Transitions into Black Box Views *
— The NetBill Protocol Revisited —

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Abstract

System specification by state machines together with property
specification and verification by temporal logics are by now standard
techniques to reason about the control flow of hardware components
and embedded systems. The techniques to reason about the dataflow
within loosely coupled systems, however, are less well developed.

In this contribution, we propose a formalism for the verification of
systems with asynchronously communicating components. The com-
ponents themselves are specified as state machines, while the dataflow
between components is described as a relation over the input and out-
put histories of a system. Communication history properties are de-

erived from temporal logic properties of the component state machines.
The history properties can then be used to deduce global properties
of a complete system.

To demonstrate our approach, we model the NetBill protocol for
micropayments in the Internet and prove some correctness properties.

1 Introduction

State machines have become a popular technique to specify software and
hardware systems. They are often described by various incarnations of state
transition diagrams, which are a suggestive notation for component design

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or implementation documents. Both in their graphical and in their non-
graphical—such as B, VDM or Z—variants, state-based specification tech-
niques have a precise semantics and clear operational models. Effects of
state transitions can be analyzed by Hoare-like triples with pre- and post-
conditions.

More abstract properties of state machines can be formulated with tem-
poral logics to express invariance or liveness properties. Proofs in temporal
logic often follow the operational intuition behind state machines: Invariance
properties, for example, are typically shown using induction over the machine
transitions.

Temporal logics are less well suited, however, to express properties of
the data flow between loosely coupled components that communicate asyn-
chronously via buffered communication channels. For such systems, black
box views relating input and output communication histories of data flow
components and systems are better suited. Such relations can be concisely
formulated in the style of Focus [9, 10, 1]; they are inherently modular and
allow easy reasoning about the global system behavior. In [2, 3, 5], we intro-
duced a formalism for the verification of black box properties of systems with
asynchronously communicating state machine components. The formalism
builds on work by Manfred Broy [8]. In [6] tool support on the basis of
Isabelle/HOL [12] is described.

In this paper, we demonstrate our approach with a model of the NetBill
protocol. We specify the protocol in an operational way that is easy to
implement but nevertheless abstract enough for verification purposes. We
formulate essential properties at different abstraction levels, and sketch their
formal proofs.

Section 2 contains a brief summary of black box and state machine speci-
cification techniques. Section 3 contains a state machine specification of the
NetBill protocol for electronic payments in the Internet. In Section 4 we
formalize correctness properties of the protocol as history relations, and
show that the state machine specification indeed satisfies these properties.
Section 5 contains a short discussion of our specification and verification ap-
proach. The conclusion in Section 6 summarizes the results and contains an
outlook on future work.

2 Component and System Specifications

Our system model is a variant of the system model of Focus [9, 10, 1]. It is
described in detail in [2]. We model a system by describing its components,
its interface with respect to the system’s environment, and its behavior. The
components are connected via directed channels. The system’s interface is described by the communication channels with the types of the message that are sent on them. The communication along all channels is modeled by finite or infinite message streams. The behavior of a system is characterized by a relation between the input and output streams, that we described in either of two different abstraction levels: black-box specifications and state machines.

2.1 Streams
The communication history between components is modeled by streams. A stream is a finite or infinite sequences of messages. Finite streams can be enumerated, for example: \( \langle 1, 2, 3, \ldots, 10 \rangle \); the empty stream is denoted by \( \langle \rangle \). For a set of messages \( \text{Msg} \), the set of finite streams over \( \text{Msg} \) is denoted by \( \text{Msg}^* \), that of infinite streams by \( \text{Msg}^\omega \). By \( \text{Msg}^\omega \) we denote \( \text{Msg}^* \cup \text{Msg}^\omega \).

Given two streams \( s, t \) and \( j \in \mathbb{N} \), \#\( s \) denotes the length of \( s \). If \( s \) is finite, \#\( s \) is the number of elements in \( s \); if \( s \) is infinite, \#\( s \) = \( \infty \). We write \( s \circ t \) for the concatenation of \( s \) and \( t \). If \( s \) is infinite, \( s \circ t = s \). We write \( s \subseteq t \), if \( s \) is a prefix of \( t \), i.e. if \( \exists u \in \text{Msg}^\omega \cdot s \circ u = t \). The \( j \)-th element of \( s \) is denoted by \( s.j \), if \( 1 \leq j \leq \#s \); it is undefined otherwise. \( \text{ft.}\ s \) denotes the first element of a stream, i.e. \( \text{ft.}\ s = s.1 \), if \( s \neq \langle \rangle \). For \( A \subseteq \text{Msg} \) we denote by \( A\text{S}s \) the subsequence that results from \( s \) by removing all elements not in \( A \). For singleton sets we often just write \( a\text{S}s \) instead of \( \{a\}\text{S}s \).

2.2 Black-Box Specifications
A black-box specification is an abstract description in the sense that it does not relate to any internals of the system, but just describes the external, visible behavior.

The behavior relation is defined by formulas \( \Phi \) where the free variables range over the input and output streams. The streams fulfilling these predicates describe the allowed black-box-behavior of our system. We can use all the operators on streams to formulate the predicates.

As a very simple example, consider a component Identity that just copies messages from one input channel \( i \) to one output channel \( o \). Its black-box behavior specification is defined by the formula \( o = i \).

