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Yukawa Hierarchies and the See-saw Mechanism

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Abstract:
The strong hierarchies in the Yukawa couplings allow, via the principle of Minimal Flavour Violation, to explain the smallness of quark and charged lepton flavour changing neutral interactions in new physics models. We study the renormalization group evolution of the Minimal Flavour Violation ansatz in the MSSM as well as of the more specific aligned 2HDM, ascertaining that flavour changing neutral interactions are within experimental bounds for large portions of the parameter space. In models with right-handed neutrinos it is natural to postulate also hierarchical neutrino Yukawa eigenvalues. However, this requires specific solutions for the mixing angles and right-handed neutrino masses in order to obtain the observed neutrino parameters via the see-saw mechanism. In the MSSM we show that, barring cancellations, the non-observation of $\mu \to e\gamma$ implies the existence of an upper bound on the smallest neutrino Yukawa eigenvalue and the lightest right-handed neutrino mass. Moreover we discuss the implications of these bounds on leptogenesis. In the general 2HDM, there is a generic way to obtain the observed mild neutrino mass hierarchy. We show that this holds true even in the decoupling limit of the second Higgs where all problems usually associated to the general 2HDM are alleviated.

Zusammenfassung:
Die stark hierarchischen Yukawa-Kopplungen ermöglichen es mit Hilfe des Prinzips der Minimalen Flavour-Verletzung zu erklären, warum keine aus Modellen neuer Physik resultierenden Flavour-ändernden neutralen Wechselwirkungen entdeckt wurden. Wir betrachten die Renormierungsgruppen-Effekte im Minimal Flavour-verletzenden MSSM und im spezielleren aligned 2HDM. Dabei bestätigen wir, dass diese Wechselwirkungen für einen großen Teil des Parameterbereichs innerhalb der experimentellen Schranken sind. In Modellen mit rechtshändigen Neutrinos ist die Annahme natürlich, dass auch die Neutrino-Yukawa-Eigenwerte hierarchisch sind. Allerdings erfordert dies spezielle Parameter im See-saw-Mechanismus, um mit den gemessenen Neutrino-Daten im Einklang zu sein. Im MSSM zeigen wir, dass, abgesehen von einer möglichen Aufhebung von Termen, die Nicht-Entdeckung von $\mu \to e\gamma$ eine obere Schranke sowohl für den kleinsten Neutrino-Yukawa-Eigenwert als auch die leichteste rechtshändige Neutrinomasse impliziert und diskutieren deren Bedeutung für Leptogenese. Im allgemeinen 2HDM kann die beobachtete milde Hierarchie in Neutrinomassen generisch erzeugt werden. Wir stellen fest, dass dies selbst im Grenzfall gilt, in dem das zweite Higgs entkoppelt und die Probleme geringer werden, die mit dem allgemeinen 2HDM üblicherweise assoziiert werden.

Contents

1 Introduction

1.1. Preface

With the copious production of the top quark at the Tevatron it became possible to access all charged fermions of the Standard Model (SM) directly in experiments. This led to quite a good determination of all of the SM Yukawa couplings [1] – apart, maybe, from the theoretical uncertainties in the light quark masses. From the theoretical side several features of the Yukawa couplings have been noticed and tried to be explained in several models. For example, the ratio of τ lepton to bottom quark mass can be nicely explained under the assumption of a Grand Unified Theory (GUT) [2, 3], and also the approximate relations of the down and strange to the electron and muon masses may be understood in a model with grand unification [4].

The most conspicuous feature of the Yukawa couplings, however, is the existence of strong mass hierarchies between the three families, in the up-quark, down-quark and the charged lepton sector. These hierarchies vary between about 20 and a few hundred. Again, several explanations have been proposed, most notably the Froggatt-Nielsen mechanism [5].

In this work we will not try to explain any feature of the Yukawa couplings. Instead, we will take a more pragmatic approach, namely projecting the feature of hierarchical flavour couplings onto extensions of the SM. This approach is based on the expectation that there is some principle organizing the SM Yukawa couplings, and on the hope that the same principle governs in an analogous way additional flavour structures in extensions of the SM.

There are good reasons to expect new physics to appear at the electroweak scale. However, in this case some organizing principle needs typically to be invoked also in order to protect the model from too large Flavour Changing Neutral Currents (FCNCs). This can be achieved efficiently by invoking the same mechanism for the suppression of FCNCs in the SM and its extension. In such a scenario the new physics model might obey the principle of Minimal Flavour Violation (MFV), which relates any flavour violating matrix to the Yukawa couplings. Although this principle can be applied rather generally, its theoretical motivation depends on the new physics model considered.

Indeed, depending on the kind of new physics model assumed, the expectations that one could extract from the SM, can vary strongly. A nice example is the see-saw model that will be introduced in more detail in section 3.2: In analogy to the SM quark sector one might infer the existence of three right-handed neutrinos and a corresponding Yukawa matrix. Naively this would lead to Dirac neutrino masses. Moreover, the fact that neutrinos are much lighter than the lightest charged fermions, would indicate the failure of the analogy between charged and neutral fermions. Therefore, in order to explain the observed smallness of neutrino masses, it is crucial to realize that in contrast to the SM fermions, right-handed neutrinos are not charged under the SM gauge symmetry, thus naturally obtaining very large Majorana masses.

Furthermore, in analogy to quarks and charged leptons one expects, in models with three right-handed neutrinos, also the neutrino Yukawa eigenvalues to exhibit strong hierarchies. Due to the see-saw mechanism this does not conflict with the mild hierarchy of atmospheric to solar mass splitting. However, strong Yukawa hierarchies require special relations amongst the see-saw parameters. This fact might help in the reconstruction of the see-saw from low scale observables. Discarding this possibility as being unnatural, it points to extensions of the SM see-saw.

1.2. Outline

As this thesis deals with several ideas and topics, here we have decided to outline only the main ideas. Of course, more detailed introductions are included in the following chapters at places we found more appropriate.

Chapter 2 is devoted mainly to the quark sector, where the failure to find experimental deviations from the SM predictions on FCNCs is at present one of the most challenging constraints on new physics models. The focus of chapter 3 is on the neutrino mass generation via the see-saw mechanism.

We will examine two new physics models: The Minimal Supersymmetric Standard Model (MSSM) and the Two Higgs Doublet Model (2HDM) but some of the results can also be applied to more general classes of supersymmetric models or to multi Higgs doublet models respectively.

As the flavour sector of the aforementioned models as well as their motivation is quite different, also different suppression mechanisms may be suited better to suppress FCNCs. The necessity of FCNC suppression is illustrated in sections 2.1.1 and 2.1.2 with two examples. A possible solution is provided by the principle of MFV, a very efficient, predictive and reasonable ansatz both in the MSSM and the 2HDM, cf. sections 2.1.3, 2.2.1 and 2.3.1.

If the principle of MFV holds at one energy scale, it does at all scales. Still, the Renormalization Group (RG) evolution is important in order to obtain reasonable values for the free parameters of the MFV ansatz at low scales. This study is performed for the MSSM in section 2.2, where also the different behaviour of MFV violating terms in the RG evolution is contrasted.

In the 2HDM, tree-level FCNCs are absent if the Yukawa couplings are aligned. This ansatz, however, is not stable under the RG evolution. Instead, all terms obeying MFV are generated. In section 2.3.2 we study the induced deviations from alignment both analytically and numerically, as well as their implications on tree-level mediated FCNC processes. Lastly, we emphasize in section 2.3.3 that in the particular case of the 2HDM also more general flavour structures are permissible if the second Higgs is decoupled up to the TeV scale.

Chapter 3 starts with a short introduction to neutrinos and the see-saw mechanism in sections 3.1 and 3.2. We will point out how difficult, if not impossible, a test of the the see-saw is within the SM. Projecting the feature of hierarchical Yukawa eigenvalues to the neutrino sector, the measured neutrino mass splittings require special relations for the right-handed neutrino masses and mixings.

With the help of processes of charged Lepton Flavour Violation (cLFV) additional information on the see-saw might be obtained in the MSSM and the 2HDM, cf. sections 3.3.1 and 3.4.1. However, the large number of parameters in the see-saw makes impossible to extract any model independent prediction. We adopt the conjecture of hierarchical neutrino Yukawa eigenvalues, and work out its consequences in the MSSM in section 3.3.2 from the non-observation of cLFV in present and possibly future searches.

Next, we analyse see-saw-induced cLFV also in the aligned 2HDM in section 3.4.2.

In contrast to the aforementioned cases, in the general 2HDM a mechanism exists to understand the puzzle of why the neutrino mass hierarchy is so mild: this can be explained by the radiative generation of the second-to-heaviest mass eigenvalue. The inspection of this possibility is carried out in section 3.4.3, where we discuss in some detail the preconditions of this mechanism and the expectations on the neutrino reactor mixing angle.

Finally, the conclusions are presented in chapter 4.

Many ideas exposed in this thesis have already been published [6–9]. However, also several discussions are included which have not been presented before.

2 FCNCs in New Physics Models

2.1. On the reach of precision experiments

The SM including neutrino masses is very successful in describing particle physics up to the electroweak scale. At this scale new physics is expected to enter, mainly because of the hierarchy problem. Currently the LHC explores this scale directly and will either confirm this expectation or refute it with rather high confidence.

On the other hand, precision experiments have already constrained models beyond the SM up to much higher energy scales. The most far-reaching bound is obtained from negative searches for proton decay, strongly constraining GUTs even at about 10^{16} GeV. However, new physics models can generically evade this bound if they do not break baryon number. Therefore, in order to constrain a more general set of new physics models with precision experiments, one may look at processes which are not forbidden by any symmetry of the SM but suppressed, in order to facilitate a precise determination. Following this guideline one is led to consider processes suppressed in the SM by small Yukawa couplings, in particular mediated by FCNCs or processes exhibiting CP violation. For example, precision measurements of meson-antimeson mixing do not permit arbitrary flavour violating structures even for new physics at the PeV scale. Therefore, new physics at the electroweak scale is only viable if it implies some mechanism of suppression of CP violation and FCNCs. In the following we will illustrate the impact of quark FCNCs with some examples. Charged LFV in specific extensions of the SM will be subject of sections 3.3 and 3.4, an example of a CP violation observable will be given in section 2.3.3.

2.1.1. Meson-antimeson mixing

In meson-antimeson oscillations some flavour number is changed by two. The effective Hamiltonian describing $\Delta F = 2$ processes can be written as

$$\mathcal{H}_{\text{eff}}^{\Delta F=2} = \sum_{i,a} C_i^a Q_i^a \,, \tag{2.1}$$

where C_i^a are Wilson coefficients and Q_i^a are dimension six four quark operators which change some flavour number by two. The value of the Wilson coefficients is calculated most conveniently at or above the electroweak scale. The strong effects from the RG evolution down to the mass scale of the meson system have been derived in [10, 11]. In the remainder of this thesis we will be interested mostly in the B_s^0–$\bar{B}_s^0$ system and the scalar mediated operators

$$Q_2^{LR} = (\bar{b}P_L s)(\bar{b}P_R s) , \qquad Q_1^{SLL} = (\bar{b}P_L s)(\bar{b}P_L s) . \tag{2.2}$$

The mass difference can be calculated as [12]:

$$\Delta m_{B_s} = 2 \left| \langle \bar{B}_s^0 \right| \mathcal{H}_{\text{eff}}^{\Delta F=2} \left| B_s^0 \rangle \right| . \tag{2.3}$$

Finally, the matrix elements $\langle \bar{B}_s^0 | Q_i^a | B_s^0 \rangle$ can be calculated [12], using results from lattice QCD [13–15]. Taking together these results and focussing on the operators given in eq. (2.2) we find for the mass difference [16]:

$$\begin{aligned}\Delta m_{B_s} \simeq \Bigg| \Delta m_{B_s}^{\text{SM}} + \frac{2}{3} m_{B_s} f_{B_s}^2 \Bigg[& \Big(2.8 + 0.54 \log \frac{\mu}{\mu_W} \Big) C_2^{LR}(\mu) \\ & - \Big(1.2 + 0.12 \log \frac{\mu}{\mu_W} \Big) C_1^{SLL}(\mu) \Bigg] \Bigg| ,\end{aligned} \tag{2.4}$$

where $\Delta m_{B_s} = (117.0 \pm 0.8) \cdot 10^{-13}\,\text{GeV}$ and $m_{B_s} \simeq 5.37\,\text{GeV}$ [1]. For the decay constant we take $f_{B_s} = 250\,\text{MeV}$ [15] and μ_W is some scale $\mathcal{O}(100\,\text{GeV})$.

The SM prediction $\Delta m_{B_s}^{\text{SM}} = (125 \pm 13) \cdot 10^{-13}\,\text{GeV}$ [17] (cf. also [18]) agrees quite well with the experimental value. To estimate the allowed new physics contribution, we require it to be smaller than $\sim 13 \cdot 10^{-13}\,\text{GeV}$ which corresponds to the uncertainty in the SM prediction. Assuming one operator dominance and $\mu \sim 1\,\text{TeV}$, this yields the impressive constraints:

$$|C_2^{LR}| \lesssim (8 \cdot 10^5\,\text{GeV})^{-2} , \qquad |C_1^{SLL}| \lesssim (5 \cdot 10^5\,\text{GeV})^{-2} . \tag{2.5}$$

2.1.2. Leptonic B decays

Very strong experimental bounds have also been derived on leptonic B decays $B^0 \to \bar{l}_i l_j$ for the leptons being electrons or muons. The decay into τ leptons on the other hand is not very constrained due to the difficulties in reconstructing τ events. In specific new physics models the decay into electrons might be suppressed compared to the decay into muons. Therefore, we will stick to the bound [19]

$$\text{BR}(B_s^0 \to \mu^+\mu^-) < 1.08 \cdot 10^{-8} . \tag{2.6}$$

The leptonic branching ratios of B_d^0 are also tightly constrained by experiment. But in generic new physics models one might expect $b \to d$ transitions to be stronger suppressed than $b \to s$.

As we will be interested in scalar mediation only and as the matrix element $\langle B_s^0 | \bar{b}s |0\rangle$ vanishes because of parity, the relevant effective Hamiltonian is given by

$$\mathcal{H}_{\text{eff}}^{b\to s\bar{\mu}\mu} = \left[C^S(\bar{\mu}\mu) + C^P(\bar{\mu}\gamma_5\mu) \right] (\bar{b}\gamma_5 s) . \tag{2.7}$$

The branching ratio can be approximately calculated [17]:

$$\text{BR}(B_s^0 \to \mu^+\mu^-) \simeq \frac{f_{B_s}^2 m_{B_s}^5}{8\pi m_b^2 \Gamma_{B_s}} \left(|C^S|^2 + |C^P|^2 \right) . \tag{2.8}$$

The SM contribution, $\text{BR}(B_s^0 \to \mu^+\mu^-) \simeq 3.3 \cdot 10^{-9}$ [17], is well below the experimental bound. Taking $m_b = 4.2\,\text{GeV}$ and $\Gamma_{B_s} = 4.5 \cdot 10^{-13}\,\text{GeV}$ [1] we obtain therefore

$$|C^S|^2 + |C^P|^2 \lesssim (1 \cdot 10^5\,\text{GeV})^{-4} . \tag{2.9}$$

2.1.3. Suppression mechanisms

In the SM the rates for FCNCs are efficiently suppressed by the GIM mechanism [20], which combines the loop suppression with a suppression due to small mass differences induced by the small Yukawa couplings. In extensions of the SM, in addition to the suppressions by loops, small couplings and small mass splittings, it is conceivable also a suppression by heavy masses of the particles mediating flavour violation.

As in the SM, the flavour violating couplings in extensions of the SM are typically free parameters of the model. They might be generated at some high scale from the same flavour model that yields the SM Yukawa couplings. In this way the smallness of the flavour violating couplings might be explained but predictions are model dependent. In 2HDMs a rather large class of such models can be approximately described with the Cheng and Sher ansatz [21] which we will employ in section 2.3.3. Another hypothesis is Minimal Flavour Violation (MFV) [22, 23] which we will present in the following.

Minimal Flavour Violation

In the limit of vanishing Yukawa couplings, the SM is invariant under unitary transformations of the fermion flavours [24]. The symmetry group is $G_{\text{flavour}} \times \mathrm{U}(1)^5$ where

$$G_{\text{flavour}} = \mathrm{SU}(3)_Q \times \mathrm{SU}(3)_u \times \mathrm{SU}(3)_d \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_e \,. \tag{2.10}$$

The SM Yukawa couplings break this group apart from three U(1) factors which can be identified as hypercharge and baryon and lepton number. In any case the U(1) factors act identically on all flavours and are therefore irrelevant for the definition of MFV. The invariance under flavour transformations can be recovered formally by treating the Yukawa couplings as the vacuum expectation values of some 'spurion' fields which transform as

$$\boldsymbol{Y_u} \sim (3, \bar{3}, 1)_{\mathrm{SU}(3)_Q \times \mathrm{SU}(3)_u \times \mathrm{SU}(3)_d} \,, \tag{2.11a}$$

$$\boldsymbol{Y_d} \sim (3, 1, \bar{3})_{\mathrm{SU}(3)_Q \times \mathrm{SU}(3)_u \times \mathrm{SU}(3)_d} \,, \tag{2.11b}$$

$$\boldsymbol{Y_e} \sim (3, \bar{3})_{\mathrm{SU}(3)_L \times \mathrm{SU}(3)_e} \,. \tag{2.11c}$$

With these preliminaries it is now possible to define any effective theory to be Minimal Flavour Violating [22, 23] if all (higher dimensional) operators can be written in terms of the Yukawa couplings such that they are formally invariant under G_{flavour}, and if all CP violation originates from the Yukawa couplings. Any extension of the SM therefore obeys the principle of MFV if its effective description is Minimal Flavour Violating. A generalization of this definition to include non-vanishing neutrino masses is possible but depends on the mechanism responsible for neutrino mass generation [25–27], see also sections 3.4.1 and 3.4.2.

Phenomenologically, new physics models obeying MFV are very attractive as they inherit the efficient suppression of FCNC and CP violation from the SM. From a theoretical point of view, however, the spurion construction is not very plausible, as G_{flavour} is anomalous.[1] Nevertheless, some subgroup of G_{flavour} might indeed be a fundamental symmetry of nature and broken only spontaneously. Depending on this subgroup the low energy

[1] The anomalies might be removed by the introduction of additional heavy fermions, cf. [28].

effective theory might be Minimal Flavour Violating or might show deviations. Independently of any spurion argument, the MFV framework applies to all models where there are no flavour mixing matrices apart from the Yukawa couplings. A prominent example for this kind of models is the Constrained MSSM.

2.2. Suppressing FCNCs in the MSSM

2.2.1. The virtues of MFV

In the MSSM a number of new flavour matrices are introduced in addition to the SM Yukawa couplings: in the quark sector three soft squark mass matrices and two trilinear couplings. As the supersymmetric partners should be as light as possible in order for supersymmetry to solve the hierarchy problem, clearly a tension arises from the fact that no FCNCs have been observed yet beyond the SM.

On the other hand all of these flavour matrices originate in the breaking of SUSY. Hence the problems with FCNCs can be easily ameliorated by imposing flavour blindness of the SUSY breaking mechanism. This corresponds to the widely studied case of the Constrained MSSM (CMSSM). However, SUSY breaking is typically assumed to happen at some very high scale and the RG evolution will induce flavour off-diagonal terms that obey MFV. Therefore, the MFV hypothesis is the most conservative possibility at the weak scale.

Though, the MFV ansatz is not restricted to the CMSSM scenario. More generally it applies to scenarios where the Yukawa couplings are the only relevant flavour matrices in the superpotential (and the Kähler) at the scale of SUSY breaking. This includes scenarios where the Higgs fields contribute to the mediation of SUSY breaking, or the MFV structure might result from some broken flavour symmetry.

Applying the principle of MFV to the MSSM quark sector yields the following decomposition of soft masses:

$$\boldsymbol{m^2_Q} = \alpha_1\,\mathbb{1} + \beta_1\,\boldsymbol{Y_u^\dagger Y_u} + \beta_2\,\boldsymbol{Y_d^\dagger Y_d} + \beta_3\,\boldsymbol{Y_d^\dagger Y_d Y_u^\dagger Y_u} + \beta_3\,\boldsymbol{Y_u^\dagger Y_u Y_d^\dagger Y_d}\,, \tag{2.12a}$$
$$\boldsymbol{m^2_{\bar u}} = \alpha_2\,\mathbb{1} + \beta_5\,\boldsymbol{Y_u Y_u^\dagger}\,, \tag{2.12b}$$
$$\boldsymbol{m^2_{\bar d}} = \alpha_3\,\mathbb{1} + \beta_6\,\boldsymbol{Y_d Y_d^\dagger}\,, \tag{2.12c}$$
$$\boldsymbol{A_u} = \alpha_4\,\boldsymbol{Y_u} + \beta_7\,\boldsymbol{Y_u}\,\boldsymbol{Y_d^\dagger Y_d}\,, \tag{2.12d}$$
$$\boldsymbol{A_d} = \alpha_5\,\boldsymbol{Y_d} + \beta_8\,\boldsymbol{Y_d}\,\boldsymbol{Y_u^\dagger Y_u}\,, \tag{2.12e}$$
$$\boldsymbol{A_e} = \alpha_e\,\boldsymbol{Y_e}\,. \tag{2.12f}$$

The conventions in this thesis are based on [29] in the discussion of supersymmetric models. In particular $\boldsymbol{m^2_Q}$ and $\boldsymbol{m^2_{\bar u}}$, $\boldsymbol{m^2_{\bar d}}$ are the soft mass matrices for left- and right-handed squarks, $\boldsymbol{A_u}$, $\boldsymbol{A_d}$, $\boldsymbol{A_e}$ denote the trilinear couplings. Here the term $\boldsymbol{A_e}$ has been included for later convenience. The parameters α_i, β_i carry mass dimension of one or two. Note that we have truncated the MFV series at some power of the Yukawa couplings. This, however, is a good approximation: In the limit of vanishing Yukawa eigenvalues of the first two families, any higher order term in the series is proportional to a term in the truncated series [22]. For example, in the basis of diagonal $\boldsymbol{Y_d}$:

$$\boldsymbol{Y_u}\boldsymbol{Y_d}^\dagger\boldsymbol{Y_d}\boldsymbol{Y_u}^\dagger \simeq \boldsymbol{V_{CKM}}\,\mathrm{diag}(0,0,y_t^2 y_b^2)\boldsymbol{V_{CKM}}^\dagger \simeq y_b^2\boldsymbol{Y_u}\boldsymbol{Y_u}^\dagger\ .$$

Here, the error made in the truncation of the MFV series is therefore of the order of the squared Yukawa eigenvalue of the charm quark.

The MFV ansatz offers a natural way to suppress new physics contributions to many flavour observables [30, 31]. Still, the MFV terms can provide interesting effects. For example, the FCNC processes $B \to X_s\gamma$ are known to be possibly strongly enhanced compared to the SM. Thereby the coefficient β_1 plays an important role: The leading contributions are due to Higgs loops, where β_1 leads to the necessary mass splitting in the up squark masses in order to overcome the GIM suppression, and due to gluino loops where it can provide the necessary flavour-off diagonal term [32]. Therefore, we will use this parameter to illustrate the evolution of MFV coefficients in fig. 2.1.

2.2.2. The MFV ansatz under the renormalization group evolution

As the spurion construction of MFV is scale independent, one can expect the principle of MFV to hold at any mass scale if it holds at one mass scale. This is true although G_{flavour} is anomalous. The reason is that the anomaly does not affect the Renormalization Group Equation (RGE). We have checked this also numerically: Starting with MFV soft terms at the scale of grand unification, we integrate the RGE numerically with the use of a modified version of the programme package SOFTSUSY [33], and decompose the result at low scales into the ansatz, eq. (2.12). Of course the parameters α_i, β_i change due to the running. However, deviations from the MFV ansatz show up only at the percent level and as a result of the truncation of the MFV expansion to the powers in the Yukawa couplings given in eq. (2.12). Further details to this study can be found in appendix A.1.

After having confirmed the scale independence of the validity of MFV, it is interesting to study the evolution of the MFV parameters under the RG evolution. First, this will yield a simple tool to understand the dependencies between high and low scale parameters. But moreover it turns out that in large portions of the parameter space the values of β_i are driven towards fixed points. Therefore the values of β_i at low scales are quite insensitive to their initial values at the high scale. This fact considerably simplifies phenomenology of FCNC phenomena in MFV models, essentially reducing it to the case of the CMSSM. Values far from the fixed points and therefore large deviations from the CMSSM case, however, can still be obtained in low scale SUSY breaking.

The fact that the soft masses tend to get aligned due to the running is well known [34–38]. However, we have quantified it for the first time in the framework of MFV [6]. The case of large $\tan\beta$ was included shortly after [39].

The β-functions of the MFV coefficients can be calculated by inserting the truncated MFV series, eq. (2.12), into the MSSM β-functions which have been derived in [40] (cf. also [41]). A subtlety is that one has to take into account also the running of the Yukawa couplings when matching the different terms into the truncated MFV series to read off the result. Neglecting the Yukawa couplings of the first two generations, we find for the flavour blind terms:

$$16\pi^2\frac{\mathrm{d}\alpha_1}{\mathrm{d}t} = -\frac{32}{3}g_3^2|M_3|^2 - 6g_2^2|M_2|^2 - \frac{2}{15}g_1^2|M_1|^2 + \frac{1}{5}g_1^2 S\ , \tag{2.13a}$$

$$16\pi^2\frac{\mathrm{d}\alpha_2}{\mathrm{d}t} = -\frac{32}{3}g_3^2|M_3|^2 - \frac{32}{15}g_1^2|M_1|^2 - \frac{4}{5}g_1^2 S\ , \tag{2.13b}$$

$$16\pi^2\frac{\mathrm{d}\alpha_3}{\mathrm{d}t} = -\frac{32}{3}g_3^2|M_3|^2 - \frac{8}{15}g_1^2|M_1|^2 + \frac{2}{5}g_1^2 S\ , \tag{2.13c}$$

$$16\pi^2\frac{\mathrm{d}\alpha_4}{\mathrm{d}t} = 12\alpha_4 y_t^2 + 10\beta_7 y_t^2 y_b^2 + 2\beta_8 y_t^2 y_b^2 + \frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15}g_1^2 M_1\ , \tag{2.13d}$$

$$16\pi^2\frac{\mathrm{d}\alpha_5}{\mathrm{d}t} = 12\alpha_5 y_b^2 + 10\beta_8 y_t^2 y_b^2 + 2\beta_7 y_t^2 y_b^2 + \frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1 + 2\alpha_e y_\tau^2\ , \tag{2.13e}$$

and for the MFV specific parameters:

$$\begin{aligned}16\pi^2\frac{\mathrm{d}\beta_1}{\mathrm{d}t} &= 2m_{H_u}^2 + 2\alpha_4^2 + 2\beta_8^2 y_t^2 y_b^2 + 2\alpha_1 + 2\alpha_2 - 10\beta_1 y_t^2 + 2\beta_5 y_t^2 \\ &+ \beta_1\left(\frac{32}{3}g_3^2 + 6g_2^2 + \frac{26}{15}g_1^2\right)\ ,\end{aligned} \tag{2.14a}$$

$$\begin{aligned}16\pi^2\frac{\mathrm{d}\beta_2}{\mathrm{d}t} &= 2m_{H_d}^2 + 2\alpha_5^2 + 2\beta_7^2 y_t^2 y_b^2 + 2\alpha_1 + 2\alpha_3 - 10\beta_2 y_b^2 - 2\beta_2 y_\tau^2 + 2\beta_6 y_b^2 \\ &+ \beta_2\left(\frac{32}{3}g_3^2 + 6g_2^2 + \frac{14}{15}g_1^2\right)\ ,\end{aligned} \tag{2.14b}$$

$$\begin{aligned}16\pi^2\frac{\mathrm{d}\beta_3}{\mathrm{d}t} &= 2\alpha_4\beta_7 + 2\alpha_5\beta_8 - 12\beta_3 y_t^2 - 12\beta_3 y_b^2 - 2\beta_3 y_\tau^2 \\ &+ \beta_3\left(\frac{64}{3}g_3^2 + 12g_2^2 + \frac{8}{3}g_1^2\right)\ ,\end{aligned} \tag{2.14c}$$

$$\begin{aligned}16\pi^2\frac{\mathrm{d}\beta_5}{\mathrm{d}t} &= 4m_{H_u}^2 + 4\left(\alpha_4 + \beta_7 y_b^2\right)^2 + 4\alpha_1 + 4\alpha_2 + 4\beta_1 y_t^2 + 4\beta_2 y_b^2 + 8\beta_3 y_t^2 y_b^2 \\ &+ \beta_5\left(-8y_t^2 - 2y_b^2 + \frac{32}{3}g_3^2 + 6g_2^2 + \frac{26}{15}g_1^2\right)\ ,\end{aligned} \tag{2.14d}$$

$$\begin{aligned}16\pi^2\frac{\mathrm{d}\beta_6}{\mathrm{d}t} &= 4m_{H_d}^2 + 4\left(\alpha_5 + \beta_8 y_t^2\right)^2 + 4\alpha_1 + 4\alpha_3 + 4\beta_1 y_t^2 + 4\beta_2 y_b^2 + 8\beta_3 y_t^2 y_b^2 \\ &+ \beta_6\left(-2y_t^2 - 8y_b^2 - 2y_\tau^2 + \frac{32}{3}g_3^2 + 6g_2^2 + \frac{14}{15}g_1^2\right)\ ,\end{aligned} \tag{2.14e}$$

$$16\pi^2\frac{\mathrm{d}\beta_7}{\mathrm{d}t} = 2\alpha_5 + \beta_7\left(-12y_b^2 - 2y_\tau^2 + \frac{32}{3}g_3^2 + 6g_2^2 + \frac{14}{15}g_1^2\right)\ , \tag{2.14f}$$

$$16\pi^2\frac{\mathrm{d}\beta_8}{\mathrm{d}t} = 2\alpha_4 + \beta_8\left(-12y_t^2 + \frac{32}{3}g_3^2 + 6g_2^2 + \frac{26}{15}g_1^2\right)\ . \tag{2.14g}$$

Here g_1, g_2, g_3 denote the gauge couplings, M_1, M_2, M_3 the corresponding gaugino masses, y_t, y_b, y_τ the Yukawa couplings to the third family and $m_{H_u}^2$, $m_{H_d}^2$ the soft squared mass of the Higgs doublets. Finally S has been defined as

$$S = m_{H_u}^2 - m_{H_d}^2 + \mathrm{Tr}\Big[\alpha_1 \mathbb{1} + \beta_1 \boldsymbol{Y_u^\dagger Y_u} + \beta_2 \boldsymbol{Y_d^\dagger Y_d} + 2\beta_3 \boldsymbol{Y_d^\dagger Y_d Y_u^\dagger Y_u} \\ - 2\alpha_2 \mathbb{1} - 2\beta_5 \boldsymbol{Y_u Y_u^\dagger} + \alpha_3 \mathbb{1} + \beta_6 \boldsymbol{Y_d Y_d^\dagger} - \boldsymbol{m_L^2} + \boldsymbol{m_{\bar{e}}^2}\Big]\,, \tag{2.15}$$

with $\boldsymbol{m_L^2}$, $\boldsymbol{m_{\bar{e}}^2}$ being the left- and right-handed slepton mass matrices.

