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HYDROELASTIC ANALYSIS OF PONTOON-TYPE VERY LARGE FLOATING STRUCTURES IN RANDOM SEAS

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The hydroelastic response of very large floating structures (VLFS) is obtained by resolving the interaction between the surface waves and the floating elastic body. We carry out the analysis in the frequency domain, assuming that the surface waves can be described by a directional wave spectrum. Applying the modal expansion method, we obtain a discrete representation of the required transfer matrices for a finite number of frequencies, while the influence of the wave direction is obtained by numerical integration of the directional components of the spectrum. The boundary element method is used to solve the Laplace equation together with the fluid boundary conditions for the velocity potential, whereas the finite element method is adopted for solving the deflection of the floating plate. Moreover, we compute the variance of the response for two different cases of mean wave angles.

Keywords: hydroelasticity, VLFS, directional spectrum, random vibration, response statistics.

1 Introduction

Pontoon-type very large floating structures (VLFS) are relatively flexible structures that behave like giants plates resting on the sea surface. The response of such structures is obtained by resolving the interaction between the surface waves and the floating elastic body. Several methods have been proposed for the hydroelastic analysis of VLFS (Watanabe et al. 2004). However, usually the response is obtained for distinct wave frequencies and wave angles. Hamamoto (1995) derived analytical expressions for the response of large circular floating structures subject to a spectrum of wave frequencies. Chen et al. (2004 and 2006) studied the influence of second-order effects of the structural geometry and wave forces on the response of VLFS under two irregular wave systems coming from different directions.

In this paper, we develop a method for hydroelastic analysis of VLFS subject to a directional wave spectrum. The analysis is done in the frequency domain by application of the modal expansion method (Newman 1994). The fluid domain is discretized by the boundary element method, while for the structure we use the finite element method derived from Mindlin plate theory. Due to the latter, the effects of transverse shear deformation and rotary inertia are accounted for. The derived linear system allows for the application of linear random vibration theory for the evaluation of the response spectrum.

2 Hydroelastic Analysis of the Floating Plate

Figure 1 shows a schematic diagram of the VLFS. The VLFS has length L, width B, and height h and is assumed to be perfectly flat with free edges. Moreover, zero draft is assumed for simplicity. The plate domain is denoted by Ω. The free and undisturbed water surface is at $z = 0$ while the seabed is found at $z = -H$, where z is the out-of-plane coordinate. Assuming that an incident wave ϕ_I of frequency ω, height $2A$ and wave angle θ enters the computational domain, the plate will deform in a steady state harmonic motion in the same frequency ω. The deflection w is measured from the free and undisturbed water surface.

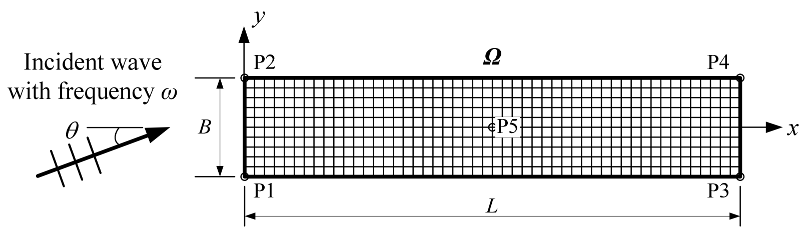

Figure 1. Schematic diagram of the floating plate

The Mindlin plate theory is used to model the VLFS. The plate material is assumed to be isotropic and described by Hooke's law. The equation of motion of the plate structure is established using the finite element method and takes the following form (after omitting the time factor $e^{-i\omega t}$)

$$\left(-\omega^2\left[M\right]+\left[K\right]\right)\left\{w\right\}=\int_{\Omega}\left\{N\right\}\left\{N\right\}^T\left\{p\right\}d\Omega \tag{1}$$

where $[M]$ and $[K]$ are mass and stiffness matrices, $\{w\}$ is the displacement vector that includes vertical displacements and two rotations, $\{N\}$ the vector of global shape functions and $\{p\}$ is the vector containing the wave pressures at the nodes with hydrostatic and hydrodynamic components.