2.3 State Machines
The behavior of a system can also be specified by a state transition system (STS), formalized by the tuple \( \mathcal{S} = (I, O, A, I, T) \). The names of the input and output channels are contained in \( I \) and \( O \), respectively. The set \( A \)
contains for each $i \in I$ a variable $i^*$ (a prefix of $i$) denoting the sequence of messages already consumed by $S$. Additionally, $A$ may contain variables to represent local data, as e.g. a variable $\sigma$ for the control state. A state of the system consists of a variable valuation that assigns values of the appropriate type to all variables. Channel variables are evaluated to streams containing the history of messages sent. The system starts in a state fulfilling the predicate $I$, and $T$ is the set of transitions.

State transition systems can be described in various ways, for example by state transition diagrams [2], by tables [10], or by the notation as used in this paper. All techniques have a common technique to describe a transition by four parts: A precondition, a set of input statements, a set of output statements and a postcondition. The informal meaning of a transition is as follows: If the available messages in the input channels can be matched with Inputs, the precondition is and the postcondition can be made true by assigning proper values to the primed variables, the transition is enabled. If it is chosen, the inputs are read, the outputs are written and the postcondition is made true.

![Figure 1: Identity Component](image)

The component Identity (Figure 1) just needs one transition, called transmit, that is always enabled, reads some value $x$ from $i$, and immediately sends it on $o$, without causing other changes in the components state.

Transitions can be schematically translated into logical formulas; see [2] for details.

### 2.4 Composition

Systems can be composed of several components by identifying channels with the same names. The composition can be graphically illustrated by structure diagrams, as used in Figure 4. The behavior of a composite system is completely defined by the behaviors of its components. Two components $S_i$
and \( S_2 \) can only be composed to \( S = S_1 \otimes S_2 \) if they are compatible (defined in [2]), meaning essentially that they do not control the same variables.

Using black-box specifications, the behavior of the composed system is defined as the conjunction of the component behavior predicates. For state machines, a transition of the composed system consist of a transition of one component together with an environment transition of the others.

### 2.5 From State Machines to Black-Box Views

While the state machines represent an operational view on system's behavior, the black-box specifications can be seen as properties of the system. Therefore, it is crucial to be able to make a formal connection between both abstraction layers, since this allows us to prove that a (implemented) system has certain black-box properties, e.g. show that the component Identity with one transition transmit indeed fulfills the property \( o = i \).

In [2], we used temporal logic to establish the connection between both abstraction levels. Properties can be split into a safety and progress part, that read in our simple example as

\[
\square o = i^v \\
\square ((\# o = k \land \# o < \#i) \Rightarrow \Diamond \# o \geq \#i)
\]

The properties express that the output is correct in all reachable states (i.e. equal to the consumed messages on \( i \)) and that the output will be eventually extended as long as there is still buffered input left. Invariance is proved as usual by showing that the invariant is valid initially, and stays valid for all transitions. Output extension can be shown by finding helpful transitions that extend the output, and that are enabled, and therefore will be taken due to some fairness properties that are assumed in the execution model of the state machines. In [6] we suggest how verification diagrams and mechanized proof support assist the verification of properties of the above format.

Invariance properties form the basis of safety properties on the black box level. Invariants are also black box safety properties, if their free variables refer only to history variables \((I \cup I^e \cup O)\) and if they are admissible [16] with respect to these variables.

Progress properties are derived from schemata similar to the property above; in general, the output channel length is compared with an arbitrary continuous expression over the length of the input channels. More details can be found in [2].
3 NetBill Specification


Figure 2 show a sample transaction of the NetBill protocol. The customer process receives an order for electronic goods $g$ at a merchant $m$ from the environment. It generates a unique transaction number $t$ which is used to identify the transaction in the subsequent message exchanges, and forwards the order to the merchant $m$. The merchant returns an invoice, which consists of a price statement and the encrypted goods. The customer process then issues a cheque to the merchant, which states that it is willing to pay the price for the goods. This cheque is digitally clearsinged: Every participant in the protocol can read it, but it is impossible for anyone to change the information in it. This cheque, together with the key for decrypting the goods, is forwarded to the bank. The bank returns a receipt and the key to the merchant, which forwards it to the customer. With this key, the customer process decrypts the goods received earlier and delivers them to the user.

Figures 3(a) and 3(b) show a transaction from the point of view of the
customer and merchant, respectively. The state structures are explained in more detail in Sections 3.2 and 3.3.

In this section, we give a formal specification of a simplified NetBill system. In Section 3.1 we define the state and message types used in the transaction protocol; Sections 3.2 to 3.4 contain component specifications for the customers, merchants and the bank.

3.1 General Definitions

Basic types. Figure 4 shows the architecture of a NetBill system. It consists of an arbitrary number of customers, an arbitrary number of merchants and the centralized bank server. Customers are identified by elements from a set $CID$ of customer identifiers; similarly, we assume a set $MID$ of merchant identifiers.

In contrast to other NetBill formalizations, we allow an arbitrary number of overlapping transactions between each customer and each merchant, i.e. a customer may order goods even if another transaction is not yet finished. To identify the various transactions, we assume a set $TID$ of transaction IDs that consist of a pair of the concerned customer and a unique serial number, i.e. $TID \subseteq CID \times \mathbb{N}$.