For practical purposes and in order to better understand the behaviour of these β-functions, we have also derived numerically linear approximations to the running MFV coefficients. In doing so, we parametrize the flavour blind terms by a universal gaugino mass $m_{1/2}$, a universal soft squared mass m_0^2 and a universal trilinear mass parameter A as in the CMSSM, thus fixing the parameters α_i at the high scale. The MFV specific terms β_i, however, are varied. Details can again be found in appendix A.1. As input scale we take the scale of grand unification and choose to work with an intermediate value of $\tan\beta = 10$. With these assumptions we find at the low scale for the parameters in our MFV decomposition, eq. (2.12):

$$\alpha_1|_{\mathrm{low}} = +0.94 m_0^2 + 5.04 m_{1/2}^2\,, \tag{2.16a}$$
$$\alpha_2|_{\mathrm{low}} = +0.95 m_0^2 + 4.72 m_{1/2}^2\,, \tag{2.16b}$$
$$\alpha_3|_{\mathrm{low}} = +0.95 m_0^2 + 4.61 m_{1/2}^2\,, \tag{2.16c}$$
$$\alpha_4|_{\mathrm{low}} = -2.00 m_{1/2} + 0.32 A\,, \tag{2.16d}$$
$$\alpha_5|_{\mathrm{low}} = -3.23 m_{1/2} + 0.98 A\,, \tag{2.16e}$$
$$\beta_1|_{\mathrm{low}} = -0.41 m_0^2 - 0.96 m_{1/2}^2 + 0.16 A m_{1/2} - 0.04 A^2 + 0.27\beta_1 - 0.03\beta_5\,, \tag{2.16f}$$
$$\beta_2|_{\mathrm{low}} = -0.43 m_0^2 - 1.38 m_{1/2}^2 + 0.57 A m_{1/2} - 0.15 A^2 - 0.02\beta_1 + 0.1\beta_2 + 0.01\beta_5\,, \tag{2.16g}$$
$$\beta_3|_{\mathrm{low}} = +0.13 m_{1/2}^2 - 0.13 A m_{1/2} + 0.04 A^2 + 0.02\beta_1 + 0.03\beta_3 - 0.01\beta_5 \\ - (0.01\beta_7 + 0.04\beta_8) A + (0.03\beta_7 + 0.08\beta_8) m_{1/2}\,, \tag{2.16h}$$
$$\beta_5|_{\mathrm{low}} = -0.83 m_0^2 - 1.96 m_{1/2}^2 + 0.32 A m_{1/2} - 0.09 A^2 - 0.07\beta_1 + 0.24\beta_5\,, \tag{2.16i}$$
$$\beta_6|_{\mathrm{low}} = -0.86 m_0^2 - 2.57 m_{1/2}^2 + 0.94 A m_{1/2} - 0.25 A^2 - 0.07\beta_1 + 0.01\beta_5 \\ + 0.12\beta_6 - 0.14 A\beta_8 + 0.25 m_{1/2}\beta_8\,, \tag{2.16j}$$
$$\beta_7|_{\mathrm{low}} = +0.51 m_{1/2} - 0.27 A + 0.10\beta_7\,, \tag{2.16k}$$
$$\beta_8|_{\mathrm{low}} = +0.27 m_{1/2} - 0.14 A + 0.30\beta_8\,. \tag{2.16l}$$

There are mainly two effects leading to the already mentioned insensitivity of low energy phenomena to the initial values of the β_i: First, the soft terms at low scales are dominated by the RGE contribution of the gluino which is of course flavour blind. A contribution to the coefficients β_i is generated from the squark masses and trilinear couplings independently of their initial values. Therefore, unless $m_{1/2}^2$ is smaller than m_0^2 or A^2, the generated flavour off-diagonal terms are subleading. Secondly, the MFV specific terms $\beta_i|_{\mathrm{low}}$ are dominated by the RG contribution and not by their initial values. Indeed, the β_i terms carry comparatively small coefficients – with the exception of $\beta_8|_{\mathrm{low}}$ where also the gaugino contribution is rather small. In terms of the β-functions eqs. (2.14) the reason can be traced back to the large contribution from the strong gauge coupling that

Point	1a	1b	2	3	4	5
m_0 [Gev]	100	200	1450	90	400	150
$m_{1/2}$ [Gev]	250	400	300	400	300	300
A [Gev]	-100	0	0	0	0	-1000
$\tan\beta$	10	30	10	10	10	5

Table 2.1.: Parameters of several SPS points [42].

suppresses any occurrence of the β_i. This suppression originates in the running of the Yukawa couplings, i.e. the coefficients of the β_i terms are small as the Yukawa couplings are smaller at the high input scale thus needing a larger parameter β_i in front to provide the same deviation from flavour blindness.

Let us next illustrate this behaviour with some exemplary benchmark points. We have chosen to utilise the so-called SPS points [42]. Those have been designed to represent as different points in the CMSSM parameter space as possible but being each point in a region of parameter space of special interest. For convenience we list them in table 2.1. For our MFV analysis we complement these points with non-vanishing values of the β_i already at the scale of grand unification. For each SPS point we show the running of the ratio β_1/α_1 in fig. 2.1 and β_6/α_3 in fig. 2.2. Here, the coefficient β_1 has been chosen as it is important in the exotic contribution to $B \to X_s\gamma$, while β_6 just provides a different example. The trilinear couplings are less suited for illustrative purposes because a normalization to the coefficients α_4, α_5 is not reasonable, as α_4, α_5 can become quite small at low scales if the RGE contribution cancels the initial value.

The solid lines correspond to the case where the only non-vanishing MFV complement is the one plotted, i.e. for fig. 2.1

$$\forall i \neq 1 : \beta_i = 0 \,, \tag{2.17}$$

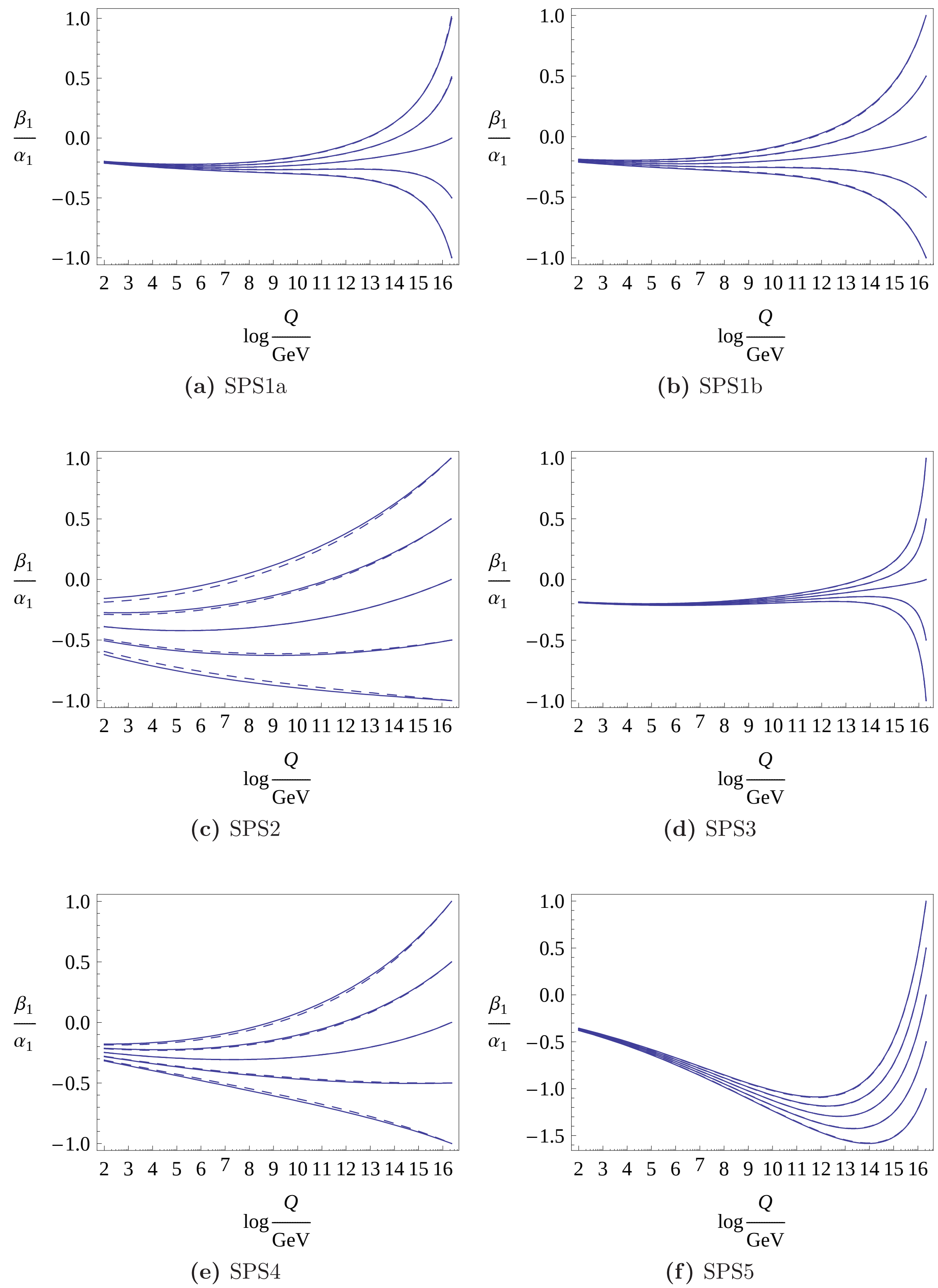
and the same condition but with $i \neq 6$ for fig. 2.2. The dashed lines correspond to universal β_i. In this case the initial values of the β_i coefficients, normalized to the corresponding flavour blind term, equal each other:

$$\frac{\beta_1}{m_0^2} = \frac{\beta_2}{m_0^2} = \frac{\beta_3}{m_0^2} = \frac{\beta_5}{m_0^2} = \frac{\beta_6}{m_0^2} = \frac{\beta_7}{A} = \frac{\beta_8}{A} \,. \tag{2.18}$$

Though the initial conditions eq. (2.17) and eq. (2.18) are quite different, for most of the SPS points the solid and dashed lines differ only slightly. Also the fixed point behaviour can be seen clearly, which is especially pronounced for large $m_{1/2}$. Both effects have been expected from the small coefficients in front of the β_i in eqs. (2.16) and have been explained already above.

2.2.3. Beyond MFV

Even if the dominant mediation mechanism of SUSY breaking yields Minimal Flavour Violating soft terms, there might exist contributions that deviate from MFV. On the



(a) SPS1a (b) SPS1b

(c) SPS2 (d) SPS3

(e) SPS4 (f) SPS5

Figure 2.1.: The running of $\frac{\beta_1}{\alpha_1}$. For the solid curve only β_1 is non-zero while for the dashed curve all β_i are universal at the high scale.

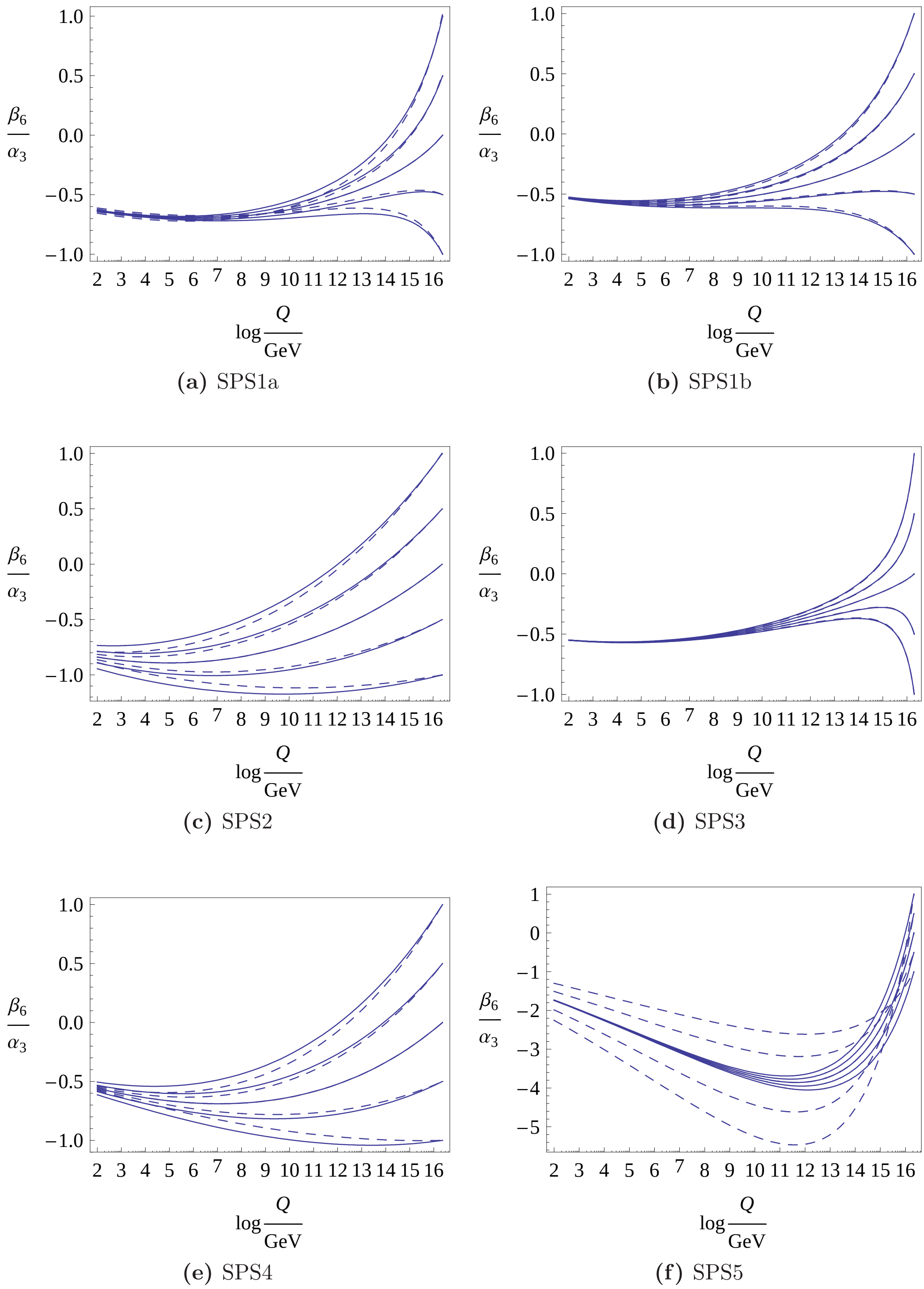


Figure 2.2.: The running of $\frac{\beta_6}{\alpha_3}$. For the solid curve only β_6 is non-zero while for the dashed curve all β_i are universal at the high scale.

other hand, also the dominant mediation mechanism might induce deviations from MFV. If the soft masses are organized by some broken flavour symmetry, exact MFV can only be expected if for each Yukawa matrix there is exactly one spurion, analogous to the construction in section 2.1.3. In view of the large hierarchies in the Yukawa couplings it is, however, reasonable to assume the existence of several spurions or a more involved generation of the Yukawa couplings. Still, the soft masses might exhibit hierarchies similar to the MFV ansatz [43].

In any case, even if the soft terms do not obey the principle of MFV, it makes sense to distinguish between MFV and non-MFV terms. MFV terms will be generated in any case, even if SUSY breaking is flavour blind. On the other hand, the RG evolution will generate only tiny deviations from the truncated MFV series, eqs. (2.12). Therefore, deviations from MFV must definitely originate in new flavour physics. Define the non-MFV terms by:

$$\boldsymbol{\Delta m^2_f} = \boldsymbol{m^2_f} - (\boldsymbol{m^2_f})^{\mathrm{MFV}} \qquad \text{where } \boldsymbol{f} = \boldsymbol{Q}, \boldsymbol{\bar{u}}, \boldsymbol{\bar{d}}\,, \tag{2.19a}$$

$$\boldsymbol{\Delta A_f} = \boldsymbol{A_f} - \boldsymbol{A_f}^{\mathrm{MFV}} \qquad \text{where } \boldsymbol{f} = \boldsymbol{u}, \boldsymbol{d}\,. \tag{2.19b}$$

Here $(\boldsymbol{m^2_f})^{\mathrm{MFV}}$, $\boldsymbol{A_f}^{\mathrm{MFV}}$ can be decomposed as in eqs. (2.12) with some parameters α_i, β_i. The non-MFV terms $\boldsymbol{\Delta m^2_f}$, $\boldsymbol{\Delta A_f}$ are determined unambiguously by the demand of orthogonality to the MFV terms with respect to the scalar product

$$\langle \boldsymbol{m_1}, \boldsymbol{m_2} \rangle = \mathrm{Tr}(\boldsymbol{m_1^\dagger m_2})\,. \tag{2.20}$$

For example $(\boldsymbol{m^2_{\bar{u}}})^{\mathrm{MFV}}$ can be decomposed into a term proportional to $\mathbb{1}$ and one proportional to $\boldsymbol{Y_u Y_u^\dagger}$. Therefore $\boldsymbol{\Delta m^2_{\bar{u}}}$ obeys:

$$\mathrm{Tr}(\boldsymbol{\Delta m^2_{\bar{u}}}) = \mathrm{Tr}(\boldsymbol{\Delta m^2_{\bar{u}} Y_u^\dagger Y_u}) = 0\,. \tag{2.21}$$

This orthogonality simplifies the β-functions for the non-MFV terms. They can be derived by inserting the decomposition eqs. (2.19) into the usual β-function for MSSM soft terms [41]. We already know that MFV terms generate only MFV terms. Thus each term in the β-function of the non-MFV terms has to be proportional to a non-MFV term $\boldsymbol{\Delta m^2_f}$ or $\boldsymbol{\Delta A_f}$. On the other hand, the orthogonality relations guarantee that any term in the β-function proportional to exactly one non-MFV term is again non-MFV – of course up to corrections suppressed by small squared Yukawa couplings. In summary, the β-functions of any MSSM soft term can be decomposed into a MFV and a non-MFV β-function which mix only with terms proportional to small Yukawa couplings or the square of $\boldsymbol{\Delta A_f}$. They read:

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{\Delta m^2_{\bar{u}}} = 2\boldsymbol{\Delta m^2_{\bar{u}} Y_u Y_u^\dagger} + 4\boldsymbol{Y_u \Delta m^2_Q Y_u^\dagger} + 2\boldsymbol{Y_u Y_u^\dagger \Delta m^2_{\bar{u}}} + 4\boldsymbol{\Delta}(\boldsymbol{A_u A_u^\dagger})\,, \tag{2.22a}$$

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{\Delta m^2_{\bar{d}}} = 2\boldsymbol{\Delta m^2_{\bar{d}} Y_d Y_d^\dagger} + 4\boldsymbol{Y_d \Delta m^2_Q Y_d^\dagger} + 2\boldsymbol{Y_d Y_d^\dagger \Delta m^2_{\bar{d}}} + 4\boldsymbol{\Delta}(\boldsymbol{A_d A_d^\dagger})\,, \tag{2.22b}$$

$$\begin{aligned} 16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t} \boldsymbol{\Delta m^2_Q} &= \boldsymbol{\Delta m^2_Q}\left(\boldsymbol{Y_u^\dagger Y_u} + \boldsymbol{Y_d^\dagger Y_d}\right) + \left(\boldsymbol{Y_u^\dagger Y_u} + \boldsymbol{Y_d^\dagger Y_d}\right)\boldsymbol{\Delta m^2_Q} \\ &\quad + 2\boldsymbol{Y_u^\dagger \Delta m^2_{\bar{u}} Y_u} + 2\boldsymbol{Y_d^\dagger \Delta m^2_{\bar{d}} Y_d} + 2\boldsymbol{\Delta}(\boldsymbol{A_u^\dagger A_u}) + 2\boldsymbol{\Delta}(\boldsymbol{A_d^\dagger A_d})\,, \end{aligned} \tag{2.22c}$$

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\Delta A_u} = \boldsymbol{\Delta A_u}\left[3\,\mathrm{Tr}(\boldsymbol{Y_u Y_u^\dagger}) + 5\boldsymbol{Y_u^\dagger Y_u} + \boldsymbol{Y_d^\dagger Y_d} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right] + \boldsymbol{Y_u}\left[4\boldsymbol{Y_u^\dagger \Delta A_u} + 2\boldsymbol{Y_d^\dagger \Delta A_d}\right] , \tag{2.22d}$$

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\Delta A_d} = \boldsymbol{\Delta A_d}\left[\mathrm{Tr}(3\boldsymbol{Y_d Y_d^\dagger} + \boldsymbol{Y_e^\dagger Y_e}) + 5\boldsymbol{Y_d^\dagger Y_d} + \boldsymbol{Y_u^\dagger Y_u} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right] + \boldsymbol{Y_d}\left[4\boldsymbol{Y_d^\dagger \Delta A_d} + 2\boldsymbol{Y_u^\dagger \Delta A_u}\right] . \tag{2.22e}$$

where $\boldsymbol{\Delta}(\boldsymbol{A_f A_f^\dagger}) = \boldsymbol{\Delta A_f A_f^\dagger} + \boldsymbol{A_f \Delta A_f^\dagger} + (\boldsymbol{\Delta A_f \Delta A_f^\dagger})^{\mathrm{non-MFV}}$ and $(\boldsymbol{\Delta A_f \Delta A_f^\dagger})^{\mathrm{non-MFV}}$ is a non-MFV term in the sense of $\boldsymbol{\Delta m_f^2}$ in eq. (2.19a). As already mentioned, the MFV β-functions, eqs. (2.13), (2.14), are only modified by the appearance of terms proportional to the square of non-MFV terms, more precisely proportional to $(\boldsymbol{\Delta A_f \Delta A_f^\dagger})^{\mathrm{MFV}}$. Therefore, as long as the trilinear couplings do not deviate strongly from MFV, the evolution of the MFV terms is independent of the non-MFV terms (and vice versa). This feature could help to disentangle different contributions to the soft masses.

Finally let us comment on the qualitative behaviour of the non-MFV terms under the renormalization group. If $\boldsymbol{\Delta A_u}$, $\boldsymbol{\Delta A_d}$ are small, the non-MFV terms $\boldsymbol{\Delta m_f^2}$ obtain only contributions proportional to Yukawa couplings which are thus rather small. The only exception is the is the entry $(\boldsymbol{\Delta m_f^2})_{33}$ which runs proportional to the top Yukawa coupling with a rather large coefficient. However, $(\boldsymbol{\Delta m_f^2})_{33}$ yields only an irrelevant correction to the MFV terms. Therefore, for small $\boldsymbol{\Delta A_u}$, $\boldsymbol{\Delta A_d}$, the non-MFV scalar squared masses are quite stable under the RG evolution.

This is not true for the running of $\boldsymbol{\Delta A_u}$, $\boldsymbol{\Delta A_d}$. Here the dominant effect is due to the strong gauge coupling and leads to a scaling that can be as large as a factor of 3. Additionally, also the top Yukawa coupling influences the evolution substantially and reduces the scaling for the last row and column of $\boldsymbol{\Delta A_u}$.

We conclude that the RG evolution of terms orthogonal to the truncated MFV series is qualitatively very different to the one of MFV terms. While the former terms are rather stable under the RGE, or – in the case of trilinear couplings – get somewhat enhanced, the MFV terms are typically dominated by the RGE contribution. Therefore, FCNCs are remarkably insensitive to the MFV parameters at high scale.

2.3. Suppressing FCNCs in the 2HDM

2.3.1. Overview

There is no fundamental reason to stick to only one Higgs doublet in the SM. However, though the 2HDM is a simple and straightforward extension, it can modify the SM phenomenology substantially. A major drawback compared to the SM is certainly the possibility of having a vacuum that breaks electric charge. At least, there is no charge breaking vacuum once there is a charge conserving one [44]. Therefore the vacuum is stable against charge breaking. Another feature is the existence of new sources of CP violation. This may be seen as a benefit, reopening the chance of electroweak baryogenesis [45–47] (see also [48] for a more recent investigation), or as a potential problem with CP violation observables. Lastly, the second Higgs may contribute significantly to the oblique parameters S, T, U [49, 50] to cLFV and FCNCs.

On the other hand, the introduction of additional Higgs fields can also help to understand features of some SM parameters: First, the introduction of a few Higgs doublets improves the unification behaviour of gauge couplings. Secondly, in section 3.4.3 it will be demonstrated that if there is more than one Higgs doublet, the mild hierarchy in neutrino masses can be explained in spite of hierarchical neutrino Yukawa couplings.

In this section we will be mainly concerned with FCNCs – a task independent of the Higgs potential V, apart from the Higgs masses. Therefore the restrictions in order to have electric charge unbroken and bounds from measurements of the oblique parameters are completely independent.

The Lagrangian of any 2HDM can be written as:

$$\mathcal{L} = \mathcal{L}^{\text{kin}} + \mathcal{L}^{\text{Yuk}} - V \, , \tag{2.23}$$

where $\mathcal{L}^{\text{kin}}$ contains the kinetic terms and $\mathcal{L}^{\text{Yuk}}$ is the Yukawa Lagrangian. In the discussion of 2HDM our notation is based on [51]. The two Higgs doublets Φ_1, Φ_2 carry the same weak hypercharge $Y = \frac{1}{2}$. Thus, without any additional (discrete) symmetry the two Higgs doublets cannot be distinguished. Therefore one is always free to rotate the Higgs basis by any angle α

$$\begin{pmatrix} \Phi_1' \\ \Phi_2' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \, . \tag{2.24}$$

Here we assume the vacuum expectation values of both Higgs to be real.[2] It should be stressed that in our notation the definition of $\tan\beta$ is inverse to the one often encountered in the context of the MSSM regarding the enumeration of the vacuum expectation values:

$$\langle \Phi_a \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_a \end{pmatrix} \, , \qquad \tan\beta = \frac{v_2}{v_1} \, . \tag{2.25}$$

Thus β is the angle rotating into the basis where one Higgs has no vacuum expectation value:

$$\begin{pmatrix} \Phi_{\text{vev}} \\ \Phi_{\text{no vev}} \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \qquad \text{where} \quad \begin{array}{l} \sqrt{2}\,\langle \Phi_{\text{vev}} \rangle = (0, v)^T \\ \langle \Phi_{\text{no vev}} \rangle = (0, 0)^T \end{array} \tag{2.26}$$

The most general Yukawa Lagrangian, often denoted as 2HDM type III, can be written as

$$-\mathcal{L}^{\text{Yuk}} = \bar{Q}_i \boldsymbol{Y}^a_{\boldsymbol{u}\,ij} u_j \tilde{\Phi}_a + \bar{Q}_i \boldsymbol{Y}^a_{\boldsymbol{d}\,ij} d_j \Phi_a + \bar{L}_i \boldsymbol{Y}^a_{\boldsymbol{e}\,ij} e_j \Phi_a + \text{h.c.} \, , \tag{2.27}$$

where a is the Higgs index, i, j are flavour indices and $\tilde{\Phi}_a = i\tau_2 \Phi_a^*$. Note in particular the difference $\boldsymbol{Y_f} \leftrightarrow \boldsymbol{Y_f}^\dagger$ to the notation used for the MSSM.

[2] This is always possible. However, in the case of spontaneous CP violation, some terms in the Lagrangian become complex by this choice.

This Lagrangian generically leads to large FCNCs. Therefore, historically it is more popular to introduce a discrete symmetry under which the two Higgs doublets are charged differently thus preventing the right-handed fermions to couple to both Higgs doublets. If Φ_2 does not couple at all to the fermions, the model is called type I, if Φ_2 does not couple to u and Φ_1 not to d, e it is of type II.

Another frequently used ansatz for the Yukawa couplings was introduced by Cheng and Sher [21] and turned out to cover various ansätze for the Yukawa couplings. However, precision measurements of flavour violation have revealed that this ansatz is only viable if most of the coefficients are small [52] (and [53] for a partial update), or the second Higgs is rather strongly decoupled. As the 2HDM becomes indistinguishable from the SM if the second Higgs is sufficiently decoupled, this case has not gained so much interest yet. Still, while a decoupled particle cannot modify SM predictions, it can have impact on the SM parameters allowing for new interpretations. Therefore we will discuss FCNCs in the decoupling limit in section 2.3.3 and get back to the decoupled 2HDM explaining how it might impact neutrino masses in section 3.4.3.

To suppress FCNCs stronger it can again be applied the principle of MFV [22, 54]. Apart from flavour symmetries, a Yukawa Lagrangian of MFV type could emerge for example from breaking of the discrete symmetry in type I or II. In this case the MFV specific terms are typically suppressed, as forbidden by the broken symmetry. Still, in the case of large $\boldsymbol{Y_d^2}$, spectacular effects are possible [22]. A special case of the MFV hypothesis that is even more conservative regarding FCNCs is the aligned 2HDM.

2.3.2. Radiative corrections to the aligned 2HDM

The assumption of alignment can be viewed as the MFV hypothesis with the terms linear in the Yukawa couplings being dominant. It can be defined by:

$$\boldsymbol{Y_f^1} = \cos\psi_f \boldsymbol{Y_f} \;, \qquad \boldsymbol{Y_f^2} = \sin\psi_f \boldsymbol{Y_f} \;, \qquad \text{where } f = u, d, e \;. \tag{2.28}$$

Here ψ_f are arbitrary complex numbers and $\boldsymbol{Y_f}$ are matrices. We have chosen to introduce the angles ψ_f to obtain results manifestly invariant under Higgs basis rotations, eq. (2.24). Under such a rotation the Yukawa matrices change by $\boldsymbol{Y_f}^{1,2} \to \cos\alpha \boldsymbol{Y_f}^{1,2} \pm \sin\alpha \boldsymbol{Y_f}^{2,1}$, but in the introduced notation only the angle is shifted, $\psi_f \to \psi_f - \alpha$ and $\boldsymbol{Y_f} \to \boldsymbol{Y_f}$.

Recently the aligned 2HDM has received some attention [55], cf. also [56]. It was also constructed an explicit model in which the Yukawa alignment is a consequence of breakdown of a discrete symmetry in a multi Higgs doublet model [57]. In this model the only Higgs doublets that are allowed to couple to the fermions are assumed to be rather heavy. These couplings are transferred to the two light Higgs doublets by integrating out the heavy states. Large Yukawa couplings can be obtained if the vacuum expectation value of the field that breaks the discrete symmetry is of order of the heavy Higgs mass.

However, the main motivation for the aligned 2HDM is phenomenological. The alignment condition allows for large Yukawa couplings of both Higgs doublets to all fermions, still forbidding strictly FCNCs at tree level. This opens up the possibility of new CP violating phases in the Yukawa sector [55, 58]. Moreover, the aligned 2HDM is a generalization of 2HDMs with discrete symmetries as the types I, II. Barring cancellations, the aligned 2HDM yields the minimal amount of FCNCs without imposing discrete symmetries at the weak scale. The FCNCs induced by charged Higgs loops have been considered in [59] and found to be below present experimental bounds even for $|\tan\psi_d|$, $|\tan\psi_e|$ of a few tens.

The alignment condition presumably holds at some high cut-off scale Λ, much larger than the electroweak scale. Via the RG evolution, one expects therefore deviations from the alignment condition which are of the MFV type. These are loop suppressed but enhanced by the logarithm of the ratio of the scale Λ to the low scale. We have analysed the induced MFV terms and their impact on FCNCs in [8] and will reproduce the discussion in the following.

