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Efficiency and Pricing in Combinatorial Auctions and the Impact of Side Constraints

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Abstract

Different pricing rules have been developed and discussed for iterative combinatorial auctions. We analyze some famous combinatorial auction designs concerning efficiency and discuss the advantages and boundaries of their pricing rules.

First we examine the possibility for full efficiency and equilibrium strategies with linear prices. So far, equilibrium strategies have only been found for combinatorial auctions with non-linear and personalized prices for very restricted sets of bidder valuations. We provide an extension of the Combinatorial Clock auction and proof that it actually leads to efficient outcomes in an ex-post equilibrium for general valuations with linear ask prices, which is not obvious given the negative results on linear competitive equilibrium prices in the literature. A theoretical analysis on the worst case efficiency of the Combinatorial Clock auction highlights the problems in valuations, in which the auction is inefficient. We complement the analysis with numerical simulations involving realistic value models which reflect the impact both of our modifications and deviations of our assumptions.

Second we perform a worst case analysis concerning efficiency on the PAUSE format and show how modifications lead to full efficiency. While the computational complexity of determining the winners is shifted to the bidders in that format we show that the bidders' bid complexity becomes $\mathcal{NP}$-hard. Simulation results show advantages over the Combinatorial Clock auction.

Third we analyze how existing combinatorial auctions deal with side constraints. Side constraints have mostly been ignored in the design of iterative combinatorial auctions, but are often requisite for the participants to express preferences and thus for auctions to yield the results envisaged. We define winning and deadness levels as a general pricing rule, which can be used in any auction format independent of the bidding language and side constraints.

We establish positive results concerning efficiency and equilibrium strategies for an auction format that uses deadness levels. A discussion how such ask prices relate to other efficient auction designs reveals that deadness levels have advantages compared to other simpler pricing rules, because they allow for higher ask price increases.

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Chapter 1

Introduction

The fast growing development of the Internet allows for the exchange of complex preference profiles and lays the foundation for the design of new market mechanisms. The promise of these mechanisms is to increase allocative efficiency and thus lead to higher economic welfare by allowing market participants to reveal more comprehensive information about cost structures or utility functions. In recent years, a growing body of literature is devoted to the design of such smart markets (Gallien and Wein, 2005; McCabe et al., 1991), with combinatorial auctions (CAs) emerging as a pivotal example (Cramton et al., 2006b). In CAs, multiple items are sold simultaneously. Nowadays, CAs are being used for the sale of spectrum licenses in Europe and the US (Cramton, 2009a), for transportation (Caplice, 2006), and in industrial procurement (Bichler et al., 2006; Sandholm and Begg, 2006). Much recent research is dedicated to the design and analysis of CAs and respective decision support tools (Adomavicius and Gupta, 2005; Bapna et al., 2007; Bichler et al., 2009; Guo et al., 2007; Scheffel et al., 2011; Xia et al., 2004).

1.1 Research Question

Unfortunately there is no "one size fits all" CA that satisfies all the requirements of real world applications. While researchers were able to design efficient CAs over the last 15 years, these auctions have serious drawbacks in practice. On the other hand the few CAs that have been used in practice have almost no theoretical desirable properties. As a consequence the outcome of such auctions is unpredictable and the strategic considerations for bidders are high as an equilibrium strategy is unknown.

We try to give answers to the following questions:

- What is needed to make practical CAs efficient?
- Can we make efficient CAs more practical?
- Side constraints are almost never considered in the design of CAs but they are indispensable in real worl applications. How do side constraints impact the existing CA designs?
- Can we define generic pricing rules that can both handle side constraints and support efficient CAs with a strong solution concept?

1.2 Research Goals

Finding efficient auctions with strong incentive properties turns out to be very hard for CAs with general valuations. While strategy-proofness might not be possible, researchers have been trying to find iterative CAs which still satisfy a strong solution concept, such as an ex-post equilibrium, for restricted types of valuations. For the design of electronic multi-item markets it is of significant interest whether such auction designs exist at all, and which assumptions they require.

Mathematical models of auctions and markets have often been criticized as unrealistic, as some of the assumptions are too strong and do not hold in practical applications (Rothkopf and Harstad, 1994). For example, the celebrated Arrow-Debreu model assumes continuous, monotonic, and strictly concave utility functions and was heavily criticized for being unrealistic (Georgescu-Roegen, 1979). Existing game-theoretical models of iterative CA formats such as the Ascending Proxy Auction (APA) (Ausubel and Milgrom, 2006a), iBundle(3) (Parkes and Ungar, 2000) or dVSV (de Vries et al., 2007) require also unrealistic assumptions in order to achieve an ex-post equilibrium strategy. Nevertheless, the models are significant contributions to the literature, not necessarily for their immediate practical applicability with human bidders, but because they show under which conditions full efficiency with a strong solution concept can be achieved in an iterative CA, and that this is possible at all.

Therefore we analyze exisiting CA theoretically to better understand their performances in the lab and the field and improve them or at least show what is necessary for full efficiency.

But we need also to aim for satisficing solutions, i.e., auction designs which provide high levels of efficiency even if bidders are restricted in the number of bids that they can reasonably submit. Game-theoretical models of CAs describe situations in which the auction is efficient. They do not provide an understanding of their efficiency when bidders deviate from their equilibrium bidding strategy or use heuristics in package selection. Lab experiments can provide insights into how bidders behave in CAs, but they are costly and limited to a small number of experiments. We argue that in addition to game-theoretical modeling, computational experiments provide an important complement for understanding the robustness of theoretical results on CAs against strong assumptions, which are often required in theory. Such a sensitivity analysis is important for applications in the field and respective systems, but often beyond what can be achieved with formal models. Computational experiments should not be used instead of, but in addition to formal models.

Therefore we ran several computational experiments to analyze the impact of deviations of auction rules and bidding strategies.

We further show the impact of side constraints on existing CAs and develope pricing mechanism that can handle them and even lead to efficient results in an CA framework with a strong solution concept.

1.3 Contribution

We provide new results on two famous CAs and how modifications can lead to full efficiency with a strong solution concept. We show in addition the impact of realistic side constraints also on these formats and propose new pricing rules that can handle those side constraints.

Ausubel et al. (2006) write that "in environments with complementary goods, a clock auction with a separate price quoted for each individual item cannot by itself generally avoid inefficiency." It has been shown that anonymous and linear competitive equilibrium prices are not rich enough to yield efficient outcomes with general biddere valuations (Kelso and Crawford, 1982). For the Combinatorial Clock (CC) auction we make an interesting observation: The ask prices in the CC auction are not necessarily what the winners have to pay, as a winner determination can be performed. In other words, ask prices in the CC auction are not competitive equilibrium prices. This is a way around the negative results on CAs with linear competitive equilibirum prices, which might allow for full efficiency. We show under which circumstances the CC auction can achieve full efficiency and even allow for an ex-post equilibrium strategy.

We first get an understanding, in which situations the CC auction is inefficient, and show that an extended version of the CC auction with an appropriate price update and a Vickrey payment rule (called CC+ auction) can achieve full efficiency with an ex-post equilibrium. Our theoretical analysis sheds light on the reasons for inefficiency in the CC auction and shows that, in contrast to the widespread belief, also linear-price CAs can lead to full efficiency. We do not propose the CC+ auction as a new auction format, but use this term to refer to modifications in the CC auction format, which are necessary for full efficiency.

In our extension of the CC auction, the CC+ auction, bidders submit bids on all packages with a positive valuation in each round as an ex-post equilibrium strategy. Even though there is a strong solution concept and incentives to follow the equilibrium strategy, we can not assume that bidders are able to submit enough bids or follow enough auction rounds, such that the auctioneer can always determine an efficient solution.

Therefore, we also describe the results of computational experiments with different value models from the *Combinatorial Auction Test Suite* (CATS) (Leyton-Brown et al., 2000) and analyze the impact of bidding strategies, which we observed in the lab in a large number of computational experiments. For example, we look at bidders who are limited in the number of bids they can provide in each round, or such who randomly select some packages from those with the highest payoff. We show that all auction formats achieve high levels of efficiency beyond 90% in smaller value models. It is interesting to focus on the comparison of the CC and the CC+ auction and those strategies in which bidders are heavily restricted in the number of bids (up to 10) they can submit in each round. For smaller value models with bidders interested in up to 129 packages, the CC+ auction yields a significantly higher efficiency than the CC auction, beyond 98%. In larger value models with bidders interested in 443 or 32,767 packages, this advantage vanishes and there is no longer a significant difference between the efficiency of the CC and the CC+ auction. Still, the average efficiency in this Real Estate 5x3 model is beyond 92% with restricted bidders. This explains the high efficiency results observed in lab experiments

(Kagel et al., 2010; Porter et al., 2003; Scheffel et al., 2011), but it also highlights that such results do not necessarily carry over to auctions with more than ten items.

The simulations also highlight some virtues of the CC and the CC+ auction compared to non-linear and personalized price auctions such as iBundle. The number of auction rounds of the CC and the CC+ auction are similar, but much lower than those of the iBundle auction. The number of bids submitted in iBundle is orders of magnitude higher than in the CC+ auction, although the efficiency is not worse. This might well make a difference in practical applications. We also simulated the Clock-Proxy auction with similar assumptions, but assumed that bidders submitted bids on all packages with a positive payoff at final clock prices in the Proxy phase. Due to this restriction, the Clock-Proxy auction was not fully efficient. The much larger number of package bids in the Proxy phase led to a modest increase in efficiency, but at the cost of a separate core-selecting auction phase.

Another interesting CA format is PAUSE. There is only little work in the literature on decentralized auctions, therefore, we study PAUSE theoretically and experimentally. Our theoretical analysis shows the growing complexity for the bidders in PAUSE and gives a worst case bound concerning efficiency, if bidders follow a certain strategy. The determination of a lower bound in CAs has to our knowledge not been done and published yet, but it reveals important insights what can go wrong concerning bidder behavior, value models and auction rules. In this context we analyze PAUSE with computational experiments. Mendoza and Vidal (2007, 2008) developed some sophisticated bidding strategies for distributed auctions, however, in our experiments we focus on more simple strategies in which bidders reveal as little as possible about their valuations. Further, we use another value model with more items. To compare and benchmark we run computational experiments with the CC auction, which is known for its sparse need of solving the *Combinatorial Allocation Problem* (CAP) (Porter et al., 2003). With a few modifications also PAUSE can be provable fully efficient but in that case PAUSE reduces to a version of iBundle.

Side constraints are almost always present in real world application, but we do not know their impact on existing CA formats. We show for example that the CC auction and PAUSE have several difficulties to incorporate side constraints in their design, but that iBundle is able to retain its properties even if a certain class of side constraints is required. The simple price update rules in iBundle, APA, or dVSV are the reason for many auction rounds required in these auctions. There is actually no strong reason for unit price updates in these auction formats.

Therefore we describe game-theoretical properties of generic pricing rules, which may serve as a foundation for practical auction designs. We show that CAs with this pricing rule satisfy an ex-post equilibrium. We compare this CA to the family of efficient CAs. This pricing rule complements the existing theory on ascending CAs, as it indicates how high the price increment can be in each round without losing the strong solution concept that these auction formats provide.

Although, we do not attempt to propose a practical auction format, we show that the new pricing rule can actually save auction rounds and reduce the communication effort. This benefit comes at the expense of computational complexity.

In other words we extend the theory on CAs to account for allocation constraints. We describe when bidders have strong incentives to reveal their preferences truthfully, thus leading to less speculation, more predictable outcomes and higher efficiency. This is an important baseline for any practical auction design.

1.4 Outline

This book has the following structure:

- Chapter 2 introduces the relevant theoretical concepts and gives an overview of the relevant CA formats.
- Chapter 3 deals with the Combinatorial Clock auction. We analyze worst case bounds on efficiency, introduce new rules to obtain 100% efficiency with a strong solution concept for general valuations and validate our theoretical results with computational experiments.
- Chapter 4 describes the PAUSE auction format, the advantages of multiple stages, the growing bid complexity and a worst case bound on efficiency. Also here new rules can lead to full efficiency. A comparison in computational experiments with the Combinatorial Clock auction shows the strength and weaknesses of PAUSE.

- In Chapter 5 we analyze the impact of side constraints on the existing auction formats and show the relevance of such constraints for real world applications.

- Chapter 6 presents new pricing rules for iterative CAs which are so general that they can handle different bidding languages and side constraints. We show also that they might be superior to existing pricing rules of efficient auction designs.

- Chapter 7 concludes by summarizing the results of our research and giving an outlook on the future work.

Parts of the following publications[1] have gone directly or indirectly into this work. In particular, parts from Chapter 2 are based on Ziegler (2007). Chapter 3 is based on Bichler et al. (2011), which was written with my colleagues Pasha Shabalin and Martin Bichler. Chapter 4 is based on Ziegler and Scheffel (2011), which was written with my colleague Tobias Scheffel. Chapter 5 and Chapter 6 are based on Ziegler et al. (2011), which was written with my colleagues Ioannis Petrakis and Martin Bichler. Chapter 1 and 2 contain also parts of these works.

[1] some of them are still under review

Chapter 2

Theoretical Background

Over centuries humans are trading goods or services using different kinds of mechanisms. This chapter gives an overview of different market mechanisms, the desired goals and necessary background information for the book at hand.

2.1 Market Mechanism

A *market mechanism* is the process of automatic pricing by the interplay of supply and demand in markets with a variety of suppliers and buyers in the market. The three market factors of supply, demand and price go hand in a close reciprocal relationship, so that changes from one of these factors result in changes in each of the other two factors. A special role in this interplay of market factors has the price, since it balances supply and demand in the market and thus causing an equilibrium state.

The design of market mechanism is an exciting field of economics, mathematics and computer science. It is concerned with the development of rules to control the interactions between suppliers and buyers for a marketplace and to set incentives to promote trading opportunities. This is achieved through the implementation of an overarching structure (design) in which the participants receive an incentive to ensure that they behave according to these rules.

An auction is a special case of a market mechanism with the goal to determine an unknown equilibrium price. Therefore prospective buyers submit their bids for goods or services to the auctioneer. The auction mechanism determines which bids become winning, and defines the payment flows between the participants. Background of this pricing are information asymmetries in the market. A supplier (auctioneer) does not know the buyers willingness to pay. If he sets too high prices, he can not sell his goods. If he sets his prices too low, he does not exhaust the potential revenue. Buyers on the other hand know their respective willingness to pay. In this situation an auction is able to provide a flexible pricing mechanism that in an ideal case results in the sale at the current market price and the optimal allocation of the goods.

Single item auctions are well studied for over 50 years. But due to the development of the Internet and the growing computational power auction designs are possible in which several goods are traded and bidders are able to reveal more complex preference profiles. The promise of these mechanisms is to increase economic welfare by allowing market participants to reveal more comprehensive information about cost structures or utility functions. Figure 2.1 shows the different kinds of multidimensional auctions.

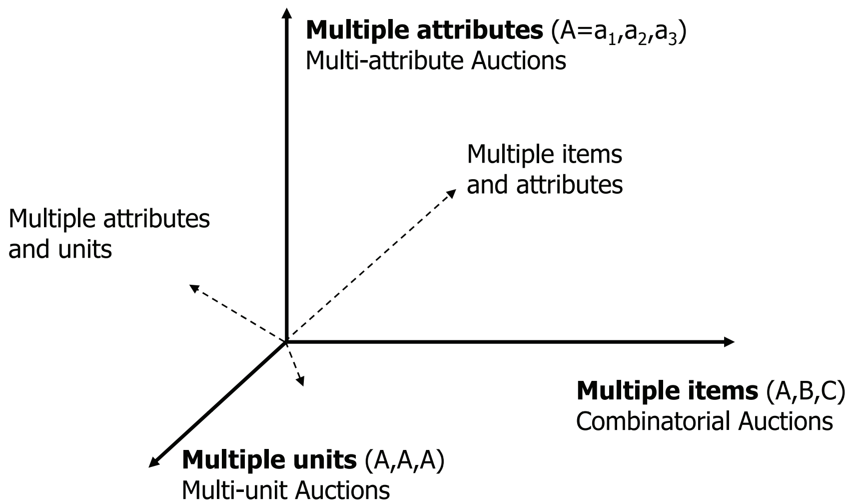

FIGURE 2.1: Multidimensional Auctions (Bichler et al., 2002).

In a *multi-unit auction* several homogenous goods are sold, and the total quantity can be split between winners. Items in a *multi-attribute auction* have important attributes other than only the price. Bidders need to specify values for each attribute, which can include product properties as well as conditions of the transaction. In a combinatorial auction many heterogenous goods are traded simultaneously. We focus on that kind of auctions in the following and give a detailed description and definition in Section 2.5.

2.2 Relevant Gametheoretical Concepts

Coalitional game theory and equilibrium theory are strongly related to the theory of auction design. There are many aspects in these theories that help to understand the architecture and outcomes of auctions. Game theory as well as auction theory study a system of self-interested players/bidders in conditions of strategic interaction. The theory of the *core* in the coalitional game can be transferred to the auction theory as bidders are players and the auctioneer is either one of them or the bank etc. As in game theory bidders follow strategies which may result in desired equilibria. The auction design with its rules may restrict bidders to certain strategies to end up in a certain outcome. To know the strategies and its weaknesses is an important research field for bidders and auctioneers not to end up with undesired results. Certain strategies can lead to efficient outcomes and an auction design goal is to emphasize these strategies as bidders shall bid straightforward (truthful demand revelation in response to prices). This section states the relevant concepts from game- and equilibrium theory and shows the relations between them.

2.2.1 Solution Concepts

Definition 1. *A solution concept is a formal rule for predicting how a game will be played.*

We basically look at three different solution concepts:

- **Dominant Strategy Equilibrium.** *Each player's strategy is a best response, regardless of the strategies of the other players. It is robust to uncertainty about the strategies adopted by the other players and their private information.*

- **Ex-Post Nash Equilibrium.** *Each player's equilibrium strategy remains an equilibrium even after learning the realization of each player's private information. It is robust to the distribution of private information.*

- **Bayesian Nash Equilibrium.** *Each player plays a best response to the strategies of the other players. Best responses are evaluated after a player learns his private information, but before he learns the private information of the other players.*

Although the *Nash Equilibrium* is a fundamental concept in game theory which states that in an equilibrium every player selects the payoff maximizing strategy given the strategies of the other players. It makes strong assumptions on the knowledge of informations about the other players and loses its advantages in games with multiple equilibria (Parkes, 2001).

Example 1. *Two radio stations (a and b) have to choose formats for their broadcasts. There are three possible formats: Pop Music, Rock Music or News. The disjoint audiences for the three formats are 50%, 30%, and 20%, respectively. If they choose the same formats they will split the audience for that format equally, while if they choose different formats, each will get the total audience for that format. Audience shares are proportionate to payoffs. The payoffs (audience shares) are in Table 2.1: There are two Nash-Equilibria - the*

		b		
		Pop	*Rock*	*News*
a	*Pop*	$25, 25$	$\mathbf{50, 30}$	$50, 20$
	Rock	$\mathbf{30, 50}$	$15, 15$	$30, 20$
	News	$20, 50$	$20, 30$	$10, 10$

TABLE 2.1: Multiple equilibria.

upper middle cell and the middle-left one, in both of which one station chooses Pop and gets a 50 market share and the other chooses Rock and gets 30. But it does not matter which station chooses which format. The total payoff is the same in both cases, namely 80. Both are efficient, in that there is no larger total payoff than 80.
Multiple Nash-Equilibria creates a danger. The danger is that both stations will choose the more profitable Pop format - and split the market, getting only

25 each. Actually, there is an even worse danger that each station might assume that the other station will choose Pop, and each choose Rock, splitting that market and leaving each with a market share of just 15.
More generally, the problem for the players is to figure out which equilibrium will in fact occur. In still other words, a game of this kind raises a coordination problem: how can the two stations coordinate their choices of strategies and avoid the danger of a mutually inferior outcome such as splitting the market?

This raises the question whether there exist an appropriate solution concept and which one is it. A Bayes Nash Equilibrium resigns the assumption of full information. Thus it is an extension of the Nash equilibrium to games with incomplete information. Each player plays a best response to the strategies of the other players. Best responses are evaluated after a player learns his private information, but before he learns the private information of the other players. Hence, the players strategy maximizes his expected utility given his private information, the joint distribution of others private information, and the strategies of the other players. Private information is drawn from a common joint distribution and beliefs about strategies are consistent.

Definition 2. *(Bayes Nash Equilibrium) A strategy profile* $s^* = (s_1^*, ..., s_n^*)$ *is a Bayes Nash equilibrium iff*

$$E\left[u_i(s^*, t)\right] \geq E\left[u_i(s_i^{'}, s_{-i}^*, t)\right] \quad \forall s_i^{'}, t, i \tag{2.1}$$

The informational assumptions in a Bayesian equilibrium are that every bidder $i \in \mathcal{I}$ knows the probability of other bidders being a particular type t. However, there is a huge number of types which makes the analysis of Baysian equilibria in complex auctions as CAs almost impossible.

A stronger solution concept is the one of a dominant strategy equilibrium. A dominant strategy is given if a player follows the same payoff maximizing strategy independently from the strategies of other players. Mechanisms with a dominant strategy equilibrium are called strategy proof, since no assumptions about the information available to players about each other are made, and every player selects his own optimal strategy without requiring the others to act rational.

Definition 3. *(Dominant strategy equilibrium) A strategy profile* $s^* = (s_1^*, ..., s_n^*)$ *is a dominant strategy equilibrium iff*

$$u_i(s_i^*, s_{-i}, t) \geq u_i(s_i^{'}, s_{-i}, t) \quad \forall s, t, i \tag{2.2}$$

The classical example of a dominant strategy is the game of the prisoners' dilemma.

Example 2. *The classical prisoners' dilemma is as follows:*
Two suspects, a and b, are arrested. The police has insufficient evidence for a conviction, and, having separated both prisoners, visit each of them to offer the same deal: if one testifies for the prosecution against the other and the other remains silent, the betrayer goes free and the silent accomplice receives the full ten-year sentence. If both stay silent, both prisoners are sentenced to only three years in jail for a minor charge. If each betrays the other, each receives a seven-year sentence. Each prisoner must make the choice of whether to betray the other or to remain silent. However, neither prisoner knows for sure what choice the other prisoner will make. So this dilemma poses the question: How should the prisoners act? If you knew the other prisoner would stay silent,

		Prisoner b	
		betray	*silent*
	betray	$\mathbf{-7,-7}$	$0,-10$
Prisoner a	*silent*	$-10,0$	$-3,-3$

TABLE 2.2: Prisoners' Dilemma.

your best move is to betray as you then walk free instead of receiving the minor sentence. If you knew the other prisoner would betray, your best move is still to betray, as you receive a lesser sentence than by silence. Betraying is a dominant strategy. The other prisoner reasons similarly, and therefore also chooses to betray. Yet by both defecting they get a lower payoff than they would get by staying silent. This demonstrates that a dominant strategy equilibrium need not be a Pareto optimum viz. need not support an efficient allocation.

Theorem 1. *Every dominant strategy equilibrium is also a Nash-Equilibrium.*

Proof. The proof follows from the definitions of both equilibria directly. □

A third type of equilibrium is the most relevant one for our analysis of iterative CAs - the *ex-post Nash equilibrium.*

Definition 4. *(Ex-post Nash equilibrium) A strategy profile $s^* = (s_1^*, ..., s_n^*)$ is an ex-post Nash equilibrium iff the utility functions u_i apply to*

$$u_i(s^*, t) \geq u_i(s_i^{'}, s_{-i}^*, t) \quad \forall s_i^{'}, t, i$$

Ex-post Nash assumes that the strategies are known, but there is uncertainty about the other bidders' types. In other words, truthful bidding in every round of an auction is an ex-post (Nash) equilibrium, if for every bidder $i \in \mathcal{I}$, if all other bidders follow the truthful bidding strategy, then bidder i maximizes his payoff in the auction by following the truthful bidding strategy independent of the type t of other bidders (Mishra and Parkes, 2007).

Ex-post equilibria in particular avoid speculation about other bidders' valuations and could therefore reduce the strategic complexity for bidders considerably, leading to higher efficiency, and also an increased adoption of iterative CAs. Note that this is weaker than a dominant strategy equilibrium, where bidders do not have to speculate about other bidders' valuations and strategies. In contrast to dominant strategy and ex-post equilibria, Bayes-Nash equilibria do always exist, but they require bidders to speculate on both, the type and the strategy of others. We refer to dominant and ex-post equilibria as *strong solution concepts*. For iterative CAs we focus on ex-post equilibria, as preference elicitation in an indirect mechanism typically does not allow for dominant strategy equilibria (Conitzer and Sandholm, 2002).

2.2.2 The Core

Another auction design goal is to end up with an outcome, no coalition is willing to renege once the result is announced - the core outcome.

Some further notation is required. There is a set $\mathcal{K}$ of m indivisible items indexed with k, which are auctioned among n bidders. Let $i, j \in \mathcal{I}$ denote the bidders and $v_i : S \to \mathbb{R}$ denote a value function of bidder i, which assigns a real value to every subset $S \subseteq \mathcal{K}$ of items. An allocation $X \in \Gamma$ of the m items among bidders is $X = \{X_1, ..., X_n\}$, with $X_i \cap X_j = 0$ for every $i \neq j$. X_i is the package of items assigned to bidder i. The social welfare of an allocation X is $\sum_{i \in \mathcal{I}} v_i(X_i)$, and an efficient allocation X^* maximizes social welfare among all allocations X, such that $\forall X, \sum_{i \in \mathcal{I}} v_i(X_i^*) \geq \sum_{i \in \mathcal{I}} v_i(X_i)$. Let $p_i(S)$ denote the ask price on package $S \subseteq \mathcal{K}$ by bidder i viz. the price bidder i has to pay for package S. By $\Pi(X, \mathcal{P})$ the auctioneer revenue/payoff at the current allocation X and prices $\mathcal{P}$ is denoted and $\pi_i(S, \mathcal{P})$ denotes the payoff to bidder i on package S with:

$$\pi_i(S, \mathcal{P}) = v_i(S) - p_i(S) \tag{2.3}$$

Definition 5 (Coalitional Value Function)**.** *The coalitional value function* $\mathfrak{w}$: $\mathfrak{P}(\mathcal{I}) \to \mathbb{R}$ *is defined as:*

$$\mathfrak{w}(C) = \begin{cases} \max\limits_{X\in\Gamma} \sum\limits_{i\in C} v_i(S_i) & \text{, } \textit{if auctioneer } \in C \subseteq \mathcal{I} \\ 0 & \text{, } \textit{else} \end{cases} \tag{2.4}$$

Subsets C of $\mathcal{I}$ are called coalitions.

The *coalitional value function* $\mathfrak{w}$ is zero if the auctioneer is not part of the coalition C, otherwise it is defined as the maximum total value created from the trade among these bidders and the auctioneer.

The overall utility or payoff, which the bidders and the auctioneer must share, is $\mathfrak{w}\left(\mathcal{I}\right) \geq \Pi + \sum_{i\in\mathcal{I}} \pi_i$. Obviously, every bidder i and the auctioneer want their portion to be as big as possible. A kind of Nash-Equilibrium is a payoff vector (Π, π), which cannot be improved for all bidders and auctioneer simultaneously, thus cannot be dominated.

Theorem 2. *All payoff vectors (Π, π), which satisfy the inequation*

$$\mathfrak{w}\left(C\right) \leq \Pi + \sum_{i\in C} \pi_i \quad \forall C \subseteq \mathcal{I} \tag{2.5}$$

are not dominated.

Proof. : A dominated payoff vector must satisfy (particularly for a coalition C) the requirement

$$\mathfrak{w}\left(C\right) > \Pi + \sum_{i\in C} \pi_i. \tag{2.6}$$

This does not apply to the payoff vectors, which satisfy inequation 2.5. □

Definition 6 (The Set of Core Payoffs)**.** *The set of core payoffs is defined as:*

$$Core\left(\mathcal{I}, \mathfrak{w}\right) = \left\{ (\Pi, \pi) : \Pi + \sum_{i\in\mathcal{I}} \pi_i = \mathfrak{w}\left(\mathcal{I}\right), \mathfrak{w}\left(C\right) \leq \Pi + \sum_{i\in C} \pi_i \left(\forall C \subset \mathcal{I}\right) \right\} \tag{2.7}$$

If any payoff vector (Π, π) is not in the core, then there is a coalition C for which the total payoff $\mathfrak{w}\left(C\right)$ is higher than the members' total payoffs in (Π, π). So there is some way to share the difference so that all members of C are strictly better off. In this case the payoff vector (Π, π) is said to be blocked by coalition C.

Remark 1. *Note the dominant strategy equilibrium in Example 2 is not in the core, since coordinating their testimonies, the prisoners could be better off and thus blocking the outcome illustrated.*

Definition 7 (Competitive Equilibrium, CE)**.** *Prices $\mathcal{P}$ and allocation $X^* = (S_1^*, ..., S_n^*)$ are in competitive equilibrium if:*

$$\pi_i\left(S_i^*, \mathcal{P}\right) = \max_{S \subseteq \mathcal{K}} \left[0, v_i\left(S\right) - p_i\left(S\right)\right] \qquad \forall i \in \mathcal{I} \tag{2.8}$$

$$\Pi\left(X^*, \mathcal{P}\right) = \max_{X \in \Gamma} \sum_{i \in \mathcal{I}} p_i\left(S_i\right) \tag{2.9}$$

In CE the allocation X^* maximizes the payoff of every bidder and the auctioneer at the given prices $\mathcal{P}$. The auction ends effectively because bidders are not willing to change the allocation by submitting any further bids. Allocation X^* is said to be supported by prices $\mathcal{P}$ in CE.

Theorem 3. *: If allocation X is supported in a competitive equilibrium then it is an efficient allocation.*

A proof of Theorem 3 can be found in Bikhchandani and Ostroy (2006). CE prices always exist, as prices $p_i = v_i$ satisfy the CE conditions.

Theorem 4. *:* (Π, π) $\in$*Core*$(\mathcal{I}, \mathfrak{w})$ *if and only if there exists prices $\mathcal{P}$ such that constraints 2.8 and 2.9 are satisfied.*

Theorem 4 was shown by Bikhchandani and Ostroy (2002) and states that there is an equivalence between the core of the coalitional game and the set of CE prices. All core outcomes can be priced, and all CE outcomes are in the core.

