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# EFFICIENT ALGORITHMS AND MODELS FOR BAYESIAN UPDATING IN STRUCTURAL RELIABILTY

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Bayesian updating enables one to consistently combine models with observations. In the context of structural reliability, it enables computing the conditional probability of system failure given measurement or monitoring results. Despite the huge potential for such updating, e.g. in geotechnical engineering or in the management of deteriorating structures, the method has seen limited applications in engineering practice. To a large extent, this was due to limitations of computational algorithms for performing Bayesian updating in the context of structural reliability, which have now been partly overcome by recent developments. This paper introduces the general concept of Bayesian updating in structural reliability and then summarizes a novel method that strongly enhances the versatility and efficiency of algorithms for computing the reliability conditional on observations. The method is illustrated by two examples.

*Keywords*: Bayesian updating, structural reliability, measurements, monitoring, geotechnical reliability.

## 1 Introduction

With advances in information and sensor technology, increasing amounts of information on performances of engineering systems are collected and stored; examples include data on deformations and dynamic properties of structural systems, or data on ambient factors influencing deterioration. This information should be used to reduce the uncertainty in engineering models and, ultimately, to optimize the management of these systems. As an example, a smart structure should use sensor information to automatically trigger actions like detailed inspections or system shut-downs. Because system predictions typically remain uncertain even with new information, such decisions should be made reliability- and risk-based. This motivates the use of Bayesian updating in structural reliability, which enables the computation of the reliability conditional on new information.

Bayesian updating in the context of structural reliability has been considered since the 1970s (e.g. Tang 1973) and in the 1980s several computational methods were proposed for this purpose (Madsen 1987; Schall et al. 1988). However, these methods have limitations with respect to accuracy and convergence, which are a main reason for the limited success of these methods in practice. Recently, the author has proposed a novel formulation of the Bayesian updating problem in structural reliability in (Straub 2011a). As opposed to previous methods, this formulation allows the use of any standard structural reliability method to compute the conditional probability of failure given any type of information. With this approach, reliability updating can now be performed more accurately and efficiently as shown in some follow-up publications (Straub 2010; Papaioannou & Straub 2012). This approach is the main topic of this paper.

Alternative approaches have been developed, which perform Bayesian updating by first updating the distribution of the basic random variables of the problem and performing reliability analysis with these updated distributions, e.g. in (Papadimitriou et al. 2001). As discussed later in this paper, this approach can be difficult to implement in the general case when the posterior distribution cannot be modeled (or approximated) by an analytical distribution. This is critical for all problems involving more than just a few random variables because numerical higher-dimensional descriptions of the posterior distribution are computationally (too) expensive. Ching and Hsieh (2009) present an alternative method that overcomes the problems associated with this high-dimensional description, but conversely has limitations on the number of observations that can be considered. A completely different approach is proposed in (Straub & Der Kiureghian 2010a,b), where a procedure for combining structural reliability methods with Bayesian networks (BN) is developed. The resulting so-called enhanced BN (eBN) is highly efficient and versatile for probabilistic updating with any kind of information, but to establish these models requires significant efforts. The method has been successfully applied e.g. for updating of deterioration models with inspection information (Straub 2009).

In this contribution, an introduction to Bayesian updating in structural reliability is given. This is followed by a presentation of the method developed in (Straub 2011a), and finally two example applications are presented; the first application is monitoring of a geotechnical construction site, the second is an academic example that illustrated the accuracy, flexibility and efficiency of the method. It is demonstrated that the method can be both robust and efficient, which are necessary conditions for near-real-time applications of Bayesian updating in structural reliability.

## 2 Bayesian Updating in Structural Reliability

### 2.1 *Structural reliability*

In structural reliability, the interest is in computing the probability of failure of an engineering system or component, which is described by a model whose input is a set of random variables $\mathbf{X} = (X_1, X_2, \dots, X_n)$. By means of the model it is possible to divide the outcome space of **X** into a failure and a safe domain. By convention, the failure domain $\Omega_F$ is defined in terms of continuous limit state functions $g(\mathbf{x})$. If there is only a single limit state function, it is

$$\Omega_F = \{g(\mathbf{x}) \leq 0\}. \tag{1}$$

A classic example is the limit state function $g(r, s) = r - s$, where $r$ is the capacity and $s$ is the load: Failure occurs when $r \leq s$, i.e. when $g(r, s) \leq 0$. In the general case, $\Omega_F$ is defined in terms of several limit state functions (e.g., Der Kiureghian 2005), corresponding to systems of components that are defined by limit state functions. For the purpose of the present paper, the formulation in Eq. (1) is sufficiently general; extension to the system application is straightforward (Straub 2011a).