The β-functions of the Yukawa couplings in the 2HDM are listed in appendix A.2.1 for convenience. To understand the qualitative behaviour, we study them analytically within the leading-log approximation,

$$\boldsymbol{Y}^a_{\boldsymbol{f}}(m_H) \approx \boldsymbol{Y}^a_{\boldsymbol{f}}(\Lambda) + \frac{1}{16\pi^2}\beta_{\boldsymbol{Y}^a_{\boldsymbol{f}}}(\Lambda)\log\left(\frac{m_H}{\Lambda}\right) , \tag{2.29}$$

but a numerical check will also be presented below. Analogously to the case of the MSSM, section 2.2, all possible MFV terms are generated. Thus, at the electroweak scale the Yukawa couplings can be written in the form:

$$\boldsymbol{Y}^a_{\boldsymbol{u}}(m_H) \simeq k^a_u \boldsymbol{Y_u} + \epsilon^a_u \boldsymbol{Y_d}\boldsymbol{Y_d}^\dagger \boldsymbol{Y_u} + \delta^a_u \boldsymbol{Y_u}\boldsymbol{Y_u}^\dagger\boldsymbol{Y_u} , \tag{2.30a}$$

$$\boldsymbol{Y}^a_{\boldsymbol{d}}(m_H) \simeq k^a_d \boldsymbol{Y_d} + \epsilon^a_d \boldsymbol{Y_u}\boldsymbol{Y_u}^\dagger \boldsymbol{Y_d} + \delta^a_d \boldsymbol{Y_d}\boldsymbol{Y_d}^\dagger\boldsymbol{Y_d} , \tag{2.30b}$$

where the expressions for k^a_f, ϵ^a_f and δ^a_f can be found in appendix A.2.2. For our purposes it is convenient to work in the basis where only one Higgs doublet Φ_{vev} has a vacuum expectation value, cf. eq. (2.26). The quark masses are proportional to the Yukawa coupling $\boldsymbol{Y}^{\text{vev}}_{\boldsymbol{f}}$ of the Higgs with non-vanishing vacuum expectation value. It reads:

$$\boldsymbol{Y}^{\text{vev}}_{\boldsymbol{f}}(m_H) = \cos\beta \boldsymbol{Y}^1_{\boldsymbol{f}}(m_H) + \sin\beta \boldsymbol{Y}^2_{\boldsymbol{f}}(m_H) . \tag{2.31a}$$

The other Higgs couples with

$$\boldsymbol{Y}^{\text{no vev}}_{\boldsymbol{f}}(m_H) = -\sin\beta \boldsymbol{Y}^1_{\boldsymbol{f}}(m_H) + \cos\beta \boldsymbol{Y}^2_{\boldsymbol{f}}(m_H) . \tag{2.31b}$$

Lastly we go into the basis of diagonal quark mass matrices. Introduce therefore unitary matrices $\boldsymbol{V^L_f}$, $\boldsymbol{V^R_f}$ such that

$$\frac{v}{\sqrt{2}}\boldsymbol{V^{L\dagger}_u}\boldsymbol{Y}^{\text{vev}}_{\boldsymbol{u}}(m_H)\boldsymbol{V^R_u} = \boldsymbol{M_u} \equiv \text{diag}(m_u, m_c, m_t) , \tag{2.32a}$$

$$\frac{v}{\sqrt{2}}\boldsymbol{V^{L\dagger}_d}\boldsymbol{Y}^{\text{vev}}_{\boldsymbol{d}}(m_H)\boldsymbol{V^R_d} = \boldsymbol{M_d} \equiv \text{diag}(m_d, m_s, m_b) . \tag{2.32b}$$

FCNCs are mediated at tree level only by $\Phi_{\text{no vev}}$ which couples to the quark mass eigenstates $\bar{Q}'$, u', d' by

$$-\mathcal{L}^{\text{Yuk}} \supset \bar{Q}'_i\sqrt{2}\boldsymbol{\Delta}_{\boldsymbol{u}ij}u'_j\tilde{\Phi}_{\text{no vev}} + \bar{Q}'_i\sqrt{2}\boldsymbol{\Delta}_{\boldsymbol{d}ij}d'_j\Phi_{\text{no vev}} + \text{h.c.} , \tag{2.33}$$

where

$$\sqrt{2}\boldsymbol{\Delta_f} = \boldsymbol{V_f^{L\dagger}} \boldsymbol{Y_f}^{\text{no vev}}(m_H) \boldsymbol{V_f^R} \;, \tag{2.34}$$

which is in general not diagonal. It can be calculated easily in the leading-log approximation using a little trick. First rewrite

$$\boldsymbol{\Delta}_f = \frac{1}{v}(\boldsymbol{V_f^{L\dagger}} \boldsymbol{Y_f}^{\text{no vev}} (\boldsymbol{Y_f}^{\text{vev}})^{-1} \boldsymbol{V_f^L}) \boldsymbol{M_f} \;. \tag{2.35}$$

The last term $\boldsymbol{M_f}$ corresponds to the measured quark mass matrix. At the cut-off scale $\boldsymbol{Y_f}^{\text{no vev}}$ is aligned to $\boldsymbol{Y_f}^{\text{vev}}$. Therefore, the product $\boldsymbol{Y_f}^{\text{no vev}}(\boldsymbol{Y_f}^{\text{vev}})^{-1}$ is proportional to the unit matrix at zeroth order and the contribution from the evolution of $\boldsymbol{V_f^L}$ cancels at first order. As long as the higher order terms in the Yukawa couplings are subleading, $\boldsymbol{Y_f}^{\text{vev}}$ can be inverted by

$$(\boldsymbol{Y_u}^{\text{vev}})^{-1}(m_H) \approx (k_u^{\text{vev}})^{-2} (\boldsymbol{Y_u})^{-1} \left(k_u^{\text{vev}} \mathbb{1} - \epsilon_u^{\text{vev}} \boldsymbol{Y_d} \boldsymbol{Y_d}^\dagger - \delta_u^{\text{vev}} \boldsymbol{Y_u} \boldsymbol{Y_u}^\dagger \right) \;, \tag{2.36a}$$

$$(\boldsymbol{Y_d}^{\text{vev}})^{-1}(m_H) \approx (k_d^{\text{vev}})^{-2} (\boldsymbol{Y_d})^{-1} \left(k_d^{\text{vev}} \mathbb{1} - \epsilon_d^{\text{vev}} \boldsymbol{Y_u} \boldsymbol{Y_u}^\dagger - \delta_d^{\text{vev}} \boldsymbol{Y_d} \boldsymbol{Y_d}^\dagger \right) \;, \tag{2.36b}$$

being

$$k_f^{\text{vev}} = \cos\beta k_f^1 + \sin\beta k_f^2 \;, \quad \epsilon_f^{\text{vev}} = \cos\beta \epsilon_f^1 + \sin\beta \epsilon_f^2 \;, \quad \delta_f^{\text{vev}} = \cos\beta \delta_f^1 + \sin\beta \delta_f^2 \;. \tag{2.37}$$

We are mainly interested in the off-diagonal entries of $\boldsymbol{\Delta_u}$, $\boldsymbol{\Delta_d}$ which lead to FCNCs. The result can be decomposed,

$$\boldsymbol{\Delta_f}^{\text{off-diag.}} \simeq E_f \boldsymbol{Q_f} \qquad\qquad \text{where } f = u, d \;, \tag{2.38}$$

into the flavour matrices $\boldsymbol{Q_u}$, $\boldsymbol{Q_d}$ which are determined by the measured CKM matrix and quark masses:

$$\boldsymbol{Q_u} \equiv \frac{1}{v^3} \left(\boldsymbol{V_{CKM}} \left(\boldsymbol{M_d} \right)^2 \boldsymbol{V_{CKM}^\dagger} \boldsymbol{M_u} \right)^{\text{off-diag.}} \;, \tag{2.39a}$$

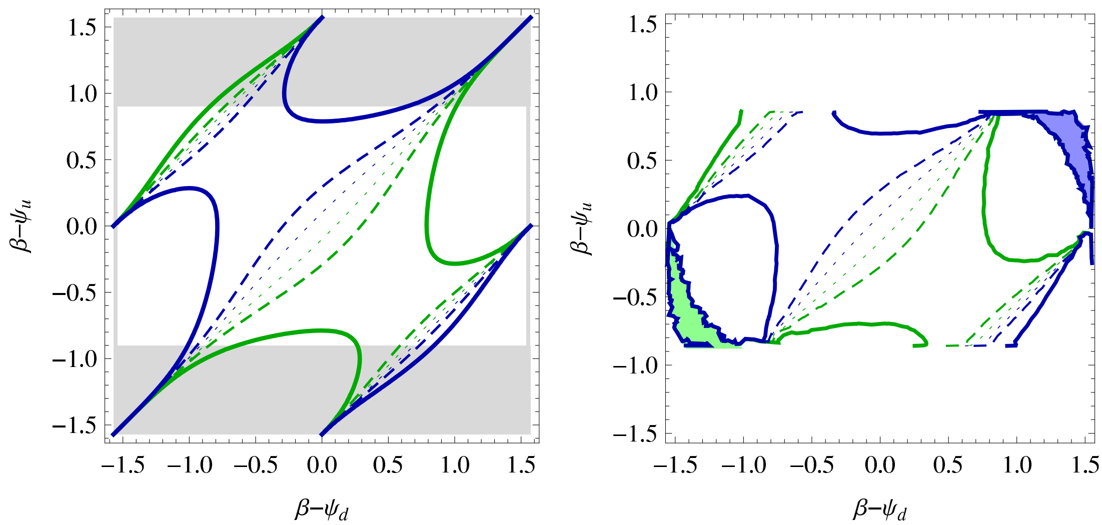
$$\boldsymbol{Q_d} \equiv \frac{1}{v^3} \left(\boldsymbol{V_{CKM}^\dagger} \left(\boldsymbol{M_u} \right)^2 \boldsymbol{V_{CKM}} \boldsymbol{M_d} \right)^{\text{off-diag.}} \;, \tag{2.39b}$$

and coefficients E_u, E_d that depend on the parameters of the 2HDM. For real ψ_u, ψ_d we find:

$$-E_d = E_u \equiv \frac{1}{8\pi^2} \frac{\sin(2(\psi_u - \psi_d))}{\cos^2(\beta - \psi_u)\cos^2(\beta - \psi_d)} \log\left(\frac{m_H}{\Lambda}\right) \;. \tag{2.40}$$

Apart from the scale, E_u, E_d depend only on the two parameters $\beta - \psi_u$ and $\beta - \psi_d$. As those are differences of angles, E_u, E_d are manifestly invariant under rotations of the Higgs basis, eq. (2.24). It is sufficient to look only at the parameter range $-\frac{\pi}{2} < \beta - \psi_{d,u} < \frac{\pi}{2}$ as E_u, E_d are invariant under a shift $\beta - \psi_{d,u} \to \beta - \psi_{d,u} + \pi$. There are two cases where tree level FCNCs vanish: Either $\psi_u = \psi_d$ or $\psi_u = \psi_d \pm \frac{\pi}{2}$. These cases correspond to type I and II 2HDMs which becomes evident in the basis $\psi_u = 0$.

Contours of E_d are plotted in fig. 2.3. As will be discussed below, E_d is permitted be $\mathcal{O}(1)$, still not being in conflict with bounds from FCNCs. The reason is of course the huge MFV suppression factor obtained from $\boldsymbol{Q_d}$.



(a) Analytical approximation of E_d **(b)** Numerical result for $2.5E_d$

Figure 2.3.: Contour plots of E_d for $\Lambda = 10^{19}$ GeV. The left figure corresponds to the analytic formula, eq. (2.40). Solid/dashed/dotted lines correspond to the absolute values of 1/0.3/0.1, blue lines correspond to negative values, green lines to positive ones. The right figure shows $2.5\boldsymbol{\Delta}_{\boldsymbol{d}23}/\boldsymbol{Q}_{\boldsymbol{d}23}$ where $\boldsymbol{\Delta}_{\boldsymbol{d}23}$ has been obtained by numerically solving the RGE. The rescaling was done in order to make the comparison to the analytical result easier.

We have chosen $\Lambda = 10^{19}$ GeV to maximise the deviations of the leading-log approximation to the numerical calculation. In the left plot we show contours of E_d as given by eq. (2.40). In the grey shaded regions at least one Yukawa coupling becomes non-perturbative below the cut-off scale. In the right plot it is shown the numerical solution of a one-loop integration for $\boldsymbol{\Delta}_{\boldsymbol{d}23}$, normalized to $\boldsymbol{Q}_{\boldsymbol{d}23}$. We have rescaled this plot by a factor of 2.5 to gain a better comparison to the analytic result. This factor is due to large, almost flavour independent RGE effects from the strong coupling constant and the top Yukawa that are underestimated in the leading-log approximation. Although the analytic expression resembles the numerical result quite well for almost the entire parameter space, there is also a new feature: At the top right and bottom left there are regions which have flipped sign and are shown shaded. In these regions, $\boldsymbol{Y}_{\boldsymbol{d}}^{\text{vev}}$ is much smaller than $\boldsymbol{Y}_{\boldsymbol{d}}^{\text{no vev}}$ and changes sign in the RG evolution. Diagonalizing the quark masses, eq. (2.32), this sign is transferred to $\boldsymbol{\Delta}_{\boldsymbol{d}}$. Equation (2.36b) fails in these regions, thus explaining why they are absent in the analytical approximation.

Comparison to experimental bounds

Next let us inspect how large E_u, E_d may be without violating experimental bounds. It turns out that a very powerful constraint comes from B_s^0–$\bar{B}_s^0$ oscillation. In the 2HDM, meson-antimeson oscillations can be mediated at tree level. The B_s^0 system is well measured and the $b \leftrightarrow s$ transition is the one with smallest MFV suppression as $\boldsymbol{Q}_{\boldsymbol{d}23} \sim 2\cdot 10^{-4}$ is still comparatively large. On the other hand $\boldsymbol{Q}_{\boldsymbol{d}32}$ is additionally suppressed by the strange Yukawa coupling. Thus the only relevant operators are Q_1^{SLL} and Q_2^{LR}, see section 2.1.1. Note that we have kept Q_2^{LR} although it is proportional to $\boldsymbol{Q_{d}}_{23}\boldsymbol{Q_{d}}_{32}$ as Q_1^{SLL} is not invariant under $\mathrm{SU(2)_L} \times \mathrm{U(1)}$ and thus suppressed if the mass of the heavy Higgs is much larger than the electroweak breaking scale.

If the Higgs potential conserves CP, the neutral Higgs fields can be expressed in terms of the mass and CP eigenstates h, H, A and G, where h, H are scalars, A is the physical pseudoscalar and G the Goldstone mode. Here we are interested in FCNCs. These are mediated only by $\Phi_{\text{no vev}}$,

$$\sqrt{2}\Phi^0_{\text{no vev}} = \sin(\alpha-\beta)H + \cos(\alpha-\beta)h + iA\,. \tag{2.41}$$

Using eq. (2.33) this yields for the corresponding Wilson coefficients:

$$C_1^{SLL} = -\frac{(\boldsymbol{\Delta_{d}}^*_{23})^2}{2}\left(\frac{\sin^2(\alpha-\beta)}{m_H^2} + \frac{\cos^2(\alpha-\beta)}{m_h^2} - \frac{1}{m_A^2}\right)\,, \tag{2.42a}$$

$$C_2^{LR} = -\boldsymbol{\Delta_{d}}^*_{23}\boldsymbol{\Delta_{d}}_{32}\left(\frac{\sin^2(\alpha-\beta)}{m_H^2} + \frac{\cos^2(\alpha-\beta)}{m_h^2} + \frac{1}{m_A^2}\right)\,. \tag{2.42b}$$

Inserting the running quark mass values for $\boldsymbol{Q_d}$ from [60] and taking the experimental bounds on the Wilson coefficients from evaluating equation (2.4) at the electroweak scale, $\mu \sim \mu_W$, one obtains the approximate bounds:

$$\left|\frac{\sin^2(\alpha-\beta)}{m_H^2} + \frac{\cos^2(\alpha-\beta)}{m_h^2} - \frac{1}{m_A^2}\right| |E_d|^2 \lesssim \frac{1}{(70\,\text{GeV})^2}\,, \tag{2.43a}$$

$$\left|\frac{\sin^2(\alpha-\beta)}{m_H^2} + \frac{\cos^2(\alpha-\beta)}{m_h^2} + \frac{1}{m_A^2}\right| |E_d|^2 \lesssim \frac{1}{(20\,\text{GeV})^2}\,. \tag{2.43b}$$

Thus even for light Higgs masses $|E_d| \gtrsim 1$ is allowed. Note that in eq. (2.43a) large cancellations can be expected to happen within the absolute value bars if the Higgs fields are close in mass or if the non-SM Higgs fields decouple, $\cos(\alpha-\beta) \to 0$, and $m_H \approx m_A$ which is a direct result of a heavy second Higgs mass, cf. section 2.3.3. In this case, eq. (2.43b) becomes relevant and simplifies to

$$|E_d| \lesssim \frac{1}{\sqrt{2}}\frac{m_H}{20\,\text{GeV}}\,. \tag{2.44}$$

For large leptonic Yukawa couplings a stronger bound can be obtained from the decay $B_s^0 \to \mu^+\mu^-$. The Wilson coefficients read neglecting $\boldsymbol{\Delta_{d}}_{32}$, cf. section 2.1.2:

$$\begin{aligned} C^S = &\frac{1}{4}\cos(\alpha-\beta)\boldsymbol{\Delta_{d}}_{23}\frac{1}{m_h^2}\left(-\frac{m_\mu}{v}\sin(\alpha-\beta) + \cos(\alpha-\beta)\boldsymbol{\Delta_{e}}_{22}\right) \\ &+ \frac{1}{4}\sin(\alpha-\beta)\boldsymbol{\Delta_{d}}_{23}\frac{1}{m_H^2}\left(\frac{m_\mu}{v}\cos(\alpha-\beta) + \sin(\alpha-\beta)\boldsymbol{\Delta_{e}}_{22}\right)\,, \end{aligned} \tag{2.45a}$$

$$C^P = \frac{1}{4}\boldsymbol{\Delta_{d}}_{23}\frac{1}{m_A^2}\boldsymbol{\Delta_{e}}_{22}\,. \tag{2.45b}$$

The conservative assumption $v\boldsymbol{\Delta}_{\boldsymbol{e}22} \ll m_\mu$ leads to a bound on $\boldsymbol{\Delta}_{\boldsymbol{d}23}$ only. However, this bound turns out to be weaker than the one obtained from B_s^0–$\bar{B}_s^0$ oscillation. In the other limit $v\boldsymbol{\Delta}_{\boldsymbol{e}22} \gg m_\mu$, we find from eq. (2.9)

$$\sqrt{\frac{1}{m_A^4} + \left|\frac{\sin(\alpha-\beta)^2}{m_H^2} + \frac{\cos(\alpha-\beta)^2}{m_h^2}\right|^2} |\boldsymbol{\Delta}_{\boldsymbol{e}22} E_d| \lesssim \frac{1}{(800 \text{ GeV})^2} \ . \tag{2.46}$$

Hence for Higgs masses of $\mathcal{O}(100\,\text{GeV})$ and E_d of $\mathcal{O}(1)$ there is only a conflict for $v\boldsymbol{\Delta}_{\boldsymbol{e}22} \gtrsim 40 m_\mu$.

To summarize, the aligned 2HDM escapes bounds from tree-level FCNCs for large portions of the parameter space. However, depending mainly on the up-type quark Yukawa couplings, our bounds yield interesting constraints if the down-type quark Yukawa matrix is much larger than in the SM.

2.3.3. The decoupled 2HDM

The decoupling limit is defined as the limit where one of the Higgs fields h is light and behaves like the SM Higgs, but the others, H, A, $H^\pm$ are much heavier than the electroweak scale. To be more concrete, we introduce the potential:

$$\begin{aligned} V = &\, m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] \\ &+ \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ &+ \left[\frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \text{h.c.}\right] \ . \end{aligned} \tag{2.47}$$

If $|\lambda_i| \lesssim 1$, the necessary and sufficient conditions for decoupling in terms of the Lagrangian are [51]:

$$m_{11}^2 + m_{22}^2 \gg v^2 \ , \tag{2.48a}$$

$$(m_{11}^2 + m_{22}^2)^2 \gg -m_{11}^2 m_{22}^2 + m_{12}^2 > 0 \ . \tag{2.48b}$$

Virtues and Drawbacks

As argued already in section 2.3.1, the 2HDM has in general problems in avoiding too large CP violation, FCNCs and contributions to the oblique parameters. These effects are either mediated by the second Higgs doublet or are proportional to its vacuum expectation value. If the second Higgs doublet is heavy, automatically its vacuum expectation value must be small. This is naively clear but will also be shown below. Therefore, all of these problems are alleviated in the decoupling limit. Also, it turns out that for similar reasons the danger of breaking electric charge with the vacuum expectation value of the second Higgs is absent in the decoupling limit. Again, we will come back to this point below.

On the other hand, introducing a new mass scale to the model is a drawback for the 2HDM to be a *minimal* extension of the SM. However, we believe that this possibility is still simpler than solving each of the aforementioned problems by a suited choice of parameters. Such a heavy Higgs also enforces the hierarchy problem by the radiative contributions to the light Higgs mass quadratic in the heavy mass scale. Lastly, from a phenomenological point of view it does not make sense to study models that are indistinguishable from the SM. This argument can of course be circumvented if the strength of decoupling is such that the 2HDM effects are invisible for present experiments but just around the corner. Furthermore, we will argue that in order to hide the 2HDM from present experiments, it is enough a heavy Higgs mass of a few TeV, at least if there are no new sources of CP violation contributing to ϵ_K. This mass scale is very close to the electroweak scale and low enough not to introduce large quadratic divergences. Therefore, in the few TeV range none of the arguments against the decoupling limit apply.

In any case, it should be noted that the RGE effects of the second Higgs, in particular the improvement of gauge coupling unification and the possible explanation of the mild neutrino mass hierarchy, section 3.4.3, depend only logarithmically on the mass scale of the decoupling. Therefore these virtues still apply in the decoupling limit.

In the following we will outline in what extent decoupling to the TeV scale is sufficient to guarantee conservation of electric charge and to suppress exotic contributions to the oblique parameters below present limits. It will turn out that generically more stringent bounds are obtained from FCNCs.

Conservation of electric charge

In the SM there is only one Higgs doublet, thus SU(2) $\times$ U(1) can be broken down in the Higgs mechanism only to U(1). Technically, this can be understood by the fact that any vacuum expectation value of a SU(2) Higgs doublet can be put into the lower SU(2) doublet component, cf. eq. (2.25), with the help of a gauge transformation. Obviously there is still the freedom for U(1) rotations of the upper component. However this freedom is broken if there exists another Higgs with a vacuum expectation value of its first component.

In general the two Higgs doublets can be rewritten as:

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi_1 \end{pmatrix} , \qquad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma \\ \varphi_2 \end{pmatrix} , \tag{2.49}$$

where σ and φ_2 are complex and φ_1 is a real field. In this notation electric charge is certainly unbroken if there does not exist any minimum of the potential with $\langle\sigma\rangle \neq 0$. We will show that indeed this is the case in the decoupling limit. For convenience we will work in the Higgs basis of $m_{12}'^2 = 0$, which can be achieved by a basis rotation eq. (2.24) and a phase rotation. For definiteness we equip in this basis any parameters of the Lagrangian with a prime. We will make use of the following axioms:

- The potential is unbounded from below.
- The quartic couplings are not arbitrarily large, e.g. $|\lambda_i'| \lesssim 1$.

  This is necessary anyway by perturbativity. The actual upper bound will shift the decoupling scale needed in order to guarantee absence of charge breaking.

- The second Higgs is decoupled.

  The definition of decoupling, eq. (2.48), is not suited because we may not assume anything about the minima, as it is done in eq. (2.48a). Instead, we will make use of two conditions that depend only on parameters of the potential and are necessary for decoupling (and turn out also to be sufficient). They read, choosing Φ_1 to be the SM-like Higgs:

$$m'^2_{11} < 0 \qquad\qquad m'^2_{22} \gg \frac{|m'^2_{11}|}{\lambda'_1} \tag{2.50}$$

Note that the only condition that is not imposed usually also on the SM, is the last one.

Consider now fixed ratios of the fields $\varphi_2 = a\varphi_1$, $\sigma = b\varphi_1$. In this direction the potential reads:

$$V_{a,b}(\varphi_1) = \frac{1}{2}m'^2_{11}|\varphi_1|^2 + \frac{1}{2}m'^2_{22}(|a|^2 + |b|^2)|\varphi_1|^2 + \text{const.} \times |\varphi_1|^4 \,. \tag{2.51}$$

Clearly, if V has a minimum at $\varphi_1 = \langle\varphi_1\rangle$, $\varphi_2 = \langle\varphi_2\rangle$, $\sigma = \langle\sigma\rangle$, then $V_{a,b}$ also has a minimum at $\varphi_1 = \langle\varphi_1\rangle$ for the parameters $a = \langle\varphi_2\rangle / \langle\varphi_1\rangle$ and $b = \langle\sigma\rangle / \langle\varphi_1\rangle$. As we require the potential to be bounded from below, the quartic part in eq. (2.51) is non-negative. Therefore $V_{a,b}$ can only have a minimum if the quadratic part is negative, thus

$$|\,\langle\varphi_2\rangle\,|^2 + |\,\langle\sigma\rangle\,|^2 < \frac{|\langle\varphi_1\rangle|^2}{m'^2_{22}}|m'^2_{11}| \,. \tag{2.52}$$

The vacuum expectation value can now be determined as in the SM by differentiation of V with respect to φ_1. The quartic terms with $\lambda'_{i>1}$ are proportional to at least one $\langle\varphi_2\rangle$ or $\langle\sigma\rangle$ and therefore negligible compared to the λ'_1 term.

$$|\langle\varphi_1\rangle|^2 = \frac{2|m'^2_{11}|}{\lambda'_1}\left[1 + \mathcal{O}\left(\sqrt{\frac{|m'^2_{11}|}{\lambda'_1 m'^2_{22}}}\right)\right] \,. \tag{2.53}$$

Varying now V with respect to σ it is straightforward to verify that electric charge is unbroken

$$\langle\sigma\rangle = 0 \,. \tag{2.54}$$

To complete the analysis of the minima, differentiate with respect to φ_2, yielding

$$\langle\varphi_2\rangle \simeq -\langle\varphi_1\rangle\frac{\lambda'^*_6\langle\varphi_1\rangle^2}{2m'^2_{22}} \,. \tag{2.55}$$

Thus $\langle\varphi_2\rangle$ vanishes in the decoupling limit and the SM vacuum is recovered.

Finally let us comment on the necessary decoupling. Strictly speaking the condition eq. (2.50) is not enough. Instead, eq. (2.53) tells us that it is necessary $\sqrt{m'^2_{22}} \gg \sqrt{|m'^2_{11}|/\lambda'_1}$ which points to the TeV scale. A more accurate estimate is presented in appendix A.3. Note also that the square root in eq. (2.53) appears only because of the λ_6 term. If this is negligible, the decoupling required in order to forbid the existence of charge breaking minima is even weaker.

Electroweak precision tests

The exotic contributions of the 2HDM to the oblique parameters S, T, U have been calculated first in [61] for the CP conserving case. Later it has been generalized to general multi Higgs models [62] and the decoupling limit was exploited in [63]. They are most conveniently written down in the Higgs basis of $\langle \Phi_2 \rangle = 0$:

$$S \simeq \frac{\lambda_4 v^2}{24\pi m_{22}^2} \,, \tag{2.56a}$$

$$T \simeq \frac{\lambda_4^2 - |\lambda_5|^2}{48\pi e^2} \frac{v^2}{m_{22}^2} \,, \tag{2.56b}$$

$$U \simeq S \tan^2 \theta_W \,. \tag{2.56c}$$

Experimentally, those values have been determined with a precision of about ± 0.1 with a central value in agreement with the SM for a rather light Higgs [1]. Therefore, in the decoupling limit, the bounds on the oblique parameters pose no additional constraint on the potential [64].

Flavour Changing Neutral Currents and CP violation

To estimate the impact of FCNC bounds we look again at tree level B_s^0–$\bar{B}_s^0$ mixing. A popular way to parametrize the unknown Yukawa couplings of the second Higgs is given by the Cheng and Sher ansatz [21],

$$\boldsymbol{\Delta_f}_{ij} = \frac{\lambda_{ij}}{v^2} \sqrt{\boldsymbol{M_f}_{ii} \boldsymbol{M_f}_{jj}} \,, \tag{2.57}$$

with parameters λ_{ij} that are expected to be $\mathcal{O}(1)$ from flavour model building (cf. e.g. [65]).

From section 2.3.2 we know already that in the decoupling limit the operator Q_2^{LR} dominates. The Wilson coefficient is given by eq. (2.42b). Comparing with the bound presented in section 2.1.1 and taking the quark masses at the TeV scale from [60], one obtains:

$$|\boldsymbol{\Delta_d}^*_{23} \boldsymbol{\Delta_d}_{32}| \frac{1}{m_H^2} \lesssim \frac{m_s m_b}{v^2} \frac{1}{(2\,\mathrm{TeV})^2} \,. \tag{2.58}$$

Note that a bound of comparable strength can be obtained also from $\mu \to e\gamma$, see section 3.4.1.

If the 2HDM breaks CP explicitly, a much stronger bound can be obtained from CP violation in kaon decay. From the analysis [53] we infer that in the Cheng and Sher ansatz the mass difference in the kaon system yields a bound on the decoupling scale similar to the one obtained in B_s^0–$\bar{B}_s^0$ mixing. The Kaon mass difference Δm_K is governed by the real part of $\langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta F=2} | K^0 \rangle$, while ϵ_K by the imaginary part [12]:

$$\Delta m_K = 2 \,\mathrm{Re} \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta F=2} | K^0 \rangle \,, \tag{2.59a}$$

$$\epsilon_K = \frac{\exp(i\frac{\pi}{4})}{\sqrt{2}\Delta m_K} \,\mathrm{Im} \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta F=2} | K^0 \rangle \,. \tag{2.59b}$$

Long distance contributions may contribute up to 30% to Δm_K and are difficult to estimate [66, 67]. Therefore, the experimental precision in Δm_K does not help in constraining new physics.

On the other hand, the imaginary part can be computed to high accuracy, and ϵ_K has been determined experimentally to a precision of $\mathcal{O}(10^{-5})$ [1]:

$$\epsilon_K = (2.23 \pm 0.01) \cdot 10^{-3} \, . \tag{2.60}$$

Therefore, if we allow for arbitrary complex phases, the necessary decoupling scale is raised by a few orders of magnitude.

We conclude that a decoupled 2HDM with a heavy Higgs scale of a few TeV is free from the problems typically associated with a 2HDM. The only additional phenomenological requirement is the absence of new contributions to CP violating observables.

3 Solving challenges of the see-saw

3.1. Neutrino data

In the last 15 years neutrino experiments have made formidable progress. Most importantly they have shown the existence of small neutrino masses. This allowed to explain the long-standing puzzle of the solar neutrino deficit [68–70]. Moreover they have revealed several properties:

The existence of three light neutrinos

The precise determination of the Z decay width has proven the number of active light neutrinos to be three with high accuracy [1]. However, there might exist also additionally light sterile neutrinos without weak interactions. The number of neutrinos can be constrained by the radiation content of the universe, in particular from requiring successful Big Bang Nucleosynthesis (BBN). Nevertheless, present data allows for one sterile neutrino and maybe for two if they are not fully thermalized [71–73]. Most of the oscillation experiments are in good agreement with the existence of no sterile neutrino. Still, there are some experimental results in conflict with the three neutrino picture [74–76]. A simultaneous explanation of all neutrino laboratory data, though, requires at least two sterile neutrinos with masses in conflict to cosmology [77].

Therefore we will stick in this thesis to the conservative assumption of three active and no light sterile neutrinos, and parametrize their masses by

$$\boldsymbol{\mathcal{M}_\nu} = \boldsymbol{U^*_{MNS}}\,\mathrm{diag}(m_1, m_2, m_3)\boldsymbol{U^\dagger_{MNS}}\,. \tag{3.1}$$

An upper bound on the overall scale of neutrino masses

The best laboratory-experimental bound on the absolute scale of neutrino masses is obtained from the endpoint of the electron spectrum in the tritium beta decay. At present the bound is around 2 eV [78, 79], but can be narrowed to approximately 0.2 eV within the next few years at the KATRIN experiment [80]. One can infer more stringent bounds from cosmology through constraints on the radiation content. These bounds vary depending on the exact cosmological model assumed but are roughly of the order 0.5 eV [81–85]. Again this bound can be expected to improve in the future by several experiments [86].

Determining the neutrino nature – Dirac or Majorana

The only known way to discriminate between Dirac and Majorana neutrinos is the search for neutrinoless double β decay. This process would lead to a narrow peak beyond the endpoint of the two electron spectrum as the electrons carry the full released energy. As this decay violates lepton number it is only allowed if neutrinos are Majorana particles. In this case the decay rate is proportional to the square of the effective Majorana mass

$$\langle m_{ee} \rangle = \left| \sum_i \boldsymbol{U_{MNS}}_{1i} m_i \right| . \tag{3.2}$$

Present bounds are located somewhere below one eV with large uncertainties from unknown nuclear matrix elements [1]. Evidence in favour for the existence of neutrinoless double β decay was found by members of the Heidelberg-Moscow [87] experiment, while the other part of the collaboration disagreed [88]. Also most of the community is sceptical about this claim, criticizing mainly the background estimation [89–91]. Certainly this issue will be answered by the many experiments currently running or being prepared [92]. Finally, the sensitivity to neutrinoless double β decay is expected to improve with present techniques down to the order of 10^{-2} eV for one ton experiments and ten years of exposure time.

Measurement of neutrino oscillation parameters

By measurements of neutrinos oscillations the squared mass splittings $\Delta m^2_{ij} = m^2_i - m^2_j$ and the mixing angles can be determined. The mixing angles are defined by [1]:

$$\begin{aligned} \boldsymbol{U_{MNS}} = &\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \\ &\times \mathrm{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}) . \end{aligned} \tag{3.3}$$

Here the Majorana phases α_{21}, α_{31} can be non-vanishing only in the case of Majorana neutrinos and cannot be determined by neutrino oscillation experiments anyway.

Neutrino oscillations have been studied by lots of experiments in the last decade. This has led to a determination of two squared mass splittings and two mixing angles with high precision.

The best determination of the solar parameters has been obtained by a combination of solar neutrino experiments and the KamLAND reactor neutrino experiment [93]:

$$\Delta m^2_{21} \simeq 7.59^{+0.20}_{-0.21} \cdot 10^{-5}\,\mathrm{eV}^2 , \qquad \sin^2 2\theta_{12} \simeq 0.86^{+0.03}_{-0.02} . \tag{3.4}$$

The atmospheric angle θ_{23} was found to be very close to maximal mixing by the Super-Kamiokande Collaboration [94]. Lastly the atmospheric mass splitting was measured at the MINOS detector utilizing the NuMI neutrino beam from Fermilab [95]:

$$|\Delta m^2_{32}| \simeq 2.43^{+0.13}_{-0.13} \cdot 10^{-3}\,\mathrm{eV}^2 , \qquad \sin^2 2\theta_{23} > 0.90 . \tag{3.5}$$

As the bounds on the overall mass scale are still far above the mass scales indicated by neutrino oscillations, at present it is not clear whether neutrinos exhibit a normal hierarchical spectrum $m_1 < m_2 < m_3$ with larger mass splittings between heavier neutrinos or an inverted hierarchical spectrum $m_3 < m_1 < m_2$. In the latter case the atmospheric mass splitting is negative.

Currently, the most of the effort is put into the determination of θ_{13}. Nevertheless the most stringent δ-independent upper bound is still set by the CHOOZ reactor experiment and reads [1, 96]:

$$\sin^2 2\theta_{13} < 0.15 \; . \tag{3.6}$$

For the case of normal hierarchy the MINOS collaboration claims a somewhat better bound which however depends on δ [97]. More interestingly, there are a few hints for a sizeable θ_{13}, the strongest one coming from electron neutrino appearance at T2K [98], an off-axis μ neutrino beam experiment, but cf. also [99, 100]. As the T2K analysis was carried out only with a small fraction of the expected data, large improvements can be expected [101]. However, the most precise determination of the oscillation angle will be provided by the new reactor experiments Double CHOOZ [102], Daya Bay [103] and RENO [104] all of which combine identical near and far detectors to reduce the normalization error. Completion of the last detectors is planned for 2012 [105–107]. The goal is the determination of $\sin^2 2\theta_{13}$ to a precision of a few percent in the next few years.

