 25$. The opposite case, $\Delta_{e\,22} \ll \frac{m_\mu}{v}$, results in a bound on E_d and the Higgs masses only. However, for generic Higgs masses the bound coming from $B_s - \bar{B}_s$ mixing is stronger.

To conclude, let us remark that the flavour violating (FV) couplings calculated here yield the absolute lower bound on the size of exotic contributions to FCNCs in any 2HDM, barring cancellations and the existence of symmetries.

3.5 $\mathcal{Z}_2$, $\mathrm{U}(1)_{\mathrm{PQ}}$ or alignment?

Note that our alignment model in sec. 3.4 can be seen as a limiting case of the MFV construction in eq. (3.37) where the MFV expansion is truncated after the first term:

$$Y^{(2)}_{u,d} = \eta_{u,d} Y^{(1)}_{u,d} \tag{3.90}$$

In the framework of *general MFV* [47], where the breaking of the flavour group is decoupled from the breaking of CP, the coefficients $\eta_{u,d}$ can be complex. In [94] 2HDMs with MFV/alignment and 2HDMs protected by discrete or Peccei-Quinn symmetries were studied. Consider first the case of a 2HDM protected from FCNCs by $\mathrm{U}(1)_{\mathrm{PQ}}$: The Peccei-Quinn symmetry must be broken explicitly because otherwise it would be broken by the vev of ϕ_1 and the theory would contain a massless Goldstone boson. But then the FCNC protection is not stable under radiative corrections. FCNCs can then get quite large. This happens for example in the MSSM with generic soft breaking terms [95, 96]. The FCNC coupling given in [94] is:

$$\mathcal{L}^{\mathrm{FCNC}} = \frac{\epsilon_d}{\cos\beta} (\tilde{\Delta}_d)_{ij} \bar{d}^i_L d^j_R \frac{\sin(\alpha-\beta)H + \cos(\alpha-\beta)h + iA}{\sqrt{2}} \tag{3.91}$$

where ϵ_d is a loop suppression of $\mathcal{O}(10^{-2})$ and $\tilde{\Delta}_d$ is the off-diagonal part of a generic flavour-breaking matrix with $\mathcal{O}(1)$ entries. This effective coupling is well above experimental bounds.

A discrete symmetry can remain unbroken and e.g. for a type II 2HDM the couplings

$Y_d^{(2)}$ and $Y_u^{(1)}$ remain exactly zero. However, there are higher-dimensional operators of the type [94]:

$$\begin{aligned}\mathcal{L}^{\text{dim. }6} = & \frac{c_1}{\Lambda^2}\bar{q}_L Y_{u1}^{\text{dim. }6} u_R \phi_2 |\phi_1|^2 + \frac{c_2}{\Lambda^2}\bar{q}_L Y_{u2}^{\text{dim. }6} u_R \phi_2 |\phi_2|^2 \\ & + \frac{c_3}{\Lambda^2}\bar{q}_L Y_{d1}^{\text{dim. }6} d_R \phi_1 |\phi_1|^2 + \frac{c_4}{\Lambda^2}\bar{q}_L Y_{d2}^{\text{dim. }6} d_R \phi_1 |\phi_2|^2\end{aligned} \tag{3.92}$$

with $c_i \sim \mathcal{O}(1)$. For $\Lambda \sim \mathcal{O}(1 \text{ TeV})$ are above experimental bounds, too. A 2HDM with a discrete symmetry is thus only protected from too large FCNCs if the neutral Higgs masses are well above LHC energies.

The situation is different for MFV/alignment models because the MFV structure of the Yukawa couplings, eq. (3.37), is renormalization group invariant, see eq. (3.52). As we have seen in 3.4 alignment/MFV models thus provide sufficient protection against FCNCs, also when radiative corrections are taken into account.

Chapter 4

Warped Extra-Dimensional Models

4.1 The Randall-Sundrum spacetime

Let us consider a model with only one extra-dimension that is compactified by the following equivalence relations for the fifth dimension y:

$$y \sim y + 2\pi \qquad\qquad y \sim -y \tag{4.1}$$

The first equivalence relation is that of a circle, S_1. The second, Z_2, identifies opposite points on the circle, see figure 4.1. The resulting orbifold is called S_1/Z_2 [28]. Now assume that there is a cosmological constant Λ in the fifth dimension, i.e. it is curved. The extra-dimension is also called the *bulk* and its 4D boundaries are called *branes*. We want the branes to remain static and flat. The induced metric at every point along the fifth dimension must therefore be the flat Minkowski metric $\eta_{\mu\nu}$ and the components of the 5D metric can only depend on y. We write the following ansatz for the 5D metric:

$$\mathrm{d}s^2 = e^{-A(y)}\mathrm{d}x^\mu \mathrm{d}x^\nu \eta_{\mu\nu} - \mathrm{d}y^2 \tag{4.2}$$

The factor $e^{-A(y)}$ is called the *warp factor* since it determines the amount of curvature along the extra dimension. We need to solve the Einstein equations in order to find $A(y)$. To do so, we make a coordinate transformation to go to a coordinate system where there is a pre-factor in front of all the coordinates. This can be achieved by a coordinate transformation $z = z(y)$ where

$$e^{-A(z)/2}\mathrm{d}z = \mathrm{d}y \tag{4.3}$$

The metric is then:

$$\mathrm{d}s^2 = e^{-A(z)}(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2) \tag{4.4}$$

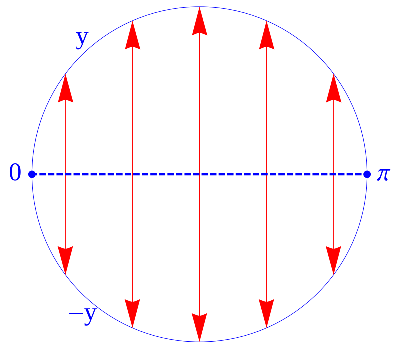


Figure 4.1: S_1/Z_2

This metric is called *conformally flat* since a conformal transformation connects it to the flat metric:

$$g_{MN} = e^{-A(z)}\eta_{MN} \tag{4.5}$$

where $M, N = 0 \ldots 4$. Plugging this into the Einstein equation, we find:

$$e^{-A(z)} = \frac{1}{(kz+1)^2} \tag{4.6}$$

where $k^2 = -\Lambda/12M_*^3$, Λ is the cosmological constant in the extra-dimension and M_* is the 5D-Planck scale. We must furthermore take into account that we are on a S_1/Z_2 orbifold and that thus $z \sim -z$. The result is then:

$$\mathrm{d}s^2 = \frac{1}{(k|z|+1)^2}(\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu - \mathrm{d}z^2) \tag{4.7}$$

Going back to our initial 5th coordinate y we must solve:

$$e^{-A(z)/2}\mathrm{d}z = \frac{\mathrm{d}z}{k|z|+1} = \mathrm{d}y \qquad \Rightarrow \qquad e^{-2k|y|} = \frac{1}{(k|z|+1)^2} \tag{4.8}$$

Our final result, the *Randall-Sundrum (RS) metric* reads [24, 30, 97, 98]:

$$\mathrm{d}s^2 = e^{-2k|y|}\mathrm{d}x^\mu\mathrm{d}x^\nu\eta_{\mu\nu} - \mathrm{d}y^2 \tag{4.9}$$

The RS space-time consists thus of a slice of 5-dimensional Anti-de Sitter space (the bulk, usually abbreviated as AdS_5) wedged in between two flat 4D boundaries, called the *Planck* or *UV brane* and the *TeV* or *IR brane*, see fig. 4.2. An Anti-de Sitter space is a maximally symmetric spacetime of *negative* curvature just as the surface of a sphere is a maximally symmetric space of *positive* curvature. k is a measure of the curvature of the fifth dimension.

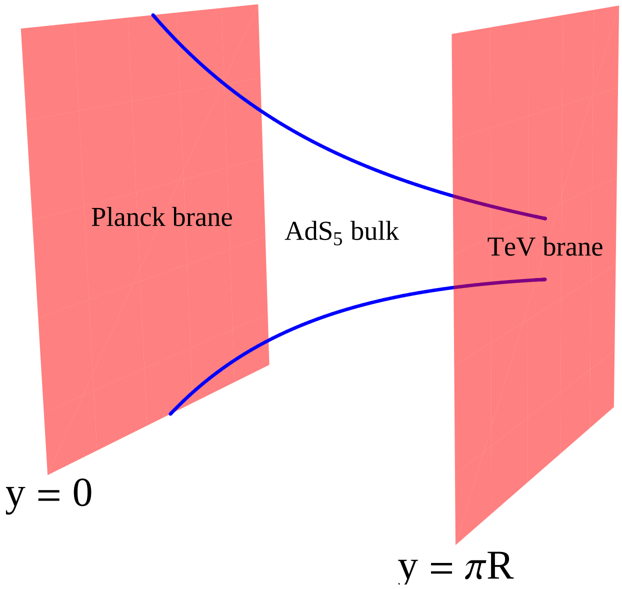


Figure 4.2: *The Randall-Sundrum spacetime. In blue, the warp factor along the extra-dimension.*

It has to be noted that in order to have a 4D low energy effective theory with (almost) vanishing 4D cosmological constant, the energy density on the branes needs to be fine-tuned against the bulk cosmological constant. This is a manifestation of the cosmological constant problem, see section 1.3 [98, 99].

4.1.1 Solving the Hierarchy Problem

Let us consider the Higgs to be localized on the TeV brane. The induced metric on the TeV brane, i.e. at $y = \pi R$, is:

$$g_{\mu\nu}^{\text{ind.}} = e^{-2\pi kR}\eta_{\mu\nu} \tag{4.10}$$

where R is the radius of the extra-dimension. The Higgs action on the TeV brane is:

$$S_{\text{Higgs}} = \int \mathrm{d}^4x\, \sqrt{-g^{\text{ind.}}}\left\{g^{\mu\nu}_{\text{ind.}}\partial_\mu\phi^\dagger\partial_\nu\phi - \frac{\lambda}{2}\left(|\phi|^2 - v_0^2\right)^2\right\} \tag{4.11}$$

where $g^{\text{ind.}}$ is the determinant of the metric. Plugging in (4.10), we get:

$$\begin{aligned} S_{\text{Higgs}} &= \int \mathrm{d}^4x\; e^{-4\pi kR}\left\{e^{2\pi kR}\eta^{\mu\nu}\partial_\mu\phi^\dagger\partial_\nu\phi - \frac{\lambda}{2}\left(|\phi|^2 - v_0^2\right)^2\right\} \\ &= \int \mathrm{d}^4x\; e^{-2\pi kR}\eta^{\mu\nu}\partial_\mu\phi^\dagger\partial_\nu\phi - e^{-4\pi kR}\frac{\lambda}{2}\left(|\phi|^2 - v_0^2\right)^2 \end{aligned} \tag{4.12}$$

Obviously, the kinetic term of the Higgs field, $\eta^{\mu\nu}\partial_\mu\phi^\dagger\partial_\nu\phi$, is not canonically normalized. We therefore need to make a transformation

$$e^{-\pi kR}\phi \to \phi \tag{4.13}$$

The vacuum expectation value v_0 then gets "warped" down on the TeV brane:

$$v = e^{-\pi kR}v_0 \tag{4.14}$$

Thus, if v_0 is of the order of the Planck scale, we need $kR \sim \mathcal{O}(10)$ to get the physical Higgs vev $v = 246$ GeV. The electroweak hierarchy problem is thus solved. v is radiatively stable, since cut-off scales get warped down near the TeV brane as well [30, 98]. What happens to gravity? As it turns out, the 4D graviton $h_{\mu\nu}(x)$ is embedded into the 5D metric as:

$$\mathrm{d}s^2 = e^{-2k|y|}\left[\eta_{\mu\nu} + h_{\mu\nu}(x)\right]\mathrm{d}x^\mu \mathrm{d}x^\nu - dy^2 \tag{4.15}$$

and the 5D Ricci tensor $R^{(5)}_{\mu\nu}$ contains the 4D Ricci tensor $R^{(4)}_{\mu\nu}$ calculated from $h_{\mu\nu}(x)$. The Einstein-Hilbert action therefore contains the following term:

$$S = -M_*^3\int \mathrm{d}^5x\ \sqrt{-g}R^{(5)} \supset -M_*^3\int \mathrm{d}^5x\ e^{-4k|y|}e^{2k|y|}R^{(4)} \tag{4.16}$$

We can therefore read off the 4D effective Planck scale:

$$M^2_{\text{Planck}} = M_*^3\int_{y=0}^{y=\pi R} e^{-2k|y|}\mathrm{d}y = \frac{M_*^3}{2k}\left(1 - e^{-2\pi kR}\right) \tag{4.17}$$

We see that if we take all the fundamental scales, v_0, M_*, k of the order of the Planck scale, the scale of gravity will remain of the order of the Planck scale while the scale of the Higgs vev gets exponentially suppressed. We can thus introduce the large hierarchy in between the Planck and the weak scale with a moderate $kR \sim \mathcal{O}(10)$!

It can be shown that the graviton's wave function is peaked around the Planck brane. That means that gravity is so weak compared to the other fundamental forces because we live at a point in the five-dimensional space-time that is far away from the localization of gravity (i.e. on the TeV brane) [98].

4.2 The Randall-Sundrum Model as an EFT

The mass dimensions of the fields in five dimensions are different from their mass dimensions in four dimensions since in 5D all terms in the Lagrangian must have mass dimension 5 in order for the 5D action

$$S = \int \mathcal{L}\ \mathrm{d}^5x \tag{4.18}$$

to be dimensionless. Looking at the kinetic terms in the 5D Lagrangian:

$$\mathcal{L}_{\text{kin.,fermions}} = \bar{\psi}\not{\partial}\psi \tag{4.19}$$

$$\mathcal{L}_{\text{kin.,scalars}} = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) \tag{4.20}$$

$$\mathcal{L}_{\text{kin.,vectors}} \supset (\partial_\mu A_\nu)(\partial^\mu A^\nu) + \ldots \tag{4.21}$$

we see that the fermions must have mass dimension 2 and the scalars and vectors must have mass dimension 3/2. Now look at the term describing the interaction of the fermions and the gauge bosons, however:

$$\mathcal{L} \supset g\bar{\psi}\gamma^\mu A^a_\mu \frac{\sigma^a}{2}\psi \tag{4.22}$$

For this term to be dimensionless we need the gauge coupling g to have $[m]^{-1/2}$. The 5D gauge coupling has negative mass dimension! As already mentioned in sec. 2.2 this means that the theory is non-renormalizable. This is no reason to dismiss it however, since it can be regarded as an EFT of a UV-complete theory that has yet to be determined.

On the other hand, this means that there is no good reason not to include *all* possible higher dimensional operators[1], such as:

$$\frac{1}{M^2}\bar{\psi}_i\psi_j\bar{\psi}_k\psi_l \tag{4.23}$$

$$\frac{1}{M}\nu\nu\phi\phi \tag{4.24}$$

Operator (4.23) yields FCNCs and proton decay, while operator (4.24) gives masses to the left-handed neutrinos. If these operators are unsuppressed, rates for proton decay and FCNCs are much to high and the neutrinos get too high masses. Therefore one usually assumes that the mass scale M that suppresses these operators is high enough for the EFT to be in agreement with experiments. But in the RS model, all masses

[1]Higher dimensional here refers to the mass dimension of the operator and has nothing to do with the fact that we are in a 5D spacetime!

get "warped down" on the TeV brane by $e^{-k\pi R}$ and so does M:

$$\frac{1}{M^2}\bar{\psi}_i\psi_j\bar{\psi}_k\psi_l \rightarrow \frac{1}{(Me^{-k\pi R})^2}\bar{\psi}_i\psi_j\bar{\psi}_k\psi_l \tag{4.25}$$

$$\frac{1}{M}\nu\nu\phi\phi \rightarrow \frac{1}{Me^{-k\pi R}}\nu\nu\phi\phi \tag{4.26}$$

This looks quite catastrophic but fortunately there is a way out, as we shall see in sec. 4.3.

4.3 Split Fermion Models

In order for the solution to the hierarchy problem to work (see sec. 4.1.1) only the Higgs field needs to be confined to the TeV brane. All the other particles - quarks, leptons and gauge fields - can propagate in the bulk [100, 101, 102, 103, 104, 105, 106, 107].