The probability of failure can then be computed by integrating $f(\mathbf{x})$, the joint probability density function (JPDF) of **X,** over the failure domain:

$$\Pr(F) = \int_{\mathbf{x} \in \Omega_F} f(\mathbf{x}) \mathrm{d}\mathbf{x}. \tag{2}$$

Structural reliability methods such as FORM, SORM, importance sampling (IS) or subset simulation (SuS) have been developed to efficiently compute such integrals (Ditlevsen & Madsen 1996, Rackwitz 2001, Der Kiureghian 2005, Melchers 1999, Au & Beck 2001).

### 2.2 *Bayesian updating of input parameters*

In some instances, measurements on model inputs **X** are available, which allow to reduce the uncertainty in the model. As an example consider a measurement $r_m$ of material strength $R$. If the measurement is exact, the random variable $R$ can be replaced by the deterministic $r_m$. In the general case, however, the measurement will be associated with uncertainty. In this case, the measurement can be represented by the likelihood function, which corresponds to the probability of making the observation for a given value of $R$:

$$\begin{aligned} L(r) &\propto \Pr(R_m = r_m | R = r) \\ &\propto f_\epsilon(r - r_m). \end{aligned} \quad (3)$$

Here, $f_\epsilon(\epsilon)$ is the probability distribution function (PDF) of an additive measurement error $\epsilon$. (For other types of measurement errors, e.g. multiplicative errors, the second line in Eq. (3) must be modified accordingly.) The likelihood function is a well-known concept from classical statistics (e.g. Coles 2007).

Bayesian updating combines the a-priori PDF of $R$, $f(r)$, with the measurement outcome, which is represented through the likelihood function $L(r)$. According to Bayes' rule, the updated PDF of $R$, denoted by $f''(r)$, is

$$f''(r) \propto L(r) f(r). \quad (4)$$

The proportionality constant in Eq. (4) can be obtained by normalization, since it must hold that $\int_{-\infty}^{\infty} f''(r)\mathrm{d}r = 1$, i.e. the proportionality constant $\alpha$ is

$$\alpha = \frac{1}{\int_{-\infty}^{\infty} L(r) f(r)\mathrm{d}r}. \quad (5)$$

Bayesian updating of an individual random variable according to Eq. (4) is illustrated in Figure 1.

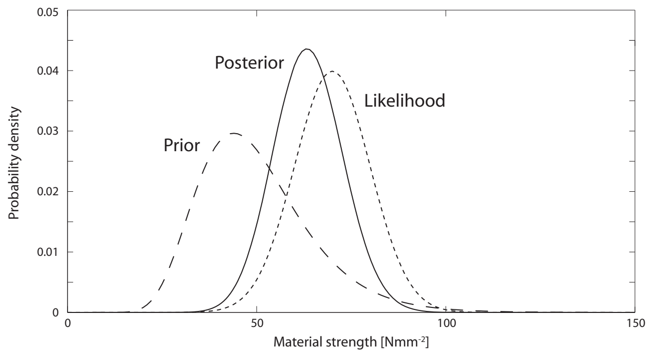


Figure 1. Bayesian updating of a random variable $R$ with measurement $r_m$.

Often several measurements are available, with corresponding likelihood functions $L_1(r), L_2(r), \ldots, L_n(r)$. In the common case that measurement errors are independent, the measurements can be combined into a single likelihood function $L(r)$ by

$$L(r) = \prod_{i=1}^{n} L_i(r). \quad (6)$$

If measurements are dependent due to common factors in the measurement (e.g. when making spatially distributed measurements), the likelihood function is the joint distribution of the measurements given the model parameter.

The above formulas can be extended from the case of a single random variable $R$ to multiple random variables **X**, simply by replacing the argument $r$ with **x**. For larger number of random variables in **X**, say larger than 5, the integration required for computing the proportionality constant $\alpha$ become prohibitive (the integral in Eq. (5) becomes an $n$-fold integral, with $n =$ number of random variables in **X**). In these cases, an alternative is provided through Markov Chain Monte Carlo (MCMC) methods, which allow to generate samples from $f''(\mathbf{x})$ without knowing $\alpha$ (Gilks et al. 1996). However, it is noted that the joint updating of multiple input random variables **X** is generally impractical in the context of structural reliability, due to prohibitively large computational requirements. Simpler and more efficient approaches are available, as presented later.