3.2. The see-saw mechanism

An inspection of the SM particle content at first sight reveals an asymmetry. In the quark sector there are three $\mathrm{SU}(2)_\mathrm{L}$ doublets and six $\mathrm{SU}(2)_\mathrm{L}$ singlets with charges such that any quark can obtain Dirac masses via the Higgs vacuum expectation value. Also in the lepton sector there are three $\mathrm{SU}(2)_\mathrm{L}$ doublets. However, its electrically neutral component, i.e. the left-handed neutrino, does not have a $\mathrm{SU}(2)_\mathrm{L}$ singlet counterpart. Therefore, even without any explanation of the SM particle content, one is led to the introduction of right-handed neutrinos by this naive symmetry argument. On a more fundamental level the introduction of right-handed neutrinos allows to put the SM particle content into three families of 16-plets of SO(10). The most general Lagrangian now includes

$$-\mathcal{L}^\nu = \bar{L}_i \boldsymbol{Y_\nu}_{ij} \nu_j \tilde{\Phi} - \frac{1}{2} \bar{\nu}_i^C \boldsymbol{M}_{ij} \nu_j + \text{h.c.} \tag{3.7}$$

The Majorana mass $\boldsymbol{M}$ is peculiar to the right-handed neutrinos as they are the only fermions not charged under the SM gauge group. As this mass is independent of electroweak symmetry breaking it can be very heavy, e.g. of the scale of SO(10) breakdown. Then the right-handed neutrinos can be integrated out yielding the Weinberg operator that is the only dimension five operator allowed in the SM

$$-\mathcal{L}^{\nu,\text{ low}} = \frac{1}{2} (\bar{L}_i \tilde{\Phi}) \boldsymbol{\kappa}_{ij} (\tilde{\Phi}^T L_j^C) + \text{h.c.} \; , \qquad \text{where } \boldsymbol{\kappa} = \boldsymbol{Y_\nu} \boldsymbol{M}^{-1} \boldsymbol{Y_\nu}^T \; . \tag{3.8}$$

After electroweak symmetry breaking the left-handed neutrinos obtain masses that are the lighter the heavier the right-handed Majorana mass is. This is the renowned see-saw mechanism, which was proposed already in the late 70s [108–110].

There are two objections concerning the heavy mass scale involved. First, if there are particles much heavier than the electroweak scale, they will contribute quadratically to the Higgs mass by radiative corrections. These corrections are typically many orders of magnitude larger than the electroweak scale, thus requiring huge cancellations in order to obtain the correct electroweak symmetry breaking scale. This is the hierarchy problem which is not special to the see-saw but inherent to any extension of the SM that requires large mass scales. A possible solution is the introduction of SUSY.

Secondly, the high scale see-saw mechanism is very difficult, if not impossible, to test. Certainly, the high mass scale cannot be accessed directly by experiment. Neither it is possible to check correlations with other effective operators, as those are suppressed by more powers of $\boldsymbol{M}$. The same holds true also for the non-unitarity of the leptonic mixing matrix, that is predicted to exist but to be incredibly small. Even worse, the see-saw makes no prediction on the flavour structure of $\boldsymbol{\kappa}$ – a fact that can be directly inferred from eq. (3.8). Nevertheless there are some chances to rule out the see-saw as presented below. Further investigations in extensions of the SM are topic of section 3.3 and 3.4.

3.2.1. Confronting the see-saw with neutrino data

In the following we will discuss why the neutrino data supports the idea of the see-saw mechanism and how the see-saw could be excluded. Therefore we will follow the same points listed in section 3.1. The question of how to embed the neutrino flavour structure into the see-saw, however, will be discussed mainly in the next section.

No sterile neutrinos

Sterile neutrinos are not charged under the gauge group of the SM. Thus in the spirit of the see-saw they should be very heavy. Vice versa, if sterile neutrinos with mass in the eV scale turned out to exist, the see-saw mechanism as an explanation for the below eV mass scale would be superfluous.

The sub eV mass scale

Of course the smallness of neutrino masses supports the see-saw mechanism. But one can go further. As the τ and bottom Yukawa couplings are of the same order one might infer by analogy that the largest neutrino Yukawa coupling is of the order of the top Yukawa coupling. This is even mandatory in GUTs [111]. Assuming the atmospheric mass splitting to set the mass scale of the heaviest of the three light neutrinos, the see-saw formula, eq. (3.8), yields, one of the heavy right-handed neutrinos to have a mass around 10^{15} GeV. This is very pleasing as it is close to the scale of grand unification where new physics is expected anyway. In addition it supports the idea of SO(10) unification which forbids neutrino Majorana masses above the scale of grand unification that is expected to be $\sim 10^{16}$ GeV from gauge coupling unification.

The Majorana nature of neutrinos

The see-saw model unambiguously predicts neutrinos to be Majorana particles. If the opposite was found, clearly the see-saw would be ruled out. However, this proof can be very challenging for experiments, as $\langle m_{ee}\rangle$ can be very small, depending on m_1. For example, in order to rule out Majorana neutrinos with the desirable future bound $\langle m_{ee}\rangle < 10^{-2}\,\text{eV}$ it is required the sum of neutrino masses to be at least more than $0.1\,\text{eV}$ [1]. Therefore, experiments that are now in the R&D phase are just sufficient to rule out the inverted hierarchy case for Majorana neutrinos. If however $m_1 \ll m_2$, another improvement of one order of magnitude in sensitivity would be necessary [91] to prove the Dirac nature. The case $m_1 \sim m_2$ even allows for vanishing $\langle m_{ee}\rangle$ due to cancellations. In this unfortunate case it seems to be impossible to decide the nature of neutrinos.

The flavour structure

Under the assumption of at least three right-handed neutrinos the matrix $\boldsymbol{\kappa}$, eq. (3.8), has full rank and can accommodate any complex 3×3 matrix. Hence it is impossible to rule out the see-saw with the help of the oscillation parameters. On the other hand, from analogy to the charged fermion sector one might have expected large mass hierarchies and small mixings. Surprisingly the oscillation parameters revealed a completely different picture. How this can be accommodated in the see-saw will be discussed in the following.

3.2.2. The top-down parametrization and the flavour surprise

Before comparing the lepton and quark flavour sectors we want to recapitulate two features of the neutrino mass matrix eq. (3.1). First, the mass hierarchy between the two heavier neutrinos is rather mild. It is maximal if $m_1 \ll m_2$ in which case it is given by the ratio of the atmospheric to the solar mass splitting:

$$\left|\frac{m_3}{m_2}\right| \lesssim \sqrt{\frac{|\Delta m_{32}^2|}{\Delta m_{21}^2}} \approx 6\;. \tag{3.9}$$

Of course in the inverted hierarchy case m_3 is the lightest neutrino mass and m_1, m_2 are quasi degenerate.

Secondly the leptonic mixing matrix can be approximated by a tri-bi-maximal matrix [112]:

$$\boldsymbol{U_{MNS}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \times \operatorname{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})\;. \tag{3.10}$$

Both features are strange in view of the Yukawa couplings of charged fermions: the mass hierarchies vary between 17 in the lepton sector and over 100 in the up-quark sector and the off-diagonal entries of the quark mixing matrix $\boldsymbol{V_{CKM}}$ are suppressed by powers of 0.2. However, big discrepancies between the neutrino and the charged fermions sector are not surprising in the see-saw mechanism as the neutrino mass is not just proportional to the neutrino Yukawa coupling but, eq. (3.8)

$$\boldsymbol{\mathcal{M}_\nu} = \frac{v^2}{2}\boldsymbol{\kappa} = \frac{v^2}{2}\boldsymbol{Y_\nu}\boldsymbol{M}^{-1}\boldsymbol{Y_\nu}^T \ . \tag{3.11}$$

The matrices $\boldsymbol{Y_\nu}$, $\boldsymbol{M}$ are often parametrized in terms of the left-handed neutrino mass matrix $\boldsymbol{\mathcal{M}_\nu}$ [113]. This has the advantage that for any value of the parameters the light neutrino mass is in accordance with experiments. Here we are interested in a qualitative discussion about the kind of left-handed neutrino mass matrix one might expect in the see-saw and under which assumptions. Therefore we parametrize $\boldsymbol{Y_\nu}$, $\boldsymbol{M}$ in terms of parameters with physical meaning at the high scale: the heavy Majorana masses, the Yukawa coupling strengths and the left- and right-handed mixing matrices. One can always choose to work in a basis of diagonal $\boldsymbol{M}$ and then decompose $\boldsymbol{Y_\nu}$

$$\boldsymbol{Y_\nu} = \boldsymbol{V_L}\boldsymbol{D_Y}\boldsymbol{V_R}^\dagger \ , \qquad \boldsymbol{D_Y} = \mathrm{diag}(y_1, y_2, y_3) \ , \qquad \boldsymbol{M} = \mathrm{diag}(M_1, M_2, M_3) \ , \tag{3.12}$$

being $\boldsymbol{V_L}$, $\boldsymbol{V_R}$ unitary matrices and all entries in $\boldsymbol{D_Y}$, $\boldsymbol{M}$ real and non-negative. This parametrization has also the advantage that the comparison to the charged fermion sector becomes transparent: the left-handed mixing matrix $\boldsymbol{V_L}$ is the lepton analogue to the CKM matrix $\boldsymbol{V_{CKM}}$ and the Yukawa couplings $\boldsymbol{D_Y}$ can be directly compared to the diagonalized Yukawa matrices of charged fermions. For the sake of this discussion we will therefore assume $\boldsymbol{D_Y}$ to be hierarchical as it is the case for all known Yukawa couplings. On the other hand the conclusion that also $\boldsymbol{V_L}$ should exhibit small mixing is less clear as there is only one analogue in the charged fermion sector, namely $\boldsymbol{V_{CKM}}$. Consequently this will also play a minor role in the discussion below. The right-handed mixing matrix $\boldsymbol{V_R}$ and the heavy Majorana mass $\boldsymbol{M}$ have no counterparts. In this parametrization the light neutrino mass reads:

$$\boldsymbol{\mathcal{M}_\nu} = \frac{v^2}{2}\boldsymbol{V_L}\boldsymbol{D_Y}\boldsymbol{V_R}^\dagger\boldsymbol{M}^{-1}\boldsymbol{V_R}^*\boldsymbol{D_Y}\boldsymbol{V_L}^T \ . \tag{3.13}$$

If the combination $\boldsymbol{V_R}^\dagger\boldsymbol{M}^{-1}\boldsymbol{V_R}^*$ was a flat matrix without any remarkable hierarchies, which is the normal case for a "random" $\boldsymbol{V_R}$ independently of the heavy masses $\boldsymbol{M}$, the eigenvalues of $\boldsymbol{\mathcal{M}_\nu}$ would behave at least as $\boldsymbol{D_Y}^2$. If, instead, $\boldsymbol{V_R}$ was very close to the unit matrix, the eigenvalues of $\boldsymbol{\mathcal{M}_\nu}$ would be proportional to the products $y_i^2 M_i^{-1}$. Again, unless hierarchies in the neutrino Yukawa coupling exactly cancel those from the Majorana masses, one expects large hierarchies between light neutrino masses. However, this expectation is known to fail at least for the two heavier neutrinos, eq. (3.9). Moreover, in both of the aforementioned cases typically large mixing angles in $\boldsymbol{V_L}$ are necessary in order to induce the large mixing angles in $\boldsymbol{U_{MNS}}$. Of course this contradicts the preferred situation of CKM-like small mixing angles in $\boldsymbol{V_L}$. Therefore if we don't want to give up the assumption of hierarchical Yukawa couplings $\boldsymbol{D_Y}$, we are restricted to special cases. To make this statement more precise, next we derive a necessary condition on $\boldsymbol{V_R}$ using arguments similar to the ones of [114].

Instead of working with $\boldsymbol{\mathcal{M}_\nu}$ it turns out to be useful to derive inequalities for the squared neutrino mass eigenvalues m_i^2 from inspection of $\boldsymbol{V_L}^\dagger\boldsymbol{\mathcal{M}_\nu}\boldsymbol{\mathcal{M}_\nu}^\dagger\boldsymbol{V_L}$. Those eigenvalues are automatically real and positive. First note that the largest eigenvalue is at least as large as the largest element on the diagonal. This can be used together with eq. (3.13):

$$\max\{m_i^2\} \geq \left(\boldsymbol{V_L}^\dagger \boldsymbol{\mathcal{M}_\nu} \boldsymbol{\mathcal{M}_\nu}^\dagger \boldsymbol{V_L}\right)_{33} \geq \frac{v^4}{4} y_3^4 \left|\boldsymbol{V_R}^T \boldsymbol{M}^{-1} \boldsymbol{V_R}\right|_{33}^2 . \tag{3.14}$$

On the other hand, each eigenvalue has to be smaller than the trace. Applying this to the inverse yields

$$\begin{aligned}\max\left\{\frac{1}{m_i^2}\right\} &< \mathrm{Tr}\left(\boldsymbol{V_L}^\dagger (\boldsymbol{\mathcal{M}_\nu}^\dagger)^{-1} \boldsymbol{\mathcal{M}_\nu}^{-1} \boldsymbol{V_L}\right) \\ &\leq \frac{4}{v^4}\frac{1}{y_1^4}\,\mathrm{Tr}\left(\boldsymbol{V_R}^\dagger \boldsymbol{M} \boldsymbol{V_R}^* \boldsymbol{V_R}^T \boldsymbol{M} \boldsymbol{V_R}\right) = \frac{4}{v^4}\frac{1}{y_1^4}(M_1^2 + M_2^2 + M_3^2) .\end{aligned}$$

Finally note that the ratio of the two heavier neutrino masses can be computed with the help of the determinant $\det(\boldsymbol{\mathcal{M}_\nu}\boldsymbol{\mathcal{M}_\nu}^\dagger) = m_1^2 m_2^2 m_3^2$:

$$\begin{aligned}6 > \frac{\max\{m_i^2\}\sqrt{\min\{m_i^2\}}}{m_1 m_2 m_3} &> \frac{y_1^2 y_3^4 \left|\boldsymbol{V_R}^T \boldsymbol{M}^{-1} \boldsymbol{V_R}\right|_{33}^2}{\sqrt{M_1^2 + M_2^2 + M_3^2}} \frac{M_1 M_2 M_3}{y_1^2 y_2^2 y_3^2} \\ &= \frac{y_3^2}{y_2^2} \frac{M_1 M_2 M_3 \left|\boldsymbol{V_R}^T \boldsymbol{M}^{-1} \boldsymbol{V_R}\right|_{33}^2}{\sqrt{M_1^2 + M_2^2 + M_3^2}} .\end{aligned} \tag{3.15}$$

It follows that in order to obtain the observed mild hierarchy, it must necessarily hold $\left|\boldsymbol{V_R}^T \boldsymbol{M}^{-1} \boldsymbol{V_R}\right|_{33}^2 \ll 1/(M_1 M_2)$. This can be accommodated more easily with hierarchical heavy Majorana masses $\boldsymbol{M}$, but in any case requires particular patterns for $\boldsymbol{V_R}$. A simple example is given by $\boldsymbol{V_R} \simeq \mathbb{1}$, $m_i \simeq y_i^2/M_i$. Another example without cancellations in $\boldsymbol{V_R}^T \boldsymbol{M}^{-1} \boldsymbol{V_R}$ but with a less strong hierarchy in the right-handed neutrino masses, $M_i \propto y_i$, was found in [114]. It requires sizeable $\boldsymbol{V_R}_{21}$ but tiny $\boldsymbol{V_R}_{13}$, $\boldsymbol{V_R}_{23}$.

3.2.3. Leptogenesis

Leptogenesis [115] is a mechanism able to produce the Baryon Asymmetry of the Universe (BAU). A very appealing feature is that it comes for free once the see-saw mechanism is introduced. Moreover, in order to produce a sufficient baryon asymmetry, the see-saw parameters have to fulfil some constraints. For example in the simplest version the lightest right-handed neutrino mass is bounded from below.

The see-saw includes all necessary requirements for successful generation of a lepton (L) asymmetry [116]: The Majorana masses break lepton number, the universe goes out of equilibrium once the right-handed neutrinos freeze out and CP is violated in the decay of right-handed neutrinos. As the lepton asymmetry is produced well above the electroweak scale, sphaleron processes will convert it partially into a baryon (B) asymmetry [117]. Here we will describe only the generic case of the leptogenesis mechanism, leaving aside the possibilities of nearly degenerate heavy Majorana neutrinos, neglecting flavour and sticking only to the classical Maxwell-Boltzmann description. For a recent and more comprehensive review see [118]. In this case the lepton asymmetry produced in the decays of the two heavier right-handed neutrinos is almost completely washed out by processes involving the lightest right-handed neutrino. Then the generated BAU depends only on four neutrino parameters [119] and can be conveniently written [120] as

$$\eta_B = \frac{3}{4}\frac{a_{\rm sph}}{f}\epsilon_1\kappa_f(M_1, m_1, \widetilde{m}_1) \ . \tag{3.16}$$

Here we have introduced several constants and functions.

The baryon-to-photon ratio $\eta_B \simeq 6 \cdot 10^{-10}$ [1] has been determined from CMB. To account for the dilution between leptogenesis and recombination the factor f is introduced which yields in the SM $f \simeq 28$. The quantity $a_{\rm sph}$ is the ratio of baryon asymmetry to lepton asymmetry once the B+L violating sphaleron processes are in equilibrium. Again for the case of the SM it is given by 28/79 [121].

Next we turn to the four see-saw parameters. First, the CP asymmetry in the decay of the lightest right-handed neutrino

$$\epsilon_1 = \frac{\Gamma_{\nu_{R1}\to lh} - \Gamma_{\nu_{R1}\to \bar{l}h^*}}{\Gamma_{\nu_{R1}\to lh} + \Gamma_{\nu_{R1}\to \bar{l}h^*}} \tag{3.17}$$

was found at the one-loop level to obey the inequality [122]

$$|\epsilon_1| \leq \frac{3}{8\pi}\frac{M_1}{v^2}(\max\{m_i\} - \min\{m_i\}) \ , \tag{3.18}$$

which allows for constraints on the neutrino masses, as will be sketched at the end of this section.

The factor κ_f is normalized to $0 \leq \kappa_f \leq 1$ and represents the efficiency of the production of the lepton asymmetry. It depends strongly on the coupling of the lightest right-handed neutrino to the thermal bath, which is encoded in the effective neutrino mass [123]

$$\widetilde{m}_1 = \frac{v^2}{2}\frac{(\mathbf{Y}_{\boldsymbol{\nu}}^\dagger \mathbf{Y}_{\boldsymbol{\nu}})_{11}}{M_1} \ . \tag{3.19}$$

If $\widetilde{m}_1 \gg 10^{-3}\,\text{eV}$ the decays proceed very fast. Therefore the departure from thermal equilibrium is small and the lepton asymmetry production is not efficient. On the other hand, if $\widetilde{m}_1 \ll 10^{-3}\,\text{eV}$ the neutrino Yukawa couplings are too small to bring the lightest right-handed neutrino into thermal equilibrium before its freeze-out. In this case, another mechanism might be invoked to generate a sufficient abundance of right-handed neutrinos. For rather large values of m_1, the $\Delta L = 2$ washout processes with heavy right-handed neutrinos in the s- or t-channel become relevant. This allows in principle to derive an upper bound on the lightest neutrino mass m_1 [119]. But taking properly into account flavour effects, this bound disappears [124].

The absolute lower bound on the lightest right-handed neutrino mass, however, is independent of the flavour structure of the neutrino Yukawa couplings [125, 126]. It is obtained from inserting eq. (3.18), which is maximized for $\min\{m_i\} \to 0$, into eq. (3.16) assuming optimal efficiency [122]:

$$M_1 \gtrsim 4 \cdot 10^8\,\text{GeV} \ . \tag{3.20}$$

This bound is very conservative. For a vanishing initial abundance of right-handed neutrinos the efficiency factor cannot be maximal and the bound is stronger by one order of magnitude [126]. Comparing eq. (3.20) with the estimate $M_1 \sim 10^{15}\,\mathrm{GeV}$ from the assumption of a large neutrino Yukawa eigenvalue, leptogenesis allows for a mass hierarchy $M_3/M_1 \lesssim 10^6$. This is a slightly stronger hierarchy than y_t/y_u but somewhat weaker than y_τ^2/y_e^2. In any case it does not conflict with the more natural solutions to the observed mild neutrino mass hierarchy, section 3.2.2.

A tension, however, arises in the supersymmetric version of leptogenesis. Qualitatively, leptogenesis works completely analogously within SUSY. Due to the approximately doubled particle content in the MSSM compared to the SM, differences arise in the decay parameters as well as in the evolution of the universe. These modifications however largely cancel each other and there are no major numerical differences compared to the SM case [118]. In order to populate the universe with right-handed neutrinos after inflation, the reheating temperature has to be somewhat larger than M_1. At such high temperatures however, in local SUSY gravitinos are copiously produced and may lead to problems. If SUSY breaking is mediated at very high energies – as for example in gravity mediation – the gravitino is unstable but long-lived. The decay process can easily upset the successful predictions of the BBN, leading to a model dependent bound on the reheating temperature that can be as small as $10^6\,\mathrm{GeV}$ [127]. This bound can be circumvented if the gravitinos have already decayed before BBN, for which a very large gravitino mass $\gtrsim 10\,\mathrm{TeV}$ is required.

In addition, the gravitino abundance is constrained by the observed dark matter density [128, 129]. If the gravitino is unstable but R-parity is conserved, this bound depends on the mass of the lightest supersymmetric particle χ_1^0, which is ultimately produced in the gravitino decay chain. An upper bound is given by [127]

$$T_{\text{reheat}} \lesssim 2 \cdot 10^{10}\,\mathrm{GeV}\,\frac{100\,\mathrm{GeV}}{m_{\chi_1^0}}\,. \tag{3.21}$$

If gravitinos are light compared to gluinos this bound becomes even stronger. This leads to a lower bound of several GeV for the gravitino mass [130, 131] – under the assumption of stable gravitinos, which is more reasonable in this mass range. Note also that the bound eq. (3.21) becomes stronger if the mass scale of the SUSY spectrum is raised in order to satisfy BBN bounds.

A possible way to circumvent any bound on the reheating temperature is by considering very light gravitinos as they can be obtained from low scale SUSY breaking, e.g. in gauge mediation. In this case the gravitino coupling can be large enough to bring the gravitinos into thermal equilibrium [132]. If the gravitino was light, however, it would constitute a warm dark matter component, which is constrained by cosmology. Therefore in this scenario an upper bound on the gravitino mass, $m_{3/2} < 16\,\mathrm{eV}$ can be derived [133] and additional particles must be invoked for dark matter.

3.3. Constraints on the see-saw in the MSSM

3.3.1. Approaching the see-saw via slepton masses

In SUSY there is the chance to obtain more information about the see-saw. This is possible as there exist additional lepton flavour matrices in the soft SUSY Lagrangian:

$$-\mathcal{L}_{\text{soft}} \supset \widetilde{L}_i^* \boldsymbol{m}^{\mathbf{2}}_{\boldsymbol{L}ij} \widetilde{L}_j \ + \widetilde{e}^*_{Ri} \boldsymbol{m}^{\mathbf{2}}_{\widetilde{\boldsymbol{e}}ij} \widetilde{e}_{Rj} + \left(\widetilde{e}^*_{Ri} \boldsymbol{A}_{\boldsymbol{e}ij} H_d \widetilde{L}_j + \text{h.c.} \right) \ . \tag{3.22}$$

These leptonic soft masses obtain radiative contributions from the see-saw. However, in order to allow for a connection between low energy determinations of the soft masses and the see-saw parameters, two obvious preconditions must be met.

Preconditions

First, the radiative contributions have to be sizeable. This excludes in particular the possibility of mediation of SUSY breaking below the mass scale of right-handed neutrinos. In such a scenario, the Kähler potential would still obtain radiative corrections due to the presence of right-handed neutrinos which in principle manifest in corrections to the soft masses. But they would be suppressed by the ratio of the right-handed neutrino mass to the mass scale of SUSY breaking mediation. Therefore, sizeable corrections to the soft masses require SUSY breaking to be mediated above the mass of the lightest right-handed neutrino. This, however, is not a strong restriction on the model. In most SUSY scenarios and in particular in the case of gravity mediation the scale of SUSY breaking mediation is very high. A high mediation scale is also a consequence of the lower bound on the gravitino mass $m_{3/2} \gtrsim 5\,\text{GeV}$ obtained in leptogenesis, cf. the discussion below eq. (3.21): This bound translates into a lower bound on the scale of spontaneous SUSY breaking $|F| = \sqrt{3} M_P\, m_{3/2} \gtrsim (5 \cdot 10^9\,\text{GeV})^2$. On the other hand, SUSY breaking is transmitted to the observable sector by messenger particles with mass M_{mes}, inducing soft masses which approximately read:

$$m^2_{\text{soft}} \sim c \frac{|F|^2}{M^2_{\text{mes}}} \ ,$$

where $10^{-4} \lesssim c \lesssim 1$ is a constant which depends on the mediation mechanism. For $m_{\text{soft}} \sim \text{TeV}$, this implies a messenger scale larger than $10^{14}\,\text{GeV}$ or $10^{16}\,\text{GeV}$ respectively. In view of the expected large hierarchy in right-handed neutrino masses, cf. section 3.2.2, this suggests at least one right-handed neutrino to be lighter than the mediation scale.

Secondly, in order to disentangle the radiatively corrections induced by the see-saw, of course the uncorrected values of the soft masses have to be known. This requires the SUSY breaking mechanism to be simple enough to be pinned down with the experimental examination of the quark sector. As discussed already in section 2.2.1, the absence of new physics contributions to FCNCs at the level of present experiments, points indeed to such, flavour blind breaking mechanisms. At the mediation scale Λ the soft masses read in this case:

$$\boldsymbol{m}^{\mathbf{2}}_{\boldsymbol{L}} = m_L^2 \mathbb{1} \ , \qquad \boldsymbol{m}^{\mathbf{2}}_{\widetilde{\boldsymbol{e}}} = m_e^2 \mathbb{1} \ , \qquad \boldsymbol{m}^{\mathbf{2}}_{\boldsymbol{\nu}} = m_\nu^2 \mathbb{1} \ , \qquad \boldsymbol{A}_{\boldsymbol{e}} = A_e \boldsymbol{Y}_{\boldsymbol{e}} \ , \qquad \boldsymbol{A}_{\boldsymbol{\nu}} = A_\nu \boldsymbol{Y}_{\boldsymbol{\nu}} \ . \tag{3.23}$$

where $\boldsymbol{m^2_\nu}$ ($\boldsymbol{A_\nu}$) is the soft squared mass (trilinear coupling) of right-handed sneutrinos. The RG evolution yields at energies below the scale of right-handed neutrinos:

$$\left(\boldsymbol{m^2_L}\right)_{ij} \simeq -\frac{1}{8\pi^2}(m_L^2 + m_\nu^2 + m_{H_u}^2 + |A_\nu|^2)\mathbf{P}_{ij} \,, \tag{3.24a}$$
$$\left(\boldsymbol{m^2_{\bar{e}}}\right)_{ij} \simeq 0 \,, \tag{3.24b}$$
$$(\boldsymbol{A_e})_{ij} \simeq -\frac{1}{8\pi^2}A_\nu \boldsymbol{Y_e}_{ii}\mathbf{P}_{ij} \,, \tag{3.24c}$$

where $i \neq j$,

$$\mathbf{P}_{ij} = \sum_k \boldsymbol{Y^*_\nu}_{ki} \log\left(\frac{\Lambda}{M_k}\right)\boldsymbol{Y_\nu}_{kj} \,, \tag{3.25}$$

and the sum runs only over $k : M_k < \Lambda$ because of the first precondition.

Testing the see-saw with slepton masses?

In the situation described above and under the idealistic assumption that all soft masses could be determined separately by experiment, eq. (3.24) presents a simple test. If, however, the predicted correlation between $\boldsymbol{m^2_L}$, $\boldsymbol{m^2_\nu}$, $\boldsymbol{A_e}$ will be disproved, the see-saw is still viable once additional new physics in the lepton sector is added. On the other hand, a confirmation of eq. (3.24) would give some confidence. Even a full determination of $\mathbf{P}$ together with the neutrino mass matrix would not be sufficient to over-constrain the see-saw [134, 135]. However it would allow for a determination of all see-saw parameters. Thus it is in principle possible to check any prejudice about parameters of the see-saw.

Determination of slepton masses from experiment

Presently the most promising way to detect flavour violation is via precision experiments. In supersymmetric models the radiative decays $l_i \to l_j\gamma$ give the strongest constraints on the off-diagonal soft masses. The experimental bounds read [1, 136]:

$$\mathrm{BR}(\tau \to \mu\gamma) < 4.4 \cdot 10^{-8} \,, \tag{3.26a}$$
$$\mathrm{BR}(\tau \to e\gamma) < 3.3 \cdot 10^{-8} \,, \tag{3.26b}$$
$$\mathrm{BR}(\mu \to e\gamma) < 2.4 \cdot 10^{-12} \,. \tag{3.26c}$$

An approximate relation of these bounds to the parameters of the MSSM Lagrangian reads:

$$\mathrm{BR}(l_i \to l_j\gamma) \simeq \frac{\alpha^3}{G_F^2}\frac{|\boldsymbol{m^2_L}_{ij}|^2}{m_S^8}\tan^2\beta\,\mathrm{BR}(l_i \to l_j\nu_i\bar{\nu}_j) \,, \tag{3.27}$$

where m_S is a mass scale of the order of typical SUSY masses [137] and $\mathrm{BR}(\mu \to e\nu_\mu\bar{\nu}_e) \simeq 1$, $\mathrm{BR}(\tau \to \mu\nu_\tau\bar{\nu}_\mu) \simeq 0.17$, $\mathrm{BR}(\tau \to e\nu_\tau\bar{\nu}_e) \simeq 0.18$ [1]. For a discussion of the accuracy of the above formula see [137, 138].

A major improvement in the constraints on rare tau decays down to several 10^{-9} can be expected from Super-B factories [139, 140]. For the case of muon flavour violation several improvements can be expected. First, the experiment MEG, which has derived the limit above, is still running and will be sensitive to $\mathrm{BR}(\mu \to e\gamma)$ at the level of a few 10^{-13}. If a signal will be found, one may use polarized muons in order to determine the chirality dependence of this process [141] and thus to discriminate between $\boldsymbol{m}_{\boldsymbol{L}}^{\boldsymbol{2}}$ and $\boldsymbol{m}_{\boldsymbol{\tilde{e}}}^{\boldsymbol{2}}$. While this search is limited by the sensitivity of the detectors and new technologies have to be developed in order to reduce the background to a significantly lower level, searches for muon conversion are currently limited by the available muon beam intensities. The corresponding experiments mu2e [142] and COMET [143] are not yet fully approved. Both aim at a sensitivity of 10^{-16}. The most ambitious plan is the PRISM/PRIME project which tries to constrain muon conversion down to a rate of 10^{-18} [144]. In SUSY this process is typically dominated by penguin diagrams in which case its rate is related to $\mathrm{BR}(\mu \to e\gamma)$ [145, 146]:

$$\mathrm{R}(\mu\mathrm{Ti} \to e\mathrm{Ti}) \simeq \alpha\, \mathrm{BR}(\mu \to e\gamma)\ . \tag{3.28}$$

In a target medium different from titanium the conversion rate can differ by a factor of two [147].

Finally, note that phases in the slepton mass matrix can be probed by lepton electric dipole moments [148]. The flavour conserving corrections to the soft mass might be determined directly by measuring the slepton mass splittings [149]. Even on flavour violating couplings, LHC bounds might become stronger than those from rare decays [150, 151].

Some see-saw conjectures

Lots of ideas have been published trying to make predictions on low energy parameters under different assumptions on the see-saw parameters. A review is clearly out of the scope of this thesis and we will confine ourselves to listing some ideas in connection with rare leptonic decays. In addition to the assumptions presented below of course also the preconditions outlined in the beginning of this section must be met.

Constraining the see-saw only with neutrino data, it is impossible to make any definite statement on rare leptonic decays, see the above discussion about testing the see-saw with slepton masses. Due to the large number of free parameters, scatter plots can easily miss regions that are fine-tuned (only) in the chosen parametrization, or they can produce wrong correlations [152]. The bottom-up approach, however, is well suited to identify interesting points in the parameter space [113, 153].

Under the assumption of leptogenesis, in principle predictions for weak scale quantities are possible. However, several parameters such as m_1 or the individual phases might be difficult or even impossible to determine. Thus in general there is no measurable prediction apart from $\tau \to e\gamma$ which cannot be too large [154].

Other works aim to predict rates or correlations of cLFV processes for general classes of see-saw parameters. For example, [155] assumes sequential dominance of right-handed neutrinos, [156] discusses also some case where no right-handed neutrino dominates. A very economic approach is to work with only two right-handed neutrinos [157, 158], assuming the third to be irrelevant for neutrino masses and cLFV. Correlations between cLFV processes of different families have also been derived, e.g. under the assumption of the absence of cancellations [159]. Large differences in the total rates of the different rare lepton decays were found for leptogenesis being realized with hierarchical neutrino Yukawa couplings and right-handed masses as well as small mixings [160]. Again, analyses using the bottom-up parametrization [113] have to be taken with some care as the latter hides the physical meaning of the parameter points [161–163]. For an overview see [164].

In the spirit of unification one large neutrino Yukawa coupling [111] and strong Yukawa hierarchies are often assumed. Additionally, one might impose CKM like mixings in $\boldsymbol{V_L}$ or large ones as in the leptonic mixing matrix [145, 146, 165, 166]. One finds that in the case of CKM like $\boldsymbol{V_L}$ the cLFV bounds can still be met easily [111, 167].

Predictive schemes can be obtained by flavour symmetries or by assuming specific textures of the neutrino Yukawa matrix [168–175]. However, predictions for cLFV vary of course from model to model.