Let us focus on the fermions [108, 109]: In order to write the bulk Lagrangian we must find the gamma matrices in the RS geometry:

$$[\Gamma^M, \Gamma^N] = 2g^{MN} \tag{4.27}$$

We can take $\Gamma^M = e^M_A\gamma^A$ where e^M_A is the *vielbein* defined by: $g^{MN} = e^M_A e^N_B \eta^{AB}$. The important thing to note here is that with this set of gamma matrices, γ^5 is part of the Dirac algebra. Therefore, unlike in four dimensions, Lorentz invariant terms cannot depend only on γ^5. We thus cannot write the fermions as left- and right-handed Weyl spinors but must write them as four-component Dirac spinors containing both left- and right-handed components. The bulk Lagrangian for fermions is then:

$$\mathcal{L}_{\text{bulk, fermions}} = \sqrt{-g}\left[i\bar{\Psi}_i\Gamma^M D_M\Psi_i + kC_{ij}\bar{\Psi}_i\Psi_j\right] \tag{4.28}$$

where D_M is the covariant derivative in the RS space-time and i, j are generation indices. We see that in the bulk the fermions have a vector-like mass term $k\bar{\Psi}C\Psi$, called the *bulk mass*. We can write the Dirac spinors in terms of left- and right-handed components:

$$\Psi = \begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix} \qquad \Psi_+ = \begin{pmatrix}\psi_+ \\ 0\end{pmatrix} \qquad \Psi_- = \begin{pmatrix}0 \\ \psi_-\end{pmatrix} \tag{4.29}$$

and solve the equations of motion corresponding to (4.28) by making a separation of

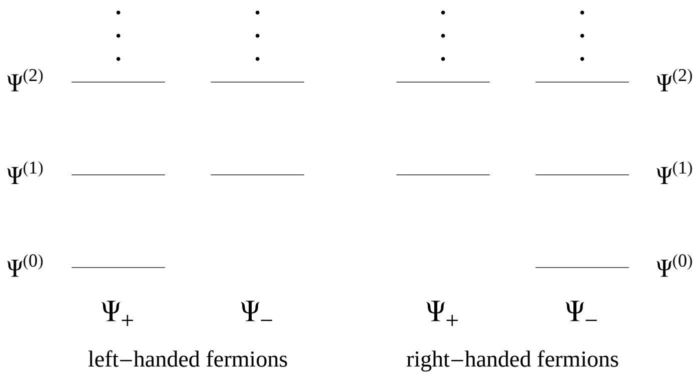


Figure 4.3: *Embedding the SM Weyl spinors into 5D Dirac spinors.*

variables ansatz:

$$\Psi_{\pm,i}(x^\mu, y) = \sum_{n=0}^{\infty} \Psi^{(n)}_{\pm,i}(x^\mu) \tilde{f}^{(n)}_{\pm,i}(y) \tag{4.30}$$

where the Kaluza-Klein modes $\Psi^{(n)}_\pm$ solve the Dirac equation $\gamma^\mu \partial_\mu \Psi^{(n)}_\pm = -m_n \Psi^{(n)}_\pm$ and $\tilde{f}^{(n)}_\pm(y)$ are the wave function profiles along the fifth dimension. First, consider the fermion zero-modes, $m_n = 0$: The general solution to the e.o.m. is given by:

$$\tilde{f}^{(0)}_{\pm,i}(y) = d^{(0)}_\pm \; e^{(2\mp c_i)ky} \tag{4.31}$$

where $d^{(0)}_\pm$ are arbitrary constants and c_i are the eigenvalues of the matrix C. The variation of the action must vanish also at the boundaries of the fifth dimension. It can be shown [108] that this is the case if we have Dirichlet boundary conditions for the fields $\Psi_\pm$:

$$\Psi^{(0)}_-\Big|_{y=0,\pi R} = 0 \qquad \text{or} \qquad \Psi^{(0)}_+\Big|_{y=0,\pi R} = 0 \tag{4.32}$$

Therefore, we can only have a left- or a right-handed zero-mode, but not both at the same time. Either $\Psi^{(0)}_+$ or $\Psi^{(0)}_-$ is killed by the boundary conditions and 4D chirality is recovered from the vector-like bulk! If we choose the left-handed field Ψ_+ and plug it into the first term in (4.28) we get:

$$\mathcal{L} \supset e^{2(1/2-c_i)ky} \; \bar{\Psi}^{(0)}_{+,i} \gamma^\mu \partial_\mu \Psi^{(0)}_{+,i} + \ldots \tag{4.33}$$

Hence with respect to the flat metric the zero-mode profile is:

$$\tilde{f}^{(0)}_+(y) \propto e^{(1/2-c)ky} \tag{4.34}$$

If instead we had chosen Ψ_- to be non-vanishing, we would have got a right-handed

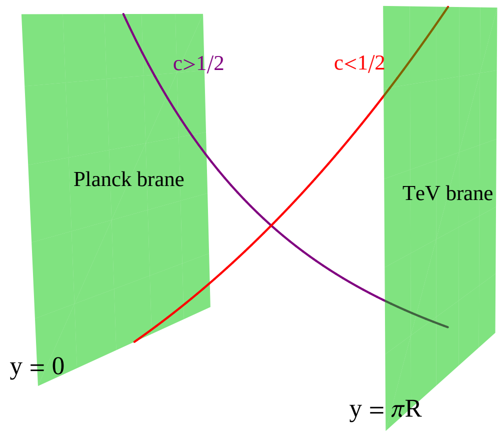


Figure 4.4: *Fermion wave function profiles in the bulk.*

zero-mode and we would need to replace $c \to -c$ in eq. (4.34). We show in fig. 4.3 how the SM fermions can be embedded in the RS framework. For every Weyl spinor ψ of the SM we must introduce a 5D Dirac spinor Ψ. Only its right- or its left-handed component has a zero-mode, however.
Solving the e.o.m. also for the KK excitations, one finds that they are Dirac states with masses:

$$m_n \simeq \left(n + \frac{|c \pm 1/2|}{2} - \frac{1}{4}\right)\pi k e^{-\pi k R} \tag{4.35}$$

We can do the same analysis also for vector, graviton and scalar fields[2] and get the following profiles for the zero-modes[3]:

field	**zero-mode profile**
scalars	$e^{\left(1 \pm \sqrt{4+a}\right)ky}$
fermions	$e^{(1/2 \pm c)ky}$
vectors	1
graviton	e^{-ky}

4.3.1 Solving the Flavour Puzzle

As $\tilde{f}_+^{(0)}(y) \propto e^{(1/2-c)ky}$, the left-handed fermions' wave function is localized towards the UV brane for $c > 1/2$ and towards the IR brane for $c < 1/2$[4], see fig. 4.4. The fermions

[2]At this stage the latter is only of academic interest since the only scalar in nature - the Higgs - does NOT propagate into the bulk.
[3]a is defined via: $m_\phi^2 \equiv ak^2$ and $\mathcal{L} \supset \sqrt{-g}m_\phi^2|\phi|^2$.
[4]For right-handed fermions replace $c \to -c$

have Yukawa couplings to the Higgs that is confined to the IR brane[5] [108, 109]:

$$\mathcal{L} \supset \sqrt{-g}\ \delta(y-\pi R)\left[\bar{q}_{Li}(\tilde{Y}_u^{5D})_{ij}u_{Rj}\tilde{\phi} + \bar{q}_{Li}(\tilde{Y}_d^{5D})_{ij}d_{Rj}\phi\right] \tag{4.36}$$

where $\tilde{Y}_{u,d}^{5D}$ is the dimensionful ($[\tilde{Y}_{u,d}^{5D}] = -1$) 5D Yukawa coupling. We can transform $\tilde{Y}_{u,d}^{5D}$ to a dimensionless coupling by factoring out k: $Y_{u,d}^{5D} = k\tilde{Y}_{u,d}^{5D}$. The effective Lagrangian on the IR brane is then:

$$\mathcal{L}_{\text{eff.4D}} \supset \bar{q}_{Li}^{(0)}(Y_u^{4D})_{ij}u_{Rj}^{(0)}\tilde{\phi} + \bar{q}_{Li}^{(0)}(Y_d^{4D})_{ij}d_{Rj}^{(0)}\phi \tag{4.37}$$

The effective 4D Yukawa matrices $Y_{u,d}^{4D}$ are given as:

$$Y_{u,d}^{4D} = F_q^{\dagger} Y_{u,d}^{5D} F_{u,d} \tag{4.38}$$

The matrices $F_{q,u,d}$ project out the zero-modes of the quarks on the IR brane:

$$F_q = \text{diag.}\left[f(c_{Q^1}), f(c_{Q^2}), f(c_{Q^3})\right] \tag{4.39}$$

$$F_u = \text{diag.}\left[f(c_{u^1}), f(c_{u^2}), f(c_{u^3})\right] \tag{4.40}$$

$$F_d = \text{diag.}\left[f(c_{d^1}), f(c_{d^2}), f(c_{d3})\right] \tag{4.41}$$

where

$$f(c) = \sqrt{\frac{1-2c}{1-e^{k\pi R(2c-1)}}} \tag{4.42}$$

In order to solve the flavour puzzle, see sec. 1.2, all we need to do is to make sure that for light particles $f(c)$ take small values and for heavy particles $f(c)$ take high values. Thanks to the exponential function in (4.42), small hierarchies in the bulk mass eigenvalues c are sufficient to have huge hierarchies in $f(c)$ and thus in the fermion masses. We can also understand this from fig. 4.4: Since the Higgs lives on the TeV brane, its wave functions' overlap with the wave function of particles localized towards the Planck brane is small. Since it is the Higgs that gives masses to these particles, they are light. Light particles such as up, down an strange quarks must therefore be localized towards the Planck brane. On the other hand heavy particles such as the top are localized towards the TeV brane and thus have a large overlap with the Higgs.

In summary, the hierarchies in the fermion masses can be generated with anarchic Yukawa couplings in the RS model. There is no need to assume that the Yukawa matrices have a large hierarchy [110, 111]. This is nice, since it seems much more natural. Such models are called *split fermion models*, since different fermions have

[5]For definiteness, we restrict ourselves to the quarks here.

different eigenvalues c of the bulk mass matrix.

4.3.2 Higher Dimensional Operators in Split Fermion Models

We said in 4.2 that putting the fermions in the bulk would solve the problem with the unsuppressed higher dimensional operators. And indeed it does. This is due to the fact that the constraints are mainly due to experiments involving light fermions. Their overlap with the higher dimensional operators on the TeV brane is small and the resulting processes are thus suppressed.

Consider the four-fermion operator in eq. (4.23). In split fermion models, the effective 4D suppression is given for light fermions (i.e. $1/2 \lesssim c \lesssim 1$) [106]:

$$\frac{1}{M_{4,\text{eff.}}^2} \sim \frac{k}{M_5^3} e^{(4-c_i-c_j-c_k-c_l)k\pi R} \tag{4.43}$$

where the 5D suppression scale M_5 is of the order of the Planck scale. As far as proton decay is concerned, we would need $c_i \gtrsim 1$ for all c's to have sufficient suppression. Unfourtunately $c_i \gtrsim 1$ would lead to too small fermion masses. So we still need a discrete symmetry but there is some suppression and thus no need to prevent operators of mass dimension higher than four. As far as FCNCs are concerned the suppression in (4.43) is sufficient in order to be in agreement with experiment for values of c that lead to the correct fermion masses [101, 112, 113].

4.4 FCNCs via KK gluon exchange

There is another source of FCNCs in RS models with fermion in the bulk, however: the exchange of KK excitations of gauge bosons [110, 114]. This can be understood as follows: Since q_L, u_R and d_R are independent fields we can rotate them to a basis where all the bulk mass matrices C are diagonal at the same time:

$$\mathcal{L} \supset \bar{q}_L C_q q_L + \bar{u}_R C_u u_R + \bar{d}_R C_d d_R \tag{4.44}$$

where

$$C_q = \text{diag.}\left(c_{q^1}, c_{q^2}, c_{q^3}\right) \quad C_u = \text{diag.}\left(c_{u^1}, c_{u^2}, c_{u^3}\right) \quad C_d = \text{diag.}\left(c_{d^1}, c_{d^2}, c_{d^3}\right) \tag{4.45}$$

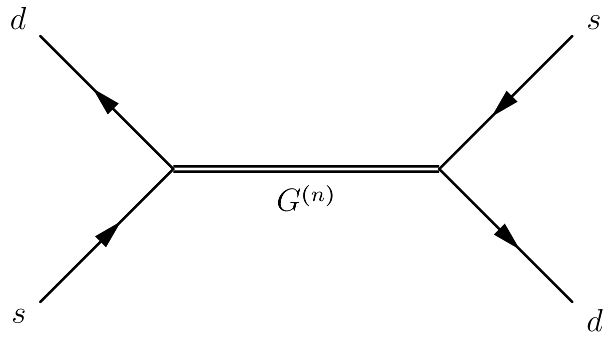


Figure 4.5: *FCNCs induced by KK gluon exchange.*

The matrices $F_{q,u,d}$ that project out the zero modes are then also diagonal. For definiteness let us consider the gluons. They couple to the quarks via:

$$\mathcal{L} \supset g_{*s}(\bar{q}_L q_L + \bar{u}_R u_R + \bar{d}_R d_R)G \tag{4.46}$$

The effective coupling to the quark zero modes is then:

$$g_{s*} F_x^\dagger F_x \tag{4.47}$$

for $x = q, u, d$. This matrix is diagonal but not universal, i.e. it is not proportional to the identity matrix. This is due to the fact that in order to explain the hierarchic quark masses, we need non-degenerate eigenvalues c_i for the bulk mass matrices C_i. As a consequence, $f(c_i)$ are then also non-degenerate.
On the other hand, consider the effective 4D Yukawa coupling (cf. (4.38)):

$$Y_{u,d}^{4D} = F_q^\dagger Y_{u,d}^{5D} F_{u,d} \tag{4.48}$$

In general, this will not be diagonal. To go to the mass basis one must thus make bi-unitary transformations on the quark fields:

$$u_L \to V_{uL} u_L \qquad u_R \to V_{uR} u_R \tag{4.49}$$

$$d_L \to V_{dL} d_L \qquad d_R \to V_{dR} d_R \tag{4.50}$$

The Yukawa matrices are then diagonal in this basis:

$$(Y_{u,d}^{4D})^{\text{diag.}} = V_{(u,d)L} F_q^\dagger Y_{u,d}^{5D} F_{u,d} V_{(u,d)R}^\dagger \tag{4.51}$$

However, when the coupling to the KK gluon, eq. (4.47), is rotated to this new basis, it is not diagonal anymore since rotating a diagonal but non-universal matrix yields a non-diagonal matrix:

$$g_{x,(R,L)} \simeq g_{s*} V_{x(R,L)} F_x^\dagger F_x V_{x(R,L)}^\dagger \neq \text{diag.}(...) \tag{4.52}$$

The exchange of KK gluons can thus induce FCNCs at tree level, see fig. 4.5. Such processes lead to a lower bound on the KK mass of ~ 10^3 TeV. The exchange of the KK excitations of other gauge bosons also generates FCNCs. Their rates are smaller however, due to the smaller gauge coupling.

4.4.1 The RS-GIM mechanism

The situation is alleviated by the *RS-GIM mechanism*, however [106, 109, 115]: Near the TeV brane, where the KK gluons are localized, the wave function of light quarks is suppressed and thus their overlap with the KK gluons is small. Near the Planck brane on the other hand, the wave function of the KK gluons is constant. Since for the fermions there is an orthonormality condition:

$$\int_0^{\pi R} \mathrm{d}y\ e^{-3ky} f_\pm^{(n)} f_\pm^{(n)} = \delta_{mn} \tag{4.53}$$

this means that the coupling of fermions to KK gluons is universal near the Planck brane.

In conclusion, there is everywhere an additional suppression of the flavour-violating couplings of light fermions to the KK gluons. Since experimental bounds are mostly due to processes involving light quarks, this improves the situation quite a bit.

However, residual contributions to ϵ_K [116, 117, 118, 119, 120, 121] and neutron electric dipole moment [109, 115, 122] still lead to significant constraints on the model. The resulting lower bound on the KK scale, which emerges from the effective 4D description of the theory, is then of the order of $\mathcal{O}(20\text{ TeV})$ [118, 119, 120].

4.4.2 The Higgs in the bulk?

Allowing the Higgs to propagate into the bulk helps to further suppress contributions to ϵ_K [123, 124]. This is because then the light fermions can be moved even further towards the Planck brane and the overall Yukawa scale can be increased while maintaining perturbativity. The Higgs vev's profile must be chosen to peak near the TeV brane in order to save the solution to the hierarchy problem, see sec. 4.1.1 [125].

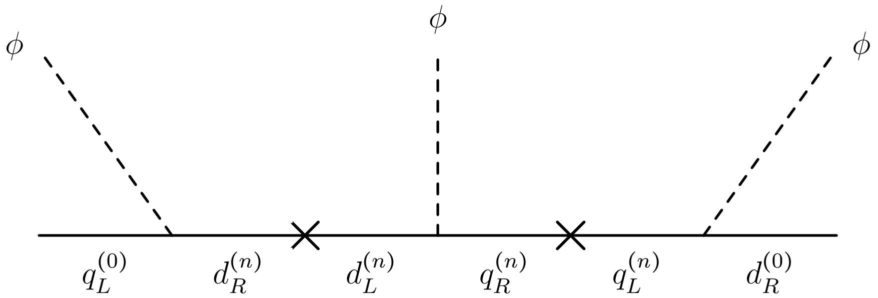


Figure 4.6: *Shift in masses and Yukawa couplings due to higher dimensional operators.*

4.5 FCNCs via Higgs exchange

In split fermion models, the couplings of the Higgs to the fermions are also misaligned with the fermion mass matrix and thus lead to FCNCs at tree level [126]: In the down sector, consider the following dimension 6 operators:

$$\lambda_{ij}\frac{\phi^2}{M^2}\phi\bar{q}_{Li}d_{Rj} \qquad k_{ij}^d\frac{\phi^2}{M^2}\bar{d}_{Ri}\partial\!\!\!/ d_{Rj} \qquad k_{ij}^q\frac{\phi^2}{M^2}\bar{q}_{Li}\partial\!\!\!/ q_{Lj} \tag{4.54}$$

When the electroweak symmetry is broken, these terms yield corrections to the fermion kinetic and mass terms:

$$v\left([Y_d^{4D}]_{ij}+\lambda_{ij}\frac{v^2}{2M^2}\right)\bar{q}_{Li}d_{Rj} \tag{4.55}$$

$$\left(\frac{\delta_{ij}}{2}+k_{ij}^d\frac{v^2}{2M^2}\right)\bar{d}_{Ri}\partial\!\!\!/ d_{Rj} \qquad \left(\frac{\delta_{ij}}{2}+k_{ij}^q\frac{v^2}{2M^2}\right)\bar{q}_{Li}\partial\!\!\!/ q_{Lj} \tag{4.56}$$

On the other hand, the couplings to the physical Higgs scalar h are:

$$\left([Y_d^{4D}]_{ij}+3\lambda_{ij}\frac{v^2}{2M^2}\right)\frac{h}{\sqrt{2}}\bar{q}_{Li}d_{Rj} \tag{4.57}$$

$$\left(2k_{ij}^d\frac{v}{\sqrt{2}M^2}\right)\frac{h}{\sqrt{2}}\bar{d}_{Ri}\partial\!\!\!/ d_{Rj} \qquad \left(2k_{ij}^q\frac{v}{\sqrt{2}M^2}\right)\frac{h}{\sqrt{2}}\bar{q}_{Li}\partial\!\!\!/ q_{Lj} \tag{4.58}$$

These couplings are not diagonal in the same basis as (4.55) and (4.56). We show the Feynman diagrams corresponding to the corrections to the masses, Yukawa couplings and kinetic terms in fig. 4.6 and 4.7. The factor of 3 in front of λ_{ij} in eq. (4.57) and the factor of 2 in front of k_{ij} in eq. (4.58) are due to the fact that there are three (two) ways to set one of the Higgses to its vev in diagram 4.6 (4.7).

The main contribution to FCNCs is due to diagram 4.6. Including thus only the

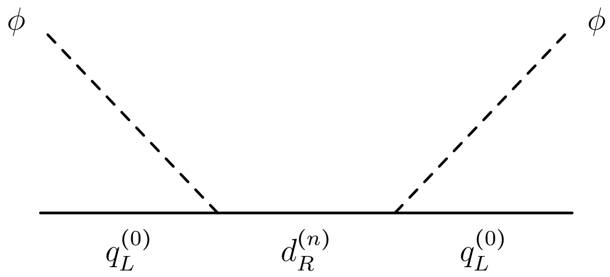


Figure 4.7: *Correction to the kinetic term. An analogous diagram exists for RH quarks.*

corrections (4.55) and (4.57), the effective 4D mass matrix is:

$$M^{\text{4D eff.}} = F_Q Y_d^{5D} F_d v - F_Q Y_d^{5D} Y_d^{5D\dagger} Y_d^{5D} F_d \frac{v^2}{M^2} v \tag{4.59}$$

and the effective 4D Yukawa coupling is:

$$Y^{\text{4D eff.}} = F_Q Y_d^{5D} F_d - 3 F_Q Y_d^{5D} Y_d^{5D\dagger} Y_d^{5D} F_d \frac{v^2}{M^2} \tag{4.60}$$

The FCNCs are then proportional to the off-diagonal elements of the difference

$$\Delta^d = M^{\text{4D eff.}} - Y^{\text{4D eff.}} v = 2 F_Q Y_d^{5D} Y_d^{5D\dagger} Y_d^{5D} F_d \frac{v^2}{M^2} v \tag{4.61}$$

The calculation for the up-sector is completely analogous.