Once all random variables in **X** are updated with their respective measurements, structural reliability analysis can be performed with the a-posteriori JPDF $f''(\mathbf{x})$ as the input. The resulting reliability is conditional on the measurements.

### 2.3 *Bayesian updating when measuring structural response or system performance*

In many instances, measurements of the system are available that do not directly correspond to the input random variables **X**, but rather to some model output variables **Y**. Such an example is illustrated in Section 3 of this paper, considering measurements of deformations of a geotechnical structure. Generally, any observed response of an engineering system falls in this category (deformations, vibrations, stresses) as well as other indicators on the condition state (e.g. measurements of chloride concentration in concrete as an indicator of corrosion, Straub 2011b).

In the above examples, the observed (measured) quantities **Y** can be described through functions of the random variables **X** as $\mathbf{Y} = h(\mathbf{X})$. Obtaining improved estimates of **X** from observations $\mathbf{y}_m$ corresponds to solving an inverse problem, which can be done with the Bayesian approach (e.g. Gelman 2004). Let $\boldsymbol{\epsilon}_m$ be additive measurement errors and therefore $\mathbf{Y} = \mathbf{y}_m - \boldsymbol{\epsilon}_m$. It follows that the likelihood function for **X** is

$$\begin{aligned} L(\mathbf{x}) &\propto \Pr(\mathbf{Y} = \mathbf{y_m} | \mathbf{X} = \mathbf{x}) \\ &\propto f_\epsilon(h(\mathbf{X}) - \mathbf{y}_m). \end{aligned} \tag{7}$$

It is, in principle, possible to update the distribution of **X** by means of this likelihood function following Eqs. (4) and (5). However, as pointed out earlier, this approach becomes computationally inefficient (or impossible) as the number of random variables **X** increases. In particular, the updated probability distribution of **X**, $f''(\mathbf{x})$, will no longer have an analytical form and can be described only numerically. Additionally, $f''(\mathbf{x})$ cannot be described by the product of its marginal distributions, because even if the **X** are independent a-priori, they will be dependent a-posteriori because of the joint likelihood function. It follows that the description of $f''(\mathbf{x})$ becomes cumbersome for higher dimensions. It is therefore desirable to avoid the explicit computation of $f''(\mathbf{x})$. This can be achieved through the methods of structural reliability, as summarized in the next section.

### 2.4 *Structural reliability approach to Bayesian updating*

Any set of measurements or observations corresponds to an event, which we here describe by $Z$. In structural reliability, this observation event can be described by a domain, similarly to the failure domain $\Omega_F$ describing a failure event, as outlined in section 2.1.

Each observation can be characterized by a continuous limit state function $h_i(\mathbf{x})$, where $\mathbf{X} = (X_1, X_2, \dots, X_n)$ are the basic random variables. The information contained in the observation is said to be of the inequality type if the corresponding domain can be written as

$$\Omega_Z = \{h(\mathbf{x}) \leq 0\}, \tag{8}$$

and it is said to be of the equality type if it can be written as

$$\Omega_Z = \{h(\mathbf{x}) = 0\}. \tag{9}$$

Bayesian updating in the context of structural reliability corresponds to computing the conditional probability of failure given the observations $\Pr(F|Z)$. This can be determined from the definition of the conditional probability as:

$$\Pr(F|Z_1 \cap \dots \cap Z_m) = \frac{\Pr(F \cap Z)}{\Pr(Z)} = \frac{\int_{\mathbf{x}\in\{\Omega_F \cap \Omega_Z\}} f(\mathbf{x})\mathrm{d}\mathbf{x}}{\int_{\mathbf{x}\in\Omega_Z} f(\mathbf{x})\mathrm{d}\mathbf{x}}. \tag{10}$$

This is illustrated in Figure 2 for the case of an observation $Z$ of the inequality type.