The water is assumed to be an ideal fluid (inviscid and incompressible) and has an irrotational flow so that a velocity potential exists. Thus the single frequency velocity potential ϕ of the water must satisfy the Laplace equation:

$$\nabla^2\phi = 0 \tag{2}$$

Based on linear potential theory, the vector of velocity potentials $\{\phi\}$ at the nodal points may be separated into

$$\left\{\phi\right\}=\left\{\phi_I\right\}+\left\{\phi_S\right\}+\left\{\phi_R\right\} \tag{3}$$

where $\{\phi_I\}$, $\{\phi_S\}$ and $\{\phi_R\}$ are the vectors of incident, scattering and radiation potentials, respectively. The elements $\{\phi_I\}_j$ of the incident potential can be expressed as follows:

$$\left\{\phi_I\right\}_j = A\frac{g}{\omega}\frac{\cosh\left(k\left(z+H\right)\right)}{\cosh kH}e^{ik\left(x_j\cos\theta+y_j\sin\theta\right)} \tag{4}$$

where x_j, y_j are the coordinates of each node, k is the wave number which can be expressed in terms of ω through the dispersion relation, and $i = \sqrt{-1}$. At the fluid–structure interface, the boundary conditions for $\{\phi_S\}$ and $\{\phi_R\}$ are as follows (Wang et al. 2008)

$$\frac{\partial\{\phi_S\}}{\partial z} = -\frac{\partial\{\phi_I\}}{\partial z} \tag{5}$$

$$\frac{\partial\{\phi_R\}}{\partial z} = -i\omega[\mathbf{I}]\{w_z\} \tag{6}$$

where $[\mathbf{I}]$ is the identity matrix representing unit vertical amplitude at each node and $\{w_z\}$ is the vector of vertical displacements of the plate. In order to decouple this interaction problem into a hydrodynamic problem in terms of the velocity potential and a fluid–plate vibration problem in terms of the generalized displacement, we adopt the modal expansion method as proposed by Newman (1994). According to this method, the displacement vector of the plate $\{w\}$ is expressed as a finite sum of products of the most significant free vibration modes $[\psi_w]$ and the corresponding complex amplitudes $\{\varsigma_w\}$, i.e.

$$\{w\} = [\psi_w]\{\varsigma_w\} \tag{7}$$

The radiation potential $\{\phi_R\}$ is expanded as

$$\{\phi_R\} = [\Phi_R]\{\varsigma_\phi\} \tag{8}$$

where the elements $[\Phi_R]_{(j,\,l)}$ of the matrix $[\Phi_R]$ indicate the value of the radiation potential on the jth node for unit vertical motion on the lth node, and $\{\varsigma_\phi\}$ is the vector of corresponding complex amplitudes. Following Newman (1994), we assume that the complex amplitudes $\{\varsigma_\phi\}$ are equal to the amplitudes of the plate motion $\{\varsigma_w\}$. Hence, substitution of Eq. (8) to Eq. (6) leads to:

$$\frac{\partial[\Phi_R]}{\partial z} = -i\omega[\mathbf{I}]\{\psi_w\} \tag{9}$$

The nodal pressures $\{p\}_j$ can be evaluated from the linearized Bernoulli equation:

$$\{p\}_j = -\rho g\{w_z\}_j + i\omega\rho\{\phi\}_j \tag{10}$$

where ρ is the fluid density. Combining Eqs. (1), (3), (8) and (10), we can derive the final equation for the hydroelastic response of the structure:

$$\left(-\omega^2[M] - \omega^2[M_w] - i\omega[C_w] + [K] + [K_w]\right)\{w\} = \{f\} \tag{11}$$

where $[M_w]$, $[C_w]$, $[K_w]$ are the added mass, radiation damping and hydrostatic stiffness matrices, respectively, and the wave force vector $\{f\}$ is expressed in terms of the vector of incident potentials as:

$$\{f\} = i4\pi\omega\rho[\bar{\Phi}_R]\{\phi_I\} \tag{12}$$

The matrices $[M_w]$, $[C_w]$ and $[\bar{\Phi}_R]$ are obtained by resolution of the boundary integral equations applying the Green function method (Wang et al. 2008).