4.6 Constraints from EWPT

There are severe experimental constraints on New Physics models coming from the measurements at LEP. These are usually referred to as *electroweak precision tests (EWPT)*. To analyze a BSM theory in the light of these constraints one usually deploys an EFT [127]. Of special importance are operators that do not contain any fermion fields (sometimes referred to as *oblique* operators). At dimension 6 there are two such operators[6]:

$$\mathcal{O}_S = \phi^\dagger \tau_a \phi F_{\mu\nu}^a B^{\mu\nu} \tag{4.62}$$

$$\mathcal{O}_T = |\phi^\dagger D_\mu \phi|^2 \tag{4.63}$$

[6] There is none at dimension 5.

where $F^a_{\mu\nu}$ and $B_{\mu\nu}$ are the field strengths' of the $SU(2)$ and $U(1)$ gauge bosons, respectively. The effective Lagrangian is then:

$$\mathcal{L}_{\text{eff.}} = \mathcal{L}_{SM} + a_S \mathcal{O}_S + a_T \mathcal{O}_S \tag{4.64}$$

The operator $\mathcal{O}_T$ in eq. (4.63) is especially dangerous since it breaks the *custodial symmetry* of the SM [128]: In the $g' \to 0$ limit, the Higgs sector of the SM possesses an accidental global $SU(2)_L \times SU(2)_R$ symmetry. This symmetry is broken $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ by the Higgs vev[7]. The three broken generators of $SU(2)_L \times SU(2)_R$ give rise to the three Goldstone bosons eaten by the $W^\pm$ and Z bosons. Still in the limit of $g' \to 0$, $SU(2)_{L+R}$ remains unbroken, $W^\pm$ and Z form a triplet of an unbroken symmetry and their masses are equal:

$$M_W^2 = \frac{1}{4} g v^2 \tag{4.65}$$

$$M_Z^2 = \frac{1}{4} (g + g') v^2 \tag{4.66}$$

Radiative corrections to the ρ-parameter, defined as:

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos\theta_W} = 1 \tag{4.67}$$

where $\cos\theta_W \equiv \frac{g^2}{g^2+g'^2}$ must therefore be proportional to g'. The SM is thus protected from large radiative corrections to ρ. Not so in NP models, as we have seen in eq. 4.63. Since experimental results are in good agreement with the SM prediction, this can cause severe problems.

Bounds on the Wilson coefficients a_S and a_T in eq. (4.64) can be deduced from experimental values. On the other hand the Wilson coefficients can be calculated for any NP model and compared to these bounds. To do so, it is common to introduce the *Peskin-Takeuchi parameters* [129]:

$$S = \frac{4scv^2}{\alpha} a_S \qquad T = -\frac{v^2}{2\alpha} a_T \tag{4.68}$$

where $s = \sin\theta_W$, $c = \cos\theta_W$ and α is the fine-structure constant.

In RS models, there are contributions to T coming from integrating out KK towers of particles, see fig. 4.8. The Peskin-Takeuchi parameter T thus violates experimental

[7] The *diagonal* subgroup $SU(2)_{L+R}$ corresponds to simultaneous $SU(2)_L$ and $SU(2)_R$ transformations with $L = R$. (L, R being the transformation matrices of $SU(2)_L$ and $SU(2)_R$, respectively.) Sometimes it is $SU(2)_{L+R}$ that is referred to as the custodial symmetry.

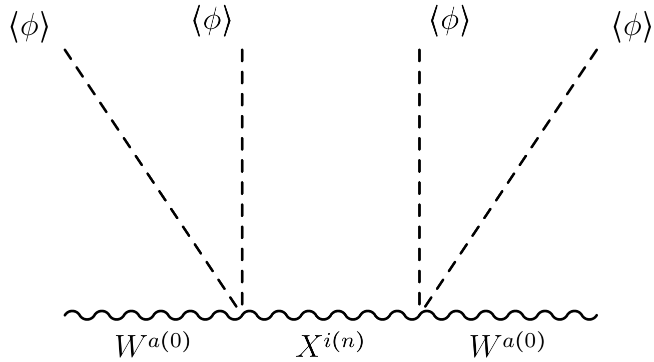


Figure 4.8: *Contribtuions to T in RS. $X^{i(n)}$ are the KK excitations of any field that couples to $W^{\pm}$.*

bounds in RS models [130, 131, 132, 133]. There is a way out, however: Models with a $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$ *gauge* symmetry in the bulk are in agreement with experiment for KK mass scales as low as 3 - 5 TeV [134, 135]. In order to recover the usual $SU(3)_c \times SU(2)_L \times U(1)_Y$ symmetry, $SU(2)_R$ is broken to $U(1)_R$ with orbifold boundary conditions on the Planck brane. $U(1)_R \times U(1)_X$ is then broken spontaneously to $U(1)_Y$ on the Planck brane, where $Y = T_{3R} + X$ is the usual hyper charge. The custodial symmetry in the bulk usually also includes a discrete LR symmetry in order to protect the $Z\bar{b}_L b_L$-coupling.

4.7 Flavour symmetries and alignment in RS

4.7.1 Previous models with suppressed FCNCs

Several models that exploit flavour symmetries in order to suppress the contribution to ϵ_K and thus allow for a lower mass scale of the KK gluons have been proposed.

In [136] the flavour symmetry $U(3)_{q_L} \times U(3)_{u_R} \times U(3)_{d_R}$ is imposed in the bulk. This symmetry implies that the bulk masses for fermions with given quantum numbers are precisely equal. The beautiful solution to the flavour puzzle, sec. 4.3.1, is thus lost. On the IR, the flavour symmetry is broken by the Yukawa couplings. However, the diagonal subgroup of the three $U(3)$'s is conserved. The flavour mixing necessary to reproduce the CKM matrix is then introduced via kinetic mixing terms on the UV brane. Flavour violating higher-dimensional operators are then forbidden everywhere but on the UV brane where they are suppressed by a high (not warped-down) cut-off scale. The difference in the masses of quarks of the different generations is also generated on the UV brane. This is certainly the biggest drawback of this model since the flavour puzzle then cannot be elucidated at the electroweak scale or at the LHC.

Another approach consists in aligning the 5D down Yukawa couplings with the bulk masses [137, 138, 139, 140]. FCNCs in the down sector are then suppressed and the constraints coming from ϵ_K can be satisfied for a lower KK scale. The advantage of these models is that one does not need to give up on addressing the flavour puzzle at the electroweak scale.

In [137], a "horizontal" $U(1)$ symmetry is used to align the bulk masses and 5D Yukawas in the down sector: To do so, we must embed each generation of quarks in four 5D multiplets of the bulk gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$:

$$Q_u^i \ (\mathbf{3},\mathbf{2},\mathbf{2})_{2/3} \qquad Q_d^i \ (\mathbf{3},\mathbf{2},\mathbf{2})_{-1/3} \tag{4.69}$$

$$U^i \ (\mathbf{3},\mathbf{1},\mathbf{1})_{2/3} \qquad D^i \ (\mathbf{3},\mathbf{1},\mathbf{1})_{-1/3} \tag{4.70}$$

Now impose a $U(1)_d$ symmetry on the down-type quark fields with charges (d_1, d_2, d_3) where $d_i \neq d_j$ for $i \neq j$ (i.e. the charges are different for every generation but the same for Q_d and D in each generation). This symmetry forces the bulk masses C_{Q_d}, C_d and the 5D Yukawas of the down sector Y_d^{5D} to be diagonal in the same basis, i.e. aligned[8]. Moreover, it forbids off-diagonal kinetic terms on the IR brane involving Q_d and D. This symmetry is valid in the bulk and on the IR brane but must be broken on the UV brane in order to allow for mixed boundary conditions of the fields Q_u and Q_d. This ensures that the two bulk fields Q_u, Q_d host only one zero mode Q_L.

One sees that it would not be possible to impose the $U(1)$ symmetry if there were only one left-handed bulk field: To allow for Yukawa couplings in the up sector we would need to assign $U(1)$ charges to U as well. Then, C_{Q_L}, C_d, C_u, Y_d and Y_u would be all aligned and there would be no CKM matrix.

A second horizontal symmetry $U(1)_Q$ is introduced in order to avoid flavour violation due to the mixed boundary conditions on the UV brane. This symmetry is valid in the bulk and on the UV brane but broken on the IR, in order to allow for a non-diagonal up-type Yukawa coupling.

The only source of flavour violation in the down sector are then off-diagonal kinetic terms on the IR brane:

$$\mathcal{L} \supset Re^{-6\pi kR} i\delta \left(\bar{Q}_u \tilde{K}_Q \partial\!\!\!/ Q_u + \bar{U} \tilde{K}_u \partial\!\!\!/ U \right) \delta(y - \pi R) \tag{4.71}$$

[8]There cannot be off-diagonal elements in C_{Q_d}, C_d and Y_d^{5D} because this would mix two fields with different $U(1)$ charges.

where $\tilde{K}_{Q,u}$ are non-diagonal matrices with entries of $\mathcal{O}(1)$ and δ is a loop-suppressed dimensionless coefficient. K_Q leads to kinetic mixing of the left-handed down quarks and thus to flavour violation in the down sector.

In this model, the leading flavour constraints come from flavour violation in the up-sector, notably $D^0 - \bar{D}^0$-mixing. These constraints are satisfied for KK scales down to 3 TeV.

In [138], for each generation of quarks there are four quark multiplets under the bulk gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$:

$$Q^i(\mathbf{3},\mathbf{2},\mathbf{2})_{2/3} \qquad Q^i_u(\mathbf{3},1,1)_{2/3} \qquad Q^i_d(\mathbf{3},1,\mathbf{3})_{2/3} \qquad \tilde{Q}^i_d(\mathbf{3},\mathbf{3},1)_{2/3} \tag{4.72}$$

Only the $T_{3R} = -1/2$ component of Q, Q_u and the $T_{3R} = -1$ component of Q_d have zero modes in order to reproduce the SM quark fields q_L, u_R and d_R. A $U(3)$ symmetry under which the fields Q_d and $\tilde{Q}_d$ transform as triplets and all the other fields are singlets is assumed[9]. In the basis where the bulk masses are diagonal and the Yukawa couplings are not, the couplings of the quark zero modes to the KK gluons,

$$\bar{u}^{(0)}_L g^{(n)}_{u_L} u^{(0)}_L \not{G}^{(n)} + \bar{d}^{(0)}_L g^{(n)}_{d_L} d^{(0)}_L \not{G}^{(n)} + \bar{u}^{(0)}_R g^{(n)}_{u_R} u^{(0)}_R \not{G}^{(n)} + \bar{d}^{(0)}_R g^{(n)}_{d_R} d^{(0)}_R \not{G}^{(n)} \tag{4.73}$$

where $G^{(n)}_\mu = T^a G^{(n)a}_\mu$, are diagonal but not flavour universal, except for g_{d_R} which is flavour universal due to the $U(3)$ flavour symmetry. Thus no flavour violating RH currents are generated in the down sector. Loop-suppressed brane kinetic terms reintroduce some FCNCs in the RH down sector, however. In this model, a KK scale of 3 – 4 TeV can be achieved.

An RS model with 5D MFV was proposed in [139, 140]. With 5D MFV the bulk masses have the general form:

$$C_Q = \alpha_Q \mathbb{1} + \beta_Q Y_d Y_d^\dagger + \gamma_Q Y_u Y_u^\dagger \tag{4.74}$$

$$C_u = \alpha_u \mathbb{1} + \gamma_u Y_u^\dagger Y_u \tag{4.75}$$

$$C_d = \alpha_d \mathbb{1} + \beta_d Y_d^\dagger Y_d \tag{4.76}$$

where $Y_{u,d}$ are anarchic 5D Yukawa matrices. In the limit where $\gamma_Q \to 0$ in eq. (4.74), C_Q, C_d and Y_d are diagonal in the same basis. Flavour violation is then completely absent in the down sector. If the alignment is not complete, i.e. if γ_Q is small but

[9]In [138] this symmetry is stated to be global. However, as remarked in [140], this symmetry should be gauged since the bulk physics includes quantum gravity effects that generically violate any global symmetries.

non-zero, the alignment is quite a bit less than naively expected. This is due to the fact that the suppression of FCNCs is due to the alignment of the *traceless* parts of C_Q and Y_d. The misalignment is then proportional to the ratio of the traceless parts of $Y_u Y_u^\dagger$ and $Y_d Y_d^\dagger$. On the other hand, the size of the respective traceless contributions is determined by the size of the non-degeneracy of the eigenvalues of C_u and C_d. Due to the hierarchy in the masses and CKM mixing angles, the degeneracy is much stronger in C_d than in C_u. Thus, in order to be in agreement with experimental bounds on ϵ_K for KK scales ~ 3 TeV, we need a rather high degree of alignment, $\gamma_Q \sim \mathcal{O}(10^{-2})$ [140]. Such a high degree of alignment can best be explained by a symmetry. In [140] in order to achieve full alignment a subgroup of the MFV flavour group $SU(3)_Q \times SU(3)_u \times SU(3)_d$ is imposed in the bulk. The authors of [140] present two different models:

- Model A: bulk flavour symmetry $SU(3)_Q \times SU(3)_d$
- Model B: the flavour symmetry is only the diagonal subgroup $SU(3)_{Q+d}$

This symmetry is then broken on the UV brane and the symmetry breaking effect is transmitted through the bulk by bulk scalar fields that transform under the flavour symmetry as bi-fundamentals $Y_d(\mathbf{3}_Q, \bar{\mathbf{3}}_d)$ (Model A) or adjoints $A_d(\mathbf{8}_{Q+d})$ (Model B) (*"shining"*).

In model A, the bulk mass parameters are of the form:

$$C_Q = \alpha_Q \mathbb{1} + \beta_Q Y_d Y_d^\dagger \tag{4.77}$$

$$C_d = \alpha_d \mathbb{1} + \beta_d Y_d^\dagger Y_d \tag{4.78}$$

$$m_{ij}^d = \frac{v}{\sqrt{2}} f_{Qi} Y_{dij} f_{dj} \tag{4.79}$$

For Model B one gets:

$$C_Q = \alpha_Q \mathbb{1} + \beta_Q A_d \tag{4.80}$$

$$C_d = \alpha_d \mathbb{1} + \beta_d A_d \tag{4.81}$$

$$m_{ij}^d = \frac{v}{\sqrt{2}} f_{Qi} \left(\alpha_Y + \beta_Y A_d\right)_{ij} f_{dj} \tag{4.82}$$

In both cases, obviously, the bulk masses and Yukawas are aligned in the down sector. On the IR brane there are however higher dimension operators that feed Y_u into Y_d such that the effective 4D down sector Yukawa coupling is misaligned with C_Q and C_d:

$$Y_d^{\text{eff.}} \sim Y_d \left(1 + \frac{N_{KK}^2}{4\pi^2} Y_u Y_u^\dagger\right) \tag{4.83}$$

where N_{KK} is the number of weakly coupled KK modes. The suppression factor for the misalignment is then given by:

$$\epsilon = \frac{N_{KK}^2 |Y_u|^2}{4\pi^2} \tag{4.84}$$

where $|Y_u|$ is the average value of the entries of Y_u. In order to suppress the tree-level CP violation in the D-system and to obtain a sufficiently large top mass, the Yukawas must be at the verge of perturbativity. Thus, such loop effects can get quite large. Possible solutions to this drawback are the introduction of a second Higgs doublet in order to have separate Higgses for the up and down sector or to raise the average values of the entries of Y_d. The latter would imply some amount of tuning in order to get the correct vev $\langle Y_d \rangle$ on the IR brane. In this model, just as in the two previous models, the up-sector is anarchic.

4.7.2 A new way to alignment

We now present a new, more efficient alignment model in RS [2]. In this model, we assume the usual custodial symmetry in the bulk in order to lower the bound from electroweak precision tests to the range of 3-5 TeV [134, 135]. The bulk gauge group is thus $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_X}$, and it is broken to $\mathrm{SU(2)_L \times U(1)_Y}$ on the UV brane. The first key idea is to separate the LH quark doublets into two sets of up and down doublets. There are several ways to choose the representations of the fields, and we choose the following:

$$Q_u^i(\mathbf{2},\mathbf{2})_{+\frac{2}{3}} \qquad Q_d^i(\mathbf{2},\mathbf{2})_{-\frac{1}{3}} \qquad U^i(1,1)_{+\frac{2}{3}} \qquad D^i(1,1)_{-\frac{1}{3}} \qquad H(\mathbf{2},\mathbf{2})_0\,, \tag{4.85}$$

where i is a generation index. The bulk flavour group is $\mathrm{SU(3)_{Q_u} \times SU(3)_{Q_d} \times SU(3)_u \times SU(3)_d}$. The fermion fields have bulk masses

$$\mathcal{L} \supset \overline{Q}_u C_{Q_u} Q_u + \overline{Q}_d C_{Q_d} Q_d + \overline{U} C_u U + \overline{D} C_d D\,, \tag{4.86}$$

and Yukawa couplings to the Higgs[10]

$$\mathcal{L} \supset \overline{Q}_u^i \left(Y_u^{5\mathrm{D}}\right)_{ij} U^j H + \overline{Q}_d^i \left(Y_d^{5\mathrm{D}}\right)_{ij} D^j H\,. \tag{4.87}$$

[10] In extra dimensional theories it is useful to transform dimensionful couplings into dimensionless form by factoring out the appropriate power of some natural scale (the AdS curvature scale k in our case). Here and below we will carelessly use the same notation for both versions of the various couplings.

Note that due to the choice of representations in eq. (4.85), Q_u cannot have a Yukawa coupling to D and Q_d cannot couple to U. The Higgs can either be localized on the IR brane or allowed to spread in the bulk. The latter choice enables to suppress the flavour constraints to some extent (see e.g. [124, 125]), but for simplicity we choose the former and rely on our flavour alignment mechanism to lower the bound. Another set of important couplings of the fermions is to the gluon field,

$$\mathcal{L} \supset g_{s*} \left(\overline{Q}_u Q_u + \overline{Q}_d Q_d + \overline{U} U + \overline{D} D \right) G \,, \tag{4.88}$$

where g_{s*} is the 5D coupling of the gluon field G.