The electronic goods handled by the protocol are taken from a set $GOODS$; for each good there is a unique identifier in the set $GID$. We use a bijective
function

\[ shelf : \text{GID} \to \text{GOODS} \]

to map IDs to the corresponding goods. Monetary values for prices are modeled by elements from a set \( \mathbb{M} \). The prices of the goods are yielded by the function

\[ price : \text{GID} \to \mathbb{M} \]

**Encryption.** An essential part of any e-commerce protocol is encryption of messages. The NetBill transaction protocol uses both symmetric and public key cryptography. We abstract from the underlying algorithms, and just assume that for each message set \( M \) there exists a set of encrypted messages \( \overline{M} \) and two functions

\[ \text{ENCRYPT} : Key \to M \to \overline{M} \quad \text{and} \quad \text{DECRYPT} : Key \to \overline{M} \to M \]

For symmetric encryption, we demand that

\[ \text{DECRYPT} \ k \ (\text{ENCRYPT} \ k \ m) = m \]

For public key encryption, we demand that the public keys of the customers are freely accessible by a function

\[ \text{PUBKEY} : \text{CID} \to Key \]

and a message that is signed by customer \( c \) (with private key \( k_c \)) can be decrypted with the public key:

\[ \text{DECRYPT} \ \text{PUBKEY}.c \ (\text{ENCRYPT} \ k_c \ m) = m \]

This latter requirement is satisfied, for example, by the well known RSA algorithm.
Message types. We define a number of complex data types to be used for messages on the communication channels.

- Environment/Customer: The only messages from the environment to the customer process are goods requests parameterized by the ID of the goods ordered, and the ID of the merchant from which the good is ordered:

  \[ T_{ec} ::= \text{Request}(\text{MID} \times \text{GID}) \]

  The only messages returned to the environment are the goods:

  \[ T_{ce} ::= \text{Delivery}(\text{MID} \times \text{GOODS}) \]

- Customer/Merchant: The customer sends two kinds of messages to the merchant: Orders of a certain good, and signed cheques which the merchant then forwards to the bank for further processing.

  \[ T_{cm} ::= \text{Order}(\text{TID} \times \text{GID}) \\
  \quad \text{\mid Cheque}(\text{TID} \times (\text{CID} \times \text{MID} \times \text{GID} \times \text{M})) \]

  The merchant sends two kinds of messages to the customer:

  \[ T_{mc} ::= \text{Invoice}(\text{TID} \times \text{GOODS} \times \text{M}) \\
  \quad \text{\mid Receipt}(\text{TID} \times \text{KEY}) \]

- Merchant/Bank: The merchant forwards the cheques to the bank together with the key to decrypt the involved goods.

  \[ T_{mb} ::= \text{KeyCheque}(\text{TID} \times \text{KEY} \times (\text{CID} \times \text{MID} \times \text{GID} \times \text{M})) \]

  The bank sends a receipt to the merchant (to be forwarded to the customer) as a signal that the money transfer has succeeded.

  \[ T_{bm} ::= \text{Receipt}(\text{TID} \times \text{KEY}) \]

We sometimes form message sets by replacing parameters of a message constructor with the placeholder “\(\cdot\)”. For example, we write

\[ \text{Receipt}(t, \cdot) \text{ for } \bigcup_{k \in \text{KEY}} \{\text{Receipt}(t, k)\} \]
3.2 Customer

A customer is identified by its ID. The state space of each customer consists of its private key, and a mapping from transaction IDs to a CustTrans record. This record holds the control state of the transition according to Figure 3(a) as well as the goods ID, the merchant ID, the encrypted goods, the price of the goods, and the decryption key:

\[
\text{CustTrans} := \begin{align*}
\text{record} & \\
\text{phase} & : \{\text{Idle, Ordered, Confirmed, Done}\} \\
\text{gid} & : \text{GID} \\
\text{mid} & : \text{MID} \\
\text{goods} & : \text{GOODS} \\
\text{price} & : \text{M} \\
\text{key} & : \text{KEY}
\end{align*}
\]

Transactions IDs are determined by the customer; our specification uses a rather simple allocation scheme based on a variable nexttid, which holds the next free ID.

Thus, a customer \( c \in \text{CID} \) is specified in Figure 5. Initially, all transactions are idle. Each transaction is processed on the customer side as shown in Figure 3(a). A transaction gets activated by the transition order that is always enabled. The customer process receives an order consisting of a goods ID and a merchant ID to describe what should be bought from which merchant. A new transaction number is generated, all required data are stored in \( \text{st},t \), and the order is forwarded to the merchant.

The transition pay accepts the encrypted goods, and generates a signed cheque that is sent to the merchant. This transition is only enabled if the price the merchant offers is less than the price the customer expects. Note that the customer cannot be sure if he got the correct goods, since they are encrypted.

Finally, the customer process gets the decryption key from the merchant (transition accept). If the goods are the goods it expected, they are sent to the system environment, and the transaction status is set to Done.