Theorem 4 is only valid in the case of one auctioneer/seller and invalid for combinatorial exchange markets where more than just one auctioneer/seller is involved. In that case only the direction that CE prices correspond to core payoffs can be proven but not vice versa (Bikhchandani and Ostroy, 2002).

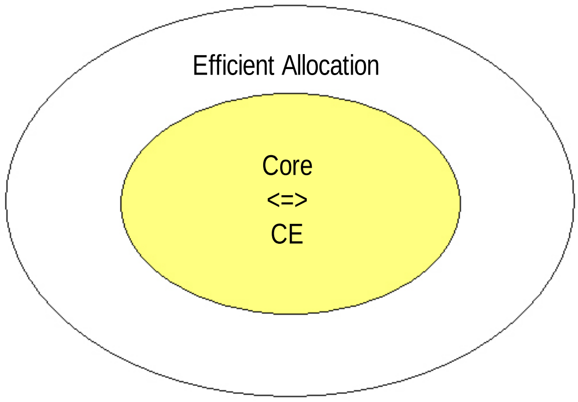

FIGURE 2.2: Venn-Diagram: Relation between Competitive Equilibrium, Core Outcome and Efficient Allocation.

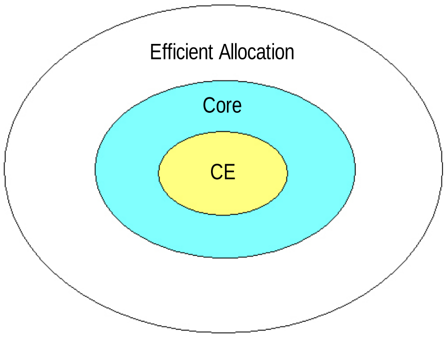

FIGURE 2.3: Venn-Diagram (general case): Relation between Competitive Equilibrium, Core Outcome and Efficient Allocation.

2.3 Design Goals

In auction theory the following four traditional design goals are relevant for the design of an auction. Unfortunately it is shown that not all of these goals can be fulfilled in one CA design.

- **Efficiency.** *In an efficient auction, the social welfare is maximized.*
- **Strategy Proofness.** *In an auction with strategy proofness, misreporting one's valuation for items never gives an advantage.*
- **Core Property.** *Given the prices, no coalition of bidders and the auctioneer can form a mutually beneficial renegotiation among themselves.*

- **Individual Rationality.** *Each bidder expects a non–negative payoff for participating.*

Knowing that an auction mechanism has a certain property would help participants to decide on their response strategies and provide information about the bidding process and the expected results. For example, in an auction with strategy proofness such as the *Vickrey-Clarke-Groves (VCG)* mechanism a wise bidder would truthfully report his valuation for packages as this is the dominant strategy for each bidder. As for an auction with core property, it would not be a good idea to resell winning items in a subgroup of participants aiming at a better payoff.
Furthermore, several auction designs can achieve an equilibrating status as a result of gaming. This property is interesting because participants of an auction are able to predict the result of their bidding and the possible actions of other bidders, which helps them to decide on their individual strategies.

The single-item ascending Clock auction (aka. Japanese auction) achieves all these four desirable economic properties. It is individually rational, efficient, strategy-proof, and the payoff vector is in the core. When all bidders know their private valuations, truthfully revealing one's demand is a dominant strategy. It would be desirable to achieve such properties for CA designs as well. Unfortunately the VCG design is the unique mechanism that satisfies individual rationality, efficiency, and strategy-proofness (Ausubel and Milgrom, 2006b). However, its results can be outside the core, which leads to a number of problems in practical settings (Ausubel and Milgrom, 2006b; Rothkopf, 2007).

Although package bidding in CAs yield higher levels of efficiency in the lab in the case of complementarities compared to simultaneous auctions without package bids, equilibrium strategies are unknown for many CA formats used in the field. The exponential number of possible package bids leads to high strategic complexity for bidders and the bidding strategies observed in the lab are diverse, with some bidders bidding on many and others bidding on only a few packages of interest in each round (Goeree and Holt, 2010; Scheffel et al., 2011). Finding efficient CA designs which satisfy a strong game-theoretical solution concept can help reduce the strategic complexity and lead to higher efficiency as a result. Even if the assumption for such equilibrium strategies is not given in particular applications, it helps to understand the sources of inefficiency observed in the lab or in the field.

In the case in which the allocation rule seeks to maximize efficiency, the payments that correspond to an incentive compatible bidding strategy are the famous VCG payments. This statement is proved by the following theorem, first appeared in Holmstrom (1979):

Theorem 5. *Suppose that for each bidder his set of types, θ^i is smoothly path connected and that for each decision outcome X, the value to bidder i of outcome X, $v_i(X_i, t_i)$ is differentiable in its second argument. Then any efficient, incentive compatible direct mechanism is a VCG mechanism.*

While this initially appears to be the silver bullet for the design of CAs, VCG mechanisms turned out to be impractical in most applications (Ausubel and Milgrom, 2006b; Rothkopf, 2007). For situations with multiple items but unit demand (Demange et al., 1986) and for multiple homogeneous goods with marginal decreasing values (Green and Laffont, 1979; Holmstrom, 1979), it has been shown that there are generalizations which can be used to implement efficient, strategy-proof mechanisms.

2.4 Challenges

Besides the traditional design goals there is a set of additional challenges an auction design has to face. In most cases those challenges are direct consequences of bidders' irrationality and the complexity of the real world. For example, in real applications it is possible that bidders only have limited budgets and fail to bid at their true valuations. Several common problems are listed in the following.

- **Exposure Problem.** *In case of complementarities, bidders run the risk of winning only a part of a complementary collection of items in an auction without package bids.*
- **Threshold Problem.** *Allowing package bids may favor bidders seeking larger packages, because small bidders do not have the incentive or capability to top the tentative winning bid of the large bidder.*
- **Coordination Problem.** *Even if small bidders have the incentive and capability to top the tentative winning bid of the large bidder, they still need to coordinate their bids in order to outbid the larger bidder.*

- **Tacit Collusion.** *Bidders use signalling such as jump bidding to cooperate in an auction.*
- **Budget Binding.** *Bidders can be influenced by budget-constrained bidders who e.g. bid for items directly at their budgets and cause high prices for every bidder.*
- **Parking.** *In order to maintain eligible bidders temporarily bid for packages they are not interested in.*
- **Waivers.** *In some auctions bidders are allowed to withdraw their bids, which may lead to more complex situations.*
- **Hold Up.** *Bidders signalize their strong willingness of winning an item and reselling it afterwards to prevent others from competing with them.*

It is hardly possible for an auction mechanism to avoid all those problems, but normally some of them can be diminished or eliminated in one design. For example, all mechanisms that allow for package bidding are supposed to solve the exposure problem, since bidders are able to express their complementary or substitute valuations for items. In some designs, e.g. the CC auction, prices are increased by the auctioneer at a certain amount, so that tacit collusion by jump bidding is prevented. However, if bidders use other methods for signalling like bidding for a group of specific items to coordinate with other bidders, CC is not able to avoid this kind of tacit collusion. Some of those problems are direct consequences of specific auction designs. For example, parking is a unique problem in auctions with activity rules or waivers exist only in auctions allowing bidders to withdraw their bids.

2.5 Combinatorial Auctions

In Cramton et al. (2006b) CAs are referred to as those auctions in which bidders can place bids on combinations of items, called packages, rather than just on individual items. If we consider this as definition of CAs we are removing from consideration the *Simultaneous Ascending Auction (SAA)* (Cramton et al., 1998) which is widely used in practice to allocate collections of items. Since we intend to use in our analysis these and similar auction formats we consider in this book the following definition of CAs:

Definition 8. *A combinatorial auction is a mechanism by which m items are allocated among n players, called bidders, who have values for subsets of items and there exists at least one bidder i such that $v_i(S) \neq \sum_{k \in S} v_i(k)$ for some $S \subseteq \{1, \ldots, m\}$. The allocation is done in such a way to optimize some relevant performance measure or objective.*

The performance measurements encountered in the literature as well as in practical applications are efficiency, i.e., finding an allocation which maximize social welfare, or, equivalently, the sum of the estimates of the bidders true values, and sellers revenue. We concentrate on efficiency in this book.

The enlargement of allowing to bid on packages also increases strategic and computational complexity. The three main problems of CAs are the following:

- A large challenge for CAs is the **Combinatorial Allocation Problem (CAP)**, also referred to as the *Winner Determination Problem (WDP)*, which computes an allocation of packages to bidders that maximizes the auctioneer revenue. The CAP can be interpreted as a weighted set packing problem (Lehmann et al., 2006) and thus is $\mathcal{NP}$-complete (Rothkopf et al., 1998). Therefore no polynomial time algorithm can be expected for solving this problem. It can be mathematically formulated as an integer problem:

$$\max_{x_i(S)} \sum_{S \subseteq \mathcal{K}} \sum_{i \in I} x_i(S) v_i(S)$$

$$s.t.$$

$$\begin{aligned} \sum_{S:k \in S} \sum_i x_i(S) &\leq 1 & \forall k \in \mathcal{K} \\ x_i(S) &\in \{0,1\} & \forall i, S \end{aligned}$$

  If package S is allocated to bidder i, $x_i(S)$ equals 1, otherwise 0. The value of the objective function is the maximum of the auctioneer's revenue. In the CAP a set of items is to be allocated across a set of bidders.

  In Figure 2.4 are nine items pictured as black squares and three bidders. The framed items illustrate packages with values to bidder i as shown. There are no other valued packages as these ones. For example bidder 1 values the package identified (a) with $v_1(a) = 2$ and (b) with $v_1(b) = 8$. The optimal solution, in which the total value across all bidders is maximized, is to allocate package (a) to bidder 1 and package (e) to bidder 3, with no allocation to bidder 2, for a total of 11.

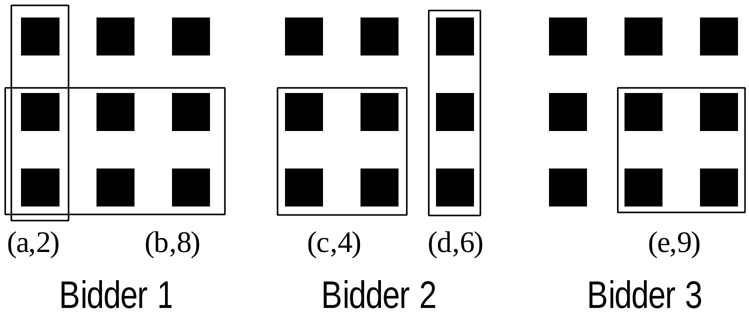

FIGURE 2.4: An example of the combinatorial allocation problem.

The CAP in which $v_i(S) = \sum_{k \in S} v_i(k)$ for all i and all $S \subseteq \{1, \ldots, m\}$ is obviously equivalent to solve to m independent, single-item auctions, a topic well researched and well understood. See Milgrom (2004) and references within for an account of the main results in single-item auction theory.

- The **Preference Elicitation Problem (PEP)** includes the *valuation problem*, in which a bidder has to figure out, which packages he is interested in, and value these from an exponential set of possible packages. Which of the $2^m - 1$ packages should a bidder bid on? Furthermore, various new possibilities of bidding strategies occur. What is the best strategy respectively the optimal bid price?
- The problem of **Communication Complexity** is closely related to PEP and deals with the question, how many bids must be submitted to the auctioneer to calculate an efficient allocation. As communication might be of exponential size, sensible *bidding languages* may address this problem by providing a compact representation of the bidder's preferences.

The challenge of the auction mechanism is to make it as simple as possible concerning computational, strategic, valuation and communication complexity without compromising on desired economic properties (such as efficiency).

As indicated in Definition 8 CAs cannot offer a better expressiveness of pure additive valuation functions. They make only sense, if one of the following conditions hold for at least one bidder i:

- **Complementarities** (aka *super-additivity*):

$$v_i(S \cup T) > v_i(S) + v_i(T) \qquad S, T \subset \mathcal{K}, S \cap T = \emptyset \tag{2.10}$$

- **Substitutabilities** (aka *sub-additivity*):

$$v_i(S \cup T) < v_i(S) + v_i(T) \qquad S, T \subset \mathcal{K}, S \cap T = \emptyset \tag{2.11}$$

Condition 2.10 appears if two items or packages are more worth than the sum of them alone, e.g. a bidder wants to purchase his first computer. He needs the computer plus a monitor, as they are just working together and singly being useless to the bidder. Condition 2.11 describes the opposite, e.g. a bidder is interested in buying *one* TV set and auctioned are many. He certainly values a package of many TV sets lower than the sum of the single values.

2.5.1 Assumptions

Achieving efficiency on markets when economic agents strategically pursue their individual self-interest is a fundamental problem in Economics. General equilibrium models show that in classical convex economies with multiple products, the Walrasian price mechanism verifies the efficiency of a proposed allocation (Arrow and Debreu, 1954) while communicating as few real variables as possible (see Mount and Reiter (1974) and Hurwicz (1977)). Furthermore, Jordan (1982) shows that the Walrasian mechanism is a unique voluntary mechanism with this property. However, these results assume that all production sets and preferences are convex and do not apply to non-convex economies with indivisible goods, such as CAs.

Bikhchandani and Mamer (1997) show that without convexity assumptions full efficiency cannot be achieved with linear CE prices for general valuations (see Nisan and Segal (2006) for an overview).

The following assumptions hold for our analysis of CAs if not otherwise stated:

- **independent private valuations**: The values of each bidder do not depend on the private information of the other bidders;
- **free disposal**: The value function satisfies $v(S \cup T) \geq v(S)$ for all combinations S and T. In particular, disposing an item from a combination cannot increase the combination value;
- **quasilinear utility**: The utility or payoff of any bidder i on a package S is given by $\pi_i(S) = v_i(s) - b_i(S)$;

- **zero auctioneer valuations**: The auctioneer values all items at zero. His revenue is the total payment he receives at a price;
- **items are indivisible**;

2.5.2 Performance Measures

The next two definitions give *performance measures*, which can be used to analyze an auction outcome.
Allocative efficiency can be measured as the ratio of the value of the final allocation X to the value of the efficient allocation X^* (Kwasnica et al., 2005).

Definition 9 (Allocative Efficiency). *Allocative efficiency is defined as:*

$$E\left(X\right) := \frac{\sum_{i\in\mathcal{I}} v_i\left(\bigcup_{S\subseteq\mathcal{K}:x_i(S)=1} S\right)}{\sum_{i\in\mathcal{I}} v_i\left(\bigcup_{S\subseteq\mathcal{K}:x_i^*(S)=1} S\right)} \tag{2.12}$$

Note that the efficiency depends only on the final allocation, and not on the final prices.

Another measure is the *revenue distribution*, which reveals in what way the overall gain is distributed between the bidders and the auctioneer. If the auction terminates with an inefficient allocation a part of the overall gain is lost. The revenue distribution calculates the fraction the auctioneer gains compared to the overall gain the final allocation X creates.

Definition 10 (Revenue Distribution). *Revenue distribution is defined as:*

$$R\left(X\right) := \frac{\sum_{S\subseteq\mathcal{K}}\sum_{i\in\mathcal{I}} x_i\left(S\right)p_i\left(S\right)}{\sum_{i\in\mathcal{I}} v_i\left(\bigcup_{S\subseteq\mathcal{K}:x_i^*(S)=1} S\right)} \tag{2.13}$$

Thus it is possible to have two auction outcomes with significantly different auctioneer revenues while the allocative efficiency is the same.

Figue 2.5 gives an overview how the social welfare can be split among the participants.

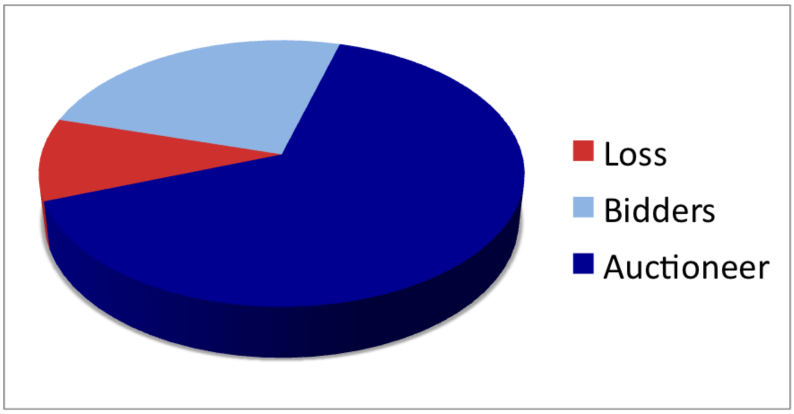

FIGURE 2.5: Revenue Distribution among bidders, auctioneer and losses.

2.5.3 Bidding Languages

This section addresses the issue of the representation of bids in CAs. Every implementation of a CA must determine the formal specifications on how bidders submit their bids to the auctioneer.

Formal specifications particularly define how bids can be expressed in terms of logical connectives. The space of possible bids in CAs is usually huge, as large as the space of possible valuations. Specifying a valuation in a CA of m items, requires providing a value for each of the possible $2^m - 1$ non-empty subsets. A naive representation would thus require $2^m - 1$ real numbers to represent each possible bid. The main aim of a bidding language is to be both expressive and simple. When attempting to choose one there is always a trade off between expressiveness and simplicity. On the one hand the language should express important valuations well, and on the other hand it should be as simple as possible.
The reader is referred to Nisan (2006) for a detailed description. For the work at hand it is sufficient to discuss two different basic bidding languages - OR and XOR.

- **OR-Bids**: Each bidder can submit an arbitrary number of bids. Implicit here is that he is willing to obtain any number of disjoint bids for the sum of their respective prices. Not all valuations can be represented in the OR-bidding language. It is easy to verify that the following proposition completely characterizes the descriptive power of OR-bids.

Proposition 1. *OR-bids can represent all bids that do not have any substitutabilities, i.e., those where for all $S \cap T = \emptyset$ the inequality*

$$v(S \cup T) \geq v(S) + v(T)$$

holds, and only them.

- **XOR-Bids**: Each bidder can submit an arbitrary number of bids. Implicit here is that he is willing to obtain at most one of these bids.

  Proposition 2. *XOR-bids can represent all valuations.*

  XOR-bids can represent everything that can be represented by OR-bids, as well as some valuations that cannot be represented by OR-bids − those with substitutabilities. Yet, the representation need not be succinct: There are valuations that can be represented by very short OR-bids and yet the representation by XOR-bids requires exponential size.

Proposition 3. *Any additive valuation on m items can be represented by OR-bids of size m. The simple additive valuation requires XOR-bids of size 2^m.*

2.5.4 Iterative Combinatorial Auctions

Iterative combinatorial auctions (ICAs) allow bidders to submit multiple bids during an auction and provides iteratively information feedback on the ongoing bidding process, in order to assist and guide bidders in expressing and finding their valuations on the items auctioned. ICAs have several advantages over sealed-bid auctions. They are by now the most promising way of addressing the PEP. In contrast to sealed-bid auctions such as the VCG auction bidders do not need to submit the entire valuation function at once and thus are not forced to compute the entire valuations before the auction starts. ICAs allow bidders to provide informations in an incremental way, they only need to reveal partial and indirect information about their valuations. During the auction process bidders learn about other bidders valuations, which may help defining the own and may produce better results in case of correlated values (Milgrom and Weber, 1982). Besides, *transparency* is another practical concern in CAs. It is important for bidders to verify and validate the outcome of an auction. Especially for losing bidders it is desirable to understand why

they lost.
Though all these possible advantages favor ICAs, they offer also new opportunities to bidders for manipulation. That is why the biggest challenge in the ICA design is to ensure focused bidding without allowing bidders to compromise the economic goals of efficiency. ICA designs are confronted with the following challenges:

- Use a sensible **ask price calculation** to guide the bidder;
- Mitigate the **threshold problem**;
- Prevent **collusive bidding** and signaling through code or jump bidding;
- Ensure auction progress by **activity rules** or sensible improvement margins;
- Ensure **termination** but avoid premature termination;
- Diminish the possibility to delay bidding until late in the auction by using sensible **eligibility rules**;
- Use a simple but **expressive bidding language** to lower the communication costs;

In this book we concentrate and analyze the three different ICAs depicted in Table 2.3:

Auction	Type	Price Feedback	Bidding language
CC	centralized	linear, anonymous	OR/XOR
PAUSE	decentralized	none	OR
iBundle	centralized	non-linear, personalized	XOR

Table 2.3: Overview of the three main ICAs analyzed in this book.

2.5.4.1 Typical Process in an ICA

The typical process in an ICA is illustrated in Figure 2.6. Over a predefined period of time, usually called an auction round, the following three steps are gone through:

- bid submission;
- bid evaluation;
- feedback to the bidders;

The information feedback to the bidders includes usually the provisional allocation and possibly a price feedback which can be used as minimal ask prices for the next round to guide the bidders through the auction. Bid improvement rules may also require a minimal percentage improvement over the current highest bid to either ensure or speed up auction progress. The auction terminates either at a fixed time or after a certain stopping rule is satisfied (e.g. no new bids are submitted). For a detailed description of the design space for ICAs the reader is referred to Parkes (2006).

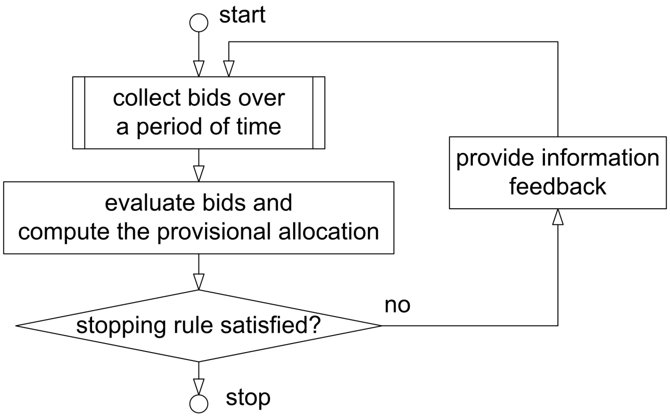

FIGURE 2.6: Process of an Iterative Combinatorial Auction (Bichler et al., 2009).

Auction designs in the field of ICAs can basically be broken down into *price-based* and *non priced-based* approaches. Whereas price-based auctions provide ask prices as information feedback to coordinate the bidding process, non price-based auctions however do not require that bidders submit bids in response to ask prices. Their underlying structure is fundamentally different and their elicitation process is based on richer query models. In general, there are three classes of non price-based approaches (cf. Parkes (2006) for further information):

- **decentralized approaches**: The CAP is moved to the bidders, who are responsible for submitting bids and also allocations of items respectively packages with a high revenue given existing bids. PAUSE (cf. Chapter 4) is such an auction.

- **proxy auctions**: Proxy agents provide an interface between bidders and the auction. Bidders provide incremental value information to the proxy agents and the agents submit bids on behalf of the bidders through a predetermined bidding procedure. Proxy agents may query bidders actively.

- **direct elicitation approaches**: The auctioneer formulates explicit queries, and a bidder's strategy determines how to respond to these queries.

2.5.4.2 Priced-based ICAs

Using price-based approaches, in which the auctioneer provides ask prices to coordinate the bidding process, the questions arises, what pricing schemes do exist and how can these prices be calculated. The following pricing schemes have been discussed in the literature:

Definition 11 (Linear and Anonymous Prices). *A set of prices $p_i(S), i \in \mathcal{I}, S \subseteq \mathcal{K}$ is called:*

- *linear (or additive), if*

$$\forall i, S : p_i(S) = \sum_{k \in S} p_i(k) \tag{2.14}$$

- *anonymous, if*

$$\forall i \neq j, S : p_i(S) = p_j(S) \tag{2.15}$$

Prices are *linear* if the price of a package is equal to the sum of prices of its items, and prices are *anonymous* if prices of the same package are equal for every bidder. *Non-anonymous* ask prices are also called *discriminatory* or *personalized* prices. A combination results in four possible sets of ask prices:

- a set of linear and anonymous prices $\mathcal{P} = \{p(k)\}$

- a set of linear and personalized prices $\mathcal{P} = \{p_i(k)\}$
- a set of non-linear and anonymous prices $\mathcal{P} = \{p(S)\}$
- a set of non-linear and personalized prices $\mathcal{P} = \{p_i(S)\}$

Many ICAs are designed to converge to CE prices or even to a minimal CE price set. To terminate with minimal CE prices is a desirable property, since they correspond to VCG payments for a restricted class of valuations. Termination with CE prices that support VCG payments brings straightforward bidding in an ex-post equilibrium (Parkes, 2006). Besides, implementing minimal CE prices avoids the problems of the VCG auction when VCG payments are not supported by minimal CE prices (Ausubel and Milgrom, 2006a). A minimal CE price set is equivalent to a *bidder-optimal core outcome.*

Definition 12 (Minimal Competitive Equilibrium Prices)**.** *Minimal CE prices minimize the auctioneer revenue* $\Pi\left(X^*, P\right)$ *on the efficient allocation across all CE prices.*

Minimal CE prices can be derived by solving the dual of the relaxation of the CAP:

$$\begin{array}{rcllr}
\min\limits_{p(i),p(k)} & \left(\sum\limits_i p(i) + \sum\limits_k p(k)\right) & & & \\
s.t. & & & & \textbf{(CAP-DLP)} \quad (2.16) \\
p(i) + \sum\limits_{k\in S} p(k) & \geq & v_i(S) & \forall i, S & \\
p(i), p(k) & \geq & 0 & \forall i, k &
\end{array}$$

The values of the dual variables quantify the monetary cost of not awarding the item to whom it has been provisionally allocated, viz. $p(k)$ can be interpreted as anonymous and linear prices, the term $\sum\limits_{k\in S} p(k)$ is the price on package S and

$$p(i) := \max_S \left\{ v_i(S) - \sum_{k\in S} p(k) \right\}$$

is the maximal payoff of the bidder i at the prices $p(k)$.
The relaxation might cause an overestimation of the item prices, since the feasible polytope may increase. It is sufficient and almost necessary that valuations satisfy the *goods are substitutes* condition, so that exact linear CE prices exist and the relaxed CAP equals CAP (Kelso and Crawford, 1982).

Definition 13 (Goods are Substitutes, GAS). *Goods are substitutes when increasing the price of one item does not reduce the demand for the other item.*

However this condition precludes the possibility of items with *complementary values*, thus is very restrictive and rare in real-world scenarios as most known applications of CAs rather deal with *complementary goods*. CE prices must be sometimes non-linear and non-anonymous.

Bikhchandani and Ostroy Bikhchandani and Ostroy (2002) show that minimal CE prices provide an upper bound on VCG payments and in special cases when *bidders are substitutes* is given they are equivalent.

Definition 14 (Bidders are Substitutes Condition, BAS). *For any subset of bidders $C \subseteq \mathcal{I}$, $\mathfrak{w}(C)$ equals the value of the efficient allocation for $CAP|_C$. This amount would be the social surplus if only the bidders in C were present. The bidders are substitutes condition requires:*

$$\mathfrak{w}(\mathcal{I}) - \mathfrak{w}(\mathcal{I} \setminus C) \geq \sum_{i \in C} [\mathfrak{w}(\mathcal{I}) - \mathfrak{w}(\mathcal{I} \setminus i)] \qquad \forall C \subseteq \mathcal{I} \tag{2.17}$$

If BAS fails, the VCG payments are not supported in any price equilibrium and truthful bidding is not an equilibrium strategy.

A number of ICAs require a slightly stronger condition to terminate with minimal CE prices.

Definition 15 (Bidder Submodularity Condition, BSM). *BSM requires that for all $C \subseteq C' \subseteq \mathcal{I}$ and all $i \in \mathcal{I}$ there is:*

$$\mathfrak{w}(C \cup \{i\}) - \mathfrak{w}(C) \geq \mathfrak{w}(C' \cup \{i\}) - \mathfrak{w}(C') \tag{2.18}$$

If BSM holds, ascending price CAs implement VCG payments and thus minimal CE prices Ausubel and Milgrom (2006a).
GAS valuations implies BSM and is almost necessary. And clearly a BSM coalitional value function also satisfies BAS.

As a result it can be stated that if one bidder does not satisfy GAS, BSM is not necessarily satisfied and thus implementation of minimal CE prices cannot be guaranteed.

So far, the APA, iBundle(3), and the dVSV auction are the only known ICAs which achieve full efficiency for restricted types of bidder valuations. If the coalitional value function satisfies the BSM condition, straightforward bidding is a best-response strategy which leads to an ex-post equilibrium and the auction results in the VCG outcome (Ausubel and Milgrom, 2002). Straightforward bidding means that bidders only bid on those packages that maximize their payoff based on current ask prices in each round. These auction formats are based on non-linear and personalized prices and can be modeled as an algorithm (primal-dual or subgradient) to solve the corresponding linear program. We refer to these auction formats as *non-linear personalized price auctions (NLPPAs)* in the following.

If the bidders' valuations in an NLPPA are not buyer submodular, bidders have an incentive to shade their bids and not follow the straightforward strategy. Even if bidders knew that their valuations are buyer submodular and they would not need to speculate about other bidders' types, it is not obvious that other bidders are able to follow the straightforward strategy in such an environment. Both computational and lab experiments have illustrated the large number of auction rounds necessary for these NLPPAs (Schneider et al., 2010), in which nearly all valuations have to be elicited to achieve efficiency.

As an alternative, linear-price CAs have been suggested resembling the fictitious Walrasian tâtonnement. Linear prices are desirable for their simplicity and the reduced communication complexity in real world applications. One line of research is based on a restricted dual of the relaxed CAP, in which the pseudo-dual variables are used as ask prices in the auction (Bichler et al., 2009; Kwasnica et al., 2005; Rassenti et al., 1982). Fluctuations of the ask prices and the complexity of the ask price calculation are problems of this approach for some applications.

In contrast, Porter et al. (2003) suggest a simple mechanism with ascending linear ask prices, called the CC auction. This mechanism has achieved high levels of efficiency in the lab (Kagel et al., 2010; Porter et al., 2003; Scheffel et al., 2011) and has a number of obvious advantages. It maintains strictly ascending, linear ask prices, and limits the computational burden on the auctioneer as he only has to solve the $\mathcal{NP}$-complete CAP in the last rounds if there is excess supply. Also, the information revelation between rounds makes it quite robust against collusion and limits the bidder's possibilities for signaling. For these reasons, the Netherlands and the UK have recently started to use a version of the CC auction for price discovery in the sale of spectrum licenses (Cramton, 2009a). It is also being used in electricity markets and other high-stakes auctions, in which anonymous linear prices are often an important requirement (Cramton et al., 2006a). Unfortunately, no equilibrium strategy is known, and it is unclear for bidders which strategy they should follow.