3 Stochastic formulation

3.1 *Directional wave spectrum*

Assuming that the irregular (random) wind waves can be described by a zero mean stationary Gaussian process, they can be completely specified by the directional wave spectrum $S(\omega, \theta)$, which represents the distribution of the wave energy in the frequency domain ω as well as in direction (wave angle) θ. The directional spectrum is generally expressed in terms of the one-dimensional frequency spectrum $S(\omega)$ as

$$S(\omega,\theta) = S(\omega) D(\theta \mid \omega) \tag{13}$$

where $D(\theta \mid \omega)$ is the directional spreading function and represents the directional distribution of wave energy for a given frequency ω. In this study, we use the following (one-sided) one-dimensional frequency spectrum (Mitsuyasu 1970):

$$S_{BM}(\omega) = 2516.7 H_{1/3}^2 T_{1/3}^{-4} \omega^{-5} \exp\left[-1605.3\left(T_{1/3}\omega\right)^{-4}\right] \tag{14}$$

where $H_{1/3}$ is the significant wave height and $T_{1/3}$ is the significant wave period. Also, we assume independence of the directional distribution on the wave frequency and adopt the following directional spreading function (Pierson et al. 1953):

$$D(\theta \mid \omega) = D(\theta) = \begin{cases} \dfrac{2}{\pi}\left(\cos\left(\theta-\bar{\theta}\right)\right)^2 & \text{for } \left|\theta-\bar{\theta}\right| \le \dfrac{\pi}{2} \\ 0 & \text{for } \left|\theta-\bar{\theta}\right| > \dfrac{\pi}{2} \end{cases} \tag{15}$$

where $\bar{\theta}$ is the mean wave angle and $\int_{-\pi}^{\pi} D(\theta)\,d\theta = 1$.

3.2 *Stochastic response*

The stochastic hydroelastic response of the VLFS is obtained by applying linear random vibration theory. Following the approach adopted for the solution for a single frequency and wave angle, we first obtain the elements of the cross-spectral matrix $[S_{II}(\omega)]$ of the vector of incident potentials $\{\phi_I\}$ as follows:

$$\left[S_{II}(\omega)\right]_{j,l} = \int_{-\pi}^{\pi} \left|H_I(\omega)\right|^2 e^{-ik\left(x_{lj}\cos\theta + y_{lj}\sin\theta\right)} S(\omega,\theta)\,d\theta \tag{16}$$

where x_{lj} and y_{lj} denote the difference of the x and y coordinates of the locations corresponding to the lth and jth node, respectively. The function $H_I(\omega)$ is the transfer function from the water surface elevation to the incident potential, given by

$$H_I(\omega) = \frac{g}{\omega}\frac{\cosh\left(k\left(z+H\right)\right)}{\cosh kH} \tag{17}$$

Furthermore, we obtain the cross-spectral matrix of the force vector as follows

$$\left[S_{FF}(\omega)\right]=\left[H_F(\omega)\right]\left[S_{II}(\omega)\right]\left[H_F(\omega)\right]^* \tag{18}$$

where $[\ ]^*$ denotes the conjugate transpose operator and the complex transfer matrix $[H_F(\omega)]$ is obtained by Eq. (12), as follows:

$$\left[H_F(\omega)\right]=i4\pi\omega\rho\left[\bar{\Phi}_R\right] \tag{19}$$

Finally, the cross-spectral matrix of the response is obtained as follows

$$\left[S_{ww}(\omega)\right]=\left[H_w(\omega)\right]\left[S_{FF}(\omega)\right]\left[H_w(\omega)\right]^* \tag{20}$$

where the response transfer matrix $[H_w(\omega)]$ is given by Eq. (11) as

$$\left[H_w(\omega)\right]=\left(-\omega^2[M]-\omega^2\left[M_w\right]-i\omega\left[C_w\right]+[K]+\left[K_w\right]\right)^{-1} \tag{21}$$

It should be noted that the inversion in Eq. (21) is trivial, since the solution approach utilizes the uncoupled modes of the system. A discrete representation of the matrix $[S_{ww}(\omega)]$ is obtained using a finite number of frequencies. The variance of the response can then be estimated by numerical integration of the diagonal entries of $[S_{ww}(\omega)]$.

4 Numerical Example

The VLFS considered by Sim and Choi (1998) is used as an example for this study. The length, width and height of the floating plate are 300, 60 and 2 m, respectively. The following material properties of the plate are assumed: Poisson's ratio $\nu = 0.13$, Young's modulus $E = 1.19 \times 10^{10}$ N/m^2, and the mass density of the plate $\rho_p = 256.25$ kg/m^3. The water density is $\rho = 1025$ kg/m^3 and a water depth $H = 20$m. The finite element mesh of the plate is shown in Figure 1.