3.3 Merchant

On the merchant side, each transaction is processed as shown in Figure 3(b). Each merchant must store for each transition the control state from Figure 3(b) as well as goods ID, customer ID, price and the decryption key:
Figure 5: Customer Specification
Figure 6: Merchant Specification
\[Merch\, Trans \ ::= \ \textbf{record} \]
\[
\begin{align*}
\text{phase} & : \ \{\text{Idle, Delivered, Cashing, Done}\} \\
gid & : \ \text{GID} \\
cid & : \ \text{CID} \\
price & : \ M \\
key & : \ \text{KEY}
\end{align*}
\]
\[\textbf{end}\]

A merchant can recognize a new order since the corresponding transaction \(t\) is in the phase \texttt{Idle}. We do not model malicious behaviors, so we do not check if the transaction number really corresponds to the customer who sent the order, encoded by the name of the channel on which the order was received. The transition \texttt{deliverGoods} immediately sends the encrypted goods to the customer, and remembers relevant information in \texttt{st.t}. The transition \texttt{cashCheque} examines the cheque of the customer, and forwards it to the bank. If the bank confirms the receipt, the key will be sent to the customer and the transaction is completed. Merchants are specified formally in Figure 6.

### 3.4 Bank

The bank state consists of an account for each customer and each merchant, and a store of transaction descriptions that is modeled as a partial function from transaction IDs to description tuples.

The bank, specified in Figure 7, has only one transition that is enabled if a received cheque is correctly signed from the customer. The bank then transfers the money from the customers account to the merchants account, and sends a receipt to the merchant. It stores the information for eventual later requests.

### 4 NetBill Verification

In this section we show that our NetBill specification satisfies the following properties:

- \textit{Guaranteed Delivery}: All goods ordered by the customer are delivered. Conversely, goods are only delivered if they were ordered. This is a typical black box property of the system; we use the verification techniques of \([2, 5]\) to derive it.
**Bank**

**inputs**

\[ m \cdot b : T_{mb}, \text{ for all } m \in \text{MID} \]

**outputs**

\[ b \cdot m : T_{bm}, \text{ for all } m \in \text{MID} \]

**state**

\[\begin{align*}
\text{MACCOUNT} & : \text{MID} \rightarrow \text{M} \\
\text{CACCOUNT} & : \text{CID} \rightarrow \text{M} \\
\text{keystore} & : \text{TID} \rightarrow (\text{KEY} \times \text{CID} \times \text{MID} \\
& \quad \times \text{GID} \times \text{M})
\end{align*}\]

**initialization**

\[\text{keystore} = \emptyset\]

**transition relation**

\[\text{transfer}[m, t]\]

---

**transfer[m, t]**

**pre**

\[\begin{align*}
\text{let } (cid, mid, gid, pr) &= \text{DECRIPT(Pubkey, fst.t, ch)} \\
\text{in } cid &= fst.t \\
\text{end}
\end{align*}\]

**input**

\[m \cdot b? \text{KeyCheque}(t, k, ch)\]

**output**

\[b \cdot m! \text{Receipt}(t, k)\]

**post**

\[\begin{align*}
\text{let } (cid, mid, gid, pr) &= \text{DECRIPT(Pubkey, fst.t, ch)} \\
\text{in } &\text{MACCOUNT,mid} := \text{MACCOUNT,mid} + pr \\
&\text{CACCOUNT.cid} := \text{CACCOUNT.cid} - pr \\
\text{end}
\end{align*}\]

\[\text{keystore} := \text{keystore} \cup \{t \mapsto (k, cid, mid, gid, pr)\}\]

---

Figure 7: Bank Specification
• **Guaranteed Payment:** For all goods ordered by a customer, the money amount corresponding to the good’s price is subtracted from the customer account.

• **Money Atomicity:** The sum of the customer and merchant accounts remains invariant. Because of the centralized NetBill bank server, this property is quite obvious, it follows immediately from a simple formula in predicate logic.

In Section 4.1, we state and prove some invariants of the NetBill protocol. They are used in Section 4.2 to prove a number of basic safety and progress properties of the system components. In Sections 4.3 and 4.4 we show the correctness statements mentioned above.

### 4.1 Invariants

For the property proofs, we make use of two invariants. One invariant describes the NetBill behavior from the point of view of a single transaction, the other describes it from the point of view of a given customer.

In this section, we frequently refer to a given customer $c$ and merchant $m$. For readability, we sometimes abbreviate $Customer[c]$ and $Merchant[m]$ by $C$ and $M$, respectively.

#### 4.1.1 Transaction View.

The transaction view describes the system state for all eight phases of a single NetBill transaction $t$ between a customer $c$ and a merchant $m$ (see Figure 2). It is visualized in Figure 8. For example, the first row represents the state in which the transaction is still inactive: Customer and merchant are still idle, and no messages have been sent along the channels. Two subsequent rows describe a transition of either the customer, merchant or bank. For example, the second state reflects the system state after an order transition of the customer: The control state of the customer changes to ORDERED, and a new message *Order* is produced on channel $c . m$. The other channels remain unchanged. In the table, newly generated messages are highlighted by boxes. A difference of the number of messages on a channel $x$ and $x^*$ means that there are unread messages available on that channel. Note that the changes in the control state directly follow the state machine structures in Figures 3(a) and 3(b).