2.5.5 Simultanous Ascending Auction

The simultaneous ascending auction (SAA) (Cramton et al., 1998) was first developed for the U.S. Federal Communications Commission's (FCC) spectrum auctions and has subsequently been adopted with slight modifications for many spectrum auctions worldwide.

Items are auctioned simultaneously during discrete rounds, where no sale takes place until the bidding is concluded on all items. The auction proceeds with an unspecified number of bidding rounds. At the end of each round, the highest bid becomes the leading bid, and the results are made available to all bidders before the start of the next round. At the end of the last round the leading bidder on each item is designated the sole winner on that item. Cramton et al. (1998) provides a detailed description.
Key aspects are (Kelly and Steinberg, 2000):
Activity Rules: *Eligibility Requirements*: Each round, a bidder is designated active on a particular item, if either he has the leading bid from the previous round or has submitted an acceptable improving bid in the current round. Bidders are required to remain active on items covering an amount which is at least A_i percent of the total amount for which they wish to remain eligible to bid. These eligibility requirements are intended to thwart the *deception effect*, whereby a bidder might bid cautiously, waiting to see how the others bid while not revealing his own interests until late in the auction.
Bid Increments: In order to be acceptable, a bid must improve the previous leading bid by at least the specified minimum amount set by the auction authority. This helps maintain the speed of the auction.
Bid Waivers: The activity rules are balanced by an allocation of a small number of *bid waivers* to each bidder to be used at will to maintain eligibility for a round without meeting the eligibility criteria. Bid waivers may be viewed as an effort to increase bidder flexibility.
Bid Withdrawals: A leading bidder is permitted to withdraw his bid during the course of the auction, but is penalized by being required to pay the difference between his bid and the price for which the package is ultimately sold; a winning bidder withdrawing after the close of the

auction suffers an extra penalty. *Bid withdrawals* may be viewed as an effort to reduce the *exposure risk* to bidders attempting to realize their synergies.

The process yields a CE in simple settings, but when competition is weak it suffers from collusive bidding strategies. Besides if some items are complements the SAA has to deal with the *exposure problem*, which occurs when a bidder wins some, but not all, of a complementary collection of items in an auction without package bids.

2.5.6 Vickrey-Clarke-Groves Auction

This section gives a brief overview of the *generalized Vickrey auction* (aka the *Vickrey-Clarke-Groves (VCG) mechanism*), its auction rules, advantages and disadvantages. For further description and proofs the reader is referred to Ausubel and Milgrom (2006b).
The generalized Vickrey auction is a *sealed-bid auction* (aka one-shot auction), which means bidders submit their bids on all packages S, they are interested in, at once and the auctioneer calculates the allocation X and the prices $\mathcal{P}$. Thus after one round the auction terminates.

Auction rules: The bidders generating the highest overall revenue win. Thus the efficient allocation is calculated by the CAP. Winning bidders pay what they bid, but receive a *VCG discount*:

$$p_i(S_i^*) = b_i(S_i^*) - (\mathfrak{w}(\mathcal{I}) - \mathfrak{w}(\mathcal{I} \setminus \{i\})) \tag{2.19}$$

where $(\mathfrak{w}(\mathcal{I}) - \mathfrak{w}(\mathcal{I} \setminus \{i\}))$ denotes the VCG discount and $b_i(S)$ represents the bid price submitted by bidder i on package S. The difference $b_i(S_i^*) - (\mathfrak{w}(\mathcal{I}) - \mathfrak{w}(\mathcal{I} \setminus \{i\}))$ is denoted as the *VCG payment.*

Example 3. *: $\mathcal{K} = (A, B)$ and $\mathcal{I} = (1, 2)$*
*The efficient allocation is indicated by the *, bidder 1 receiving item (A) and*

	A	B	AB
b_1	8^*	9	12
b_2	6	8^*	14

Table 2.4: VCG mechanism.

bidder 2 item (B). Bidder 1 has to pay a price of $p_1(A) = [8-((8+8)-14)] = 6$ *for (A) and bidder 2 a price of* $p_2(B) = [8-(16-12)] = 4$ *for (B).*

Advantages:

- Truthful reporting in the VCG mechanism is a dominant strategy, as the bidder do not have to pay their bid prices. Bidding below the valuations makes no sense as the risk increases not winning the package, while no better chances of a higher payoff is given. Bidding above the valuations makes also no sense, since a bidder could end up with a negative payoff.

- Assuming bidders report their true valuations, the outcome is efficient, since the auctioneer knows all valuations and can thus compute the efficient allocation with CAP.

- Losing bidders pay zero.

Theorems by Green and Laffont (1979), Holstrom (1979) show that, under weak assumptions, the VCG mechanism is the unique mechanism with these three properties Ausubel and Milgrom (2006b).

Disadvantages: Probably the most important disadvantage of the VCG mechanism is that auctioneer revenues can be very low or even zero, even when the item that were sold are valuable. This problem is related to the VCG discount. The following example illustrates the problem:

Example 4. *:* $\mathcal{K} = (A, B)$ *and* $\mathcal{I} = (1, 2)$
*The efficient allocation is indicated by the *, bidder 1 receiving item (A) and*

	A	B	AB
b_1	8^*	0	10
b_2	0	8^*	10

Table 2.5: VCG mechanism: auctioneer revenue.

bidder 2 item (B). Bidder 1 has to pay a price of $p_1(A) = [8-((8+8)-10)] = 2$ *for (A) and bidder 2 a price of* $p_2(B) = [8-(16-10)] = 2$ *for (B). Resulting in an auctioneer revenue of 4. If both bidders would value the package (AB) with just 8 the auction would terminate with the same efficient allocation but the auctioneer revenue would decrease to zero.*

Another eminent defect of the auction design is the *monotonicity problem*, that occurs when increasing competition by adding bidders might cause a reduction of the auctioneer revenue.

Example 5. *:* $\mathcal{K} = (A, B)$ *and* $\mathcal{I} = (1, 2, 3)$
Adding bidder 3 reduces auctioneer revenues from 10 to 4. This is problematic

	A	B	AB
b_1	0	0	12
b_2	10	10	10
b_3	10	10	10

Table 2.6: VCG mechanism: monotonicity problem.

in two ways. First, the seller might seek to exclude bidder 3 or disqualify his bid. Thus the auctioneer has to be trusted. Second, bidder 2 could profitably sponsor a fake bidder 3, thus the auction is vulnerable to shill bidding and collusion.

Other problems are a high valuation complexity since complete information about all packages is required, computing the VCG payments is $\mathcal{NP}$-hard and transparency about the dominant strategy is often not given to bidders.

Although the disadvantages of the VCG mechanism are eminent, it is nevertheless an important theoretical construct that provides insight into fundamental properties of auction mechanisms in general. VCG auctions are often used as reference point to derive meaningful statements about other auction designs.

2.5.7 iBundle/ Ascending Proxy Auction

iBundle uses personalized and non-linear prices. It calculates a provisional revenue maximizing allocation at the end of every round and increases the prices based on the bids of non-winning (unhappy) bidders. Parkes and Ungar (2000) suggest different versions of iBundle called iBundle(2), iBundle(3), and iBundle(d). iBundle(3), which will be used in this book, maintains personlized package prices throughout the auction. That means in every round the prices for every unhappy bidder are increased by the increment ϵ for every package on which he has submitted a bid. The auction terminates if no bidder is unhappy[1]. For a detailed description we refer to (Parkes and Ungar (2000)). iBundle has some desirable properties as efficiency and a strong solution concept when the BSM condition holds.

The *ascending proxy auction (APA)* has been proposed in the context of the FCC spectrum auction design (Ausubel and Milgrom, 2006a). The APA is similar to the *iBundle* design by Parkes (2001), except from the fact that it emphasizes proxy agents, which essentially lead it to a sealed-bid auction format.

Ausubel and Milgrom concentrate on the case of transferable utility. In particular, they focus their attention on an APA in which winning bidders pay what they bid and losing bidder pay zero. Each proxy bidding strategy is to bid the minimum increment straightforwardly, i.e., to select the package that has the highest potential payoff and bid the minimum increment on it. They state:

Theorem 6. *In the transferable utility model, the payoff imputation determined by the APA is a core imputation with respect to the reported preferences.*

This theorem tells little about the effciency of the outcome. In fact, only in the case in which bidders report truthfully their preferences to the proxy agent the auctions outcome is efficient. And only in the case in which the BSM condition is satisfied truthful reporting is an ex -post Nash equilibrium strategy profile of the APA. In the general case, Ausubel and Milgrom prove the following

Theorem 7. *In the transferable utility model, given any pure strategy profile for the other bidders, bidder i has a best reply that is a profit-target strategy.*

A profit-target or semi-sincere strategy is a strategy in which bidder i reports a value for each S equal to $\max\{0, v_i(S) - \pi_i\}$, i.e., each package true value is reduced by the same amount π_i , which is the bidders minimum profit target. Although of theoretical interest this result is of limited use in practice since in order to compute his optimal profit target a bidder needs to have full information about other bidders valuations. Ausubel and Milgrom proves also the following

[1] To assure that every bidder is happy at termination bidders are able to bid a zero amount on the empty package ($b_i(\varnothing) = 0$), which can be allocated to them.

Theorem 8. *In the transferable utility model, for every bidder-optimal point π in the core, the strategy profile in which each bidder i plays its π_i profit-target strategy is a Nash equilibrium with associated profit vector π.*

Chapter 3

Combinatorial Clock Auction

The Combinatorial Clock (CC) auction has become very popular for its simplicity and for its highly usable price discovery, derived by the use of linear prices. Unfortunately, no equilibrium bidding strategies are known. Given the importance of the CC auction in the field, it is highly desirable to understand whether there are efficient versions of the CC auction, providing a strong game theoretical solution concept. So far, equilibrium strategies have only been found for CAs with non-linear and personalized prices for very restricted sets of bidder valuations. We provide an extension of the CC auction, the CC+ auction, and show that it actually leads to efficient outcomes in an ex-post equilibrium for general valuations with only linear ask prices. We also provide a theoretical analysis on the worst case efficiency of the CC auction, which highlights problems in the valuations, in which the CC is very inefficient. As in all other theoretical models of CAs, bidders in the field might not be able to follow the equilibrium strategies suggested by the game-theoretical predictions. Therefore, we complement the theoretical findings with results from computational experiments using realistic value models. This analysis helps to understand the impact of deviations from the equilibrium strategy and the robustness of such auctions. The experimental analysis shows that the CC auction and its extensions have a number of virtues in practical applications, in particular a low number of auction rounds and bids submitted compared to auction designs with non-linear and personalized ask prices.

Apart from a few lab experiments, little theoretical research has focused on the CC auction as of yet. Ausubel et al. (2006) argue that anonymous and linear prices are not generally rich enough to yield efficient outcomes. The arguments are based on Ausubel and Milgrom (2002), who show that with linear prices bidders have an incentive to engage in demand reduction to favorably impact prices, which implies that the auction outcome is not fully efficient. Therefore, the version of the CC auction used for spectrum auctions in Europe and the Clock-Proxy design extend the clock auction by an additional phase, in which sealed bids can be submitted (Ausubel et al., 2006; Cramton, 2009a) and a payment rule is defined with the intention of providing incentives for truthful bidding. So far, no formal equilibrium analysis for such two-phased auctions has been available and the theoretical efficiency results only consider the auction format in the second phase, where the bids are typically restricted by an activity rule and the bids submitted in the Clock phase.

3.1 Related Theory and Definitions

So far, only ICA designs with non-linear and personalized prices have been shown to be fully efficient. Bikhchandani and Ostroy (2002) prove that only with personalized non-linear prices does a CA always achieves a CE.

In most practical applications of ICAs, linear and anonymous ask prices are essential. For example, day-ahead markets for electricity sacrifice efficiency for the sake of having linear prices (Meeus et al., 2009). Also, the main auction formats which have been used or discussed for selling spectrum in the USA use linear prices (Brunner et al., 2010). The CC auction is probably the most widespread ICA format, but the negative results by Gul and Stacchetti (1999) seem to indicate that there is no hope of making the CC auction fully efficient for general valuations.

A notable difference between the CC auction and auctions with pseudo-dual linear prices or the efficient ICAs (APA, iBundle, dVSV), is that bidders need not pay the ask prices of the final round. The winner determination in the final round can select a bid and the corresponding ask price from a previous round, so that there is a distinction between final ask prices and payments. This distinction opens up the possibility of achieving efficiency with linear ask prices and a strong game-theoretical solution concept for general valuations in the CC auction. The latter is important, as any restriction on the valuations is typically unknown.

Definition 16. *Final ask prices are the ask prices of the last round of an iterative auction.*

Definition 17. *A payment is the amount of money a bidder has to pay for his winning items.*

We show conditions, under which the CC auction with linear ask prices satisfies an ex-post equilibrium for general valuations and provide sensitivity analysis to understand, how robust the CC auction is against violations of these conditions. A CA design needs to elicit at least all losing bids, in order to be fully efficient. We first show problems of the CC auction with the straightforward bidding strategy, which is typically being assumed in related equilibrium analyses. Then we adapt the rules of the CC auction to avoid such efficiencies and derive a bidding strategy, which leads to an ex-post equilibrium.

We focus on linear-price CAs, in which an ask price p_k for each of the m items is available; the price of a package S is the sum of the prices of the items in this package. We assume that the *demand* of each bidder are the packages which maximize his utility.

Definition 18. *(Blumrosen and Nisan, 2007) For a given bidder valuation v_i and given item prices $p_1, ..., p_m$, a package $T \subseteq \mathcal{K}$ is called a demand of bidder i if for every other package $S \subseteq \mathcal{K}$ we have that $v_i(S) - \sum_{k \in S} p_k \leq v_i(T) - \sum_{k \in T} p_k$.*

A feasible allocation X and a price vector p_k are in CE when the allocation maximizes the payoff of every bidder and the auctioneer given the prices. A *Walrasian equilibrium* can then be described as a vector of item prices.

Definition 19. *A Walrasian equilibrium is a set of nonnegative prices $p_1, ..., p_m$ and an allocation X if for every bidder i, X_i is the demand of bidder i at those prices and for any item k that is not allocated $p_k = 0$.*

Simple examples illustrate that Walrasian equilibria do not exist for general valuations in CAs if goods are indivisible; in other words, for certain types of bidder valuations it is impossible to find linear CE prices which support the efficient allocation X^* (Blumrosen and Nisan, 2007). Let us assume that bidder 1 has a value of 10 for the items (1), (2), and also for the package (1, 2), and bidder 2 has only a positive valuation of 12 for the package, but not for the singletons. The optimal allocation is to allocate the package (1, 2) to bidder 2. The prices will be 10 for each item, otherwise bidder 1 would demand one of the items, and consequently 20 for the package (1, 2). Bidder 2 will, however, not demand the package at a price of 20, and no equilibrium exists.

The economic GAS property is a sufficient condition for the existence of Walrasian equilibrium prices (Kelso and Crawford, 1982). Later, Gul and Stacchetti (1999) proved that for all bidders it is almost necessary that GAS to ensure efficiency with linear CE prices. Intuitively, this property implies that every bidder continues to demand the items which do not change in price, even if the prices on other items increase. Overall, the GAS condition is very restrictive as most known practical applications of CAs deal more with complementary goods.

Actually, Gul and Stacchetti (2000) show that even if bidders' valuation functions satisfy the goods are substitutes condition, no ascending CA exists that uses anonymous and linear prices and arrives at the VCG solution. This means that bidders may have an incentive to demand smaller packages of items in order to lower their payments.

3.2 The CC Auction

We concentrate on the CC auction as introduced by Porter et al. (2003) and give a precise description in Algorithm 1. Prices for all items are initially zero. In every round bidders identify a package of items, or several packages, which they offer to buy at current prices. If two or more bidders demand an item then its price is increased by a fixed bid increment in the next round. It is schematically illustrated in Figure 3.1. This process iterates. The bids

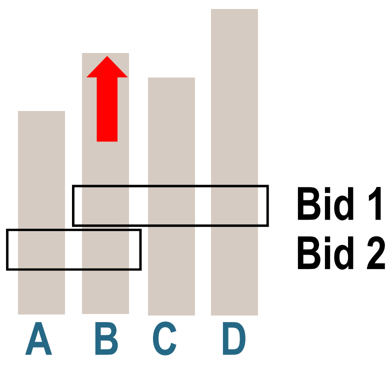

FIGURE 3.1: Overdemand and price increase in the CC auction.

which correspond to the current ask prices are called *standing*, and a bidder is standing if he has at least one standing bid. In a simple scenario in which supply equals demand, the auction terminates and the items are allocated according to the standing bids. If at some point there is excess supply for at least one item and no item is over-demanded, the auctioneer determines the winners to find an allocation of items that maximizes his revenue by considering all submitted bids. If the solution displaces a standing bidder, the prices of items in the corresponding standing bids rise by the bid increment and the auction continues. The auction ends when no prices are increased and bidders finally pay their bid prices for winning packages. We analyze a version that uses an XOR bidding language.

3.2.1 Efficiency of the CC Auction

We analyze the worst-case efficiency of the CC auction with bidders following the straightforward strategy, which is typically assumed in game-theoretical models of ICAs. We also evaluate a powerset strategy, which describes the situation in which bidders reveal all packages with a positive valuation at the current prices. We draw on this strategy in subsequent sections.

Definition 20. *A straightforward bidder bids only for his* demand *in each round at the current ask prices* $p_1, ..., p_m$.

Note that a straightforward bidder might bid on several packages in a round if they apply to the definition of demand (cf. Definition 18).

Definition 21. *The powerset bidder bids on all packages* S *with a non-negative value* $v_i(S) - \sum_{k \in S} p_k \geq 0$ *at the current set of ask prices* $p_1, ..., p_m$.

We show that if all bidders follow the straightforward strategy, the efficiency of the CC auction can be as low as 0%. For this, we refer to a recent theorem by Kagel et al. (2010) on the efficiency of auctions which maximize the auctioneer's revenue based on bid prices.

A *standard* package auction is defined such that it selects an allocation $\overline{X}$ to maximize the auctioneer's revenue $\overline{X} \in argmax_X \sum_{i \in \mathcal{I}} b_i(X_i)$ and has bidder i pay $b_i(\overline{X}_i)$. $b_i(X_i)$ denotes the highest price that i bids for a package X_i during the course of the auction.

A standard package auction can be modeled as a cooperative game with transferable utility, in which the payoff vector or imputation is given by the auctioneer's revenue $\Pi = \sum_{i \in \mathcal{I}} b_i(X_i)$, and bidder i's payoff $\pi_i = v_i(X_i) - b_i(X_i)$.

Data: package bids $b_i(S)$
Result: allocation $\overline{X}$ and prices $p_i(\overline{X}_i)$
initialization
  for *k=1* **to** m **do** $p_k \leftarrow 0$
  for *i=1* **to** n **do** $X_i \leftarrow \emptyset$
repeat
    overdemand $\leftarrow FALSE$; *oversupply* $\leftarrow FALSE$
    for *i=1* **to** n **do**
      bidders submit bids $\beta_i(S)$
    for *k=1* **to** m **do**
      if ≥ 2 *bidders* $i \neq j$ *demand item* k **then**
        $p_k \leftarrow p_k + \epsilon$
        overdemand $\leftarrow$ TRUE
      end
      if *item* k *is not part of a bid* $p_i(S)$ **then**
        oversupply $\leftarrow$ TRUE
      end
    if overdemand = *TRUE* **then** exit iteration
    else if oversupply = *FALSE* **then** exit loop
    else
      for *k=1* **to** m **do**
        Assign $p_i(S)$ with $k \in S$ to the set of standing bids $\mathcal{B}$
        Calculate $\overline{X}$ based on all bids submitted in the auction
          if *a bidder holding a bid in* $\mathcal{B}$ *is displaced, i.e. no bid by this bidder is in* $\overline{X}$*,* **then**
            foreach *item* k *which was displaced:* **do**
              $p_k \leftarrow p_k + \epsilon$
            end
          else $\overline{X}$ is the final allocation
      end
until *stop*

Algorithm 1: CC auction.

The value of a coalition including the auctioneer and the bidders in $C \subseteq \mathcal{I}$ is $\mathfrak{w}(C) = \sum_{i \in C} v_i(X_i^*|_C)$.

Theorem 9. *(Kagel et al., 2010) In a standard package auction, let $\mathcal{B}$ denote the set of final bids and $\overline{X}$ the final allocation in the auction. If for all bidders i, $v_i(X_i) - b_i(X_i) \leq v_i(\overline{X}_i) - b_i(\overline{X}_i)$, then the allocation $\overline{X}$ is efficient: $\sum_{i \in \mathcal{I}} v_i(\overline{X}_i) = \mathfrak{w}(\mathcal{I})$. If the efficient allocation is unique, then the condition*

$v_i(X_i) - b_i(X_i) \leq v_i(\overline{X}_i) - b_i(\overline{X}_i)$ is necessary as well as sufficient for $\overline{X}$ to be efficient.

To promote these results, the auction mechanism must encourage bidders to bid aggressively all the way up to their full values ($b_i(X_i) = v_i(X_i)$) for *efficiency-relevant packages*, i.e., packages that may become winning packages.

3.2.2 Worst-case Efficiency with Straightforward Bidders

	$b_{(1)}$	$b_{(2)}$	$b_{(3)}$	...	(1)	(2)	(3)	...	(1, 2)	(1, 3)	...
v_1					10*						
v_{2_a}						4*			10		
v_{2_b}									10		
v_{3_a}							4*			10	
v_{3_b}										10	
...								...			...
$r = 1$	1	1	1	...	1_1				$2_{2_a,2_b}$	$2_{3_a,3_b}$	...
$r = 2$	2	2	2	...	2_1				$4_{2_a,2_b}$	$4_{3_a,3_b}$	...
$r = 3$	3	3	3	...	3_1				$6_{2_a,2_b}$	$6_{3_a,3_b}$	...
$r = 4$	4	4	4	...	4_1				$8_{2_a,2_b}$	$8_{3_a,3_b}$	...
$r = 5$	5	5	5	...	5_1				$10_{2_a,2_b}$	$10_{3_a,3_b}$	...
$r = 6$	6	6	6	...	6_1						
$r = 7$	7	6	6	...	7_1						
...											
$r = 10$	10	6	6	...	10_1						

TABLE 3.1: Example of a demand masking set of bidder valuations and CC auction process assuming straightforward bidders.

If a bidder follows the straightforward strategy in the CC auction, he does not bid on all relevant packages in the course of the auction. The example in Table 3.1 illustrates a characteristic situation that we refer to as *demand masking set*. The upper part of the table describes valuations of $2m - 1$ bidders for m items, while the lower part shows both ask prices for items and corresponding package bids in individual rounds r. The indices of the bid prices for different packages indicate which straightforward bidder submits the bid on the respective package. There is one bidder called bidder 1 and for each $h \in \{2, ..., m\}$ there are two bidders h_a and h_b. Bidder 1 values item ($\mathtt{1}$) at a value of 10 and does not value any other item. For $h = 2, ..., m$, bidders h_a and h_b value the package ($\mathtt{1}$,h) at 10 and bidders h_a the item (h) at 4, and are not interested in any other package. Without loss of generality, we assume a bid increment of 1. Straightforward bidders h_a and h_b demand the package ($\mathtt{1}$,h) until round 6, at which point they demand nothing. After round 6 there is excess supply and the auctioneer solves the winner determination problem, which displaces the sole remaining standing bidder, who bids on item ($\mathtt{1}$). Thus the price on item ($\mathtt{1}$) further increases until bidder 1 wins item ($\mathtt{1}$) in round 10, and the auction terminates with a social surplus of 10. However, the efficient allocation assigns item ($\mathtt{1}$) to bidder 1, and item (h) to bidder h_a for a social welfare of $10 + 4(m - 1)$. $10/(10 + 4(m - 1))$ converges to 0 as m approaches infinity.

We provide a formal definition of a demand masking set and derive a worst-case bound for these situations as a function of m.

Definition 22. *A* demand masking *set of bidder valuations is given if the following properties are fulfilled. There is a set of bidders $\mathcal{I}$ with $|\mathcal{I}| \geq 3$, a set of items $\mathcal{K} = \{1, ..., m\}$ with $T \subseteq \mathcal{K}$ and a partition $\mathcal{H}$ of $\mathcal{K} \backslash T$. Let S_h be the elements of $\mathcal{H}$ with $h \in \{2, ..., |\mathcal{H}| + 1 = g\}$. For each S_h there are two bidders h_a and h_b. Bidder 1 values package T with ξ. For $h \in \{2, ..., g\}$ bidders h_a value the packages S_h with ν_h and $T \dot{\cup} S_h$ with μ and bidders h_b value only package $T \dot{\cup} S_h$ with μ. No bidders are interested in the other packages, i.e., the marginal value of winning any additional item to the positively valued packages is zero.*

	T	$\{S_h\}$	$\{T \dot{\cup} S_h\}$
v_1	ξ	0	ξ
$\{v_{h_a}\}$	0	ν_h	μ
$\{v_{h_b}\}$	0	0	μ

Table 3.2: Demand masking set of bidder valuations.

Note that the valuations of zero as shown in Table 3.2 do not need to be strictly zero, but rather sufficiently small so as not to influence the economy.

Theorem 10. *If bidder valuations are demand masking and all bidders follow the straightforward strategy in the CC auction, then the efficiency converges to* $\frac{2}{m+1}$ *in the worst case.*

Proof. The following proof is provided for two or more items for sale and $2m-1$ bidders. With less than $2m-1$ bidders and XOR bidding efficiency can only increase. Without loss of generality, we assume item-level bid increments of $\epsilon = 1$ in each round $r \in \mathcal{R} \subset \mathbb{N}$. We consider the value μ as given and determine ξ and ν_h such that efficiency decreases to the worst case of 0%.

case a) $\mu \geq \xi + \sum_h \nu_h$:

The efficient solution is to sell $T \cup S_h$ to one of the bidders h_a or h_b. The CC auction terminates with the efficient outcome in this case.

case b) $\mu < \xi + \sum_h \nu_h \wedge \xi = \mu$:

The proof is by showing that a straightforward bidder h_a cannot bid on S_h throughout the auction in a demand masking set of valuations. For this, the payoff $\pi_{h_a}(T\dot{\cup}S_k)$ must be higher than $\pi_{h_a}(S_h)$ for each bidder h_a in each round of the auction $r \in \mathcal{R}$:

$$v_{h_a}(T\dot{\cup}S_h) - p_{h_a,r}(T\dot{\cup}S_h) > v_{h_a}(S_h) - p_{h_a,r}(S_h) \quad \forall h \in \{2, ..., g\}, \forall r \in \mathcal{R} \tag{3.1}$$

Since we know that $v_{h_a}(T\dot{\cup}S_h) = v_{h_b}(T\dot{\cup}S_h) = \mu$, and all bidders bid straightforward, we know that the price for all the items in $\mathcal{K}$ rise in each round by ϵ. Therefore, inequality (1) can be rewritten as

$$\mu - |T\dot{\cup}S_h|t\epsilon > \nu_h - |S_h|t\epsilon \overset{\epsilon=1}{\Longrightarrow} r < \frac{\mu - \nu_h}{|T|} \quad \forall h \in \{2, ..., g\}, \forall r \in \mathcal{R} \tag{3.2}$$

Inequality (2) shows that as long as t is smaller than the right-hand side, a straightforward bidder always bids on the package $T\dot{\cup}S_h$. We can now determine a round $r_{min} = \min\{r | r \geq \frac{\mu - \nu_h}{|T|}, \forall h_a\}$, in which the payoff $\pi_{h_a}(T\dot{\cup}S_h)$ is for the first time smaller or equal to the payoff $\pi_{h_a}(S_h)$. We call r_{min} the *decisive round*. If either the right side or both sides of inequality (1) become negative in round r_{min}, bidder h_a cannot bid on S_h or the auction ends for bidder h_a as also the ask price for $T\dot{\cup}S_h$ is higher than $v_{h_a}(T\dot{\cup}S_h)$. If straightforward bidder h_a does not reveal his preferences for S_h throughout the auction, then the auctioneer selects any of the other bids with a revenue of μ, resulting in an efficiency of $\mu/(\xi + \sum_h \nu_h)$.

We determine maximal ν_h such that in round r_{min} the payoff of bidder h_a on package S_h is negative, which minimizes efficiency. We know that as long as bidder h_a's payoff is negative in the decisive round r_{min}, i.e., $\nu_h - |S_h| r_{min} < 0$, then bidder h_a does not bid on S_h. We also know that $r_{min} = \lceil (\mu - \nu_h)/|T| \rceil$ is the decisive round. We can now maximize ν_h such that $\nu_h - |S_h| \lceil (\mu - \nu_h)/|T| \rceil < 0$, resulting in $\nu_{h_{max}} = \max\{\nu_h | \nu_h < |S_h| \mu/(|T| + |S_h|)\}$. In order to maximize $\sum_h \nu_h$ and so minimize the efficiency $\mu/(\xi + \sum_h \nu_h)$ we set $|T| = 1$ and $|S_h| = 1$ for all $h \in \{2, ..., g\}$. This results in an efficiency of $E(X) = \mu/(\xi + \sum_h (\frac{\mu}{2} - \rho))$ with $\rho > 0$. With $\rho \to 0$ and $\xi = \mu$ efficiency decreases to $2/(g+1)$ which is $2/(m+1)$ in the worst case. Note that it does not matter if ξ is smaller or larger than $\sum_h \nu_h$.

case c) $\mu < \xi + \sum_h \nu_h \wedge \mu \neq \xi$:

Efficiency can only increase compared to case b) considering the worst case. Either the enumerator of $E(X) = \frac{\max\{\xi, \mu\}}{\max\{\xi + \sum_h \nu_h, \mu + \sum_h \nu_h\}}$ increases or the denominator decreases.