On the UV brane both LH quark fields have the same quantum numbers, and thus can mix. Consequently, the LH part of the bulk flavour group $\mathrm{SU}(3)_{\mathrm{Q_u}} \times \mathrm{SU}(3)_{\mathrm{Q_d}}$ is enhanced to $\mathrm{SU}(6)_{\mathrm{Q}}$. This mixing can be used to get rid of the excess of LH states, via coupling them to a set of UV brane localized states $\tilde{Q}$,

$$\left(\overline{Q}_u, \overline{Q}_d \right)^i M_{ij} \tilde{Q}^j \,, \tag{4.89}$$

where M is a 6×3 mass matrix, naturally of order k, composed of the two 3×3 matrices M_u and M_d:

$$(\overline{Q}_u, \overline{Q}_d) \begin{pmatrix} M_u \\ M_d \end{pmatrix} \tilde{Q} = (\overline{Q}_u M_u + \overline{Q}_d M_d) \tilde{Q} \,. \tag{4.90}$$

As a result, three linear combinations of the six states (Q_u, Q_d) obtain masses of order k, and three other combinations remain massless. The former therefore decouple from the effective 4D theory, while the latter correspond to the physical SM LH states.

To see how this works explicitly, we diagonalize the UV mass matrix M by

$$V M V_R^\dagger \,, \tag{4.91}$$

where V and V_R are 6×6 and 3×3 unitary matrices, respectively. Without loss of generality, the lines of V can be ordered such that the first three eigenvalues are zero and the last three are non-zero. We can thus parameterize:

$$\begin{pmatrix} Q_L \\ Q_H \end{pmatrix} \equiv V \cdot \begin{pmatrix} Q_u \\ Q_d \end{pmatrix} \equiv \begin{pmatrix} A_u & A_d \\ B_u & B_d \end{pmatrix} \begin{pmatrix} Q_u \\ Q_d \end{pmatrix} \,, \tag{4.92}$$

with Q_L and Q_H representing the massless and massive states, respectively. Note that the zero eigenvalue states are defined only up to a unitary transformation U from left.

In other words, the following rotation should not have any physical significance:

$$V \to \begin{pmatrix} U & 0 \\ 0 & \mathbf{1} \end{pmatrix} \times V = \begin{pmatrix} UA_u & UA_d \\ B_u & B_d \end{pmatrix} . \tag{4.93}$$

The relation in eq. (4.92) can be inverted to express the original states as

$$Q_{u,d} = A^\dagger_{u,d} Q_L + B^\dagger_{u,d} Q_H \,. \tag{4.94}$$

As mentioned before, the massive states Q_H are too heavy to be of any physical relevance, and therefore can be omitted in the discussion below.

Going back to the bulk by plugging eq. (4.94) into eq. (4.86), we find that the effective bulk mass term for the physical LH quark states is

$$\overline{Q}_L \left(A_u C_{Q_u} A_u^\dagger + A_d C_{Q_d} A_d^\dagger \right) Q_L \equiv \overline{Q}_L C_Q Q_L \,. \tag{4.95}$$

The bulk mass C_Q can then be diagonalized by a unitary transformation $Q_L \to U_C Q_L$. This makes it evident that any rotation of the form of Eq. (4.93) would be absorbed by U_C , such that it is unphysical.

The Yukawa couplings in Eq. (4.87) are now written as

$$\overline{Q_L} U_C A_u Y_u^{\rm 5D} U + \overline{Q_L} U_C A_d Y_d^{\rm 5D} D \,. \tag{4.96}$$

After performing KK decomposition for the quark fields, we can write the effective 4D Yukawa couplings for the zero-mode quarks as

$$\begin{aligned} Y_u^{\rm 4D} &= F_Q^\dagger U_C A_u Y_u^{\rm 5D} F_u \,, \\ Y_d^{\rm 4D} &= F_Q^\dagger U_C A_d Y_d^{\rm 5D} F_d \,. \end{aligned} \tag{4.97}$$

The matrices $F_{Q,u,d}$ project the zero-modes of the quarks on the IR brane. Their eigenvalues, which we denote by f_{Q^i,u^i,d^i} , are functions only of the corresponding bulk mass eigenvalues (denoted by c_{Q^i,u^i,d^i}), cf eq. (4.42). In order to go to the mass basis, the Yukawa matrices need to be diagonalized:

$$\begin{aligned} (Y_u^{\rm 4D})_{\rm mass} &= V_{uL} F_Q^\dagger U_C A_u Y_u^{\rm 5D} F_u V_{uR}^\dagger \,, \\ (Y_d^{\rm 4D})_{\rm mass} &= V_{dL} F_Q^\dagger U_C A_d Y_d^{\rm 5D} F_d V_{dR}^\dagger \,. \end{aligned} \tag{4.98}$$

The CKM matrix is then given by

$$V^{\text{CKM}} = V_{uL} V_{dL}^{\dagger} \,. \tag{4.99}$$

Finally, we are interested in studying the main FCNC sources in the RS framework, i.e. the KK modes of the gluon. In order to calculate the size of the flavour violating couplings, we rewrite eq. (4.88) after KK decomposition in terms of the zero-mode quarks in the mass basis and the first KK gluon G^1. Since the KK states are very close to being IR-localized, the overlap of the zero-mode quark wave functions and the KK gluon is given by the F matrices to a reasonable approximation. Consequently, the couplings of the quark zero modes to the first KK gluon are

$$g_{uL}^{(1)} = g_{s*} V_{uL} F_Q^{\dagger} U_C (A_d A_d^{\dagger} + A_u A_u^{\dagger}) U_C^{\dagger} F_Q V_{uL}^{\dagger} \,, \tag{4.100}$$
$$g_{dL}^{(1)} = g_{s*} V_{dL} F_Q^{\dagger} U_C (A_d A_d^{\dagger} + A_u A_u^{\dagger}) U_C^{\dagger} F_Q V_{dL}^{\dagger} \,, \tag{4.101}$$
$$g_{uR}^{(1)} = g_{s*} V_{uR} F_u^{\dagger} F_u V_{uR}^{\dagger} \,, \tag{4.102}$$
$$g_{dR}^{(1)} = g_{s*} V_{dR} F_d^{\dagger} F_d V_{dR}^{\dagger} \,, \tag{4.103}$$

for left-handed up- and down-type quarks and right-handed up- and down-type quarks, respectively. Note that we omit here and below the universal part of the KK gluon couplings, which is irrelevant for our discussion on FCNCs. The Lagrangian in eq. (4.88) then reads:

$$\mathcal{L} \supset \left(\bar{u}_L \, g_{uL}^{(1)} \, u_L + \bar{d}_L \, g_{dL}^{(1)} \, d_L + \bar{u}_R \, g_{uR}^{(1)} \, u_R + \bar{d}_R \, g_{dR}^{(1)} \, d_R \right) G \tag{4.104}$$

Another source of flavour violation in RS is the Higgs, stemming from the misalignment between its Yukawa couplings and the SM fermion masses, see sec. 4.5. The leading FCNC spurion in our model is of the form

$$\frac{4v^2}{M_{KK}^2} F_Q^{\dagger} U_C A_{d,u} \left(Y_{d,u}^{5\text{D}} \right)^3 F_{d,u} \,, \tag{4.105}$$

where M_{KK} is the KK scale and $v = 174$ GeV.

Flavour Alignment

The next step in achieving alignment is to enforce the flavour symmetry of the model. It is actually not important in this context how this is performed, but one can do so for instance by gauging these symmetries. In appendix B we write the most general renormalizable scalar potential consistent with the flavour symmetry for the spurion fields $Y^{5D}_{u,d}$ and $C_{Q_u,Q_d,u,d}$ on the UV brane as well as in the bulk. As is shown there, at the minimum of the potential the following pairs of objects are aligned, such that they can be taken as simultaneously diagonal[11]:

$$C_{Q_u} \leftrightarrow M_u M_u^\dagger \qquad C_{Q_d} \leftrightarrow M_d M_d^\dagger \qquad C_u \leftrightarrow (Y_u^{5D})^\dagger Y_u^{5D} \qquad C_d \leftrightarrow (Y_d^{5D})^\dagger Y_d^{5D} \,. \tag{4.106}$$

Note that our model relies solely on the flavour symmetry to achieve an alignment of Yukawa couplings and bulk masses, while previous alignment models [136, 137, 138, 139, 140] need MFV as a second ingredient.

In order for the model described above to exhibit suppression of FCNCs compared to the typical anarchic scenario, one more ingredient is required, however: we assume that the typical size of the eigenvalues of the down UV mass matrix, M_d, is suppressed by a factor of ϵ compared to the eigenvalues of the up UV mass matrix, M_u. The universal parameter ϵ is technically natural, so in principle it could be taken to be arbitrarily small. We now analyze the effect of the above assumptions, demonstrating the resulting suppression of FCNCs as a function of ϵ.

Starting from a basis where the up-type masses are diagonal, without loss of generality, the UV masses are written as:

$$M_u = \begin{pmatrix} m_{u_1} & 0 & 0 \\ 0 & m_{u_2} & 0 \\ 0 & 0 & m_{u_3} \end{pmatrix}, \qquad M_d = \epsilon \begin{pmatrix} m_{d_1} & 0 & 0 \\ 0 & m_{d_2} & 0 \\ 0 & 0 & m_{d_3} \end{pmatrix} \times V_X \,, \tag{4.107}$$

where the various eigenvalues m_i are of order k and V_X is some unitary matrix. A simple calculation shows that diagonalizing the UV mass matrix, the matrices $A_{u,d}$ defined in Eq. (4.92) can be written in the form:

$$A_d \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ 0 & 1 & \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix}, \qquad A_u \sim \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \end{pmatrix}, \tag{4.108}$$

after expanding in ϵ and omitting for simplicity spurious $\mathcal{O}(1)$ factors (which will be reinstated at a later stage). This is of course not a unique representation, following Eq. (4.93), but it is a convenient one.

Next we analyze the resulting structure of $C_Q = (A_u C_{Q_u} A_u^\dagger + A_d C_{Q_d} A_d^\dagger)$, recalling that

[11] Since the bulk masses are adjoints of the corresponding parts of the bulk flavour group, we write also the matrices $M_{u,d}$ and $Y^{5D}_{u,d}$ in their adjoint form.

in the chosen basis both C_{Q_u} and C_{Q_d} can be taken as diagonal. It is simple to see then that for the term $A_u C_{Q_u} A_u^\dagger$ all matrix entries are of order ϵ^2, while for the term $A_d C_{Q_d} A_d^\dagger$ the diagonal is $\mathcal{O}(1)$ times the eigenvalues of C_{Q_d} and the off-diagonal elements are $\mathcal{O}(\epsilon^2)$. Consequently, the eigenvalues of C_Q are dominated by those of C_{Q_d} plus $\mathcal{O}(\epsilon^2)$ corrections[12] that depend on both C_{Q_u,Q_d}. Additionally, the diagonalizing matrix U_C is close to a unit matrix with $\mathcal{O}(\epsilon^2)$ corrections for all elements. To sum this up, we have

$$c_{Q^i} \sim c_{Q_d^i} + \mathcal{O}(\epsilon^2)\,, \qquad\qquad U_C \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & 1 \end{pmatrix}. \tag{4.109}$$

Following the assumption of alignment, $F_{Q,u,d}$ and $Y_{u,d}^{5\mathrm{D}}$ can be taken as diagonal at once, and the only non-diagonal matrices in this basis are $A_{u,d}$ and U_C. Thus it is easy to see the structure of the physical Yukawas in eq. (4.97). The eigenvalues of $Y_u^{4\mathrm{D}}$, denoted by $y_{u^i}^{4\mathrm{D}}$, are all of order ϵ because of A_u in Eq. (4.108):

$$y_{u^i}^{4\mathrm{D}} \sim f_{Q^i} y_{u^i}^{5\mathrm{D}} f_{u^i} \epsilon\,, \tag{4.110}$$

where $y_{u^i}^{5\mathrm{D}}$ stands for the eigenvalues of the 5D Yukawa $Y_u^{5\mathrm{D}}$. This suggests that ϵ cannot be taken too small, since in that case it would be hard to reproduce the top mass. Moreover, because of the anarchic structure of A_u, the diagonalizing matrices of the up Yukawas $V_{uL,uR}$ in Eq. (4.98) have the same form as in the anarchic scenario,

$$(V_{uL})_{ij} \sim f_{Q^i}/f_{Q^j} \;\; (i<j) \qquad (V_{uR})_{ij} \sim f_{u_i}/f_{u_j} \;\; (i<j)\,. \tag{4.111}$$

In contrast, due to the structure of A_d in Eq. (4.108) and U_C in Eq. (4.109), the eigenvalues of $Y_d^{4\mathrm{D}}$ are not proportional to ϵ,

$$y_{d^i}^{4\mathrm{D}} \sim f_{Q^i} y_{d^i}^{5\mathrm{D}} f_{d^i}\,, \tag{4.112}$$

while all the off-diagonal entries in this basis are suppressed by ϵ^2. This leads to the interesting observation that the off-diagonal elements of $V_{dL,dR}$ have an ϵ^2 factor, in addition to the typical structure, that is

$$(V_{dL})_{ij} \sim \epsilon^2 f_{Q^i}/f_{Q^j} \;\; (i<j) \qquad (V_{dR})_{ij} \sim \epsilon^2 f_{d_i}/f_{d_j} \;\; (i<j)\,. \tag{4.113}$$

[12] These corrections are actually important due to the exponential dependence of $f(c)$ on c, and so they are included in our full calculation.

The standard form of the CKM matrix in the anarchic RS scenario,

$$V_{ij}^{\rm CKM} \sim f_{Q_i}/f_{Q_j}\ (i<j)\,, \tag{4.114}$$

is reproduced thanks to the structure of V_{uL} in Eq. (4.111)[13].

We now reach to the main result of the our analysis, which is the suppression of FCNCs mediated by the KK gluon. Starting from the couplings of the KK gluon to LH quarks, eqs. (4.100) and (4.101), it can be seen that the inner part $U_C(A_dA_d^\dagger + A_uA_u^\dagger)U_C^\dagger$ is similar to a unit matrix with ϵ^2 corrections for all elements (see Eqs. (4.108) and (4.109)). The surrounding F_Q matrices then make it of the form $f_{Q^i}f_{Q^j}$ with ϵ^2 suppressions for the off-diagonal elements. Now for the up sector, the matrix V_{uL}, Eq. (4.111), washes out all the ϵ^2 terms and leaves the standard RS-GIM form $g_{uL}^{(1)} \sim f_{Q^i}f_{Q^j}$. However, in the down sector the structure of V_{dL}, Eq. (4.113), maintains the ϵ^2 suppression for the off-diagonal elements. For the right-handed quarks, it is simple to see that $g_{uR}^{(1)}$ in eq. (4.102) is again just the RS-GIM one, while for $g_{dR}^{(1)}$ in Eq. (4.103) there is a suppression of ϵ^2 in the off-diagonal elements. To conclude, we find:

$$g_{uL}^{(1)} \sim g_{s*}\begin{pmatrix} f_{Q^1}^2 & f_{Q^1}f_{Q^2} & f_{Q^1}f_{Q^3} \\ f_{Q^1}f_{Q^2} & f_{Q^2}^2 & f_{Q^2}f_{Q^3} \\ f_{Q^1}f_{Q^3} & f_{Q^2}f_{Q^3} & f_{Q^3}^2 \end{pmatrix} \qquad g_{uR}^{(1)} \sim g_{s*}\begin{pmatrix} f_{u^1}^2 & f_{u^1}f_{u^2} & f_{u^1}f_{u^3} \\ f_{u^1}f_{u^2} & f_{u^2}^2 & f_{u^2}f_{u^3} \\ f_{u^1}f_{u^3} & f_{u^2}f_{u^3} & f_{u^3}^2 \end{pmatrix} \tag{4.115}$$

$$g_{dL}^{(1)} \sim g_{s*}\begin{pmatrix} f_{Q^1}^2 & f_{Q^1}f_{Q^2}\epsilon^2 & f_{Q^1}f_{Q^3}\epsilon^2 \\ f_{Q^1}f_{Q^2}\epsilon^2 & f_{Q^2}^2 & f_{Q^2}f_{Q^3}\epsilon^2 \\ f_{Q^1}f_{Q^3}\epsilon^2 & f_{Q^2}f_{Q^3}\epsilon^2 & f_{Q^3}^2 \end{pmatrix} \tag{4.116}$$

$$g_{dR}^{(1)} \sim g_{s*}\begin{pmatrix} f_{d^1}^2 & f_{d^1}f_{d^2}\epsilon^2 & f_{d^1}f_{d^3}\epsilon^2 \\ f_{d^1}f_{d^2}\epsilon^2 & f_{d^2}^2 & f_{d^2}f_{d^3}\epsilon^2 \\ f_{d^1}f_{d^3}\epsilon^2 & f_{d^2}f_{d^3}\epsilon^2 & f_{d^3}^2 \end{pmatrix} \tag{4.117}$$

We see the RS-GIM mechanism in the up-type quark couplings and the additional ϵ^2 suppression factor in the down-type FCNC couplings, which constitutes the down alignment of the model.

An important point is that this suppression is stable under radiative corrections, since it is protected by the flavour symmetry. This is an advantage of our model with respect to [140]. As we saw in sec. 4.7.1, eq. (4.83), there, loops and higher dimensional oper-

[13]Note that the entries on the main diagonal of V_{dL} are not suppressed by ϵ.

ators feed $Y_u^{5\mathrm{D}}$ into $Y_d^{5\mathrm{D}}$ via terms proportional to $\overline{Q}Y_u^{5\mathrm{D}}(Y_u^{5\mathrm{D}})^\dagger Y_d^{5\mathrm{D}}DH$, thus spoiling the alignment. Since the theory is on the verge of being perturbative, such effects can get quite large. Note that such terms do not appear in our model since $Y_u^{5\mathrm{D}}$ and $Y_d^{5\mathrm{D}}$ transform under different left-handed flavour groups.

Another difference between our model and the one described in [140] is that their model relies on an MFV ansatz plus an additional symmetry to realize the alignment, whereas in our case this follows automatically from the most general potential invariant under the flavour symmetry, plus one "accidentally" small parameter.

Finally, in our case the FCNC suppression in the down sector at the level of the cross section is ϵ^4 compared to the anarchic scenario, which means that the resulting bound on the KK scale is suppressed by ϵ^2. This is in contrast to the approximate alignment in [140], described by

$$C_Q = \alpha_Q \mathbf{1} + \beta_Q Y_d^{5\mathrm{D}}(Y_d^{5\mathrm{D}})^\dagger + \gamma_Q Y_u^{5\mathrm{D}}(Y_u^{5\mathrm{D}})^\dagger\,, \tag{4.118}$$

with $\gamma_Q \ll \beta_Q$. In this case the KK scale is only linearly suppressed by the small parameter γ_Q/β_Q.