The chosen parameters for the spectrum of Eq. (14) are $H_{1/3} = 2$ m and $T_{1/3} = 6.298$ sec. We consider two cases for the mean wave angle $\bar{\theta}$, namely 0° and 30°. Figure 2 shows the response spectra for five selected points (P1 to P5, as shown in Figure 1) for the two cases considered.

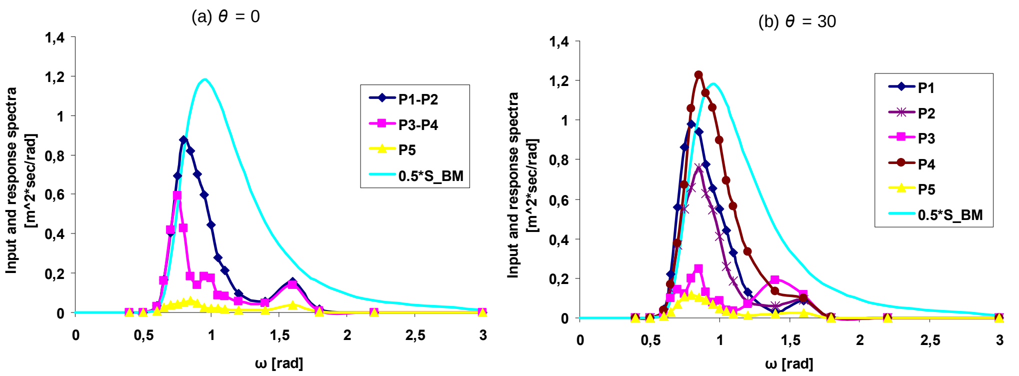

Figure 2. Input spectrum and response spectra at 5 selected points for the two mean wave angles considered

In the first case, we obtain symmetric response spectra with respect to the horizontal axis, since the directional spreading function is symmetric about the wave angle of zero degrees and therefore the effect of oblique wave angles is balanced. Moreover, larger responses are observed in the front and rear end compared to the middle parts. In the second case, the spectra are not symmetric, and the largest response is obtained at the upper corner of the rear end, corresponding to the point P4. This is due to the fact that in this case the directional spectrum includes a larger number of wave angles that trigger the twisting vibration modes of the plate.

In Figure 3, the standard deviation of the vertical displacement is plotted for the two mean wave angles considered. It is shown that larger standard deviations are obtained in the second case, where more waves that approach the weak axis of the plate are included. This effect can only be captured if a directional spectrum is considered. For the example case of mean wave angle of 30°, neglecting the probability of occurrence of larger oblique wave angles would lead to significantly smaller variances.

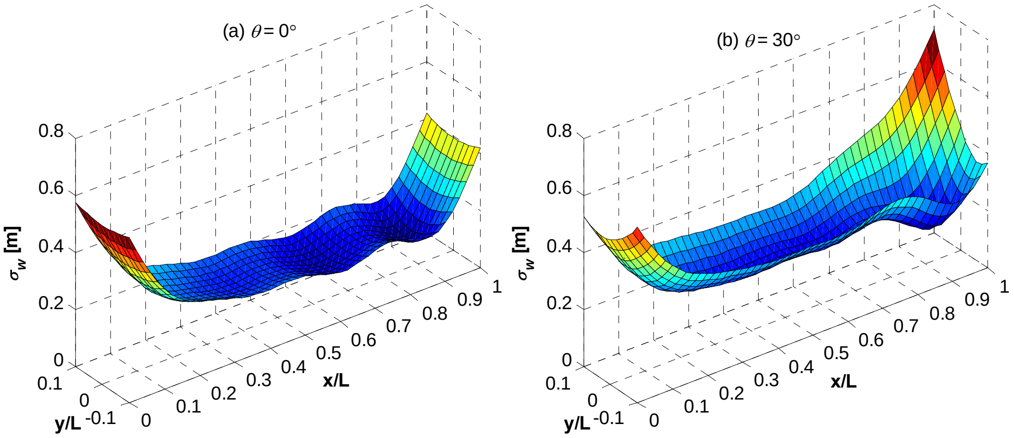

Figure 3. Standard deviation of the response for two different mean wave angles.

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