- $\xi > \mu$: $\Rightarrow E(X) = \frac{\xi}{\xi + \sum_h \nu_h} = \frac{\mu + \delta}{\mu + \delta + \sum_h \nu_h}$ with $\delta > 0$ is always greater than the efficiency $E(X)$ in case b).
- $\xi < \mu$: $\Rightarrow$
  - either $E(X) = \frac{\mu}{\xi + \sum_h \nu_h}$ which is greater than $E(X) = \frac{\mu}{\mu + \sum_h \nu_h}$ the efficiency of case b).
  - or $E(X) = \frac{\mu}{\mu + \sum_{h=2}^{g-1} \nu_h}$ which is also greater than $E(X) = \frac{\mu}{\mu + \sum_{h=2}^{g} \nu_h}$ the efficiency of case b).

□

In the example of Table 3.1, ν_h is smaller than 5 for all h. With $m = 3$ and $\nu_h = \nu = 5 - \rho$ for all h, efficiency is approximately $50\% = 10/(10 + \nu(m-1))$, which is equal to $2/(m+1)$ in the worst case. Obviously if the number of items m and the corresponding number of bidders increases to fulfill the requirements of a demand masking set, efficiency converges to 0% in the worst case. While such a situation that leads to 0% efficiency can be considered a degenerated case that does not happen that often in practice, we found regular situations in simulations with realistic value models in which the case of $m = 2$ or $m = 3$ occurred, which still leads to efficiencies of 67% or 50% in the worst case. Note that these are not necessarily the only characterizations of value models in which such low efficiency can occur.

3.2.3 Worst-Case Efficiency with Powerset Bidders

One of the reasons for the popularity of ascending auctions is that they require only partial revelation of the private information (Blumrosen and Nisan, 2007). In a CA this is less of an advantage, as it is still necessary to elicit all valuations except those of the winning bids in the efficient allocation in the worst case. This means that if there are z winning package bids in an efficient allocation, $n2^m - z$ valuations need to be elicited by the auctioneer to guarantee full efficiency. For example, ascending auctions with non-linear and personalized prices such as iBundle, the APA, or dVSV are protocols that in each round elicit the demand set of each bidder and provably find an efficient solution at the expense of an exponential number (in m) of auction rounds (Blumrosen and Nisan, 2007). In such an NLPPA with straightforward bidders at least all valuations of all losing bidders are elicited.

As an alternative to straightforward bidding, the auctioneer can try to encourage bidders to bid on many packages from the start. In the best case, bidders reveal all packages with positive payoff, i.e., they follow a powerset strategy. Unfortunately, even if bidders follow the powerset strategy, the CC auction does not necessarily terminate with an efficient solution.

Theorem 11. *If all bidders follow the powerset strategy, the efficiency of the CC auction converges to 0% in the worst case.*

Proof. Since efficiency cannot be negative it is sufficient to present an example, in which the efficiency is almost 0%. Assuming two bidders and three items for sale. The two bidders have valuations for packages as shown in Table 3.3. They value all other packages with zero. The final ask prices are $p_{(1)} = 2$, $p_{(2)} = 2$ and $p_{(3)} = 1$, and the final allocation assigns package $(1, 2)$ to bidder 1, which is inefficient if $\mu > 4$. Efficiency decreases to 0% if $\mu \to \infty$.

□

We assume no free disposal concerning the valuations in Table 3.3. Otherwise, bidder 1 has a valuation of μ also for package $(1, 2, 3)$, and this would get sold

	(1, 2)	(2, 3)
v_1	4	μ
v_2	2	0

Table 3.3: Valuations that lead to inefficiencies in the CC auction with assuming powerset bidders.

to bidder 1 for a price of 5. The payoff for bidder 1 in this allocation would be $\mu - 5$, which would be efficient, as the sum of the bidders' payoffs and the auctioneer revenue gets maximized. Free disposal can lead to situations, in which powerset bidding drives up prices to very high levels and reduces bidders' utility. It can also lead to high inefficiency (see 3.2.3.1). Consequently, powerset bidding is even more unlikely in a CC auction with free disposal.

Inefficiencies in the CC auction with powerset bidders occur if there are two overlapping packages by the winning bidder, and there is only competition on the package with the lower valuation. This drives up the prices only on the lower valued package, which is finally sold, although the bidder has a higher valuation for the other package, for which he cannot increase his bid.

3.2.3.1 Powerset Strategies with Free Disposal

In the following, we describe an economy with powerset bidders and free disposal. We show that the CC auction leads to very high prices, thus reducing the bidders' utility, even in cases where there is no competition. The example shows that the inefficiency in these situations can be almost as low as 50%.

	(1)	(2)	...	(m)
v_1	μ	0	...	0
v_2	0	$(\mu/m) - \epsilon$	...	0
...	...	...	...	...
v_m	0	0	...	$(\mu/m) - \epsilon$

Table 3.4: Valuations in an economy with powerset bidders and free disposal.

Given the valuations in Table 3.4 and an economy without free disposal, the bidders would all bid on a single item only, and the CC auction would stop after the first round at a price of the minimum bid increment ϵ. In an example, assume $m = n = 100$, $\epsilon = 1$, and $\mu \geq 200$. The allocation assigning bidder i item (i) is efficient and would maximize overall welfare. Bidder 1 would get a payoff of $\mu - 1$, while all other bidders achieve a payoff of $(\mu/100) - 1$. With an auctioneer revenue of 100, the social welfare is $199\pi/100$. If we assume m is the number of items, then the social welfare would be maximized at $(2m-1)\mu/m$.

Now, with free disposal, bidder 1 would bid on all $2^{(m-1)}$ packages that enfold the item (1) in each round until a price of μ/m is reached and he wins all items. His payoff would be 0 and the auctioneer would make a revenue of μ, which is inefficient. With $m \to \infty$ efficiency converges to 50%.

3.2.4 Modifications

The analysis in Section 3.2 shows that even if bidders reveal all profitable packages in each round, the CC auction can be inefficient. However, a small change in the price update rule allows all losing package valuations to be elicited and makes the CC auction fully efficient with powerset bidders.

Definition 23. *A* partial revelation price update rule *in the CC auction also increases prices for each overdemanded item and in addition for each item of a standing bid which is displaced by the winner determination.*

The difference to the original price update rule is very small. While the original CC auction terminates if all bidders holding a standing bid get any package in the final allocation (not necessarily one of their standing bids), the partial revelation price update rule requires a bidder to get exactly his standing bid allocated. Thus one bidder holding two or more standing bids causes prices to increase and the auction to continue.

Corollary 1. *If all bidders follow the powerset strategy, the CC auction with the partial revelation price update rule and sufficiently small bid increments terminates with an efficient outcome.*

Proof. Based on the statement of Theorem 9, we only need to show that the valuations of relevant packages get revealed with powerset bidders in the modified CC auction. By construction of the partial revelation price update rule powerset bidders, who are not part of the efficient allocation, reveal all their valuations. But the rule also ensures that all the bidders in the efficient allocation reveal their valuations on all packages except the ones that are in the winning allocation. As long as a bidder bids on more than one package the auction continues as each bidder can only win one package. As long as a bidder bids on a package that is not winning, prices increase and he can keep bidding. Thus the CC auction with the partial price update rule elicits all valuations except the ones of winning packages and terminates with an efficient allocation. □

The auction can still suffer from small inefficiencies due to the minimal bid increment. *Last-and-final bids* have been suggested as means to get rid of these inefficiencies (Parkes, 2006). They allow bidders to submit a final bid on a package which is above the ask price of the previous round, but below the current ask price for a package. For the sake of clarity, we omit this rule in our analysis.

3.3 The CC+ Auction

Even if the powerset strategy leads to full efficiency in a modified CC auction with linear ask prices, it is not obvious why a bidder should follow the powerset strategy. We show that the powerset strategy is an ex-post equilibrium, but that it requires an even stronger price-update rule and a VCG payment rule (Ausubel and Milgrom, 2006b). We refer to this auction design as a CC+ auction. A description of the CC+ auction with powerset bidders is provided in Algorithm 2. Modifications to the original CC auction are underlined.

Definition 24. *A* full revelation price update rule *in the CC+ auction increases prices on items as long as at least a single bidder bids on the item.*

We aim for a strong game-theoretical solution concept. A desirable property is a profile of strategies with an ex-post equilibrium, in which a bidder does not regret his bid even when he is told what everyone's type is after the auction. Note that we are not attempting to achieve a dominant strategy equilibrium, as preference elicitation in an indirect mechanism can invalidate dominant strategy equilibria existing in a single-step version of a mechanism (Conitzer and Sandholm, 2002). We discuss the types of speculation that are possible in a CC+ auction with full information in Section 3.3.1.1. It illustrates that ex-post equilibria are not as strong as dominant strategy equilibria, but they are much stronger than Bayesian Nash equilibria, because they do not require agents to speculate on other bidders' types or valuations. When iterative preference elicitation is used to implement a mechanism which is a dominant-strategy direct-revelation mechanism in a sealed-bid version, then each agent's best (even in hindsight) strategy is to act truthfully if the other agents act truthfully (Conen and Sandholm, 2001).

Definition 25. *Truthful bidding in every round of an auction is an* ex-post equilibrium *if for every bidder $i \in \mathcal{I}$; if bidders in $\mathcal{I}_{-i}$ follow the truthful bidding strategy, then bidder i maximizes his payoff in the auction by following the truthful bidding strategy (Mishra and Parkes, 2007).*

```
Data: package bids b_i(S)
Result: efficient allocation X* and prices p_i(X_i*)
initialization
  for k=1 to m do p_k ← 0
  for i=1 to n do X_i ← ∅
repeat
    overdemand ← FALSE; oversupply ← FALSE
    for i=1 to n do
      submit a bid β_i(S) on each package S,
      which applies to v_i(S) − Σ_{k∈S}(p_k) ≥ 0
    for k=1 to m do
      if ≥ 1 bidders demand item k then
            p_k ← p_k + ε
            overdemand ← TRUE
      end
      if item k is not part of a bid p_i(S) then
            oversupply ← TRUE
      end
    if overdemand = TRUE then exit iteration
    else if oversupply = FALSE then exit loop
    else
       Calculate the final allocation X* based on all submitted bids
       exit loop
until true
Calculate VCG prices p*_VCG based on all submitted bids
```

Algorithm 2: CC+ auction with powerset bidding.

3.3.1 Efficiency and Incentive Compatibility

We show that the CC+ auction maintains linear ask prices and achieves an efficient solution, while being incentive compatible. Note that we do not need to make any restrictive assumptions on the bidders' valuations. To prove the efficiency, already the slightly weaker partial revelation price update rule is sufficient (cf. proof for Corollary 1).

Corollary 2. *A powerset strategy is an ex-post equilibrium in the CC+ auction with the full revelation price update rule.*

Proof. The proof for the ex-post equilibrium strategy is from the VCG mechanism. Let t_j denote the type of bidder j. We look at the bidder j and assume all other bidders follow the truth revealing powerset strategy. Bidder j receives a payment of $\sum_{i \neq j} u_i(X, t'_i) - \sum_{i \neq j} u_i(X_{-j}, t'_i)$ from the center. The final payoff to bidder j reporting type t' and an allocation X and a VCG payment rule is $u_j(X, t_j) + \sum_{i \neq j} u_i(X, t'_i) - \sum_{i \neq j} u_i(X_{-j}, t'_i)$. A bidder in this payment rule cannot affect the choice of X_{-j}. Hence, j can focus on maximizing $u_j(X, t_j) + \sum_{i \neq j} u_i(X, t'_i)$, i.e., his utility and the sum of the other's utilities. As the auction will maximize $\sum_i u_i(X, t'_i)$, j's utility will be maximized, if $t'_j = t_j$. The partial revelation price update rule is not sufficient for an ex-post equilibrium: In the example in Table 3.5, the CC+ auction with a partial revelation price update rule ends up with final ask prices of $p_{(1)} = 3$ and $p_{(2)} = 4$, before the VCG prices are calculated. If the auctioneer calculates VCG prices based on the submitted bids, then bidder 2 pays $3 - (7 - 5) = 1$ for the item (1). If bidder 2 knew $v_3(2)$, he could have bid up to 6 on item (2). This would increase the final ask price for (2) to 7, and lead to a new VCG price of $3 - (10 - 7) = 0$ for (1) for bidder 2. In a VCG mechanism, bidder 2 could not influence the bid submission of bidder 3 in a similar way, which is why the VCG mechanism has a dominant strategy. Therefore, in the CC+ auction with a partial revelation price update rule, the strategy of bidder 2 is not independent of other bidders' types. Even if the other bidders bid truthfully, a bidder could improve his payoff by deviating from a truth revealing powerset strategy, if he knew the other bidders' types and the other bidders truthfully follow the powerset strategy.

□

As all bidders reveal all valuations, a bidder cannot improve his payoff by unilaterally deviating from the truthful powerset strategy in a respective CC+

	(1)	(2)
v_1	0	3
v_2	3*	0
v_3	2	7*

TABLE 3.5: Valuations that do not lead to an ex-post equilibrium with powerset bidders when using the partial revelation price update rule in the CC+ auction.

auction, or influence whether the other bidders reveal their valuations truthfully. Therefore, the bidder's truthful powerset strategy is independent of the other bidders' types. This result shows what types of price update and payment rules are sufficient for a powerset strategy to satisfy an ex-post equilibrium. While the partial revelation price update rule is sufficient for efficiency, when all bidders follow a powerset strategy, a full revelation price update rule is necessary to achieve an ex-post equilibrium.

However the type of speculation that the partial revelation price update rule allows is extremely unlikely in practical situations and would only make sense under full information which is never the case in auctions.

3.3.1.1 Ex-Post Equilibrium

Does the CC+ auction satisfy a dominant strategy or an ex-post equilibrium? In the single-unit case, there has been an interesting recent discussion on the types of ascending auctions that actually satisfy a dominant strategy equilibrium. Isaac et al. (2007) have shown that while the clock version of an ascending single-item auction has a dominant strategy, the widespread English auction, which allows jump bids, has not.

The CC+ auction can be seen as a multi-item generalization of the ascending clock auction. Also, the VCG auction can be thought of as a single-round version of the CC+ auction, in which the bidder's dominant strategy is to bid truthfully on all possible packages, similar to a powerset strategy. Both auctions satisfy a dominant strategy equilibrium. Does the CC+ auction also satisfy a dominant strategy, or is it restricted to an ex-post equilibrium? In the following, we provide an example in which signals revealed throughout the CC+ auction can make it beneficial for a bidder to deviate from his truth-telling powerset strategy when also others deviate from this strategy.

	(1)	(2)	(1, 2)
v_1	2*	0	0
v_2	0	3*	0
v_3	0	0	4

TABLE 3.6: Example of the difference between the VCG auction and the CC+ auction.

The valuations for three bidders and two items are given in Table 3.6. The VCG price of bidder 1 is $2-(5-4)=1$ for item (1), and his payoff is 1. Now, assume that bidder 1 knows that bidder 2 will increase his bid on (2) to 4, if the ask price for (1) was 3. In round 2, the price clock ticks to 2 for each item and all three bidders signal demand at these prices. In round 3, prices are 3 for both items and again bidders 1 and 2 will signal demand. This will encourage bidder 2 to signal demand even in round 4 for item (2), when bidder 1 drops out. Now, bidder 1 gets a VCG price of $3+(7-4)=0$ and consequently increased his true payoff from 1 to 2. Bidder 2 learns through the course of the CC+ auction that there is a demand for (1) at a price of 3, which would not be possible in a direct revelation VCG auction.

This cannot happen in a clock auction with only a single item, as the bidders can only drop out or continue to signal demand on a single item. This illustrates that the dominant strategy equilibrium does not extend from the single-item clock auction to its multi-item generalization. The powerset strategy in a multi-item CC+ auction is therefore an ex-post equilibrium and not a dominant strategy equilibrium.

3.3.2 Communication Complexity

Nisan and Segal (2006) show that determining an optimal allocation requires an exponential number of queries from the auctioneer to the bidders. There are subtle differences, however, in the amount of information that is elicited by different auction formats. A VCG auction and a CC+ auction ask bidders to reveal all $n2^m$ valuations to the full extent. In a CC+ auction, a bidder sees the price clock increase on various items and learns at which prices nobody demands a particular item any more. In a VCG auction, bidders only know that a bid on a particular package was lost. In both cases, the auctioneer learns all valuations of all bidders. Using the partial revelation price update rule in the CC+ auction with z winning bids, only $n2^m - z$ losing valuations are elicited.

In NLPPAs such as the APA, iBundle(3), or dVSV, the auctioneer elicits $n2^m - z$ preferences in the worst case. It might also be that the winners do not need to reveal all valuations on losing packages. However, a strong solution concept is only satisfied if buyer submodularity is given. Clearly, communication complexity will always remain a stumbling block for any of the theoretical models in situations with more than a few items only. The assumption of following a straightforward strategy in exponentially many auction rounds only holds in automated settings with proxy agents. The same is true for the powerset strategy, even if the number of auction rounds is much lower. We address this issue and the robustness of the efficiency results with respect to deviations from the powerset strategy in Section 3.4.

Similar to work on NLPPAs, the CC+ auction is, however, of theoretical value as it shows sufficient rules and assumptions to design an ascending CA that uses linear ask prices and achieves an efficient outcome with a strong solution concept for general valuations. This provides a theoretical foundation for CC auctions.

3.3.3 Alternative Payment Rules

The CC+ auction suffers from some of the problems of the VCG design, in particular that the outcome might not be in the core (Ausubel and Milgrom, 2006b). In other words, there are some bidders who could make a counteroffer to the auctioneer that both sides would prefer to the VCG outcome. In such situations, the auctioneer can increase his sales revenue by excluding certain bidders, which is also referred to as revenue non-monotonicity. The bidders could also increase their payoff through shill bidding. These vulnerabilities of VCG outcomes are considered serious problems for applications in the field. In some settings, it is sufficient to have a mechanism which is in the core, but which is as close to incentive compatibility as possible.

Day and Raghavan (2007) have recently suggested bidder-Pareto-optimal prices in the core as an alternative to VCG prices. An outcome of an auction is bidder-Pareto-optimal in the core if no Pareto improvement is possible within the core. This means that if we lower one bidder's payment, some other bidder's payment must increase to remain in the core. Such an outcome minimizes the total payments within the core.

Definition 26. *(Day and Raghavan, 2007) An outcome is bidder-Pareto-optimal if there is no other core outcome weakly preferred by every bidder and strictly preferred by at least one bidder in the winning coalition.*

Note that if items are complements, core prices exceed VCG prices strictly. Day and Milgrom (2007) show that a core-selecting auction provides minimal incentives for bidders to deviate from truthful reporting, if it chooses a bidder-Pareto-optimal outcome. Day and Raghavan (2007) also describe a constraint generation approach that generates bidder-Pareto-optimal core prices rapidly for sealed bid auctions. The payment scheme minimizes the total availability of gains from unilateral strategic manipulation. The final bids of each bidder on all packages in a CC+ auction can also be used to calculate bidder-Pareto-optimal core prices.

Corollary 3. *The CC+ auction with powerset bidders terminates with a core outcome if it charges bidder-Pareto-optimal prices as payments instead of VCG prices.*

Proof. Since the CC+ auction elicit all valuations from all bidders and the algorithm from Day and Raghavan (2007) calculates core prices upon the submitted bids the statement is shown. □

Note that even with the weaker partial revelation price update rule, Corollary 3 holds. In contrast to the Clock-Proxy auction (Ausubel et al. (2006)), bidders in the CC+ auction do not need to type in valuations to a proxy agent after the CC auction has finished, and the bidder-Pareto-optimal prices are calculated right away.

3.4 Computational Experiments

Computational experiments provide additional insight and complement the game theoretical analysis of the first sections. They can show the robustness of a design against deviations from equilibrium strategies. In the previous section, we show that the powerset strategy leads to efficiency in the CC+ auction. Powerset bidding is typically not viable for bidders except for small CAs. So far, only a few papers provide results on individual bidding behavior in CAs. Scheffel et al. (2011) report that lab subjects submit around 10 to 12 bids per round in linear-price auctions independent of the number of packages with a positive valuation. Kagel et al. (2010) report that bidders bid only on a fraction of the profitable packages in the CC auction. Global bidders bid between 12 and 14 percent of the profitable packages in one treatment with six items and 21 to 28 percent in a treatment with four items.

This section describes the results of computational experiments and analyzes efficiency, revenue, number of auction rounds, and the number of submitted bids with artificial bidders in the CC and variations of the CC+ auction with respect to deviations from the powerset strategy. The bidding agents follow either the straightforward or the powerset strategy, plus we also implement agents with restrictions on the number of packages submitted in each round.

3.4.1 Experimental Setup

The experimental setup is based on three treatment variables, namely the auction formats, the value model and the bidding strategy.

3.4.1.1 Auction Formats.

Apart from the CC and the CC+ auction, we analyze iBundle and Clock-Proxy auction formats in our experiments. Our implementation of iBundle follows the description in Parkes and Ungar (2000). iBundle is fully efficient given that bidders follow a straightforward bidding strategy. The Clock-Proxy auction has been described in (Ausubel et al., 2006). It consists of a CC auction in the first phase with an XOR bidding language, and a second stage sealed-bid phase. The second phase is then implemented following the rules of iBundle or the Ascending Proxy Auction (Ausubel and Milgrom, 2006a), with automated proxy bidders, who follow a straightforward bidding strategy.

In our implementation, the Clock phase of the Clock-Proxy auction terminates as soon as there is no overdemand in an auction round any more. In contrast, the standalone CC auction will not terminate after a round with excess supply. If the winner determination displaces a bidder who was active in the last round, the auction continues. The winner determination after the Clock phase of the Clock-Proxy auction is necessary to determine the minimum bid prices for proxy bids. These prices are set to the prices of the winning bids of the Clock phase. All bids submitted in the Clock phase are automatically submitted to the Proxy phase independent of the prices. Note that we assume, bidders in the

Proxy phase actually submit bids on all possible packages with a positive payoff given the Clock prices. This is a very favorable assumption, since we assume that bidders are restricted in the Clock phase in some treatments. However, this provides a reasonable upper bound on the efficiency of the Clock-Proxy auction. Of course, restrictions on the number of package bids submitted will also hold in the Proxy phase in any but small value models.

3.4.1.2 Value Models.

Since there are hardly any real-world CA data sets available, we base our experiments on synthetic valuations generated with the Combinatorial Auctions Test Suite (CATS) (Leyton-Brown et al., 2000).

The ***Transportation*** value model uses the *Paths in Space* model from the CATS. It models a nearly planar transportation graph in Cartesian coordinates, in which each bidder is interested in securing a path between two randomly selected vertices (cities). The items traded are edges (routes) of the graph. Parameters for the Transportation value model are the number of items (edges) m and graph density η, which defines an average number of edges per city, and is used to calculate the number of vertices as $(2m)/\eta$. The bidder's valuation for a path is defined by the Euclidean distance between two nodes multiplied by a random number, drawn from a uniform distribution. Consequently only a limited number of packages, which represent paths between both selected cities, are valuable for the bidder. This allows the consideration of even larger transportation networks in a reasonable time. In this work we use a value model with 25 items and 15 bidders. Every bidder has interest in 16 different packages on average.

The ***Real Estate 3x3*** value model is based on the *Proximity in Space* model from the CATS. Items sold in the auction are the real estate lots k, which have valuations $v(k)$ drawn from the same normal distribution for each bidder. Adjacency relationships between two pieces of land p and q (e_{pq}) are created randomly for all bidders. Edge weights $r_{pq} \in [0, 1]$ are then generated for each bidder, and they are used to determine package valuations of adjacent pieces of land:

$$v(S) = (1 + \sum_{e_{pq}:p,q \in S} r_{pq}) \sum_{k \in S} v(k)$$

In this work we use the *Real Estate 3x3* value model with nine lots for sale. Individual item valuations have a normal distribution with a mean of 10 and a variance of 2. There is a 90% probability of a vertical or horizontal edge, and an 80% probability of a diagonal edge. Edge weights have a mean of 0.5 and a variance of 0.3. All experiments with the Real Estate 3x3 value model are conducted with five bidders, who are interested in a maximum package size of 3, because large packages are always valued more highly than small ones. This is also motivated by real-world observations by An et al. (2005), in which bidders typically have an upper limit on the number of items they are interested in. Without this limitation, the auction easily degenerates into a scenario with a single winner for the package containing all items.

In order to analyze a value model with many items, a very large number of possible packages for each bidder, and the impact of the threshold problem, we also use a ***Real Estate 3x5*** value model. This model contains two different bidder types one big bidder interested in all 15 items, and five smaller bidders. Each small bidder is interested in a randomly determined preferred item, all horizontally and vertically adjacent items and the items adjacent to those. This means that a small bidder is typically interested in six to eleven items with local proximity to their preferred item. For each bidder we draw the baseline item valuation $v_i(k)$ from a uniform distribution separately. Complementarities occur upon vertical and horizontal adjacent items based on a logistic function to determine package valuations: $v_i(S) = \sum_{M \in P} \left(\left(1 + \frac{a}{100\left(1+e^{b-|M|}\right)} \right) * \sum_{k \in M} v_i(k) \right)$, with P being the partition of S containing maximal connected packages M. For our simulations we choose $a = 340$ and $b = 8$ for the big bidder and $a = 160$ and $b = 4$ for all small bidders, and draw the baseline valuations for the big bidder on the range $[3, 9]$ and for the small bidders on the range $[3, 20]$.

The size of a value model describes the number of possible bids which a bidder can evaluate. While in the Transportation value model bidders are interested in only 16 packages on average and in the Real Estate 3x3 value model in 129, small bidders in the Real Estate 3x5 value model are interested in 443 packages on average and the big bidder is interested in $2^{15} - 1 = 32,767$ packages. We find that the size of the value model has an impact on the average efficiency achieved if bidders do not reveal all their valuations throughout the auction, as is the case with a straightforward strategy in iBundle or a powerset strategy in the CC+ auction.

Since we find similar results in other models, we concentrate only on the ones described above for clarity and move the others to Appendix A.1.

3.4.1.3 Bidding Agents.

In our theoretical analysis, we introduce the straightforward and the powerset strategies. The ***powerset*** bidder evaluates all possible packages in each round, and submits bids for all packages which are profitable given current prices. In addition to the powerset bidder, we analyze bidders which are restricted to bid only on the best six or the best ten packages in each round, similar to bidders in the lab. These bidders choose those packages with the highest payoff. Inspired by observations in the lab, we also model a ***heuristic 5of20*** bidder. This bidder randomly selects five out of his 20 best packages based on his payoff in a round. This bidder allows the evaluation of the robustness of the auction against randomness in the bidding strategies.

In contrast, the ***straightforward*** bidder only bids on his demand in each round, i.e., on those package(s) that maximize his payoff given current prices.

3.4.1.4 Treatment Structure

We use a $7 \times 7 \times 5$ factorial design (cf. Table 3.7), in which all value models are analyzed in different auction formats with all of the above bidding strategies. Each treatment is repeated 50 times with different random seeds for value models and bidding strategies, resulting in 11,750 auctions. The auctions use a minimum increment of 1 and the XOR bidding language. The results on the Transportation large, Pairwise Synergy and Airports value models are moved to the Appendix A.1.

Value Model		Auction Format		Bidding Strategy
Transportation		CC		Straightforward
Real Estate 3x3		CC+ (partial, Core)		Heuristic 5 of 20
Real Estate 5x3		CC+ (full, Core)		Powerset6
Transportation large (A.1)	×	CC+ (partial, VCG)	×	Powerset10
Pairwise Synergy low (A.1)		CC+ (full, VCG)		Powerset
Pairwise Synergy high (A.1)		iBundle		
Airports (A.1)		Clock-Proxy		

TABLE 3.7: Treatment factors.

3.4.2 Experimental Results

We present the aggregate results of our computational experiments with the three different value models[1]. We evaluate straightforward and powerset bidders, but also bidders following heuristic bidding strategies in order to provide an indication of the impact of heuristic bidding strategies as they can be found with human bidders in the lab or in the field on efficiency.

The results are presented in Tables 3.8 to 3.10 and in Appendix A.1. We measure mean and minimum efficiency, and mean revenue to characterize the auction outcome. Furthermore, we compare number of rounds and total number of bids submitted by the bidders. iBundle leads to a very large number of auction rounds in all but small value models. For the Real Estate 5x3 value model the computation time was such that only a single auction took over 60 hours and 500 rounds as the winner determination takes increasing amounts of time. We decided not to report iBundle results on this value model, as such auctions would not be conducted with human bidders in the field. For similar reasons we also do not report on the results of the Clock-Proxy auction in the the Real Estate 5x3 value model, in which essentially the Proxy phase is equivalent to iBundle with powerset bidders.

Result 1. *(Mean efficiency across auction formats and bidder types) The mean efficiency for the CC and the CC+ auction is higher than 96.9% for all restricted bidder types (Heuristic 5of20, Powerset6, and Powerset10) and all tested value models, except the Real Estate 5x3 value model, where the bidders were interested in a very large number of packages. In the Real Estate 5x3 model, the CC and the CC+ auction yielded an average efficiency of 91.9-94% for restricted bidder types, which is due to the fact that a smaller proportion of the valuations are elicited in larger value models if bidders are restricted to less than ten bids per round. If all bidders follow a powerset strategy, the CC+ auction is almost fully efficient. Small inefficiencies of* $< 0.3\%$ *in some cases are due to the minimum bid increment. With an* ϵ *bid increment and* m *items, the outcome of a CC+ auction without last-and-final bids can be* $(m-1)\epsilon$ *away from full efficiency. An unrestricted powerset strategy in the CC auction leads to 96.8% efficiency on average for all value models that we analyzed, illustrating the robustness of this simple auction format.*

[1] We applied the nonparametric Wilcoxon rank sum test for testing the difference between the treatments: $\sim$ is used to indicate an insignificant order, $\succ^{*}$ indicates significance at the 10% level, $\succ^{**}$ indicates significance at the 5% level, and $\succ^{***}$ indicates significance at the 1% level.