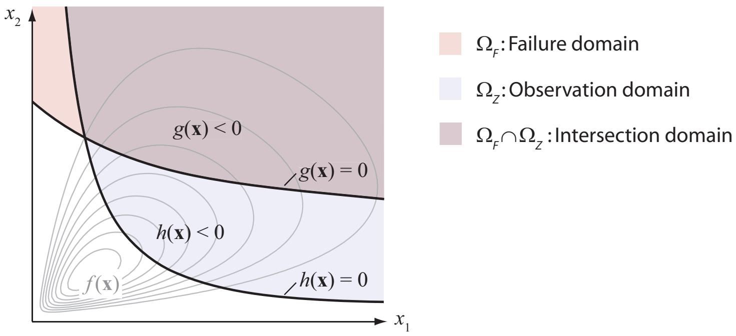


Figure 2. Illustration of Bayesian updating in structural reliability: Computation of the conditional probability of failure given an observation $Z$ is performed through $\Pr(F|Z) = \Pr(F \cap Z) / \Pr(Z)$; whereby $\Pr(F \cap Z)$ is obtained from the integral over the area $\{\Omega_F \cap \Omega_Z\}$ and $\Pr(Z)$ is obtained by an integral over the area $\Omega_Z$.

If observations are exclusively of the inequality type, Eq. (8), evaluation of the above integrals is straightforward using any of the available structural reliability methods (SRM). However, if the observation event $Z$ is of the equality type, the integrals result in zero, since this event has zero probability a-priori. Direct application of SRM is thus not possible in this case. Solutions to overcome this problem have been suggested by (Madsen 1987) and the group of Rackwitz (e.g. Schall et al. 1988). Madsen's solution is based on De L'Hôpital's rule. The solutions of the Rackwitz group are based on computing surface integrals, using first- or second order approximations of the surfaces $h_i(\mathbf{x}) = 0$. These solutions are implemented in the Strurel software (Gollwitzer et al. 2006). Both Madsen's and Rackwitz' methods are efficient and can often represent a sufficiently accurate approximation. However, in cases where FORM/SORM solutions are not sufficiently accurate or in which it is difficult to identify the joint design point, these methods should not or cannot be used. For this reason, (Straub 2011a) developed a method for transforming equality into inequality information, which is summarized in the following section.

### 2.5 *A transformation for facilitating Bayesian updating*

This section presents a slightly modified summary of the method introduced in (Straub 2011a) for transforming Bayesian updating in structural reliability. The method is based on representing observations through the likelihood function $L(\mathbf{x})$, Eq. (7). Since any observation can be expressed through $L(\mathbf{x})$, the method is applicable independent of whether the information is of the equality or inequality type.

The central idea of the method is the introduction of a limit state function that represents the likelihood function. This limit state function is denoted by $h_E(\mathbf{x})$ and it is of the inequality type, Eq. (8). The derivation of $h_E(\mathbf{x})$ is summarized in the following. Firstly, a random variable $P$ with uniform distribution in the range $[0,1]$ is introduced, together with a constant $c$ that is selected so that $0 \leq cL(\mathbf{x}) \leq 1$ for all $\mathbf{x}$. The following identity holds for any value of $\mathbf{X} = \mathbf{x}$:

$$L(\mathbf{x}) = \frac{\Pr[P \leq cL(\mathbf{x})]}{c}. \tag{11}$$

Following Eq. (7), the likelihood function is defined as $L(\mathbf{x}) \propto \Pr(Z|\mathbf{X} = \mathbf{x})$. (In Eq. (7) the observation event is $Z = \{\mathbf{Y} = \mathbf{y_m}\}$.) Let $\alpha$ denote the corresponding proportionality constant. By combining with Eq. (11), we obtain:

$$\Pr(Z|\boldsymbol{X} = \boldsymbol{x}) = \alpha L(\mathbf{x}) = \frac{\alpha}{c}\Pr[P \leq cL(\mathbf{x})]. \tag{12}$$

It follows from the total probability theorem that the probability of the information event $Z$ is

$$\Pr(Z) = \int_{\mathbf{X}} \Pr(Z|\boldsymbol{X} = \boldsymbol{x})\, f(\mathbf{x})\mathrm{d}\mathbf{x} = \frac{\alpha}{c}\int_{\mathbf{X}} \Pr[P \leq cL(\mathbf{x})]\, f(\mathbf{x})\mathrm{d}\mathbf{x}. \tag{13}$$