In addition to the control state and data flow information of Figure 8, the system accumulates information about the ordered goods, keys, and price in the customer and merchant data state variables. For each of the eight phases
<table>
<thead>
<tr>
<th></th>
<th>#Order(*,$)</th>
<th>#Invoice(*,$)</th>
<th>#Cheque(*,$)</th>
<th>#Receipt(*,$)</th>
<th>#KeyCheque(*,$)</th>
<th>#KeyReceipt(*,$)</th>
<th>Cstr.phase</th>
<th>Mstr.phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma_1 )</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>Idle</td>
<td>Idle</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_2 )</td>
<td>1 0 0 0 0</td>
<td>1 0 0 0 0</td>
<td>1 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>Ordered</td>
<td>Idle</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_3 )</td>
<td>1 1 0 0 0</td>
<td>1 1 0 0 0</td>
<td>1 1 0 0 0</td>
<td>0 0 0 0 0</td>
<td>0 0 0 0 0</td>
<td>Confirmed</td>
<td>Delivered</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_4 )</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>Confirmed</td>
<td>Cashing</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_5 )</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>Confirmed</td>
<td>Done</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_6 )</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>Confirmed</td>
<td>Done</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_7 )</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>Confirmed</td>
<td>Done</td>
<td></td>
</tr>
<tr>
<td>( \Gamma_8 )</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>1 1 1 1 1</td>
<td>Confirmed</td>
<td>Done</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8: Transaction View: Control Invariant
\[ \Delta_1 \overset{a}{=} \text{true} \]
\[ \Delta_2 \overset{a}{=} \Delta_1 \land \# \text{Order}(t, g) \oplus \text{c} \cdot \text{m} = 1 \land \text{c.st.t.id} = g \]
\[ \Delta_3 \overset{a}{=} \Delta_2 \land \# \text{Invoice}(t, \text{ENCRYPT}(k, \text{shelf.g}), \text{price.g}) \oplus \text{m} \cdot \text{c} = 1 \land \text{M.st.t.id} = g \land \text{M.st.t.key} = k \]
\[ \Delta_4 \overset{a}{=} \Delta_3 \land \# \text{Cheque}(t, \text{ENCRYPT C.pk}(c, m, \text{c.st.t.id}, \text{price.g})) \oplus \text{c} \cdot \text{m} = 1 \land \text{c.st.t.goods} = \text{ENCRYPT}(k, \text{shelf.g}) \]
\[ \Delta_5 \overset{a}{=} \Delta_4 \land \# \text{KeyCheque}(t, \text{M.st.t.key}, \text{ENCRYPT C.pk}(c, m, \text{c.st.t.id}, \text{price.g})) \oplus \text{m} \cdot \text{b} = 1 \]
\[ \Delta_6 \overset{a}{=} \Delta_5 \land \# \text{Receipt}(t, \text{M.st.t.key}) \oplus \text{b} \cdot \text{m} = 1 \]
\[ \Delta_7 \overset{a}{=} \Delta_6 \land \# \text{Receipt}(t, \text{M.st.t.key}) \oplus \text{m} \cdot \text{c} = 1 \]
\[ \Delta_8 \overset{a}{=} \Delta_7 \]

Figure 9: Transaction View: Data Invariant

In the protocol, we formulate a property \( \Delta_i \) that relates the components’ data state and message contents. The data invariant is shown in Figure 9. Note that in our example, the set of properties becomes stronger in every phase, since all transitions just set internal values that were undefined before. We only state a subset of the valid properties that is sufficient for the correctness proofs later; it is straightforward to extend them with pricing information.

Together, Figures 8 and 9 illustrate the following invariant for the NetBill system:

\[ \forall c, m, t \cdot \exists g, k \cdot \bigvee_{i=1}^{8} (\Gamma_i \land \Delta_i) \]

The proof of the invariant proceeds along the typical inductive proof structure for invariants: It holds initially, and is not violated by any system transition.

### 4.1.2 Customer View.

While the transaction invariant describes the system states for a given customer \( c \), merchant \( m \) and transaction \( t \), we now formalize an invariant that hides the transactions and just refers to the goods a customer ordered.

The customer issues orders to the NetBill system and accepts the delivered goods. Each order is handled by a single transaction. Transactions that did not yet lead to delivery of a good are called pending. The set of pending transactions for an order of a good \( g \) by the customer \( c \) at the merchant \( m \)
is denoted by $\mathcal{P}_{c, m, g}$ and defined as follows:

$$
\mathcal{P}_{c, m, g} \overset{\Delta}{=} \{ t \in TID \mid \text{Customer}\{c\}.st.t.phase \not\in \{\text{Idle, Done}\} \land \\
\text{Customer}\{c\}.st.t.gid = g \} 
$$

From the customer point of view, the number of orders for a good $g$ should equal the number of deliveries of this good plus the number of pending transactions for $g$. This invariant is formally expressed as

$$
\forall c, m, g \cdot \#\text{Delivery}(m, \text{shelf}.g) \circ e + |\mathcal{P}_{c, m, g}| = \#\text{Request}(m, g) \circ e \circ e^* 
$$

It is easy, though tedious, to prove this invariant. The proof structure is visualized by the verification diagram in Figure 10, which splits the invariant into the two cases $|\mathcal{P}_{c, m, g}| = 0$ and $|\mathcal{P}_{c, m, g}| > 0$. Each of the diagram nodes represents one case. The free variables $c, m, g$ are fixed by skolemizing the quantifiers of the invariant above.

The opaque dot in the upper diagram node means that initially the upper case of the invariant holds; this is easy to see from the customer specification. The arrows show the verification obligations for the inductive step: Each new order increases $|\mathcal{P}_{c, m, g}|$ and the number of processed Request messages; each delivery of a good identified by $g$ decreases $|\mathcal{P}_{c, m, g}|$, but increases the number of Delivery messages at the same time. Orders and deliveries of a different good $\tilde{g}$ as well as the customer payment transitions leave the invariant unchanged. Moreover, although not represented in the diagram, it is easy to see that the invariant is also left unchanged by transitions of other customers, other merchants or the bank.