One can conclude that there are many ideas but no widely accepted prediction for the rate of cLFV. With the following constraints we try to improve this situation by applying conjectures we find particularly well motivated.

3.3.2. Probing leptogenesis with μ conversion

Constraining the lightest neutrino Yukawa eigenvalue with cLFV

In this section we will derive another connection between see-saw parameters and rare leptonic decays, following [7]. To be more specific, we will derive an upper bound on the smallest neutrino Yukawa eigenvalue. We believe that this bound is interesting on its own as a constraint on a fundamental physical constant. Moreover, y_1 is a crucial parameter in leptogenesis [154]. In addition, the same reasonings employed in this section will be also applied to constrain the lightest right-handed neutrino mass M_1 from above, thus allowing to probe leptogenesis, cf. eq. (3.20).

To this end, we will make use of the following plausible assumptions:

- hierarchical neutrino Yukawa eigenvalues $y_1 \ll y_2 \ll y_3$,
- the absence of cancellations and
- non-vanishing $|\boldsymbol{\mathcal{M}_\nu}_{12}| \sim m_2$.

As emphasized already above, the requirement of the absence of cancellations is very delicate in the absence of a flavour model. However, we will argue that the cancellations required to spoil our result cannot be provided by a flavour model. Additionally, we will indicate when we make use of this assumption.

The size of $\boldsymbol{\mathcal{M}_\nu}_{12}$ is presently unknown. For hierarchical light neutrino masses $m_1 \ll m_2$ the relation $\boldsymbol{\mathcal{M}_\nu}_{12} \simeq \frac{1}{3}\sqrt{\Delta m_{21}^2}e^{-i\alpha_{21}} - \frac{\sin\theta_{13}}{\sqrt{2}}e^{-i\delta-i\alpha_{31}}\sqrt{\Delta m_{32}^2}$ holds. Therefore $\boldsymbol{\mathcal{M}_\nu}_{12}$ can vanish if $\sin^2 2\theta_{13} \simeq 0.03$ and $e^{i\alpha_{21}-i\delta-i\alpha_{31}} \simeq 1$. This possibility might be ruled out with improved bounds on θ_{13}, cf. section 3.1

First we will construct a model yielding the minimal amount of cLFV. Later on we will show that indeed this choice yields the minimal rate for $\mu \to e\gamma$ barring accidental cancellations.

First note that eq. (3.27) is valid only approximately. The correct formula is obtained from summing the contributions of several particles in the loops. Though some amount of cancellations is certainly possible, it is difficult to conceive a complete neutralisation because of the involved dependence on the high energy parameters.

According to eqs. (3.24), (3.27) the off-diagonal entries of the matrix $\mathbf{P}$ are the relevant quantities for rare leptonic decays. In the case of hierarchical neutrino Yukawa couplings the rare leptonic decays can be suppressed if leptonic mixing is only in the right-handed sector, thus $\boldsymbol{V_L} = \mathbb{1}$. In this case it is easy to calculate $\mathbf{P}_{12}$:

The see-saw formula eq. (3.13) can always be rewritten (mind the different notations in this thesis for SM and SUSY):

$$\boldsymbol{V_R}^T \boldsymbol{M}^{-1} \boldsymbol{V_R} = \frac{2}{\langle H_u^0 \rangle^2} \boldsymbol{D_Y}^{-1} \boldsymbol{V_L}^T \boldsymbol{\mathcal{M}_\nu} \boldsymbol{V_L} \boldsymbol{D_Y}^{-1} . \tag{3.29}$$

If $(\boldsymbol{V_L}^T \boldsymbol{\mathcal{M}_\nu} \boldsymbol{V_L})_{11}$ is sizeable, the entry proportional to y_1^{-2} is the largest on the right-hand side and can therefore be identified with $1/M_1$.

$$M_1 \simeq \frac{\langle H_u^0 \rangle^2}{2} \frac{y_1^2}{|\boldsymbol{V_L}^T \boldsymbol{\mathcal{M}_\nu} \boldsymbol{V_L}|_{11}} , \qquad \boldsymbol{V_R}_{11} \simeq e^{\frac{i}{2} \arg(\boldsymbol{V_L}^T \boldsymbol{\mathcal{M}_\nu} \boldsymbol{V_L})_{11}} . \tag{3.30}$$

The next to largest entry is then proportional to $y_1^{-1} y_2^{-1}$ and determines the mixing [154, 176–178]:

$$\boldsymbol{V_R}_{12} \simeq \frac{y_1}{y_2} \frac{(\boldsymbol{V_L}^T \boldsymbol{\mathcal{M}_\nu} \boldsymbol{V_L})_{12}}{(\boldsymbol{V_L}^T \boldsymbol{\mathcal{M}_\nu} \boldsymbol{V_L})_{11}} e^{\frac{i}{2} \arg(\boldsymbol{V_L}^T \boldsymbol{\mathcal{M}_\nu} \boldsymbol{V_L})_{11}} . \tag{3.31}$$

Now we have derived the necessary ingredients to write down $\mathbf{P}_{12}$, which reads for $\boldsymbol{V_L} = \mathbb{1}$:

$$\mathbf{P}_{12} = \left(\boldsymbol{Y_\nu}^\dagger \log \frac{\Lambda}{\boldsymbol{M}} \boldsymbol{Y_\nu} \right)_{12} \simeq y_1^2 \frac{\boldsymbol{\mathcal{M}_\nu}_{12}}{\boldsymbol{\mathcal{M}_\nu}_{11}} \log \frac{M_2}{M_1} . \tag{3.32}$$

A more careful derivation of this result can be found in [7]. Generically this yields

$$\mathbf{P}_{12}^{\min} \simeq y_1^2 \log \frac{M_2}{M_1} . \tag{3.33}$$

This is indeed the smallest value $\mathbf{P}_{12}$ can attain barring accidental cancellations. To show this, note that strong cancellations in $\mathbf{P}_{12}$ amongst the terms proportional to a different $\log M_i$ are difficult to achieve as the logarithm spoils possible connections between $\boldsymbol{M}$ and $\boldsymbol{V_R}$.

$$\mathbf{P}_{12} \simeq \max_{i=1,2,3} |(\boldsymbol{Y_\nu}^\dagger)_{1i} \boldsymbol{Y_\nu}_{2i}| \log H , \tag{3.34}$$

where H represents some typical hierarchy of right-handed neutrino masses or the ratio of right-handed neutrino masses to the cut-off scale. We will assume $\log H$ to be of the order of a few. Thus, in order to subvert the bound eq. (3.33) it has to hold for any i:

$$|(\boldsymbol{Y_\nu}^\dagger)_{1i} \boldsymbol{Y_\nu}_{2i}| \ll y_1^2 .$$

On the other hand we demand

$$m_2 \sim |\boldsymbol{\mathcal{M}_\nu}_{12}| = \frac{\langle H_u^0 \rangle^2}{2} \left| \boldsymbol{Y_\nu}^T \frac{1}{\boldsymbol{M}} \boldsymbol{Y_\nu} \right|_{12} \lesssim \frac{\langle H_u^0 \rangle^2}{2} \frac{1}{M_1} \max_{i=1,2,3} |(\boldsymbol{Y_\nu}^\dagger)_{1i} \boldsymbol{Y_\nu}_{2i}| . \tag{3.35}$$

This is in tension with the last equation: If already y_1 was too large to yield the correct scale for the neutrino mass, then y_2, y_3 won't do better. More mathematically this can be seen from

$$m_2 \frac{2}{\langle H_u^0 \rangle^2} \gtrsim \sqrt{(\boldsymbol{V_L^T \mathcal{M}_\nu \mathcal{M}_\nu^\dagger V_L^*})_{11}} = y_1 \sqrt{\sum_i |\boldsymbol{V_R^T M^{-1} V_R D_Y}|^2_{1i}} > \frac{y_1^2}{M_1} \,. \tag{3.36}$$

This argument can be circumvented with (partial) degeneration of right-handed neutrino masses thus allowing for $\log H = 0$. However, this possibility is difficult to obtain with hierarchical Yukawa eigenvalues [114]. Additionally, large left-handed mixing angles cannot be prevented in this case and would have to cancel each other to yield small $\mathbf{P}_{12}$. We have not checked whether such a scenario is compatible with tiny $\mathbf{P}_{12}$.

Finally we discuss the meaning of the lower bound, eq. (3.33), on rare decays. Given any expectation on the smallest neutrino Yukawa eigenvalue y_1 this will lead to a minimal rate of $\mu \to e\gamma$ which can be estimated using eq. (3.27). The converse argument is to obtain an upper bound on y_1 from the upper bound on rare muon decay, given of course supersymmetric soft masses and $\tan\beta$. We find

$$y_1 \lesssim 2 \cdot 10^{-2} \left(\frac{\mathrm{BR}(\mu \to e\gamma)}{2.4 \cdot 10^{-12}} \right)^{1/4} \left(\frac{m_S}{200\,\mathrm{GeV}} \right) \left(\frac{\tan\beta}{10} \right)^{-1/2} \,. \tag{3.37}$$

where we have assumed $\log(M_2/M_1) \simeq 5$ and $m_0 \sim A_0 \sim m_S$. We have also checked that approximately the same number is obtained by analysing the MSSM benchmark points SPS1a and SPS 1b, cf. table 2.1. Here we have presented the bound in a form suited to compare with the currently experimentally best constrained process. It can be transformed using bounds on muon conversion with the help of eq. (3.28):

$$y_1 \lesssim 2 \cdot 10^{-2} \left(\frac{\mathrm{R}(\mu\mathrm{Ti} \to e\mathrm{Ti})}{2 \cdot 10^{-14}} \right)^{1/4} \left(\frac{m_S}{200\,\mathrm{GeV}} \right) \left(\frac{\tan\beta}{10} \right)^{-1/2} \,. \tag{3.38}$$

We admit that presently this bound is just strong enough to allow for the ingoing assumption $y_1 \ll y_2 \ll y_3$. Depending on SUSY parameters, however, non-observation of cLFV in future experiments can tighten this bound. Still it is much larger than the up-quark Yukawa coupling. However this bound does not rely on grand unification.

Constraining the lightest right-handed neutrino mass

The results of the last section can easily be translated into an upper bound on the lightest right-handed neutrino mass. In a scenario with hierarchical neutrino Yukawa eigenvalues it holds, cf. eq. (3.30):

$$M_1 \lesssim \frac{\langle H_u^0 \rangle^2}{2} \frac{y_1^2}{|\boldsymbol{V_L^T \mathcal{M}_\nu V_L}|_{11}} \,. \tag{3.39}$$

Therefore an upper bound on y_1, eq. (3.33) implies an upper bound on M_1:

$$M_1 \lesssim \frac{\langle H_u^0 \rangle^2}{2|\boldsymbol{V_L^T \mathcal{M}_\nu V_L}|_{11}} \frac{\mathbf{P}_{12}}{\log(M_2/M_1)} \,. \tag{3.40}$$

Unless $|\boldsymbol{V_L}^T \boldsymbol{\mathcal{M}_\nu} \boldsymbol{V_L}|_{11} \ll m_2$ this represents an interesting constraint. In [7] we have argued that the case $(\boldsymbol{V_L}^T \boldsymbol{\mathcal{M}_\nu} \boldsymbol{V_L}|)_{11} \sim 0$ generically leads to large $\boldsymbol{V_L}_{11}$, $\boldsymbol{V_L}_{12}$ thus greatly enhancing the minimal rate for $\mu \to e\gamma$.

Here we will follow the arguments given in eqs. (3.34), (3.35). The two equations can be combined:

$$M_1 \lesssim \frac{\langle H_u^0 \rangle^2}{2|\boldsymbol{\mathcal{M}_\nu}_{12}|} \frac{\mathbf{P}_{12}}{\log H} . \tag{3.41}$$

This has the advantage that the dependence on $\boldsymbol{V_L}$ disappeared. Note also that we had to assume sizeable $\boldsymbol{\mathcal{M}_\nu}_{12}$ from the beginning in order to guarantee some amount of $\mu \leftrightarrow e$ transitions.

Numerically this bound can be written, taking $\log H = 5$ and using eq. (3.27):

$$M_1 \lesssim 5 \cdot 10^{12}\,\mathrm{GeV} \left(\frac{\mathrm{BR}(\mu \to e\gamma)}{2.4 \cdot 10^{-12}} \right)^{1/2} \left(\frac{\sqrt{\Delta m_{21}^2}}{3|\boldsymbol{\mathcal{M}_\nu}_{12}|} \right) \left(\frac{m_S}{200\,\mathrm{GeV}} \right)^2 \left(\frac{10}{\tan\beta} \right) . \tag{3.42}$$

Of course one may rewrite this bound using muon conversion instead, cf. eqs. (3.28).

Implications for leptogenesis

This upper bound should be compared with the lower bound on the right-handed neutrino mass $M_1 \gtrsim 4 \cdot 10^8$ GeV, eq. (3.20) leaving an allowed window of four orders of magnitude for the lightest right-handed neutrino mass. Thus, in order to rule out leptogenesis it is required an improvement in sensitivity of eight orders of magnitude for the rare muon decay or a corresponding improvement in muon conversion. This is probably not feasible in this nor the next decade, cf. section 3.3.1. However, if there is no signal for $\mu \leftrightarrow e$ transitions in future experiments, this can drastically reduce the allowed see-saw parameter space for thermal leptogenesis.

Beyond that, note that in deriving this bound on the rate of muon conversion we have made very conservative assumptions. In particular, there is no reason to assume the absence of left-handed mixing. Therefore, in general it can be expected $\mathbf{P}_{12}$ to be much larger than y_1^2. Also, there might be other sources for cLFV, e.g. if SUSY breaking is not flavour blind. Thus, being our bound very conservative, it is very favourable for observing a signal in future searches for cLFV.

We show in figure 3.1 the impact of the bound, eq. (3.42), on the parameter space of thermal leptogenesis, presuming non-observation of muon conversion with a projected sensitivity of the planned PRISM/PRIME project. In the plot we have assumed $m_S \simeq 200\,\mathrm{GeV}$, $\tan\beta \simeq 10$ and $|\boldsymbol{\mathcal{M}_\nu}_{12}| \simeq \sqrt{\Delta m_{21}^2}/3$.

The yellow/orange region corresponds to the allowed region found by Blanchet and di Bari, which is shown in fig. 1 of [125]. As

$$\widetilde{m}_1 \geq \frac{\langle H_u^0 \rangle^2}{2} \frac{1}{M_1} |\boldsymbol{Y_\nu}^T \boldsymbol{Y_\nu}|_{11} \geq \boldsymbol{\mathcal{M}_\nu}_{11} , \tag{3.43}$$

we expect $\widetilde{m}_1 > \sqrt{\Delta m_{21}^2}/3$ corresponding to the region marked orange in the plot. The solid lines bound the allowed region for certain leptogenesis scenarios: thermal (thin) vs. zero initial abundance of right-handed neutrinos (thick) and maximal (right) vs. no

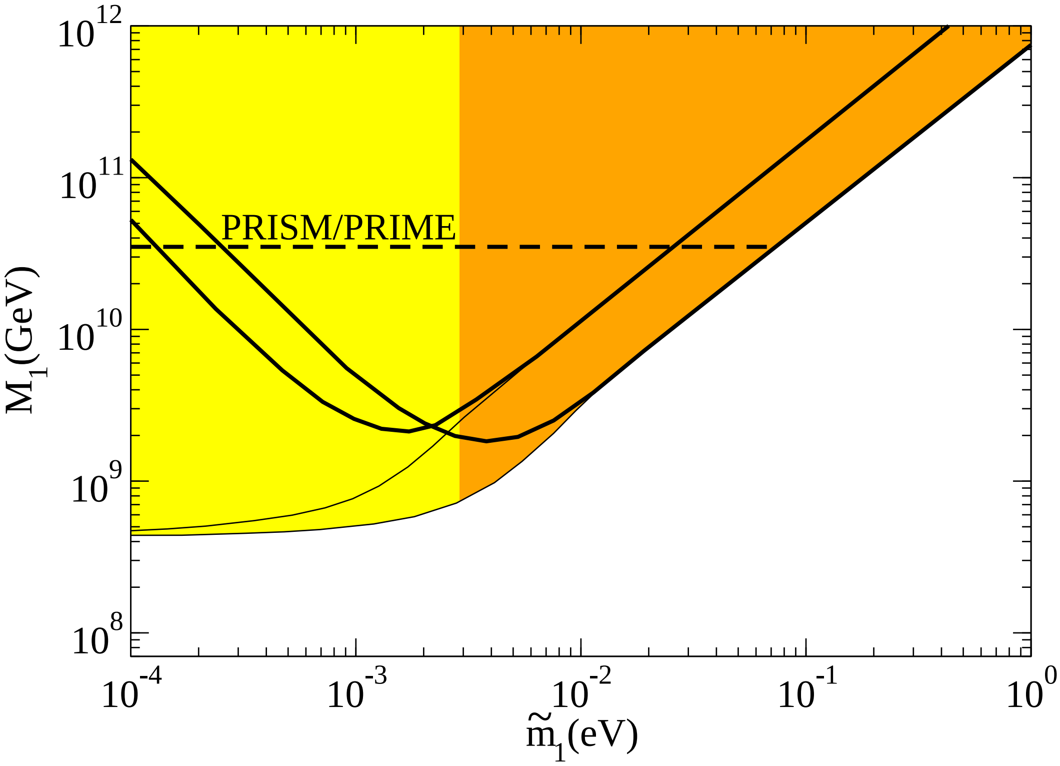


Figure 3.1.: Allowed parameter space of leptogenesis adopted from [125], including the constraint on M_1 from the projected non observation of muon conversion at PRISM/PRIME, under the assumption of hierarchical neutrino Yukawa eigenvalues and barring accidental cancellations. The orange region corresponds to the preferable range of $\widetilde{m}_1 > \sqrt{\Delta m^2_{21}}/3$. In this plot it is assumed $m_S \simeq 200\,\mathrm{GeV}$, $\tan\beta \simeq 10$ and $|\boldsymbol{\mathcal{M}}_{\boldsymbol{\nu}12}| \simeq \sqrt{\Delta m^2_{21}}/3$.

flavour effects (left).

We conclude that already with present data on $\mu \to e\gamma$ large values for M_1 may be ruled out. However, the window for M_1 can be closed, even with the projected sensitivity of PRISM/PRIME, only for very favourable SUSY parameters and zero initial right-handed neutrino abundance. Nevertheless, this analysis shows that at least some rate for $\mu \leftrightarrow e$ transitions must be expected in SUSY if leptogenesis is the source of the baryon asymmetry of the universe. Also, if $\mu \leftrightarrow e$ transitions are not detected this will guide us to very special situations for the see-saw parameters. Once more data on neutrino masses, SUSY and rare decays is available, a more refined analysis might be able to set more stringent bounds. Therefore, if SUSY will be detected, searches for cLFV are a very promising tool to probe our conjectures on (minimal) leptogenesis.

3.4. Some aspects in 2HDM

3.4.1. Rare leptonic decays and the see-saw

Rare leptonic decays

Similarly to the problems of the general 2HDM with quark FCNCs , charged LFV processes also have to be suppressed in order to satisfy present experimental bounds.

Although $l_i \to l_j \bar{l}_k l_k$ is mediated at tree level, stronger constraints are obtained by the loop suppressed process $l_i \to l_j\gamma$, as the former is suppressed by the small Yukawa coupling of the outgoing light lepton l_k. Semi-leptonic processes $\tau \to \mu$ meson(s) yield also interesting bounds [179–181] but they necessarily depend on the unknown quark Yukawa couplings to the second Higgs. A notable exception is again muon conversion: If the quark Yukawa couplings to the second Higgs vanish, the dipole dominates these processes, yielding a suppressed rate compared to $\mu \to e\gamma$, eq. (3.28) that is still interesting in view of the possible experimental improvements on muon conversion. Additionally, if those Yukawa couplings do not vanish, muon conversion can also proceed via tree level Higgs exchange [182]. Therefore, the upper bounds on flavour violating couplings presented in this section from the non-observation of $\mu \to e\gamma$ can easily be translated into bounds from muon conversion by eq. (3.28).

Before presenting the branching ratio for $\mu \to e\gamma$, recall the notation of section 2.3.2 and adapt it to the leptonic sector

$$- \mathcal{L}^{\text{Yuk}} \supset \bar{L}'_i \sqrt{2} \boldsymbol{\Delta}_{\boldsymbol{e}ij} e'_j \Phi_{\text{no vev}} + \text{h.c.} , \tag{3.44}$$

where $\langle \Phi_{\text{no vev}} \rangle = 0$ and the lepton fields are mass eigenstates:

$$\boldsymbol{M_e} \equiv \text{diag}(m_e, m_\mu, m_\tau) . \tag{3.45}$$

In the process $l_i \to l_j\gamma$ the strong suppression from chirality flips is superseded by the two loop Barr-Zee contribution [183]. The leading contribution comes from the top quark loop unless $\boldsymbol{\Delta}_{\boldsymbol{u}33}$ is small [182]. In the decoupling limit the Barr-Zee contribution to the branching ratio reads:

$$\frac{\mathrm{BR}(l_i \to l_j \gamma)}{\mathrm{BR}(l_i \to l_j \nu_i \bar{\nu}_j)} \simeq \frac{8\alpha^3}{3\pi^3} \frac{v^2 |\boldsymbol{\Delta}_{\boldsymbol{e}ji}|^2}{\boldsymbol{M}_{\boldsymbol{e}ii}^2} \left| f\!\left(\frac{m_t^2}{m_h^2}\right) \cos(\alpha-\beta) - \frac{v \boldsymbol{\Delta}_{\boldsymbol{u}33} m_t}{m_H^2} \log^2 \frac{m_t^2}{m_H^2} \right|^2 . \tag{3.46}$$

Here, the function $f(z)$ is defined in [182] and evaluates $f(2) \approx 1$ and $\alpha - \beta$ is the mixing angle between the mass eigenstates of the neutral CP even Higgs scalars and the basis Φ_{vev}, $\Phi_{\mathrm{no\,vev}}$, cf. eq. (2.41). Note that $\cos(\alpha-\beta)$ scales with m_H^{-2} in the decoupling limit [51].

Thus, the experimental bound on $\mathrm{BR}(\mu \to e\,\gamma)$, eq. (3.26), can be evaded for the concrete flavour structure $v\boldsymbol{\Delta}_{\boldsymbol{e}12} = \sqrt{m_e m_\mu}$ and $v\boldsymbol{\Delta}_{\boldsymbol{u}33} = m_t$, only if the mass of the heavy Higgs $m_H \gtrsim 3\,\mathrm{TeV}$. If instead $\cos(\alpha-\beta) = v^2/m_H^2$ and $\boldsymbol{\Delta}_{\boldsymbol{u}33}$ is negligible, a decoupling $m_H \gtrsim 800\,\mathrm{GeV}$ is sufficient.

For the radiative τ decays, the one loop contribution is also important if $v\boldsymbol{\Delta}_{\boldsymbol{e}33} \gg m_\tau$. However, in the ansatz by Cheng and Sher [21], $v\boldsymbol{\Delta}_{\boldsymbol{e}ij} = \sqrt{\boldsymbol{M}_{\boldsymbol{e}ii}\boldsymbol{M}_{\boldsymbol{e}jj}}$, the constraints from $\mu \to e\gamma$ are much more stringent.

This analysis completes the inspection of the decoupled 2HDM, section 2.3.3, showing that also cLFV is under control once one accepts the rather strong decoupling.

The see-saw

The Weinberg operator in 2HDM is a matrix in lepton flavour and Higgs space:

$$-\mathcal{L}^{\nu,\ \mathrm{low}} = +\frac{1}{2}(\bar{L}_i \tilde{\Phi}_a)\boldsymbol{\kappa}_{ij}^{ab}(\tilde{\Phi}_b^T L_j^C) + \mathrm{h.c.}\ . \tag{3.47}$$

In the basis of diagonal charged lepton masses, the neutrino mass therefore reads:

$$\boldsymbol{\mathcal{M}}_{\boldsymbol{\nu}} = \frac{v^2}{2} \boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{L}\dagger}[\boldsymbol{\kappa}^{11}\cos^2\beta + (\boldsymbol{\kappa}^{12} + \boldsymbol{\kappa}^{21})\cos\beta\sin\beta + \boldsymbol{\kappa}^{22}\sin^2\beta]\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{L}*}\ , \tag{3.48}$$

where the unitary matrix $\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{L}}$ is introduced in order to rotate into the mass eigenstate basis of charged leptons, cf. section 2.3.2:

$$\frac{v}{\sqrt{2}} \boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{L}\dagger}[\cos\beta \boldsymbol{Y}_{\boldsymbol{e}}^1 + \sin\beta \boldsymbol{Y}_{\boldsymbol{e}}^2]\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{R}} = \boldsymbol{M}_{\boldsymbol{e}}\ . \tag{3.49}$$

In the see-saw,

$$-\mathcal{L} \supset \bar{L}_i \boldsymbol{Y}_{\boldsymbol{\nu}\,ij}^a \nu_j \tilde{\Phi}_a + \bar{L}_i \boldsymbol{Y}_{\boldsymbol{e}\,ij}^a e_j \Phi_a - \frac{1}{2}\bar{\nu}_i^C \boldsymbol{M}_{ij}\nu_j + \mathrm{h.c.}\ , \tag{3.50}$$

the Weinberg operator reads at tree level:

$$\boldsymbol{\kappa}^{ab} = \boldsymbol{Y}_{\boldsymbol{\nu}}^a \boldsymbol{M}^{-1} \boldsymbol{Y}_{\boldsymbol{\nu}}^{bT}\ . \tag{3.51}$$

Connecting the see-saw with rare decays

A priori no connection exists between see-saw parameters and $\boldsymbol{\Delta}_{\boldsymbol{e}}$. Therefore it is in general impossible to constrain the see-saw with rare decays. This situation is completely analogous to the supersymmetric case where the connection arises only if one assumes flavour blind SUSY breaking. Note that – again as in the SUSY case – additional assumptions on the flavour structure are welcome anyway to attenuate problems with FCNCs and cLFV.

It has been proposed to generalize the MFV principle to the leptonic sector in order to connect cLFV and neutrino masses [25]. We know already from the SUSY case that such a connection is not even possible in the Constrained MSSM, which is a minimal example of a MFV model, cf. also [26]. Thus in [25] it is assumed degenerate right-handed neutrinos and absence of CP violation in the right-handed sector.

In the following section we will elaborate on the conjecture of aligned Yukawa couplings. This constitutes a minimal MFV model in exact analogy to the Constrained MSSM: The only sources of flavour violation in the leptonic sector are the neutrino Yukawa matrix and the mass matrix of right-handed neutrinos. And, at some high scale there is no cLFV at tree level.

3.4.2. Lepton flavour violation in the aligned 2HDM

It is straightforward to generalize the alignment condition, eq. (2.28), to the see-saw:

$$\boldsymbol{Y_f^1} = \cos\psi_f \boldsymbol{Y_f} \;, \qquad \boldsymbol{Y_f^2} = \sin\psi_f \boldsymbol{Y_f} \;, \qquad \text{where } f = u, d, e, \nu \;. \tag{3.52}$$

Note that this definition is independent of whether $\boldsymbol{M}$ is assumed to be diagonal and note that it is meaningful only at a scale Λ larger than the heaviest right-handed neutrino mass.

Between Λ and the mass of the right-handed neutrinos, the RG evolution transmits the flavour violation into the charged lepton Yukawa. The analytic derivation of the cLFV coupling $\boldsymbol{\Delta_e}$ in the leading-log approximation is completely analogous to the quark sector. Using the formulas of appendix A.2.2 the result reads:

$$\boldsymbol{\Delta_e}^{\text{off-diag.}} = \frac{1}{8\pi^2 v} \frac{\cos(\psi_\nu - \psi_e)\sin(\psi_e - \psi_\nu^*)}{\cos^2(\beta - \psi_e)} \boldsymbol{Y_\nu} \log\frac{\boldsymbol{M}}{\Lambda} \boldsymbol{Y_\nu^\dagger} \boldsymbol{M_e} \;. \tag{3.53}$$

However, $\boldsymbol{Y_\nu}$ is a priori unknown. At tree level it is connected to the left-handed neutrino mass matrix via, cf. eqs. (3.48), (3.51):

$$\boldsymbol{\mathcal{M}_\nu} = \frac{v^2}{2}\cos^2(\beta - \psi_\nu) \boldsymbol{Y_\nu} \boldsymbol{M}^{-1} \boldsymbol{Y_\nu^T} \;. \tag{3.54}$$

For the leading-log approximation to $\boldsymbol{\mathcal{M}_\nu}$ see appendix A.2.2.

Therefore, the connection of low energy physics to the see-saw parameters is exactly analogous to the CMSSM. With rare decays it is in principle possible to determine $\boldsymbol{Y_\nu} \log\frac{\boldsymbol{M}}{\Lambda}\boldsymbol{Y_\nu^\dagger}$ if the angle ψ_ν can be deduced somehow from the charged fermion sector. This combination of see-saw parameters is complementary to the information gained by neutrino masses, cf. [134]. Therefore it is impossible to make any definite prediction on rare decays even if $\boldsymbol{\mathcal{M}_\nu}$, ψ_f and the Higgs potential were known. On the other hand, inspections of additional conjectures carried out in the CMSSM, e.g. section 3.3.2, can also be applied to the aligned 2HDM case.

Here we will confine ourselves to estimating the possible rates for rare decays. From eq. (3.46) it follows

$$\frac{\mathrm{BR}(l_i \to l_j \gamma)}{\mathrm{BR}(l_i \to l_j \nu_i \bar{\nu}_j)} \simeq 5 \cdot 10^{-12} \left| \frac{\cos(\psi_\nu - \psi_e)\sin(\psi_e - \psi_\nu^*)}{\cos^2(\beta - \psi_e)} \right|^2 \left| \boldsymbol{Y_\nu} \log \frac{\boldsymbol{M}}{\Lambda} \boldsymbol{Y_\nu}^\dagger \right|^2_{ji} \times$$
$$\left| f\!\left(\frac{m_t^2}{m_h^2}\right) \cos(\alpha - \beta) - \frac{v \boldsymbol{\Delta_{u}}_{33} m_t}{m_H^2} \log^2 \frac{m_t^2}{m_H^2} \right|^2 . \quad (3.55)$$

In contrast to the quark sector, the logarithm from the RG evolution is typically not large. Now assume, in analogy to the discussion in the quark sector, the expression in the first absolute value bars to be $\mathcal{O}(1)$ to $\mathcal{O}(10)$. Then, the present bounds on rare leptonic decays, eqs. (3.26), can be met easily, even for large mixing angles, if the second Higgs is somewhat decoupled.

3.4.3. Explaining mild neutrino mass hierarchy in decoupled 2HDM

We have seen in section 3.2.2 that the plausible assumption of hierarchical neutrino Yukawa couplings can be accommodated in the SM see-saw only in special cases. From analogy to the charged fermions, one might for example assume the largest neutrino Yukawa eigenvalue y_3 to be about $100y_2$. Then, the most natural solution to the observed mild neutrino mass requires a hierarchy in the right-handed neutrino mass $M_3/M_2 \sim 10^4$. Whereas such large hierarchies in the right-handed neutrino mass are certainly possible, it is difficult to conceive that the large hierarchy in the Yukawa $\boldsymbol{Y_\nu}$ almost retracts the one of the right-handed neutrino mass $\boldsymbol{M}$ in the generation of left-handed neutrino masses. This situation does not change in SUSY nor the aligned 2HDM. However, in the general 2HDM there are two independent neutrino Yukawa couplings potentially with a large eigenvalue each. Therefore the (6×6-dimensional) Weinberg operator $\boldsymbol{\kappa}_{ij}^{ab} \propto \boldsymbol{Y}^a_{\boldsymbol{\nu}ik} \boldsymbol{M}^{-1}_{kl} \boldsymbol{Y}^{bT}_{\boldsymbol{\nu}lj}$ naturally possesses two large eigenvalues. Still at tree level a generically hierarchical left-handed neutrino mass $\boldsymbol{\mathcal{M}_\nu}$ is obtained, see the discussion below. On the other hand, radiative corrections lead to the generation of a second neutrino mass eigenvalue that is suppressed by a loop factor but enhanced by a large logarithm compared to the heaviest one generated at tree level [184]. Thus the observed mild neutrino mass hierarchy arises completely naturally.

After a short discussion on the tree level neutrino mass, we will revisit in this section this mechanism of tree level plus radiative neutrino mass generation in the general 2HDM. We will pursue a renormalization group approach, as we did in [9]. Rather than being motivated by finding signatures of new physics, we will construct a simple and natural framework capable to explain the smallness of neutrino masses as well as the observed mild mass hierarchy, cf. section 3.2.2. Finally we will argue that, as in the see-saw, the successes of the SM are automatically preserved, once the new particles are sufficiently heavy.