To conclude this section, we note that the Higgs FCNC spurion in eq. (4.105) enjoys the same type of suppression as the KK gluon couplings, that is an anarchic-like structure in the up sector and an ϵ^2 suppression in the down sector. However, in this case the cross section scales as M_{KK}^{-4}, so that the final bound on M_{KK} is only linearly proportional to ϵ. We discuss the implication of this issue in the next section.

FCNC Suppression in Practice

We are now interested in studying the implication of the FCNC suppression of the model, or in other words estimate the value required for ϵ in order for the flavour constraints to be in line with other sources of constraints, such as electroweak precision tests.

In order to perform this task, we should first ask whether the simple picture presented in the previous section is accurate enough, or if a more careful analysis which keeps track of all the "order 1" factors is needed. As we now show, these factors indeed lead to dangerous enhancements, compared to the naive estimate above. We focus here on $(g_{dL}^{(1)})_{12}$ and $(g_{dR}^{(1)})_{12}$, which are responsible for $s_L \to d_L$ and $s_R \to d_R$ transitions, respectively, since their multiplication gives the model's contribution to ϵ_K.

First we define the matrices $\tilde{A}_{u,d}$ to be composed of the coefficients of the leading power of ϵ in the elements $A_{u,d}$ in Eq. (4.108), respectively. More precisely,

$$(\tilde{A}_u)_{ij} \equiv (A_u)_{ij}/\epsilon\,, \quad (\tilde{A}_d)_{ii} \equiv \left[1 - (A_d)_{ii}\right]/\epsilon^2\,, \quad (\tilde{A}_d)_{ij} \equiv (A_d)_{ij}/\epsilon^2 \quad (i < j)\,. \tag{4.119}$$

Note that the leading terms on the diagonal of A_d are all 1, so we defined $(\tilde{A}_d)_{ii}$ to be the coefficients of the ϵ^2 correction to these terms. This is required in order to maintain consistency in our ϵ expansion, without losing any terms which contribute at the same order. All the elements of $\tilde{A}_{u,d}$ defined in eq. (4.119) are therefore $\mathcal{O}(1)$.

Using these definitions, the full expression for $(g^{(1)}_{dL})_{12}$ is

$$(g^{(1)}_{dL})_{12} \simeq g_{s*} f_{Q^1} f_{Q^2} \epsilon^2 \Bigg[(\tilde{A}_u)_{11}(\tilde{A}_u)_{21} + (\tilde{A}_u)_{12}(\tilde{A}_u)_{22} + (\tilde{A}_u)_{13}(\tilde{A}_u)_{23} - \frac{c_{Q^2_d}(\tilde{A}_d)_{12} + c_{Q^1_u}(\tilde{A}_u)_{11}(\tilde{A}_u)_{21} + c_{Q^2_u}(\tilde{A}_u)_{12}(\tilde{A}_u)_{22} + c_{Q^3_u}(\tilde{A}_u)_{13}(\tilde{A}_u)_{23}}{c_{Q^1_d} - c_{Q^2_d}} \Bigg] \tag{4.120}$$

A very similar expression is also found for $(g^{(1)}_{dR})_{12}$:

$$(g^{(1)}_{dR})_{12} \simeq g_{s*} f_{d^1} f_{d^2} \epsilon^2 \frac{y^{5\text{D}}_{d^1}}{y^{5\text{D}}_{d^2}} \times \frac{c_{Q^2_d}(\tilde{A}_d)_{12} + c_{Q^1_u}(\tilde{A}_u)_{11}(\tilde{A}_u)_{21} + c_{Q^2_u}(\tilde{A}_u)_{12}(\tilde{A}_u)_{22} + c_{Q^3_u}(\tilde{A}_u)_{13}(\tilde{A}_u)_{23}}{c_{Q^1_d} - c_{Q^2_d}} \tag{4.121}$$

The first three terms in the squared brackets of eq. (4.120) are all $\mathcal{O}(1)$. However, the last term, which also appears in Eq. (4.121), contains a factor of $(c_{Q^1_d} - c_{Q^2_d})$ in its denominator. This factor is potentially very small, since the bulk masses needed to reproduce the masses of the d and s quarks are both typically close to 0.6: because of the exponential dependence of the wave function's overlap with the IR brane on the bulk mass, eq. (4.42), a small difference between the two bulk masses is enough to obtain the hierarchy between the quark masses.

We can estimate the typical magnitude of $(c_{Q^1_d} - c_{Q^2_d})$ via the following procedure. First we use the fact that up to $\mathcal{O}(\epsilon^2)$ corrections, $c_{Q^i_d}$ is equal to c_{Q^i}, as shown in eq. (4.109). Next, for c larger than (and not too close to) 0.5, we can approximate $f(c) \sim \sqrt{2c-1}\exp\left(k\pi R\,(1/2 - c)\right)$ (see (4.42)). Plugging this into Eq. (4.112) yields

$$y^{4\text{D}}_{d^i} \sim y^{5\text{D}}_{d^i} f_{d^i} \sqrt{2c_{Q^i} - 1}\, \exp\left[k\pi R\left(\frac{1}{2} - c_{Q^i}\right)\right]. \tag{4.122}$$

We now need to invert this relation to isolate c_{Q^i}. Since the dominant dependence of c_{Q^i} on the other factors $y^{4\mathrm{D}}_{d^i}$ etc. is logarithmic and since we are only interested in the difference $(c_{Q^1_d} - c_{Q^2_d})$, we can also omit the factor $\sqrt{2c_{Q^i}-1}$ from eq. (4.122). Then it is easy to invert eq. (4.122) and obtain

$$c_{Q^1_d} - c_{Q^2_d} \sim -\frac{1}{k\pi R}\left[\log\left(\frac{y^{4\mathrm{D}}_{d^1}}{y^{5\mathrm{D}}_{d^1} f_{d^1}}\right) - \log\left(\frac{y^{4\mathrm{D}}_{d^2}}{y^{5\mathrm{D}}_{d^2} f_{d^2}}\right)\right] \simeq \frac{1}{k\pi R}\log\left(\frac{f_{d^1}}{f_{d^2}}\frac{m_s}{m_d}\right), \qquad (4.123)$$

where in the last equality we replaced the ratio of 4D Yukawas by the ratio of physical quark masses and neglected a factor of $\log(y^{5\mathrm{D}}_{d^1}/y^{5\mathrm{D}}_{d^2})$, as the 5D Yukawas are all of the same order.

It is now possible to estimate the actual FCNC suppression in eqs. (4.120) and (4.121) compared to the naive estimate in the previous section. We only keep the last term in the squared brackets in Eq. (4.120), since it is the dominant one and the relative sign/phase between the various terms cannot be determined in general. For the remaining term, we pull out a factor of 0.6 from the numerator to represent the value of the different c's, and assume that the rest is $\mathcal{O}(1)$. In eq. (4.121) we ignore the ratio of 5D Yukawas, which is generally $\mathcal{O}(1)$. This leads us to

$$\frac{(g^{(1)}_{dL})_{12}}{g_{s*} f_{Q^1} f_{Q^2}\epsilon^2} \sim \frac{(g^{(1)}_{dR})_{12}}{g_{s*} f_{d^1} f_{d^2}\epsilon^2} \sim 0.6\,\frac{k\pi R}{\log\left(\frac{f_{d^1}}{f_{d^2}}\frac{m_s}{m_d}\right)} \simeq 7-13\,, \qquad (4.124)$$

using $k\pi R = 35$, which reproduces the hierarchy of Planck to TeV scales, and values in the range of 1 to 1/4 for f_{d^1}/f_{d^2} (yielding 7 to 13, respectively). We thus find that as suspected, the FCNCs are quite significantly enhanced compared to the naive estimation made before. It should be noted that the same type of enhancement with the same structure as for the KK gluon and of the same magnitude also appears in the Higgs flavour violating couplings.

Regarding transitions that involve the third generation, the enhancement is much less significant. This is because the denominator contains $(c_{Q^{1,2}_d} - c_{Q^3_d})$, and typically we have $c_{Q^3_d} < 0.5$, which makes this difference larger.

In order to substantiate this troubling enhancement, we performed a numerical scan of our model. We randomly generated a large number of points in the parameter space that roughly reproduce the correct quark masses and CKM mixing angles. Then we calculated the full KK gluon coupling matrices for each point. The results were obtained by averaging over all the points (median values were also calculated and found to be close to the mean). We learn the following:

- The transitions $s_L \to d_L$ and $s_R \to d_R$ are enhanced on average by factor of 5-9 compared to the expected suppressions of $f_{Q_1} f_{Q_2} \epsilon^2$ and $f_{d_1} f_{d_2} \epsilon^2$, respectively. As may have been expected from Eq. (4.124), both have relatively wide distributions, such that for a certain point in the parameter space this enhancement can be much bigger or much smaller.

- All the other down FCNCs are enhanced by a factor of 3 or less compared to expected, with relatively narrow distributions.

- The distributions of the transitions $c_L \to u_L$ and $c_R \to u_R$ are rather wide, but they are in general centered around the corresponding anarchic values.

- Other up FCNCs are similar to the anarchic scenario or below.

The range given for the above enhancement factors is for varying ϵ in the range 0.1-0.3. This dependence is explained by noting that changing ϵ also changes the typical bulk mass eigenvalues that give the correct masses and mixing angles, thus affecting the results.
We are now in a position to estimate the value for ϵ that would reduce the bound from flavour constraints, such that it would not be the most severe bound on the theory. The lower bound on the KK scale from EWPT is estimated to be 3-5 TeV [141, 142, 143]. Hence we choose to aim at ~3 TeV as the flavour bound.
As already mentioned, the strongest constraint on the RS framework typically comes from ϵ_K where the largest contribution is due to the operator

$$\mathcal{O}_4^{ds} = (\bar{s}_R d_L)(\bar{s}_L d_R) \tag{4.125}$$

(cf. eq. (2.13)). For KK gluon exchange the Wilson coefficient of this operator is:

$$C_4^{ds}(M_{KK}) = -\frac{\left[g_{dR}^{(1)}\right]_{21}\left[g_{dL}^{(1)}\right]_{12}}{M_{KK}^2} \sim -\frac{g_{s*}^2}{M_{KK}^2}\epsilon^4 f_{Q^1} f_{Q^2} f_{d^1} f_{d^2} \tag{4.126}$$

The eigenvalues of the 4D and 5D Yukawas are related by (cf. eq. (4.112)):

$$y_{d^1}^{4D} = \frac{m_d}{v} \sim f_{Q^1} y_{d^1}^{5D} f_{d^1} \tag{4.127}$$

$$y_{d^2}^{4D} = \frac{m_s}{v} \sim f_{Q^2} y_{d^2}^{5D} f_{d^2} \tag{4.128}$$

We can therefore write eq. (4.126) as:

$$C_4^{ds}(M_{KK}) = -\frac{g_{s*}^2}{M_{KK}^2}\epsilon^4 \frac{m_d m_s}{v^2 |Y_d^{5D}|^2} \tag{4.129}$$

where $|Y_d^{5D}|$ is the typical 5D down Yukawa scale.We require that this contribution to $|\epsilon_K|$ should be at most 60% of the experimental value [144, 145] and evaluate the resulting suppression scale and quark masses at 3 TeV. We use $g_{s*} = 3$ for the 5D gluon coupling after one loop matching [124], $m_d/v = 1.342 \cdot 10^{-5}$, $m_s/v = 2.718 \cdot 10^{-4}$ (from [146], evolved up to 3 TeV), an upper bound on C_4^{ds} of $2.38 \cdot 10^{-11}$ TeV^{-2} [116] and include a correction factor of 1.7 for the overlap of the KK gluon and the quarks (see *e.g.* [118]). We then find for the lower bound on the mass scale of the KK gluons:

$$M_{\rm KK} \gtrsim \frac{48\, x\, \epsilon^2}{|Y_d^{5D}|}\,{\rm TeV}\,, \tag{4.130}$$

where x is the numerical enhancement factor discussed above. Taking $x = 7$ as a representative value and choosing $|Y_d^{5D}| = 2.4$, we see that with $\epsilon = 0.15$ the bound is reduced to

$$M_{\rm KK} \gtrsim 3.2\ {\rm TeV}\,. \tag{4.131}$$

This should be compared with $M_{\rm KK} \gtrsim 20$ TeV for the standard anarchic scenario ($x = \epsilon = 1$). For larger values of $|Y_d^{5D}|$ the bound from the Higgs FCNC contribution becomes the dominant one, since it depends linearly on $|Y_d^{5D}|$ (and is also more weakly suppressed within our model, as mentioned in the previous section). Allowing the Higgs to propagate in the bulk can significantly reduce the above bound (or alternatively allow for a larger ϵ), but it is interesting to see that even in the IR Higgs case, the flavour constraints can be significantly ameliorated with reasonable parameters.

Some Collider Implications

The model presented above features a suppression of flavour violation in the down sector, while maintaining the anarchic contributions to FCNCs in the up sector. Furthermore, it relies on the flavour symmetries in order to achieve that. Both these aspects lead to some potentially observable phenomena, as discussed in this section.

First, it is interesting to note that reducing the bound on the KK scale coming from the down sector to the 3 TeV range means that up sector constraints are right around the corner. In order to make this argument concrete, we analyze the bound from CP violation in $D^0-\overline{D^0}$ mixing: The two neutral D-meson mass eigenstates $|D_1\rangle$ with mass m_1 and $|D_2\rangle$ with mass m_2 are linear combinations of the interaction eigenstates $|D_0\rangle$ (with quark content $c\bar{u}$) and $|\bar{D}^0\rangle$ (with quark content $\bar{c}u$) [147, 148, 149, 150, 151]:

$$|D_1\rangle = p\,|D^0\rangle + q\,|\bar{D}^0\rangle \qquad |D_2\rangle = p\,|D^0\rangle - q\,|\bar{D}^0\rangle \tag{4.132}$$

The difference in mass in between the two mass eigenstates is given as

$$x_{12} \equiv \frac{m_2 - m_1}{\Gamma} \tag{4.133}$$

where $\Gamma = \frac{\Gamma_1+\Gamma_2}{2}$ is the average width. The decay amplitudes to a final state f are defined as follows:

$$A_f = \langle f|\mathcal{H}|D^0\rangle \qquad \bar{A}_f = \langle f|\mathcal{H}|\bar{D}^0\rangle \tag{4.134}$$

It is then convenient to define a complex dimensionless parameter λ_f:

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f} \tag{4.135}$$

For the decays $D^0 \to K^+\pi^-$, $\bar{D}^0 \to K^-\pi^+$, $D^0 \to K^+K^-$ and $\bar{D}^0 \to K^+K^-$ we can then write [152]:

$$\lambda^{-1}_{K^+\pi^-} = r_d\left|\frac{p}{q}\right|e^{-i(\delta_{K\pi}+\phi_{12})} \qquad \lambda_{K^-\pi^+} = r_d\left|\frac{q}{p}\right|e^{-i(\delta_{K\pi}-\phi_{12})} \qquad \lambda_{K^+K^-} = -\left|\frac{q}{p}\right|e^{i\phi_{12}} \tag{4.136}$$

where r_d is a real, positive dimensionless parameter, δ_f is a strong (i.e. CP conserving) and ϕ_{12} is a weak (i.e. CP violating) phase. We use the most updated results of the heavy flavour averaging group for these parameters [153]:

$$x_{12} \in [0.25, 0.99]\%\,, \qquad \phi_{12} \in [-8.4^\circ, 24.6^\circ]\,, \tag{4.137}$$

both at 95% confidence level. We take the upper bounds of these ranges and assume maximal phases. Choosing for simplicity $|Y_u^{5D}| = |Y_d^{5D}| = 2.4$ as in the previous section, we obtain[14]

$$M_{\rm KK} \gtrsim 2.8 \text{ TeV}\,, \tag{4.138}$$

which is very close to the bound in Eq. (4.131). We can turn this calculation around and ask what is the required improvement in the experimental constraints in order for the $D^0 - \overline{D^0}$ measurement to be the dominant one in setting the bound. This is presented in Fig. 4.9, where it is evident that a 40% improvement in $x_{12}\sin\phi_{12}$ would be enough for such a turn of events. If we adopt the bound in Eq. (4.131) as the actual KK scale, then an observable signal of CP violation in $D^0 - \overline{D^0}$ mixing may be expected soon.

Another interesting implication of the anarchic structure in the up sector is a potentially

[14] The strongest constraint in this case comes from the Higgs exchange rather than the KK gluon. It can be reduced to ~2.2 TeV by taking $|Y_u^{5D}| = 1.9$, which corresponds to equal constraints from Higgs and KK gluon exchanges.

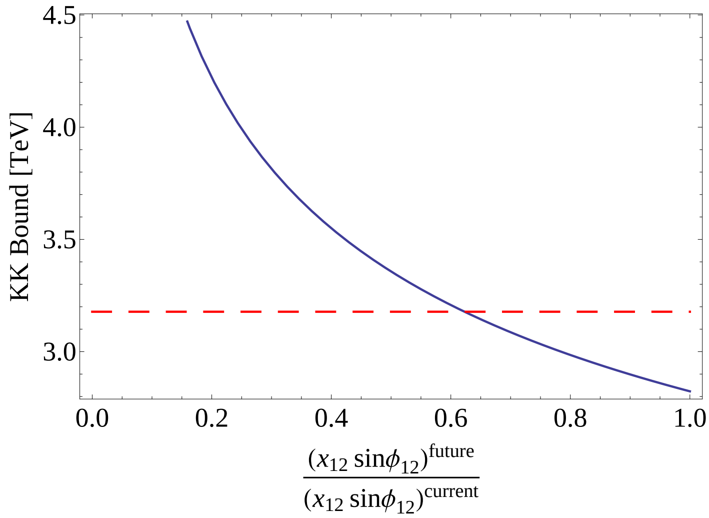


Figure 4.9: *The bound from CP violation in $D^0 - \overline{D^0}$ mixing (solid blue) compared to the bound from ϵ_K (dashed red), for a future improvement of the experimental determination of the combination of parameters $x_{12}\sin\phi_{12}$.*

large contribution to direct CP violation in charm decays via a dipole operator of the form $\bar{u}\sigma_{\mu\nu}T^a(1\pm\gamma_5)c\,G_a^{\mu\nu}$. As shown in [154], such a contribution may account for the recent observation of a large CP asymmetry difference in the decays $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ by LHCb [155] and CDF [156]:

$$\Delta a_{CP} \equiv a_{K^+K^-} - a_{\pi^+\pi^-} = -(0.67\pm 0.16)\% \tag{4.139}$$

where

$$a_f \equiv \frac{\Gamma(D^0\to f) - \Gamma(\bar{D}^0\to f)}{\Gamma(D^0\to f) + \Gamma(\bar{D}^0\to f)} \tag{4.140}$$

Since our model is based on imposing the flavour symmetries in order to achieve alignment of the relevant objects, it is reasonable to assume that these symmetries are gauged. The consequence of that is the existence of flavour gauge bosons, which may be observable at the LHC.