The event $\{P \leq cL(\mathbf{x})\}$ can be defined through the limit state function

$$h_e(\mathbf{x}, p) = p - cL(\mathbf{x}), \tag{14}$$

and the corresponding domain $\Omega_{Ze} = \{h_e(\mathbf{x}, p) \leq 0\}$. This has the same form as the domains describing inequality information, Eq. (8). Equation (13) can now be rewritten to

$$\Pr(Z) = \frac{\alpha}{c}\int_{\mathbf{x},p\in\Omega_{Ze}} f(p)f(\mathbf{x})\mathrm{d}\mathbf{x}\,\mathrm{d}p = \frac{\alpha}{c}\int_{\mathbf{x},p\in\Omega_{Ze}} f(\mathbf{x})\mathrm{d}\mathbf{x}\,\mathrm{d}p. \tag{15}$$

The second identity follows from $f(p) = 1$. Similarly, the probability of $\{F \cap Z\}$ is obtained as

$$\begin{aligned}\Pr(F \cap Z) &= \int_{\mathbf{X}} \Pr(Z|\boldsymbol{X} = \boldsymbol{x}) \Pr(F|\boldsymbol{X} = \boldsymbol{x})\, f(\mathbf{x})\mathrm{d}\mathbf{x} \\ &= \frac{\alpha}{c}\int_{\mathbf{x},p\in\{\Omega_F\cap\Omega_{Ze}\}} f(\mathbf{x})\mathrm{d}\mathbf{x}\,\mathrm{d}p.\end{aligned} \tag{16}$$

The conditional probability of $F$ given $Z$ is therefore

$$\Pr(F|Z) = \frac{\int_{\mathbf{x},p\in\{\Omega_F\cap\Omega_{Ze}\}} f(\mathbf{x})\mathrm{d}\mathbf{x}\,\mathrm{d}p}{\int_{\mathbf{x},p\in\Omega_{Ze}} f(\mathbf{x})\mathrm{d}\mathbf{x}\,\mathrm{d}p}. \tag{17}$$

Here, the proportionality constant $\alpha$ cancels out. The denominator in Eq. (17) corresponds to a component reliability problem, the nominator to a parallel system reliability problem. Since all domains are now of the inequality type, both integrals in Eq. (17) can be computed using any SRM, including FORM/SORM as well as any sampling method.

Note that the above solution is directly applicable for several observations $Z = \{Z_1 \cap \ldots \cap Z_m\}$. Either, a separate limit state function $h_{e,i}$ is formulated for each observation $Z_i$, or all observations are combined into a single likelihood function following Eq. (6). The latter solution is typically simpler and is used in the second example presented below.

## 3 Examples

Two examples are presented in the following for illustration. Additional examples can be found in (Straub 2011a) on updating fatigue crack growth reliability and in (Straub 2011b) on spatial updating of corrosion reliability.

### 3.1 *Updating geotechnical reliability*

This application is originally presented in (Papaioannou & Straub 2012). Consider the geotechnical construction site shown in Figure 3. Here, failure is considered to be the event of the horizontal deformation of the sheet pile wall at top of the trench $u_x$ exceeding an allowable deformation of 0.1m. The limit state function for failure therefore is

$$g(\mathbf{x}) = 0.1\text{m} - u_x(\mathbf{x}), \tag{18}$$

where $u_x(\mathbf{x})$ is obtained as a the solution of a non-linear finite element (FE) computation. The random variables **X** describe the soil properties (specific weight, young's modulus, friction angle) through random fields. Because of this random field representation, the problem has a total of 432 random variables. The unconditional probability of failure (without measurement) is computed as $\Pr(F) = 0.0014$.

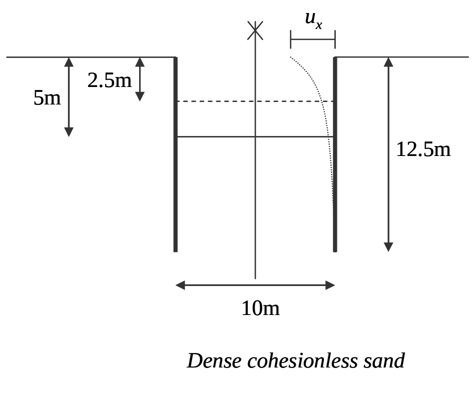


Figure 3. Sheet pile wall in sand (from Papaioannou & Straub 2012).