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	99.48	97.02	97.38	96.96	96.83
	CC+ (partial)	99.52	99.87	99.86	99.85	99.92
	CC+ (full)	99.62	99.88	99.84	99.87	99.93
	iBundle	100.00	93.74	97.54	97.89	97.22
	Clock-Proxy	99.96	99.93	99.95	99.95	99.96
Min. Efficiency in %	CC	94.81	84.65	86.71	83.22	83.15
	CC+ (partial)	94.81	96.69	96.69	96.69	98.60
	CC+ (full)	94.81	96.69	96.69	96.69	98.60
	iBundle	100.00	74.72	85.71	89.44	74.56
	Clock-Proxy	97.90	98.60	98.60	98.60	99.48
Mean Rounds	CC	29.10	25.36	25.22	25.10	24.96
	CC+ (partial)	29.04	31.22	31.40	31.10	30.90
	CC+ (full)	44.78	37.80	37.94	37.50	37.26
	iBundle	77.08	277.84	193.48	130.44	75.86
	Clock-Proxy*	24.24	23.54	23.30	23.30	23.02
Mean # of Bids	CC	295.18	452.50	479.92	562.50	805.88
	CC+ (partial)	295.12	475.72	505.56	586.30	828.92
	CC+ (full)	332.88	471.36	501.08	582.22	825.44
	iBundle	7785.48	5791.42	5051.68	4941.52	6440.32
Mean Revenue in %	CC	69.43	83.74	83.30	84.23	84.30
	CC+ (partial, Core)	55.34	58.80	58.23	58.67	58.77
	CC+ (full, Core)	55.02	56.31	55.83	56.33	56.31
	CC+ (partial, VCG)	49.19	52.77	52.64	52.78	52.76
	CC+ (full, VCG)	46.32	47.29	46.87	47.41	47.40
	iBundle	59.58	56.01	53.84	54.18	54.14
	Clock-Proxy	58.74	58.42	58.40	58.40	58.24

TABLE 3.8: Transportation with 25 items and 15 bidders (VCG bidder gain 37.25%). *Clock-Phase only.

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	96.52	98.00	96.99	97.97	99.03
	CC+ (partial)	96.37	99.04	97.47	98.15	100.00
	CC+ (full)	96.47	99.04	97.47	98.15	100.00
	iBundle	100.00	93.92	98.91	99.30	43.02
	Clock-Proxy	97.87	99.16	98.35	98.35	100.00
Min. Efficiency in %	CC	71.85	85.63	80.64	75.18	90.02
	CC+ (partial)	71.85	93.17	80.64	75.18	99.90
	CC+ (full)	71.85	93.17	75.18	75.18	99.90
	iBundle	100.00	81.05	92.74	96.04	14.40
	Clock-Proxy	71.85	93.17	75.18	75.18	99.95
Mean Rounds	CC	288.44	270.04	269.88	268.44	264.36
	CC+ (partial)	291.44	293.64	295.10	291.70	287.38
	CC+ (full)	329.02	299.14	300.00	295.72	294.82
	iBundle	1537.18	19951.04	16176.36	9756.16	1.00
	Clock-Proxy*	274.76	268.42	264.26	264.26	263.24
Mean # of Bids	CC	1914.74	5421.78	6397.74	10061.70	80269.48
	CC+ (partial)	1922.86	5484.46	6467.54	10127.52	80337.72
	CC+ (full)	1995.94	5484.72	6465.74	10126.56	80340.70
	iBundle	484560.06	452946.94	4022475.72	414268.22	645.00
Mean Revenue in %	CC	87.12	96.02	94.38	95.71	97.07
	CC+ (partial, Core)	68.02	84.20	75.40	82.21	86.60
	CC+ (full, Core)	67.80	83.46	74.77	81.56	85.91
	CC+ (partial, VCG)	56.68	82.98	71.87	80.45	85.89
	CC+ (full, VCG)	55.84	81.52	70.55	79.45	84.79
	iBundle	86.07	81.46	83.72	84.14	0.00
	Clock-Proxy	73.60	83.49	82.00	82.00	85.92

TABLE 3.9: Real Estate 3x3 with 9 items and 5 bidders (VCG bidder gain 15.31%). *Clock-Phase only.

iBundle achieves full efficiency with straightforward bidders as predicted by the

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	83.01	93.80	90.19	92.09	99.29
	CC+ (partial)	83.17	93.93	90.09	91.93	99.87
	CC+ (full)	83.09	93.90	90.09	91.93	99.86
Min. Efficiency in %	CC	60.78	74.32	74.32	74.32	89.61
	CC+ (partial)	60.78	74.32	74.32	74.32	99.07
	CC+ (full)	60.78	74.32	74.32	74.32	99.07
Mean Rounds	CC	42.34	40.98	40.04	39.58	38.40
	CC+ (partial)	42.62	42.64	43.10	42.78	42.58
	CC+ (full)	44.70	43.36	43.66	43.40	43.06
Mean # of Bids	CC	247.74	919.22	1099.54	1780.52	368823.10
	CC+ (partial)	248.10	929.68	1109.38	1795.28	369040.14
	CC+ (full)	251.16	929.74	1109.30	1795.32	369040.14
Mean Revenue in %	CC	78.79	89.67	85.94	88.29	96.93
	CC+ (partial, Core)	64.93	77.47	74.09	76.86	87.50
	CC+ (full, Core)	64.75	77.36	73.99	76.76	87.35
	CC+ (partial, VCG)	59.49	70.16	67.88	70.41	84.23
	CC+ (full, VCG)	58.25	69.90	67.82	70.23	83.89

Table 3.10: Real Estate 5x3 with 15 items and 5+1 bidders (VCG bidder gain 15.5%).

theory. With heuristic 5of20, powerset6, and powerset10 bidders the average efficiency results are in most instances significantly worse, but in the Real Estate 3x3 value model also better than the CC+ auction for Powerset6 and Powerset10 bidders.

The Clock-Proxy auction achieves the highest levels of efficiency regardless of the bidding strategy. This is due to the fact that that the bidding agents were not restricted in the Proxy phase and submitted bids on all packages, for which their valuation exceeded the ask prices of the Clock phase. Not only that the second core-selecting Proxy phase requires an elaborate software infrastructure, bidders in the field are not likely to submit as many bids in the second phase, for the same reasons as we assumed restricted bidding in the Clock phase. The reason that the Clock-Proxy auction was not 100% efficient is that bidders cannot submit bids on all possible packages, but just on those where the valuation is higher than the bid price in the last clock round. Note that the efficiency gains over the CC+ auction were small. We did not report on experiments with restricted bidders in the Proxy phase, because this would require additional assumptions. However, if we enforced also strong restrictions in the Proxy phase, the efficiency of the Clock-Proxy auction was at or below the level of the CC+ auction.

In the following, we refer to the Real Estate 5x3 value model as a large value model, because bidders are interested in 443 or even 32,767 packages. All other value models are referred to as small. Note that in realistic applications, we do not expect bidders to have several hundred or thousands of positive valuations for packages, and the "small" value models describe realistic problem sizes with up to 129 packages with a positive valuation. Note that we do not report on iBundle and the Clock-Proxy auction for the large Real Estate 5x3 value model, because the computation time for the proxy phase of this large value model exceeded several days for a single auction.

Result 2. *(Efficiency of the CC and the CC+ auction in small and large value models) In the small value models, the CC+ auction achieves significantly higher efficiency than the CC auction (CC+ $\succ^{***}$ CC or CC+ $\succ^{**}$ CC, depending on the value model and on the type of the powerset bidder). In the large RealEstate 5x3 value model with powerset bidders, the CC+ auction has significantly higher efficiency (CC+ $\succ^{***}$ CC), but there are no significant differences for restricted bidding strategies.*

In small value models, powerset6 and powerset10 bidders reveal a larger proportion of their valuations, which has a positive effect on the efficiency. In the larger RealEstate 5x3 value model, a smaller proportion of the valuations are revealed in each round and the advantages of the CC+ auction compared to the CC auction vanish. Note that even for such a large value model, the mean efficiency is around 92% even for powerset bidders, who are restricted to six or ten bids per round, and almost 94% for heuristic 5of20 bidders.

From our theoretical treatment, we know that the efficiency of the CC auction can be almost 0% in the worst case. In the following, we take a look at the lowest efficiency, which has been achieved in experiments with different CATS value models.

Result 3. *(Minimum efficiency for restricted and unrestricted powerset bidders) In Airport, Transportation and Pairwise Synergy value models, the CC auction and iBundle have significantly lower minimum efficiency than the CC+ auction for restricted and unrestricted powerset bidders (see also Appendix A.1). In the Real Estate 3x3 and 5x3 value models, the minimum efficiency goes down to 74% for restricted bidders. With powerset bidders, the minimum efficiency in the CC auction, was always significantly lower than in the CC+ auction which was almost fully efficient also in the worst case (CC+ $\succ^{***}$ CC). Despite straightforward bidding strategies in some value models, in which the Clock-Proxy auction outperforms the CC+ auction, minimum efficiency is similiar in both auction formats.*

Result 4. *(Number of rounds and bids) The difference in the number of rounds and bids between CC and CC+ is always significant (CC+ $\succ^{***}$ CC), but* *rather small. Note that the number of rounds of a CC+ auction with full revelation price update rule is not necessarily higher than in the CC+ auction with a partial revelation price update rule, since prices on more items are increased by the CC+ auction with full revelation price update rule. The number of rounds in iBundle is orders of magnitude higher than in the CC or the CC+ auction. With powerset bidders iBundle terminates prematurely due to the termination rule, which can result in low revenue and number of rounds, as for example in the Real Estate 3x3 value model. In this value model, there were several thousand auction rounds in iBundle. The rounds of the Clock-Proxy auction were below that of the CC auction, because the auction stops as soon as there is no overdemand on any of the items. The statistic does not reflect the number of rounds in the Proxy phase.*

We always used a minimum bid increment of 1. Clearly, the number of auction rounds can be decreased by increasing the bid increment, but at the expense of efficiency. Note that the valuations in the Real Estate 3x3 value model were determined in a very different way to the Real Estate 5x3 model, as was explained in Section 3.4.1. The valuations for items and packages were on different levels, leading to a different number of auction rounds and a different number of bids submitted.

Result 5. *(Average revenue) The average auctioneer revenue is the highest in the CC auction and decreases significantly with the introduction of bidder-Pareto-optimal core prices and even more so with VCG prices (CC $\succ^{***}$ CC+ (Core) $\succ^{***}$ CC+ (VCG)). The revenue of iBundle with straightforward bidders is significantly higher than that of the CC+ auction with powerset bidders and a VCG rule. The revenue generated by the Clock-Proxy auction is similar to that of the CC+ auction with bidder optimal core prices, except from straightforward bidding agents, where the Clock-Proxy auction achieves higher revenue.*

Note that iBundle is always in the core with straightforward bidders, while the VCG mechanism is not, which can lead to lower revenue if valuations are not buyer submodular.

Result 6. *(iBundle) iBundle needs significantly more rounds and bids per auction than the CC and CC+ auction. But the auctions achieve high levels of efficiency and revenue on average regardless of the bidder types. However the outliers are much more and bigger, which makes the auction outcome less predictable.*

3.4.3 Summary

As theory predicts, iBundle is fully efficient in the computational experiments with straightforward bidders, and so is the CC+ auction with powerset bidders. This full efficiency comes at a cost in both auction formats. The number of auction rounds in the CC+ auction is only slightly increased compared to the CC auction, but the number of bids submitted by powerset bidders was much higher, in particular with the large Real Estate 5x3 value model. Note that the number of bids revealed in iBundle with straightforward bidders was an order of magnitude higher than the number of bids submitted by a fully efficient CC+ auction with powerset bidders in all value models. For example, in the Transportation value model, the fully efficient CC+ auction led to 825.44 bids on average, whereas iBundle led to 7785.48 bids. This was even worse in the case of the Real Estate 3x3 value model (80,340.70 in CC+ vs. 484,560.00 bids on average in iBundle), and the Real Estate 5x3 value model, where more than 4.8 million bids were submitted in the experiments that we run.

If bidders are not able to follow such equilibrium strategies, either for the number of rounds or the number of bids that need to be submitted, and are restricted in the number of bids submitted in each round, full efficiency can no longer be guaranteed. To gain an understanding of how such restrictions impact the efficiency, we have run simulations with the heuristic 5of20, powerset6, and powerset10 bidders. Interestingly, the auctions still yield fairly high levels of efficiency on average, mostly higher than 90%. Note, however, that the number of rounds and the number of bids submitted in iBundle is much higher than in the CC or the CC+ auctions. In most applications with human bidders, more than fifty auction rounds would not be acceptable, and the auctioneer would have to increase the minimum bid increment significantly in iBundle, which can lead to additional inefficiencies. The Clock-Proxy auction yielded higher levels of efficiency, but only under the assumption that bidders submitted bids on all packages with positive payoff in the second phase. In the field, bidders will likely bid on a subset of these packages. We found that in simulations where the restrictions of the Clock phase also hold in the Proxy phase, the efficiency of the CC+ auction was higher than that of the Clock-Proxy auction.

3.5 Conclusion on CC

Combinatorial auctions have led to a substantial amount of research and found a number of applications in high-stakes auctions for industrial procurement, logistics, energy trading, and the sale of spectrum licenses. Anonymous linear ask prices are very desirable and sometimes even essential for many of these applications (Meeus et al., 2009). Unfortunately, Walrasian equilibria with linear prices are only possible for restricted valuations. Already Kelso and Crawford (1982) showed that the "goods are substitutes" property (aka gross substitutes) is a sufficient and an almost necessary condition for the existence of linear competitive equilibrium prices. Later, Gul and Stacchetti (2000) found that even if bidders' valuation functions satisfy the restrictive "goods are substitutes" condition, no ascending VCG auction exists that uses anonymous linear prices. Bikhchandani and Ostroy (2002) show that personalized non-linear competitive equilibrium prices always exist. Several auction designs are based on these fundamental theoretical results and use non-linear personalized prices. While these NLPPAs achieve efficiency, they only satisfy an ex-post equilibrium if the valuations meet buyer submodularity conditions, and they lead to a very large number of auction rounds requiring bidders to follow the straightforward strategy throughout.

These theoretical results assume ask prices throughout the auction to be equivalent to the final competitive equilibrium prices and the payments of bidders. The CC auction differentiates, which is also a way around the negative theoretical results. Still, the CC auction (Porter et al., 2003) cannot be fully efficient. We provide worst-case bounds on the efficiency of the CC auction with straightforward bidders, and propose an extension of the CC auction, the CC+ auction design, which achieves full efficiency with bidders following a powerset strategy. This design modifies the price update rule of the CC auction and adds a VCG payment rule. We show that with such a VCG payment rule, a powerset strategy leads even to an ex-post equilibrium. Note that there are no restrictions on the type of valuations of bidders, which is important for any application. The discussion also shows that the number of ask prices that need to be communicated by the auctioneer, as well as the number of bids required by bidders, is significantly lower than in NLPPAs.

Clearly, a powerset strategy is prohibitive for any but small combinatorial auctions and some other auction rules of the CC+ auction are impractical for real world applications. Actually, the CC+ auction is almost equivalent to a VCG auction, except that bidders learn the highest valuations of items throughout the auction, which they do not in a sealed-bid auction. Since the CC+ auction is iterative, however, we step back from dominant strategies and limit ourselves to an ex-post equilibrium. This is in line with previous results on the VCG auction (Green and Laffont, 1979; Holmstrom, 1979), which show that any efficient mechanism with the dominant strategy property are equivalent to the VCG mechanism, always leading to identical equilibrium outcomes. Later, Williams (1999) found that all Bayesian mechanisms that yield efficient equilibrium outcomes and in which losers have zero payoffs lead to the same expected equilibrium payments as the VCG mechanism. So it is not surprising that the CC+ auction also uses a VCG payment rule to satisfy an ex-post equilibrium.

The CC+ auction is of theoretical and practical relevance. We show that with a simple change in the price update rule, the efficiency of the CC auction can be increased. From a theoretical point of view, we show under which conditions full efficiency with a strong solution concept for general valuations is possible with a clock auction. This helps understand possible sources of inefficiency in the field. We ran sensitivity analysis to investigate how robust the CC+ auction is against deviations from the equilibrium strategies. Interestingly, even if the number of bids submitted in each round is severely restricted or bidders heuristically select some of their "best" bids in each round, both the CC and the CC+ auction achieve very high efficiency levels. The results also explain some of the high efficiency and robustness results of the CC auctions in the lab. However, we also show that the efficiency decreases with an increase in the number of packages of interest to bidders, which can be explained by communication complexity being a fundamental problem in all combinatorial auctions (Nisan and Segal, 2001).

Apart from the CC auction, some authors suggested CAs with pseudo-dual linear prices (Bichler et al., 2009; Kwasnica et al., 2005). Such prices are determined based on the restricted dual of the linear programming relaxation of the winner determination problem. As of yet, there is no formal equilibrium analysis for these auction formats, and the complicated price calculation would make such an analysis very challenging. Note that the efficiency of the CC auction in the simulations in Bichler et al. (2009) was close to the best auction formats with pseudo-dual linear prices.

Chapter 4

PAUSE

A major issue of many CAs is the requirement to optimally solve the $\mathcal{NP}$-hard CAP. To release a centralized auctioneer from that computational burden he can shift it to the bidders. One of the few discussed decentralized CAs is PAUSE, in which bidders suggest new allocations to the auctioneer. In our theoretical analysis we examine the bidders' bid complexity and determine a worst case bound concerning efficiency, if bidders follow a profit maximizing strategy. Based on these results we conduct computational experiments with different bidding and computation strategies, and analyze their impact on efficiency, auctioneer's revenue and auction runtime. Surprisingly, even if agents deviate from the optimal bid price calculation, PAUSE still achieves high levels of efficiency and auctioneer's revenue compared to the Combinatorial Clock auction.

4.1 The PAUSE Auction

Decentralizing the CAP is the approach of the ***P****rogressive* ***A****daptive* ***U****ser* ***S****election* ***E****nvironment (PAUSE)* auction proposed by Kelly and Steinberg (2000). In PAUSE bidders submit not only their own bids, the desired packages of items and the price, but have to propose a new allocation including their new bids and existing bids, being better than the current provisional allocation. Checking bid validity and publishing accepted bids remains the auctioneer's only tasks. Another simplification for the auctioneer is that there is no need for a price calculation mechanism in the iterative process like in most other ICAs.

PAUSE especially concentrates on achieving the following properties: It should permit bidders to submit any combinatorial bid they choose (*fully combinatorial*) and allow losing bidders to clearly see why they lost (transparent). Furthermore it should allow the auctioneer to determine the winner easily for auctions of any size and achieve high auctioneer payoffs. It should also prevent jump bidding and mitigate the threshold problem.

Definition 27 (Composite Bid)**.** *A composite bid (denoted by X^{CB}) is a set of disjoint package bids (including bidder's own bids) that covers all items in the auction, but can include prior bids by any of the bidders. However, prior bids by another bidder j that are included in the composite bid of bidder i must have been submitted by j during a single round of the auction. The bid price $p(X^{CB})$ of a composite bid is the sum of its package bid prices.*

A *composite bid* consists of the following informations that are registered in a database; the database is accessible to all bidders:

- the total bid price of the composite bid;
- for each package bid in the composite bid:
  - the price of the package bid;
  - the identity of the bidder of the package bid;
  - the specification of the package viz. the items that make up the package;

In general a bidder has positive valuations on only a subset of items in the auction - and in any given round, he is interested perhaps only in a subset of these. However, for the items he has no interest on, the bidder fills his composite bid by using prior bids by any of the bidders.
The following example illustrates how composite bids are build. Lets assume there are five items in the auction, (A), (B), (C), (D), (E) where Stage 1 ended with a bid of 2 on each item by bidders 1, 2, 3, 4, 5 respectively; thus with a revenue to the auctioneer from these five bids totaling 10. In stage 2 composite bids have to be submitted with package sizes up to two items. Bidder 1 has a high valuation for the package (AD), and submits a composite bid consisting his own bid of $b_1(AD) = 10$ on the package (AD), together with the prior bids of 2 each on (B), (C), and (E) from bidders 2, 3, and 5, with revenue to the auctioneer of $p(X^{CB}) = [10 + 2 + 2 + 2] = 16$ from bidder 1's composite bid.

In the subsequent round bidder 2 forms a composite bid, including a bid from himself of $b_2(BC) = 8$ on the package (BC), together with previous bids. Here he has two possibilities to fill his own bid with (A), (D) and (E). Either he takes the prior bids of 2 on each item from bidders 1, 4, and 5 or the bids of 10 on (AD) and 2 on (E) from bidders 1 and 5. The second possibility produces the higher revenue to the auctioneer of $p(X^{CB}) = [8 + 10 + 2] = 20$ comparing to $\Pi(\hat{X}, \mathcal{P}) = [8 + 2 + 2 + 2] = 14$ taking the first possibility.
In PAUSE only the second possibility is available since a monotonic increase of the value of a composite bid is requested $[14 \not> 16]$.

Definition 28 (Validity of a Composite Bid)**.** *A composite bid is valid if the following properties hold:*

1. *The composite bid is a collection of disjoint package bids covering all items in the auction.*
2. *The value of the composite bid is increased by at least the minimum increment ϵ but no more than 2ϵ.*

Definition 29 (A Round in the PAUSE Auction Format)**.** *In each round all valid composite bids are registered in the database and the highest composite bid is accepted by the auctioneer (aka winning composite bid). A round ends when bidding ends.*

PAUSE is a multi-round, multi-stage CA decentralizing the CAP. A PAUSE auction with m items has m stages.

Stage 1 consists of a SAA (cf Section 2.5.5) on all items. During this stage bidders can only place individual bids on items - no package bidding is allowed. The stage ends when bidding ends and the auctioneer determines the provisional allocation by simply choosing the best bid on every item.

In each round of a successive stage $h = 2, 3, \ldots, m$ a bidder participate in an ascending price auction and is required to submit a composite bid, which covers all items and includes only disjoint package bids each of maximum cardinality of h. Bidders are allowed to use bids that other agents have placed in previous rounds. For each new package bid in a composite bid, the bidder has to outbid the currently winning composite bid by at least ϵ. After each round the auctioneer declares the highest composite bid as the provisional allocation and registers the highest submitted package bids in the database. A

Data: item bids $b_i(k)$ and composite bids X_i^{CB}
Result: allocation $\overline{X}$ and prices $b_i(\overline{X}_i)$
initialization
  for *k=1* **to** m **do** $b_k \leftarrow 0$
  for *i=1* **to** n **do** $X_i \leftarrow \emptyset$
stage 1: SAA
  repeat
    for *i=1* **to** n **do**
      bidders submit bids $b_i(k)$
    for *k=1* **to** m **do**
      $b_k \leftarrow \max_i b_k$
    $V \leftarrow \sum_k b_k$
    $S \leftarrow (b_{k=S}, i)$
  until *no new bids*
stage $h \geq 1$
  for *h=2* **to** m **do**
    repeat
      for *i=1* **to** n **do**
        bidders submit composite bids X_i^{CB}
        auctioneer checks bid validity:
        if $\exists k \notin X_i^{CB}$ **then** exit iteration
        else if $b(S), b(T) \in X_i^{CB} : S \cap T \neq \emptyset$ **then** exit iteration
        else if $b(S) \in X_i^{CB} : |S| > h$ **then** exit iteration
        else if X_i^{CB} *includes non-exisiting bids of other bidders* **then** exit iteration
        else if $p(X^{CB}) < V + \epsilon$ **then** exit iteration
        else if $p(X^{CB}) > V + 2\epsilon$ **then** exit iteration
        else $V \leftarrow \max\{p(X^{CB}), V\}$
          $\overline{X} \leftarrow argmax_{X_i^{CB}} V$
          $S \leftarrow (b_S, i)$
    until *no new bids*

Algorithm 3: PAUSE auction.

stage ends when bidding finishes. At the end of each stage h, all agents know the best bid for every subset of size h or less so far.

For our theoretical analysis we assume bidders follow a straightforward strategy, by bidding on the package which yields the highest possible payoff at current prices. Since there is no known equilibrium bidding strategy in PAUSE this assumption is justified by the typical use in game theoretical analysis and as it seems natural since bidders reveal as little information as possible keeping the chance for high profits. We assume further that the straightforward bidders do not consider a combination of their package bids, since they are able to bid on those combinations in a single package bid in later stages, thus avoiding a possible exposure problem, which would leave a bidder winning a package of items at prices he is not willing pay.

4.1.1 Rationality for Multiple Stages

The idea might arise that all stages $h < m$ are just for transparency and to simplify the auction for the bidder.
The following example clarifies the opposite.

Example 6. *: $\mathcal{K} = (A, B)$ and $\mathcal{I} = (1, 2)$*

	A	B	AB
v_1	5	10^*	16
v_2	10^*	5	16

TABLE 4.1: Rationality for multiple stages.

If the auction would start in stage 2 both bidders would only bid on the package (AB), since that would maximize their payoff and at the end one bidder would receive it by paying $p_i(AB) = 16$. But it is obvious, that the efficient allocation indicated by the * would be achieved, if PAUSE starts with Stage 1 and bidders are incentivized to participate in every stage. To achieve this aim strict eligibility rules are necessary.

Using this restriction of the size of packages in every stage PAUSE partly solves the efficiency problem of the CC auction with demand masking sets of valuations. Still this problem can also arise in PAUSE (cf Section 4.1.3).

Another thought might be why is it necessary to allow bidders in stgae h also to bid on packages that are of size smaller than h.

Conjecture 1. *It is necessary to allow bidders to bid in Stage h on packages S with $|S| < h$.*

Example 7. *Suppose bidder i has at the end of stage 2 the highest bid on package (AB). In stage 3 bidder j bids on (ABC) and the package is part of the current winning composite bid. Since bidder i still demands (AB) he must be able to increase his bid on this package.*

4.1.2 Bidder Complexity

Each bidder i has a demand set:

$$\mathcal{D}_{i,h} := \left\{ S : v_i\left(S\right) \geq \max_j b_j\left(S\right), i \neq j \wedge |S| \leq h \right\} \tag{4.1}$$

A demand set contains all packages S for which bidder i has a higher valuation than the price of the current highest bid from another bidder j ($\max_j b_j\left(S\right)$) and the cardinality of S must not be greater than h. If bidders want to determine the ask price for a package S, they have to calculate the price ($p\left(X^{CS}\left(S\right)\right)$) of a set of complement disjoint bids, not overlapping with S and covering all items in $\mathcal{K}\backslash S$.

Kelly and Steinberg (2000) designed PAUSE under the premises of an OR-bidding language, meaning a bidder can win more than just one of his bids, and super-additive valuation functions. We adopted these assumptions in our analysis of the *Bid Determination Problem* (BDP) and the worst case efficiency bound.

Definition 30. *The Bid Determination Problem: To maximize bidder i's current payoff $\pi_i \in \mathbb{R}_0^+$, he has to bid on the package(s) S determined by:*

$$\max_{S \in \mathcal{D}_{i,h}} \left(v_i\left(S\right) - p\left(X^{CB}\right) + p\left(X^{CS}\left(S\right)\right) - \epsilon \right) \geq \pi_i \tag{4.2}$$

The inequation ensures that bidder i bids on package(s) S only, if the prospective payoff will not be less than his current payoff. The optimal determination of $p\left(X^{CS}(S)\right)$ is $\mathcal{NP}$-hard, as it is a CAP on the complementary set, which has to be calculated for every package $S \in \mathcal{D}_{i,h}$ to determine the straightforward bid.

4.1.3 Efficiency

The efficiency of PAUSE can be as low as 0% if bidders bid straightforward.

Consider a special case of a demand masking set of bidder valuations: For each item, there is one bidder. Each i-th bidder values the big package which contains all items with $V_{\mathcal{K}}$ and the i-th single item with V. All other package valuations are zero. We set $mV > V_{\mathcal{K}} > V$ so that at the efficient allocation every bidder wins a single item.

	item $\mathfrak{i}$	$\mathcal{K}$	item $\mathfrak{i}' \neq \mathfrak{i}$
i-th bidder	V	$V_{\mathcal{K}}$	0

TABLE 4.2: Special case of a demand masking set of bidder valuations.

The following example in Table 4.3 shows a demand masking set of valuations of bidders in $\mathcal{I} = \{1, 2\}$ for the items in $\mathcal{K} = \{\mathfrak{1}, \mathfrak{2}\}$ and sketches the PAUSE auction process with straightforward bidders. PAUSE does not achieve the efficient allocation indicated by the asterisks, but terminates with 51.5% efficiency.

	$\mathfrak{1}$	$\mathfrak{2}$	$\mathfrak{1}, \mathfrak{2}$	$p(X^{CB})$	π_1	π_2
v_1	100*	0	103			
v_2	0	100*	103			
Stage1	1_1	1_2	0	2	99	99
Stage2	0	0	3_1	3	100	0
	0	0	4_2	4	0	99
	...					
	Termination		103_1	103	0	0

TABLE 4.3: Bidders' valuations and auction process - an example of low efficiency in PAUSE.

Theorem 12. *PAUSE terminates with an allocation that is at least* $1/m$ *efficient, if all bidders follow the straightforward strategy and have super-additive valuations.*

Proof: The proof leans towards the example in Table 4.3. Given the premises stated in the theorem, inefficiencies can only occur in PAUSE, if the auction terminates allocating big packages, although disjoint subsets of them would support the efficient allocation.

Lets assume stage 1 terminates with bids

$$b_i(\mathfrak{i}) \quad = \quad \max_{i \neq j} v_j(\mathfrak{i}) + \epsilon \quad \forall i \in \mathcal{I} \tag{4.3}$$

W.l.o.g. these bids can be considered to support the efficient allocation. The current auctioneers revenue $\Pi_{h=1}$ would be $\sum_i b_i(\mathfrak{i})$.

In order to terminate with another allocation we demand no improvement on any of these individual bids. That means once any of these bids $p_i(\mathfrak{i}) \notin X^{CB} \Rightarrow \exists S \in \mathcal{K}$ which applies to

$$\begin{aligned} & v_i(S) - (p(X^{CB})) > v_i(\mathfrak{i}) - (p(X^{CB}) - p(X^{CS}(\mathfrak{i}))) \\ \wedge \quad & |S| \leq h \end{aligned} \tag{4.4}$$

i.e. bidder i has a better alternative than bidding on the individual item $\mathfrak{i}$ once his provisional payoff drops to zero.