Finally, we note that the model proposed here enables to reduce the KK scale without forcing the bulk masses of the first two generation quarks to be degenerate. Since the dependence of the masses of the KK quarks on the bulk mass parameters is roughly linear, one may wonder whether the LHC would be able to distinguish between the KK quarks of the first two generations. However, studies of KK quark signals at the LHC show that they are very difficult to detect [157, 158], and that only the third generation partners have a chance to be observed (see *e.g.* [159] and refs. therein).

4.8 Other aspects of the RS model

4.8.1 AdS/CFT

Thanks to the AdS/CFT correspondence [160] we can find a 4D model that is dual to the 5D RS model. The AdS/CFT correspondence says that a type IIB string theory on $AdS_5 \times S^5$ is dual to a four dimensional $SU(N)$ gauge theory with 4 supersymmetry generators, the $d = 4$ $\mathcal{N} = 4$ super-Yang-Mills theory. It can furthermore be shown that the super-Yang-Mills theory is a *conformal field theory (CFT)*, i.e. it is invariant under dilatation (scale invariance) and the special conformal transformations.
Now, a string theory on $AdS_5 \times S^5$ is not exactly the set-up of RS. RS however represents the effective low energy description of a string theory when the radius R of the extra-dimension is much larger than the length of the strings l_s. Since they are related by:

$$\frac{R^4}{l_s^4} = 4\pi g_{YM}^2 N \tag{4.141}$$

where g_{YM} is the gauge coupling of the dual super-Yang-Mills theory, this means that $g_{YM} N \gg 1$, i.e. the dual CFT is strongly coupled [108].
Due to the the scale invariance of the conformal symmetry, there are no mass scales in a CFT. However in the RS setup, we do not consider an infinite AdS_5 but one that is bounded by two four-dimensional branes. Adding the branes and their associated mass scales breaks the conformal symmetry.
There is an "AdS/CFT dictionary" relating the 4D and 5D theories. Using this dictionary one can show that RS with the Higgs confined to the IR brane is dual to a 4D theory with a composite Higgs. The fermions are mixtures of elementary and composite states. Heavy fermions such as the top quark, that are localized towards the IR brane, are predominantly composite while light fermions such as the electron are almost elementary.

4.8.2 Dark Matter

It is clear that in a warped fifth dimension there is no translational invariance. The fifth component of momentum is thus not conserved and there is a priori no KK parity in RS models (as opposed to UED models, cf. 1.4.2). It is however possible to construct warped extra-dimensional models with KK parity:
In [161] a model with a warp factor that is symmetric with respect to reflection about the midpoint of the extra-dimension was considered. Such a setup can be achieved by gluing together two slices of AdS_5. The slices can either be glued in the UV or the IR

region: For the setup IR-UV-IR we have the metric

$$ds^2 = e^{-2k|y|}dx^\mu dx^\nu \eta_{\mu\nu} - dy^2 \tag{4.142}$$

where $y \in [-\pi R, \pi R]$. In the setup UV-IR-UV the metric is

$$ds^2 = e^{2(k|y|-\pi R)}dx^\mu dx^\nu \eta_{\mu\nu} - dy^2 \tag{4.143}$$

In other words, the warp factor has a minimum at the midpoint. In both cases it is then assumed that $y \to -y$ is an exact symmetry of the theory. For any given level in the KK decomposition there are even modes whose profiles are symmetric under reflexion around the midpoint of the extra dimension and odd modes whose profiles are anti-symmetric. Odd modes then can couple only in pairs to even modes. The lightest odd mode is thus stable and provides a DM candidate. In [161] this is the lightest KK partner of the Z boson. Its predicted relic abundance is in the correct range.

Another benefit of this model is that if the lightest KK states are odd, they do not contribute at tree level to operators that are constrained by EWPT and flavour physics. They can thus be rather light. This in turn ameliorates the *little hierarchy problem*: the fact that to avoid large contributions to the Higgs mass NP states should be present already at the weak scale (i.e. sub-TeV) but that on the other hand in order to be consistent with EWPT and flavour observables, one must require the NP scale to be *higher* that a few TeV.

This requires a splitting in between the odd and even KK modes, however. As shown in [161], in the IR-UV-IR setting this can only be achieved by very large IR-brane kinetic terms that on the other hand create a certain tension with perturbativity. In the UV-IR-UV setup the splitting of odd and even KK modes is automatic. This setup is however not stable gravitationally.

A similar ansatz was made in [162, 163]. In this model, there is a non-trivial warping with two IR boundaries. No UV brane is put in by hand, but a dynamical UV brane is generated in the middle of the extra dimension. In the limit where this UV brane becomes infinitely thin, this model corresponds to the IR-UV-IR setup in [161]. The DM candidate is the first KK mode of the radion.

In such models KK modes are produced in pairs at colliders and there is missing transverse energy from the DM candidate.

A DM candidate also emerges when considering GUTs in warped extra dimensions [164, 165, 166]. GUTs are feasible in RS models since due to the logarithmic running of gauge couplings in AdS_5 gauge coupling unification is possible when the gauge bosons are placed in the bulk [167]. In [164, 165] a gauged baryon symmetry is imposed in the bulk[15] in order to avoid proton decay due to four fermion operators without having to move the fermions too close to the UV brane and thus suppress their effective 4D Yukawa couplings too much. This baryon symmetry can be consistent with a GUT if the unified gauge group is broken by boundary conditions such that the SM quarks and leptons are obtained from different multiplets. For example for SO(10) we have the multiplets:

$$\begin{pmatrix} \boldsymbol{u_L\ d_L} \\ u'^c_R \\ d'^c_R \\ \nu'_L\ e'_L \\ e'^c_R \\ \nu'^c_R \end{pmatrix}_{B=1/3} \qquad \begin{pmatrix} u'_L\ d'_L \\ \boldsymbol{u^c_R} \\ \boldsymbol{d^c_R} \\ \nu'_L\ e'_L \\ e'^c_R \\ \nu'^c_R \end{pmatrix}_{B=-1/3} \qquad \begin{pmatrix} u'_L\ d'_L \\ u^{c\prime}_R \\ d^{c\prime}_R \\ \boldsymbol{\nu_L\ e_L} \\ \boldsymbol{e^c_R} \\ \boldsymbol{\nu^c_R} \end{pmatrix}_{B=0} \tag{4.144}$$

where only the boldface fields have zero modes. Furthermore, the Z_3 symmetry

$$\Phi \to e^{2\pi i \left(B - \frac{n_c - \bar{n}_c}{3}\right)} \Phi \tag{4.145}$$

where n_c ($\bar{n}_c$) is the number of colour (anti-colour) indices is imposed. While the SM fields are not charged under this Z_3, the gauge bosons of SO(10), lepton-like states in the multiplets that carry baryon number and quark-like states that carry non-standard baryon number are charged. As a consequence, the lightest Z_3 charged particle cannot decay to SM particles, i.e. is stable. As it turns out this particle has the quantum numbers of a RH neutrino and is a viable DM candidate.

An *exchange symmetry* [168, 169] can also be used in order to have a suitable DM candidate [170, 171]. The exchange symmetry is constructed as follows: a bulk field Φ is replaced by a pair of fields Φ_1 and Φ_2 and the symmetry $\Phi_1 \leftrightarrow \Phi_2$ is imposed. The even linear combination $\Phi_+ \equiv (\Phi_1 + \Phi_2)/\sqrt{2}$ is identified with the original field. We can then assign the multiplicative charge -1 to the orthogonal combination $\Phi_- \equiv (\Phi_1 - \Phi_2)/\sqrt{2}$ and the charge +1 to Φ_+. It can be shown that this symmetry is exact. The lightest KK excitation with negative charge under the exchange symmetry is therefore stable. Since we are looking for a DM candidate we will of course double a field that has the right properties for being DM, i.e. an electric and colour charge neutral field. We need more fields that are odd under the exchange symmetry, however: If the DM candidate

[15]This baryon symmetry is broken on the UV brane in order to avoid a massless gauge boson.

were the only odd particle it could couple only via non-renormalizable interactions to the other fields. Its annihilation rate would then be extremely small and its relic density unacceptably large.

4.8.3 Neutrinos

At first sight, it seems that warped extra-dimensional models cannot explain the smallness of neutrino masses, since operators such as eq. (4.24) are suppressed by a warped down scale only and the induced masses of left-handed neutrinos would be far above experimental bounds. This operator can be forbidden by imposing lepton number conservation, however.

In split fermion models, a small Dirac mass term for the left-handed neutrinos can be induced [104]: A sterile bulk fermion is included and the boundary conditions are chosen such that there is a right-handed zero mode with wave function $F(c_{\nu_R})$ on the TeV brane. The effective 4D Yukawas for the left-handed neutrinos are then[16]:

$$Y_\nu^{4D} = Y_\nu^{5D} F(c_{\nu_R}) \tag{4.146}$$

Upon EWSB a Dirac mass term $\bar{\nu}_L m_\nu \nu_R$ is induced for the neutrinos, where $m_\nu = vY_\nu^{4D} = vY_\nu^{5D} F(c_{\nu_R})$. Thus if the right-handed neutrinos are localized towards the UV brane, i.e. $|F(c_{\nu_R})| << 1$, small neutrino masses can be generated. In this model there is no breaking of lepton number[17].

In [172] it was shown however that the neutrino masses can be purely Dirac even in the presence of an explicit breaking of lepton number via a Majorana mass term on the UV brane. This is achieved by having the right-handed neutrino localized on the IR brane. Thus the right-handed neutrino is separated physically from the source of lepton number violation and the effective low energy theory conserves lepton number. In this model, since the physical low energy neutrinos are Dirac particles there is no neutrinoless double beta decay or other lepton number violating processes such as $K^+ \to \pi^- \mu^+ \mu^-$.

In [173] it is shown that the see-saw mechanism can indeed be implemented in RS: A right-handed neutrino ν_R is introduced and lepton number is broken only by its Majorana mass term on the Planck brane. As is shown in [173], this setup leads to the correct masses for the light left-handed neutrinos without any small parameters.

[16] Here we suppose for simplicity that the left-handed neutrinos live on the TeV brane but it also works for bulk left-handed neutrinos.

[17] A global lepton number symmetry is imposed which is however problematic given that a consistent theory of quantum gravity cannot conserve any global symmetries.

The discrepancy between the Majorana and Planck scales is explained by exponential suppressions of bulk wave functions.

However, in this model, in order to have realistic masses for the charged leptons and the neutrinos at the same time, values as small as 10^{-4} are needed for the entries of the Yukawas [174]. Therefore, in [174] a model is constructed that generates small Majorana masses for the left-handed neutrinos using only parameters of order unity. This is achieved by coupling the lepton doublets to a $SU(2)$ triplet scalar. The smallness of the light neutrino masses is then due to the small overlap of the triplet, a bulk singlet zero mode that is localized close to the UV brane and the Higgs, located as usual on the IR brane.

Another peculiarity of neutrinos apart from their very small masses are their non-hierarchical mixing angles: two of the observed neutrino mixing angles are close to maximal. In [175] an A4 discrete symmetry is used in order to achieve the correct neutrino mixing pattern. The hierarchy in the charged lepton masses is achieved as usual via their localization in the bulk, while the much milder hierarchy in neutrino masses is explained by $\mathcal{O}(1)$ factors in the neutrino Majorana mass matrix.

A flavour symmetry is also used in [176] in order to explain the large neutrino mixing angles: In the spirit of minimal flavour violation the authors assume that the Yukawa couplings are the only sources of breaking of the flavour symmetry in the lepton sector, $U(3)_l \times U(3)_e \times U(3)_\nu$. The bulk masses are then given as:

$$C_e = a_e \mathbb{1} + b_e Y_e^\dagger Y_e \tag{4.147}$$

$$C_\nu = a_\nu \mathbb{1} + c_\nu Y_\nu^\dagger Y_\nu \tag{4.148}$$

$$C_l = a_l \mathbb{1} + b_l Y_e Y_e^\dagger + c_l Y_\nu Y_\nu^\dagger \tag{4.149}$$

The light neutrino masses are assumed to arise from a see-saw mechanism involving heavy right-handed neutrinos. In order to prevent the seesaw operator to arise also from higher-dimensional operators that are not sufficiently suppressed due to the warp factor, lepton number symmetry is imposed. It is broken on the UV brane. Now consider the mixing angles: in the quark sector, we saw that the entries of the CKM matrix are given as the ratio of the zero mode wave function on the IR brane, $V_{ij}^{CKM} \sim f_{Q^i}/f_{Q^j}$, cf. eq. (4.114). Small mixing angles thus arise when the wave functions are non-degenerate. In the neutrino sector we thus need nearly degenerate wave functions in order to reproduce the large mixing angles. Indeed, the left-handed neutrino wave functions are fairly degenerate if the $U(3)_l$ symmetry is not broken or broken only by a small amount in the bulk. It is then primarily broken by the 5D Yukawa coupling in the IR.

4.8.4 Collider phenomenology

In contrast to MSSM particles or KK excitations in UED new particles in the RS model are not produced in pairs and there is no missing transverse energy at colliders. That is unless there is some kind of symmetry and a DM candidate as in the models presented in 4.8.2.

The best channel to probe the RS framework at the LHC is the production of KK gluons[18] [177, 178]. It is challenging to discover the KK gluon since

1. due to the fermion profiles in the bulk its couplings to light fermions (i.e. light quarks as in protons) are suppressed and
2. it decays mainly to third generation quarks with a large width (especially $t\bar{t}$ with 95% branching ratio).

The dominant production mechanism is through $u\bar{u}$ and $d\bar{d}$ annihilation. At the LHC, the background is mainly due to $t\bar{t}$ production from gluon fusion. This cross-section is comparable to $t\bar{t}$ production from KK gluons. However, while in the SM an equal amount of left- and right-handed $t\bar{t}$-pairs is produced, this is not the case for $t\bar{t}$ from KK gluons. There should be a large bias towards RH tops and thus a LR asymmetry should be measured if KK gluons are indeed produced. This is due to the different localization of the LH and RH third generation quarks in the bulk. The chirality information is encoded in the decay products since the top decays before it hadronizes. If an excess in the $t\bar{t}$ cross section *and* an LR asymmetry are measured, the KK gluon can thus be discovered at the LHC for masses of a few TeV.

In the SO(10) model of [164, 165] (cf. sec. 4.8.2) the lightest KK fermion is lighter than the KK gauge bosons. Since the lightest KK fermion is the one with the smallest bulk mass c, it will come from the multiplet that contains the top quark [178]. If light KK quarks are produced at the LHC, they could be detected via multi-W final states [179].

Another prediction are TeV KK gravitons. They lead to spin 2 resonances spaced according to the roots of the first Bessel function [178].

[18] The production rate of the EW KK gauge bosons is suppressed by g_Z/g_{QCD} with respect to the KK gluon production.

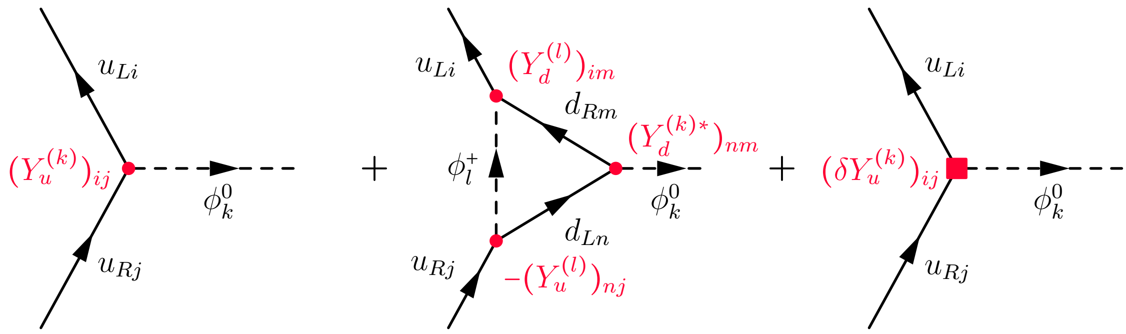
Appendices

Appendix A

Renormalized Yukawas in a 2HDM

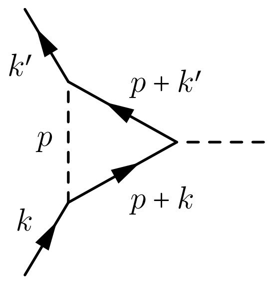
A.1 Vertex renormalization (one loop)

A.1.1 Up-type Yukawa couplings



+ terms from gauge interactions (the same as in the standard model)

We calculate the one-loop Feynman diagram:



$$= \sum_{l=1,2} (-i)^3 (Y_d^{(l)})_{im} (Y_d^{(k)*})_{nm} (-Y_u^{(l)})_{nj}$$

$$\times \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{i}{p^2 - m_\phi^2} \cdot \frac{i\left[\not{p} + \not{k} + m_q\right]}{(p+k)^2 - m_q^2} \cdot \frac{i\left[\not{p} + \not{k}' + m_q\right]}{(p+k')^2 - m_q^2}$$

we can work in the limit of massless particles, so that we get:

$$= -\sum_{l=1,2} Y_d^{(l)} Y_d^{(k)\dagger} Y_u^{(l)} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{(\not{p}+\not{k})(\not{p}+\not{k}')}{p^2(p+k)^2(p+k')^2}$$

The superficial degree of divergence D ($D \equiv$ (power of p in numerator)-(power of p in denominator)) is zero, meaning that we have a logarithmic divergence only. To calculate the coefficient of this divergence we can therefore set all momenta other than the loop momentum p equal to zero. Because of the divergence we evaluate the integral in d dimensions (dimensional regularization):

$$\sim -\sum_{l=1,2} Y_d^{(l)} Y_d^{(k)\dagger} Y_u^{(l)} \int \frac{\mathrm{d}^d p}{(2\pi)^d} \frac{\not{p}\not{p}}{p^2 p^2 p^2} = -\sum_{l=1,2} Y_d^{(l)} Y_d^{(k)\dagger} Y_u^{(l)} \int \frac{\mathrm{d}^d p}{(2\pi)^d} \frac{1}{(p^2)^2}$$

The solutions of integrals of this type are given in (A.44) in [5]:

$$\int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \frac{1}{(\ell^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}} \frac{\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \tag{A.1}$$

we therefore get the result:

$$\sim \frac{-i}{(4\pi)^{d/2}} \sum_{l=1,2} Y_d^{(l)} Y_d^{(k)\dagger} Y_u^{(l)} \Gamma\left(2-d/2\right) \left(\frac{1}{\Delta}\right)^{(2-\frac{d}{2})}$$

where Δ is a combination of the momenta k, k' we set to zero in the previous calculation. To calculate the β function we can set $\Delta = M^2$ where M is the renormalization scale:

$$\sim -i \sum_{l=1,2} Y_d^{(l)} Y_d^{(k)\dagger} Y_u^{(l)} \frac{\Gamma\left(2-d/2\right)}{(4\pi)^{d/2}} \left(\frac{1}{M^2}\right)^{(2-\frac{d}{2})}$$