Hierarchies in the tree level neutrino mass matrix

At tree level the left-handed neutrino mass matrix reads, eqs. (3.48), (3.51):

$$\boldsymbol{\mathcal{M}}_{\boldsymbol{\nu}} = \frac{v^2}{2}(\cos\beta \boldsymbol{Y}_{\boldsymbol{\nu}}^1 + \sin\beta \boldsymbol{Y}_{\boldsymbol{\nu}}^2)\boldsymbol{M}^{-1}(\cos\beta \boldsymbol{Y}_{\boldsymbol{\nu}}^1 + \sin\beta \boldsymbol{Y}_{\boldsymbol{\nu}}^2)^T \, . \tag{3.56}$$

In order to obtain more than one non-vanishing light neutrino mass eigenvalue, more than one right-handed neutrino necessarily has to contribute significantly to the see-saw. For a very hierarchical right-handed neutrino mass $\boldsymbol{M}$, the observed mild neutrino mass hierarchy can only be obtained if hierarchies in $(\cos\beta \boldsymbol{Y}_{\boldsymbol{\nu}}^1 + \sin\beta \boldsymbol{Y}_{\boldsymbol{\nu}}^2)$ retract the hierarchies in $\boldsymbol{M}$. This situation is not advantageous compared to the SM.

On the other hand, for rather degenerate right-handed neutrinos it seems possible to obtain the observed mild hierarchy if $|\boldsymbol{Y}_{\boldsymbol{\nu}}^1| \sim |\boldsymbol{Y}_{\boldsymbol{\nu}}^2|$ and $\cos\beta \sim \sin\beta$. However, this requires that the largest coupling in $\boldsymbol{Y}_{\boldsymbol{\nu}}^2$ acts on different left- and right-handed neutrinos compared to $\boldsymbol{Y}_{\boldsymbol{\nu}}^1$. For example, if ${\boldsymbol{Y}_{\boldsymbol{\nu}}^1}_{ij} \sim \delta_{i3}\delta_{j3}$, then at least one of ${\boldsymbol{Y}_{\boldsymbol{\nu}}^2}_{ij}$, $i, j \leq 2$ must be $\mathcal{O}(1)$ in order to yield the observed mild mass hierarchy. In such a case, if the renormalization group mixes $\boldsymbol{Y}_{\boldsymbol{\nu}}^1$ and $\boldsymbol{Y}_{\boldsymbol{\nu}}^2$, the hierarchies in $\boldsymbol{Y}_{\boldsymbol{\nu}}^1$, $\boldsymbol{Y}_{\boldsymbol{\nu}}^2$ are erased. This nullifies our principal assumption, namely to have strong hierarchies in the neutrino Yukawa eigenvalues in analogy to the Yukawa couplings in the charged fermion sector.

We conclude that also in the general 2HDM only in very special cases the observed mild neutrino mass hierarchy can be accommodated at tree level with hierarchical neutrino Yukawa couplings. Generically – at tree level – one would expect the left-handed neutrino masses to exhibit strong hierarchies.

Radiative generation of the second neutrino mass

To emphasize the main features of the quantum corrections to the neutrino mass matrix, we will simplify the notation by assuming without loss of generality $\cos\beta = 1$, i.e. $\langle\Phi_2\rangle = 0$. Moreover we will concentrate in what follows on a see-saw model with just one right-handed neutrino with mass M. Embedded in a more realistic model with three right-handed neutrinos it corresponds to the most important one in the see-saw, e.g. to the one with largest Yukawa coupling or to the lightest neutrino mass eigenstate. Then, with this assumption, the neutrino Yukawa couplings Y_ν^a are 3-vectors.

With only one right-handed neutrino, at tree level the neutrino mass matrix must have rank one since it is the outer product of two vectors:

$$\boldsymbol{\mathcal{M}}_{\boldsymbol{\nu}}^{\text{tree}} = \frac{v^2}{2}[\boldsymbol{\kappa}^{11}]^{\text{tree}} = \frac{v^2}{2}\frac{\boldsymbol{Y}_{\boldsymbol{\nu}}^1 \boldsymbol{Y}_{\boldsymbol{\nu}}^{1T}}{M} \, . \tag{3.57}$$

Of course the existence of only one non-vanishing eigenvalue is in conflict with the two observed neutrino mass splittings.

However, in order to compare the predictions of the model with low energy experiments one must take into account radiative corrections. Therefore we will make use of the β-functions given in appendix A.2.1, to run the effective coupling $\boldsymbol{\kappa}^{ab}$ from some high scale $\Lambda \geq M$ to the scale of the second Higgs doublet m_H. Even if $m_H \gg m_Z$ the running below m_H will not introduce any new qualitative feature but will only modify the values of the neutrino mass by a small factor, proportional to the squared τ Yukawa coupling [185].

On the other hand, the effects of the RG evolution above m_H can be written, in the leading-log approximation:

$$\boldsymbol{\delta\kappa}^{11} \simeq \boldsymbol{B}_{1a}\frac{\boldsymbol{Y}_{\boldsymbol{\nu}}^{a}\boldsymbol{Y}_{\boldsymbol{\nu}}^{1T}}{M} + \frac{\boldsymbol{Y}_{\boldsymbol{\nu}}^{1}\boldsymbol{Y}_{\boldsymbol{\nu}}^{aT}}{M}\boldsymbol{B}_{1a}^{T} + b\frac{\boldsymbol{Y}_{\boldsymbol{\nu}}^{2}\boldsymbol{Y}_{\boldsymbol{\nu}}^{2T}}{M} \ , \tag{3.58}$$

where $\boldsymbol{B}_{1a}$ denote 3×3 matrices which can be found in appendix A.2.3. Here we are mainly interested in the complex number b which turns out to be the relevant parameter in the radiative generation of a second neutrino mass eigenvalue. It is easy to derive from the 2HDM β-function eq. (A.8)

$$b = -\frac{1}{16\pi^2}2\lambda_5 \log\frac{M}{m_H} \ , \tag{3.59}$$

which depends linearly on the quartic coupling $\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2$, cf. eq. (2.47). Note that there cannot be any contribution from the RG evolution from above M: Technically, a term proportional to $\boldsymbol{Y}_{\boldsymbol{\nu}}^{2}\boldsymbol{Y}_{\boldsymbol{\nu}}^{2T}$ appears in the leading-log approximation of $(\boldsymbol{Y}_{\boldsymbol{\nu}}^{1}\boldsymbol{Y}_{\boldsymbol{\nu}}^{1T})(M)$ at second order in the logarithm. Moreover, it is clear that at the scale M, the rank of $\boldsymbol{\kappa}^{11}(M)$ is one, as it is proportional to the outer product $\boldsymbol{Y}_{\boldsymbol{\nu}}^{1}(M)\boldsymbol{Y}_{\boldsymbol{\nu}}^{1T}(M)$.

The neutrino mass matrix $\boldsymbol{\mathcal{M}_\nu} = \boldsymbol{\mathcal{M}}_{\boldsymbol{\nu}}^{\text{tree}} + \boldsymbol{\delta\mathcal{M}_\nu}$ can be diagonalized using perturbation theory, giving as a result the eigenvalues $m_i = m_i^{\text{tree}} + \delta m_i$ with m_i^{tree} the eigenvalue at tree level and δm_i the first order correction.

At lowest order in perturbation theory, the only non-vanishing neutrino mass eigenvalue and the associated column of the leptonic mixing matrix can be read off from eq. (3.57)

$$m_3^{\text{tree}} = \frac{v^2}{2M}|\boldsymbol{Y}_{\boldsymbol{\nu}}^{1}|^2 \ , \tag{3.60a}$$

$$\boldsymbol{U}_{\boldsymbol{MNS}\,i3}^{\text{tree}} = \frac{\boldsymbol{Y}_{\boldsymbol{\nu}i}^{1*}}{|\boldsymbol{Y}_{\boldsymbol{\nu}}^{1}|} \ , \tag{3.60b}$$

while the first two columns are undefined, due to the degeneracy of the corresponding neutrino mass eigenvalues. In this expression we have used the Frobenius norm, $|\boldsymbol{Y}_{\boldsymbol{\nu}}^{a}| = (\sum_i |\boldsymbol{Y}_{\boldsymbol{\nu}i}^{a}|^2)^{1/2}$.

The correction to the neutrino mass eigenvalues due to the perturbation $\boldsymbol{\delta\kappa}^{11}$ is given by:

$$\delta m_i = \frac{v^2}{2}\,\text{Re}[(\boldsymbol{U}_{\boldsymbol{MNS}}^{\text{tree}\,T}\,\boldsymbol{\delta\kappa}^{11}\,\boldsymbol{U}_{\boldsymbol{MNS}}^{\text{tree}})_{ii}] \ , \tag{3.61}$$

which slightly modifies the value of the heaviest neutrino mass eigenvalue:

$$\delta m_3 = \frac{v^2}{2M}\,\text{Re}\left[2(\boldsymbol{Y}_{\boldsymbol{\nu}}^{1\dagger}\boldsymbol{B}_{1a}\boldsymbol{Y}_{\boldsymbol{\nu}}^{a}) + b\frac{(\boldsymbol{Y}_{\boldsymbol{\nu}}^{1\dagger}\boldsymbol{Y}_{\boldsymbol{\nu}}^{2})^2}{|\boldsymbol{Y}_{\boldsymbol{\nu}}^{1}|^2}\right] \ . \tag{3.62}$$

More importantly, at first order also a non-vanishing value for m_2 is generated. Note that this does not happen in the SM extended with a single right-handed neutrino, as the RG evolution does not change the rank of the Weinberg-operator.

According to eq. (3.61), in order to determine the radiatively generated δm_2 it is necessary to know the second row of the leptonic mixing matrix $\boldsymbol{U}^{\rm tree}_{\boldsymbol{MNS}i2}$.

Since the matrix $\boldsymbol{U}^{\rm tree}_{\boldsymbol{MNS}}$ is unitary, the second column $\boldsymbol{U}^{\rm tree}_{\boldsymbol{MNS}i2}$ has to be orthogonal to the third one and has to be normalized. The only vector constructed from the vectors $\boldsymbol{Y}^1_{\boldsymbol{\nu}}$ and $\boldsymbol{Y}^2_{\boldsymbol{\nu}}$ satisfying those properties reads:

$$\boldsymbol{U}^{\rm tree}_{\boldsymbol{MNS}i2} = \frac{1}{N_2}\left[\boldsymbol{Y}^{2*}_{\boldsymbol{\nu}i} - \frac{\boldsymbol{Y}^{2\dagger}_{\boldsymbol{\nu}}\boldsymbol{Y}^1_{\boldsymbol{\nu}}}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|}\frac{\boldsymbol{Y}^{1*}_{\boldsymbol{\nu}i}}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|}\right] e^{-\frac{i}{2}\arg(-\lambda_5)} , \tag{3.63}$$

where the phase has been chosen to yield m_2 real and positive and

$$N_2 = \left[\boldsymbol{Y}^{2\dagger}_{\boldsymbol{\nu}}\boldsymbol{Y}^2_{\boldsymbol{\nu}} - \frac{|\boldsymbol{Y}^{2\dagger}_{\boldsymbol{\nu}}\boldsymbol{Y}^1_{\boldsymbol{\nu}}|^2}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|^2}\right]^{1/2} . \tag{3.64}$$

Substituting into the expression for δm_2 we find:

$$m_2 = \frac{1}{16\pi^2}\frac{|\lambda_5|v^2}{M_{\rm maj}}\left[|Y^2_\nu|^2 - \frac{|Y^{2\dagger}_\nu Y^1_\nu|^2}{|Y^1_\nu|^2}\right]\log\frac{M_{\rm maj}}{m_H} . \tag{3.65}$$

At this level the lightest neutrino remains massless. However, it receives tiny finite corrections arising from two-loop diagrams involving W bosons [186–188].

It is interesting to note that, under some well motivated assumptions, the hierarchy between the tree level mass m_3 and the radiatively generated neutrino mass m_2 can be fairly mild. For instance, taking the typical values $|\lambda_5| \sim 1$, $M \sim 10^{11}\,\text{GeV}$ and $m_H \sim 1\,\text{TeV}$ and assuming non-aligned neutrino Yukawa couplings with $|\boldsymbol{Y}^2_{\boldsymbol{\nu}}| \sim |\boldsymbol{Y}^1_{\boldsymbol{\nu}}|$ one obtains for the ratio between the two heaviest neutrino mass eigenvalues:

$$\frac{m_2}{m_3} \simeq \frac{|\lambda_5|}{8\pi^2}\frac{|\boldsymbol{Y}^2_{\boldsymbol{\nu}}|^2}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|^2}\log\frac{M}{m_H} \sim 0.2 , \tag{3.66}$$

which yields a mild mass hierarchy, in qualitative agreement with the experimental data. Note that, whereas the overall scale of the light neutrino masses depends linearly on the inverse of the heavy right-handed neutrino mass, the ratio between the two heaviest neutrino mass eigenvalues depends only logarithmically on the masses of the new particles. As a consequence the mild mass hierarchy is fairly insensitive to the exact values of the masses of right-handed neutrinos and the second Higgs doublet.

In the previous analysis we have assumed for simplicity that only one right-handed neutrino participates in the neutrino mass generation. If there are several right-handed neutrinos, the tree level contributions to all neutrino mass eigenvalues will be non-vanishing. As discussed above however, the neutrino mass hierarchy generated at tree level is expected to be much larger than the one inferred from experiments. Therefore, the radiatively generated contribution to the next-to-lightest neutrino mass will dominate over the tree level contribution, and the conclusions presented above will still hold.

Lastly, note that in order to obtain a non-vanishing radiative contribution, eq. (3.65), the Yukawa couplings $\boldsymbol{Y}^1_{\boldsymbol{\nu}}$, $\boldsymbol{Y}^2_{\boldsymbol{\nu}}$ have to be misaligned. Though, in contrast to the case of tree level generation of a mild mass hierarchy, only one right-handed neutrino is involved. Therefore, even when extending this model with several right-handed neutrinos, the Yukawa coupling hierarchies are stable under the renormalization group. On the other hand, the misalignment induces flavour violating neutral interactions. If the misalignment is only in the neutrino sector, we found that the radiatively induced cLFV can be well below experimental bounds even if the second Higgs is rather light [9]. The more natural choice, which is having misalignment also in the charged fermion sector, requires rather strong decoupling of the second Higgs as will be discussed at the end of this section.

A parametrization in terms of low energy observables

In the model of radiative generation of the next-to-lightest neutrino mass, the neutrino mass matrix can be written in the leading-log approximation, eq. (3.58):

$$\boldsymbol{\mathcal{M}_\nu} \simeq \frac{v^2}{2}\left[(\boldsymbol{Y_\nu^1}+\boldsymbol{B}_{1a}\boldsymbol{Y_\nu^a})\frac{1}{M}(\boldsymbol{Y_\nu^1}+\boldsymbol{B}_{1a}\boldsymbol{Y_\nu^a})^T+\boldsymbol{Y_\nu^2}\frac{b}{M}\boldsymbol{Y_\nu^{2T}}-\mathcal{O}(\boldsymbol{B}^2)\right] . \tag{3.67}$$

Neglecting the term $\mathcal{O}(\boldsymbol{B}^2)$, this is formally identical to the neutrino mass generation in the SM with two right-handed neutrinos with masses M and M/b. In this analogy, $(\boldsymbol{Y_\nu^1}+\boldsymbol{B}_{1a}\boldsymbol{Y_\nu^a})$ and $\boldsymbol{Y_\nu^2}$ correspond to the different columns of the neutrino Yukawa coupling acting on the different right handed neutrinos. This correspondence allows to apply the results of [113] to write explicit expressions for the Yukawa couplings in terms of the low energy neutrino parameters and one complex angle $\hat{\theta}$:

$$\boldsymbol{Y_\nu^2} = \frac{\sqrt{2}}{v}\sqrt{\frac{M}{b}}\left(-\sqrt{m_2}\sin\hat{\theta}\,\boldsymbol{U}^*_{\boldsymbol{MNS}i2} \pm \sqrt{m_3}\cos\hat{\theta}\,\boldsymbol{U}^*_{\boldsymbol{MNS}i3}\right) , \tag{3.68a}$$

$$\boldsymbol{Y_\nu^1}+\boldsymbol{B}_{1a}\boldsymbol{Y_\nu^a} = \frac{\sqrt{2}}{v}\sqrt{M}\left(+\sqrt{m_2}\cos\hat{\theta}\,\boldsymbol{U}^*_{\boldsymbol{MNS}i2} \pm \sqrt{m_3}\sin\hat{\theta}\,\boldsymbol{U}^*_{\boldsymbol{MNS}i3}\right) . \tag{3.68b}$$

Note, however, that this parametrization may fail if terms $\mathcal{O}(\boldsymbol{B}^2)$ cannot be neglected.

Corrections to the mixing angles and discussion of $\sin\theta_{13}$

As radiative corrections are very important in the generation of two neutrino mass eigenvalues from one right-handed neutrino, it is interesting to investigate also the impact of the RG evolution on the mixing angles. Indeed we will find that the corrections can be sizeable. Therefore, once a certain flavour model is applied at high energy, the radiative corrections have to be taken into account in order to predict the left-handed neutrino mass. Here we will not assume any flavour model but we will argue that in this model generically rather large θ_{13} is obtained by the corrections even if $\boldsymbol{U}_{\boldsymbol{MNS}13}$ vanished at high scale due to a flavour symmetry.

The leptonic mixing matrix $\boldsymbol{U_{MNS}}$ receives radiative corrections from two different origins. First, the RG evolution of the Weinberg operator $\boldsymbol{\kappa}$[11] (and, above the right-handed neutrino mass M, the evolution of $\boldsymbol{Y_\nu^1}\boldsymbol{Y_\nu^{1T}}$) generates a correction to the leptonic mixing matrix given by

$$\boldsymbol{\delta U^\kappa_{MNS}} = \boldsymbol{U^{\mathrm{tree}}_{MNS}}\boldsymbol{T} , \tag{3.69}$$

where

$$\boldsymbol{T}_{ii} \equiv -\frac{i[\boldsymbol{U}_{\boldsymbol{MNS}}^{\mathrm{tree\,T}}\boldsymbol{\delta\mathcal{M}_\nu}\boldsymbol{U}_{\boldsymbol{MNS}}^{\mathrm{tree}}]_{ii}}{2m_i^{\mathrm{tree}}}, \qquad \text{if } m_i^{\mathrm{tree}} \neq 0, \tag{3.70a}$$

$$\boldsymbol{T}_{ij} \equiv \frac{m_i^{\mathrm{tree}}[\boldsymbol{U}_{\boldsymbol{MNS}}^{\mathrm{tree\,T}}\boldsymbol{\delta\mathcal{M}_\nu}\boldsymbol{U}_{\boldsymbol{MNS}}^{\mathrm{tree}}]_{ij} + m_j^{\mathrm{tree}}[\boldsymbol{U}_{\boldsymbol{MNS}}^{\mathrm{tree\,T}}\boldsymbol{\delta\mathcal{M}_\nu}\boldsymbol{U}_{\boldsymbol{MNS}}^{\mathrm{tree}}]_{ij}^*}{(m_j^{\mathrm{tree}})^2 - (m_i^{\mathrm{tree}})^2}, \quad \text{if } m_i^{\mathrm{tree}} \neq m_j^{\mathrm{tree}}. \tag{3.70b}$$

Here we are particularly interested in the correction to the last column of the leptonic mixing matrix, which in general yields a non-vanishing contribution to $\sin\theta_{13}$ and the atmospheric mixing. It can be written using eq. (3.58):

$$\begin{aligned}\boldsymbol{\delta U}^{\boldsymbol{\kappa}}_{\boldsymbol{MNS}i3} = \frac{\boldsymbol{Y}^{1*}_{\boldsymbol{\nu}i}}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|}\Bigg[&-\frac{\mathrm{Re}(\boldsymbol{Y}^{1\dagger}_{\boldsymbol{\nu}}\boldsymbol{B}_{1a}\boldsymbol{Y}^a_{\boldsymbol{\nu}})}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|^2} + \frac{i}{2}\frac{\mathrm{Im}(b^*(\boldsymbol{Y}^{2\dagger}_{\boldsymbol{\nu}}\boldsymbol{Y}^1_{\boldsymbol{\nu}})^2)}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|^4}\Bigg] \\ &+ \frac{(\boldsymbol{B}^*_{1a}\boldsymbol{Y}^{a*}_{\boldsymbol{\nu}})_i}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|} + \left(\boldsymbol{Y}^{2*}_{\boldsymbol{\nu}i} - \boldsymbol{Y}^{1*}_{\boldsymbol{\nu}i}\frac{(\boldsymbol{Y}^{2\dagger}_{\boldsymbol{\nu}}\boldsymbol{Y}^1_{\boldsymbol{\nu}})}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|^2}\right) b^*\frac{\boldsymbol{Y}^{2\dagger}_{\boldsymbol{\nu}}\boldsymbol{Y}^1_{\boldsymbol{\nu}}}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|^3}\ .\end{aligned} \tag{3.71}$$

Secondly, the RGE evolution also modifies the structure of the charged lepton Yukawa coupling. In a basis of diagonal $\boldsymbol{Y}^1_{\boldsymbol{e}}$ at the high scale Λ, this introduces an additional correction to the leptonic mixing matrix $\boldsymbol{U}_{\boldsymbol{MNS}}$, cf. eqs. (3.48), (3.49). Summing up both corrections, the leptonic mixing matrix reads at low energies:

$$\boldsymbol{U}_{\boldsymbol{MNS}} = \boldsymbol{V}^{\boldsymbol{L}T}_{\boldsymbol{e}}\boldsymbol{U}^{\mathrm{tree}}_{\boldsymbol{MNS}} + \boldsymbol{\delta U}^{\boldsymbol{\kappa}}_{\boldsymbol{MNS}}\ . \tag{3.72}$$

The matrix $\boldsymbol{V}^{\boldsymbol{L}}_{\boldsymbol{e}}$ can be explicitly calculated from the β-functions of the charged lepton Yukawa couplings, eqs. (A.6), (A.9) Using

$$(\boldsymbol{Y}^1_{\boldsymbol{e}}\boldsymbol{Y}^{1\dagger}_{\boldsymbol{e}})(m_H) \simeq \boldsymbol{Y}^1_{\boldsymbol{e}}\boldsymbol{Y}^{1\dagger}_{\boldsymbol{e}} - \frac{\log\frac{M}{m_H}}{16\pi^2}\left[\beta_{\boldsymbol{Y}^1_{\boldsymbol{e}}}\boldsymbol{Y}^{1\dagger}_{\boldsymbol{e}} + \boldsymbol{Y}^1_{\boldsymbol{e}}\beta^\dagger_{\boldsymbol{Y}^1_{\boldsymbol{e}}}\right] - \frac{\log\frac{\Lambda}{M}}{16\pi^2}\left[\beta^\nu_{\boldsymbol{Y}^1_{\boldsymbol{e}}}\boldsymbol{Y}^{1\dagger}_{\boldsymbol{e}} + \boldsymbol{Y}^1_{\boldsymbol{e}}\beta^{\nu\dagger}_{\boldsymbol{Y}^1_{\boldsymbol{e}}}\right], \tag{3.73}$$

we obtain

$$\boldsymbol{V}^{\boldsymbol{L}}_{\boldsymbol{e}ij} \simeq -\frac{1}{16\pi^2}\frac{\left[\beta_{\boldsymbol{Y}^1_{\boldsymbol{e}}}\boldsymbol{Y}^{1\dagger}_{\boldsymbol{e}} + \mathrm{h.c.}\right]_{ij}\log\frac{M}{m_H} + \left[\beta^\nu_{\boldsymbol{Y}^1_{\boldsymbol{e}}}\boldsymbol{Y}^{1\dagger}_{\boldsymbol{e}} + \mathrm{h.c.}\right]_{ij}\log\frac{\Lambda}{M}}{(\boldsymbol{Y}^1_{\boldsymbol{e}})^2_j - (\boldsymbol{Y}^1_{\boldsymbol{e}})^2_i}\ , \qquad i \neq j\ . \tag{3.74}$$

A quantity of particular interest is the angle θ_{13}, which is constrained by present experiments to be small. Adding up the corrections eqs. (3.71), (3.74), using the expressions in appendices A.2.1 and A.2.3 and neglecting terms cubic in the charged lepton Yukawa couplings as well as terms proportional to $\boldsymbol{U}^{\mathrm{tree}}_{\boldsymbol{MNS}13}$, we find for the radiatively induced value

$$\begin{aligned}\boldsymbol{\delta U}_{\boldsymbol{MNS}13} = -\frac{\boldsymbol{Y}^{2*}_{\boldsymbol{\nu}1}}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|}\Bigg\{&\left[\mathrm{Tr}(3\boldsymbol{Y}^{1\dagger}_{\boldsymbol{u}}\boldsymbol{Y}^2_{\boldsymbol{u}} + 3\boldsymbol{Y}^1_{\boldsymbol{d}}\boldsymbol{Y}^{2\dagger}_{\boldsymbol{d}} + \boldsymbol{Y}^{1\dagger}_{\boldsymbol{\nu}}\boldsymbol{Y}^2_{\boldsymbol{\nu}}) + 2\boldsymbol{Y}^{1\dagger}_{\boldsymbol{\nu}}(\boldsymbol{Y}^1_{\boldsymbol{e}})^{-1}\boldsymbol{Y}^{2\dagger}_{\boldsymbol{e}}\boldsymbol{Y}^1_{\boldsymbol{\nu}}\right]\frac{\log\frac{\Lambda}{M}}{16\pi^2} \\ &+ \left[3\,\mathrm{Tr}(\boldsymbol{Y}^{1\dagger}_{\boldsymbol{u}}\boldsymbol{Y}^2_{\boldsymbol{u}} + \boldsymbol{Y}^1_{\boldsymbol{d}}\boldsymbol{Y}^{2\dagger}_{\boldsymbol{d}}) + 2\lambda^*_6 + 2\lambda^*_5\frac{\boldsymbol{Y}^{2\dagger}_{\boldsymbol{\nu}}\boldsymbol{Y}^1_{\boldsymbol{\nu}}}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|^2}\right]\frac{\log\frac{M}{m_H}}{16\pi^2}\Bigg\} \\ + \frac{(\boldsymbol{Y}^{1\dagger}_{\boldsymbol{\nu}}(\boldsymbol{Y}^1_{\boldsymbol{e}})^{-1}\boldsymbol{Y}^{2\dagger}_{\boldsymbol{e}})_1}{|\boldsymbol{Y}^1_{\boldsymbol{\nu}}|}&\left\{\mathrm{Tr}(\boldsymbol{Y}^{2\dagger}_{\boldsymbol{\nu}}\boldsymbol{Y}^1_{\boldsymbol{\nu}})\frac{\log\frac{\Lambda}{M}}{16\pi^2} + 3\,\mathrm{Tr}(\boldsymbol{Y}^{2\dagger}_{\boldsymbol{u}}\boldsymbol{Y}^1_{\boldsymbol{u}} + \boldsymbol{Y}^2_{\boldsymbol{d}}\boldsymbol{Y}^{1\dagger}_{\boldsymbol{d}})\frac{\log\frac{\Lambda}{m_H}}{16\pi^2}\right\}.\end{aligned} \tag{3.75}$$

The first line might vanish if $\Lambda = M$; the last line depends on the flavour structure of $\boldsymbol{Y_e^2}$. However, the second line yields a contribution to $\sin\theta_{13}$ which is, as the ratio m_2/m_3, suppressed by the loop factor but enhanced by the large logarithm of the ratio of the Majorana mass over the heavy Higgs mass. As a result, the radiatively generated θ_{13} can be as large as ~ 0.2 if any of the entries in the bracket is $\mathcal{O}(1)$. Conversely, if $\sin\theta_{13}$ turns out to be tiny, this would require vanishing λ_6 and $\lambda_5 \boldsymbol{Y_\nu^{2\dagger}} \boldsymbol{Y_\nu^1}$ in the absence of accidental cancellations.[1]

Quantum effects also induce corrections to the atmospheric mixing angle, leading to deviations to the maximal mixing even if $\theta_{23} = \pi/4$ at tree level. It is interesting that if the charged lepton Yukawa couplings are subdominant in the corrections to the leptonic mixing matrix, eqs. (3.71), (3.74), then a correlation arises between the deviations of $\boldsymbol{U_{MNS}}_{23}/\boldsymbol{U_{MNS}}_{33}$ and $\boldsymbol{U_{MNS}}_{13}$ from their corresponding values at the cut-off scale.

In this limit, the radiative corrections to the last column of the leptonic mixing matrix are proportional to $\boldsymbol{Y_\nu^1}$ and $\boldsymbol{Y_\nu^2}$, eq. (3.71). It can be schematically written as:

$$\boldsymbol{U_{MNS}}_{i3} = (1+\epsilon_3)\, \boldsymbol{U^{\rm tree}_{MNS}}_{i3} + \epsilon_2\, \boldsymbol{U^{\rm tree}_{MNS}}_{i2}\ . \tag{3.76}$$

It can be checked that to first order the ratio $\boldsymbol{U_{MNS}}_{23}/\boldsymbol{U_{MNS}}_{33}$ does not depend on ϵ_3. Then, using this equation for $\boldsymbol{U_{MNS}}_{13}$ to eliminate ϵ_2 it follows that:

$$\frac{\boldsymbol{U_{MNS}}_{23}}{\boldsymbol{U_{MNS}}_{33}} - \frac{\boldsymbol{U^{\rm tree}_{MNS}}_{23}}{\boldsymbol{U^{\rm tree}_{MNS}}_{33}} \simeq \frac{\boldsymbol{U^{\rm tree}_{MNS}}_{22}\boldsymbol{U^{\rm tree}_{MNS}}_{33} - \boldsymbol{U^{\rm tree}_{MNS}}_{32}\boldsymbol{U^{\rm tree}_{MNS}}_{23}}{(\boldsymbol{U^{\rm tree}_{MNS}})^2_{33}}\, \frac{\boldsymbol{U_{MNS}}_{13} - \boldsymbol{U^{\rm tree}_{MNS}}_{13}}{\boldsymbol{U^{\rm tree}_{MNS}}_{12}}\ . \tag{3.77}$$

Concretely, in the case when at the cut-off scale the atmospheric mixing angle is exactly maximal and θ_{13} vanishes, at low energy scales the elements of the leptonic mixing matrix approximately satisfy

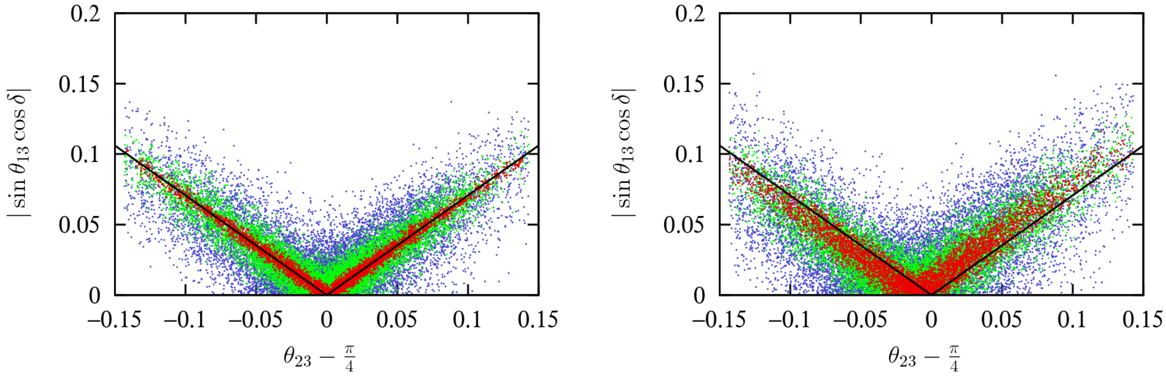
$$\frac{\boldsymbol{U_{MNS}}_{23}}{\boldsymbol{U_{MNS}}_{33}} - 1 \simeq 2\sqrt{2}\boldsymbol{U_{MNS}}_{13}\ , \tag{3.78}$$

which can be recast as

$$\tan\theta_{23} \simeq |1 + 2\sqrt{2}\sin\theta_{13} e^{-i\delta}| \qquad \text{or} \qquad \theta_{23} - \frac{\pi}{4} \simeq \sqrt{2}\sin\theta_{13}\cos\delta\ . \tag{3.79}$$

If there are additional sources of lepton flavour violation, then the low energy values of θ_{23} and θ_{13} are expected to deviate from this relation. This is illustrated in the scatter plots shown in fig. 3.2, which have been obtained by the numerical one loop integration of the RGE of the 2HDM extended by one right-handed neutrino. We assume in the plot $m_H = 3\,\text{TeV}$, $M = 10^{14}\,\text{GeV}$ and tri-bi-maximal mixing at a cut-off scale, which we choose $\Lambda = M$ in the left and $\Lambda = 10^{18}\,\text{GeV}$ in the right plot.

[1] We have derived eq. (3.75) in the basis $\langle\Phi_2\rangle = 0$, i.e. under a condition that needs to be applied at a low energy scale. It is more reasonable to employ the ansatz $\boldsymbol{U^{\rm tree}_{MNS}}_{13} = 0$ in a basis that is defined at the scale Λ. Then, the contributions to eq. (3.75) due to the Higgs self-energy, in particular the traces in the first two lines, might cancel when going at low energies into the basis $\langle\Phi_2\rangle = 0$. Therefore we do not claim vanishing $\mathrm{Tr}(\boldsymbol{Y_u^{1\dagger}}\boldsymbol{Y_u^2} + \boldsymbol{Y_d^1}\boldsymbol{Y_d^{2\dagger}})$ to be necessary for tiny θ_{13}. On the other hand, if $\mathrm{Tr}(\boldsymbol{Y_f^{1\dagger}}\boldsymbol{Y_f^2})$ was sizeable for any f, then λ_6 is generated at one loop [189] and we expect $\sin\theta_{13}$ to be generated at second order in the leading-log approximation.