### 4.2 Black Box Properties

From the invariance and enabledness properties of the previous two sections, we now derive black box safety and liveness properties. The liveness properties are of a special form, which we call progress: They express that all external input to a component is processed.

#### 4.2.1 Safety Properties.

From the transition view invariant, we can derive a number of black box safety properties for a fixed transaction $t$, customer $c$, merchant $m$ and the bank:
To see that each of these properties indeed holds for the systems black box view, note that each property is an equality of continuous functions on streams. Moreover, each property is a state machine invariant of the Net-Bill system, which can be seen by comparing the entries for the left and right hand side of each equation in the transition view invariant table shown in Figure 8. For example, the third and the sixth column have the same entries, which means the the number of Cheque- and Invoice—messages on $c \cdot m$ and $m \cdot c^*$ are equal in all phases of a transaction. This proves the customer property.

In addition to the property above, the customer satisfies two other black box safety properties:

$$\# \text{Order}(\cdot, \cdot) \subseteq c \cdot m = \# \text{Receive}(\cdot, \cdot) \subseteq m \cdot c$$

$$\# \text{Delivery}(m, \cdot) \subseteq c \cdot m = \# \text{Receive}(\cdot, \cdot) \subseteq m \cdot c$$
These properties do not refer to given goods or transactions. They are based on invariants which again are easy to show using standard invariant verification techniques.

4.2.2 Progress Properties.

The black box properties above relate the consumed input and the produced output of each component; they are pure safety properties, derived from state machine invariants like the transition invariant of Figure 8. The NetBill system enjoys also black box properties that relate the external input and the consumed input for each component:

**Customer:**

\[
\# \text{Request}(\cdot, \cdot) \in \text{e} \cdot \text{c}^* = \# \text{Request}(\cdot, \cdot) \in \text{e} \cdot \text{c}
\]

\[
\# \text{Invoice}(t, \cdot) \in \text{m} \cdot \text{c}^* = \# \text{Invoice}(t, \cdot) \in \text{m} \cdot \text{c}
\]

\[
\# \text{Receipt}(t, \cdot) \in \text{m} \cdot \text{c}^* = \# \text{Receipt}(t, \cdot) \in \text{m} \cdot \text{c}
\]

**Merchant:**

\[
\# \text{Order}(t, \cdot) \in \text{m} \cdot \text{c}^* = \# \text{Order}(t, \cdot) \in \text{m} \cdot \text{c}
\]

\[
\# \text{Cheque}(t, \cdot, \cdot) \in \text{m} \cdot \text{c}^* = \# \text{Cheque}(t, \cdot, \cdot) \in \text{m} \cdot \text{c}
\]

\[
\# \text{Receipt}(t, \cdot) \in \text{b} \cdot \text{m}^* = \# \text{Receipt}(t, \cdot) \in \text{b} \cdot \text{m}
\]

**Bank:**

\[
\# \text{KeyCheque}(t, \cdot, \cdot) \in \text{m} \cdot \text{b}^* = \# \text{KeyCheque}(t, \cdot, \cdot) \in \text{m} \cdot \text{b}
\]

These properties are immediate consequences of the five black box properties \( \text{e} \cdot \text{c}^* = \text{e} \cdot \text{c}, \text{m} \cdot \text{c}^* = \text{m} \cdot \text{c}, \text{c} \cdot \text{m}^* = \text{c} \cdot \text{m}, \text{b} \cdot \text{m}^* = \text{b} \cdot \text{m} \) and \( \text{m} \cdot \text{b}^* = \text{m} \cdot \text{b} \).

For the customer property \( \text{m} \cdot \text{c}^* = \text{m} \cdot \text{c} \), it is sufficient to show that \( \# \text{m} \cdot \text{c}^* \geq \# \text{m} \cdot \text{c} \), since \( \text{m} \cdot \text{c}^* \subseteq \text{m} \cdot \text{c} \) is by construction an invariant. This length property can be reduced to the following temporal logic property:

\[
\Box \left( \# \text{m} \cdot \text{c}^* = k \land \# \text{m} \cdot \text{c} > k \Rightarrow \Diamond \left( \# \text{m} \cdot \text{c}^* > k \right) \right)
\]

This is an example of an output extension property. To prove is, it is sufficient to find a helpful transition \( \tau \) such that:

\[
\# \text{m} \cdot \text{c}^* = k \land \# \text{m} \cdot \text{c} > k \Rightarrow \text{En}(\tau) \quad \text{and}
\]

\[
\# \text{m} \cdot \text{c}^* = k \land \# \text{m} \cdot \text{c} > k \land \tau \Rightarrow \# (\text{m} \cdot \text{c}^*)' > k
\]

Assume now that the system is in a state where \( \# \text{m} \cdot \text{c}^* = k \land \# \text{m} \cdot \text{c} > k \). Then we know that there is at least one unprocessed message in \( \text{m} \cdot \text{c} \). In other words, there exists a message \( x \) such that

\[
\text{m} \cdot \text{c}^* \vdash (x) \subseteq \text{m} \cdot \text{c}
\]

According to the type definition of \( \text{T}_m \cdot \text{c} \) in Section 3.1, the message \( x \) is either an invoice, or a receipt with a key; in any case, it refers to a transaction \( t \):
1. $x \in Invoice(t, \cdot)$: Then $\#Invoice(t, \cdot) \preceq m \cdot c^* < \#Invoice(t, \cdot) \preceq mc$. From Figure 8, we now that the system must then be in a state characterized by $\Gamma_3$, and thus by $\Gamma_3 \land \Delta_3$.