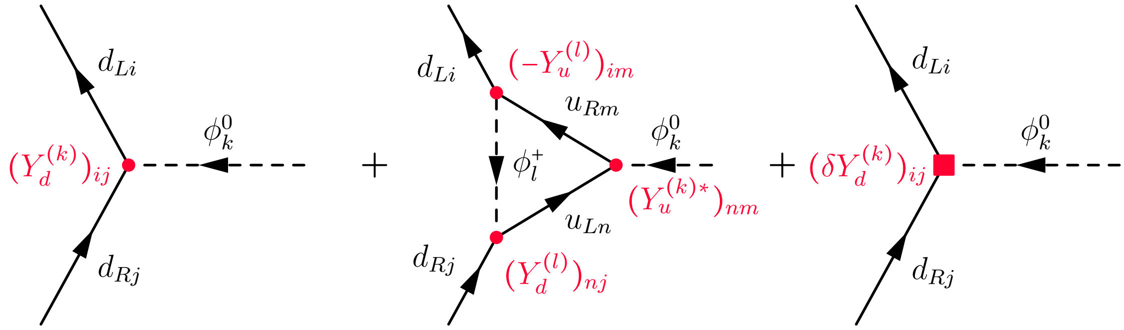
The loop divergence must be cancelled by the counterterm diagram

$$-i\delta Y_u^{(k)}$$

Therefore, we see immediately that the counterterm must be:

$$\delta Y_u^{(k)} = -\sum_{l=1,2} Y_d^{(l)} Y_d^{(k)\dagger} Y_u^{(l)} \frac{\Gamma\left(2-d/2\right)}{(4\pi)^{d/2}} \left(\frac{1}{M^2}\right)^{(2-\frac{d}{2})} \tag{A.2}$$

A.1.2 Down-type Yukawa couplings



\+ terms from gauge interactions (the same as in the standard model)

The calculation of the loop integral is completely analogous to the up-type Yukawa coupling just discussed. The counterterm is therefore:

$$\delta Y_d^{(k)} = -\sum_{l=1,2} Y_u^{(l)} Y_u^{(k)\dagger} Y_d^{(l)} \frac{\Gamma(2-d/2)}{(4\pi)^{d/2}} \left(\frac{1}{M^2}\right)^{(2-\frac{d}{2})} \tag{A.3}$$

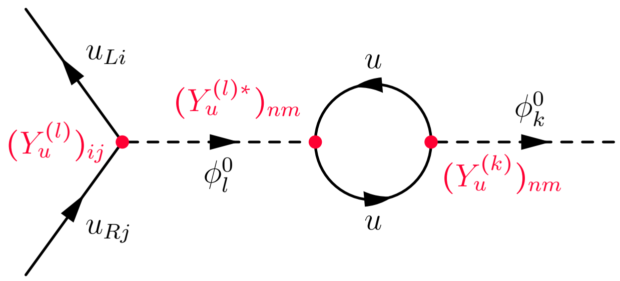
A.1.3 Lepton Yukawa couplings

Since in this model there are no right-handed neutrinos, there are no vertex corrections for Y_e other than those coming from gauge interactions (which do not lead to off-diagonal couplings).

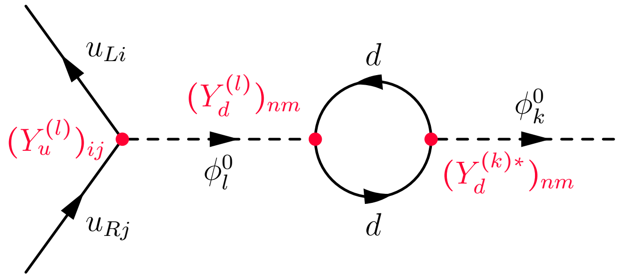
A.2 Wave function renormalization

A.2.1 Higgs wave function renormalization

The one-loop correction of the Yukawa coupling $Y_u^{(k)}$ due to the Higgs wave function renormalization can be written as:



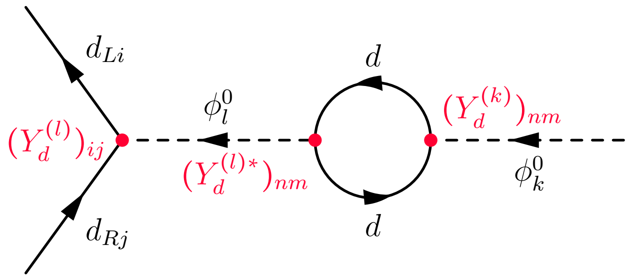
$$\propto \sum_{l=1,2} 3(Y_u^{(l)})_{ij}(Y_u^{(l)*})_{nm}(Y_u^{(k)})_{nm} = \sum_{l=1,2} 3Y_u^{(l)}\mathrm{Tr}(\mathrm{Y}_u^{(l)\dagger}\mathrm{Y}_u^{(k)})$$



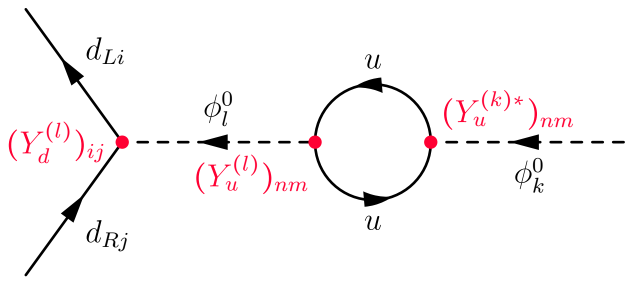
$$\propto \sum_{l=1,2} 3(Y_u^{(l)})_{ij}(Y_d^{(l)})_{nm}(Y_d^{(k)*})_{nm}$$
$$= \sum_{l=1,2} 3Y_u^{(l)}\mathrm{Tr}(\mathrm{Y}_\mathrm{d}^{(l)}\mathrm{Y}_\mathrm{d}^{(k)\dagger})$$

where the factors of three are due to colour. The structure for leptons in the loop is exactely the same, again we just need to replace $u \to \nu$, $d \to e$ and to drop the factor of three.

The one-loop correction of the Yukawa coupling $Y_d^{(k)}$ due to the Higgs wave function renormalization can be written as:



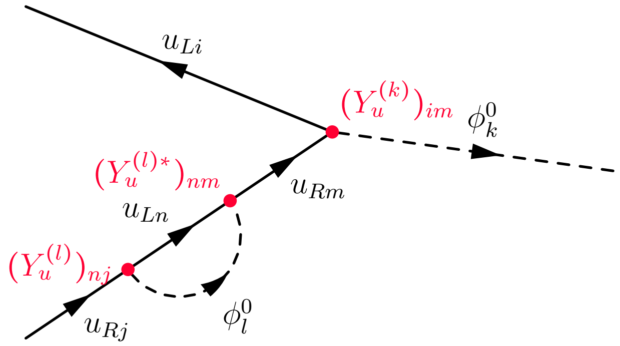
$$\propto \sum_{l=1,2} 3(Y_d^{(l)})_{ij}(Y_d^{(l)*})_{nm}(Y_d^{(k)})_{nm}$$
$$= \sum_{l=1,2} 3Y_d^{(l)}\mathrm{Tr}(\mathrm{Y}_\mathrm{d}^{(l)\dagger}\mathrm{Y}_\mathrm{d}^{(k)})$$



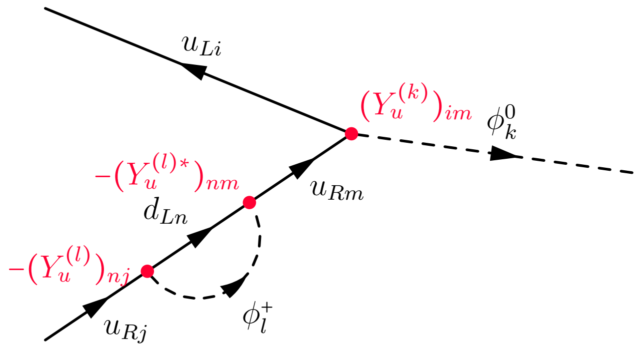
$$\propto \sum_{l=1,2} 3(Y_d^{(l)})_{ij}(Y_u^{(l)})_{nm}(Y_u^{(k)*})_{nm}$$
$$= \sum_{l=1,2} 3Y_d^{(l)}\mathrm{Tr}(\mathrm{Y}_\mathrm{u}^{(l)}\mathrm{Y}_\mathrm{u}^{(k)\dagger})$$

A.2.2 Fermion wave function renormalization

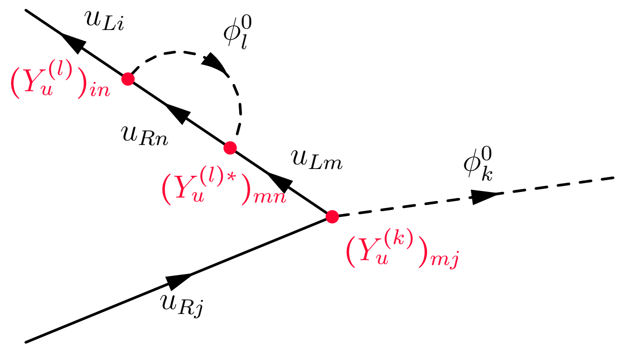
The one-loop correction of the Yukawa coupling $Y_u^{(k)}$ due to the fermion wave function renormalization can be written as:



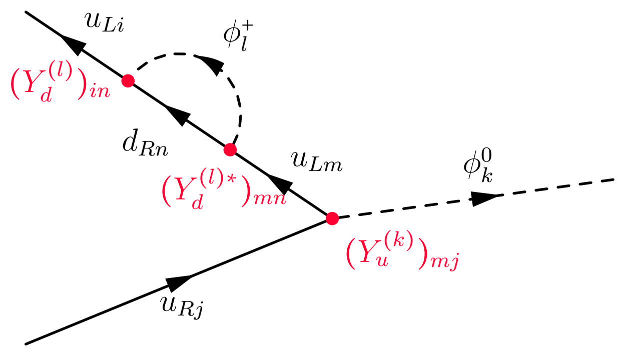
$$\propto \sum_{l=1,2} (Y_u^{(k)})_{im}(Y_u^{(l)*})_{nm}(Y_u^{(l)})_{nj}$$
$$= \sum_{l=1,2} Y_u^{(k)} Y_u^{(l)\dagger} Y_u^{(l)}$$



$$\propto \sum_{l=1,2} (Y_u^{(k)})_{im}(Y_u^{(l)*})_{nm}(Y_u^{(l)})_{nj}$$
$$= \sum_{l=1,2} Y_u^{(k)} Y_u^{(l)\dagger} Y_u^{(l)}$$

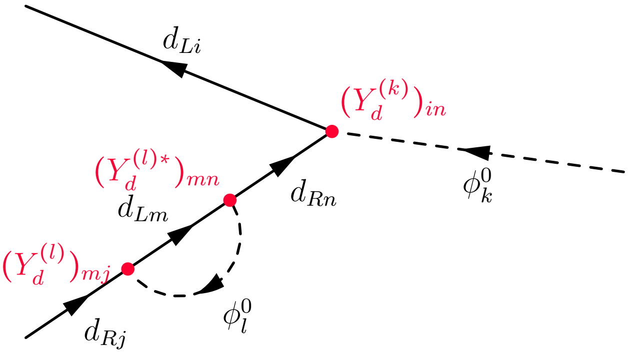


$$\propto \sum_{l=1,2} (Y_u^{(l)})_{in}(Y_u^{(l)*})_{mn}(Y_u^{(k)})_{mj}$$
$$= \sum_{l=1,2} Y_u^{(l)} Y_u^{(l)\dagger} Y_u^{(k)}$$

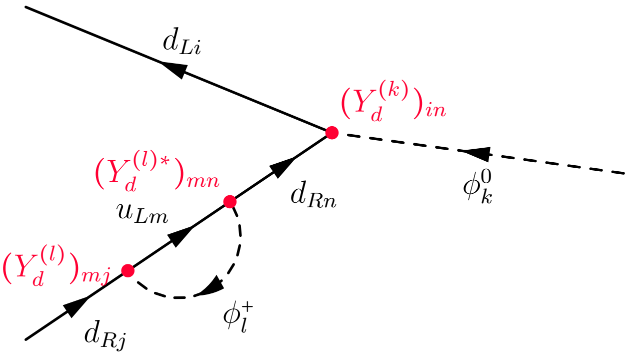


$$\propto \sum_{l=1,2} (Y_d^{(l)})_{in}(Y_d^{(l)*})_{mn}(Y_u^{(k)})_{mj}$$
$$= \sum_{l=1,2} Y_d^{(l)} Y_d^{(l)\dagger} Y_u^{(k)}$$

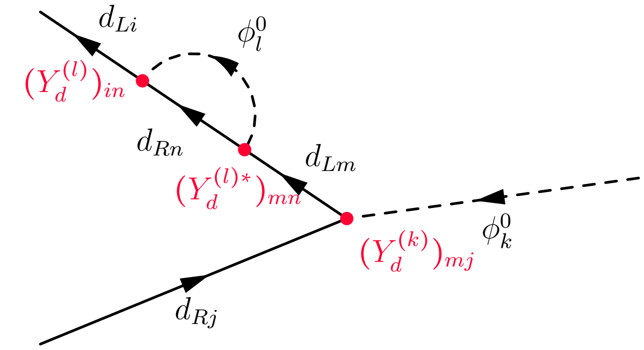
The one-loop correction of the Yukawa coupling $Y_d^{(k)}$ due to the fermion wave function renormalization can be written as:



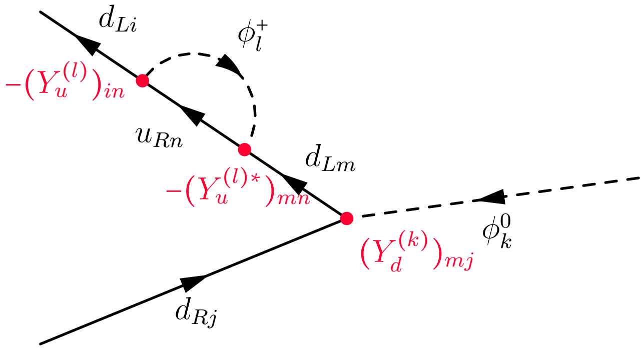
$$\propto \sum_{l=1,2} (Y_d^{(k)})_{in}(Y_d^{(l)*})_{mn}(Y_d^{(l)})_{mj}$$
$$= \sum_{l=1,2} Y_d^{(k)} Y_d^{(l)\dagger} Y_d^{(l)}$$



$$\propto \sum_{l=1,2} (Y_d^{(k)})_{in}(Y_d^{(l)*})_{mn}(Y_d^{(l)})_{nj}$$
$$= \sum_{l=1,2} Y_d^{(k)} Y_d^{(l)\dagger} Y_d^{(l)}$$



$$\propto \sum_{l=1,2} (Y_d^{(l)})_{in}(Y_d^{(l)*})_{mn}(Y_d^{(k)})_{mj}$$
$$= \sum_{l=1,2} Y_d^{(l)} Y_d^{(l)\dagger} Y_d^{(k)}$$



$$\propto \sum_{l=1,2} (Y_u^{(l)})_{in}(Y_u^{(l)*})_{mn}(Y_d^{(k)})_{mj}$$
$$= \sum_{l=1,2} Y_u^{(l)} Y_u^{(l)\dagger} Y_d^{(k)}$$

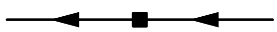
For the fermion wave function renormalization there are also contributions from gauge interactions. These are however not significant for our purposes as they are flavour diagonal. The diagrams for the leptonic Yukawa couplings are again completely analogous to those for the quarks.

A.3 The complete β-functions

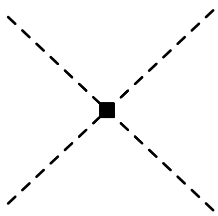
We can now calculate the complete diagrams and find the counterterms. For a general Yukawa Lagrangian

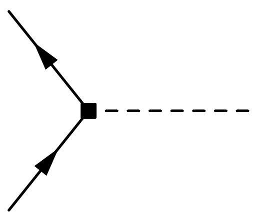
$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\not{\partial} - m_\psi)\psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{\lambda}{4!}\phi^4 - Y\bar{\psi}\psi\phi \\ &+ \bar{\psi}(i\delta Z_\psi\not{\partial} - \delta m_\psi)\psi + \frac{1}{2}\delta Z_\phi(\partial_\mu\phi)^2 - \frac{1}{2}\delta m_\phi\phi^2 - \frac{\delta_\lambda}{4!}\phi^4 - \delta Y\bar{\psi}\psi\phi\end{aligned} \tag{A.4}$$

these can be found using the Feynman rules:

Fermion propagator:  $i(\not{p}\delta Z_\psi - \delta m_\psi)$

scalar propagator:  $i(p^2\delta Z_\phi - \delta m_\phi)$

four-scalar interaction  $-i\delta_\lambda$

Yukawa vertex:  $-i\delta Y$

Then, with the help of the counterterms, the two- and three-point Greens functions can be calculated and the β-function and the anomalous dimension γ (due to the wave function renormalization) can be found by solving the Callan-Symanzik equation.