During the construction process, observations of deformations at the intermediate excavation stages can be made. These observations are traditionally used by the geotechnical engineer to verify the design. Here, this so-called "observational method" is formalized by computing the reliability of the final excavation stage conditional on the observed horizontal deformation $u_{x,m}$ at the intermediate stage, i.e. when the excavation is at 2.5m depth. Following Eq. (14), the limit state function describing the observation is:

$$h_e(\mathbf{x}, p) = p - c f_{\epsilon,m}\left(u_{x,m} - u_{x_{0.5}}(\mathbf{x})\right). \quad (19)$$

Here, $P$ is a uniform random variable and $c$ a constant as described in Section 2.5. $f_{\epsilon,m}$ is the probability density function of the measurement error $\sigma_{\epsilon,m}$ and $u_{x_{0.5}}(\mathbf{x})$ is the computed deformation at the intermediate stage for given values of the random variables.

For illustration, Figure 4 shows results of the FE model for the intermediate and the final excavation stage.

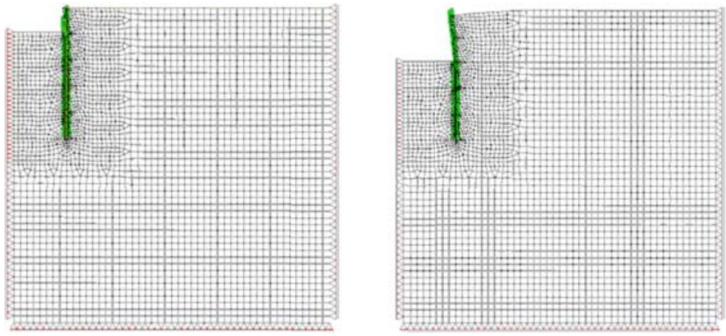

Figure 4. Finite element model (FEM) showing the magnified deformed construction site at the intermediate and the final excavation step (computed with mean values).

The conditional reliability of the excavation site is computed using the methods presented in Section 2.5. Due to the large number of random variables, FORM is not effective for this example, and instead subset simulation (Au & Beck 2001) is employed. Figure 5 shows the resulting probability of failure for different values of the measured deformation. Clearly, the probability of failure increases with increasing observed deformations.

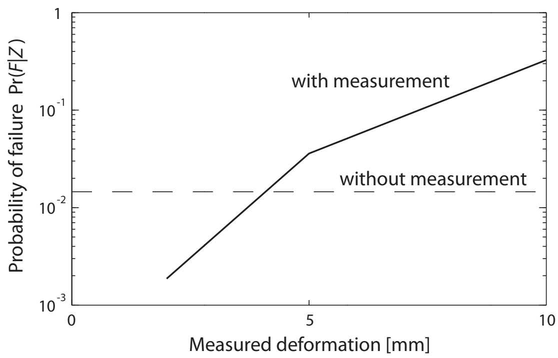


Figure 5. Probability of failure at the final construction stage, conditional on different deformation measurement results at the intermediate stage (from Papaioannou & Straub 2012).

Results such as those presented in Figure 5 may serve for defining thresholds on measurement results that trigger modifications or revisions of the geotechnical design. They can support the traditionally applied observational method in geotechnical engineering by backing it up with a quantitative assessment.

### 3.2 *Updating of a Gaussian random process*

This example is included for illustrational purposes, since it has an analytical solution that enables demonstrating the accuracy of the proposed method for Bayesian updating in combination with importance sampling. In addition, the example shows the applicability of the method to problems involving spatially distributed observations and limit state functions.

Limit states functions are defined at 100 locations along a vector $\mathbf{t} = [1; 2; \dots; 100]$. The problem involves only one random vector $\mathbf{X} = [X(1); X(2); \dots; X(100)]$, which is modeled as stationary Gaussian process with mean $\mu_X = 5$ and autocovariance function

$$\mathrm{Cov}[\mathrm{X}(t), \mathrm{X}(t + \Delta t)] = \exp\left(-\frac{\Delta t}{10}\right). \tag{20}$$

The limit state function at all locations **t** is:

$$g\big(\mathrm{X}(t)\big) = \mathrm{X}(t) - 2, \qquad t = 1,2, \dots, 100. \tag{21}$$

For this simple problem, the design point of the FORM solution is $X_t^* = 2$ for all $t$ and the unconditional probability of failure is $\Pr\big(F(t)\big) = \Phi(-(5-2)) = 0.0013$. Here, $\Phi$ is the standard Normal CDF. Next, we assume that the following measurements $\mathbf{X}_m$ are made at locations $\mathbf{t}_m = [10; 20; \dots; 90]$:

Table 1. Measurements made of the process $X(t)$ at different locations $t_m$.