If $v_i(S)$ is part of the final allocation, we want $p(X^{CS}(S))$ to be as small as possible considering the worst case. Thus we determine $S = \mathcal{K}$. That means as long as $h < m$ every bid $b_i(\mathfrak{i})$ for all i is part of the composite bid, which further means that no new bids are submitted before stage m. In stage m the following must apply:

$$\exists i \in \mathcal{I} \text{ with } v_i(S) - (\Pi_m) > \pi_i \tag{4.5}$$

Since in this case bidder i bids on the package S, all other bidders $j \in \mathcal{I} \setminus \{i\}$ have a current payoff $\pi_j = 0$ and thus also the following inequation must hold:

$$v_j(S) - (\Pi_m + \epsilon) > v_j(\mathfrak{j}) - (b_j(\mathfrak{j}) + p(X^{CS}(\mathfrak{j})))\forall j \neq i \tag{4.6}$$

Efficiency is then calculated by

$$E(X^{CB}) = \frac{\max_i v_i(S)}{\sum_i v_i(\mathfrak{i})} \tag{4.7}$$

To determine the worst case efficiency we need to minimize the numerator and maximize the denominator. Thus we can determine w.l.o.g. $v(S) = v_i(S)$ and $v(\mathfrak{i}) = v_i(\mathfrak{j}) \forall i$.

Since the most strict condition on $v_i(S)$ is

$$v_i(S) \quad > \sum_j b_j(\mathfrak{j}) + v_i(\mathfrak{i}) - b_i(\mathfrak{i}) + \epsilon \tag{4.8}$$

the worst case efficiency results in:

$$\begin{aligned} \min_v E(X^{CB}) &= \min_v \frac{v_i(S)}{\sum_i v_i(\mathfrak{i})} \\ &\overbrace{=}^{\epsilon=1} \frac{m+v(\mathfrak{i})+1}{m \cdot v(\mathfrak{i})} \\ &\overbrace{=}^{v(\mathfrak{i})\to\infty} \frac{1}{m} \end{aligned} \tag{4.9}$$

Note assuming a bid increment $\epsilon = 1$ the equations 4.9 only apply if the valuation $v(\mathfrak{i})$ is sufficiently large, i.e. depending on m this valuation must be greater than 2 or 1 respectively. □

While such situations which lead to $1/m$ efficiency can be considered degenerated cases that will not happen too often in practice, it is very likely to achieve high efficiency on average with more realistic value models.

4.2 The PAUSE+ Auction

PAUSE with straightforward bidders is obviously not efficient. But it needs only a slight modification concerning the requirements of the composite bids in order to gain provable full efficiency with straightforward bidding. If bidders need not to outbid the current composite bid in order to get their own bids transferred to the database the PAUSE auction reduces to a special kind of iBundle. In other words bidder may submit an invalid composite bid but made of valid bids on each package, i.e. higher than the incumbent bid on each package. This basically means that each of the bidder in turn submits his composite bid and the database is updated with all valid package bids from among all these bids submitted and then the composite bid is checked for validity by the auctioneer and if valid becomes the new provisional allocation if its total price is higher than the current allocation. With this relaxation we call the new auction format PAUSE+.

The implications of the new rule are that straightforward bidders do not only maximize their potential payoff considering their composite bid but also based

Data: item bids $b_i(k)$ and composite bids X_i^{CB}
Result: allocation $\overline{X}$ and prices $b_i(\overline{X}_i)$
initialization
  for *k=1* **to** m **do** $b_k \leftarrow 0$
  for *i=1* **to** n **do** $X_i \leftarrow \emptyset$
stage 1: SAA
  repeat
    for *i=1* **to** n **do**
      bidders submit bids $b_i(k)$
    for *k=1* **to** m **do**
      $b_k \leftarrow \max_i b_k$
    $V \leftarrow \sum_k b_k$
    $S \leftarrow (\beta_{k=S}, i)$
  until *no new bids*
stage $h \geq 1$
  for *h=2* **to** m **do**
    repeat
      for *i=1* **to** n **do**
        bidders submit composite bids X_i^{CB}
        auctioneer checks bid validity:
        if $\exists k \notin X_i^{CB}$ **then** exit iteration
        else if $b(S), b(T) \in X_i^{CB} : S \cap T \neq \emptyset$ **then** exit iteration
        else if $b(S) \in X_i^{CB} : |S| > h$ **then** exit iteration
        else if X_i^{CB} *includes non-exisiting bids of other bidders* **then** exit iteration
        else if $p(X^{CB}) < V + \epsilon$ **then** <u>continue</u>
        else if $p(X^{CB}) > V + 2\epsilon$ **then** <u>continue</u>
        else $V \leftarrow \max\{p(X^{CB}), V\}$
          $\overline{X} \leftarrow argmax_{X_i^{CB}} V$
          $S \leftarrow (b_S, i)$
    until *no new bids*

Algorithm 4: PAUSE+ auction; differences to PAUSE are underlined.

on their single package, i.e. straightforward bidders have to compare their potential payoff by their bids in the database with the potential payoff resulting from outbidding the highest package bids in the database.

Theorem 13. *If bidders with super-additive valuation functions following the straightforward strategy in PAUSE+ and at least the last composite bid is op-*

timal concerning efficiency given the bids in the database then the auction terminates with the efficient solution.

Proof. Given the new rule PAUSE+ reduces to iBundle except from the restriction of each stage. That means that in every stage PAUSE+ collects all necessary bids to allocate the items efficiently considering only the allowed package size of that stage. Since this statement applies also to the last stage and bidder valuations are super-additive it applies to the whole auction. □

Since iBundle is already impractical for real world settings except for very small settings PAUSE+ becomes even more impractical due to the m stages. Because of this argument and the fact that PAUSE+ gives up some of the nice characteristics of PAUSE (e.g. to be an ascending auction) we intentionally did not implement that version for the analysis in our computational experiments.

4.3 Computational Experiments

To analyze the impact of our theoretical results on the outcome of the PAUSE auction in realistic settings, we conduct computational simulations, which consists of three main components. A value model, which defines valuations of all packages for each bidder, auction formats, which define the rules, and bidding agents, who follow certain strategies.

4.3.1 Value Model

We use a 3 x 6 ***Real Estate*** value model that is based on the *Proximity in Space* model from the Combinatorial Auction Test Suite (CATS) in Leyton-Brown et al. (2000). Our model contains two different bidder types, one big (national) bidder, interested in all items, and five smaller (regional) bidders. Each small bidder is interested in a randomly determined preferred item, all horizontal and vertical neighbors and their respective neighbors. This means small bidders are interested in 6 to 11 items with local proximity to their preferred item. Two examples are shown in Table 4.4, in which the preferred items of small bidders are Q and K, and all gray shaded items in the proximity of the preferred item have a positive valuation.

A	B	C	D	E	F
G	H	I	J	K	L
M	N	O	P	Q*	R

A	B	C	D	E	F
G	H	I	J	K*	L
M	N	O	P	Q	R

TABLE 4.4: Examples of preferred items Q and K of two small bidders. All their positive valued items are shaded.

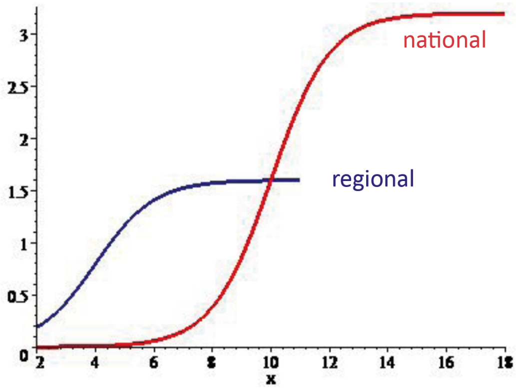

FIGURE 4.1: Functions for calculating their complementarities of the small (regional) and the big (national) bidders.

For each bidder we draw the baseline item valuation $v_i(k)$ from a uniform distribution separately. We assume that bidders experience only low complementarities on small packages, but complementarities increase heavily with a certain amount of adjacent items. We further assume that adding items to already large packages do not increase the complementarities anymore. The explanation for these assumptions is the lack of economies of scale with small packages and a saturation of this effect with larger packages. Therefore, complementarities arise based on a logistic function, which assigns a higher value to larger packages than to smaller ones. Complementarities occur upon vertical and horizontal adjacent items based on a logistic function to determine package valuations: $v_i(S) = \sum_{M \in P} \left(\left(1 + \frac{a}{100\left(1+e^{b-|M|}\right)} \right) * \sum_{k \in M} v_i(k) \right)$, with P being the partition of S containing maximal connected packages M. This complementarity structure takes the lack of economies of scale with small packages and a saturation effect with larger packages into account. For our experiments we choose $a = 320$ and $b = 10$ for the big bidder and $a = 160$ and $b = 4$ for all small bidders, and draw the baseline valuations for the big bidder on the range $[3, 9]$ and for the small bidders on the range $[3, 20]$.

4.3.2 Auction Formats

We analyze two different auction formats in our economic environment. The PAUSE auction, as described in Section 4.1 with a minimum increment of 3 and the CC auction as described in Chapter 3.

4.3.3 Bidding Agents

In PAUSE we use two different bidding strategies and two different approaches to determine the bid price. As introduced in Section 4.1 we implement the straightforward (*BR*) bidding strategy, and a *Greedy* bidding strategy that allows the agents to reduce their demand set to one package calculated by $\max(v_i(S)/|S|), \forall S \in \mathcal{D}_{h,i}$ in every stage. As shown by our theoretical analysis the optimal calculation of the corresponding complement set $X^{CS}(S)$ is $\mathcal{NP}$-hard, therefore we explore two different types of calculating it, an optimal (*oCS*) and a heuristic (*hCS*) approach. We propose the following heuristic, with $k\left(X^{CS}\right)$ denoting the set of items covered by the bids in X^{CS}:

1) $X^{CS} := \emptyset$
2) `while` $k\left(X^{CS}\right) \neq \mathcal{K} \backslash S$

$$X^{CS} = X^{CS} \cup \arg \max_{b_i(T)|_{T \subseteq \mathcal{K} \backslash (S \cup k(X^{CS}))}} b_i(T)$$

We start with an empty complement set X^{CS}, determine all active bids not overlapping the current considered package S, choose the bid with the highest price and add it to our complement set X^{CS}. Then we determine the next bid, not overlapping S and $k(X^{CS})$ with the highest bid price. We repeat until our complement set covers all items of $\mathcal{K} \backslash S$.

For our experiments with the CC auction we use the straightforward bidder and a *heuristic* bidder (*5of20*) bidding on 5 of his 20 best packages in every round, more details to this in Bichler et al. (2009). Additionally we implemented a *preselect* bidder (*pres10*) who determines his 10 most valuable packages before the auction starts, and bids in each round on all of them applying to $v_i(S) \geq p(S)$.

4.3.4 Results

We run 50 simulations for every of the 4 bidding agents in PAUSE and for the 3 different bidding strategies in CC. All experiments run on an Intel Core2Duo processor with 2.67 GHz, 4 GB of RAM, Windows Vista and the open source IP solver "lp_solve".

	PAUSE			
	BR_{oCS}	BR_{hCS}	$Greedy_{oCS}$	$Greedy_{hCS}$
∅ Efficiency in %	97.71	97.52	90.54	91.01
∅ Auctioneers' revenue in %	88.02	88.44	73.62	73.54
∅ Bidders' revenue in %	9.69	9.08	16.92	17.48
∅ Rounds	126.98	127.74	101.48	101.32
∅ Unsold items	0.00	0.00	0.00	0.00
∅ Auction runtime in sec.	22714.29	2166.12	26.85	25.37
∅ Number of final bids	54.95	55.07	33.33	33.14
∅ Size of winning packages	5.03	5.60	2.87	2.75

TABLE 4.5: Summary of simulation results of PAUSE.

	CC		
	BR	$5of20$	$pres10$
∅ Efficiency in %	81.81	91.70	90.95
∅ Auctioneers' revenue in %	76.22	87.96	88.68
∅ Bidders' revenue in %	5.59	3.74	2.27
∅ Rounds	43.14	47.02	44.88
∅ Unsold items	3.96	1.66	1.50
∅ Auction runtime in sec.	44.81	45.34	11.10
∅ Number of final bids	35.49	145.35	52.63
∅ Size of winning packages	6.50	5.34	10.58

TABLE 4.6: Summary of simulation results of CC.

As expected by our theoretical analysis, straightforward bidding in PAUSE with more items and higher competition leads to a better efficiency than the lower bound. We find that BR_{oCS} agents achieve in many cases a solution near the efficient one (Figure 4.2) and a high auctioneer's revenue (Table 4.5). In PAUSE all considered agents are able to find a highly efficient solution, even

Greedy agents, who generate only ~60% final bids compared to BR agents. Surprisingly, calculating the complement set X^{CS} with our heuristic (hCS) leads only to a small deviation in all measures (except the runtime) from the results with agents calculating X^{CS} optimally.

Result 7. *Determining the complement set X^{CS} suboptimally has only a small impact on the auction outcome.*

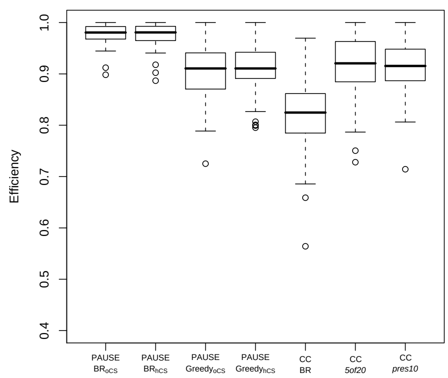

FIGURE 4.2: Auction efficiency with different bidding strategies and auction formats

In contrast to PAUSE, the CC auction mostly ends in allocations with lower efficiency and auctioneers' revenue. We suspect mainly the high number of unsold items (Table 4.6) to lead to such inefficiencies, together with the bigger size of winning packages (∅6.5 with BR agents vs. ∅5.03 with BR_{oCS} agents vs. ∅5.45 in efficient solutions) and the lower number of final bids. To analyze the pure impact of unsold items we ran additional simulations with CC auctions, in which we enforce the agents to bid in the first round on all items they are interested in and found, that the efficiency increases to 89.93% on average with BR agents.

Result 8. *An auction mechanism forcing agents to bid also on smaller packages, guides them in solving their coordination problem.*

Result 9. *CC needs fewer rounds to clear than PAUSE auctions.*

This results from the only moderate increasing of the allowed package size and from the package increment vs. the linear item increment in CC auctions.

Concerning bidders' calculation complexity shows Figure 4.3 that with an increasing number of items a small BR_{oCS} agent is interested in, the required

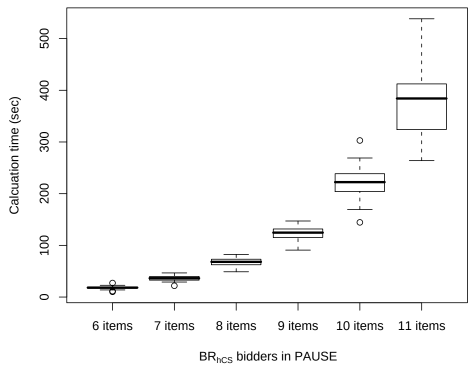

FIGURE 4.3: Bidders' required time over the auction dependent on the number of items they are interested in.

calculation time in the auction increases exponentially. We omit the result of the big bidder, who needs around six hours (particularly ∅ 21.451 sec.) per auction. The boxplot further exhibits, with 7 items or more of interest, a single BR_{oCS} agent in PAUSE requires more calculation time than the complete CC auction process.

Result 10. *Determining the straightforward bid in PAUSE drastically increase the bidders' complexity.*

Comparing the BR bidders in PAUSE with the $pres10$ bidders in CC or the $Greedy$ bidders in PAUSE with the BR bidders in CC we find the following result.

Result 11. *With a similar number of active bids, PAUSE leads to higher efficiency.*

PAUSE collects package bids of every size due to the restrictions of the package size in every stage. This helps to find allocations with high revenue, while in CC more bigger sized package bids are collected which often overlap with each other and so prohibit a "good" allocation.

4.4 Conclusion on PAUSE

We provide a deeper theoretical insight in the decentralized PAUSE auction and present experimental results of two different auction mechanisms. We analyze effects of the straightforward bidding strategy in PAUSE. First we discover following this strategy leads to a growing bid determination complexity, as bidders are not allowed to submit new package bids without embedding them in a new allocation. Secondly if all bidders follow the straightforward strategy, we determine a worst case bound of $1/m$ efficiency.

Since our theoretical analysis promises better efficiency and auctioneer's revenue by the use of more realistic value models, we conducted computational experiments to verify this prediction. We used an agent-based system to compare different bidding strategies and auction mechanisms and find straightforward bidding with optimal bid price determination in PAUSE leads to very high efficiency and auctioneer revenue. Surprisingly, deviating from the optimal bid price determination does not have a significant impact on the auction outcomes, while the auction runtime is reduced drastically. The comparison to the CC auction exhibits that PAUSE is a better guide solving the bidders' coordination problem since it collects different sizes of package bids. A slight modification of the validity of bids leads to efficient auction results if bidders follow the straightforward strategy, but this relaxation makes PAUSE even more unrealistic for real world implementation.

PAUSE shows some desirable properties, however, before taking it to the field it needs further research concerning bidder behavior and auction rules.

Chapter 5

Impact of Side Constraints

In real auctions various side constraints exist for different purposes. They are important in many domains (Bichler et al., 2006; Sandholm and Suri, 2006; Sandholm, 2003). Those constraints have various impacts on the final allocation and auctioneer revenue in different auction mechanisms. It is essential for all participants to know the consequences of deciding whether to use or not to use certain side constriants.

Spectrum auctions, which have been the driving application for much research in the area of CAs, regularly face spectrum caps (max # items/bidder). Such limits are often used to avoid monopoly. For example, the German 3G spectrum auction in 2000/01 allowed each bidder to win from two to three licenses (Seifert and Ehrhart, 2005). Truckload transportation services auctions require a broad range of allocation constraints, summarized in Caplice (2007). In procurement applications, side constraints are the rule rather than the exception (Bichler et al., 2006; Sandholm and Suri, 2006). Buyers need to specify lower or upper bounds on the number of suppliers overall or per group (min/max # winning bidders): lower bounds in order to hedge the risk that some suppliers fail to deliver and upper bounds in order to avoid administrative expenses. Market share constraints are defined on a group of bidders. For example, due to corporate requirements at least one minority supplier must be included in the set of winners on a particular set of items. Another example is again the European 3G spectrum auction in 2000/01, where the number of winners constraint was used to regulate the market (Seifert and Ehrhart, 2005). In Germany, the number of winners was set from four to six. A special variation of this constraint is to limit the number of winners in a specific group of bidders. For example, in the spectrum auction in the UK, one license was set aside for new entrants (Seifert and Ehrhart, 2005). Auctioneers also need to impose lower or upper bounds on the number of items (min/max # items/bidder), which can be awarded to a particular bidder or a group of bidders, or on spend.

It has also been shown in experimental analysis that OR bidding languages are often more efficient than XOR languages due to the reduced number of package bids that need to be submitted (Brunner et al., 2010). OR bids can represent only bids that do not have any substitutabilities, i.e., purely additive and super-additive valuations (Nisan, 2006). If a bidder uses an OR bidding language, it might also be useful to specify constraints on the number of items (min/max # items/bidder) or budget awarded, in order to avoid the exposure problem, which can occur with substitutes valuations (e.g., a bidder is winning packages AB and CD, but only wants two lots at a maximum) or to express his capacity constraints in case he is a supplier in a procurement auction. Disjunctive constraints are relevant to the auctioneer, when a bidder is allowed to win one set of items or another one, but not both in case of an OR bidding language. Carriers in a transportation auction use disjunctive constraints to communicate the message "give me this set of lanes, or this set of lanes, but not both" (Caplice, 2007).

Table 5.1 provides an overview of side constraints in CAs. We group side constraints in *allocation constraints* that specify limits on the allocation of the available items to the bidders, and *price constraints* which set price limits on items, packages, a bidder's budget or auctioneer revenue. Further we can divide side constraints into bidder specific ones and such constraints, which concern more than a single bidder. Bidder specific constraints might be imposed by the bidder or the auctioneer, especially when the OR bidding language is used. The XOR language can be seen as a bidder specific allocation constraint. Group specific constraints are typically important to the auctioneer.

side constraint	**allocation constraint**	**price constraint**
bidder level	min/max # items/bidder	budget
	disjunctive	
group level	min/max # winning bidders	reserve prices
	market share	budget
	disjunctive	

TABLE 5.1: Side constraints.

Having flexibility in the bidding language and the side constraints used by the auctioneer and the bidders allows for a much broader applicability of CAs, and this can be considered a prerequisite for most applications in transportation and industrial procurement.

Both CC and PAUSE would have severe problems to intigrate and deal with additional side constraints. Since CC has to run the CAP only at the end of the auction various side constraints could not be considered during the auction process leading to prices that do not reflect the market situation. In PAUSE the situation is different as either the bidders themselves have to consider the side constraints while creating their composite bids, which would increase the bid complexity even more or the auctioneer has to consider the side constraints during the bid validity check. This causes an unnecessary involvement of the auctioneer which is against the goals of PAUSE and would further alienate the bidders. It has been shown that incentive-compatible auctions are impossible in general if there are private budget limits (Dobzinski et al., 2008), and also reserve prices by the auctioneer increase expected revenue at the expense of efficiency (Myerson, 1981). From here on we focus on efficient auctions, and therefore limit ourselves to allocation constraints.

5.1 Impact of Allocation Constraints on Efficient CAs

We want to understand equilibrium strategies in CAs with allocation constraints. In order to analyze such CAs with respect to efficiency and incentive compatibility we first need to understand the impact of allocation constraints on those CA formats, which are known to be efficient with a strong game-theoretical solution concept. First, we analyze the VCG auction, which is known to be the unique CA format that is strategy proof, efficient and individually rational (Green and Laffont, 1977). Second, we focus on ascending CAs as iBundle, the APA, and dVSV in which straightforward bidding is an ex-post equilibrium for BSM valuations.

5.1.1 The VCG Mechanism

What is the impact of side constraints on the VCG mechanism and its unique properties of efficiency, individual rationality and strategy proofness? This is in particular important as the VCG outcome serves as a baseline for all other efficient auction formats. For example, under BSM valuations APA, iBundle, and dVSV terminate with VCG prices which are in the core, eliminating incentives for speculation. Core prices have the property that no coalition of bidders can renegotiate the outcome with the auctioneer in order to increase everyones payoff in this coalition. The property of terminating with VCG prices would become negligible, if VCG with side constraints loses its desirable properties.

The VCG auction is a sealed bid auction allowing for package bids on all combinations of items. Bidders place sealed XOR bids on their desired packages without getting any feedback by the auctioneer or knowing bids of other bidders. The auctioneer calculates a feasible allocation $\overline{X}$ that maximizes the sum of bid prices. Bidders payments are calculated in a second step. Winning bidders pay their bid prices $b_i(S)$ reduced by a discount which is equal to their marginal contribution to the whole economy. $p_i(S) = b_i(S) - (\mathfrak{w}(\mathcal{I}) - \mathfrak{w}(\mathcal{I} \backslash i)) \forall S \in X^*$ and zero otherwise.

While Ausubel and Milgrom (2006b) show that budget constraints can lead to inefficiency in the VCG mechanism, we show that allocation constraints do not affect its properties. However, the calculation of the VCG prices and in particular of the coalitional value from $\mathfrak{w}(C)$ with $C \subset \mathcal{I}$ has to consider the allocation constraints, as otherwise the auctioneer could suffer a negative payoff and participation would not be individually rational.

Definition 31. *(Shoham and Leyton-Brown (2009)) An environment exhibits the no-single-agent effect if* $\forall i, \forall v_{-i}, \forall X$ *there exists an allocation* X' *that is feasible without* i *and* $\sum\limits_{j \neq i} v_j(X') \geq \sum\limits_{j \neq i} v_j(X)$.

A mechanism is weakly budget balanced when it will not lose money, this means if the mechanism is not weakly budget balanced the auctioneer might be confronted with a negative payoff which would contradict individual rationality of the mechanism.

Theorem 14. *(Shoham and Leyton-Brown (2009)) The VCG mechanism is weakly budget balanced when the no single agent effect property holds.*

This theorem extends to VCG auctions with allocation constraints.

Corollary 4. *The VCG mechanism with allocation constraints is not weakly budget balanced, unless the valuations exhibit the no-single-agent effect.*

Proof. It may happen that $\mathfrak{w}(\mathcal{I}\backslash i)$ in the VCG payment computation is zero because of allocation constraints. Consider an example where the auctioneer requires 2 winning bidders and only 2 bidders participate. Bidder $B1$ bids 10 for item A and bidder $B2$ bids 10 for item B. The Vickrey payments are $p_1(A) = p_2(B) = 10 - (20 - 0) = -10 \leq 0$ such that the auctioneer revenue is negative. □

If the no-single-agent effect does not hold, the auctioneer might want to consider bids by bidders $i \notin C$ to assure a feasible allocation, while maximizing $\mathfrak{w}(C)$.

5.1.2 Efficient Ascending CAs

The recent game-theoretical research has led to a coherent theoretical framework and a family of ascending CAs (iBundle, APA, dVSV) which satisfy an ex-post equilibrium. These efficient ascending CAs use personalized and non-linear prices. They calculate a provisional revenue maximizing allocation at the end of every round and increase the prices for a certain group of bidders. The different approaches can be interpreted as implementations of primal-dual algorithms (dVSV) or subgradient algorithms (iBundle, APA) to solve an underlying linear programming problem (de Vries et al., 2007). This linear program (CAP_3) always yields integral solutions and the dual variables have a natural interpretation as non-linear and personalized ask prices (Bikhchandani and Ostroy, 2002).

We want to understand, whether additional allocation constraints have an impact on equilibrium strategies and efficiency in these auction formats. For this reason, we analyze the impact on allocation constraints on CAP_3. The original CAP_3 formulation changes with additional allocation constraints. An arbitrary allocation constraint can make certain allocations infeasible. Rather than modeling specific allocation constraints, we partition the set of all allocations in two subsets: the feasible allocations Γ and the infeasible ones Γ_u, which turn infeasible due to the violation of certain allocation constraints (e.g., the maximum number of winners). This extends CAP_3 by constraint set (LP4):

$$
\begin{array}{rclllll}
\max\limits_{x_i(S)} \sum\limits_{S \subseteq \mathcal{K}} \sum\limits_{i \in \mathcal{I}} x_i(S) v_i(S) & & & & & & \\
s.t. & & & & & & \\
\sum\limits_{S \subseteq \mathcal{K}} x_i(S) & \leq & 1 & \forall i & (\pi_i) & (LP1) & \\
x_i(S) & \leq & \sum\limits_{X: S_i \in X} y(X) & \forall i, S & (p_i(S)) & (LP2) & (5.1) \\
\sum\limits_{X \in \Gamma \cup \Gamma_u} y(X) & \leq & 1 & \forall X \in \Gamma \cup \Gamma_u & (\Pi) & (LP3) & \\
y(X) & \leq & 0 & \forall X \in \Gamma_u & (z(X)) & (LP4) & \\
x_i(S), y(X) & \geq & 0 & \forall i, S, X \in \Gamma \cup \Gamma_u & & &
\end{array}
$$

Theorem 2 in Bikhchandani and Ostroy (2002) shows that a certain allocation is a competitive equilibrium (CE), if it is an optimal solution to CAP_3 and the corresponding dual linear program. The dual to the extended CAP_3 in (5.1) is:

$$
\begin{array}{rcllll}
\min\limits_{\pi_i, \Pi} \sum\limits_{i \in \mathcal{I}} \pi_i + \Pi & & & & & \\
s.t. & & & & & \\
\pi_i + p_i(S) & \geq & v_i(S) & \forall i, S & (x_i(S)) & (DLP1) \\
\Pi - \sum\limits_{S_i \in X} p_i(S) & \geq & 0 & \forall X \in \Gamma & (y(X)) & (DLP2a) \quad (5.2) \\
\Pi + z(X) - \sum\limits_{S_i \in X} p_i(S) & \geq & 0 & \forall X \in \Gamma_u & (y(X)) & (DLP2b) \\
\pi_i, \Pi, p_i(S) & \geq & 0 & \forall i, S & &
\end{array}
$$

The proof of Theorem 2 by Bikhchandani and Ostroy (2002) is based on the resulting complementary slackness conditions. We show that additional allocation constraints causing additional infeasible solutions do not impact the theorem and the equivalence between competitive equilibrium and to optimal solution to (5.1) is still given. Infeasible allocations can be readily removed without impacting the equivalence. Let us first enumerate the complementary slackness (CS) conditions:

$$
\begin{aligned}
\left(\sum_S x_i(S) - 1\right)\pi_i &= 0 \quad \forall i && (CS1)\\
\left(x_i(S) - \sum_{S_i \in k} y(X)\right) p_i(S) &= 0 \quad \forall i, S && (CS2)\\
\left(\sum_X y(X) - 1\right)\Pi &= 0 && (CS3)\\
y(X)z(X) &= 0 \quad \forall X \in \Gamma_u && (CS4)\\
\left(\pi_i + p_i(S) - v_i(S)\right) x_i(S) &= 0 \quad \forall i, S && (CS5)\\
\left(\Pi - \sum_{S_i \in X} p_i(S)\right) y(X) &= 0 \quad \forall X \in \Gamma && (CS6)\\
\left(\Pi + z(X) - \sum_{S_i \in X} p_i(S)\right) y(X) &= 0 \quad \forall X \in \Gamma_u && (CS7)
\end{aligned}
$$

The competitive equilibrium (CE) conditions are:

$$
\begin{aligned}
\pi_i &= \max_S \left(v_i(S) - p_i(S)\right) \quad \forall i && (CE1)\\
\Pi &= \max_{X \in \Gamma} \sum_{S_i \in X} p_i(S_i) && (CE2)
\end{aligned}
$$

Lemma 1. *The optimal solution of the LP in (5.1) is integral if and only if a competitive equilibrium exists.*

Proof. We follow the proof of Theorem 2 by Bikhchandani and Ostroy (2002), but show that the additional infeasible allocations due to additional allocation constraints do not violate the equality of competitive equilibrium and optimality of the winner determination problem. (CS4) and (CS7) do always hold, as $y(X) = 0$ for the infeasible allocations (cf. (LP4) and $y(X) \geq 0$).
Sufficiency: Suppose the LP (5.1) has an integral solution (X^*) with $x_i(S) = 1$ iff $S = S^*$ and $y(X) = 1$ iff $X = X^*$. Let $(\Pi^*, \pi_i^*, p_i(S)^*, z(X)^*)$ be an optimal solution of the DLP (5.2). $z(X)^* \geq \sum_{S_i \in X} p_i(S)$ because it does not appear anywhere else than in (DLP2b) and the program minimizes Π. (CS5) and (DLP1) imply the first CE condition (CE1). (DLP2a) and (DLP2b) imply $\Pi \geq \max\left\{\max_{X \in \Gamma} \sum_{S_i \in X} p_i(S), \max_{X \in \Gamma_u} \sum_{S_i \in X} (p_i(S) - z(X))\right\}$. Due to $z(X)^* \geq \sum_{S_i \in X} p_i(S)$ the last term is always smaller or equal to zero, while the first term is always greater or equal to zero. Due to (CS6) the above inequality implies the second CE condition (CE2). Hence $(X^*, p_i(S)^*)$ is a CE.