The Renormalization Group Equation of the Yukawa couplings reads

$$16\pi^2\mu\frac{\mathrm{d}Y_f}{\mathrm{d}\mu} = \beta_{Y_f}, \tag{A.5}$$

where

$$\beta_{Y_u^{(k)}} = a_u Y_u^{(k)} + \sum_{l=1,2}\left[3\mathrm{Tr}\left(Y_u^{(k)}Y_u^{(l)\dagger} + Y_d^{(k)\dagger}Y_d^{(l)}\right) + \mathrm{Tr}\left(Y_e^{(k)\dagger}Y_e^{(l)}\right)\right]Y_u^{(l)} \tag{A.6}$$
$$+ \sum_{l=1,2}\left(-2Y_d^{(l)}Y_d^{(k)\dagger}Y_u^{(l)} + Y_u^{(k)}Y_u^{(l)\dagger}Y_u^{(l)} + \frac{1}{2}Y_d^{(l)}Y_d^{(l)\dagger}Y_u^{(k)} + \frac{1}{2}Y_u^{(l)}Y_u^{(l)\dagger}Y_u^{(k)}\right),$$

$$\beta_{Y_d^{(k)}} = a_d Y_d^{(k)} + \sum_{l=1,2}\left[3\mathrm{Tr}\left(Y_u^{(k)\dagger}Y_u^{(l)} + Y_d^{(k)}Y_d^{(l)\dagger}\right) + \mathrm{Tr}\left(Y_e^{(k)}Y_e^{(l)\dagger}\right)\right]Y_d^{(l)} \tag{A.7}$$
$$+ \sum_{l=1,2}\left(-2Y_u^{(l)}Y_u^{(k)\dagger}Y_d^{(l)} + Y_d^{(k)}Y_d^{(l)\dagger}Y_d^{(l)} + \frac{1}{2}Y_u^{(l)}Y_u^{(l)\dagger}Y_d^{(k)} + \frac{1}{2}Y_d^{(l)}Y_d^{(l)\dagger}Y_d^{(k)}\right),$$

$$\beta_{Y_e^{(k)}} = a_e Y_e^{(k)} + \sum_{l=1,2}\left[3\mathrm{Tr}\left(Y_u^{(k)\dagger}Y_u^{(l)} + Y_d^{(k)}Y_d^{(l)\dagger}\right) + \mathrm{Tr}\left(Y_e^{(k)}Y_e^{(l)\dagger}\right)\right]Y_e^{(l)} \tag{A.8}$$
$$+ \sum_{l=1,2}\left(Y_e^{(k)}Y_e^{(l)\dagger}Y_e^{(l)} + \frac{1}{2}Y_e^{(l)}Y_e^{(l)\dagger}Y_e^{(k)}\right),$$

where a_f $(f = u, d, e)$ stands for contributions due to gauge interactions, which are flavour-diagonal [85]:

$$a_u = -8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2, \tag{A.9}$$
$$a_d = -8g_s^2 - \frac{9}{4}g^2 - \frac{5}{12}g'^2, \tag{A.10}$$
$$a_e = -\frac{9}{4}g^2 - \frac{15}{4}g'^2, \tag{A.11}$$

where g_s, g and g' are the gauge couplings constants of $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$, respectively. The terms in the first sum in the β-function are due to the Higgs wave function renormalization, the first term in the second sum is due to the vertex renormalization (absent for leptons) and the last three (two) terms are due to the renormalization of the fermion wave function.

A.4 Yukawa couplings at the EW scale in a 2HDM with alignment

Using the leading log approximation, eq. (3.51), and plugging in the Yukawa couplings at the high scale, eqs. (3.48) - (3.50), and the β-functions, eqs. (A.6) - (A.8) we get the following formulae for the Yukawa couplings at the electroweak scale:

A.4.1 d-quarks

$$Y_d^{(k)}(m_Z) \approx k_d^{(k)}Y_d + \epsilon_d^{(k)}Y_uY_u^\dagger Y_d + \delta_d^{(k)}Y_dY_d^\dagger Y_d \tag{A.12}$$

with

$$k_d^{(1)} = \cos\psi_d + \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\Big[a_d\cos\psi_d + 3\cos\psi_u^*\cos(\psi_u-\psi_d)\mathrm{Tr}(Y_u^\dagger Y_u) + 3\cos\psi_d\mathrm{Tr}(Y_dY_d^\dagger) + \cos\psi_e\cos(\psi_e^*-\psi_d)\mathrm{Tr}(Y_eY_e^\dagger)\Big], \tag{A.13}$$

$$\epsilon_d^{(1)} = \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\left(\frac{1}{2}\cos\psi_d - 2\cos\psi_u^*\cos(\psi_u-\psi_d)\right), \tag{A.14}$$

$$\delta_d^{(1)} = \frac{3\log\frac{m_Z}{\Lambda}}{32\pi^2}\cos\psi_d, \tag{A.15}$$

$$k_d^{(2)} = \sin\psi_d + \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\Big[a_d\sin\psi_d + 3\sin\psi_u^*\cos(\psi_u-\psi_d)\mathrm{Tr}(Y_u^\dagger Y_u) + 3\sin\psi_d\mathrm{Tr}(Y_dY_d^\dagger) + \sin\psi_e\cos(\psi_e^*-\psi_d)\mathrm{Tr}(Y_eY_e^\dagger)\Big], \tag{A.16}$$

$$\epsilon_d^{(2)} = \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\left(\frac{1}{2}\sin\psi_d - 2\sin\psi_u^*\cos(\psi_u-\psi_d)\right), \tag{A.17}$$

$$\delta_d^{(2)} = \frac{3\log\frac{m_Z}{\Lambda}}{32\pi^2}\sin\psi_d; \tag{A.18}$$

A.4.2 u-quarks

$$Y_u^{(k)}(m_Z) \approx k_u^{(k)}Y_u + \epsilon_u^{(k)}Y_dY_d^\dagger Y_u + \delta_u^{(k)}Y_uY_u^\dagger Y_u \tag{A.19}$$

with

$$k_u^{(1)} = \cos\psi_u + \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\Big[a_u\cos\psi_u + 3\cos\psi_u\mathrm{Tr}(Y_u^\dagger Y_u) + 3\cos\psi_d^*\cos(\psi_d-\psi_u)\mathrm{Tr}(Y_dY_d^\dagger) + \cos\psi_e^*\cos(\psi_e-\psi_u)\mathrm{Tr}(Y_eY_e^\dagger)\Big], \tag{A.20}$$

$$\epsilon_u^{(1)} = \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\left(\frac{1}{2}\cos\psi_u - 2\cos\psi_d^*\cos(\psi_d-\psi_u)\right), \tag{A.21}$$

$$\delta_u^{(1)} = \frac{3\log\frac{m_Z}{\Lambda}}{32\pi^2}\cos\psi_u, \tag{A.22}$$

$$k_u^{(2)} = \sin\psi_u + \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\Big[a_u\sin\psi_u + 3\sin\psi_u\mathrm{Tr}(Y_u^\dagger Y_u) + 3\sin\psi_d^*\cos(\psi_d-\psi_u)\mathrm{Tr}(Y_dY_d^\dagger) + \sin\psi_e^*\cos(\psi_e-\psi_u)\mathrm{Tr}(Y_eY_e^\dagger)\Big], \tag{A.23}$$

$$\epsilon_u^{(2)} = \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\left(\frac{1}{2}\sin\psi_u - 2\sin\psi_d^*\cos(\psi_d-\psi_u)\right), \tag{A.24}$$

$$\delta_u^{(2)} = \frac{3\log\frac{m_Z}{\Lambda}}{32\pi^2}\sin\psi_u; \tag{A.25}$$

A.4.3 Leptons

As there are no vertex corrections in the leptonic sector, the coupling at the electroweak scale has a different structure:

$$Y_e^{(k)}(m_Z) \approx k_e^{(k)} Y_e + \delta_e^{(k)} Y_e Y_e^\dagger Y_e \tag{A.26}$$

with

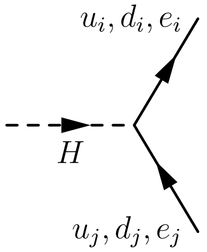
$$k_e^{(1)} = \cos\psi_e + \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\Big[a_e\cos\psi_e + 3\cos\psi_u^*\cos(\psi_u - \psi_e)\mathrm{Tr}(Y_u^\dagger Y_u) \tag{A.27}$$
$$+ 3\cos\psi_d\cos(\psi_d^* - \psi_e)\mathrm{Tr}(Y_d Y_d^\dagger) + \cos\psi_e\mathrm{Tr}(Y_e Y_e^\dagger)\Big],$$

$$\delta_e^{(1)} = \frac{3\log\frac{m_Z}{\Lambda}}{32\pi^2}\cos\psi_e, \tag{A.28}$$

$$k_e^{(2)} = \sin\psi_e + \frac{\log\frac{m_Z}{\Lambda}}{16\pi^2}\Big[a_e\sin\psi_e + 3\sin\psi_u^*\cos(\psi_u - \psi_e)\mathrm{Tr}(Y_u^\dagger Y_u) \tag{A.29}$$
$$+ 3\sin\psi_d\cos(\psi_d^* - \psi_e)\mathrm{Tr}(Y_d Y_d^\dagger) + \sin\psi_e\mathrm{Tr}(Y_e Y_e^\dagger)\Big],$$

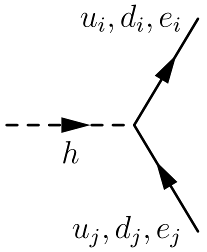
$$\delta_e^{(2)} = \frac{3\log\frac{m_Z}{\Lambda}}{32\pi^2}\sin\psi_e; \tag{A.30}$$

A.5 Feynman rules for FV Higgs couplings



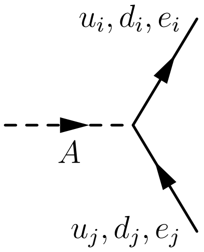
$$= -\frac{i}{2}\left[2c_{\alpha-\beta}\frac{m_{fi}}{v}\delta_{ij} + s_{\alpha-\beta}\left((\Delta_f + \Delta_f^\dagger)_{ij} + (\Delta_f - \Delta_f^\dagger)_{ij}\gamma^5\right)\right] \tag{A.31}$$

for $f = u, d, e$



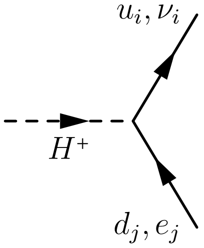
$$= -\frac{i}{2}\left[-2s_{\alpha-\beta}\frac{m_{fi}}{v}\delta_{ij} + c_{\alpha-\beta}\left((\Delta_f + \Delta_f^\dagger)_{ij} + (\Delta_f - \Delta_f^\dagger)_{ij}\gamma^5\right)\right] \tag{A.32}$$

for $f = u, d, e$



$$= \frac{1}{2}\left[(\Delta_f - \Delta_f^\dagger)_{ij} + (\Delta_f + \Delta_f^\dagger)_{ij}\gamma^5\right] \tag{A.33}$$

for $f = d, e$ and for $f = u$ the same with negative sign



$$= -\frac{i}{\sqrt{2}}\left[(-\Delta_u^\dagger V_{CKM} + V_{CKM}\Delta_d)_{ij} + (\Delta_u^\dagger V_{CKM} + V_{CKM}\Delta_d)_{ij}\gamma^5\right] \tag{A.34}$$

for quarks and $V_{CKM}\Delta_d \to \Delta_e$, $\Delta_u \to 0$ for leptons

Here $m_{f\,i}$ is the corresponding fermion mass and Δ_f is defined by eqs. (3.61), (3.62) and analogously for the leptons.

Appendix B

Scalar potential of the spurion fields

B.1 Bulk

For simplicity and without loss of generality, we work with a two-generation model (the generalization to three generations is simple). First consider the spurions in the bulk, *i.e.* C_{Q_u}, C_{Q_d}, C_u, C_d, Y_u and Y_d.[1] The bulk masses are adjoints and therefore transform as:

$$C_x \to \Omega_x C_x \Omega_x^\dagger \,, \tag{B.1}$$

where $x = u, d, Q_u, Q_d$, while the Yukawa couplings are bi-fundamentals:

$$Y_u \to \Omega_{Q_u} Y_u \Omega_u^\dagger \,, \tag{B.2}$$

$$Y_d \to \Omega_{Q_d} Y_d \Omega_d^\dagger \,. \tag{B.3}$$

We can construct invariants under the flavour symmetry from the spurion fields, see Table B.1. From these invariants we build the scalar potential V (using only renormalizable terms). For the vacuum expectation values of the spurions we make the following ansatz:

$$C_x = \begin{pmatrix} c_{x,1} & 0 \\ 0 & c_{x,2} \end{pmatrix} , \tag{B.4}$$

again with $x = u, d, Q_u, Q_d$. It is possible to choose all four bulk masses to be simultaneously diagonal, since Q_d, Q_u, d and u are independent fields. The Yukawas are a

[1]Throughout this appendix the Yukawa matrices are the 5D ones, and we omit the 5D superscript in order to simplify the notation.

mass dim. 1	mass dim. 2	mass dim. 3	mass dim. 4
$\mathrm{Tr}(C_x)$	$\det(C_x)$	$\mathrm{Tr}(C_x C_x C_x)$	$\mathrm{Tr}(C_d C_d Y_d^\dagger Y_d)$
	$\det(Y_d)$	$\mathrm{Tr}(C_d Y_d^\dagger Y_d)$	$\mathrm{Tr}(C_u C_u Y_u^\dagger Y_u)$
	$\det(Y_u)$	$\mathrm{Tr}(C_u Y_u^\dagger Y_u)$	$\mathrm{Tr}(C_{Q_d} C_{Q_d} Y_d Y_d^\dagger)$
	$\mathrm{Tr}(Y_d^\dagger Y_d)$	$\mathrm{Tr}(C_{Q_d} Y_d Y_d^\dagger)$	$\mathrm{Tr}(C_{Q_u} C_{Q_u} Y_u Y_u^\dagger)$
	$\mathrm{Tr}(Y_u^\dagger Y_u)$	$\mathrm{Tr}(C_{Q_u} Y_u Y_u^\dagger)$	

Table B.1: *Invariants of the spurions in the bulk up to mass dimension 4 ($x = u, d, Q_u, Q_d$).*

priori not diagonal in this basis, and we choose to parameterize them as

$$Y_d = \begin{pmatrix} \cos\theta_{Q_d} & \sin\theta_{Q_d} \\ -\sin\theta_{Q_d} & \cos\theta_{Q_d} \end{pmatrix} \begin{pmatrix} y_{d,1} & 0 \\ 0 & y_{d,2} \end{pmatrix} \begin{pmatrix} \cos\theta_d & -\sin\theta_d \\ \sin\theta_d & \cos\theta_d \end{pmatrix}, \tag{B.5}$$

$$Y_u = \begin{pmatrix} \cos\theta_{Q_u} & \sin\theta_{Q_u} \\ -\sin\theta_{Q_u} & \cos\theta_{Q_u} \end{pmatrix} \begin{pmatrix} y_{u,1} & 0 \\ 0 & y_{u,2} \end{pmatrix} \begin{pmatrix} \cos\theta_u & -\sin\theta_u \\ \sin\theta_u & \cos\theta_u \end{pmatrix}. \tag{B.6}$$

Differentiating the potential leads to the following relations (for the up-sector):

$$\frac{\partial V}{\partial \theta_{Q_u}} \propto \sin(2\theta_{Q_u}), \tag{B.7}$$

$$\frac{\partial V}{\partial \theta_u} \propto \sin(2\theta_u), \tag{B.8}$$

$$\frac{\partial^2 V}{\partial \theta_{Q_u}^2} \propto -2(c_{Q_u,1} - c_{Q_u,2})(y_{u,1}^2 - y_{u,2}^2)\cos(2\theta_{Q_u}), \tag{B.9}$$

$$\frac{\partial^2 V}{\partial \theta_u^2} \propto -2(c_{u,1} - c_{u,2})(y_{u,1}^2 - y_{u,2}^2)\cos(2\theta_u). \tag{B.10}$$

We thus find the minimum of the potential at $\theta_{Q_u} = \theta_u = 0$ or at $\theta_{Q_u} = \theta_u = \pi/2$, depending on the relative size of the Yukawa eigenvalues (we always have $c_{Q_u,1} > c_{Q_u,2}$ and $c_{u,1} > c_{u,2}$). In both cases the Yukawa matrices are diagonal, since having $\theta_{Q_u} = \theta_u = \pi/2$ is equivalent to the transformation

$$\begin{pmatrix} y_{u,1} & 0 \\ 0 & y_{u,2} \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} y_{u,1} & 0 \\ 0 & y_{u,2} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{B.11}$$

which just changes the order of the eigenvalues. An analogous discussion shows that the down sector is also aligned.

mass dim. 1	mass dim. 2	mass dim. 3	mass dim. 4
$\mathrm{Tr}(C_Q)$	$\mathrm{Tr}(MM^\dagger)$	$\mathrm{Tr}(C_Q C_Q C_Q)$	$\det(C_Q)$
		$\mathrm{Tr}(C_Q MM^\dagger)$	$\mathrm{Tr}(C_Q C_Q MM^\dagger)$

Table B.2: *Invariants composed of the spurions on the UV brane up to dimension 4.*

B.2 UV brane

Now consider the spurions on the UV brane, *i.e.* M_u, M_d, C_{Q_u} and C_{Q_d}. Since the flavour group is enhanced on the UV brane, we write them as:

$$M \equiv \begin{pmatrix} M_u \\ M_d \end{pmatrix}, \qquad C_Q \equiv \begin{pmatrix} C_{Q_u} & 0 \\ 0 & C_{Q_d} \end{pmatrix}. \tag{B.12}$$

M then transforms as a bi-fundamental under the enhanced flavour group

$$M \to \Omega_Q M \Omega^\dagger_{\tilde{Q}}, \tag{B.13}$$

and C_Q as an adjoint

$$C_Q \to \Omega_Q C_Q \Omega^\dagger_Q. \tag{B.14}$$

Again, we find the invariants (see Table B.2), and write down the most general potential. Since the Matrix M only appears via the combination $MM^\dagger$, and this is a Hermitian matrix (*i.e.* diagonalized by a unitary matrix), we can plug in:

$$C_Q = \begin{pmatrix} c_{Q_u^1} & 0 & 0 & 0 \\ 0 & c_{Q_u^2} & 0 & 0 \\ 0 & 0 & c_{Q_d^1} & 0 \\ 0 & 0 & 0 & c_{Q_d^2} \end{pmatrix}, \qquad MM^\dagger \equiv U \begin{pmatrix} m_{u,1} & 0 & 0 & 0 \\ 0 & m_{u,2} & 0 & 0 \\ 0 & 0 & m_{d,1} & 0 \\ 0 & 0 & 0 & m_{d,2} \end{pmatrix} U^\dagger, \tag{B.15}$$

where U is a rotation matrix depending on the six angles $\theta_{12}, \theta_{13}, \theta_{14}, \theta_{23}, \theta_{24}, \theta_{34}$. Searching for the minimum of the potential, we get the following relations

$$\left.\frac{\partial V}{\partial \theta_{ij}}\right|_{\text{all other } \theta_{kl}=0} \propto \sin(2\theta_{ij}), \qquad \left.\frac{\partial^2 V}{\partial \theta_{ij}^2}\right|_{\text{all other } \theta_{kl}=0} \propto \pm\cos(2\theta_{ij}). \tag{B.16}$$

Thus, at the minimum of the potential we have $\theta_{ij} = 0$ or $\theta_{ij} = \pi/2$. Again, $\theta_{ij} = \pi/2$ only implies that the eigenvalues are shuffled around (note that here we have the same angles θ_{ij} on the right and on the left of the diagonal matrix).
It should be noted that we do not assume any MFV relations such as

$$C_{Q_u} = \alpha_{Q_u}\mathbf{1} + \beta_{Q_u} Y_u Y_u^\dagger \,, \tag{B.17}$$

although this would be compatible with the flavour symmetry. Rather, C_{Q_u}, C_{Q_d}, C_u, C_d, M_d, M_u, Y_u and Y_d are all independent spurions. Therefore, there is no relation in between their eigenvalues.

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