(a) $\Lambda = M$ **(b)** $\Lambda = 10^4 M$

Figure 3.2.: Scatter plots showing $|\sin\theta_{13}\cos\delta|$ against $\theta_{23}-\frac{\pi}{4}$ at low energies for random choices of high energy parameters consistent with the measured neutrino oscillation parameters. We have assumed tri-bi-maximal mixing at the cut-off scale Λ, being the deviation from $\theta_{23}=\pi/4$ and $\theta_{13}=0$ at low energies only due to the radiative corrections. In the left plot it is assumed the cut-off scale to equal the mass of the right-handed neutrino, $\Lambda = M = 10^{14}$ GeV, while a much higher cut-off is presumed in the right plot. For more details see the main text.

In order to obtain tri-bi-maximal mixing we employ the ansatz

$$\boldsymbol{Y}_{\boldsymbol{\nu}}^1 = \frac{N_\nu^1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}\,, \qquad \boldsymbol{Y}_{\boldsymbol{\nu}}^2 = \frac{N_\nu^2}{\sqrt{3}}e^{i\phi}\begin{pmatrix}1\\1+y\\-1+y\end{pmatrix}\,.$$

Here $N_\nu^1, N_\nu^2 \sim 0.5$ are normalization factors which are adjusted to yield the correct neutrino squared mass differences at low scales, y is a random complex parameter which we vary in $0 \le |y| \le 0.5$ and ϕ is a random phase. We vary all random phases and angles between 0 and 2π and take only flat priors. Furthermore, we fix $|\lambda_5| = 0.5$ in order to obtain the mild neutrino mass hierarchy naturally and we take random values with $|\lambda_6| < 0.45$, where the upper bound has been chosen in order to preserve the perturbativity of the quartic couplings in the RG evolution. The remaining quartic couplings have no direct impact on the neutrino mass matrix and we fix them $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.5$, $\lambda_7 = 0$.

In order to investigate the impact of the charged lepton mixing in the correlation, we have adopted the ansatz $\boldsymbol{Y}_{\boldsymbol{e}}^2 = \boldsymbol{V}\boldsymbol{Y}_{\boldsymbol{e}}^1$, where $\boldsymbol{V}$ is a general unitary matrix with random angles and phases. Furthermore, since the effect of the charged lepton Yukawa couplings on the corrections to the leptonic mixing matrix is proportional to $\mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{u}}^1\boldsymbol{Y}_{\boldsymbol{u}}^{2\dagger})$, we have taken in the scatter plot $|\boldsymbol{Y}_{\boldsymbol{u}\,33}^2/\boldsymbol{Y}_{\boldsymbol{u}\,33}^1| \le 0.05$ (red points), $|\boldsymbol{Y}_{\boldsymbol{u}\,33}^2/\boldsymbol{Y}_{\boldsymbol{u}\,33}^1| \le 0.15$ (green points) and $|\boldsymbol{Y}_{\boldsymbol{u}\,33}^2/\boldsymbol{Y}_{\boldsymbol{u}\,33}^1| \le 0.3$ (blue points). As its role is completely analogous we have set $\boldsymbol{Y}_{\boldsymbol{d}}^2 = 0$. Points outside the experimentally preferred region $0.84 < \sin^2(2\theta_{12}) < 0.90$, $0.92 < \sin^2(2\theta_{23})$, $\sin^2(2\theta_{13}) < 0.15$ have been dismissed.

It is apparent from the plots that when the charged lepton Yukawa couplings have a negligible effect on the running (corresponding to $|\boldsymbol{Y}^2_{\boldsymbol{u}\,33}/\boldsymbol{Y}^1_{\boldsymbol{u}\,33}| \ll 1$), there is a fairly strong correlation between the radiatively generated $\sin\theta_{13}\cos\delta$ and $\theta_{23} - \frac{\pi}{4}$. When the cut-off is $\Lambda = M$, the numerical results are in good agreement with eq. (3.79), shown as a black solid line in the plot. In contrast, when $\Lambda = 10^{18}\,\mathrm{GeV}$ there is a larger spread. Besides, the points are shifted to the left of the black solid line. This effect can be traced back to the fact that non-trivial $\boldsymbol{V}^{\boldsymbol{L}}_{\boldsymbol{e}}$ is generated in the RG evolution independently of $\boldsymbol{Y}^2_{\boldsymbol{e}}$ by the neutrino Yukawa couplings, cf. eqs. (3.74), (A.9).

To summarize, from our analytical and numerical analysis, we expect in the general 2HDM with one right-handed neutrino deviations of the atmospheric angle from the maximal value and of θ_{13} from zero. In models with tri-bi-maximal mixing those are approximately correlated by eq. (3.79) with perturbations due to flavour violating couplings in the charged lepton Yukawa couplings.

Charged Lepton Flavour Violation

We have seen that the mechanism of radiative generation of a sizeable next-to-lightest neutrino mass requires the misalignment of $\boldsymbol{Y}^1_{\boldsymbol{\nu}}$ and $\boldsymbol{Y}^2_{\boldsymbol{\nu}}$. More specifically, it is necessary the same right-handed neutrino to couple with comparable strength to different left-handed lepton doublets. This condition, generalized to include all families and the charged fermions, would imply

$$v(\boldsymbol{\Delta_f})_{ij} \sim (\boldsymbol{M_f})_{jj} \qquad\qquad \text{for } f = u, d, e\,. \tag{3.80}$$

Certainly, this condition would lead to unacceptably large flavour violating neutral interactions if all Higgs particles had masses at the electroweak scale.

Note, however, that the experimental bound on $\mu \to e\gamma$ can be satisfied if the second Higgs is decoupled at the TeV scale. Employing eq. (3.46), it is for example sufficient $m_H \gtrsim 3\,\mathrm{TeV}$ for $v\boldsymbol{\Delta}_{\boldsymbol{e}12} = m_\mu$ even if $\lambda_6 = v^2/m_H^2$ or $v\boldsymbol{\Delta}_{\boldsymbol{u}33} = 0.4 m_t$.

Also the stringent bounds from the tree level contribution to meson-antimeson mixing are not strengthened compared to the Cheng and Sher ansatz, cf. section 2.3.3, as the induced meson mass difference is proportional to $\boldsymbol{\Delta}_{\boldsymbol{f}\,ij}\boldsymbol{\Delta}_{\boldsymbol{f}\,ji}$ in the decoupling limit. Therefore, the large mixing angles in the left-handed sector are compensated by the small ones in the right-handed sector. However, loop induced contributions could be dominant, thus yielding stronger bounds on the necessary decoupling and for a definite conclusion a proper analysis needs to be done.

Therefore, while the situation (3.80) is certainly not comfortable in view of flavour changing neutral interactions, it can be accommodated if the second Higgs is rather strongly decoupled.

4 Conclusions and outlook

The Yukawa couplings of the Standard Model exhibit strong hierarchies in their eigenvalues. In this thesis we have examined several possible implications of these Yukawa hierarchies in different extensions of the Standard Model.

In the first part of the thesis we were concerned with the fact that new physics models generically predict large flavour changing neutral interactions in conflict with the good agreement between experiments and SM predictions. It is well known that this tension can be alleviated if the new physics model obeys the principle of Minimal Flavour Violation (MFV). In this case any flavour matrix can be written as a power series in the Yukawa matrices. Due to the large hierarchies, however, there is only a small number of significantly different terms in the series. Therefore, it is a good approximation to consider only the first few terms in the expansion.

Within the MSSM, we have investigated the RG evolution of the truncated MFV series. In this analysis we focussed on the quark sector, for the RG effects are less pronounced in the lepton sector and the expected contribution from some see-saw mechanism is unknown. First we have checked explicitly that the validity of MFV is scale independent. Then we have provided β-functions for the coefficients of this series and illustrated their behaviour under the RGE with linear approximations for the case of universal scalar and gaugino masses and a universal trilinear mass parameter. It turns out that the rates of FCNC processes depend remarkably mild on the high scale coefficients of the MFV series. This fixed point behaviour is particularly pronounced in case of gaugino mass dominance, in which case the phenomenology of the MSSM with MFV applied at some high scale is not very different to the more restricted Constrained MSSM.

We have looked also at deviations from the truncated MFV series. Unless the trilinear couplings are large and deviate strongly from MFV, we have found the evolution of the terms obeying MFV not to be affected by the deviations. Therefore, the partition of flavour matrices into a part that can be described by the truncated MFV series and a part that describes deviation from MFV, is stable under the RG evolution. Interestingly, we found the deviations from MFV in the squark squared masses to be almost scale independent.

We have also studied the scale-dependence of a 2HDM with Yukawa couplings aligned at some high scale. This constitutes a minimal model consistent with the MFV principle. It also yields the minimal amount of FCNCs without imposing any discrete symmetries as e.g. in the type I or II 2HDM. However, the alignment condition is not stable under the RG evolution but the deviations from alignment obey the MFV principle. We have presented simple analytic formulas for the flavour violating Yukawa couplings mediating tree level FCNCs, cf. eq. (2.38) to (2.40), as well as a numerical check. The tree level induced contribution to meson-antimeson mixing turns out to be below the experimental limits for a wide range of parameters. This analysis could be extended, in analogy to our investigations in the MSSM, allowing at the high scale for arbitrary terms consistent with the principle of MFV. An inspection of the lepton sector, taking into account the see-saw, will be summarized below.

In the second part of this thesis, we have investigated the implications of hierarchical neutrino Yukawa eigenvalues in models with heavy right-handed neutrinos, in which the observed light neutrino masses are generated via the see-saw mechanism. We believe that it is very natural to assume hierarchical neutrino Yukawa eigenvalues in analogy with the three known Yukawa matrices. On the other hand, in order to obtain the mild mass hierarchy inferred from solar and atmospheric neutrino oscillations from hierarchical Yukawa couplings, the see-saw parameters are required to fall into specific classes.

In supersymmetric models the see-saw leaves its imprints in the slepton masses via the RG evolution. Any flavour violating entry of the slepton masses manifests in charged LFV processes like $\mu \to e\gamma$. A precise measurement of both, the neutrino mass and the left-handed slepton mass matrix, would allow for a determination of all see-saw parameters in the Constrained MSSM. However, as this presently does not seem to be feasible, it is sensible to restrict the see-saw additionally by theoretical considerations.

In section 3.3 we have applied the suppositions of hierarchical neutrino Yukawa eigenvalues and the absence of cancellations. We believe that both assumptions are very plausible though the absence of cancellations depends on the parametrization used. Therefore in [7] we have chosen to work in a parametrization of the see-saw where all parameters have a clear physical meaning at the high scale. Instead in this thesis, we have required the absence of cancellations amongst terms proportional to the logarithm of different right-handed neutrino masses, as the logarithm in general destroys correlations, which can be residual from a flavour symmetry. In both cases we find upper bounds on the weakest neutrino Yukawa eigenvalue y_1 and on the lightest right-handed neutrino mass M_1, which scale respectively like $\mathrm{BR}(\mu \to e\gamma)^{1/4}$ and $\mathrm{BR}(\mu \to e\gamma)^{1/2}$. Interestingly, both parameters play an important role in the generation of the baryon asymmetry via the leptogenesis mechanism. The comparison of the upper bound $M_1 \lesssim 5 \cdot 10^{12}\,\mathrm{GeV}$ obtained for typical SUSY parameters from eq. (3.42), to the lower bound required by the thermal leptogenesis scenario, leaves open a range of three orders of magnitude assuming zero initial abundance of right-handed neutrinos. Therefore, in order to rule out this scenario either an improvement of six orders of magnitude in $\mathrm{BR}(\mu \to e\gamma)$ is necessary, or a corresponding improvement in muon conversion. This is about two orders of magnitude below the reach of the planned experiment PRISM/PRIME. However, as the rate of charged LFV processes depends strongly on SUSY masses and $\tan\beta$, before deriving any stringent conclusion on the minimal rate for rare processes one should first wait for determination or further constraints on SUSY from LHC.

Next we have inspected the see-saw in the aligned 2HDM. Again, the neutrino Yukawa couplings affect the charged lepton Yukawa couplings via the RG evolution. This spoils the Yukawa alignment except for the special cases where charged LFV is forbidden by some discrete symmetry. We estimate the rate for $\mu \to e\gamma$ to be close to the present experimental bound if mixings are large and the second Higgs is not decoupled. As in the Constrained MSSM, the only additional flavour matrix compared to the SM is the neutrino Yukawa coupling, and charged LFV is induced only via the RG evolution. Therefore, also the same combination of see-saw parameters governs the neutrino mass generation and charged LFV. This allows to transfer qualitative features from investigations within the Constrained MSSM. For example, in the aligned 2HDM a precise measurement of both the neutrino mass matrix and the second charged lepton Yukawa matrix would allow in principle for a determination of all see-saw parameters.

Finally we have pointed to the possibility of accommodating the mild neutrino mass hierarchy naturally with hierarchical neutrino Yukawa eigenvalues in the general 2HDM. More specifically, if the Yukawa couplings are misaligned, a next-to-heaviest mass eigenvalue is generated radiatively which is suppressed by a loop factor but enhanced by a large logarithm. Hence this model simultaneously explains the smallness of neutrino masses via the see-saw mechanism and the mildness of the hierarchy observed between solar and atmospheric mass splittings without the need of very specific assumptions on any parameters.

The same way the next-to-heaviest neutrino mass is generated radiatively in this model, a non-vanishing value for the reactor angle θ_{13} is induced by the RG evolution even if it vanishes at high scales. Therefore, we expect hierarchies in the mixing angles not to be strong. If the leptonic mixing matrix was exactly tri-bi-maximal at the scale of the right-handed neutrino mass, a correlation may arise between θ_{23} and θ_{13}, cf. eq. (3.79). However, flavour symmetries in the general 2HDM need to be explored in order to make predictions on the tree level mixing angles.

In the RG approach employed in this thesis, the weak, logarithmic dependence of the mild neutrino mass hierarchy on the mass of the second Higgs becomes obvious. This allows to evade the constraints from charged LFV processes by decoupling of the second Higgs up to several TeV. In this mass range, the 2HDM is also free from constraints from the oblique parameters, and we have shown that electric charge is conserved automatically. Constraints from FCNCs have only been sketched here and a more careful investigation is desirable. In any case, making the heavy Higgs scale sufficiently large, all the successes of the SM can be recovered, while still allowing to explain the mild neutrino mass hierarchy.

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A Appendix

A.1. Numerical analysis of the MFV ansatz in the MSSM

A.1.1. General procedure

Here we describe in more detail the numerical procedure and the obtained results concerning the renormalization group evolution of the MFV ansatz in the MSSM. The numerical integration of the RGE was performed with SOFTSUSY 2.0.14 [33], which we have generalized to allow for the MFV ansatz, eqs. (2.12) at the input scale. For our numerical studies we restrict to $\tan\beta = 10$ and the flavour blind part to the CMSSM:

$$M_1 = M_2 = M_3 \equiv m_{1/2} \ , \tag{A.1a}$$

$$\alpha_1 = \alpha_2 = \alpha_3 = m^2_{H_u} = m^2_{H_d} \equiv m_0^2 \ , \quad \boldsymbol{m^2_{\bar{e}}} = \boldsymbol{m^2_L} = m_0^2\, \mathbb{1} \ , \tag{A.1b}$$

$$\alpha_4 = \alpha_5 \equiv A \ . \tag{A.1c}$$

These boundary conditions are applied at the scale of grand unification, which is determined by SOFTSUSY to be the scale of unification of g_1 and g_2. The low scale, where the results are extracted, corresponds to the geometric mean of the stop masses. The parameter region of the scans performed is bounded by

$$200\,\text{GeV} \leq m_{1/2} \leq 500\,\text{GeV} \ , \tag{A.2a}$$

$$100\,\text{GeV} \leq m_0 \leq 1500\,\text{GeV} \ , \tag{A.2b}$$

$$|A| \leq 1000\,\text{GeV} \ , \quad |A| \leq m_0 \ , \tag{A.2c}$$

$$|b| \leq 1 \ . \tag{A.2d}$$

where the parameter b will be defined below. All parameters are taken to be real.

After the numerical integration we perform a fit of the obtained soft masses to the truncated MFV series. This is done by minimizing the Frobenius norm of the deviation

$$|\boldsymbol{\Delta m}| = |\boldsymbol{m}^{\text{SOFTSUSY}} - \boldsymbol{m}^{\text{MFV, fit}}| = \sqrt{\sum_{i,j} |\boldsymbol{m}_{ij}^{\text{SOFTSUSY}} - \boldsymbol{m}_{ij}^{\text{MFV, fit}}|^2} \ , \tag{A.3}$$

where $\boldsymbol{m}$ stands for any of $\boldsymbol{m^2_Q}$, $\boldsymbol{m^2_{\bar{u}}}$, $\boldsymbol{m^2_{\bar{d}}}$, $\boldsymbol{A_u}$, $\boldsymbol{A_d}$ and $\boldsymbol{m}^{\text{MFV, fit}}$ can be written in the form of eqs. (2.12). The best fit defines the parameters $\alpha_i|_{\text{low}}$, $\beta_i|_{\text{low}}$ as well as the deviation from the ansatz $\boldsymbol{\Delta m}$. Note that this decomposition corresponds to the one of section 2.2.3.

A.1.2. Confirmation of the scale independence

To check the scale independence of the validity of the MFV ansatz we consider universal MFV terms

$$\beta_1 = \beta_2 = \beta_3 = \beta_5 = \beta_6 \equiv b m_0^2 \ , \quad \beta_7 = \beta_8 \equiv bA \ . \tag{A.4}$$

and perform a parameter scan according to the specifications of the last section. This ansatz is not motivated by any physical scenario but should be sufficient for a check of scale independence.

The truncation of the MFV series, eqs. (2.12) leads to an error proportional to the squared Yukawa eigenvalues. Due to the large hierarchies in the Yukawa eigenvalues this is in general a good approximation. Nevertheless one expects the RG evolution to add corrections to this truncated MFV series of the order of the omitted terms, which are further suppressed by the loop order at which they appear. In our scan we find the deviations to be not larger than

$$|\boldsymbol{\Delta m^2_Q}|/|\boldsymbol{m^2_Q}| \sim 10^{-7} \ , \quad |\boldsymbol{\Delta m^2_{\bar{u}}}|/|\boldsymbol{m^2_{\bar{u}}}| \sim 10^{-6} \ , \quad |\boldsymbol{\Delta m^2_{\bar{d}}}|/|\boldsymbol{m^2_{\bar{d}}}| \sim 10^{-5} \ ,$$
$$|\boldsymbol{\Delta A_u}|/m_{1/2} \sim 10^{-2} \ , \quad |\boldsymbol{\Delta A_d}|/m_{1/2} \sim 10^{-3} \ .$$

This corresponds to the expected deviations. The comparatively large deviations for the trilinear couplings have several reasons. First, as the trilinear couplings can be written as a sum of odd powers of the Yukawa matrices in the MFV series the Yukawa suppression is only $m_c/m_t \approx 10^{-2}$. Secondly, the omitted terms appear already at the one loop level. Lastly, as $|\boldsymbol{A_u}|$, $|\boldsymbol{A_d}|$ can become quite small at low scales if the RGE contribution cancels the initial value, we have normalized the deviations to $m_{1/2}$. This yields of course a larger ratio in the part of the parameter space where $m_{1/2} \ll |A|$.

A.1.3. Linear approximation to the running MFV coefficients

Also the approximate formulae for the soft masses at low scale, eqs. (2.16), have been obtained using a scan over the parameter region defined in appendix A.1.1. In order to separate the impact of the different MFV coefficients, for each run only one β_i is set to $\beta_i = b m_0^2$ or $\beta_i = bA$ respectively while $\beta_j = 0$ for all $j \neq i$.

After having performed this scan for each β_i a fit is performed on this dataset to obtain the dependence of $\alpha_i|_{\text{low}}$, $\beta_i|_{\text{low}}$ on the initial values. In this fit it is assumed that the low energy values depend linearly on products of $m_{1/2}$, m_0^2, A, α_i, β_i of appropriate mass dimension. Terms with negligible impact on the quality of the fit have been removed from this fitting procedure. Finally the approximations eqs. (2.16) reproduce the results of our numerical procedure to the accuracy of

$$\frac{|\alpha_i^{\text{approx}} - \alpha_i|}{|\alpha_i|} < 0.1 \ , \tag{A.5a}$$

$$\frac{|\beta_{1,2,3;5;6}^{\text{approx}} - \beta_{1,2,3;5;6}|}{|\alpha_{1;2;3}|} < 0.02 \ , \qquad \frac{|\beta_{7;8}^{\text{approx}} - \beta_{7;8}|}{m_{1/2}} < 0.02 \ . \tag{A.5b}$$

A test with random initial conditions for β_i confirms our results.

A.2. Renormalization group evolution in 2HDM

A.2.1. The β-functions of the multi HDM

The one-loop β-functions of the multi-Higgs doublet model, including the dimension five operator which yields neutrino masses, have been derived in [189]. In our conventions, eq. (2.27), the β-functions of the Yukawa couplings read:

$$\begin{aligned}\beta_{\boldsymbol{Y}_{\boldsymbol{u}}^a} = {} & a_u \boldsymbol{Y}_{\boldsymbol{u}}^a + \sum_c \left[3\,\mathrm{Tr}\left(\boldsymbol{Y}_{\boldsymbol{u}}^a \boldsymbol{Y}_{\boldsymbol{u}}^{c\dagger} + \boldsymbol{Y}_{\boldsymbol{d}}^{a\dagger} \boldsymbol{Y}_{\boldsymbol{d}}^c\right) + \mathrm{Tr}\left(\boldsymbol{Y}_{\boldsymbol{e}}^{a\dagger} \boldsymbol{Y}_{\boldsymbol{e}}^c\right)\right] \boldsymbol{Y}_{\boldsymbol{u}}^c \\ & + \sum_c \left(-2\boldsymbol{Y}_{\boldsymbol{d}}^c \boldsymbol{Y}_{\boldsymbol{d}}^{a\dagger} \boldsymbol{Y}_{\boldsymbol{u}}^c + \boldsymbol{Y}_{\boldsymbol{u}}^a \boldsymbol{Y}_{\boldsymbol{u}}^{c\dagger} \boldsymbol{Y}_{\boldsymbol{u}}^c + \frac{1}{2}\boldsymbol{Y}_{\boldsymbol{d}}^c \boldsymbol{Y}_{\boldsymbol{d}}^{c\dagger} \boldsymbol{Y}_{\boldsymbol{u}}^a + \frac{1}{2}\boldsymbol{Y}_{\boldsymbol{u}}^c \boldsymbol{Y}_{\boldsymbol{u}}^{c\dagger} \boldsymbol{Y}_{\boldsymbol{u}}^a\right) ,\end{aligned} \tag{A.6a}$$

$$\begin{aligned}\beta_{\boldsymbol{Y}_{\boldsymbol{d}}^a} = {} & a_d \boldsymbol{Y}_{\boldsymbol{d}}^a + \sum_c \left[3\,\mathrm{Tr}\left(\boldsymbol{Y}_{\boldsymbol{u}}^{a\dagger} \boldsymbol{Y}_{\boldsymbol{u}}^{c} + \boldsymbol{Y}_{\boldsymbol{d}}^{a} \boldsymbol{Y}_{\boldsymbol{d}}^{c\dagger}\right) + \mathrm{Tr}\left(\boldsymbol{Y}_{\boldsymbol{e}}^{a} \boldsymbol{Y}_{\boldsymbol{e}}^{c\dagger}\right)\right] \boldsymbol{Y}_{\boldsymbol{d}}^c \\ & + \sum_c \left(-2\boldsymbol{Y}_{\boldsymbol{u}}^c \boldsymbol{Y}_{\boldsymbol{u}}^{a\dagger} \boldsymbol{Y}_{\boldsymbol{d}}^c + \boldsymbol{Y}_{\boldsymbol{d}}^a \boldsymbol{Y}_{\boldsymbol{d}}^{c\dagger} \boldsymbol{Y}_{\boldsymbol{d}}^c + \frac{1}{2}\boldsymbol{Y}_{\boldsymbol{u}}^c \boldsymbol{Y}_{\boldsymbol{u}}^{c\dagger} \boldsymbol{Y}_{\boldsymbol{d}}^a + \frac{1}{2}\boldsymbol{Y}_{\boldsymbol{d}}^c \boldsymbol{Y}_{\boldsymbol{d}}^{c\dagger} \boldsymbol{Y}_{\boldsymbol{d}}^a\right) ,\end{aligned} \tag{A.6b}$$

$$\begin{aligned}\beta_{\boldsymbol{Y}_{\boldsymbol{e}}^a} = {} & a_e \boldsymbol{Y}_{\boldsymbol{e}}^a + \sum_c \left[3\,\mathrm{Tr}\left(\boldsymbol{Y}_{\boldsymbol{u}}^{a\dagger} \boldsymbol{Y}_{\boldsymbol{u}}^{c} + \boldsymbol{Y}_{\boldsymbol{d}}^{a} \boldsymbol{Y}_{\boldsymbol{d}}^{c\dagger}\right) + \mathrm{Tr}\left(\boldsymbol{Y}_{\boldsymbol{e}}^{a} \boldsymbol{Y}_{\boldsymbol{e}}^{c\dagger}\right)\right] \boldsymbol{Y}_{\boldsymbol{e}}^c \\ & + \sum_c \left(\boldsymbol{Y}_{\boldsymbol{e}}^a \boldsymbol{Y}_{\boldsymbol{e}}^{c\dagger} \boldsymbol{Y}_{\boldsymbol{e}}^c + \frac{1}{2}\boldsymbol{Y}_{\boldsymbol{e}}^c \boldsymbol{Y}_{\boldsymbol{e}}^{c\dagger} \boldsymbol{Y}_{\boldsymbol{e}}^a\right) ,\end{aligned} \tag{A.6c}$$

where a_f $(f = u, d, e)$ stands for contributions due to gauge interactions, which are flavour-diagonal:

$$a_u = -8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2 , \tag{A.7a}$$

$$a_d = -8g_s^2 - \frac{9}{4}g^2 - \frac{5}{12}g'^2 , \tag{A.7b}$$

$$a_e = -\frac{9}{4}g^2 - \frac{15}{4}g'^2 . \tag{A.7c}$$

Finally the β-function of the Weinberg operator $\boldsymbol{\kappa}^{ab}$, cf. eq. (3.47), is given by

$$\begin{aligned}\beta_{\boldsymbol{\kappa}^{ab}} = {} & \frac{1}{2}\sum_c \left[\boldsymbol{Y}_{\boldsymbol{e}}^c \boldsymbol{Y}_{\boldsymbol{e}}^{c\dagger} \boldsymbol{\kappa}^{ab} + \boldsymbol{\kappa}^{ab}\left(\boldsymbol{Y}_{\boldsymbol{e}}^c \boldsymbol{Y}_{\boldsymbol{e}}^{c\dagger}\right)^T\right] + 2\sum_c \left[\boldsymbol{Y}_{\boldsymbol{e}}^c \boldsymbol{Y}_{\boldsymbol{e}}^{b\dagger} \boldsymbol{\kappa}^{ac} + \boldsymbol{\kappa}^{cb}\left(\boldsymbol{Y}_{\boldsymbol{e}}^c \boldsymbol{Y}_{\boldsymbol{e}}^{a\dagger}\right)^T\right] \\ & - 2\sum_c \left[\boldsymbol{Y}_{\boldsymbol{e}}^c \boldsymbol{Y}_{\boldsymbol{e}}^{a\dagger}(\boldsymbol{\kappa}^{cb} + \boldsymbol{\kappa}^{bc}) + (\boldsymbol{\kappa}^{ac} + \boldsymbol{\kappa}^{ca})\left(\boldsymbol{Y}_{\boldsymbol{e}}^c \boldsymbol{Y}_{\boldsymbol{e}}^{b\dagger}\right)^T\right] \\ & + \sum_c \left[3\mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{u}}^a \boldsymbol{Y}_{\boldsymbol{u}}^{c\dagger} + \boldsymbol{Y}_{\boldsymbol{d}}^{a\dagger} \boldsymbol{Y}_{\boldsymbol{d}}^c) + \mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{e}}^{a\dagger} \boldsymbol{Y}_{\boldsymbol{e}}^c)\right] \boldsymbol{\kappa}^{cb} \\ & + \sum_c \boldsymbol{\kappa}^{ac} \left[3\mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{u}}^b \boldsymbol{Y}_{\boldsymbol{u}}^{c\dagger} + \boldsymbol{Y}_{\boldsymbol{d}}^{b\dagger} \boldsymbol{Y}_{\boldsymbol{d}}^c) + \mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{e}}^{b\dagger} \boldsymbol{Y}_{\boldsymbol{e}}^c)\right] \\ & - 3g^2 \boldsymbol{\kappa}^{ab} + 2\sum_{c,d} \lambda_{acbd} \boldsymbol{\kappa}^{cd} ,\end{aligned} \tag{A.8}$$

where we have introduced the tensor λ_{acbd} such that the quartic couplings in the potential can be written as $V \supset \frac{1}{2}\lambda_{abcd}(\Phi_a^\dagger \Phi_b)(\Phi_c^\dagger \Phi_d)$.