Necessity: Let $(X^*, p_i(S)^*)$ be a CE. Therefore by definition:

$$\begin{array}{lclclr} \pi_i^* & \equiv & v_i(S^*) - p_i(S^*) & = & \max_S (v_i(S) - p_i(S)) \quad \forall i & (CE1) \\ \Pi^* & \equiv & \sum_{S_i \in X^*} p_i(S) & = & \max_{X \in \Gamma} \sum_{S_i \in X} p_i(S) & (CE2) \end{array}$$

Let $x_i(S) = 1$ iff $S = S^*$ and $y(X) = 1$ iff $X = X^*$, else 0. X^* is a feasible solution to LP (5.1) since the allocation is supported in a CE equilibrium. Similarly $(\Pi^*, \pi_i^*, p_i(S)^*, z(X)^*)$ is feasible to DLP (5.2). The dual variable $t(X)$ does not impact this equivalence.

The remaining proof of integrality of the solution is identical to the proof of Theorem 2 in Bikhchandani and Ostroy (2002). □

Corollary 5. *The CAP_3 formulation with allocation constraints (5.1) yields integral solutions and thus the efficient ascending CAs (iBundle, APA, dVSV) still achieve efficient solutions if allocation constraints are present and CE prices if bidders follow the straightforward bidding strategy.*

This follows directly from Lemma 1 and the original proofs of the efficiency of iBundle (Parkes and Ungar, 2000) and dVSV (de Vries et al., 2007). Parkes and Ungar (2000) show that all complementary slackness conditions except (CS1) are satisfied in each round of the iBundle auction. (CS1) states that every bidder with a positive utility for some packages at the current prices must receive a package in the allocation. Only in the last round this condition is satisfied for all bidders. The new complementary slackness conditions (CS4) and (CS7) due to allocation constraints are trivially satisfied, because $y(X)$ is null, and do not impact the proof. While the price updates in dVSV follow a primal-dual algorithm, iBundle and APA can be considered subgradient algorithms (de Vries et al., 2007).

Ausubel and Milgrom (2006a) show that the APA (and therefore iBundle) terminates with an efficient solution and straightforward bidding is an ex-post Nash equilibrium strategy when the BSM condition holds. The proof is defined on some coalitional value function, which might be implemented by CAP_3 but also a CAP_3 with additional allocation constraints, and it is therefore not affected by allocation constraints. In summary, allocation constraints neither have an impact on the efficiency of the family of efficient ascending CAs, nor on the incentive properties. While iBundle, the APA, and dVSV allow for allocation constraints, they do not explicitly take them into account in their pricing rule, but implement simple price increments for subsets of bidders.

Chapter 6

Generic Pricing

The design of efficient CAs has drawn considerable attention (Cramton et al., 2006a), as they raise fundamental questions on pricing and efficiency in multi-object markets. A number of ascending CA designs achieve full efficiency with a strong game-theoretical solution concept. iBundle, the APA and dVSV lead to an efficient allocation, if bidders bid straightforward, which is an ex-post equilibrium as long as the BSM condition holds for all valuations. This line of work is heavily based on duality theory in linear programming. All these auction formats increase the ask price for losing bids by a minimum increment, which causes a large number of auction rounds (Schneider et al., 2010). The APA uses proxy bidders in order to cope with the large number of auction rounds, and to make sure that bidders follow a straightforward strategy, which turns the mechanism into a sealed bid auction.

The above auction formats can be described as core-selecting auctions. A problem with all core-selecting auctions is that they do not satisfy a strong solution concept (Goeree and Lien, 2010) such as a dominant strategy or an ex-post equilibrium, if the VCG outcome is outside the core. Unfortunately, this is only the case for substitutes valuations. Nevertheless core-selecting CAs have been used in recent spectrum auctions in Europe, in which bidder-optimal, and closest to Vickrey core-selecting payments for the winners are determined (Cramton, 2009b). Recent work has dealt with the issue, how core solutions can be calculated fast from a given set of bids (Day and Milgrom, 2007; Hoffman et al., 2006), assuming that bidder valuations have been revealed truthfully. Othman and Sandholm (2010) suggest envy quotes as ask prices in iterative core-selecting auctions. Optimal envy quotes are the highest ask prices, such that a bid at those prices could not change the current allocation or prices.

Instead of simple price increments as used in the auctions above, Adomavicius and Gupta (2005) introduce more general pricing rules. Winning levels (WLs) describe the lowest possible bid at which a losing bid would become winning if no other bids are submitted, whereas deadness levels (DLs) are a lower bound to bids, which still can become winning in the course of the auction. DLs and WLs describe natural bounds and interesting feedback for a losing bidder. While there is no rationale to bid below the DL, bidding at the WL could be rational in some situations, where a bidder is the only one able to outbid a winning coalition of bidders. WLs are equivalent to the minimal winning bids described by Rothkopf et al. (1998). These pricing rules are independent of the allocation rules, and laboratory experiments with respective auction formats yielded high levels of efficiency (Adomavicius et al., 2007).

The analysis in Adomavicius and Gupta (2005) is focused on a pure OR bidding language and assumes no substitutes valuations. OR bids can represent only bids that do not have any substitutabilities, i.e., purely additive and super-additive valuations (Nisan, 2006). Also, this initial work did not analyze equilibrium strategies in such auctions. It is natural to ask, how these generic pricing rules can be defined for auctions with XOR bidding languages and other types of side constraints as well, and which game-theoretical solution concepts can be satisfied. Auctions with strong game-theoretical solution concepts are strategically easier for bidders because they set no incentives for speculation, and respective auction formats are more likely to lead to high efficiency.

We introduce WLs and DLs for general CAs, which allow for XOR bids and other allocation constraints. Auctioneers use various types of allocation constraints to limit the number of winners or the number of items allocated to one bidder or to a group of bidders (cf Chapter 5). Such constraints are actually the rule rather than the exception in application domains such as industrial procurement (Bichler et al., 2006; Sandholm and Suri, 2006) or transportation (Caplice, 2007). Therefore, we extend our analysis to CAs, which consider different types of allocation constraints in the pricing rule (as is the case with WLs and DLs). This is an important extension of the existing theoretical literature on CAs, which typically neglects side constraints, and covers a much broader set of real-world applications.

While generic pricing rules for different CAs with side constraints would also have significant practical importance, we show that these pricing rules bare significant theoretical challenges. The main goal is not to introduce a new auction format, but *to define and analyze game-theoretical properties of WLs and DLs* . In particular, we want to understand which pricing rules satisfy strong game-theoretical solution concepts such as ex-post equilibria and how these pricing rules relate to the theoretical framework on efficient and ascending CAs (de Vries et al., 2007; Parkes and Ungar, 2000).

In summary, our work provides *a theoretical foundation for allocation constraints in CAs and a theoretical framework for recent contributions in this field* (Adomavicius et al., 2007; Adomavicius and Gupta, 2005), which shows *conditions when auction formats based on winning or deadness levels are efficient with a strong game-theoretical solution concept.* In other words, we describe when bidders have strong incentives to reveal their preferences truthfully in a CA, thus leading to less speculation, more predictable outcomes and higher efficiency as long as bidders act rational. This is an important *baseline for any practical auction design*, which provides reasons for inefficient outcomes one might observe in the lab.

The following example briefly illustrates a collection of important pricing rules analyzed. A formal definition will be provided in Section 6.1.

packages	AB	BC	AC	B	C
bids	22_1^*,16_2	24_3	20_4	7_5	8_6^*
P_{RAD}	22	24	14	16	8
$P_{iBundle}$	22_1, 17_2	25_3	21_4	8_5	8_6
DL	22	24	20	7	8
WL	22	30	30	30	8

TABLE 6.1: Example with six bids and different ask prices.

The upper part of Table 6.1 describes six bids from different bidders $B1$ to $B6$, submitted on subsets of three items A, B, and C, while the lower part shows the resulting prices in various auction formats. Subscripts indicate bidders, i.e. 22_1 indicates a bid of 22 from bidder $B1$. Prices have subscripts only if they are personalized. Asterisks denote the provisional winning bids. P_{RAD} gives an example of pseudo-dual linear ask prices as they are described in Kwasnica et al. (2005). Note that these linear prices can be lower than a losing bid (see the bid on AC of bidder $B4$) and that multiple price vectors can satisfy the linear program that is used to determine the ask prices. Another problem with pseudo-dual ask prices is the fact that they can also decrease if the competition shifts, and that they do not take into account side constraints, and consequently prices are biased. The same is true for linear prices in the CC auction. Bichler et al. (2009) analyze the problems in defining pseudo-dual linear prices.

iBundle increases the bids of all losing bidders by a minimum bid increment (1 in this example). DLs and WLs describe generic pricing rules, i.e., they are independent of the allocation constraints used by the auctioneer. DLs describe deadness level ask prices, which are simply the bids of all bidders in the last round in this example. Losing bidders need to bid higher than this by a minimum bid increment. With a minimum bid increment of one, the example would yield $P_{iBundle}$ ask prices. In particular, in the presence of allocation constraints, DL ask prices can be much higher than the ones of iBundle, as we will see later.

WLs describe bid prices, above which a single bid would become winning without the help of another bidder. Bidders $B4$ and $B5$ could become winning at a lower price, if they would form a coalition. In a threshold problem, the WL for a small bidder might be way too high to outbid a winning bidder unilaterally, and with only WL ask prices, it will become difficult for bidders to coordinate. The spread between DLs and WLs can often be quite large in realistic value models, providing little guidance to bidders.

In this example, there are no allocation constraints. Note that linear-price auction formats such as the CC auction do not consider such constraints and pricing can be considerably biased. Let us assume that bidders submit bids on all blocks in a CC auction, but they can win at most 2 blocks. This will drive up prices on all blocks, although there is no competition among the bidders. Bidders will have an incentive to shade their bids. There is a need for pricing rules, which consider allocation constraints adequately.

6.1 Pricing Rules

Efficient ascending CAs such as iBundle and dVSV typically use unit price updates. In particular, in the presence of side constraints, unit price updates on losing package bids do not convey the auction dynamics and might lead to a large number of unnecessary auction rounds.

We draw on WLs and DLs as very general pricing rules. The main virtues of the rules are their generality, applicability to a wide range of settings and their comprehensibility. The problem WL and DL address is that the bidders are usually not able to evaluate the potential of their bids to become part of the winning allocation. As this problem exacerbates when side constraints or XOR bidding are enforced, the necessity of enhanced bidder support emerges. WL and DL answer two natural questions of bidders: WL answers "how much should I bid at least to certainly obtain package S in the next auction iteration (provided no new bids are submitted)?" and DL "how much would I have to bid at least, in order my bid to have at least a chance to win (at some next auction iteration)?". The first question refers to the next auction iteration and not to the end of the auction, since it is impossible to know how much to bid to certainly win at the end due to unknown future bids. The second question refers to the end of the auction since it is possible to compute a lower bound on the bid value needed in order the bid to be "competitive", not "dead"', i.e. not to be certainly losing until the end of the auction, no matter which future bids may be submitted.

We first define WL and DL and introduce the necessary terminology. A subauction on itemset $S \subseteq \mathcal{K}$ refers to an auction where only the items $k \in S$ are auctioned. Auction round $r \in \mathbb{N}$ refers to the auction after the the bids of the first r rounds are submitted. $CAP_r(\mathcal{K})$ denotes the auction revenue and $\overline{X}_r \in \Gamma_r$ the revenue-maximizing allocation at round r[1]. $CAP(S)$ is the revenue of subauction S.

6.1.1 Winning Levels

Definition 32. *The WL of a package S at auction round r is the minimal price a bidder i must bid to win that package at auction round $r+1$ considering no other bids were submitted.* $WL_r(S,i) = \min\{b_i(S) : b_i(S) \in \overline{X}_{r+1}\}$.

Adomavicius and Gupta (2005) define the anonymous WL of package S at auction state r by

$$WL_r^{OR}(S) = CAP_r(\mathcal{K}) - CAP_r(\mathcal{K}\backslash S) \tag{6.1}$$

Intuitively a bid on S can only win, if the bid price together with the revenue from the complementary set of items $\mathcal{K}\backslash S$ exceeds the actual revenue of the

[1]Depending on the context we may omit the index r.

whole auction. Implicitly, the following assumptions are made: (i) OR bidding language and (ii) absence of allocation constraints. When these assumptions are relaxed, the calculation of WLs as in equation (6.1) is inappropriate. A first reason is that allocation constraints, which are not bidder specific, cannot be globally validated when solving subauction $CAP(\mathcal{K}\backslash S)$. In addition, WLs must be personalized as the following example demonstrates.

Example 8. *Consider an auction with items A,B,C, bidders B1 and B2 and the constraint that each bidder cannot win more than two items. Bidder B1 bids 5 on AB and B2 2 on C. $WL(C)$ is 7 for B1 whereas 2 for B2.*

	A	B	C
$B1$	5		0
$B2$	0		2

TABLE 6.2: Bids of two bidders on items A,B and C.

Thus, the WLs are different for each bidder. We introduce the following formula to calculate personalized WLs that takes XOR bidding and allocation constraints into account:

Proposition 4.

$$WL_r(S,i) = CAP_r(\mathcal{K}) - CAP_r(\mathcal{K}, S_i) \tag{6.2}$$

Proof. We give a self-contained proof that is based on the proof of Theorem 2 of Adomavicius and Gupta (2005). We introduce the symbol $\Gamma_r^E(S) = \{X' \in X \cup E | X \in \Gamma_r, E \in \emptyset \cup \bigcup_{i=1}^n (b_i(S) = 0) \forall S\}$ to denote the set of feasible allocations that can also include bids of zero value on S[2]. We define a binary relation $\prec$ on bid combinanations to compare the values of two allocations: $X' \prec X'' \Rightarrow v(X') < v(X'')$ where $v(X) = \sum_i b_i(X_i)$ is the value of the allocations X. $\overline{X}_r^E(S_i) = max_{\prec}\{X \in \Gamma_r^E | b_i(S) \in X\}$ represents the winning allocation of the whole auction at state r subject to the condition that bidder i wins S for free. Let $\Gamma' = \{X \in \Gamma_{r+1} | b_i(S) \in X\}$ and $\Gamma'' = \{X \in \Gamma_{r+1} | b_i(S) \notin$

[2]We need this extension since otherwise allocations where a bidder wins a package for free would not be feasible. These allocations are considered in $CAP_r(\mathcal{K}, S_i)$.

$X\}$ be the set of all allocations with and without $b_i(S)$ respectively. It holds $\Gamma' \cap \Gamma'' = \emptyset$ and $\Gamma' \cup \Gamma'' = \Gamma_{r+1}$. Therefore:

$$\overline{X}_{r+1} = \max_{\prec}\{X \in \Gamma_{r+1}\} = \max_{\prec}\{\Gamma' \cup \Gamma''\} \tag{6.3}$$

$$= \max_{\prec}\{\max_{\prec}\Gamma', \max_{\prec}\Gamma''\} \tag{6.4}$$

$$\text{and since } b_i(S) \notin X \quad \forall X \in \Gamma'' : \Gamma'' = \Gamma_r \Rightarrow \max_{\prec}\Gamma'' = \max_{\prec}\Gamma_r = \overline{X}_r \tag{6.5}$$

Furthermore:

$$\begin{aligned} max_{\prec}\Gamma' &= max_{\prec}\{X \in \Gamma_{r+1} | b_i(S) \in X\} \\ &= \{b_i(S)\} \cup \max_{\prec}\{X \backslash \{b_i(S)\} | X \in \Gamma_{r+1}, b_i(S) \in X\} \\ &= \{b_i(S)\} \cup \max_{\prec}\{X | X \in \Gamma_r | X_i \in X \cap S = \emptyset\} \\ &= \{b_i(S)\} \cup \max_{\prec}\{X \in \Gamma_r^E | b_i(S) = 0 \in X\} \\ &= \{b_i(S)\} \cup \overline{X}_r^E(S_i) \end{aligned}$$

This equation together with (6.4) and (6.5) imply:

$$\overline{X}_{r+1} = \max_{\prec}\{\overline{X}_r, \{b_i(S)\} \cup \overline{X}_r^E(S_i)\} \tag{6.6}$$

$$\text{and } b_i(S) \in \overline{X}_{r+1} \iff v(\overline{X}_r) < b_i(S) + v(\overline{X}_r^E(S_i)) \tag{6.7}$$

Thus, for a new bid $b_i(S)$ to win, its value together with the revenue from CAP which subject to the constraint that the bidder i wins S for free (we denoted this CAP as $CAP_r(\mathcal{K}, S_i)$), must exceed the actual revenue $CAP_r(\mathcal{K})$. This completes the proof. □

$CAP_r(\mathcal{K}, S_i)$ denotes the revenue of the whole auction provided that bidder i wins package S for free. Thus the auction revenue $CAP_r(\mathcal{K}, S_i)$ is raised from the items in $\mathcal{K}\backslash S$, as it is the case in $CAP(\mathcal{K}\backslash S)$. If the OR bidding language is used and no allocation constraints exist, the computation in (6.2) yields the same WLs as (6.1).

6.1.2 Deadness Levels

Definition 33. *The DL of package S at auction round r is the minimal price a bidder must bid to have a chance to win that package in any future round state.* $DL_r(S, i) = \min\{b_i(S) : \exists r' > r : b_i(S) \in \overline{X}_{r'}\}$.

DLs constitute lower bounds on bid prices and all future bids below are "dead", i.e., they cannot win any more no matter which bids are submitted in a future auction round $r' > r$. This implies that $DL_r(S, i)$ is monotonically increasing through the progress of any iterative auction.[3]

Adomavicius and Gupta (2005) define the anonymous DL of a package S by

$$DL_r^{OR}(S) = CAP_r(S) \tag{6.8}$$

In words, a bid on S cannot be part of the winning allocation if it is below the revenue of the subauction S (i.e. $CAP(S)$). For example if $S = AB$ and there are already bids $A = 10$ and $B = 15$, then any bid on AB below 25 is dead.

The DL in equation (6.8) is not valid if there are allocation constraints or any of the bidders uses an XOR bidding language or other allocation constraints. In these cases *DLs* must be personalized. We also need to understand what influence the additional allocation constraints might have in future auction rounds. For instance, consider again Example 8. If in the future auction round $r = 3$ $B1$ bids 8 on C, then his previous bid on AB cannot win due to the allocation constraint. Thus $B2$ can win AB for free and $DL(AB) = 0$ for $B2$. We say that the bid has been "blocked" due to the constraint.

A bid on package S loses in an auction with allocation constraints if at least one of the following conditions is met: (i) There exists a higher bid or bid combination in subauction S that wins in the whole auction (without violating allocation constraints). (ii) S is not part of the revenue maximizing allocation. (iii) The bid in interaction with other bids that win in the whole auction violates an allocation constraint.

The first two conditions are common for every CA, with or without constraints. The third one leads us to the definition of *blocked bids*, which we use later on; a bid on S is blocked if it does not win due to allocation constraints. In this case there is another winning bid that prevents the blocked bid to win. Both bids together cannot win due to constraints.

[3]Through our analysis, we assume no bid revocability, otherwise DLs would be always zero.

Definition 34. *A bid is blocked if it does not win due to allocation constraints. Let* $\overline{X}_{\bar{r}}^{-b'}$ *denote the winning bid combination under consideration of all bids submitted until round* $\bar{r}$ *except bid* b'*. Bid* $b_i(S)$ *is blocked at round* $\bar{r}$ *if* $\exists b'(T)$ *with* $T \subseteq \mathcal{K}\backslash S$*,* $b' \in \overline{X}_{\bar{r}}$ *and* $b \in \overline{X}_{\bar{r}}^{-b'}$*.*

We extend this definition to bid sets. In the simultaneously blocked bid set none of the bids wins due to the existence of winning bids in the complementary subauction. If these bids had not been submitted, at least one bid of the blocked bid set would have won.

Definition 35. *A bid set* $\mathcal{B}_{S,r}^{block}$ *with* $\mathcal{B}_{S,r}^{block} \cap \overline{X}_r = \emptyset$ *is simultaneously blocked if all of its bids are blocked at an auction round. Let* $\mathcal{B}_{S,r} = \{b(T)|T \subseteq S, r' \leq r\}$*. Bid set* $\mathcal{B}_{S,r}^{block} \subseteq \mathcal{B}_{S,r}$ *is simultaneously blocked if* $\exists \mathcal{B}' \subseteq \mathcal{B}_{\mathcal{K}\backslash S,r} : \mathcal{B}' \cap \overline{X}_r \neq \emptyset$ *and* $\mathcal{B}_{S,r}^{block} \cap \overline{X}_r^{-\mathcal{B}'} \neq \emptyset$*.*

We call a bid set simultaneously blockable (instead of blocked) if it can be blocked in the future, i.e. if there can be a future auction round at which the bid set will be simultaneously blocked. These definitions allow us to characterize *DLs*. Bid sets of other bidders in S that can be blocked in the future allow bidder i to win S without having to overbid these bids. Only foreign bids in S that are not dominated from bids of bidder i must be examined whether they are blockable. We say that a bid on package T is i-dominated if there is a bid of i on $T' \subseteq T$ with same or higher value. Hence, we can compute the *DL* of a package S for bidder i by finding the simultaneously blockable, non-i-dominated bid sets $\mathcal{B}_{S,r,-i}^{block}$, which are foreign to i, and removing them in turn from subauction S:

6.1.2.1 General Method to Compute DLs for arbitrary allocation constraints

$$DL_r(S,i) = \min_{\mathcal{B}_{S,r,-i}^{block}} \left\{ CAP_r(S, \mathcal{B}_{S,r} \backslash \mathcal{B}_{S,r,-i}^{block}) : \mathcal{B}_{S,r,-i}^{block} \in f(\mathcal{B}_{S,r,-i}) \right\} \tag{6.9}$$

Function f takes as argument $\mathcal{B}_{S,r,-i}$ the set of all foreign to i, non-i-dominated bids $b_j(T)$ with $T \subseteq S$, $r' \leq r$ and $j \neq i$. Function f's value defines a set of bid sets that are simultaneously blockable. Then $CAP_r(S, \mathcal{B}_{S,r} \backslash \mathcal{B}_{S,r,-i}^{block})$ is the revenue of subauction S after the removal of a simultaneously blockable bid set $\mathcal{B}_{S,r,-i}^{block}$. With an OR bidding language and without side constraints (6.9) reduces to (6.8), since without these constraints, there are no blockable bids and thus $DL_r(S,i) = CAP_r(S, \mathcal{B}_{S,r} \backslash \mathcal{B}^{block}_{S,r,-i}) = CAP_r(S, \mathcal{B}_{S,r} \backslash \emptyset) = CAP_r(S, \mathcal{B}_{S,r}) = CAP_r(S)$.

Equation 6.9 can be seen as a general two-phase method to compute $DL(S,i)$. In the first phase all simultaneously blockable, foreign non-i-dominated bid sets in S are identified. In the second phase each of the identified sets are removed consecutively and $CAP(S)$ is solved. The lowest of these CAP revenues equals to $DL(S,i)$. It is not always necessary to compute all these $CAPs$. If only bids of bidder i remain, we do not need to continue and the $DL(S,i)$ is reached.

Example 9. *Consider an auction with items A, B, C, D, bidders* $B1$ *to* $B5$ *and the XOR bidding language. The bids in AB are:* $b_{B1}(AB) = 19$ *in round 1,* $b_{B2}(A) = 5$ *in round 2,* $b_{B2}(B) = 8$ *in round 3,* $b_{B3}(A) = 10$ *in round 4,* $b_{B4}(AB) = 15$ *in round 5 and* $b_{B5}(AB) = 11$ *in round 6. We compute* $DL_6(AB, B5)$ *using the above formula. Function f's argument* $\mathcal{B}_{AB,6,-B5}$ *contains all bids except the bid of* $B5$

<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td>$B1$</td><td colspan="2">19</td><td colspan="2">0</td></tr>
<tr><td>$B2$</td><td>5</td><td>8</td><td colspan="2">0</td></tr>
<tr><td>$B3$</td><td>10</td><td colspan="3">0</td></tr>
<tr><td>$B4$</td><td colspan="2">15</td><td colspan="2">0</td></tr>
<tr><td>$B5$</td><td colspan="2">11</td><td colspan="2">0</td></tr>
</table>

TABLE 6.3: Bids of five bidders on items A,B,C and D. What is the DL of the package AB for bidder B5?

and returns all simultaneously blockable sets $\mathcal{B}^{block}_{AB,6,-B5}$ *and* $f(\mathcal{B}_{AB,6,-B5}) = \{\{1,2,3\},\{1,4\},\{1,5\},\{2,3,4\},\{2,3,5\},\{4,5\}\}$ *Bids are referred by the round of submission in this example. Bids {1,2,3} can be blocked due to XOR bidding if* $B1$ *and* $B2$ *win C and D respectively, bids {1,4} if* $B1$ *and* $B3$ *win C and D respectively, bids {1,5} if* $B1$ *and* $B4$ *win C and D respectively and so forth. We then remove each simultaneously blockable set from subauction AB (i.e. from the set of all bids in this subauction) and compute* CAP*. After removing the first set* $CAP = 15$*, after the second one* $CAP = 15$*, after the third one 18, after the forth one 19, after the fifth one 19 and after the last one 19. Minimum* CAP *equals to 15. Thus* $DL_6(AB, B5) = 15$*.* $B5$ *could win AB for 15 in a future state at which* $B1$ *wins C and* $B2$ *wins D. A lower price is impossible at this round. Note that with a simple unit price update as in iBundle and dVSV, the ask price for* $B5$ *on package AB would be 12.*

The computation of f is specific to allocation constraints and the bidding language, which determine whether a bid set in subauction S can be simultaneously blocked in a future round or not. Exploring the space of future auction rounds implies assuming possible future bids and examining their impact on $\mathcal{B}$.

6.2 Efficiency and Equilibrium Analysis

We want to understand economical characteristics of CAs and whether pricing rules such as WL and DL can also achieve 100% efficiency with a strong solution concept, and how they relate to iBundle and other efficient ascending CAs. Except from the ask price calculation (i.e., pricing rules) the following auctions (iBundle$_{WL}$ and iBundle$_{DL}$) are equivalent to iBundle and the APA. As in all other efficient ascending CAs such as iBundle, the APA, and dVSV we assume a straightforward bidding strategy where the bidders only have to reveal their demand set in each round. We show that while iBundle$_{WL}$ does not lead to an efficient solution with this bidding strategy, iBundle$_{DL}$ leads to an efficient outcome and straightforward bidding is an ex-post equilibrium.

Bidders use the XOR bidding language as it allows for full expressiveness. Ask prices start at zero for each package and are personalized and non-linear. After every round a revenue maximizing allocation is computed and ask prices are only increased for previously submitted bids of losing bidders. The auction terminates if no bidder is losing. To assure that every bidder is winning at termination, bidders are able to bid a zero bid price on the empty package which can be allocated to them.

6.2.1 iBundle$_{WL}$

In the iBundle$_{WL}$ auction losing bidders in a round get an ask price of $WL(S,i) + \epsilon$ for a package S. In each round WLs for losing bids of losing bidders have to be calculated, not for each possible package and bidder.

The efficiency of a iBundle$_{WL}$ can be as low as 0% if the bidders bid straightforward. We reuse the special case of a demand masking set of valuations as in Section 4.1.3. We first provide another example with $m = 4$, $V = 2$ and $V_{\mathcal{K}} = 5$, where iBundle$_{WL}$ is inefficient.

Example 10. *There are four bidders, $B1$ to $B4$ and four items A to D. Table 6.4 indicates the auction progress. Prices are initialized to* 0*. At the beginning, all bidders bid on the big package. When its price increases to* 3*, then the losing bidders bid also on the single items, since their payoff is* 2*, i.e. equals the payoff of the big package. Their bids on the single items are unsuccessful and the prices are updated to* 4*. These updated prices exceed their valuations $V_s = 2$, therefore they never bid again on the single items and the auction fails to reach the efficient solution.*

	iBundle$_{WL}$					
packages	A	B	C	D	ABCD	$\emptyset$
valuations	2_1	2_2	2_3	2_4	$5_1, 5_2, 5_3, 5_4$	
round 1					0_1^*, 0_2, 0_3, 0_4	
round 2					0_1, 1_2^*, 1_3, 1_4	
round 3					2_1^*, 1_2, 2_3, 2_4	
round 4		0_2	0_3	0_4	2_1, 3_2^*, 3_3, 3_4	
round 5	0_1				3_2, 4_3^*, 4_4	
round 6					5_1^*, 5_2, 4_3, 5_4	0_1, 0_2^*, 0_4^*
round 7					5_1^*, 5_2, 5_3, 5_4	0_1, 0_2^*, 0_3^*, 0_4^*
	Termination					

TABLE 6.4: iBundle$_{WL}$ process.

Theorem 15. *If bidder valuations are demand masking, the efficiency of iBundle$_{WL}$ with straightforward bidding converges to $1/m$ in the worst case with $m > 1$.*

Proof. As WLs are equivalent to the optimal straightforward bid in PAUSE, this statement is already shown by the proof of Theorem 12. □

While there might also be other bidder valuations leading to low efficiency, it is sufficient for our purposes to show that the efficiency of iBundle$_{WL}$ can actually be as low.

6.2.2 iBundle$_{DL}$

Contrary to the negative result on iBundle$_{WL}$, we show that iBundle$_{DL}$ lead to full efficiency with straightforward bidding, but it requires less rounds and less bids than iBundle and the APA.