In this case, the helpful transition is the customer transition $pay$. It is enabled because according to $\Delta_3$ the customer control state of $t$ is ORDERED, and according to the assumption the next unread message is an invoice for $t$. Moreover, since $pay$ indeed consumes the invoice message, the length of the processed input $m \cdot c^*$ is extended.

2. $x \in Receipt(t, \cdot)$: This case is similar to the one above; instead of the third phase, however, the system must be in phase 7: The system state is characterized by $\Gamma_7 \land \Delta_7$, and the helpful transition is the customer transition $accept$.

The proof for the other internal channels is analogous. The proof that $e \cdot c^* = e \cdot c$ is a bit different: Instead of appealing to the transition invariant table, we make use of the fact that there is always a free transaction ID ($nextid$), so that transition $order$ is enabled whenever there is unprocessed input on channel $e \cdot c$.

### 4.3 Guaranteed Delivery

The black box correctness property states that all ordered goods are delivered, and that all delivered goods have been ordered:

$$\forall c, m, g \bullet \#Delivery(m, shelf, g) \preceq c \cdot e = \#Request(m, g) \preceq e \cdot c$$

This property can be split into a safety and a liveness part:

$$\forall c, m, g \bullet \#Delivery(m, shelf, g) \preceq c \cdot e \leq \#Request(m, g) \preceq e \cdot c$$

$$\forall c, m \bullet \#Delivery(m, \cdot) \preceq c \cdot e \geq \#Request(m, \cdot) \preceq e \cdot c$$

The liveness part need not to refer to a goods ID. Since the correctness of the output follows already from the safety part, it is sufficient to show that enough output is produced.

#### 4.3.1 Safety.

For the safety part, we observe that

$$\#Delivery(m, shelf, g) \preceq c \cdot e$$

$$\leq \#Delivery(m, shelf, g) \preceq c \cdot e + |P_{c,m,g}| \text{ since } |P_{c,m,g} \geq 0|$$

$$\leq \#Request(m, \cdot) \preceq e \cdot c^* \text{ by customer view invariant}$$

$$\leq \#Request(m, g) \preceq e \cdot c \text{ since } e \cdot c^* \subseteq e \cdot c$$
Thus, the safety formula is by itself an invariant of the NetBill system. It is also admissible, and is thus also valid at the black box level, where all free variables range over the limits of a system execution.

4.3.2 Liveness.

For the liveness part we show the following equality sequence for all $c$ and $m$:

\[
\#Delivery(m, \cdot)\circ c\cdot e = \#Receipt(\cdot, \cdot)\circ m\cdot c^* \quad \text{by customer safety}
\]

\[
= \#Order(\cdot, \cdot)\circ c\cdot m \quad \text{see below}
\]

\[
= \#Request(m, \cdot)\circ c\cdot e^* \quad \text{by customer safety}
\]

\[
= \#Request(m, \cdot)\circ c\cdot e \quad \text{by customer progress}
\]

To show the second equality, we observe that for all $c$, $m$ and $t$:

\[
\#Receipt(t, \cdot)\circ m\cdot c^* = \#Receipt(t, \cdot)\circ m\cdot c \quad \text{by customer progress}
\]

\[
= \#Receipt(t, \cdot)\circ b\cdot m^* \quad \text{by merchant progress}
\]

\[
= \#Receipt(t, \cdot)\circ b\cdot m \
\]

\[
= \#KeyChaque(t, \cdot, \cdot)\circ m\cdot b^* \quad \text{by bank safety}
\]

\[
= \#KeyChaque(t, \cdot, \cdot)\circ m\cdot b \
\]

\[
= \#Cheque(t, \cdot, \cdot)\circ c\cdot m^* \
\]

\[
= \#Cheque(t, \cdot, \cdot)\circ c\cdot m \
\]

\[
= \#Invoice(t, \cdot, \cdot)\circ m\cdot c^* \quad \text{by customer safety}
\]

\[
= \#Invoice(t, \cdot, \cdot)\circ m\cdot c \
\]

\[
= \#Order(t, \cdot, \cdot)\circ c\cdot m^* \quad \text{by merchant safety}
\]

\[
= \#Order(t, \cdot, \cdot)\circ c\cdot m \quad \text{by merchant progress}
\]

Each of the safety and progress steps in this sequence is immediate from the black box properties of Section 4.2.

4.4 Guaranteed Payment and Money Atomicity

The previous section showed the correctness of a black box property, which only refers to the channel communication histories. The correctness proof made use of both temporal logic properties (the invariance and response properties), and of predicate logic properties (for example, to show the enabledness of a transition).
However, temporal and predicate logic are useful not only for the derivation of black box properties. There are some properties, which are best expressed by temporal logic because they refer to the internal state of a component, and properties best expressed by predicate logic, because they also refer to a given transition.