The β-functions of the multi HDM with right-handed neutrinos

Assuming the multi HDM to be augmented with right-handed neutrinos as in the see-saw mechanism, the β-functions read above the scale of decoupling of the right-handed neutrinos:

$$\beta^{\nu}_{\boldsymbol{Y}^a_{\boldsymbol{u}}} = \beta_{\boldsymbol{Y}^a_{\boldsymbol{u}}} + \sum_c \mathrm{Tr}\left(\boldsymbol{Y}^a_{\boldsymbol{\nu}} \boldsymbol{Y}^{c\dagger}_{\boldsymbol{\nu}}\right) \boldsymbol{Y}^c_{\boldsymbol{u}} \, , \tag{A.9a}$$

$$\beta^{\nu}_{\boldsymbol{Y}^a_{\boldsymbol{d}}} = \beta_{\boldsymbol{Y}^a_{\boldsymbol{d}}} + \sum_c \mathrm{Tr}\left(\boldsymbol{Y}^{a\dagger}_{\boldsymbol{\nu}} \boldsymbol{Y}^{c}_{\boldsymbol{\nu}}\right) \boldsymbol{Y}^c_{\boldsymbol{d}} \, , \tag{A.9b}$$

$$\beta^{\nu}_{\boldsymbol{Y}^a_{\boldsymbol{e}}} = \beta_{\boldsymbol{Y}^a_{\boldsymbol{e}}} + \sum_c \mathrm{Tr}(\boldsymbol{Y}^{a\dagger}_{\boldsymbol{\nu}} \boldsymbol{Y}^{c}_{\boldsymbol{\nu}}) \boldsymbol{Y}^c_{\boldsymbol{e}} + \sum_c \left(-2\boldsymbol{Y}^c_{\boldsymbol{\nu}} \boldsymbol{Y}^{a\dagger}_{\boldsymbol{\nu}} \boldsymbol{Y}^c_{\boldsymbol{e}} + \frac{1}{2} \boldsymbol{Y}^c_{\boldsymbol{\nu}} \boldsymbol{Y}^{c\dagger}_{\boldsymbol{\nu}} \boldsymbol{Y}^a_{\boldsymbol{e}}\right) \, , \tag{A.9c}$$

$$\begin{aligned} \beta^{\nu}_{\boldsymbol{Y}^a_{\boldsymbol{\nu}}} = {} & \left[-\frac{9}{4}g^2 - \frac{3}{4}g'^2\right] \boldsymbol{Y}^a_{\boldsymbol{\nu}} + \sum_c \left[3\mathrm{Tr}(\boldsymbol{Y}^a_{\boldsymbol{u}} \boldsymbol{Y}^{c\dagger}_{\boldsymbol{u}} + \boldsymbol{Y}^{a\dagger}_{\boldsymbol{d}} \boldsymbol{Y}^{c}_{\boldsymbol{d}}) + \mathrm{Tr}(\boldsymbol{Y}^a_{\boldsymbol{\nu}} \boldsymbol{Y}^{c\dagger}_{\boldsymbol{\nu}} + \boldsymbol{Y}^{a\dagger}_{\boldsymbol{e}} \boldsymbol{Y}^{c}_{\boldsymbol{e}})\right] \boldsymbol{Y}^c_{\boldsymbol{\nu}} \\ & + \sum_c \left(-2\boldsymbol{Y}^c_{\boldsymbol{e}} \boldsymbol{Y}^{a\dagger}_{\boldsymbol{e}} \boldsymbol{Y}^c_{\boldsymbol{\nu}} + \boldsymbol{Y}^a_{\boldsymbol{\nu}} \boldsymbol{Y}^{c\dagger}_{\boldsymbol{\nu}} \boldsymbol{Y}^c_{\boldsymbol{\nu}} + \frac{1}{2} \boldsymbol{Y}^c_{\boldsymbol{e}} \boldsymbol{Y}^{c\dagger}_{\boldsymbol{e}} \boldsymbol{Y}^a_{\boldsymbol{\nu}} + \frac{1}{2} \boldsymbol{Y}^c_{\boldsymbol{\nu}} \boldsymbol{Y}^{c\dagger}_{\boldsymbol{\nu}} \boldsymbol{Y}^a_{\boldsymbol{\nu}}\right) \, . \end{aligned} \tag{A.9d}$$

Finally the the anomalous dimension of the right-handed neutrino mass is

$$\gamma^{\nu}_{\boldsymbol{M}} = -\sum_c \boldsymbol{M}^{-1} \left[\left(\boldsymbol{Y}^{c\dagger}_{\boldsymbol{\nu}} \boldsymbol{Y}^c_{\boldsymbol{\nu}}\right)^T \boldsymbol{M} + \boldsymbol{M} \boldsymbol{Y}^{c\dagger}_{\boldsymbol{\nu}} \boldsymbol{Y}^c_{\boldsymbol{\nu}}\right] \, . \tag{A.10}$$

A.2.2. Leading-log approximations in the aligned 2HDM

In a 2HDM that is aligned at the scale Λ, and which is parametrized by eq. (2.28), the Yukawa couplings can be written in the leading-log approximation at the scale m_H of the heaviest Higgs

$$\boldsymbol{Y}^a_{\boldsymbol{u}}(m_H) \simeq k^a_u \boldsymbol{Y}_{\boldsymbol{u}} + \epsilon^a_u \boldsymbol{Y}_{\boldsymbol{d}} \boldsymbol{Y}^{\dagger}_{\boldsymbol{d}} \boldsymbol{Y}_{\boldsymbol{u}} + \delta^a_u \boldsymbol{Y}_{\boldsymbol{u}} \boldsymbol{Y}^{\dagger}_{\boldsymbol{u}} \boldsymbol{Y}_{\boldsymbol{u}} \, , \tag{A.11a}$$

$$\boldsymbol{Y}^a_{\boldsymbol{d}}(m_H) \simeq k^a_d \boldsymbol{Y}_{\boldsymbol{d}} + \epsilon^a_d \boldsymbol{Y}_{\boldsymbol{u}} \boldsymbol{Y}^{\dagger}_{\boldsymbol{u}} \boldsymbol{Y}_{\boldsymbol{d}} + \delta^a_d \boldsymbol{Y}_{\boldsymbol{d}} \boldsymbol{Y}^{\dagger}_{\boldsymbol{d}} \boldsymbol{Y}_{\boldsymbol{d}} \, , \tag{A.11b}$$

$$\boldsymbol{Y}^a_{\boldsymbol{e}}(m_H) \simeq k^a_e \boldsymbol{Y}_{\boldsymbol{e}} + \delta^a_e \boldsymbol{Y}_{\boldsymbol{e}} \boldsymbol{Y}^{\dagger}_{\boldsymbol{e}} \boldsymbol{Y}_{\boldsymbol{e}} \, , \tag{A.11c}$$

where we have introduced the following abbreviations:

$$\begin{aligned} k^a_u = c^a_u + \frac{\log \frac{m_H}{\Lambda}}{16\pi^2} \big[& a_u c^a_u + 3c^a_u \cos(\psi^*_u - \psi_u) \, \mathrm{Tr}(\boldsymbol{Y}^{\dagger}_{\boldsymbol{u}} \boldsymbol{Y}_{\boldsymbol{u}}) \\ & + 3c^{a*}_d \cos(\psi_d - \psi_u) \, \mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{d}} \boldsymbol{Y}^{\dagger}_{\boldsymbol{d}}) + c^{a*}_e \cos(\psi_e - \psi_u) \, \mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{e}} \boldsymbol{Y}^{\dagger}_{\boldsymbol{e}}) \big] \, , \end{aligned} \tag{A.12a}$$

$$\epsilon^a_u = \frac{\log \frac{m_H}{\Lambda}}{32\pi^2} [c^a_u \cos(\psi^*_d - \psi_d) - 4c^{a*}_d \cos(\psi_d - \psi_u)] \, , \tag{A.12b}$$

$$\delta^a_u = \frac{3 \log \frac{m_H}{\Lambda}}{32\pi^2} c^a_u \cos(\psi^*_u - \psi_u) \, , \tag{A.12c}$$

$$
\begin{aligned}
k_d^a &= c_d^a + \frac{\log\frac{m_H}{\Lambda}}{16\pi^2}\Big[\, a_d c_d^a + 3c_u^{a*}\cos(\psi_u - \psi_d)\,\mathrm{Tr}(\boldsymbol{Y_u^\dagger Y_u}) \\
&\qquad\qquad + 3c_d^a\cos(\psi_d^* - \psi_d)\,\mathrm{Tr}(\boldsymbol{Y_d Y_d^\dagger}) + c_e^a\cos(\psi_e^* - \psi_d)\,\mathrm{Tr}(\boldsymbol{Y_e Y_e^\dagger})\Big] \;, && \text{(A.12d)}\\
\epsilon_d^a &= \frac{\log\frac{m_H}{\Lambda}}{32\pi^2}\left[c_d^a\cos(\psi_u^* - \psi_u) - 4c_u^{a*}\cos(\psi_u - \psi_d)\right] \;, && \text{(A.12e)}\\
\delta_d^a &= \frac{3\log\frac{m_H}{\Lambda}}{32\pi^2} c_d^a\cos(\psi_d^* - \psi_d) \;, && \text{(A.12f)}\\
k_e^a &= c_e^a + \frac{\log\frac{m_H}{\Lambda}}{16\pi^2}\Big[\, a_e c_e^a + 3c_u^{a*}\cos(\psi_u - \psi_e)\,\mathrm{Tr}(\boldsymbol{Y_u^\dagger Y_u}) \\
&\qquad\qquad + 3c_d^a\cos(\psi_d^* - \psi_e)\,\mathrm{Tr}(\boldsymbol{Y_d Y_d^\dagger}) + c_e^a\cos(\psi_e^* - \psi_e)\,\mathrm{Tr}(\boldsymbol{Y_e Y_e^\dagger})\Big] \;, && \text{(A.12g)}\\
\delta_e^a &= \frac{3\log\frac{m_H}{\Lambda}}{32\pi^2} c_e^a\cos(\psi_e^* - \psi_e) \;. && \text{(A.12h)}
\end{aligned}
$$

Here a_f is defined as in eqs. (A.7) and we have introduced

$$
c_f^a = \begin{cases} \cos\psi_f & \text{for } a = 1 \\ \sin\psi_f & \text{for } a = 2 \end{cases} \qquad\qquad \text{where } f = u, d, e \;. \tag{A.13}
$$

Leading-log approximations in the aligned 2HDM with right-handed neutrinos

In addition to eq. (2.28) in the aligned 2HDM with right-handed neutrinos, it holds:

$$
\boldsymbol{Y_\nu^1} = \cos\psi_\nu \boldsymbol{Y_\nu} \;, \qquad\qquad \boldsymbol{Y_\nu^2} = \sin\psi_\nu \boldsymbol{Y_\nu} \;. \tag{A.14}
$$

Then the Yukawa couplings can be estimated at low scales in the leading-log approximation:

$$
\begin{aligned}
\boldsymbol{Y_u^a}(m_H) &\simeq k_u^{\nu\,a}\boldsymbol{Y_u} + \epsilon_u^a \boldsymbol{Y_d Y_d^\dagger Y_u} + \delta_u^a \boldsymbol{Y_u Y_u^\dagger Y_u} \;, && \text{(A.15a)}\\
\boldsymbol{Y_d^a}(m_H) &\simeq k_d^{\nu\,a}\boldsymbol{Y_d} + \epsilon_d^a \boldsymbol{Y_u Y_u^\dagger Y_d} + \delta_d^a \boldsymbol{Y_d Y_d^\dagger Y_d} \;, && \text{(A.15b)}\\
\boldsymbol{Y_e^a}(m_H) &\simeq k_e^{\nu\,a}\boldsymbol{Y_e} + \epsilon_e^{\nu\,a} \boldsymbol{Y_\nu} \log\frac{\boldsymbol{M}}{\Lambda} \boldsymbol{Y_\nu^\dagger Y_e} + \delta_e^a \boldsymbol{Y_e Y_e^\dagger Y_e} \;, && \text{(A.15c)}
\end{aligned}
$$

where we have introduced the following abbreviations:

$$
\begin{aligned}
k_u^{\nu\,a} &= k_u^a + \frac{1}{16\pi^2} c_\nu^a \cos(\psi_\nu^* - \psi_u)\,\mathrm{Tr}(\boldsymbol{Y_\nu} \log\frac{\boldsymbol{M}}{\Lambda}\boldsymbol{Y_\nu^\dagger}) \;, && \text{(A.16a)}\\
k_d^{\nu\,a} &= k_d^a + \frac{1}{16\pi^2} c_\nu^{a*} \cos(\psi_\nu - \psi_d)\,\mathrm{Tr}(\boldsymbol{Y_\nu} \log\frac{\boldsymbol{M}}{\Lambda}\boldsymbol{Y_\nu^\dagger}) \;, && \text{(A.16b)}\\
k_e^{\nu\,a} &= k_e^a + \frac{1}{16\pi^2} c_\nu^{a*} \cos(\psi_\nu - \psi_e)\,\mathrm{Tr}(\boldsymbol{Y_\nu} \log\frac{\boldsymbol{M}}{\Lambda}\boldsymbol{Y_\nu^\dagger}) \;, && \text{(A.16c)}\\
\epsilon_e^{\nu\,a} &= \frac{1}{32\pi^2}\left[c_e^a\cos(\psi_\nu^* - \psi_\nu) - 4c_\nu^{a*}\cos(\psi_\nu - \psi_e)\right] \;. && \text{(A.16d)}
\end{aligned}
$$

Here it is defined c_ν^a in analogy to eq. (A.13).

The leading-log result for the Weinberg operator κ^{ab} can be decomposed:

$$\begin{aligned}\kappa_\nu^{ab}(m_H) \simeq\ & k_\nu^{ab}\boldsymbol{Y_\nu}\frac{1}{\boldsymbol{M}}\boldsymbol{Y_\nu}^T + \log\frac{m_H}{\Lambda}\left(\epsilon_\nu^{ab}\boldsymbol{Y_e}\boldsymbol{Y_e}^\dagger\boldsymbol{Y_\nu}\frac{1}{\boldsymbol{M}}\boldsymbol{Y_\nu}^T + \epsilon_\nu^{ba}\boldsymbol{Y_\nu}\frac{1}{\boldsymbol{M}}\boldsymbol{Y_\nu}^T\boldsymbol{Y_e}^*\boldsymbol{Y_e}^T\right)\\ &+ \delta_\nu^{ab}\left(\boldsymbol{Y_\nu}\log\frac{\boldsymbol{M}}{\Lambda}\boldsymbol{Y_\nu}^\dagger\boldsymbol{Y_\nu}\frac{1}{\boldsymbol{M}}\boldsymbol{Y_\nu}^T + \boldsymbol{Y_\nu}\frac{1}{\boldsymbol{M}}\boldsymbol{Y_\nu}^T\boldsymbol{Y_\nu}^*\log\frac{\boldsymbol{M}}{\Lambda}\boldsymbol{Y_\nu}^T\right)\\ &+ \zeta_\nu^{ab}\boldsymbol{Y_\nu}\frac{1}{\boldsymbol{M}}\log\frac{m_H}{\boldsymbol{M}}\boldsymbol{Y_\nu}^T\\ &+ \left(\xi_\nu^{ab}\boldsymbol{Y_e}\boldsymbol{Y_e}^\dagger\boldsymbol{Y_\nu}\frac{1}{\boldsymbol{M}}\log\frac{m_H}{\boldsymbol{M}}\boldsymbol{Y_\nu}^T + \xi_\nu^{ba}\boldsymbol{Y_\nu}\frac{1}{\boldsymbol{M}}\log\frac{m_H}{\boldsymbol{M}}\boldsymbol{Y_\nu}^T\boldsymbol{Y_e}^*\boldsymbol{Y_e}^T\right)\ ,\end{aligned} \tag{A.17}$$

with

$$k_\nu^{ab} = c_\nu^a c_\nu^b + \mathcal{O}(\frac{1}{16\pi^2})\ , \tag{A.18a}$$

$$\epsilon_\nu^{ab} = \frac{1}{32\pi^2}\left[c_\nu^a c_\nu^b\cos(\psi_e - \psi_e^*) - 4c_e^{a*}c_\nu^b\cos(\psi_e - \psi_\nu)\right]\ , \tag{A.18b}$$

$$\delta_\nu^{ab} = \frac{1}{32\pi^2}c_\nu^a c_\nu^b\cos(\psi_\nu - \psi_\nu^*)\ , \tag{A.18c}$$

$$\zeta_\nu^{ab} = \frac{1}{32\pi^2}\left[3(g^2 + g'^2)c_\nu^a c_\nu^b + 4\lambda_{acbd}c_\nu^c c_\nu^d\right]\ , \tag{A.18d}$$

$$\xi_\nu^{ab} = \frac{1}{8\pi^2}\left[c_\nu^a c_e^{b*} - c_e^{a*}c_\nu^b\right]\cos(\psi_e - \psi_\nu)\ . \tag{A.18e}$$

Lastly, the neutrino mass is usually defined in the basis of diagonal charged lepton masses. Therefore it is necessary a unitary basis transformation of the leptons analogously to eq. (2.32). Only the rotation matrix $\boldsymbol{V_e^L}$ of the lepton doublets is relevant for the neutrino masses, which reads, treating the RGE contributions as corrections:

$$\boldsymbol{V^L_e}_{ij} = \begin{cases} 1\ , & i = j\ ,\\ -(\boldsymbol{Y_\nu}\log\frac{\boldsymbol{M}}{\Lambda}\boldsymbol{Y_\nu}^\dagger)_{ij}\left(\frac{\epsilon_e^{\text{vev}}}{k_e^{\text{vev}}}\right)^*\ , \quad \text{for} & i > j\ ,\\ (\boldsymbol{Y_\nu}\log\frac{\boldsymbol{M}}{\Lambda}\boldsymbol{Y_\nu}^\dagger)_{ij}\frac{\epsilon_e^{\text{vev}}}{k_e^{\text{vev}}}\ , & i < j\ ,\end{cases} \tag{A.19}$$

where

$$\frac{\epsilon_e^{\text{vev}}}{k_e^{\text{vev}}} = \frac{1}{32\pi^2}\left[\cos(\psi_\nu^* - \psi_\nu) - 4\frac{\cos(\psi_\nu - \psi_e)\cos(\beta - \psi_\nu^*)}{\cos(\beta - \psi_e)}\right]\ . \tag{A.20}$$

A.2.3. The Weinberg operator in the leading-log approximation in the general 2HDM with one right-handed neutrino

In the case of only one right-handed neutrino of mass M, $\boldsymbol{\kappa}^{11}$ can be written in the leading-log approximation in the form of eq. (3.58),

$$\boldsymbol{\kappa}^{11} \simeq \frac{\boldsymbol{Y_\nu}^1\boldsymbol{Y_\nu}^{1T}}{M} + \boldsymbol{B}_{1a}\frac{\boldsymbol{Y_\nu}^a\boldsymbol{Y_\nu}^{1T}}{M} + \frac{\boldsymbol{Y_\nu}^1\boldsymbol{Y_\nu}^{aT}}{M}\boldsymbol{B}_{1a}^T + b\frac{\boldsymbol{Y_\nu}^2\boldsymbol{Y_\nu}^{2T}}{M}\ , \tag{A.21}$$

where we have defined flavour matrices $\boldsymbol{B}_{11}$, $\boldsymbol{B}_{12}$ and the complex number b by:

$$\begin{aligned}
\boldsymbol{B}_{11} = {} & \frac{\log \frac{M}{m_H}}{16\pi^2}\left[\frac{3}{2}g^2 - \lambda_1 - 3\,\mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{u}}^1\boldsymbol{Y}_{\boldsymbol{u}}^{1\dagger} + \boldsymbol{Y}_{\boldsymbol{d}}^{1\dagger}\boldsymbol{Y}_{\boldsymbol{d}}^1) - \mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{e}}^{1\dagger}\boldsymbol{Y}_{\boldsymbol{e}}^1) - \frac{1}{2}\boldsymbol{Y}_{\boldsymbol{e}}^2\boldsymbol{Y}_{\boldsymbol{e}}^{2\dagger} + \frac{3}{2}\boldsymbol{Y}_{\boldsymbol{e}}^1\boldsymbol{Y}_{\boldsymbol{e}}^{1\dagger}\right] \\
& + \frac{\log \frac{\Lambda}{M}}{16\pi^2}\left[\frac{9}{4}g^2 + \frac{3}{4}g'^2 - 3\,\mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{u}}^1\boldsymbol{Y}_{\boldsymbol{u}}^{1\dagger} + \boldsymbol{Y}_{\boldsymbol{d}}^{1\dagger}\boldsymbol{Y}_{\boldsymbol{d}}^1) - \mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{\nu}}^1\boldsymbol{Y}_{\boldsymbol{\nu}}^{1\dagger} + \boldsymbol{Y}_{\boldsymbol{e}}^{1\dagger}\boldsymbol{Y}_{\boldsymbol{e}}^1)\right. \\
& \left. - \frac{1}{2}\boldsymbol{Y}_{\boldsymbol{e}}^2\boldsymbol{Y}_{\boldsymbol{e}}^{2\dagger} + \frac{3}{2}\boldsymbol{Y}_{\boldsymbol{e}}^1\boldsymbol{Y}_{\boldsymbol{e}}^{1\dagger} - \frac{1}{2}\boldsymbol{Y}_{\boldsymbol{\nu}}^1\boldsymbol{Y}_{\boldsymbol{\nu}}^{1\dagger} - \frac{1}{2}\boldsymbol{Y}_{\boldsymbol{\nu}}^2\boldsymbol{Y}_{\boldsymbol{\nu}}^{2\dagger}\right], \qquad \text{(A.22a)}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{B}_{12} = {} & \frac{\log \frac{M}{m_H}}{16\pi^2}\left[-2\lambda_6 - 3\,\mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{u}}^1\boldsymbol{Y}_{\boldsymbol{u}}^{2\dagger} + \boldsymbol{Y}_{\boldsymbol{d}}^{1\dagger}\boldsymbol{Y}_{\boldsymbol{d}}^2) - \mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{e}}^{1\dagger}\boldsymbol{Y}_{\boldsymbol{e}}^2) + 2\boldsymbol{Y}_{\boldsymbol{e}}^2\boldsymbol{Y}_{\boldsymbol{e}}^{1\dagger}\right] \\
& + \frac{\log \frac{\Lambda}{M}}{16\pi^2}\left[-3\,\mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{u}}^1\boldsymbol{Y}_{\boldsymbol{u}}^{2\dagger} + \boldsymbol{Y}_{\boldsymbol{d}}^{1\dagger}\boldsymbol{Y}_{\boldsymbol{d}}^2) - \mathrm{Tr}(\boldsymbol{Y}_{\boldsymbol{e}}^{1\dagger}\boldsymbol{Y}_{\boldsymbol{e}}^2 + \boldsymbol{Y}_{\boldsymbol{\nu}}^1\boldsymbol{Y}_{\boldsymbol{\nu}}^{2\dagger}) + 2\boldsymbol{Y}_{\boldsymbol{e}}^2\boldsymbol{Y}_{\boldsymbol{e}}^{1\dagger}\right], \qquad \text{(A.22b)}
\end{aligned}$$

$$b = -2\lambda_5\frac{\log \frac{M}{m_H}}{16\pi^2}\ . \qquad \text{(A.22c)}$$

A.3. Charge breaking in the decoupled multi HDM

A.3.1. Setup

A problem typically associated to multi HDMs is their potential capability of breaking SU(2) × U(1) completely instead of conserving electric charge. In this section we will show that this cannot happen once all but one Higgs doublet are sufficiently decoupled.

The potential of any multi HDM with N Higgs doublets can be written as

$$V = \sum_{a=1}^{N} m_a^2 \Phi_a^\dagger \Phi_a + \sum_{a,b,c,d=1}^{N} \lambda_{abcd} \Phi_a^\dagger \Phi_b \Phi_c^\dagger \Phi_d \ , \tag{A.23}$$

where we have chosen to work in a basis of diagonal Higgs mass terms. In order to guarantee stability of the potential the tensor λ_{abcd} has to be positive definite, i.e. there may not be Unbounded From Below nor Runaway Directions.

Without loss of generality we take Φ_1 to be the only light Higgs. Then, in order to break SU(2) × U(1),

$$m_1^2 < 0 \ . \tag{A.24}$$

On the other hand, negative squared masses for the other Higgs fields lead to large vacuum expectation values and do not allow for decoupling, thus

$$M^2 \equiv \min_{a>1} \{m_a^2\} > 0 \ . \tag{A.25}$$

A sensible measure for the strength of decoupling is given by

$$\epsilon \equiv \frac{\sqrt{-m_1^2}}{M} \sqrt{N-1} \ . \tag{A.26}$$

Here we will assume

$$\epsilon < \frac{1}{8} \frac{\lambda_{1111}}{\Lambda} \ , \qquad \text{where } \Lambda \equiv \max\{|\lambda_{abcd}|\} \ . \tag{A.27}$$

A.3.2. Proof

Step 0: A suitable notation

Using the SU(2) × U(1) gauge freedom one can always choose

$$\Phi_1 = (0, \phi_1^0)^T \ , \tag{A.28}$$

where ϕ_1^0 is a real valued field. The other fields are decomposed according

$$\Phi_a = \begin{pmatrix} \phi_{2a-1}^+ + i\phi_{2a}^+ \\ \phi_{2a-1}^0 + i\phi_{2a}^0 \end{pmatrix} \ , \qquad 1 < a \leq N \ , \tag{A.29}$$

where again ϕ_a^+, ϕ_a^0 are real valued fields. In this notation electric charge is broken exactly if there exists $\langle \phi_a^+ \rangle \neq 0$.

In terms of the real fields, the potential reads

$$V = \sum_{a=1}^{2N} \sum_{s=+,0} m_a^{2\,s} (\phi_a^s)^2 + \sum_{a,b,c,d=1}^{2N} \sum_{s,t=+,0} \lambda_{abcd}^{st} \phi_a^s \phi_b^s \phi_c^t \phi_d^t \,, \tag{A.30}$$

where $m_{2a-1}^{2\,s} = m_{2a}^{2\,s} = m_a^2$. As identical quartic terms appear more than once in the second sum, the definition of λ_{abcd}^{st} is ambiguous. A simple choice is to define them to equal the real or imaginary part of the some λ_{efgh} (times i to some power). In this case

$$\Lambda \geq \max\{|\lambda_{abcd}^{st}|\} \,, \tag{A.31}$$

which is the only property we will be using apart from $\lambda_{1111}^{00} = \lambda_{1111}$.

Lastly introduce the abbreviations

$$\Sigma = \sum_{a=1}^{2N} \,, \qquad \Sigma^{\not{1}} = \sum_{a=3}^{2N} \,. \tag{A.32}$$

Step 1: Show that v_1 is the largest eigenvalue

In the minimum of the potential some fields obtain a vacuum expectation value $v_a^s \equiv \langle \phi_a^s \rangle$. By the choice of the gauge only the neutral component of the light Higgs Φ_1 has a vacuum expectation value $v_1 \equiv v_1^0$, which is real.

We will be interested in the one-dimensional slice of field space that is parametrized by:

$$\phi_a^s = \frac{v_a^s}{\sqrt{\Sigma(v_a^+)^2 + \Sigma(v_a^0)^2}} \phi_v \,, \tag{A.33}$$

being ϕ_v a real valued field. The potential in this direction reads:

$$\begin{aligned} V_v(\phi_v) = {} & \left[\Sigma m_a^{2\,+} (v_a^+)^2 + \Sigma m_a^{2\,0} (v_a^0)^2\right] \frac{\phi_v^2}{\Sigma(v_a^+)^2 + \Sigma(v_a^0)^2} \\ & + \left[\sum_{a,b,c,d=1}^{2N} \sum_{s,t=+,0} \lambda_{abcd}^{st} v_a^s v_b^s v_c^t v_d^t\right] \frac{\phi_v^4}{[\Sigma(v_a^+)^2 + \Sigma(v_a^0)^2]^2} \,. \end{aligned} \tag{A.34}$$

By definition of ϕ_v, this function has a minimum at $\langle \phi_v \rangle = \sqrt{\Sigma(v_a^+)^2 + \Sigma(v_a^0)^2}$. This implies the quadratic term to be negative

$$-\, m_1^2 v_1^2 > \Sigma^{\not{1}} m_a^{2\,+} (v_a^+)^2 + \Sigma^{\not{1}} m_a^{2\,0} (v_a^0)^2 \geq M^2 \Sigma^{\not{1}} [(v_a^+)^2 + (v_a^0)^2] \,. \tag{A.35}$$

This inequality can be split introducing $0 \leq \delta \leq 1$ as the fraction of the vacuum expectation values that breaks electric charge charge

$$\Sigma^{\not{1}} (v_a^+)^2 < \frac{-m_1^2}{M^2} v_1^2 \, \delta \,, \qquad \Sigma^{\not{1}} (v_a^0)^2 < \frac{-m_1^2}{M^2} v_1^2 \, (1 - \delta) \,. \tag{A.36}$$

And from the inequality of arithmetic and quadratic mean

$$\Sigma^{\not{1}} |v_a^+| < \epsilon v_1 \sqrt{2\delta} \qquad \Sigma^{\not{1}} |v_a^0| \,, < \epsilon v_1 \sqrt{2 - 2\delta} \,. \tag{A.37}$$

Step 2: Constrain all vacuum expectation values

The minimum of the potential can of course be determined by setting all first derivatives to zero. In particular also the first derivative of eq. (A.34) has to vanish at the minimum, yielding

$$\langle\phi_v\rangle^2 = \frac{-\left[\Sigma m_a^{2\,+}(v_a^+)^2+\Sigma m_a^{2\,0}(v_a^0)^2\right]\left[\Sigma(v_a^+)^2+\Sigma(v_a^0)^2\right]}{2\left[\sum_{a,b,c,d=1}^{2N}\sum_{s,t=+,0}\lambda_{abcd}^{st}v_a^s v_b^s v_c^t v_d^t\right]} \;. \tag{A.38}$$

With the use of eqs. (A.35), (A.26) the nominator can be constrained

$$\begin{aligned}
-\left[\Sigma m_a^{2\,+}(v_a^+)^2+\Sigma m_a^{2\,0}(v_a^0)^2\right]\left[\Sigma(v_a^+)^2+\Sigma(v_a^0)^2\right] &< [-m_1^2 v_1^2][v_1^2+\Sigma^{\not 1}(v_a^+)^2+\Sigma^{\not 1}(v_a^0)^2] \\
&< [-m_1^2 v_1^2][v_1^2+v_1^2\epsilon^2] \;.
\end{aligned} \tag{A.39}$$

The denominator is dominated by the term $\lambda_{1111}v_1^4$. This can be seen by splitting the sum over λ_{abcd}^{st} into terms containing v_1 to different powers and assuming each quartic term to be maximal, i.e. equal to Λ, and to have opposite phase compared to the λ_{1111} term. This yields the minimal allowed denominator:

$$\begin{aligned}
&\left|\lambda_{1111}v_1^4-\sum_{a,b,c,d=1}^{2N}\sum_{s,t=+,0}\lambda_{abcd}^{st}v_a^s v_b^s v_c^t v_d^t\right| \\
&\quad< 4\Lambda v_1^3(\Sigma^{\not 1}|v_a^0|)+2\Lambda v_1^2\left[(\Sigma\not{1}|v_a^+|)^2+(\Sigma^{\not 1}|v_a^0|)^2\right]+4\Lambda v_1^2(\Sigma^{\not 1}|v_a^0|)^2+ \\
&\quad\quad+4\Lambda v_1(\Sigma^{\not 1}|v_a^0|)\left[(\Sigma^{\not 1}|v_a^+|)^2+(\Sigma^{\not 1}|v_a^0|)^2\right]+\Lambda\left[(\Sigma^{\not 1}|v_a^+|)^2+(\Sigma^{\not 1}|v_a^0|)^2\right]^2 \\
&\quad< \Lambda v_1^4\{4\sqrt{2}\epsilon+4\epsilon^2+8\epsilon^2+8\sqrt{2}\epsilon^3+4\epsilon^4\} \;.
\end{aligned} \tag{A.40}$$

where it has been used eqs. (A.36), (A.37) in the last step. Combining the bounds on nominator and denominator,

$$\langle\phi_v\rangle^2 < \frac{-m_1^2 v_1^4}{2\lambda_{1111}v_1^4}\cdot\frac{1+\epsilon^2}{1-\frac{\Lambda}{\lambda_{1111}}4\epsilon\{\sqrt{2}+3\epsilon+2\sqrt{2}\epsilon^2+\epsilon^3\}} \;, \tag{A.41}$$

which yields, taking into account the decoupling, eq. (A.27),

$$\Sigma(v_a^+)^2+\Sigma(v_a^0)^2 = \langle\phi_v\rangle^2 < 7\frac{-m_1^2}{\lambda_{1111}} \;. \tag{A.42}$$

Step 3: Constrain the charge breaking vacuum expectation values stronger

The derivative of the potential V with respect to a potentially charge breaking field ϕ_a^+ can be written as:

$$\frac{\partial V}{\partial\phi_b^+} = 2m_b^{2\,+}\phi_b^+ + \sum_{a=3}^{2N}\sum_{c,d=1}^{2N}\sum_{s=+,0}(\lambda_{abcd}^{+s}+\lambda_{bacd}^{+s}+\lambda_{cdab}^{s+}+\lambda_{cdba}^{s+})\phi_a^+\phi_c^s\phi_d^s \;. \tag{A.43}$$

Setting this to zero one can determine v_b^+. We will again estimate the quartic terms by Λ and organize the terms according to the power of ϕ_1^0:

$$\begin{aligned}|v_b^+| &< \frac{1}{2M^2}\big\{\,4\Lambda(\Sigma^{\not 1}|v_a^+|)v_1^2 + 8\Lambda(\Sigma|v_a^+|)v_1(\Sigma|v_a^0|)\\ &\qquad\qquad + 4\Lambda(\Sigma|v_a^+|)[(\Sigma|v_a^+|)^2 + (\Sigma^0|v_a|)^2]\big\}\\ &< \frac{\Lambda v_1^2}{M^2}\,\epsilon v_1\sqrt{2\delta}\big\{2 + 4\sqrt{2}\epsilon + 4\epsilon^2\big\}\ . \end{aligned} \tag{A.44}$$

The first term can be estimated using the result of the last step, eq. (A.42),

$$\frac{\Lambda v_1^2}{M^2} < \frac{\Lambda}{M^2}\left[\Sigma(v_a^+)^2 + \Sigma(v_a^0)^2\right] < 7\frac{\Lambda}{\lambda_{1111}}\frac{-m_1^2}{M^2} = 7\frac{\Lambda}{\lambda_{1111}}\frac{\epsilon^2}{N-1}\ , \tag{A.45}$$

which allows to constrain the sum of the charge breaking vacuum expectation values from eq. (A.44):

$$\Sigma^{\not 1}|v_a^+| < \epsilon v_1\sqrt{2\delta}\,14\frac{\Lambda}{\lambda_{1111}}\epsilon^2\big\{2 + 4\sqrt{2}\epsilon + 4\epsilon^2\big\} < \epsilon v_1\sqrt{2\delta}\cdot 0.7\ , \tag{A.46}$$

where we have used again our condition on decoupling eq. (A.27) in the last inequality. Equivalently, this constraint on the charge breaking vacuum expectation values can be written

$$\Sigma^{\not 1}|v_a^+| < \epsilon v_1\sqrt{2\delta}\ , \qquad\qquad \text{where } 0 \le \delta < 0.7^2\ . \tag{A.47}$$

Step 4: The charge breaking vacuum expectation values have to vanish

By going n times through step 3 it is clear that

$$\Sigma^{\not 1}|v_a^+| < \epsilon v_1\sqrt{2\delta}\ , \qquad\qquad \text{where } 0 \le \delta < 0.7^{2n} \xrightarrow{n\to\infty} 0\ . \tag{A.48}$$

Therefore the vacuum expectation values v_a^+ have to vanish and electric charge is unbroken.

A comment on the necessary decoupling

The necessary decoupling in order to *guarantee* absence of charge breaking minima depends on the quartic couplings, cf. eq. (A.27). While $\lambda_{1111} \sim 1$ in order to yield correct electroweak symmetry breaking, the other quartic couplings are a priori not restricted but can be assumed to be $\lesssim 1$ by perturbativity.

More precisely, the requirement of perturbativity should not be applied to the quartic terms λ_{abcd}^{st} as defined above, as double counting in the sum of the quartic terms can appear. Instead one may define quartic couplings λ':

$$V = \sum_{a=1}^{2N}\sum_{s=+,0} m_a^{2\,s}(\phi_a^s)^2 + \sum_{a,b,c,d=1}^{2N}\sum_{s,t=+,0}\lambda'^{st}_{abcd}\phi_a^s\phi_b^s\phi_c^t\phi_d^t\ , \tag{A.49}$$

which obey

$$\lambda'^{st}_{abcd} = 0 \qquad\qquad \text{unless } a \le b \le c \le d\ . \tag{A.50}$$

With this definition also double counting in the estimates eqs. (A.40), (A.44) can be reduced. Therefore instead of eq. (A.27) it is sufficient to demand

$$\epsilon < 0.35 \frac{\lambda_{1111}}{\Lambda'} \,, \qquad \text{where } \Lambda' \equiv \max\{|\lambda'^{st}_{abcd}|\} \,. \tag{A.51}$$

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