Lemma 2. *DL ask prices are always higher or equal to iBundle prices given the same bids.*

Proof. Equation (6.9) implies $DL_r(S,i) \geq CAP_r(S,i) + \epsilon$. Let $p_i^r(S)$ be the iBundle price after round r. iBundle uses an XOR bidding language, and $CAP_r(S,i)$ is equal to the highest bid of i in S: $CAP_r(S,i) = max\{b_i(T)|T \subseteq S, r' \leq r\}$. The price update rules in iBundle ensure that in each round $p_i^r(S) = max\{b_i(T)|T \subseteq S, r' \leq r\} + \epsilon$ (Parkes and Ungar, 2000).[4] Thus $CAP_r(S,i) + \epsilon = p_i^r(S)$ and $DL_r(S,i) \geq p_i^r(S)$. □

Lemma 3. *iBundle$_{DL}$ terminates with the efficient solution and with CE prices if bidders bid straightforward.*

Proof. We draw on the proof of optimality by Parkes and Ungar (2000) and show that optimality is not affected by the requirement to bid DLs instead of only an ϵ above $max\{b_i(T)|T \subseteq S, r' \leq r\}$. This is true since a bid $b_i(S)$ below DL would cause $x_i(S)$ to become zero and bidder i would remain unhappy until bidding above DL on a package in his demand set and winning it. More formally, Parkes and Ungar (2000) prove the efficiency of their auction by considering a primal and a dual version of CAP[5] and show that when the auction terminates, all five complementary slackness conditions are satisfied. The only modification of iBundle$_{DL}$, i.e. to quote DLs instead of simple price updates, does not change this proof with respect to their complementary slackness conditions CS-1, CS-3, CS-4 and CS-5. We only need to show that CS-6, which states that "the allocation must maximize the auctioneer's revenue at prices $p(S)$, over all possible allocations and irrespective of bids received by agents", is satisfied by iBundle$_{DL}$ too. Replace $p(S)$ by $DL(S)$. From the DL computation follows that there is always a bidder or group of bidders willing to pay $DL(S)$ for every package in the revenue-maximizing allocation X^*_{DL} that is computed based on the prices (DLs) and irrespective of the bids. For this, observe that the highest possible $DL(S)$, which is the case when no bids

[4]Due to the free disposal assumption implying that packages are priced at least as high as the greatest price of any package they contain, i.e. $p_i^r(S) \geq p_i^r(T)$ for $S \supseteq T$.

[5]This formulation, due to Bikhchandani and Ostroy (2002), is known as CAP_2 and is very similar to CAP_3. The main difference is that its prices are anonymous and it corresponds to $iBundle(2)$. The efficiency of $iBundle(3)$ or simply iBundle follows directly from $iBundle(2)$ (Parkes and Ungar, 2000). Parkes and Ungar (2000) assumes the safety condition in CAP_2, i.e., no single bidder bids on two non-overlapping packages.

are blockable, is equal to $CAP(S)$ considering all submitted bids. Hence there is always a bidder or a group of bidders willing to pay $DL(S)$. Therefore, allocation X^*_{DL} with auctioneer's revenue $\sum_{S^* \in X^*_{DL}} DL(S^*)$ can be realized by assigning each S^* to a subset of bidders $C(S^*), C(S^*) \subseteq \mathcal{I}$ with $\bigcap C(S^*) = \emptyset$, i.e. every bidder receives at most one package (since packages S^* form a feasible allocation and are obviously non-overlapping and no single bidder bids on non-overlapping packages) and hence the XOR constraint is not violated. In summary, we showed that it is always possible for the auctioneer to realize the revenue-maximizing allocation at prices $DL(S)$ irrespective of bids received, since there are always bidders willing to take these prices. □

Lemma 4. *At termination the DL ask price for a winning package by bidder i is equal to the iBundle price if bidders bid straightforward and the BSM condition holds.*

Proof. Firstly, suppose at termination $p_i(S) < DL(S,i)$ and package S is assigned to bidder i in iBundle. $S \in X^*$, because iBundle and iBundle$_{DL}$ both terminate with an efficient solution. $DL(S,i)$ cannot be higher than $p_i(S)$ as it would contradict the definition of DL. This statement is true since iBundle$_{DL}$ collects the same bids as iBundle except dead bids. Secondly, $p_i(S)$ are Vickrey prices when BSM holds, i.e. bidders receive their Vickrey payoff, which is the highest payoff over all points in the core (Ausubel and Milgrom, 2006b). Hence, if $DL(S,i) < p_i(S)$, this implies that the bidders' payoffs in the iBundle$_{DL}$ are not in the core. Thus the outcome of iBundle$_{DL}$ is not in CE. Contradiction to Lemma 3. □

Theorem 16. *iBundle$_{DL}$ is efficient if bidders follow the straightforward strategy and bidding straightforward is an ex-post Nash equilibrium if the BSM condition holds.*

Theorem 16 follows directly from Lemma 3 and 4. Theorem 16 with Lemma 4 show that both versions of price feedback result in the same bidder payoff and consequently the same auctioneer revenue.

Corollary 6. *DL ask prices are the highest ask prices possible in iBundle that lead to an ex-post Nash equilibrium if the BSM condition holds.*

Proof. Suppose bidder i bids in the last auction round ϵ above $DL(S,i)$ and wins S_i with $S \in X^*$ having payoff π_i. If $S \notin X^*$ then the solution is inefficient and hence not in equilibrium. We know that if BSM holds, every bidder's payoff in iBundle is his VCG payoff, which coincides with the unique bidder optimal point in the core (Ausubel and Milgrom, 2006b). From Lemma 4 follows that the VCG payoff of i is $v_i(S) - DL(S, i)$ and hence higher than π_i. □

Theorem 16 with Lemma 2 indicates that the iBundle$_{DL}$ can reduce the number of auction rounds, which is a considerable problem of iBundle as shown by Scheffel et al. (2011) and Schneider et al. (2010).

Corollary 7. *iBundle$_{DL}$ requires less or an equal number of rounds compared to iBundle if bidders follow their equilibrium strategy.*

Proof. Assuming the same increment ϵ both mechanisms terminate with an efficient solution and achieve the same auctioneer revenue (cf. Theorem 16). Lemma 2 shows that ask prices in iBundle$_{DL}$ are always higher or equal to iBundle ask prices. Thus iBundle$_{DL}$ cannot require more rounds to terminate than iBundle. □

We provide a simple example that iBundle$_{DL}$ can terminate with strictly less rounds than iBundle.

Example 11. *Consider items A,B,C are auctioned among bidders $B1$ to $B4$ in iBundle and iBundle$_{DL}$ using an increment of $\epsilon = 1$. Bidders bid straightforward and are single minded which means they value only one package positively and all others with zero. The exact valuations of each bidder and the auction rounds are described in Table 6.5. Ties are broken in favor of more winners.*

Also the number of bids[6] is reduced by the use of DL ask prices. Bidders do not have to submit bids that are below their respective DLs. So "dead" bids that cannot become winning bids do not have to be submitted and taken into account in the winner determination.

The example illustrated in Table 6.5 shows that iBundle$_{DL}$ reduces the number of auction rounds, the communication effort and also the computational effort. In general the reduction of auction rounds and communication effort comes at the price of higher computational effort as the $\mathcal{NP}$-hard CAP has to be solved several times to calculate DLs.

[6] We consider only bids that correspond to new ask prices, i.e. winning bids are automatically resubmitted and not counted.

	iBundle					iBundle$_{DL}$			
packages	A	B	C	ABC		A	B	C	ABC
valuations	5_1	5_2	5_3	8_4		5_1	5_2	5_3	8_4
round 1	1_1^*	1_2^*	1_3^*	1_4		1_1^*	1_2^*	1_3^*	1_4
round 2	1_1^*	1_2^*	1_3^*	2_4		1_1	1_2	1_3	4_4^*
round 3	1_1^*	1_2^*	1_3^*	3_4		2_1^*	2_2^*	2_3^*	4_4
round 4	1_1	1_2	1_3	4_4^*		2_1	2_2	2_3	7_4^*
round 5	2_1^*	2_2^*	2_3^*	4_4		3_1^*	3_2^*	3_3^*	7_4
round 6	2_1^*	2_2^*	2_3^*	5_4		3_1^*	3_2^*	3_3^*	$\emptyset_4^*$
round 7	2_1^*	2_2^*	2_3^*	6_4		Termination			
round 8	2_1	2_2	2_3	7_4^*					
round 9	3_1^*	3_2^*	3_3^*	7_4					
round 10	3_1^*	3_2^*	3_3^*	$8_4, \emptyset_4^*$					
	Termination								

TABLE 6.5: iBundle and iBundle$_{DL}$ process.

6.3 Conclusion on Generic Pricing

Designing efficient combinatorial auctions turned out to be a challenging task. A few recent papers have described efficient and ascending combinatorial auctions which satisfy strong game-theoretical solution concepts. In many applications the consideration of additional allocation constraints and flexibility in the choice of the bidding language are essential. These requirements have not been covered by the theoretical literature so far. It is important to extend the theory and address these requirements. This could extend the applicability of CAs in domains such as transportation or industrial procurement considerably and bares significant practical potential.

We consider CAs allowing for side constraints and OR as well as XOR bidding languages. We draw on the work by Adomavicius and Gupta (2005) and define winning and deadness levels (WLs and DLs) as a general pricing rule for CAs. This extension leads to a number of theoretical challenges. We show that straightforward bidding is an ex-post equilibrium in iBundle with DLs, and how this pricing rule can be integrated in the theoretical framework of efficient CAs. While both, iBundle and the iBundle$_{DL}$ allow for allocation constraints, DLs take allocation constraints into account and actually lead to a lower number of auction rounds and bids that need to be submitted. The high number of auction rounds turned out to be one of the main obstacles for efficient CAs such as iBundle, the Ascending Proxy Auction, and dVSV. *DLs* come at a computational cost, however.

Chapter 7

Conclusions and Future Work

7.1 Conclusion

Combinatorial auctions have led to a substantial amount of research and found a number of applications in high-stakes auctions for industrial procurement, logistics, energy trading, and the sale of spectrum licenses. Price feedback in iterative combinatorial auctions is an important and crucial part of the decision support for bidders. Different pricing rules have been discussed in the literature. Anonymous linear ask prices are very desirable and sometimes even essential for many applications (Meeus et al., 2009). Unfortunately, Walrasian equilibria with linear prices are only possible for restricted valuations. Already Kelso and Crawford (1982) showed that the "goods are substitutes" property is a sufficient and an almost necessary condition for the existence of linear competitive equilibrium prices. Later, Gul and Stacchetti (2000) found that even if bidders' valuation functions satisfy the restrictive "goods are substitutes" condition, no ascending VCG auction exists that uses anonymous linear prices. Bikhchandani and Ostroy (2002) show that personalized non-linear competitive equilibrium prices always exist. Several auction designs are based on these fundamental theoretical results and use non-linear personalized prices. While these NLPPAs achieve efficiency, they only satisfy an ex-post equilibrium if the valuations meet buyer submodularity conditions, and they lead to a very large number of auction rounds requiring bidders to follow the straightforward strategy throughout.

These theoretical results assume ask prices throughout the auction to be equivalent to the final competitive equilibrium prices and the payments of bidders.

The Combinatorial Clock (CC) auction differentiates, which is also a way around the negative theoretical results. Still, the CC auction (Porter et al., 2003) cannot be fully efficient. We provide worst-case bounds on the efficiency of the CC auction with straightforward bidders, and propose an extension of the CC auction, the CC+ auction design, which achieves full efficiency with bidders following a powerset strategy. This design modifies the price update rule of the CC auction and adds a VCG payment rule. We show that with such a VCG payment rule, a powerset strategy leads even to an ex-post equilibrium.

Clearly, a powerset strategy is prohibitive for any but small combinatorial auctions and some other auction rules of the CC+ auction are impractical for real world applications. Actually, the CC+ auction is almost equivalent to a VCG auction, except that bidders learn the highest valuations of items throughout the auction, which they do not in a sealed-bid auction.

Therefore we ran a sensitivity analysis to investigate how robust the CC+ auction is against deviations from the equilibrium strategies. Interestingly, even if the number of bids submitted in each round is severely restricted or bidders heuristically select some of their "best" bids in each round, both the CC and the CC+ auction achieve very high efficiency levels. The results also explain some of the high efficiency and robustness results of the CC auctions in the lab.

We also provide a deeper theoretical insight in the decentralized PAUSE auction. First we discover following a straightforward strategy leads to a growing bid determination complexity, as bidders are not allowed to submit new package bids without embedding them in a new allocation. Secondly if all bidders follow the straightforward strategy, we determine a worst case bound of $1/m$ efficiency.

We conducted computational experiments to verify the sharpness of the worst case bounds. We compare different bidding strategies and auction mechanisms and find straightforward bidding with optimal bid price determination in PAUSE leads to very high efficiency and auctioneer revenue. Surprisingly, deviating from the optimal bid price determination does not have a significant impact on the auction outcomes, while the auction runtime is reduced drastically. The comparison to the CC auction exhibits that PAUSE is a better guide solving the bidders' coordination problem since it collects different sizes of package bids. A slight modification of the validity of bids leads to efficient auction results if bidders follow the straightforward strategy, but this relaxation makes PAUSE even more unrealistic for real world implementation.

In many applications the consideration of additional allocation constraints and flexibility in the choice of the bidding language are essential. These requirements have not been covered by the theoretical literature so far. It is important to extend the theory and address these requirements. This could extend the applicability of CAs in domains such as transportation or industrial procurement considerably and bares significant practical potential.

We consider CAs allowing for side constraints and OR as well as XOR bidding languages. We draw on the work by Adomavicius and Gupta (2005) and define winning and deadness levels (WLs and DLs) as a general pricing rule for CAs. This extension leads to a number of theoretical challenges. We show that straightforward bidding is an ex-post equilibrium in iBundle with DLs, and how this pricing rule can be integrated in the theoretical framework of efficient CAs. While both, iBundle and the iBundle$_{DL}$ allow for allocation constraints, DLs take allocation constraints into account and actually lead to a lower number of auction rounds and bids that need to be submitted. The high number of auction rounds turned out to be one of the main obstacles for efficient CAs such as iBundle, the Ascending Proxy Auction, and dVSV. DLs come at a computational cost, however. These results provide a theoretical foundation for practical auction design. Such designs can leverage different pricing rules, and even combine DLs and WLs. Such hybrid designs might well lead to high efficiency in the lab and in the field.

7.2 Future Work

The work and projects of this dissertation are still in progress. Concerning the CC+ auction we are currently evaluating results of laboratory experiments, in order to understand human bidder behavior in such auctions and to investigate the incentive for demand reduction in the CC auction in a specific value model. The PAUSE auction needs more theoretical analysis and additional thoughts about specific auction rules (e.g. activity rules) before taking it to the laboratory or the field. However even if this auction format will not be applicable for laboratory or field experiments, some key aspects like the process in certain stages are worth while to be further analysed and eventually adopted in other combinatorial auction designs. The main work remains concerning generic pricing rules in cases of side constraints. While we found that the complexity to determine exact deadness levels lies in the Π_2^P-complete class, heuristics might be a good comprise, i.e. approximations to the exact computation of deadness levels could potentially be an area of future research. Both computational and laboratory experiments are required to gain knowledge on bidding behavior and efficiency in complex markets with allocation constraints and will give more insights into the drawbacks and opportunities of this new pricing rule.

Appendix A

Appendix

A.1 Further Computational Experiments

A.1.1 Value Models

We also test a ***Transportation Large*** value model with 50 items and 30 bidders. The other characteristics are as described in Section 3.4.

The ***Airports*** value model is an implementation of the *matching* scenario from CATS. It models the four largest airports in the USA, each having a predefined number of departure and arrival time slots. For simplicity there is only one slot for each time unit and airport available. Each bidder is interested in obtaining one departure and one arrival slot (i.e., item) in two randomly selected airports. His valuation is proportional to the distance between the airports and reaches maximum when the arrival time matches a certain randomly selected value. The valuation is reduced if the arrival time deviates from this ideal value, or if the time between departure and arrival slots is longer than necessary.

The ***Pairwise Synergy*** value model from An et al. (2005) is defined by a set of valuations of individual items $v(k)$ with $k \in \mathcal{K}$ and a matrix of pairwise item synergies $\{syn_{k,l} : k, l \in \mathcal{K}, syn_{k,l} = syn_{l,k}, syn_{k,k} = 0\}$. The valuation of a package S is then calculated as

$$v(S) = \sum_{k=1}^{|S|} v(k) + \frac{1}{|S| - 1} \sum_{k=1}^{|S|} \sum_{l=k+1}^{|S|} syn_{k,l}(v(k) + v(l))$$

A synergy value of 0 corresponds to completely independent items, and the synergy value of 1 means that the package valuation is twice as high as the sum of the individual item valuations. The model is very generic, as it allows different types of synergistic valuations, but it was also used to model valuations in transportation auctions (An et al., 2005). We use the Pairwise Synergy value model with seven items; item valuations are drawn for each auction independently from a uniform distribution between 4 and 12. The synergy values are drawn from a uniform distribution between 1.5 and 2.0. The auctions with the Pairwise Synergy value model have five bidders each. We use a high synergy and low synergy setting.

In the *Real Estate* and *Pairwise Synergy* value models, bidders are interested in a maximum package size of 3, because in these value models large packages are always valued more highly than small ones. This is also motivated by real-world observations (An et al., 2005), in which bidders typically have an upper limit on the number of items they are interested in. Without this limitation, the auction easily degenerates into a scenario with a single winner for the package containing all items.

A.1.2 Experimental Results

As already discussed, the exponential communication complexity remains a stumbling block (Nisan and Segal, 2001). While in NLPPAs this leads to a huge number of auction rounds, the CC+ auctions require bidders to submit a large number of bids in each round. However, while we show that winners need to reveal more information in the CC+ auction as in NLPPAs, the number of actual bids submitted by bidders in CC+ is much lower. This is due to the bid increments of packages. If the prices for 5 items increase by ϵ, then the price for the package of these 5 items increases by $5 * \epsilon$. For example, in our Real Estate (3x3) value model a bidder had 130 valuations. In the CC+ auction (with last-and-final bids) 4,128 bids were submitted in 32 rounds by powerset bidders, and only 419 bids were submitted by Powerset10 bidders. In contrast, in the same setting iBundle(3) (Parkes and Ungar, 2000) elicited 7,741 bids per bidder in 150 rounds, and in the Credit-Debit auction even 14,895 bids in 266 rounds.

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	98.48	97.94	98.04	97.86	98.07
	CC+ (partial)	98.19	99.17	99.25	99.33	99.27
	CC+ (full)	98.11	99.14	99.19	99.29	99.26
	iBundle	100.00	86.95	94.22	94.74	95.89
	Clock-Proxy	99.83	99.56	99.72	99.72	99.64
Min. Efficiency in %	CC	90.74	85.00	85.00	85.00	85.00
	CC+ (partial)	90.74	95.29	95.29	95.29	95.29
	CC+ (full)	90.74	96.10	95.35	96.47	96.13
	iBundle	100.00	69.62	76.03	76.40	84.24
	Clock-Proxy	97.69	95.18	96.47	96.47	95.18
Mean Rounds	CC	17.80	17.08	14.00	13.82	13.62
	CC+ (partial)	16.90	14.90	14.86	14.60	14.40
	CC+ (full)	21.48	18.14	18.14	18.00	17.88
	iBundle	57.18	1222.42	860.08	534.06	58.14
	Clock-Proxy*	13.68	11.66	11.78	11.78	11.52
Mean # of Bids	CC	234.04	376.20	406.68	496.66	1201.30
	CC+ (partial)	226.76	375.90	407.26	496.76	1201.30
	CC+ (full)	227.40	373.66	404.74	494.50	1199.88
	iBundle	34968.16	35673.06	31686.36	30947.04	25136.04
Mean Revenue in %	CC	83.80	89.11	89.27	89.21	89.32
	CC+ (partial, Day)	67.62	69.97	69.95	70.28	70.39
	CC+ (full, Day)	67.81	69.43	69.34	69.76	69.82
	CC+ (partial, VCG)	56.30	60.70	60.95	61.54	61.68
	CC+ (full, VCG)	54.67	57.78	58.33	58.69	58.72
	iBundle	76.92	64.32	64.91	65.43	66.99
	Clock-Proxy	75.11	74.73	74.94	74.94	73.94

TABLE A.1: Transportation Large with 50 items and 30 bidders (VCG bidder gain 37.25%). *Clock-Phase only.

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	98.62	97.18	97.23	97.10	97.17
	CC+ (partial)	98.60	98.74	98.71	98.71	98.71
	CC+ (full)	98.74	98.75	98.62	98.76	98.73
	iBundle	100.00	95.50	98.93	98.13	97.87
	Clock-Proxy	99.77	99.72	99.66	99.66	99.64
Min. Efficiency in %	CC	95.39	92.63	93.42	93.57	93.73
	CC+ (partial)	95.04	96.57	96.57	97.06	96.86
	CC+ (full)	96.10	97.06	95.74	97.06	96.86
	iBundle	100.00	88.50	96.57	93.00	92.55
	Clock-Proxy	97.92	98.96	98.60	98.60	98.60
Mean Rounds	CC	10.94	8.48	8.28	8.20	8.20
	CC+ (partial)	13.58	11.80	11.40	11.28	11.36
	CC+ (full)	17.82	11.86	11.48	11.46	11.42
	iBundle	27.30	48.98	41.58	33.82	33.68
	Clock-Proxy*	10.06	8.78	8.18	8.18	8.02
Mean # of Bids	CC	472.72	721.44	786.74	957.68	993.40
	CC+ (partial)	511.22	758.20	819.98	989.44	1025.66
	CC+ (full)	527.80	734.16	798.52	969.44	1004.22
	iBundle	6364.38	6262.72	5522.90	6364.66	7095.52
Mean Revenue in %	CC	82.75	91.30	91.33	91.67	91.75
	CC+ (partial, Core)	41.02	44.45	43.91	44.24	44.09
	CC+ (full, Core)	41.43	43.31	42.90	43.36	43.13
	CC+ (partial, VCG)	35.17	38.61	38.54	38.91	38.64
	CC+ (full, VCG)	36.35	38.04	38.08	38.54	38.38
	iBundle	49.02	48.51	47.88	46.74	47.25
	Clock-Proxy	46.10	46.98	46.40	46.40	46.34

TABLE A.2: Airports with 84 items and 40 bidders (VCG bidder gain 57.76%). *Clock-Phase only.

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	99.16	99.06	99.15	99.02	99.29
	CC+ (partial)	99.09	99.83	99.72	99.74	100.00
	CC+ (full)	99.26	99.83	99.72	99.74	100.00
	iBundle	100.00	97.93	99.33	99.14	39.52
	Clock-Proxy	99.95	99.86	99.77	99.77	100.00
Min. Efficiency in %	CC	86.52	94.22	96.32	94.22	94.22
	CC+ (partial)	86.52	97.79	97.20	96.22	99.95
	CC+ (full)	94.53	97.79	97.20	96.22	100.00
	iBundle	100.00	94.19	94.99	93.36	10.90
	Clock-Proxy	97.94	97.79	96.66	96.66	99.95
Mean Rounds	CC	323.66	321.60	325.50	322.94	321.84
	CC+ (partial)	336.72	353.24	356.12	352.24	352.00
	CC+ (full)	367.12	355.56	358.74	355.44	354.52
	iBundle	1596.30	13220.48	11009.38	6821.48	1.00
	Clock-Proxy*	311.16	320.60	321.50	321.50	321.16
Mean # of Bids	CC	2068.46	6976.56	8337.60	13368.04	63268.94
	CC+ (partial)	2106.56	7074.98	8440.68	13468.12	63377.04
	CC+ (full)	2165.84	7069.80	8429.24	13463.68	63371.72
	iBundle	332557.40	311450.98	287116.04	303880.26	315.00
Mean Revenue in %	CC	89.76	97.01	96.84	97.20	97.46
	CC+ (partial, Core)	73.42	87.68	83.20	86.81	88.53
	CC+ (full, Core)	72.87	87.06	82.54	86.13	87.95
	CC+ (partial, VCG)	69.93	86.87	81.42	85.71	87.85
	CC+ (full, VCG)	69.33	85.95	80.39	84.71	86.96
	iBundle	88.07	86.20	85.90	87.08	0.00
	Clock-Proxy	83.91	87.14	86.38	86.38	87.96

Table A.3: Pairwise Synergy High with 7 items and 5 bidders (VCG bidder gain 12.97%). *Clock-Phase only.

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	98.22	98.80	98.44	98.98	99.16
	CC+ (partial)	97.80	99.76	98.96	99.56	100.00
	CC+ (full)	97.76	99.77	98.96	99.56	100.00
	iBundle	100.00	97.66	98.78	99.28	52.88
	Clock-Proxy	99.80	99.79	99.56	99.56	100.00
Min. Efficiency in %	CC	88.28	93.18	91.81	93.18	93.18
	CC+ (partial)	88.28	96.61	91.81	96.26	99.95
	CC+ (full)	88.28	96.61	91.81	96.26	99.95
	iBundle	100.00	93.36	92.76	95.63	22.32
	Clock-Proxy	96.31	96.61	96.26	96.26	99.96
Mean Rounds	CC	368.54	349.38	351.72	350.40	348.78
	CC+ (partial)	395.12	388.88	386.72	386.70	384.04
	CC+ (full)	418.10	390.20	390.98	389.58	388.76
	iBundle	1694.72	13613.30	11696.20	6971.54	1.00
	Clock-Proxy*	345.78	347.70	348.52	348.52	347.48
Mean # of Bids	CC	2295.56	7401.86	8800.06	14070.80	68422.96
	CC+ (partial)	2381.44	7519.58	8919.26	14186.08	68541.52
	CC+ (full)	2392.08	7504.12	8903.12	14176.12	68533.64
	iBundle	358160.68	325568.06	304894.88	312606.56	315.00
Mean Revenue in %	CC	88.38	96.85	95.84	96.89	97.23
	CC+ (partial, Core)	73.14	87.11	83.27	86.70	88.26
	CC+ (full, Core)	73.29	86.42	82.34	85.81	87.44
	CC+ (partial, VCG)	69.57	86.29	81.85	85.65	87.69
	CC+ (full, VCG)	68.53	85.16	80.20	84.30	86.55
	iBundle	87.54	83.86	85.09	85.04	0.00
	Clock-Proxy	83.12	86.37	85.89	85.89	87.44

Table A.4: Pairwise Synergy Low with 7 items and 5 bidders (VCG bidder gain 13.43%). *Clock-Phase only.

Appendix B

List of Symbols

Item-related

$\mathcal{K}$ - set of items

$k, l, h \in \mathcal{K}$ - item

m - number of items

$S, T \subseteq \mathcal{K}$ - subset of items (package, aka bundle)

Bidder-related

$\mathcal{I}$ - set of bidders

$i, j \in \mathcal{I}$ - bidder

n - number of bidders

$C \subseteq \mathcal{I}$ - subset of bidders

$\mathfrak{P}(\mathcal{I})$ - power set of $\mathcal{I}$

Auction-related

$v_i(S)$ - private valuation of the bidder i for the package S

$b_i(S)$ - bid price of the bidder i for the package S

$\mathcal{B}$ - set of bids

$p_i(S)$ - ask price for bidder i for the package S

$\mathcal{P}$ - set of pay prices

$\pi_i(S, \mathcal{P})$ - payoff of the bidder i for the package S at prices $\mathcal{P}$

$\Pi(X, \mathcal{P})$ - auctioneer revenue for the allocation X at prices $\mathcal{P}$

$\mathcal{R}$ - set of auction rounds

r - round number

ϵ - minimum bid increment

Γ - set of all possible allocations

$X = (X_1, ..., X_n) = \{X_i\}$ - allocation where bidder i gets package X_i

$\overline{X}$ - revenue maximizing allocation given the bids so far

X^* - efficient allocation

$x_i(S) \in \{0; 1\}$ - binary variable which determines, whether the bidder i becomes allocated exactly the package S

$E(X) \in [0, 1]$ - allocative efficiency of the allocation X

$R(X) \in [0, E(X)]$ - auctioneer utility share in the allocation X

PAUSE-related

X^{CB} - composite bid

$p(X^{CB})$ - price of a composite bid

$X^{CS}(S)$ - allocation on the complement set of S, i.e. allocation on $\mathcal{K} \backslash S$

$p(X^{CS}(S))$ - sum of bid prices in $X^{CS}(S)$

$k(X^{CS})$ - items covered by bids in X^{CS}

Generic Pricing-related

$CAP(S)$ - revenue of subauction S

$CAP(\mathcal{K}, S_i)$ - revenue of the whole auction providing bidder i receives package S for free

$\overline{X}^{-b}$ - winning bid combination considering all bids except bid b

$\mathcal{B}_S$ - set of all bids on items or packages in S

$\mathcal{B}_S^{block}$ - a simultaneously blocked bid set

$f(\mathcal{B}_{S,r,-i})$ - the functions' value is a set of simultaneously blocked bid sets

Game Theory-related

$\mathfrak{w}(C)$ - coalitional value function on the coalition C

(Π, π) - payoff vector

$Core(\mathcal{I}, \mathfrak{w})$ - set of core payoffs

θ_i - set of types of bidder i

t - bidder type

s - strategy profile

$u_i(s,t)$ - utility function of bidder i

E - expected value

Appendix C

List of Abbreviations

APA **A**scending **P**roxy **A**uction

BAS **B**idders **A**re **S**ubstitutes condition

BDP **B**id **D**etermination **P**roblem

BSM **B**idder **S**ub**m**odularity condition

CAP **C**ombinatorial **A**llocation **P**roblem

CA **C**ombinatorial **A**uction

CATS **C**ombinatorial **A**uction **T**est **S**uite

CC **C**ombinatorial **C**lock auction

CE **C**ompetitive **E**quilibrium

DL **D**eadness **L**evel

FCC **F**ederal **C**ommunication **C**ommission

GAS **G**oods **A**re **S**ubstitutes condition

ICA **I**terative **C**ombinatorial **A**uction

NLPPA **N**on-**L**inear **P**ersonalized **P**rice **A**uction

NP **N**on/deterministic **P**olynomial time

OR additive-**OR** (bidding language)

PAUSE **P**rogressive **A**daptive **U**ser **S**election **E**nvironment

PEP **P**reference **E**licitation **P**roblem

RAD **R**esource **A**llocation **D**esign

SAA **S**imultaneous **A**scending **A**uction

TUM **T**echnische **U**niversiät **M**ünchen

VCG **V**ickrey-**C**larke **G**roves mechanism

VM **V**alue **M**odel

WDP **W**inner **D**etermination **P**roblem

WL **W**inning **L**evel

XOR e**x**clusive-**OR** (bidding language)

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