4.4.1 Temporal Logic.

This level is useful when properties about the internal states of a component are formalized. In the NetBill example, we can specify that if the customer orders a good, at a later state the price of the good will have been subtracted from the customer account.

\[
\forall x \in M, c \in CID, m \in MID, t \in TID \bullet
\]

\[
\Box \left( Bank.CAccount.c = x \land \right.
\]

\[
Customer[c].st.phase \in \{\text{Ordered, Confirmed}\}
\]

\[
\Rightarrow \Box \left( Bank.CAccount.c \leq x - price(Customer[c].st.gid) \right)
\]

Note that it would be quite difficult to precisely characterize the amount of money on the customer account, because there can be any number of active transactions being processed at any time.

4.4.2 Predicate Logic.

The predicate logic level is useful when properties of single transitions are formalized. In the NetBill example, all money related activities occur at a centralized bank server, which is specified by a single transition. The property that within the NetBill system money is neither destroyed nor created can be expressed as the following formula in first-order logic:

\[
\forall x \in M, c \in CID, m \in MID \bullet
\]

\[
(\text{Bank.CAccount}.c + \text{Bank.MAccount}.m = x) \land \text{transfer}
\]

\[
\Rightarrow (\text{Bank.CAccount}'.c + \text{Bank.MAccount}.m' = x)
\]

5 Discussion

In this contribution, we used a simplified formalization of the NetBill protocol, which differs from the description in [11] in various respects. In this section, we discuss possible extensions of our model and briefly summarize the proof methodology from Section 4.
5.1 NetBill Model

In the original description of the NetBill protocol, the customer process can query the bank for the receipt and decryption key for a given transaction. This ensures that the customer can decrypt the goods even if the network connection between bank, merchant and customer can lose messages.

For customer queries, the system structure diagram must be extended by channels that connect the customers and the bank server. The transaction state transition diagrams must be extended as shown in Figure 11. It is straightforward to define the new transitions \textit{query}, \textit{acceptBank} and \textit{dropKey}.

![Diagram of NetBill Model](image)

(a) Customer view  
(b) Merchant view

Figure 11: Customer and Merchant View of a Transaction

Moreover, the message types in [11] are more complex. In our model we removed much redundancy; we also abstracted from some details of the encryption algorithms. Finally, we do not examine malicious merchants and customers or limited customer credit lines, although of course the NetBill protocol has to be examined carefully for these cases.

5.2 Proof Methodology

Many of the proofs in Sections 4.1–4.3 are not completely formal; they cannot be, since we did not give precise translations definitions of the component state machine transitions into logic. Still, we hope that the reader can get the gist of the proof style that we used:
• Based on the message sequence chart of the NetBill protocol (Figure 2) and the transition state transition diagrams (Figure 3) the reachable states from the perspective of a single transactions could be concisely described in tabular form (Figure 8). Again, we did not give a completely formal translation of the table into a logical expression, we refer to Broy’s work on tabular specification techniques [7] for details.

• From the invariant table, a number of black box safety properties that relate input and output of each system component are immediate. Only the communication along the two channels that connect the system to the environment had to be treated specially.

• For black box liveness properties, we first proved component progress: Each component consumes all its input. For progress, we made use of temporal logic and black box verification rules from [2], which are tailored to proving the length properties typical for progress arguments. The premises of these rules—essentially enabledness statements for helpful transitions—could easily be discharged by a case split on the message types, and again a lookup in the invariant table.

• For the final black box correctness proof it is then sufficient to assemble the component black box properties in a chain of (in)equations.

Note that in this way, the correctness of the protocol is proven compositionally: History specifications in the style of Focus are a modular description technique that allows succinct reasoning with black box properties of components.

In previous examples [2, 3], we proved liveness properties directly, instead of showing progress first. The approach here seems to be more schematic and easier to scale to larger systems.

We did not yet attempt the correctness proofs for the extended NetBill version with customer queries, malicious customer or merchant behavior or limited credit lines; it remains to be seen whether and which parts of the current proof can be reused for the new aspects.

6 Conclusion

State based and I/O history based views of the system can be linked by temporal logical formulas for invariance and response. This technique is documented in more detail in [2, 8, 3, 5], where simple buffer examples are verified. In this contribution, we applied specification and proof principles
for black box properties of distributed systems to the NetBill protocol [11, 19] for electronic payments over the Internet.

There is no single language for the formulation of mathematical correctness properties of a system. Simple Hoare-like verification conditions, temporal logic formulas and history relations in the style of Focus [9, 1] allow the formulation of properties at different abstraction levels. Conversely, verification conditions for each level lead to verification conditions formulated in the lower levels languages.

The black box correctness property of the NetBill system is assembled from black box properties of the individual components. This shows how the inherent modularity of Focus specifications can be used for concise compositional proofs: Both liveness and safety arguments are reduced to simple (in)equality reasoning.

The simple structure of the temporal logic formulas needed for the verification of black box properties is well-suited for verification diagrams [14, 15] in order to structure property proofs. While the verification conditions associated with a verification diagram are simple, their sheer number requires tool support. In [6], we present a formalization of our approach in Isabelle/HOL[12]. It consists of extensions of Shankar’s PVS formalization of state machines [18] to handle liveness properties and asynchronous communication as well as verification tactics tailored to invariance and response diagrams. The Isabelle formalization and verification diagrams are documented in a technical reports [6]. The theory files and proof scripts can be accessed electronically [4].

Typically, when examining protocols for e-commerce applications, the focus is on security, and less on the safety and liveness issues handled by our approach. Further work will combine our verification techniques with Paulson’s inductive approach to protocol verification [17], and with formal models of threat scenarios [13].
References