| $t_{m,i} =$ | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
|---|---|---|---|---|---|---|---|---|---|
| $x_{m,i} =$ | 5 | 3 | 4 | 6 | 5 | 5 | 3 | 3 | 4 |

The measurement error $\epsilon_{m,i}$ is additive with normal distribution with zero mean and standard deviation $\sigma_{\epsilon,m} = 0.5$. Assuming that the $\epsilon_{m,i}$ are independent, the limit state function describing these observations is obtained following Eq. (14) as:

$$\begin{aligned} h_e(\mathbf{x}) &= p - cL(\mathbf{x}) = p - c\prod_{i=1}^{10} L_i(\mathbf{x}) \\ &= p - c\prod_{i=1}^{9} \frac{1}{\sigma_{\epsilon,m}} \varphi\left(\frac{x_{m,i} - x\big(t_{m,i}\big)}{\sigma_{\epsilon,m}}\right). \end{aligned} \tag{22}$$

Since it is necessary to compute the conditional reliability simultaneously at 100 locations, an efficient computation method that requires little intervention by the analyst is required. An adaptive importance sampling approach is selected. To evaluate the conditional reliability at location $t$, an initial sampling density with mean equal to the design point of the limit state function at $t$ is chosen. This density is then adapted following a procedure derived from (Bucher 1988). For further details on the importance sampling solution of the integrals given in Eq. (17), the reader is referred to (Straub 2010).

The results obtained with $10^4$ and $10^5$ samples are summarized in Figure 6. Since all random variables are normal and since all limit state functions describing failure are linear, an exact solution can be obtained, which is also given for comparison. It can be observed that the sampling error is negligible with $10^5$ samples; with $10^4$ samples, it is sufficiently small for most practical purposes. It is pointed out that the solution presented in this paper can be implemented for any non-linear non-normal limit state functions with equal accuracy.

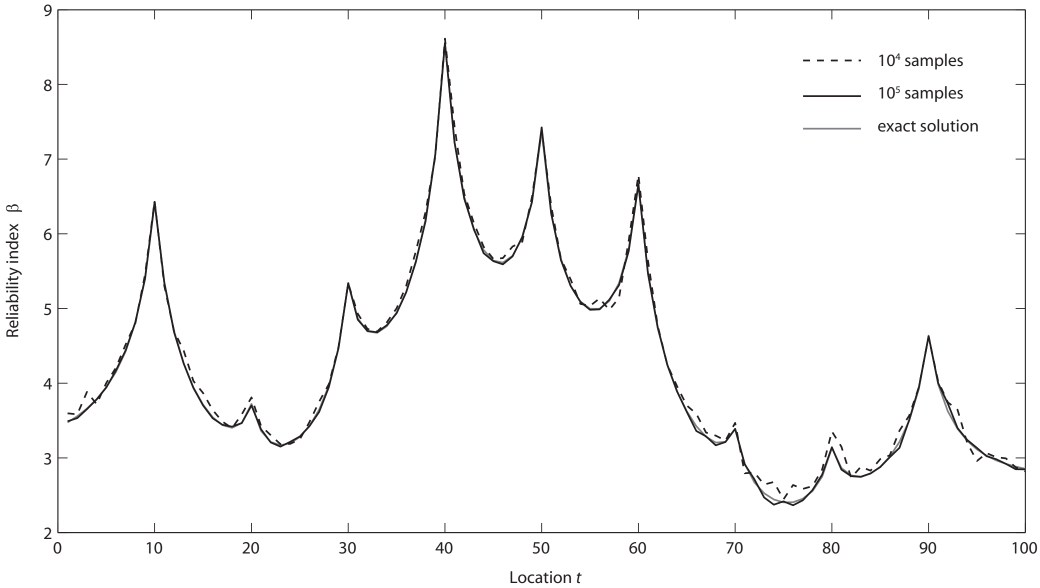


Figure 6. Reliability index at locations $\mathbf{t} = [1; 2; \ldots; 100]$, conditional on measurements of the process $\mathbf{X}(t)$ at locations $t = [10; 20; \ldots; 90]$.

## 4 Conclusions

An efficient computational procedure for Bayesian updating of the reliability of engineering structures and systems is summarized. Due to its flexibility, the procedure can be combined with any existing structural reliability method, to provide optimal computational performance for any problem. The examples provided in the paper illustrate the applicability of the procedure for Bayesian updating in finite element-based reliability analysis and in problems involving spatially distributed observations and failures.

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